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Distribution: The Evolution of its
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Abstract

The evolution of higher moments of the firm size distribution so far seems to be neglected in the empirical firm growth literature. Based on GMM-estimates, this paper introduces simple Wald tests to investigate whether the firm size distribution converges in both the second and third central moment. Using a comprehensive sample of Austrian firms, the estimation results indicate a substantial reduction in both the second and third central moment for the younger age cohorts. This effect is much less pronounced for older firms. Across age cohorts one observes an increase in variance, while the third central moment tends to vanish.

Keywords: Growth of firms; Market concentration; Moments of the firm size distribution; GMM estimation; Wald test

JEL: C21, O47

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1 Introduction

The evolution of the skewness of the firm size distribution so far seems to be neglected in the empirical firm growth literature. The main body of this research is interested in testing Gibrat's law of proportionate growth, i.e. on testing conditional β -convergence in firm size. From an economic theory point of view, the analysis of the evolution of shape of the firm size distribution forms the means to describe the evolution of market structure over time. Under an increasing variance and/or skewness of the distribution of (the log of) the firm size, markets tend to concentrate and large firms gain dominance over time (see Clarke, 1985, Martin, 2002). Economic policy, specifically competition policy, is exactly interested in this issue.

However, even if Gibrat's law of proportionate growth does not hold, the variance and/or skewness of the firm size distribution might not shrink over time. Hart (1995, 2000) among others has plausibly demonstrated that the variance of the size distribution can increase over time despite the presence of mean reversion, if the variance of the initial size distribution is not too large and the intra-distribution dynamics is strong. In this case, the ranking of firms in terms of their size may change over time. Small firms catch up and some of them are able to outgrow their larger counterparts. However, the opposite is also possible. Under the dominance of conditional β -convergence (in the absence of Gibrat's law) the ranking of firms in terms of their size tends to stay constant over time. The same argument holds true for the skewness of the size distribution as will be shown in this paper. In particular, it will be demonstrated that rejecting Gibrat's law of proportionate growth may be compatible with both an increase or a decrease in the skewness of the firm size distribution of a given age cohort.

This paper extends the previous literature on firm growth in two ways. First, a GMM estimation and testing approach is proposed to investigate the evolution of the second and third central moment of the firm size distribution, condition-

ing on a set of explanatory variables. Previously proposed tests for a change in the variance rely on the assumption of a lognormal firm size distribution (Carree and Klomp, 1997, Egger and Pfaffermayr, 2007). Under GMM, specific assumptions on the firm size distribution, such as a log normal distribution, need not be maintained. The available tests thus can easily be generalized to account for distributions which are more heavily skewed than the lognormal (see Cabral and Mata, 2003). Second, a Wald test for convergence in terms of the third moment and the coefficient of skewness of the firm size distribution is derived. Also, the relationship between these two concepts, convergence in variance and convergence in skewness, is analyzed. A Monte Carlo study shows that the proposed tests are properly sized and powerful in samples of reasonable size to detect conditional convergence in terms of variance and skewness. The proposed Wald tests are applied to a large sample of Austrian firms which survived the period 1996 to 2005. The estimation results reveal that for the young age cohorts a decrease in the skewness of the firm size distribution indeed can be found, while the distribution of older age cohorts tends to be more stable over time.

2 The evolution of the skewness of the firm size distribution

The now standard econometric specification of firm growth models employed in empirical research to test Gibrat's law of proportionate growth regresses log firm size in the final period on firm size in the initial period. Thereby, the initial period is not necessarily the size at the time of entry. In particular, these models

postulate that log firm size evolves according to ¹

$$y_{iT} = \alpha_i + \pi y_{i0} + u_i, \quad (1)$$

where y_{it} denotes the log of firm size in period $t = 0, T$. u_i is the sum of independent stochastic shocks occurring between period 0 and T , $u_i = \sum_{t=1}^T \rho^{T-t} v_{it}$, where v_{it} denotes iid disturbances. $\pi = \rho^{-T}$ is the persistence parameter in the cross section model. In almost all applications, it is assumed that log initial firm size is independent of (the sum of) the subsequent shocks as captured by u_i . At $\pi = 1$ this specification implies that log firm size follows a random walk and obeys Gibrat's law, while at $\pi < 1$ it is described by a first-order autoregressive process with some, but not perfect, persistence. In this case, a steady state firm size (given by $y_i^* = \frac{\alpha_i}{1-\pi}$) exists. Hence, the model allows for deviations from Gibrat's law of proportionate growth, i.e. the absence of a unit root. Geroski (2005) emphasizes that this specification is flexible enough to encompass the most important models of firm growth put forward by economic theory, such as adjustment costs and learning models, Penrose effects and organizational capabilities.

This view of the growth process of firms imposes restrictions on the evolution of the higher moments of the firm size distribution. To see this, denote the variance of log firm size at time $t = 0$ and T by σ_{y0}^2 and σ_{yT}^2 , respectively. Hart (1995 and 2000) among other has shown that under the maintained assumptions the model implies $\sigma_{yT}^2 = \pi^2 \sigma_{y0}^2 + \sigma_u^2$. The variance of log firm size in period T is comprised of (i) a term capturing mean reversion ($\pi^2 \sigma_{y0}^2$) and (ii) a second one for the intra-distribution dynamics (σ_u^2). It can easily be seen that under Gibrat's law ($\pi = 1$) the variance of log firm size grows without bound. In the cross-section framework pursued here, this implies that $\sigma_{yT}^2 = \sigma_{y0}^2 + \sigma_u^2 > \sigma_{y0}^2$. On the other hand, under mean reversion ($\pi < 1$) the variance of log firm size may

¹In empirical research, a number of additional exploratory variables is introduced. The most important ones are company age, possibly an interaction effect of company age and initial size and additional controls for the market characteristics like market growth, entry and exit rates.

shrink over time leading to σ -convergence in firm size if $\pi < 1 - \frac{\sigma_y^2}{\sigma_{y0}^2}$.

A similar, but widely neglected relation is given in terms of the third central moments: $\xi_{yT}^3 = \pi^3 \xi_{y0}^3 + \xi_u^3$, where ξ^3 denotes the third central moment of the respective random variables. If the initial distribution of firm size and/or that of the innovations is skewed, then also the third central moment can grow over time. Convergence to a lognormal distribution with $\xi_{y\infty}^3 = 0$ may occur for large T and $\pi < 1$. The reason is that the skewness of the distribution of time aggregated idiosyncratic shocks (ξ_u^3) vanishes according to the central limit theorem, while the impact of a skewed distribution of initial firm size disappears over time under $\pi < 1$. Under Gibrat's law, by contrast, the impact of a skewed distribution of initial firm size persists forever.²

There are some well established stylized facts concerning the evolution of the firm size distribution (see Cabral and Mata, 2003, Cabral 2005, Caves 1998 and Sutton, 1997 for an overview). Specifically, conditional on survival mean reversion is found for younger and smaller firms, while Gibrat's law of proportionate growth reasonably describes the growth process of older and larger firms. Conditional on survival and systematic determinants like market growth, the variance of log firm size tends to decrease in size (Hart, Oulton, 1996). Further, the distribution of young age cohorts shrinks over time, implying conditional σ -convergence in firm size (Pfaffermayr, 2006).

Empirical evidence on the evolution of higher moments of the firm size distribution seems hardly available. The density plots in Cabral and Mata (2003) suggest that the size distribution is more skewed for young firms than the log normal distribution implies (see also Cabral, 2005). However, for a given age cohort skewness tends to become smaller over time. Cabral and Mata (2003)

²Formally, we have $y_{iT} = \rho^T y_{i0} + \sum_{t=1}^T \rho^{T-t} v_{it}$. Under Gibrat's law, $\rho = 1$ and $y_{iT} = y_{i0} + \sum_{t=1}^T v_{it}$ or $\frac{y_{iT} - y_{i0}}{T} = \frac{1}{T} \sum_{t=1}^T v_{it}$. Hence, for large T the log difference $\sqrt{T}(\frac{y_{iT} - y_{i0}}{T})$ converges to a normal with mean 0 and variance σ_v^2 (see also Sutton 1997). Put differently, in finite samples one has the approximation $y_{iT} = y_{i0} + \sum_{t=1}^T v_{it}$ under Gibrat's law. For T large enough the average yearly shocks will be approximately normally distributed so that the skewness of size distribution persists if the initial size distribution is skewed.

emphasize that this finding is not due to selection. Rather it originates from the higher growth rates of smaller firms. Based on a sequence of tests for the normality, Lotti and Santarelli (2001 and 2004) also report that firm size distribution is fairly skewed to the right during the firms' infancy stage, whereas the distribution of log firm sizes converges towards a more symmetric distribution over time. These results are typically found in large samples of firms where the smaller firms are adequately represented. However, these contributions rely on unconditional convergence and do not control for systematic influences like age and industry specific determinants (i.e. conditional convergence).

Unfortunately, economic theory does not say too much on the evolution of the skewness of the firm size distribution. Rather, the error terms of equation (1) are often assumed to be normal, although there are some exceptions (see the tent shaped distributions of firm growth rates found by Bottazzi and Secchi, 2003). Under mean reversion one then observes that the firm size converges to a lognormal distribution.

Several theoretical models explain the evolution of the first and second moment of firm size distribution. The learning models initiated by the seminal paper of Jovanovic (1982) imply mean reversion due to learning and selection (see also the active learning models of Ericson and Pakes, 1995; Hopenhayn, 1992 and Mitchell, 2000). Firms update their beliefs about their true efficiency and successful firms adjust their size accordingly and grow. In contrast, firms finding out that they are not efficient enough shrink and eventually exit. These models imply that the firms' growth rates decrease in size at given age for the group of surviving firms. Also, the variance of the firm size distribution is higher for young firms, but it declines for a given age cohort according to these models. This leads to smaller changes in the size of the firms as they grow older.³

³However, Sutton (1997) argues that learning models can say little about the evolution of the size distribution, since it depends inter alia on unobservables such as the initial distribution of efficiency levels. Nevertheless, the distribution of initial firm size can be a useful proxy on the unobserved distribution of productivity, especially for young age cohorts. It seems plausible

Sutton (1998) introduces the independent submarket model which states that firms face independent business opportunities. These may be taken up either by incumbent or by new firms at constant probabilities. He derives a lower bound on the concentration rate which is based on the exponential distribution in the limit. This model also implies a skewed firm size distribution in the limit.

Cabral and Mata (2003) argue that age dependent financial constraints may explain the observation that the skewness of the firm size distribution vanishes over time for a given age cohort. They emphasize that this phenomenon is due to the fact that young incumbent firms grow faster on average, but not due to the selection of successful firms. In their model some of the entering firms start off very small because they do not have the financial resources to immediately achieve their efficient size. Over time, the probability of a financial constraint decreases as the entrepreneurs running the successful surviving firms accumulate wealth and creditors learn about successful firms. As a result the left tail of the firms size distribution moves towards the right and the skewness of the log firm size distribution of a given age cohort disappears over time (see also Cooley and Quadrini, 2001, and Celeменти and Hopenhayn, 2006 on the impact of borrowing constraints on firm growth). Precisely this hypothesis is at issue in the empirical analysis below. Especially, it is shown that the hypothesis of convergence in skewness can be rigorously tested in a simple GMM-framework under the inclusion of a set of conditioning control variables.

3 The GMM approach to test convergence in firm size

In a cross-section firm growth is modelled in terms of the bivariate distribution of final and initial firm size. It has to be emphasized that initial firm size is

that firm size at early stages is more dispersed and more skewed to the right than its steady state counterpart (see below).

not necessarily the size of the firm at its founding date. The econometric model explaining firm size in the initial and the final period is given by (note bold letters denote vectors or matrix)

$$\begin{aligned}\mathbf{y}_T &= \pi \mathbf{y}_0 + \mathbf{X}_T \beta_T + \mathbf{u} \\ \mathbf{y}_0 &= \mathbf{X}_0 \beta_0 + \mathbf{v}.\end{aligned}\tag{2}$$

$\mathbf{y}_t$ comprises the vector of observed log firm size measured in terms of e.g. employment at time $t = 0$ and T . The elements of the error term at T , $\mathbf{u}$, are distributed as *iid* $(0, \sigma_u^2, \xi_u^3)$; the elements of $\mathbf{v}$, v_i , are distributed as *iid* $(0, \sigma_v^2, \xi_v^3)$. It is assumed that the moments of u_i and v_i exist at least up to order 6. In addition, the assumption $E[u_i v_i] = 0$ is maintained throughout as in almost all cross-sectional studies on firm growth. These assumptions explicitly allow for deviations from the lognormal distribution and more heavily skewed firm size distributions may be considered.

With respect to the starting values, we follow Blundell and Bond (1998) and assume mean stationarity, i.e. the deviation of initial firm size from its steady state is random and uncorrelated with subsequent shocks. This is a fair approximation if the explanatory variables capture all systematic long run effects explaining both y_{iT} and y_{i0} . The explanatory variables are collected in the design matrices $\mathbf{X}_t = 0, T$. These capture systematic (steady state specific) influences on the size of a firm like its age as well as industry dummies which capture market characteristics like market growth, entry and exit rates, as well as sunk costs. In addition regional dummies capturing systematic determinants referring to the location of a firm are included.

GMM-estimation: The estimation of the firm growth model is based on a GMM-approach involving the following moment conditions (see Richardson and Smith, 1993, for a similar approach to test for the skewness of asset returns and

Bai and Ng, 2005 for a general moment based test of skewness of dependent random variables in time series applications):

$$\begin{aligned}
E[\mathbf{x}'_{i0}(y_{i0}-\mathbf{x}_{i0}\beta_0)] &= \mathbf{0} \\
E[(y_{i0}, \mathbf{x}_{iT})'(\mathbf{y}_{iT}-\pi y_{i0}-\mathbf{x}'_{i0}\beta_0)] &= \mathbf{0} \\
E[v_i^2] - \sigma_v^2 &= 0 \\
E[v_i^3] - \xi_v^3 &= 0 \\
E[u_i^2] - \sigma_u^2 &= 0 \\
E[u_i^3] - \xi_u^3 &= 0,
\end{aligned} \tag{3}$$

where $\mathbf{x}_{it}$ denotes the i -th row of the design matrix $\mathbf{X}_t$. The corresponding empirical moment conditions are given by

$$\begin{aligned}
\mathbf{m}_0^1 &= \frac{1}{N}\mathbf{X}'_0(\mathbf{y}_0-\mathbf{X}_0\beta_0) = \mathbf{0} \\
\mathbf{m}_T^2 &= \frac{1}{N}(\mathbf{y}_{0T}, \mathbf{X}_T)'(\mathbf{y}_T-\pi\mathbf{y}_{0T}-\mathbf{X}_T\beta_T) = \mathbf{0} \\
m_0^3 &= \frac{1}{N}\sum_{i=1}^N v_i^2 - \sigma_v^2 = 0 \\
m_0^4 &= \frac{1}{N}\sum_{i=1}^N v_i^3 - \xi_v^3 = 0 \\
m_T^5 &= \frac{1}{N}\sum_{i=1}^N u_i^2 - \sigma_u^2 = 0 \\
m_T^6 &= \frac{1}{N}\sum_{i=1}^N u_i^3 - \xi_u^3 = 0,
\end{aligned} \tag{4}$$

$\mathbf{v} = \mathbf{y}_0 - \mathbf{X}_0\beta_0$ and $\mathbf{u} = \mathbf{y}_T - \pi\mathbf{y}_0 - \mathbf{X}_T\beta_T$. Since the model is just identified, the moment conditions can be solved directly and the choice of the weighting matrix is irrelevant. In particular, solving the first two moment conditions $\mathbf{m}_0^1$ and $\mathbf{m}_T^2$ yields OLS-estimates $\hat{\beta}_0$ and $\hat{\pi}, \hat{\beta}_T$, while $\hat{\sigma}_v^2, \hat{\xi}_v^3, \hat{\sigma}_u^2$ and $\hat{\xi}_u^3$, can be derived from the remaining moment conditions based on the estimated residuals using

$\hat{\mathbf{v}} = \mathbf{y}_0 - \mathbf{X}_0 \hat{\beta}_0$ and $\hat{\mathbf{u}} = \mathbf{y}_T - \hat{\pi} \mathbf{y}_0 - \mathbf{X}_T \hat{\beta}_T$. The variance-covariance matrix of the estimated parameters can be consistently estimated by

$$\hat{\mathbf{V}}(\hat{\theta}) = \left[\mathbf{G}(\hat{\theta})' \hat{\mathbf{S}}^{-1} \mathbf{G}(\hat{\theta}) \right]^{-1}, \quad (5)$$

with $\hat{\theta} = (\hat{\beta}'_0, \hat{\pi}, \hat{\beta}'_T, \hat{\sigma}_v^2, \hat{\xi}_v^3, \hat{\sigma}_u^2, \hat{\xi}_u^3)$. $\mathbf{G}(\hat{\theta}) = \frac{\partial \mathbf{m}(\hat{\theta})}{\partial \theta}$ denotes the estimated gradient of the moment conditions and it is given by

$$\mathbf{G}(\hat{\theta}) = \begin{bmatrix} -\mathbf{X}'_0 \mathbf{X}_0 / N & \mathbf{0}' & 0 & 0 & 0 & 0 \\ \mathbf{0} & -\mathbf{W}'_T \mathbf{W}_T / N & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ -\frac{3}{N} \hat{\sigma}_v^2 \mathbf{x}_{\cdot 0} & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -\frac{3}{N} \hat{\sigma}_u^2 \mathbf{w}_{\cdot T} & 0 & 0 & 0 & -1 \end{bmatrix}, \quad (6)$$

with $\mathbf{W}_T = [\mathbf{y}_0, \mathbf{X}_T]$, $\mathbf{x}_{\cdot 0} = \sum_{i=1}^N \mathbf{x}_{i0}$, $\mathbf{w}_{\cdot T} = \sum_{i=1}^N \mathbf{w}_{iT}$. The vectors $\mathbf{x}_{i0}$ and $\mathbf{w}_{iT}$ represent the i -th row of $\mathbf{X}_0$ and $\mathbf{W}_T$, respectively (i.e. $\mathbf{X}_0 = [\mathbf{x}_{10}, \dots, \mathbf{x}_{N0}]'$ and $\mathbf{W}_T = [\mathbf{w}_{1T}, \dots, \mathbf{w}_{NT}]'$). The matrix $\mathbf{S}(\hat{\theta})$ denotes the variance covariance matrix of the moments. The probability limit of $\mathbf{S}(\theta)$ is given in the Appendix, which shows that $\mathbf{S}(\theta)$ can be consistently estimated by $\mathbf{S}(\hat{\theta}) = \frac{1}{N} \sum_{i=1}^N \mathbf{m}_i(\hat{\theta}) \mathbf{m}_i(\hat{\theta})'$. The theory of GMM-estimation (see e.g. Wansbeek and Meijer, 2000) implies that under the present assumption the GMM-estimates are consistent and asymptotically normal: $n^{\frac{1}{2}}(\hat{\theta} - \theta) \xrightarrow{d} \mathbf{N}(\mathbf{0}, \mathbf{V}(\theta))$, where $\mathbf{V}(\theta) = \mathbf{G}(\theta)' \mathbf{S}(\theta)^{-1} \mathbf{G}(\theta)$. It can also be shown that $\mathbf{V}(\theta)$ can consistently be estimated by $\mathbf{V}(\hat{\theta}) = \mathbf{G}(\hat{\theta})' \mathbf{S}(\hat{\theta})^{-1} \mathbf{G}(\hat{\theta})$. Hence, inference in finite samples can be based on the approximation $\hat{\theta} \sim N(\theta, \frac{1}{N} \mathbf{V}(\theta))$.

Testing for persistence in variance and skewness: The Wald tests for the persistence of firm size in terms of its in variance and skewness are based on the

following relations of the second and third moments of y_{iT} and y_{i0} :

$$\begin{aligned} E[(y_{iT} - E[y_{iT}])^2] &= \sigma_{y_T}^2 = \pi^2 \sigma_v^2 + \sigma_u^2 \\ E[(y_{iT} - E[y_{iT}])^3] &= \xi_{y_T}^3 = \pi^3 \xi_v^3 + \xi_u^3 \end{aligned} \quad (7)$$

Convergence in terms of the second and third moment of the firm size distribution implies the following non-linear restrictions, which can be tested with a standard Wald test:

$$H_0^1 : r^1(\theta) = (\pi^2 - 1)\sigma_v^2 + \sigma_u^2 = 0 \text{ vs. } H_1^1 : (\pi^2 - 1)\sigma_v^2 + \sigma_u^2 < 0 \text{ or} \quad (8)$$

$$\phi_2 = \pi^2 + \frac{\sigma_u^2}{\sigma_v^2} = 1 \text{ vs. } H_1^1 : \phi_2 < 1$$

$$H_0^2 : r^2(\theta) = (\pi^3 - 1)\xi_v^3 + \xi_u^3 = 0 \text{ vs. } H_1^2 : (\pi^3 - 1)\xi_v^3 + \xi_u^3 < 0 \text{ or} \quad (9)$$

$$\phi_3 = \pi^3 + \frac{\xi_u^3}{\xi_v^3} = 1 \text{ vs. } \phi_3 < 1$$

The economic interpretation of these non-linear restrictions is straightforward. If the forces of mean reversion as captured by terms $(\pi^2 - 1)\sigma_v^2$ and $(\pi^3 - 1)\xi_v^3$, respectively, are strong enough, and if the intra-distribution dynamics as reflected by σ_u^2 and ξ_u^3 turns out weak, the firm size distribution shrinks over time both in terms of its second and third central moment (see Hart, 1995, Egger and Pfaffermayr, 2006 and Pfaffermayr, 2006). This means that the distribution not only tends to concentrate around its mean, but also that it becomes more symmetric over time. Under a decreasing variance the Herfindahl concentration index shrinks over time and we observe a decreasing market concentration.⁴ A tendency towards a less skewed firm size distribution is observed if e.g. the credit constraint of young firms tends to soften as they grow older. On the one hand, this implies that the (initial) size distribution of young firms tends to be heavily

⁴Clarke (1985) shows that the Herfindahl concentration index can be written as $H = \frac{1}{N} \left(\frac{\sigma_{y_t}^2}{\mu_{y_t}^2} - 1 \right)$. Hence, at constant μ_{y_t} a decrease in variance implies a reduction in the concentration rate.

skewed. On the other hand, the softening of the credit constraints enables young firms to grow faster and to converge towards their long run size. As a result, the skewness in log firm size diminishes over time and the firm size distribution converges to a log normal distribution (see Cabral and Mata, 2003).

Even if both H_0^1 and H_0^2 are rejected, it is not clear whether the coefficient skewness of the firm size distribution is reduced in course of the growth process of firms. The skewness coefficient is defined as $\frac{\xi_{y_t}^3}{(\sigma_{y_t}^2)^{3/2}}$ $t = 0, T$. As a measure of skewness, it is preferable to the third central moment, because it is invariant with respect to changes in scale and location of the variable of interest. The change in the coefficient of skewness is at issue under H_0^3 . The corresponding non-linear restriction is somewhat more complicated and given by

$$H_0^3 : r^3(\theta) = \frac{\pi^3 \xi_v^3 + \xi_u^3}{(\pi^2 \sigma_v^2 + \sigma_u^2)^{\frac{3}{2}}} - \frac{\xi_v^3}{(\sigma_v^2)^{\frac{3}{2}}} = 0 \Leftrightarrow \frac{\pi^3 + \xi_u^3 / \xi_v^3}{(\pi^2 + \sigma_u^2 / \sigma_v^2)^{\frac{3}{2}}} = \frac{\phi_3}{\phi_2^{\frac{3}{2}}} = 1 \text{ or}$$

$$\phi_3 - \phi_2^{\frac{3}{2}} = 0 \text{ vs. } H_1^3 : \phi_3 < \phi_2^{\frac{3}{2}},$$

where $\phi_3 = \pi^3 + \xi_u^3 / \xi_v^3$ and $\phi_2 = \pi^2 + \sigma_u^2 / \sigma_v^2$. One observes convergence in skewness if $r^3(\theta) < 0$. Rejecting both H_0^1 and H_0^2 is not sufficient to conclude that the coefficient of skewness of the firm size distribution diminishes over time. The reduction of the third central moment has to be more pronounced than that of the variance of the firm size distribution so that H_0^3 can be rejected as well.

To derive the Wald tests for conditional convergence in variance and skewness, the derivatives of the restrictions by $\mathbf{R}^k(\theta) = \frac{\partial r^k(\theta)}{\partial \theta}$, $k = 1, 2, 3$ and $\theta = (\beta_0', \beta_T', \sigma_v^2, \xi_v^3, \sigma_u^2, \xi_u^3)$ have to be derived.

$$\begin{aligned} \mathbf{R}^1(\theta) &= \begin{bmatrix} \mathbf{0}, & 2\pi\sigma_v^2, \mathbf{0}', & \pi^2 - 1, & 0, & 1, & 0 \end{bmatrix} \\ \mathbf{R}^2(\theta) &= \begin{bmatrix} \mathbf{0}, & 3\pi^2\xi_v^3, \mathbf{0}', & 0, & \pi^3 - 1, & 0, & 1 \end{bmatrix} \\ \mathbf{R}^3(\theta) &= \begin{bmatrix} \mathbf{0}, & 3\pi^2 - 3\pi m_2^{\frac{1}{2}}, \mathbf{0}', & \frac{3}{2}m_2^{\frac{1}{2}} \left(\frac{\sigma_u^2}{\sigma_v^4} \right), & \frac{\xi_u^3}{\xi_v^6}, & -\frac{3}{2}m_2^{\frac{1}{2}} \left(\frac{1}{\sigma_v^2} \right), & \frac{1}{\xi_v^3} \end{bmatrix}, \end{aligned}$$

Inserting consistent estimates of θ into $r^k(\theta)$ and $R^k(\theta)$ leads to the following three Wald tests.

$$W_k = N r^k(\hat{\theta})^2 \left[\mathbf{R}^k(\hat{\theta})' \mathbf{V}(\hat{\theta}) \mathbf{R}^k(\hat{\theta}) \right]^{-1}, \quad k = 1, 2, 3 \quad (10)$$

Unfortunately, a closed form solution for $\mathbf{V}(\hat{\theta})$ and, therefore, for the Wald tests, does not exist. However, with the estimated parameters at hand the matrices $\mathbf{G}(\hat{\theta})$ and $\mathbf{S}(\hat{\theta})$ can easily be calculated. To get a one sided test for the alternatives $r^k(\hat{\theta}) < 0$ one can use the same test statistic W_k at the doubled nominal size of the test.⁵

Table 1 provides Monte Carlo evidence on the performance of the three Wald tests. The set-up of the experiments assumes that the error terms u_i and v_i are distributed independently as log-gamma (see Prentice, 1974). This distribution is flexible enough to produce arbitrary skewness (see Cabral and Mata, 2003). In the first step independent log-gamma random variables u_i and v_i are drawn. These are scaled so that $\sigma_v^2 = 4$, $\sigma_u^2 = 1.56$, $\xi_v^3 = 4.23$ and $\xi_u^3 = 2.21$. This parametrization implies a coefficient of skewness of about 0.53 for v_i and 1.14 for u_i , respectively. This design does not reproduce the stylized facts concerning the evolution of the firm size distribution, but it guarantees that the restrictions r_1 , r_2 and r_3 are simultaneously fulfilled at $\pi = 0.78$. Hence, one can analyze the power functions for the three Wald-tests at a single scale. To set-up the firm growth model an explanatory variable x has been generated as $G[-3, 3]$ that enters with parameter $\beta_1 = 0.5$ and is fixed in repeated samples. In addition,

⁵For a one-sided test with just one restriction, Cameron and Trivedi (2005) suggest to use the square root of W_K . This gives the Wald z -statistic which is asymptotically standard normal under H_0 and especially suited for testing a single one-sided hypothesis as the present one.

$$W_{z,k} = \frac{r^k(\hat{\theta})}{\sqrt{N^{-1} \left[\mathbf{R}^k(\hat{\theta})' \mathbf{V}(\hat{\theta}) \mathbf{R}^k(\hat{\theta}) \right]}}, \quad k = 1, 2, 3$$

there is a constant with parameter $\beta_0 = 5$. The starting values are generated as $y_{i0} = \frac{1}{1-\pi}(\beta_1 x + \beta_0) + v_i$ and the firm growth equation as $y_{iT} = \pi y_{i0} + \beta_1 x + \beta_0 + u_i$.

*** Table 1 ***

The power functions are simulated over a range of π . These are defined as the share of rejections under the nominal size of $\alpha = 0.05$ for both the one- and two-sided tests. If the respective hypothesis is true (at $\pi = 0.78$) one obtains the size of the test (i.e. share of rejections when H_0 is true), while for the other values of π it is the power of the test which is defined as share of rejections when H_0 is false. The experiments are repeated 20000 times for the sample sizes 1000, 5000, 10000 and 20000. The results of these Monte Carlo experiments indicate that the Wald tests work well. They are properly sized and equipped with enough power to detect deviations from H_0 . With respect to H_0^2 and H_0^3 we observe the well-known phenomenon that testing for higher moments is a data hungry business. Table 1 shows that at $N = 1000$ the power of W_2 and W_3 is rather low. In addition, the one-sided test for H_0^3 is oversized because of the skewness of distribution of the estimated parameters. However, at large sample sizes as used in many studies on firm growth the Wald tests are well in line with the nominal size and exhibit satisfactory high power.

4 Data and Estimation Results

The empirical analysis of the evolution of the moments of the log firm size distribution uses data from the social security records of Austrian employees. This is an administrative data set which is widely used in empirical research (see e.g. Ichino, et al., 2006; Kaniovski and Peneder 2007; Stiglbauer et. al. 2003). Our sample covers all employees of business units in the Austrian private sector between 1996 and 2005. The data contain a daily calendar of the starting date of an employment relationship of an individual at a particular business unit and the

corresponding end date (if employment spells are terminated before the end of 2005). From this data we constructed a data base which reports annual employment stocks for each business unit on May 1 of each year.

Between 1996 and 2005 the data set covers about 119219 business units covering the manufacturing and the services sector which provide usable information on their size (employment), age, their region and the industry they work in. However, as is common with administrative employment data sets, the comprehensiveness comes at the cost of scarce information on other characteristics of the business units. In addition, the definition of business units is not always precise. For this reason the data has been cleaned, using a series of plausibility checks to ensure that business units are properly defined (see Schöberl, 2004 and Stiglbauer et. al. 2003 for details). In particular, there is potential measurement error due to classification changes for administrative purposes. This problem is prevalent in almost all social security data sets. Furthermore, it is not always possible to distinguish between the plant and the company level. Some employers report (and have corresponding identifiers) for each plant separately, whereas others report employment in various plants under a single identifier. However, Stiglbauer (2003) argues that the vast majority of observations are at the enterprise level rather than at the plant level.

The data comprise only units which survived until 2005, but may have been born after 1996. Furthermore, information on firm age is only available for firms younger than 35 years. Hence, older firms have been excluded from the analysis. To obtain a cross section the first and last available observation for each firm (called initial and final observation) have been extracted and an average growth rate in terms of employment has been calculated. Following Egger and Pfaffermayr (2006) log final firms size y_T is calculated from initial size based on the one year's average growth rate. This does not distort the econometric analysis, but gives the highest power of the Wald test.

*** Table 2 ***

Table 2 reports descriptive statistics on the evolution of the unconditional moments of the firm size distribution for all firms as well as for manufacturing and services firms separately. There are a few striking features: First, we observe that the average firm size only increases for firms younger than 10 years. This holds true for both manufacturing firms and services firms. Second, the variance of firm size tends to be higher for older age cohorts, but it decreases only in the group of young firms over time. Third, the skewness of the firms size distribution tends to be smaller for older age cohorts. However, even for the cohort of firms of age 30 and older the skewness does not vanish. Also, for a given age cohort we observe virtually no decrease in skewness over time.

*** Table 3 ***

Table 3 reports the estimation results for the persistence parameter and the Wald tests on conditional convergence in variance and skewness based on the estimated firm growth equations. For the whole sample of firms, the estimated equations for both initial and final size include 5 dummies for age cohorts, 3-digit industry dummies as well as dummies for Austrian Nuts II regions. The industry dummy variables control for systematic influences like market growth as well as entry and exit barriers. In addition, the equations that explain initial firm size include three dummies for the size classes 0-25, 25-100 and 100-250 employees to control for unobserved differences in initial size (see Pfaffermayr, 2006). The estimations for individual age cohorts are based on the same design, but exclude age dummies.

The findings on persistence parameters are well in line with the literature that use large and comprehensive data sets. The estimated coefficient is near 1 and only for younger firms growth seems not to be independent of size. The Wald tests show that there is evidence on conditional σ -convergence. For all age cohorts the

hypothesis of a constant variance is rejected in favor of its decrease. However, the decrease is largest in absolute size for the younger firms confirming the learning models of firm growth. This finding is in line with previous empirical studies (see Sutton, 1997 and Cabral and Mata, 2003) and the theoretical learning models of e.g. Cabral (1995), Ericson and Pakes (1995), Hopenhayn (1992) Jovanovic, (1982) and Mitchell (2000). Hart and Oulton (1996 and 1999) report similar results for size classes of firms.

The tests for a decrease in the third central moment are well in line with the results of Lotti and Santarelli (2001 and 2004) and Cabral and Mata (2003) who analyze the unconditional evolution of the skewness of the firm size distribution. For manufacturing firms a marked decrease in the third central moment can only be found for the first two age cohorts which are comprised of firms of age 10 or below. For the services firms the Wald test for the absence of conditional convergence in the third central moment rejects in case of all size classes. However, for firms older than ten years the reduction is very small. The third hypothesis refers to the evolution of the skewness coefficient. From an empirical point of view this measure of skewness is preferable as it is invariant to changes in location and scale. However, it involves a highly non-linear hypothesis in terms of the third and second central moments. The Wald tests reject the hypotheses of a constant coefficient of skewness for all age cohorts in manufacturing and services as well as for the full sample. But again the reduction in the skewness parameter is very small for firms older than 10 years.

Table 3 also exhibits a decomposition with respect to the forces of mean reversion and the intra-distribution dynamics. We observe that the innovations shaping the intra-distribution dynamics contribute much less to the conditional central second and third moment of the firm size distribution in the final period than the mean reversion component does. Especially, the evolution of the third central moments indicates that the innovations do not come from a skewed distribution, rather they are compatible with a log normal. The mean reversion term,

in contrast, markedly decreases across age cohorts and shows that for older firms the skewness of the log firms distribution is negligible and even slightly negative for the manufacturing firms. This also confirms the results of Lotti and Santarelli (2001 and 2004) and Cabral and Cabral (2003), who found pronounced skewness for cohorts of young firms that disappeared within a few years. Overall, we observe convergence in variance and skewness of the firm size distribution despite a relatively high persistence parameter, because of the small size of the second and third moments of the innovations.

5 Conclusions

This paper proposes a GMM estimation and testing approach to investigate the evolution of the second and third central moment of the firm size distribution. In contrast to previous work this setting allows to condition on a set of explanatory variables when analyzing the evolution of the higher moments of the firm size distribution. A further advantage of this approach is that in large samples there is no need to rely on specific assumptions concerning the distribution of firm size.

In particular it is argued that the widely used firm growth models that regress the log firm size in the final period on that in the initial period imply convergence in second and third central moment in the predicted firm size if the persistence parameter is sufficiently small. In this case, the forces mean reversion are strong enough to compensate the intra-distribution dynamics arising from the innovations. This forms a non-linear hypothesis which can be tested with standard Wald tests.

The empirical analysis uses data from the social security records of employees in the Austrian private sector to construct a large cross-section of business units. The estimation results reveal high persistence in firm size in both the manufacturing and the services sector. There is convergence in both variance and skewness for the young age cohorts. In contrast, for older firms the reduction of the central

second and third moment over time is small and, in case of manufacturing firms, insignificant. Decomposing the central moments of the predicted log firms size in a mean reversion term and the innovations that reflect the intra-distribution dynamics indicates that the former by far dominates. Across age cohorts one observes an increase in variance, while the third central moments tend to vanish. Hence, for mature and older firms the log normal distribution is a valid description of their size distribution.

6 References

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Appendix

Due to the law of large numbers, we have

$$\mathbf{S}_0 = N^{-1} p \lim_{N \rightarrow \infty} \sum_{i=1}^N \mathbf{m}_i(\theta) \mathbf{m}_i(\theta)' = N^{-1} \sum_{i=1}^N E \mathbf{m}_i(\theta) \mathbf{m}_i(\theta)' =$$

$$N^{-1} \sum_{i=1}^N \begin{bmatrix} Ev_i^2 \mathbf{x}'_{0i} \mathbf{x}_{0i} & \mathbf{0}' & Ev_i^3 \mathbf{x}'_{i0} & Ev_i^4 \mathbf{x}'_{i0} & 0 & 0 \\ \mathbf{0} & Eu_i^2 \mathbf{w}'_{Ti} \mathbf{w}_{Ti} & 0 & 0 & Eu_i^3 \mathbf{w}_{iT} & Eu_i^4 \mathbf{w}_{iT} \\ Ev_i^3 \mathbf{x}_{i0} & 0 & Ev_i^5 - \sigma_v^4 & Ev_i^5 - \xi_v^3 \sigma_v^2 & 0 & 0 \\ Ev_i^4 \mathbf{x}_{i0} & 0 & Ev_i^5 - \sigma_v^2 \xi_v^3 & Ev_i^6 - \xi_v^6 & 0 & 0 \\ 0 & Eu_i^4 \mathbf{w}_{iT} & 0 & 0 & Eu_i^4 - \sigma_u^4 & Eu_i^5 - \sigma_u^2 \xi_u^3 \\ 0 & Eu_i^4 \mathbf{w}_{iT} & 0 & 0 & Eu_i^5 - \sigma_u^2 \xi_u^3 & Eu_i^6 - \xi_u^6 \end{bmatrix}$$

It can easily be seen that a consistent estimate of $\mathbf{S}_0$ is given by

$$\begin{bmatrix} \frac{\hat{\sigma}_v^2 \mathbf{x}'_{0i} \mathbf{x}_{0i}}{N} & \mathbf{0}' & \frac{\hat{\xi}_v^3 \mathbf{x}_{i0}}{N} & \frac{\hat{\kappa}_v^4 \mathbf{x}_{i0}}{N} & 0 & 0 \\ \mathbf{0} & \frac{\hat{\sigma}_u^2 \mathbf{w}'_{Ti} \mathbf{w}_{Ti}}{N} & 0 & 0 & \frac{\hat{\xi}_u^3 \mathbf{w}_{iT}}{N} & \frac{\hat{\kappa}_u^4 \mathbf{w}_{iT}}{N} \\ \frac{\hat{\xi}_v^3 \mathbf{x}_{i0}}{N} & 0 & \hat{\kappa}_v^4 - \hat{\sigma}_v^4 & \hat{\eta}_v^5 - \hat{\xi}_v^3 \hat{\sigma}_v^2 & 0 & 0 \\ \frac{\hat{\kappa}_v^4 \mathbf{x}_{i0}}{N} & 0 & \hat{\eta}_v^5 - \hat{\sigma}_v^2 \hat{\xi}_v^3 & \hat{\phi}_v^6 - \hat{\xi}_v^6 & 0 & 0 \\ 0 & \frac{\hat{\xi}_u^3 \mathbf{w}_{iT}}{N} & 0 & 0 & \hat{\kappa}_u^4 - \hat{\sigma}_u^4 & \hat{\eta}_u^5 - \hat{\sigma}_u^2 \hat{\xi}_u^3 \\ 0 & \frac{\hat{\kappa}_u^4 \mathbf{w}_{iT}}{N} & 0 & 0 & \hat{\eta}_u^5 - \hat{\sigma}_u^2 \hat{\xi}_u^3 & \hat{\phi}_u^6 - \hat{\xi}_u^6 \end{bmatrix}$$

where $\mathbf{x}_{\cdot 0} = \sum_{i=1}^N \mathbf{x}_{i0}$ and $\mathbf{w}_{\cdot T} = \sum_{i=1}^N \mathbf{w}_{iT}$. $\hat{\sigma}_z^2$, $\hat{\xi}_z^3$, $\hat{\kappa}_z^4$, $\hat{\eta}_z^5$ and $\hat{\phi}_z^6$ denote consistent estimates of the moments $E(z_i^j)$, $z_i = u_i, v_i$ and $j = 1, \dots, 6$. They are given by the empirical moments of the respective residuals.

Table 1: The size and power if the Wald test for convergence in variance and skewness in, Monte Carlo simulations (20000 replications)

$\Delta\pi$	two-sided test			one-sided test		
	W_1	W_2	W_3	W_1	W_2	W_3
N=1000						
-0.20	1.000	0.259	0.027	1.000	0.386	0.023
-0.15	0.993	0.196	0.026	0.997	0.312	0.034
-0.10	0.883	0.131	0.040	0.932	0.215	0.056
-0.05	0.400	0.074	0.062	0.516	0.123	0.083
0.00	0.058	0.046	0.089	0.065	0.048	0.124
0.05	0.449	0.088	0.120	0.000	0.010	0.155
0.10	0.973	0.326	0.163	0.000	0.000	0.213
0.15	1.000	0.784	0.207	0.000	0.000	0.259
0.20	1.000	0.989	0.269	0.000	0.000	0.330
N=5000						
-0.20	1.000	0.830	0.309	1.000	0.891	0.001
-0.15	1.000	0.685	0.167	1.000	0.788	0.003
-0.10	1.000	0.447	0.080	1.000	0.574	0.010
-0.05	0.956	0.184	0.047	0.980	0.284	0.035
0.00	0.052	0.045	0.058	0.059	0.055	0.081
0.05	0.986	0.270	0.123	0.000	0.001	0.170
0.10	1.000	0.909	0.210	0.000	0.000	0.284
0.15	1.000	1.000	0.338	0.000	0.000	0.415
0.20	1.000	1.000	0.441	0.000	0.000	0.534
N=10000						
-0.20	1.000	0.986	0.627	1.000	0.993	0.000
-0.15	1.000	0.919	0.377	1.000	0.956	0.000
-0.10	1.000	0.734	0.159	1.000	0.817	0.002
-0.05	0.999	0.315	0.058	1.000	0.441	0.020
0.00	0.050	0.049	0.055	0.047	0.050	0.069
0.05	1.000	0.494	0.134	0.000	0.000	0.203
0.10	1.000	0.997	0.282	0.000	0.000	0.367
0.15	1.000	1.000	0.479	0.000	0.000	0.573
0.20	1.000	1.000	0.664	0.000	0.000	0.741
N=20000						
-0.20	1.000	1.000	0.930	1.000	1.000	0.000
-0.15	1.000	0.998	0.699	1.000	0.999	0.000
-0.10	1.000	0.946	0.339	1.000	0.975	0.000
-0.05	1.000	0.543	0.106	1.000	0.658	0.009
0.00	0.048	0.052	0.052	0.054	0.050	0.064
0.05	1.000	0.783	0.169	0.000	0.000	0.247
0.10	1.000	1.000	0.422	0.000	0.000	0.518
0.15	1.000	1.000	0.703	0.000	0.000	0.778
0.20	1.000	1.000	0.892	0.000	0.000	0.936

Note: $\sigma_0^2=4$, $\sigma_1^2=1.56$, $\xi_0^3=4.23$ and $\xi_1^3=2.21$. The size of the tests is given at $\Delta\pi=0$ (bold figures). $\Delta\pi$ denotes the deviation from the value of π at which the H_0 holds true. W_1 - W_3 are χ^2 Wald tests.

Table 2: Descriptive statistics of log firm size by age cohorts

Age cohort	Period	All Firms				Manufacturing and construction				Services			
		Firms	mean	variance	skewness	Firms	mean	variance	skewness	Firms	mean	variance	skewness
All	Initial	119219	1.02	1.10	1.23	27053	1.41	1.24	0.90	92166	0.91	1.02	1.33
	Final	119219	1.05	1.08	1.26	27053	1.44	1.22	0.94	92166	0.93	1.01	1.35
0-5	Initial	47061	0.70	0.93	1.74	8782	0.98	1.10	1.36	38279	0.64	0.88	1.83
	Final	47061	0.75	0.92	1.74	8782	1.04	1.08	1.36	38279	0.68	0.87	1.84
5-10	Initial	27617	0.87	1.01	1.41	6002	1.16	1.16	1.09	21615	0.79	0.94	1.49
	Final	27617	0.91	0.99	1.44	6002	1.21	1.13	1.13	21615	0.83	0.93	1.52
10-15	Initial	17177	1.27	1.10	0.94	4384	1.64	1.20	0.84	12793	1.14	1.04	0.94
	Final	17177	1.28	1.09	0.97	4384	1.65	1.18	0.87	12793	1.16	1.03	0.98
15-20	Initial	12176	1.49	1.17	0.84	3329	1.91	1.23	0.70	8847	1.33	1.11	0.88
	Final	12176	1.49	1.16	0.87	3329	1.90	1.23	0.74	8847	1.34	1.10	0.91
25-30	Initial	8502	1.56	1.18	0.72	2500	1.98	1.23	0.61	6002	1.39	1.12	0.77
	Final	8502	1.56	1.17	0.76	2500	1.96	1.22	0.65	6002	1.39	1.11	0.80
30-35	Initial	6686	1.69	1.24	0.67	2056	2.03	1.25	0.53	4630	1.54	1.21	0.76
	Final	6686	1.68	1.24	0.71	2056	2.01	1.24	0.57	4630	1.53	1.21	0.80

Table 3: Wald tests for convergence in the second and third central moment

	Age	π	$\Delta\mu_2$	W_1	$\Delta\mu_3$	W_2	Δsk	W_3	$\pi^2 \sigma_v^2$	σ_u^2	σ_v^2	$\pi^3 \xi_v^3$	ξ_u^3	ξ_v^3
All Firms	All	0.981	-0.012	1113.7 ***	-0.013	3118.8 ***	-0.022	16943.8 ***	0.591	0.010	0.614	0.236	0.001	0.249
	0- 5	0.976	-0.009	156.3 ***	-0.026	1099.5 ***	-0.041	7169.9 ***	0.503	0.016	0.528	0.360	0.001	0.387
	5-10	0.975	-0.021	767.4 ***	-0.022	1419.7 ***	-0.020	4735.5 ***	0.574	0.009	0.604	0.290	0.000	0.312
	10-15	0.986	-0.014	364.2 ***	-0.005	464.7 ***	-0.010	2156.4 ***	0.657	0.005	0.676	0.115	0.000	0.121
	15-25	0.990	-0.008	127.7 ***	-0.001	58.8 ***	-0.009	227.1 ***	0.636	0.004	0.648	0.031	0.000	0.032
	25-30	0.991	-0.008	89.6 ***	0.000	1.8	-0.011	6.6	0.645	0.004	0.656	-0.007	0.000	-0.007
	30-35	0.994	-0.004	25.9 ***	0.000	1.5	-0.027	0.8	0.631	0.004	0.639	0.003	0.000	0.003
Manufacturing and construction	All	0.981	-0.016	387.8 ***	-0.007	738.9 ***	-0.018	3133.0 ***	0.683	0.010	0.710	0.129	0.000	0.137
	0- 5	0.974	-0.016	77.8 ***	-0.029	349.6 ***	-0.038	1881.0 ***	0.630	0.018	0.665	0.363	0.001	0.393
	5-10	0.971	-0.032	326.9 ***	-0.021	538.1 ***	-0.018	1231.9 ***	0.684	0.010	0.726	0.232	0.000	0.254
	10-15	0.987	-0.013	84.6 ***	0.003	93.1	-0.013	261.3 ***	0.684	0.005	0.702	-0.072	0.000	-0.075
	15-25	0.993	-0.005	15.4 ***	0.003	44.3	-0.010	244.5 ***	0.633	0.004	0.642	-0.120	0.000	-0.123
	25-30	0.992	-0.006	16.6 ***	0.004	40.2	-0.009	151.5 ***	0.643	0.004	0.653	-0.188	0.000	-0.192
	30-45	0.994	-0.003	4.9 ***	0.003	20.6	-0.009	132.4 ***	0.619	0.004	0.626	-0.160	0.000	-0.163
Services	All	0.982	-0.011	692.5 ***	-0.014	2176.8 ***	-0.023	12771.4 ***	0.563	0.010	0.584	0.268	0.001	0.283
	0- 5	0.977	-0.007	84.8 ***	-0.025	730.2 ***	-0.042	5275.8 ***	0.474	0.016	0.497	0.359	0.001	0.385
	5-10	0.977	-0.017	443.4 ***	-0.021	874.1 ***	-0.020	3481.0 ***	0.543	0.008	0.569	0.304	0.000	0.326
	10-15	0.985	-0.015	281.3 ***	-0.008	442.8 ***	-0.010	2003.3 ***	0.646	0.005	0.666	0.180	0.000	0.188
	15-25	0.989	-0.010	118.5 ***	-0.003	152.9 ***	-0.009	793.6 ***	0.635	0.004	0.649	0.086	0.000	0.089
	25-30	0.991	-0.009	75.6 ***	-0.002	65.1 ***	-0.008	377.1 ***	0.641	0.003	0.653	0.068	0.000	0.070
	30-35	0.993	-0.005	22.3 ***	-0.002	58.8 ***	-0.009	453.1 ***	0.630	0.003	0.638	0.081	0.000	0.083

Note: For the one sided Wald test one has to look at $\chi^2(1)$ -value at $\alpha=0.1$ for negative values of the tested restriction.

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