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 **WORKING PAPERS**

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# REGULARITY AND LONG RUN DYNAMICS IN CONSUMER DEMAND SYSTEMS

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Abstract:

*This paper deals with the regularity problem in the Almost Ideal Demand System (AIDS) in terms of the boundedness of the budget shares within the  $[0,1]$  interval. The violation of ‘cointegration accounting’ can be seen just as another consequence of irregularity. The discussion of modifications in the PIGLOG cost functions leads to the proposition of a modified flexible PIGLOG formulation, from which a system of budget shares can be derived, which ensures regularity and validity of ‘cointegration accounting’. A special feature of the resulting share equations is the divergence from the time path of the AIDS shares over time and therefore better budget share forecasts. This property is shown empirically with long run forecasts from demand systems estimated for consumers demand of some European economies.*

Key words: AIDS model, PIGLOG specification

JEL classification: D11, D12, C30, C32

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# 1 Introduction

Demand systems have become an important instrument of microeconomic research and are an essential part of disaggregated macroeconomic or computable general equilibrium models. Especially the empirical application of the Almost Ideal Demand System (AIDS) put forward by Deaton and Muellbauer (1980) has gained scope in the past due to the increasing use of disaggregated models for policy simulations and forecasting. The experience with these models has, however, often demonstrated the importance of regularity in empirical applications, especially at a highly disaggregated level. The main empirical problem of violated regularity conditions in an AIDS is ‘that, under large changes in real incomes, budget shares can stray outside the  $[0,1]$  interval’ (Rimmer and Powell, 1996). The work of Rimmer and Powell (1996) and of Almon (1998) can be seen as two examples, where the empirical problems with regularity of the AIDS have led to the formulation of a completely new demand system. Cooper and McLaren (1992) also take the violation of regularity in AIDS as a starting point but propose a modified PIGLOG class of preferences, from which a modified AIDS system (MAIDS) can be derived.

This study departs from the work of Cooper and McLaren (1992), who mention in their work as a side aspect of non-regularity, that ‘cointegration accounting’ is also not satisfied in the share equation system of AIDS. This is an important point in the application of the dynamic AIDS. A consistent formulation of the cointegrating equation can be seen as the prerequisite for setting up a dynamic AIDS model. Like the study of Cooper and McLaren (1992) this study deals with ‘cointegration accounting’ without entering explicitly in the field of cointegration and dynamic specification. Instead, we are interested in a side aspect of regularity: bounded budget shares on the left hand side of the equation cannot be cointegrated for the whole price-expenditure space with an unbounded income variable. As a

consequence this might lead to implausible forecasts of budget shares. This is taken as the starting point for the derivation of empirically oriented specifications of PIGLOG preferences. It is shown, that the MAIDS proposed by Cooper and McLaren (1992) satisfies the two conditions of regularity and ‘cointegration accounting’. This leads to another alternative flexible PIGLOG preferences formulation, where the budget shares are not only a positive function of real expenditure but also a negative function of the difference between actual real expenditure and the expenditure level at the starting point  $t = 0$ . The time path of the budget share for a luxury in this system is very close to a logistic function and the system also satisfies regularity and ‘cointegration accounting’. In turn that leads to more plausible forecasts of the budget shares. This formulation of the share equation system comprises the AIDS as a special case at  $t = 0$ . An empirical comparison of this flexible MAIDS with both the AIDS and the MAIDS of Cooper and McLaren (1992) is carried out on an aggregated level of consumer goods for different European countries. The resulting demand systems are used to compute long run forecasts of budget shares. The exogenous variables for these forecasts are derived from a time series forecast for total expenditure and prices.

## 2 Regularity problems in the AIDS

The starting point of the analysis is the PIGLOG-specification of the cost function  $C$  in the AIDS which is usually written as:

$$\log C(\mathbf{u}, \mathbf{p}) = (1-u) \log[a(\mathbf{p})] + u \log[b(\mathbf{p})], \quad (1)$$

where  $\mathbf{p}$  is the vector of prices. Lower case letters for  $u$  and  $c$  denote the value of the variables 'utility' and 'total nominal expenditure' respectively. In (1) the functions  $a(\mathbf{p})$  and

$b(\mathbf{p})$  are both positive linearly homogenous (i.e. hd1) functions of prices. Deaton and Muellbauer (1980) show that  $u$  is usually between 0 and 1 (for exceptions see the Appendix of their paper), which allows them to interpret  $a(\mathbf{p})$  as the cost of subsistence (if  $u$  is zero) and  $b(\mathbf{p})$  as the cost of bliss ( $u$  equal to one). For the ease of exposition it will be convenient to derive the functional form that is estimated in the AIDS using the notation of Cooper and McLaren (1992). Therefore, the functions  $a(\mathbf{p})$  and  $b(\mathbf{p})$  are replaced by two price aggregator functions  $P_1$  and  $P_2$ , which are defined as follows:

$$P_1 = a(\mathbf{p}), \tag{2}$$

and

$$P_2 = \log(b/a) = \log(b) - \log(a). \tag{3}$$

Rewriting (1) as:

$$\begin{aligned} \log C(u, \mathbf{p}) &= (1-u) \log[a(\mathbf{p})] + u \log[b(\mathbf{p})] \\ &= \log[a(\mathbf{p})] - u \log[a(\mathbf{p})] + u \log[b(\mathbf{p})] \\ &= \log[a(\mathbf{p})] + u \{\log[b(\mathbf{p})] - \log[a(\mathbf{p})]\}, \end{aligned} \tag{4}$$

and inserting (2) and (3) into (4) yields the PIGLOG cost function in terms of  $P_1$  and  $P_2$ :

$$\log C(u, \mathbf{p}) = \log(P_1) + uP_2. \tag{5}$$

In what follows, the elasticities of the price aggregator functions will be denoted as:

$$\varepsilon_{ki} = \frac{\partial \log P_k}{\partial \log p_i} \quad ; \quad \varepsilon_{kij} = \frac{\partial^2 \log P_k}{\partial \log p_i \partial \log p_j} \quad (6)$$

while the logarithm of real expenditure is written as:

$$R = \log \left( \frac{c}{P_1} \right) \quad (7)$$

The indirect utility function in the case of PIGLOG expenditures is:

$$U = (\log C(u, \mathbf{p}) - \log(P_1)) * (P_2)^{-1} \quad (8)$$

By virtue of Shepard's Lemma and this indirect utility function we get the Marshallian demands stated in terms of budget shares:

$$w_i^{PIGLOG} = \varepsilon_{1i} + \varepsilon_{2i} R \quad (9)$$

Equation (9) is the final budget share equation derived from the PIGLOG cost function. From its formulation it becomes obvious that the bounded variable  $w_i$  must at some point stray outside the  $[0,1]$  interval if  $R$  increases. Furthermore, Cooper and McLaren (1992) mention the aspect that (9) does not satisfy the rules of 'cointegration accounting' for all possible values of  $R$ , as the bounded variable  $w_i$  is a function of the non-bounded variable  $R$ . We make this point explicit and assume that the time path of  $R$  can be described by a random walk with drift process:



$$R_t = R_0 + \beta_0 t + \sum_{n=1}^t \eta_n \quad (10)$$

and

$$\Delta R_t = \beta_0 + \eta_t. \quad (11)$$

The share equations (9) can then be written in a dynamic formulation as:

$$w_{it} = w_{i0} + \varepsilon_{2i} (\beta_0 t + \sum_{n=1}^t \eta_n) \quad (12)$$

To set up the procedure of finding a cointegrating equation between  $w_i$  and  $R$  one would start with stationarity tests (see Engle and Granger, 1987). A test on stationarity of  $R$  would yield that the series is I(1) and  $\Delta R$  is I(0), i.e., that  $R$  is difference stationary. For some sample range one might also end up with the result that the budget shares  $w_i$  are I(1). Running a regression of type (9) would therefore yield the stationary linear combination of  $w_i$  and  $R$ :

$$V_t = w_{it} - \varepsilon_{1i} - \varepsilon_{2i} R_t \quad (13)$$

This could be taken as an acknowledgment for the starting hypothesis, that  $w_{it}$  and  $R_t$  are cointegrated. This procedure is currently applied in empirical work about dynamic AIDS models before setting up the dynamic specification (e.g. Attfield, 1997). From a theoretical point of view it must be noted, however, that this cointegrating relationship only exists for a certain sample range or – in economic terms – for a certain region of the expendi-

ture-price space. As  $w_i$  is bounded by the  $[0,1]$  interval,  $v_i$  cannot be stationary for the whole expenditure-price space, or to put it the other way, a forecast based on (13) will violate regularity in terms of the boundedness of  $w_i$ . This is the point where ‘cointegration accounting’ and regularity in terms of the boundedness of  $w_i$  can be seen as interrelated.

Similar to the study of Cooper and McLaren (1992) our analysis focuses on the level of the cointegrating relationship without entering explicitly in the empirical application of cointegration. Another aspect of the problem of regularity and ‘cointegration accounting’ can be seen in the violation of the homogeneity restriction, a result repeatedly found in empirical studies on estimating cointegrating AIDS models (see e.g. Ng, 1995). This aspect is also obvious from (9) and has already been mentioned by Cooper and McLaren (1992, p.658): ‘...the PIGLOG share equations do not satisfy cointegration accounting unless the homogeneity restrictions on  $\varepsilon_{1i}$  and the  $\varepsilon_{2i}$  are relaxed, a possibility that may explain the frequent rejection of homogeneity restrictions in non-regular models such as AIDS’.

The properties of AIDS which lead to the violation of regularity can also easily be seen in the derivatives of (9). The response of the budget share to growth in expenditure is

$\partial w_i / \partial \log c = \varepsilon_{2i}$ . A constant response of the budget share must at some point of the price-expenditure space lead to irregularity. The same result can be obtained by looking at the time path of the budget shares, which is driven by the linear deterministic change  $\beta_0$ , such that  $\partial w_{it} / \partial t$  can be derived from (9) and (10) as:

$$\frac{\partial w_{it}}{\partial t} = \varepsilon_{2i} \beta_0 \tag{14}$$

From (14) the link between the violation of regularity and of ‘cointegration accounting’ becomes obvious again. At some point  $t$  the budget shares  $w_{it}$  must lie outside the  $[0,1]$  interval, because the deterministic component of the random walk model  $\partial w_{it}/\partial t$  is a constant and  $\partial^2 w_{it}/\partial t^2 = 0$ .

### 3 Modified formulations of the AIDS (MAIDS)

Cooper and McLaren (1992) propose an alternative PIGLOG formulation of preferences, which allows the derivation of a system with bounded budget shares given rising expenditure. They propose the following alternative cost function (with the same notation as in (5)):

$$\log C(\mathbf{u}, \mathbf{p}) = \log P_1 + \frac{uP_2}{[C(u, p)]^\phi} \quad (15)$$

where  $P_1$  is a homogenous of degree one function in the vector of prices  $\mathbf{p}$  and  $P_2$  is a homogenous of degree  $\phi$  function in  $\mathbf{p}$ . The indirect utility function for this modified PIGLOG (MPIGLOG) specification is:

$$U(\mathbf{c}, \mathbf{p}) = [ \log (c/P_1) ] (c^\phi / P_2) \quad (16)$$

The PIGLOG specification of AIDS described in (5) and (8) can be seen as a special case of (15) and (16) whenever  $\phi = 0$ . Cooper and McLaren (1992) derive the restriction  $0 < \phi < 1$  from a discussion of regularity of the indirect utility function. We will limit ourselves in this study to discuss regularity by directly analysing the budget share equations

of a demand system. Using again the elasticities of the price aggregator functions, the solution of Cooper and McLaren (1992) for the share system is derived by applying Shephard's Lemma and Roy's identity:

$$w_{it} = \frac{\varepsilon_{1i} + \varepsilon_{2i}R_t}{1 + \phi R_t} \quad (17)$$

In (17) the restrictions  $\sum_i \varepsilon_{1i} = 1$  and  $\sum_i \varepsilon_{2i} = \phi$  apply, which follow from  $P_1 = \text{hd}1$  (homogenous of degree 1) and  $P_2 = \text{hd}\phi$  (homogenous of degree  $\phi$ ). The result derived above for the AIDS model – that the boundedness of  $w_{it}$  can be achieved at the expense of relaxing the homogeneity restrictions on  $\varepsilon_{1i}$  and  $\varepsilon_{2i}$  – becomes again obvious from (17). In the region  $c \geq P_1$  the restrictions  $\varepsilon_{1i} \geq 0$  and  $\varepsilon_{2i} \geq 0$  are sufficient to guarantee  $0 \leq w_i \leq 1$ . It also becomes obvious that  $w_i$  is not only a positive function of the I(1) variable  $R$  determined by  $\varepsilon_{2i}$  as in (9), but also a negative function of  $R$  determined by the value of  $\phi$  and the response of the budget share to growth in expenditure,  $\partial w_i / \partial \log c$ , is not a constant:

$$\frac{\partial w_{it}}{\partial \log c} = \frac{\varepsilon_{2i} - w_{it}\phi}{1 + \phi R_t}. \quad (18)$$

The insight, that the budget shares of MAIDS are bounded can also be derived from the time path of  $w_i$  compared to the time path in AIDS:

$$\frac{\partial w_{it}}{\partial t} = \frac{\varepsilon_{2i}\beta_0 - w_{it}\phi\beta_0}{1 + \phi R_0 + \phi\beta_0 t}, \quad (19)$$

where  $\partial w_{it}/\partial t$  is not a constant as in (11) and  $\partial^2 w_{it}/\partial^2 t$  has the opposite sign of  $\partial w_{it}/\partial t$ .

The time path of the budget share in AIDS (11) can again be derived as a special case when  $\phi = 0$ .

The specification of the cost function proposed in this paper picks up the idea of Cooper and McLaren (1992) in that an empirically oriented demand system can be a modification of the AIDS specification. In a more recent paper the same authors (Cooper and McLaren, 1996) show another general PIGLOG specification, from which AIDS and the linear expenditure system (LES) can be derived as special cases and which also ensures regularity. We propose a modification of AIDS, where (i) the share equations guarantee regularity as far as the boundedness of the shares is concerned, (ii) the static specification fulfills the rules of ‘cointegration accounting’, and (iii) the PIGLOG preferences formulation *as well as* the derived share equations of AIDS can be seen as a special case. We start from the following modification of the indirect utility function:

$$U_t(c_t, \mathbf{p}_t) = [ \log(c_t/P_{1,t}) ] ((c_t/c^*)^\rho / P_{2,t}) \quad (20)$$

where  $c^*$  may be seen as a reference value of expenditure similar to the minimum income level in the linear expenditure system. In (20) we choose a dynamic formulation taking into account that  $R$  is a time series following a random walk with drift process and we are interested in analysing regularity and the validity of ‘cointegration accounting’ simultaneously. The PIGLOG specification of AIDS described in (8) can as in MAIDS be seen as a special case of (20) for the trivial case that  $\rho = 0$ . The share equations can again be derived by applying Shephard’s Lemma and Roy’s identity. For this purpose we define the reference value  $c^*$  as a certain level of expenditure within the sample range. Since  $c^*$  is having the character of a minimum value we propose that it should be specified such that  $\log c^* = R_0$  and  $R_0 = \log(c_0/P_{1,0})$ . Hence, the share equations are written as:

$$w_{it} = \frac{\varepsilon_{1i} + \varepsilon_{2i}R_t}{1 + \rho R_t - \rho R_0} \quad (21)$$

where again  $\sum_i \varepsilon_{1i} = 1$ , but where the restriction for  $\varepsilon_{2i}$  depends on time now:  $\sum_i \varepsilon_{2i} = \rho (1 - R_0 / R_t)$ . As in MAIDS we derive the bounded variable  $w_{it}$  in (21) as a non-linear function of the I(1) variable  $R_t$ . There is another particular limiting case in the share equations, when AIDS nests this flexible MAIDS formulation, namely at  $R_t = R_0$ . So it is a special feature of the derived share equations that they begin to diverge from the time path of the AIDS shares from  $t = 0$  on. The difference  $(R_t - R_0)$  plays an important role in the determination of  $w_{it}$ . From (10) we know that  $R$  follows a random walk plus drift process, and hence this difference is a function of  $t$  due to the deterministic trend component  $\beta_0$ .

$$R_t - R_0 = \beta_0 t + \sum_{n=1}^t \eta_n \quad (22)$$

There is a direct feed back therefore in (22) from the deterministic trend component  $\beta_0$  on the shares as expenditure  $R_t$  grows. This feed back depends also on  $\rho$  and ensures boundedness and the validity of ‘cointegration accounting’. The response to growth in expenditure and the time path of the shares are given by:

$$\frac{\partial w_{it}}{\partial \log c} = \frac{\varepsilon_{2i} \rho w_{it}}{1 + \rho R_t - \rho R_0} = \frac{\varepsilon_{2i} \rho w_{it}}{1 + \rho \beta_0 t} \quad (23)$$

$$\frac{\partial w_{it}}{\partial t} = \frac{\varepsilon_{2i} \beta_0 - \rho w_{it} \beta_0}{1 + \rho \beta_0 t} \quad (24)$$

That means that if we look at a dataset for a luxury, which starts with very small values of the share, the time path at the beginning will be very similar to the one implied in AIDS (equation (9)). The most important feature of the resulting share equations of the proposed flexible PIGLOG formulation therefore is the process of divergence from the AIDS shares as  $t$  increases, while AIDS is the limiting case at  $t = 0$ . This is achieved at the expense that the restrictions for additivity concerning  $\varepsilon_{2i}$  become time varying.

## 4 Empirical application of demand systems

In order to allow for an empirical estimation of the derived demand systems specific functional forms of  $a(\mathbf{p})$  and  $b(\mathbf{p})$  must be chosen. Following Deaton and Muellbauer (1980) the functional forms appear as in (25) and (26) below:

$$\log a(\mathbf{p}) = \log P_1 = a_0 + \sum_{i=1}^n \alpha_i \log p_i + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^m \gamma_{ij}^* \log p_i \log p_j \quad (25)$$

and

$$\log b(\mathbf{p}) = \log a(\mathbf{p}) + b_0 \prod_{i=1}^n p_i^{\beta_i} \quad (26)$$

That is,  $\log[a(\mathbf{p})]$  is a translog-function and  $f(\mathbf{p})$  is a Cobb-Douglas type function. The elasticities of the price aggregator functions are easily derived as:

$$\varepsilon_{1i}^{AIDS} = \alpha_i + \sum_{j=1}^n \gamma_{ij} \log p_j \quad (27)$$

and

$$\varepsilon_{2i}^{AIDS} = \beta_i \quad (28)$$

In line with most empirical applications it is furthermore assumed, that the translog price index  $P_I$  can be approximated by a known price index, thereby making use of the collinearity of prices (see e.g. Deaton and Muellbauer, 1980, p. 316, or Green and Alston, 1990). For this purpose the geometric price index  $P^*$  of Stone (1954) is used:

$$\log P^* = \sum_k w_k \log p_k \quad (29)$$

It follows, that the well known budget shares equations for the AIDS are then obtained as:

$$w_i^{AIDS} = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log \left( \frac{c}{P^*} \right) \quad (30)$$

Assuming the same empirical specifications of the price aggregator function  $P_I$  and  $P_2$  the budget shares of the MAIDS of Cooper and McLaren (1992) then take the following form:

$$w_i^{MAIDS} = \left( \alpha_0 + \sum_j \gamma_{ij} \log p_j + \beta_i \log \left( \frac{c}{P^*} \right) \right) \left( 1 + \eta \left( \frac{c}{P^*} \right) \right)^{-1} \quad (31)$$



Finally, the budget shares of the dynamic modified AIDS are derived as:

$$w_i^{dynMAIDS} = \left( \alpha_0 + \sum_j \gamma_{ij} \log p_j + \beta_i \log \left( \frac{c}{P^*} \right) \right) \left( 1 + \rho \left( \frac{c}{P^*} \right) - \rho \left( \frac{c_0}{P_0^*} \right) \right)^{-1} \quad (32)$$

where  $c_0$  and  $P_0^*$  correspond to the respective values of expenditure and the Stone price index at the beginning of the sample period used for estimation.

When the budget share equations (30), (31), and (32) above shall satisfy the standard properties of demand functions, three sets of restrictions have to be implied on the estimated parameters. First, in order to satisfy the adding-up condition it must hold true that:

$$\sum_{i=1}^n \alpha_i = 1; \quad \sum_{i=1}^n \gamma_{ij} = 0; \quad \sum_{i=1}^n \beta_i = 0 \quad (33)$$

Homogeneity in prices and total expenditure is assured if:

$$\sum_{j=1}^n \gamma_{ij} = 0. \quad (34)$$

Finally, symmetry of the Slutsky equation is attained by:

$$\gamma_{ij} = \gamma_{ji}, \quad (35)$$

since  $\gamma_{ij} = \frac{1}{2}(\gamma_{ij}^* + \gamma_{ji}^*) = \gamma_{ji}$ . So far the properties of homogeneity and symmetry could be imposed on the demand system by placing restrictions on the parameters, while Additivity

will be satisfied automatically once the data add up perfectly. The Negativity condition, however, can only be checked by directly observing the elements  $s_{ij}$  of the Slutsky matrix. We estimated these demand systems at a rather aggregated level of goods (excluding energy) for different European countries using the OECD National Accounts database. The empirical estimation involves a total of five commodity groups:

- 1 Food, Beverages, Tobacco
- 2 Clothing and Footwear
- 3 Recreation and Culture
- 4 Restaurants and Hotels
- 5 Other Goods and Services

The necessary data to make estimation feasible of course involve the outlays on the respective commodity group in both nominal as well as real terms. In addition to the estimation framework outlined in equations (30), (31) and (32) above, the empirical application of the MAIDS and dynamic MAIDS involves two ‘aggregation-variables’ (see Cooper and McLaren, 1992, who find that the inclusion of variables approximating the income distribution appears to be appropriate when applying the theoretical framework to aggregated data), which were chosen to be the unemployment rate as well as the consumer price index. Both variables are added with two separate coefficients on the right hand side of equations (30-32), with the coefficients of each variable being restricted to sum up to zero over the equations to secure additivity. Demand systems have been estimated for Austria, Denmark, and Finland.

The estimation setting is the following. First, the three models as stated by (30) to (32) (with the addition of aggregation variables for the MAIDS and dynamic MAIDS as men-

tioned above) were estimated as systems (SURE estimation) using Version 4.1 of the EViews software package. All estimation is conducted by imposing the symmetry as well as homogeneity restriction on the respective systems (results are available from the authors upon request).

In order to compare the different long run dynamics of budget shares implied by the different demand systems, we carried out a simulation experiment. Our main hypothesis here is that boundedness of the budget shares and validity of 'co-integration accounting' are linked. As lined out in (10) income or total expenditure follows a random walk with drift process. Therefore we carried out a short time series analysis in order to determine the trends in income according to  $\Delta R_t = \beta_0 + \eta_t$  as formulated in (11). First of all we tested (with ADF), whether the series were I(1) and we then filtered out the corresponding trend. The same procedure was applied to the time series of prices, which were also determined as I(1) processes. After forecasting these exogenous time series up until 2050 the estimated models were used to forecast the time path of each budget share.

The following figures depict the budget shares forecasts from selected models estimated for Austria, Denmark and Finland. The projections of budget shares run up to 2050.

In turning to the models' forecasts it becomes immediately obvious, that none of the budget shares projected by the AIDS (or any other modelling framework) in any of the countries violates the [0,1] interval. This holds true even though the time horizon used for projections in this study comprises around 50 years, which can clearly be considered a long-term horizon. The biggest change projected for a budget share is that for consumption group 1 in the AIDS model for Denmark, whose value is forecast to decrease from 0.30 in 2001 down to 0.12 in the course of the fifty years. Referring to this it must not, however, be concluded, that the potential violation of the [0,1] interval is not relevant empirically. This result rather appears to be greatly influenced by the equal size-

distribution of the budget shares chosen in this study. Considering e.g. the setting for Austria in the year 2000 it becomes obvious, that four out of the five groups amount to values between 0.19 and 0.25 (with budget share 2 being the exception at 0.11). This is not an unusual setting giving the purpose of the models application, namely their incorporation within a larger modelling framework, implying that the consumption models must capture the whole range of commodities available. In contrast to this, however, empirical studies often wish to deal with some specific commodities or much smaller groups of consumption goods (such as e.g. the consumption pattern regarding different types of meat). In such a setting one is likely to start with a much more unequal distribution of the relative size of the budget shares under investigation. As the empirical results show, the inherent tendency of the AIDS to violate the  $[0,1]$  interval is not particularly relevant when the budgets shares comprise broad groups of commodities as is the case in this study – it seems to be likely, however, that the rather equal distribution of the budget shares greatly influences this result. Nevertheless the modified AIDS models reveal an advantage compared to AIDS as far as plausibility of forecasts is considered. The time path of a bounded budget share significantly differs from the time path of an unbounded budget share.

The results shown in Figure 1-3 document, that both the MAIDS as well as the dynamic MAIDS bring forth significantly different forecast of future budget shares than does the AIDS. Across countries this might best be shown by means of commodity group 1, which is of course a necessity showing decreasing relevance in the overall expenditure pattern of all countries. It was mentioned, that group 1 in the AIDS model for Denmark exhibits a rather strong decrease, but the same also holds true for Austria and Finland. The MAIDS and dynamic MAIDS, however, are clearly capable of projecting a much more plausible course of the budget share in all countries. The projection for budget share 2 in Denmark and Austria are another case in point. In the AIDS model for Austria, the share decreases

from slightly below 0.11 in 2000 to around 0.05 in 2050, the respective values for Denmark are 0.09 and 0.06. Moreover, the course of the forecast reveals, that the projected decrease is getting larger over the years. This pattern can often be observed in the AIDS models projections, consider e.g. budget share 4 in Austria's model or budget shares 3 and 5 in the one for Denmark. When comparing the MAIDS with its dynamic counterpart, it turns out that the latter reveals an even more moderate pattern in almost all cases, with the budget share 4 in Austria and Finland being the only exceptions.

## 5 Conclusions

In this study we derive empirically oriented specifications of PIGLOG preferences. It is shown, that the MAIDS proposed by Cooper and McLaren (1992) as well as our dynamic MAIDS satisfy the two conditions of regularity and 'cointegration accounting'. The time path of the budget share for a luxury in dynamic MAIDS is very close to a logistic function. It can be further be shown, that the MAIDS systems describe more general cases of demand and preferences and comprise AIDS as a special case.

We tested our model empirically on an aggregated level of consumer goods for different European countries. The resulting demand systems were used to compute long run forecasts of budget shares. In summarizing it can be said, that even though the potential violation of the  $[0,1]$  interval by the AIDS model is not an empirical problem in the setting as applied in this investigation, both the MAIDS and especially the dynamic MAIDS introduced in this paper are capable of projecting a much more plausible time path of budget shares. The dynamic MAIDS – quite expectedly, given its specification – turns out to produce even more restrained results as compared to the MAIDS. There is therefore a twofold advantage of the dynamic MAIDS over AIDS: (i) MAIDS satisfies cointegration

accounting and is therefore more appropriate from a theoretical point of view and (ii)

MAIDS guarantees more plausible forecasts of budget shares.

Figure 1: Simulation results for Austria

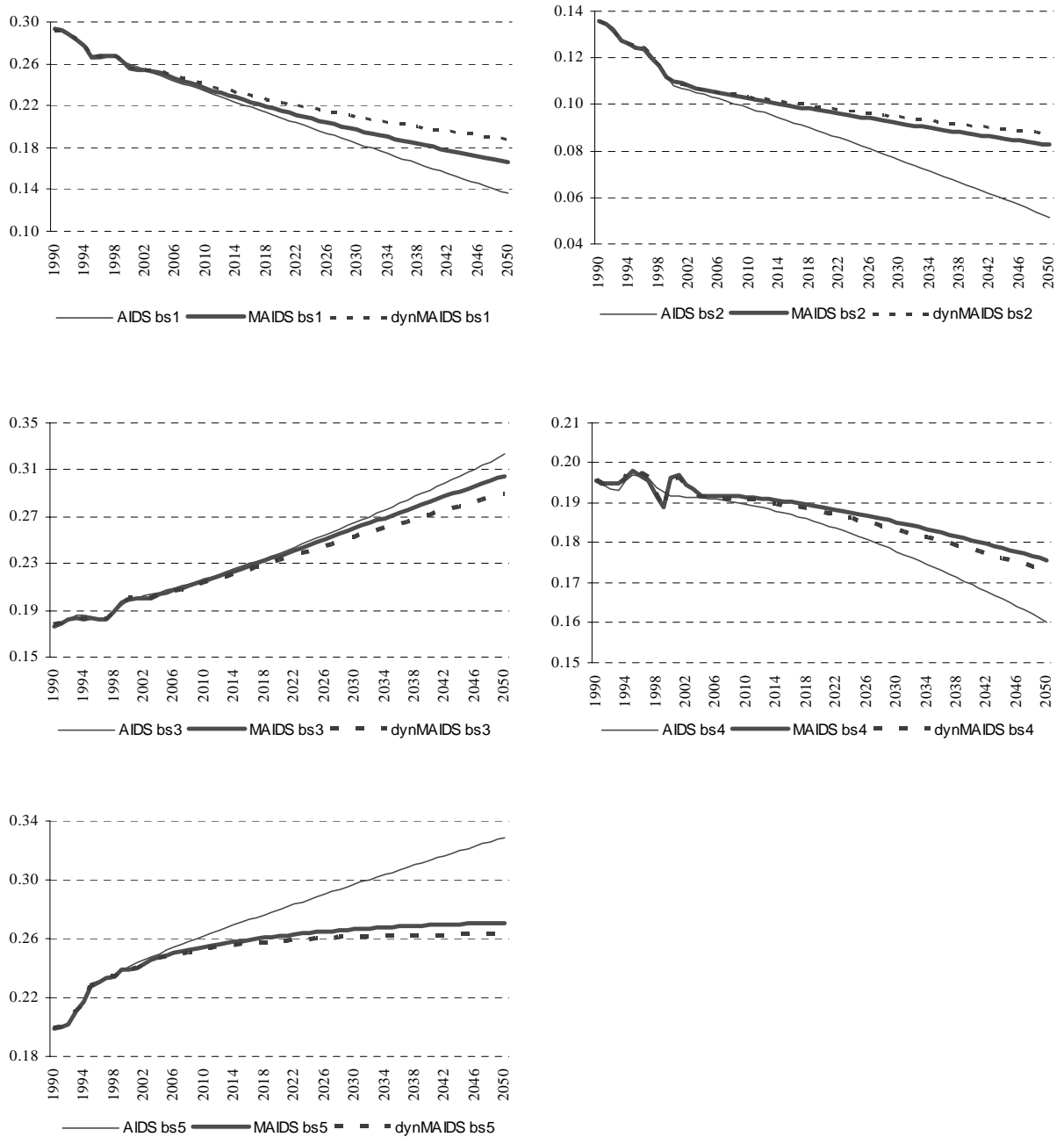


Figure 2: Simulation results for Denmark

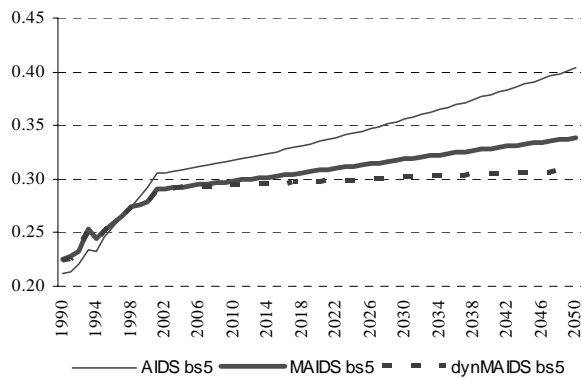
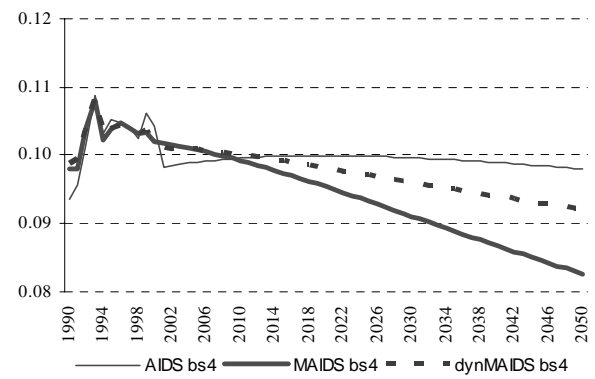
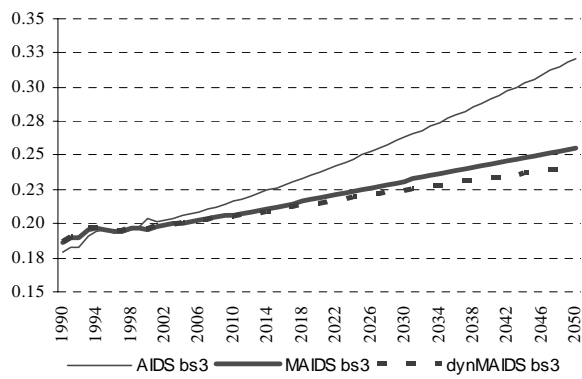
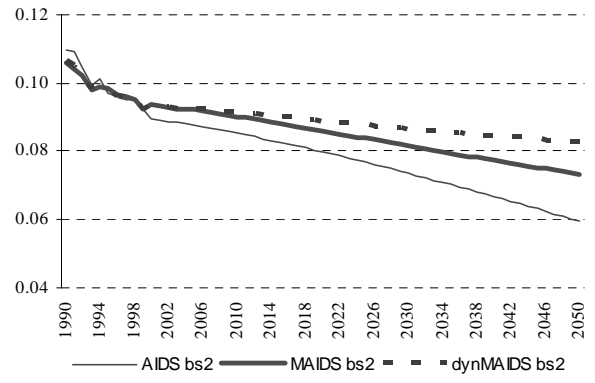
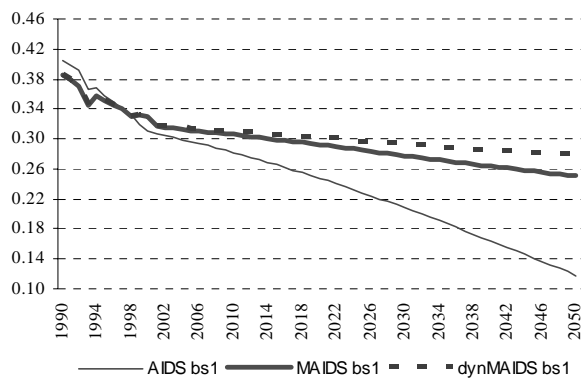
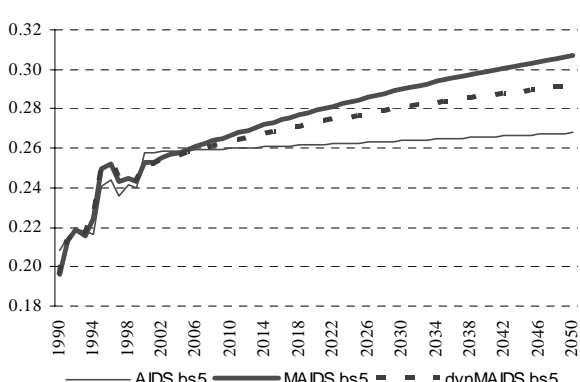
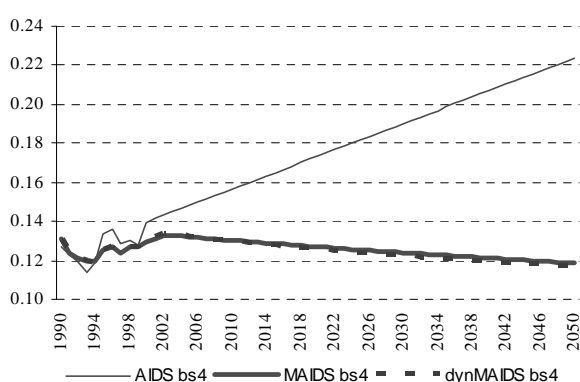
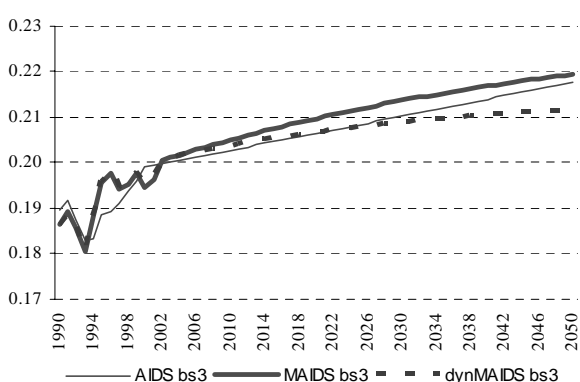
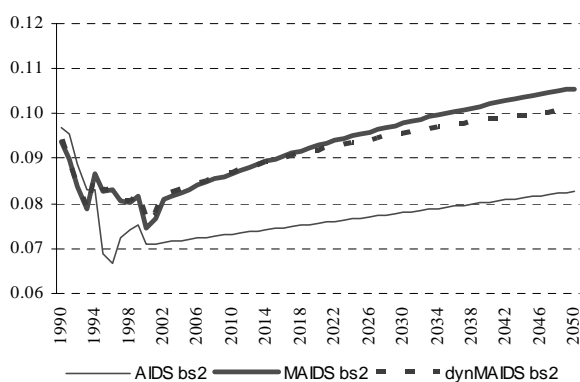
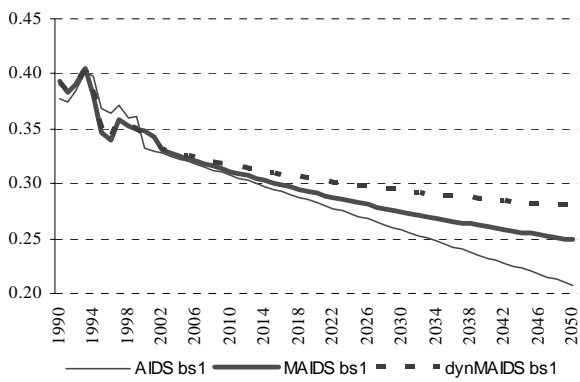




Figure 3: Simulation results for Finland



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