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**Demand Shifts, Labour Mobility and Unemployment Persistence**

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Abstract: In this study a two sector general equilibrium model with fully integrated labour and goods markets is presented. The two labour markets are segmented and represent two different ‘labour market regimes’ (Pissarides (1998)). The primary sector is an unionized high wage sector with ‘good’ and rationed jobs, in the secondary sector the wage rate is given by a competitive labour market with ‘bad’ jobs. Mobility between the two sectors takes place as in the Harris,Todaro (1970) model of migration depending on expected wage income in a sector. Goods demand depends on real household (workers) income and on relative prices given by unit costs. A simulation experiment shows that an \textit{ex ante} aggregate neutral demand shift from the secondary to the primary sector might have a positive impact on aggregate unemployment, while a demand shift from the primary to the secondary sector could have a negative impact.

Key words: demand shift hypothesis, general equilibrium, labour mobility, sectoral wages and unemployment

JEL classification: J64, J31, D58

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1. Introduction

In recent years the ‘demand shift hypothesis’ has gained in importance as an explanation for labour market problems. This hypothesis states that demand shifts have lead to a shift in sectoral unemployment rates, which under certain conditions can contribute to higher aggregate unemployment and to higher wage differentials between heterogenous types of labour. The standard theoretical framework for this kind of analysis is the sectoral model of unemployment as proposed in Jackson, Layard, Nickell (1991, chapter 6).

The actual story behind this general description of the demand shift hypothesis can be different. In most studies the two sectors represent the skilled and the unskilled labour market and the demand shift is from unskilled to skilled labour, which is to a certain extent supported by stylized facts (for an empirical assessment of this hypothesis s.: Nickell, Bell (1994)). This demand shift raises the unskilled unemployment rate and lowers the skilled unemployment rate thereby inducing wage increases for the skilled, which has a negative feedback on skilled employment with the possible outcome of a higher aggregate unemployment rate and a higher wage differential. An important point in this case is that supply responses of the skilled labour force to an increase in skilled employment are very low. In a more advanced model mobility of the labour force from the unskilled to the skilled sector can take place and depends on the costs of becoming skilled compared to the expected wage in skilled employment (Jackson, Layard, Nickell (1991)). Nickell, Bell (1994) argue that in this
enlarged model the higher wage rate and lower unemployment rate in the skilled segment of the labour market would induce training and therefore ‘migration’ from unskilled to skilled, which possibly could offset the initial effects in the long run. Behind this demand shift from unskilled to skilled labour the literature finds two main reasons: (i) increased competition for goods with high unskilled labour content due to enforced integration and (ii) skill biased technological progress, which shifts labour demand. Other studies deal with demand shifts between industries and occupations (Siebert (1997)) and also analyze the possible effects on unemployment in this categories. All these studies show on a theoretical or empirical basis the consequences of the demand shift for sectoral wages and sectoral unemployment rates.

As the standard theoretical framework for the analysis of shifts between labour markets the two sector analysis has evolved (Agenor, Aizenman (1997), Gregg, Manning (1997), Saint–Paul (1996)). The general features of such a two sector analysis are: (i) segmented labour markets (ii) differentials in wages and unemployment rates and in most but not all studies (iii) different types of labour markets in terms of wage setting, supply responses, etc. Sectoral details therefore are allowed for wage setting, labour demand and labour supply (search). It is clear that labour supply modelling and wage setting are to a certain extent alternatives, but in a two sector analysis labour supply features can be introduced via search activities or mobility of the labour force. These different labour markets in the two sector model could be labelled by the term ‘labour market regimes’ recently suggested by Pissarides (1998). In his study he sets up an identical aggregate production function with the corresponding derived labour market demand function and combines it with four different ‘labour market regimes’
(competitive labour markets, union wage bargaining, search equilibrium and efficiency wages) in terms of wage setting and labour supply.

The most important shortcoming of the two sector studies mentioned above is, that sectoral detail is only allowed for labour market variables, so that goods markets are not fully integrated in the analysis. The integration of goods markets would mean, that in sector \( i \) with type of labour \( i \) the specific product \( i \) at the wage rate and productivity of this sector is produced and sold to the market at the corresponding price. Another critical issue is that the labour markets are either totally segmented because of heterogenous labour (as in the skilled/unskilled case) or that search activities are allowed, without showing the implicit consequences in terms of shifts in the sectoral labour force. An interesting exception of this general rule are the studies, where the skilled/unskilled dimension is mixed with a sectoral dimension (Agenor, Aizenman (1997), McGregor et.al. (1998)). An important example for the integration of goods markets are the studies of Davis (1998a, 1998b). He starts from a Heckscher–Ohlin model and shows how in a global economy an increase in unemployment with minimum wages and an increasing wage differential between the sectors can be seen as a trade off. The integration of goods markets is done by the Stolper–Samuelson theorem, so that factor price and goods prices are directly linked, which is an important model feature in the case of increased competition as the cause of the demand shift. Some of the two sector (or multisectoral) studies integrate goods market in this sense and develop a general equilibrium approach (Gersbach (1999), McGregor et.al. (1998)), which can be seen as the most promising way of dealing with the different aspects of demand shifts.
In this study such a two sector general equilibrium model shall be proposed, where labour and goods market are fully integrated. The two labour markets are segmented as in the other studies and represent two different ‘labour market regimes’ in the terminology of Pissarides (1998). The idea of some of the cited studies is taken up, that one sector represents a primary sector with ‘good’ and rationed jobs and the other represents a secondary sector with ‘bad’ jobs and a competitive labour market („IBM and MacDonalds“). The primary sector is an unionized high wage sector, in the secondary sector the wage rate is in general given by a competitive labour market, but could be fixed by the government at a minimum wage level. Labour is not heterogenous as in the case of a skilled and an unskilled labour market and mobility between the two sectors takes place as in the Harris, Todaro (1970) model of migration depending on expected wage income in a sector. Goods demand depends on real household (workers) income and on relative prices given by unit costs. Ex ante aggregate neutral demand shifts in both directions can be introduced in this model as shocks to exogenous parameters in the sectoral demand functions. A simulation experiment shows that under certain conditions a demand shift from the secondary to the primary sector could have a positive impact on aggregate unemployment, while a demand shift from the primary to the secondary sector could have a negative impact.
2. A General Equilibrium Model

The explicit link between labour and goods market shall be described by a general equilibrium approach. In this model sector 1 (the primary sector) is a high wage, high productivity unionized sector. Sector 2 is a sector with a competitive full employment labour market. The behaviour of firms in both sectors can be described by profit maximizing in perfect competitive goods markets. Firms determine the derived labour demand for given wage rates. In sector 1 the wage rate is set by union wage bargaining, in sector 2 the wage rate is given by the identity of labour demand and labour supply.

The behaviour of households can be described by utility maximization in consumption, from which demand functions for the two goods can be derived. The budget constraint for total real household income (real wage income deflated by the aggregate price index) closes the model.

Mobility between the two sectors takes place depending on the differential in expected wage incomes, where the mobility parameter measures the opportunities of mobility in segmented labour markets.
2.1. Firms

Firms in the sector \( i (i = 1, 2) \) face a cost function (1), where costs depend on the level of output, \( Y_i \) and the wage rate \( w_i \) of the only input labour, \( L_i \). This cost function can be seen as the dual of a production function for \( Y_i = Y_i (L_i) \).

Factor Demand is derived via Shephard’s Lemma: \( L_i = \delta G_i / \delta w_i \) in order to get the optimal input coefficients \( L_i/Y_i \), which are assumed to take the functional form (2) for the two sectors.

\[
G_i = G_i (Y_i, w_i) 
\]

\[
\lambda_1 = (L_1/Y_1) = a_1 (1/w_1)^{\alpha_1} ; \quad \lambda_2 = (L_2/Y_2) = a_2 (1/w_2)^{\alpha_2} 
\]

with \( 0 < a_1, a_2 < 1 \) as well as \( 0 < \alpha_1, \alpha_2 < 1 \).

In a competitive product market with constant returns to scale prices equal unit costs, so that prices for the goods produced by the two sectors may be written as:
Equations (2) and (3) describe the firm behaviour and determine the level of employment and product prices for a given wage rate and given output levels. The level of output could additionally be determined by a production function. Demand and supply than had to balance out by the prices and the wage rate would adjust to the balance of (fixed) sectoral labour supply and derived labour demand as is actually done in most general equilibrium models. In this model the perfect competition condition is introduced by price = unit costs, and the level of output at any wage rate can be seen as determined by demand under the condition of the availability of sectoral labour force at full employment.

2.2. Households

Starting point for the households is the utility maximization of households, described by a expenditure function for PY (P = aggregate price level, Y = total real output) in the level of utility, \( v \) and the vector of goods prices, \( p \).
$$PY = PY(v, p)$$  \hspace{1cm} (4)

Actually such an expenditure function is the starting point for the Almost Ideal Demand System (AIDS). I propose a similar demand system, where the shares of real income and not the cost shares as in AIDS are explained by a shift parameter $\eta$ (representing tastes, preferences and other exogenous influences on demand) and cross and own price elasticity effects measured by the parameters $\varepsilon_{ij}$.

$$d_1 = \frac{Y_1}{Y} = \eta_1 + \varepsilon_{11} p_1 + \varepsilon_{12} p_2$$ \hspace{1cm} (5)

$$d_2 = \frac{Y_2}{Y} = \eta_2 + \varepsilon_{12} p_1 + \varepsilon_{22} p_2$$ \hspace{1cm} (6)

These demand functions can be directly expressed in terms of sectoral wages:

$$d_1 = \frac{Y_1}{Y} = \eta_1 + \varepsilon_{11} a_1 w_1^{1-\alpha_1} + \varepsilon_{12} a_2 w_2^{1-\alpha_2}$$ \hspace{1cm} (5a)

$$d_2 = \frac{Y_2}{Y} = \eta_2 + \varepsilon_{12} a_1 w_1^{1-\alpha_1} + \varepsilon_{22} a_2 w_2^{1-\alpha_2}$$ \hspace{1cm} (6a)

The symmetry restriction of the demand system is applied as the parameter $\varepsilon_{12}$ is the same for $p_2$ in (6) as for $p_1$ in (5). Additivity further requires that
\[ \eta_1 + \eta_2 = 1 \, , \, \varepsilon_{11} + \varepsilon_{12} = 0 \, , \, \varepsilon_{12} + \varepsilon_{22} = 0 \, . \]

In this two sector case these restrictions for the \( \varepsilon_{ij} \) also guarantee homogeneity. Total real household income is given as the sum of labour incomes deflated by the aggregate price index of the economy:

\[ Y = \left( w_1 L_1 + w_2 L_2 \right) / P \]  \hspace{1cm} (7)

\[ P = d_1 p_1 + d_2 p_2 \]  \hspace{1cm} (8)

For given labour forces in the two sectors \( N_1 \) and \( N_2 \) the unemployment rates would be given with:

\[ u_1 = 1 - \left( \lambda_1 Y_1 / N_1 \right) \]  \hspace{1cm} (9)

\[ u_2 = 1 - \left( \lambda_2 Y_2 / N_2 \right) \]  \hspace{1cm} (10)

Equations (2), (5a), (6a), (7), (8), (9) and (10) make up the goods supply and demand side and the unemployment definitions of the model at given wage rates. Demand shifts introduced by
a change in the shift parameters \( \eta \) result in a redistribution of sectoral output, employment and unemployment. Aggregate real income does not change after this kind of demand shifts. This can be proofed by writing \( w_1L_1, w_2L_2 \) as \( w_1\lambda_1d_1Y, w_2\lambda_2d_2Y \) and \( d_1p_1, d_2p_2 \) as \( w_1\lambda_1d_1, w_2\lambda_2d_2 \). The demand shift is defined as \( \delta d_1 = -\delta d_2 \) or vice versa and does not alter wage rates and sectoral productivities. Therefore for a given level of \( Y \) the percentage change in \( w_1L_1 \) will be the same as the percentage change in \( d_1p_1 \) and the same will hold for \( w_2L_2 \) and \( d_2p_2 \). This is the intuition behind the conclusion, that household income will not be affected by a demand shift.  

Such a small sectoral model serves to explain demand shifts and the consequences on sectoral employment structure of a 'demand bias' as shown in Gundlach (1994). The driving forces are changes in the exogenous shift parameters \( \eta_1 \) and \( \eta_2 \), which could come about by increased competition for goods of one sector due to enforced integration. The other case of demand shifts discussed in the literature, namely skill biased technological progress could be treated with as in Gregg, Manning (1997) by introducing a (technological) shift parameter in the labour demand equations (2). The impact on sectoral employment and unemployment of such a 'technological bias demand shift' would not be different in the model part outlined here from a shift in goods demand.

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1 The formal proof that \( \delta \log (w_1L_1 + w_2L_2) = \delta \log (d_1p_1 + d_2p_2) \) is more complicated and is left out here for reasons of simplicity.
2.3. Labour Markets

The two sectors can be described as two different 'labour market regimes' which allows to treat them as segmented labour markets. Sector 1 or the primary sector is an unionized labour market with wage bargaining given a union utility function (s.: Pissarides (1998)). The alternative to the union wage bargaining model in sector 1 would be an efficiency wage model as described in Saint – Paul (1996) and in Agenor, Aizenman (1997). The common feature of these models is that the wage rate will be above the competitive equilibrium model. The wage bargaining process distributes the gains of a job for firms between firms and employees, which face a certain reservation wage given by unemployment benefits. Usually unemployment benefits are given with a constant replacement ratio $\rho$ ($0 < \rho < 1$) of the former earned wage. The arguments of a wage setting function following from a bargaining model in sector 1 are the inverse unemployment rate and the difference between productivity as a measure of the gains of the job to firms and the reservation wage of sector 1 workers, $\rho w_1$. The competitive wage from (2) would be $w_{1,c} = (\lambda_1/a_1)^{-1/\alpha_1}$ and the wage rate following from (11) can be seen as a mark up on this competitive wage.

$$w_1 = \varpi_1 + \phi_1 (1/u_1) + \phi_1 (1/\lambda_1 - \rho_1 w_1)$$ (11)

where $\varpi_1, \phi_1, \phi_1 > 0$. 
The explicit formulation for the wage rate can be written as

\[
\frac{(w_1 + \varphi_1 (1/u_1) + \phi_1(1/\lambda_1))}{(1 + \phi_1 \rho)}
\]  

(12)

Jobs in sector 1 may be seen as rationed and workers of sector 2 as well as of sector 1 are searching for jobs in sector 1. As sector 2 is assumed to be a competitive labour market, labour supply equals labour demand in every moment. The unemployment rate is at a minimum given by frictional unemployment: \( u_2 = u_2^* \).

So for given labour force in sector 2, \( N_2 \), labour supply is \( (1 - u_2^*) N_2 \) which must equal \( L_2 \) given with \( a_2 (1/w_2)^{\alpha_2} d_2 Y \). The resulting competitive wage rate is

\[
w_2 = \left[ \frac{((1 - u_2^*) N_2) / (a_2 d_2 Y)}{\alpha_2} \right]^{-\frac{1}{\alpha_2}}
\]  

(13)

Alternatively to the wage rate determined by (13) the government could fix a level of minimum wage rate \( w_2^* \) for 'bad' jobs as described in Agenor, Aizenman (1997). If this wage rate lies above the competitive wage labour demand would again be determined by (2) and the unemployment rate in sector 2, \( u_2 \), would be above the full employment rate. The
unemployment rate in sector 1, \( u_1 = 1 - \left( \frac{\lambda_1 \cdot Y_1}{N_1} \right) \), is at given labour supply, \( N_1 \), determined by the wage rate, \( w_1 \) and sectoral output, \( Y_1 \). In this setting it is reasonable to assume, that \( u_1 \) lies above the full employment level due to wages above the competitive equilibrium in a sector with rationed jobs.

A totally different setting is the combination of the efficiency wage model with the search model as described in Agenor, Aizenman (1997) and Saint – Paul (1996). There sectoral unemployment rates are given by turnover rates in the sectors and arrival rates of workers to jobs. The primary high wage sector in an economy usually has a low turnover rate and therefore a low unemployment rate, whereas in the secondary sector supply is abundant and the turnover rate is high resulting in a relatively high unemployment rate. Saint – Paul (1996) describes a framework, where the condition that the arrival rate of workers to jobs is higher for unskilled than for skilled workers suffices to derive that \( u_2 > u_1 \). The conditions \( w_1 > w_2 \) and \( u_2 > u_1 \) are also compatible with stylized facts in the skilled/unskilled case and in the case of regional labour markets. It must be noted, that in such a setting the labour supply responses are small and mobility between the two segmented labour markets is not described explicitly.

In general in the skilled/unskilled case unemployed persons are attached to the segment of the labour force, where they come from and in unemployment still belong to. In this study the unemployed should be attached in the labour force of the segment, where they search for jobs. As Saint – Paul also notes in the sector with ‘good jobs‘ people will queue in search for work. In this sense an assumption, that the unemployment rate in the high wage sector exceeds the unemployment rate in the secondary sector seems reasonable, although this is inconsistent with our concept of measuring sectoral unemployment.
The crucial point, where this study deviates from others is the combination of the segmented labour markets model with mobility of the labour force between the sectors. Actually unions cannot prevent workers from sector 2 moving to sector 1 and accepting jobs there. I do not explicitly take into account, that the wage bargaining process in sector 1 may be influenced by migration into sector 1, as Burda, Funke (1993) have lined out. Mobility is just not ruled out by any assumption of labour market segmentation in this study and is taking place on the lines of the Harris, Todaro (1970) migration model. Sectoral unemployment rates are therefore also determined by the denominator in (9) and (10). Mobility is not perfect, but reacts with a certain elasticity to expected income differentials and this elasticity measures the actual opportunity to move. When total labour force $N$ is constant: $N = N_1 + N_2$, we have that

$$N_2 = N - N_1$$  \hspace{1cm} (14)

Migration takes place in one direction, if there is an expected wage rate differential. This differential is given with the wage rates adjusted for the probability to find a job in each sector approximated by the employment rates $L_1/N_1$ and $L_2/N_2$ as described in Burda, Funke (1993). In the Harris, Todaro (1970) study equilibrium was defined as zero migration and migration took only place from the rural (agricultural) sector to the urban (industrial) sector, i.e. only in one direction. Taking that as a starting point for this study one would expect, that in principle workers are willing to migrate to sector 1, where jobs are rationed at high wages. So the labour force in sector 1, $N_1$, is made up of a constant, $\mu_1$, and net migration into sector 1, $M_1$. 
It may be noted, that this specification does not rule out that net migration from sector 1 to sector 2 might occur, net migration is just expressed in terms of sector 1 labour force and can have a positive or negative sign. As the total labour force is given, the balance for labour force in sector 2, N_2 is always determined by (14).

\[ N_1 = \mu_1 + M_1 \]  

(15)

The migration rate is then given by the differential of expected income, where \((1 - u_i)\) stands for the probability of getting a job in sector \(i\).

\[ M_i/\mu_i = \gamma [w_1 (1 - u_1) - w_2 (1 - u_2^*)] \]  

(16)

with \(\gamma > 0\).

The equilibrium condition in Harris, Todaro (1970) for zero migration is that the expected incomes are equal in the two sectors, what implies:

\[ w_1 (1 - u_1) - w_2 (1 - u_2^*) = 0 \]  

(17)

In this trivial case of zero net migration in equilibrium the two unemployment rates are directly linked to the wage differential:
Here the unemployment rate in sector 1 is a simple increasing function of the wage $w_1$ and a decreasing function of the wage $w_2$.

Actually the zero net migration condition (17) can in the case of a positive wage differential $w_1 > w_2$ only be fulfilled, if the unemployment rate in sector 1 exceeds the unemployment rate in sector 2: $u_1 > u_2$ to compensate for the wage differential. But besides that in case of a positive income differential induced migration may be small, depending on the size of the mobility parameter $\gamma$. The stylized facts in the skilled/unskilled case and the assumptions of the theoretical studies on this issue start from the opposite conditions: $w_1 > w_2$ and $u_2 > u_1$.

The migration model presented here is only consistent with a positive differential of wage rates and a negative differential of unemployment rates between sector 1 and 2, if we introduce an additional term like the costs of training and education in (16) and (17). Actually mobility in the skilled/unskilled case is additionally restricted by the costs of becoming skilled.²

² So in this case we could rewrite (16) as: (16a) $M_1/\mu_1 = \gamma [w_1 (1 - u_1) - w_2 (1 - u_2^*) - k w_2]$ with $k$ as the costs of training and education expressed in terms of the unskilled wage rate. The zero net migration condition would then be: (17a) $w_1 (1 - u_1) - w_2 (1 - u_2^*) - k w_2 = 0$ and the costs of education compensate for the wage and unemployment differentials. So these costs are the barrier, why unskilled workers are not necessarily moving in the sector with higher wages and higher employment probability.
A definition for the aggregate unemployment rate \( u \) with \( n_1 = N_1/N \), \( n_2 = N_2/N \) complements the model:

\[
u = u_1 n_1 + u_2 n_2
\] (19)
3. Equilibrium Unemployment

The equations set up above can be combined to a small equilibrium model. The core of the model are the wage equations for \( w_1 \) and \( w_2 \) and the unemployment equation for \( u_1 \), which are determined also by mechanisms acting in the goods markets (price elasticities).

We can differentiate the two cases where (i) \( u_1 \) is given by the zero net migration condition (18) as in the Harris, Todaro (1970) study and (ii) equilibrium migration between the two sectors takes place and \( u_1 \) is determined by (9). In the first case (18) must be combined with the two wage equations and with the income equation. In the case of equilibrium migration the system must additionally contain the equation describing labour force mobility.

In the second case we would simply not enforce zero net migration as a condition for equilibrium. At first sight the treatment of mobility which implies a rate of change or a time derivative in a comparative static framework raises some problems and questions about the nature of this migration rate. Harris, Todaro (1970) avoid this problem by introducing the zero net migration condition. The dynamic point of view consistent with this condition is that at an existing expected income differential migration would never cease, but would continue until the unemployment rate \( u_1 \) has risen enough to close the expected income differential. Here I would like to assume, that an equilibrium is defined by any stable solution to the system, what could imply some ‘equilibrium‘ value for the migration rate \( \gamma [w_1 (1 - u_1) - w_2 (1 - u_2)] \).
The interpretation of this term in a comparative static framework would be the necessary once and for all mobility between the labour force in order to reach a new equilibrium. In this new equilibrium we might still have an expected income differential, but we can calculate from (16) the exact number of labour force movement induced by this differential. The labour force in sector 1 in this comparative static framework is simply given by the constant $\mu_1$ and the second term depending on relative wages and unemployment rates.

**Case 1: zero net migration**

In this case we get a system in the unknowns $u_1$, $w_1$, $w_2$ and $Y$ with the respective equations. The unemployment rate in sector 1 is totally determined now by this zero net migration condition:

\[
\begin{align*}
    w_2(1 - u_2^*) \\
    u_1 = 1 - \frac{w_2(1 - u_2^*)}{w_1}
\end{align*}
\]

(18)
Therefore employment in sector 1 becomes $L_1 = (1 - u_1) N_1$, which overwrites the labour demand equation in sector 1, so that: $\lambda_1 = L_1 / Y_1$. For a given labour supply $N_1$ without migration employment is now a function of the unemployment rate, which is restricted by zero migration (18). The labour input coefficient $\lambda_1$, that enters the wage equation for sector 1 is then for a given impact of the unemployment rate on employment determined by output $Y_1$.

We can now write the wage equations as well as the income equation as functions \{ \} of the other variables:

$$w_1 = \frac{(\omega_1 + \phi_1 (1/u_1) + \phi_1 (1/\lambda_1 \{u_1, d_1 \{\eta_1, w_1, w_2\}, Y\})}{(1 + \phi_1 \rho)} \quad (20)$$

$$w_2 = [(1 - u_2^*) (N - N_1)] / (a_2 d_2 \{\eta_2, w_1, w_2\} Y)^{-1/\alpha_2} \quad (21)$$

$$Y = \frac{[w_1 L_1 \{u_1, d_1 \{\eta_1, w_1, w_2\}, Y\} + w_2 \{d_2 \{\eta_2, w_1, w_2\}, Y\} L_2]}{[d_1 \{\eta_1, w_1, w_2\} p_1 \{w_1\} + d_2 \{\eta_2, w_1, w_2\} p_2 \{w_2\}]} \quad (22)$$
Wages and employment in sector 1 are both a function of $u_1$ and of the demand share $d_1$, which by itself is a function of $\{\eta_1, w_1, w_2\}$. Both wage rates are functions of total income. From the budget balance (22) it can be seen easily, that $Y$ cannot be expressed explicitly. In a first step therefore we could think of searching for the analytical solution of the system for a given $Y$. It can be shown however, that a simple explicit treatment for $w_1$ and for $w_2$ is also impossible given the functional forms for labour demand and prices in this model (s.: Appendix). That together with the nature of the budget balance (22) makes it plausible to treat the model as a small CGE model, where a numerical solution can be found.

Analysing the partial derivatives in the system we see directly from (18), that the unemployment rate in sector 1 in this setting is a negative function of the wage differential $w_2/w_1 : \delta u_1 / \delta (w_2/w_1) < 0$. An *ex ante* aggregate neutral demand shift in favour of sector 1 ($\delta \eta_1 > 0 = - \delta \eta_2$) ceteris paribus raises the wage rate in sector 1 and lowers the wage rate in sector 2 (for the derivation of the partial derivatives $\delta w_1/\delta \eta_1$ and $\delta w_1/\delta \eta_2$ s.: Appendix), so that $\delta (w_2/w_1) < 0$. Obviously this would have a feedback on the wage rate $w_1$ and the final outcome might be different from what one derives from the partial derivatives. If the new equilibrium is associated with a decrease of the wage differential $w_2/w_1$ the unemployment rate $u_1$ will rise as a consequence of a favourable demand shift for sector 1.

The reason is that the restriction of zero migration totally determines the results and sustaining zero migration with an increasing wage differential for sector 1 is only possible at the expense of lower employment probability in sector 1. This is a striking result for the case of a demand shift, as we would expect that a demand shift has a positive impact on
employment in the sector where demand shifts in. In the case of zero net migration employment in sector 1 is determined by the unemployment rate, which balances the increase in \( w_1 \) in order to fulfill the zero net migration condition. Therefore the zero migration condition seems too restrictive for a full analysis of the adjustment of sectoral wages, demand and labour force in reaction to a demand shift. As the unemployment rate \( u_1 \) is always directly determined by the wage differential, i.e. \( \frac{\delta u_1}{\delta (w_2/w_1)} < 0 \), a numerical solution of such a CGE model would not yield more insight in the adjustment mechanism of goods and labour markets than can be derived from the partial derivatives. An important point is that in contrast to the system made up by equations (1) to (10) it is not clear now at all, that an \textit{ex ante} aggregate neutral demand shift will not change total real household income, \( Y \). Changes in \( Y \) will come along with changes in sectoral employment, which changes simultaneously with the wage structure.

\textit{Case 2: equilibrium migration}

In this case the expression for the core variable of the system \( u_1 \) becomes more complicated and must be written as a function not only of the two wage rates, \( w_1 \) and \( w_2 \) but also of total income \( Y \), the labour force in sector 1, \( N_1 \) and the demand shift parameter \( \eta_1 \).
\[
\lambda_1 \{ w_1 \} d_1 \{ \eta_1, w_1, w_2 \} Y \\
\lambda_1 \{ w_1 \} d_1 \{ \eta_1, w_1, w_2 \} Y \\
-u_1 = 1 - \frac{1}{N_1} \\
\]

(23)

In this setting labour demand in sector 1 is still influenced by the wage rate and there is no a priori restriction on \( u_1 \), i.e. higher demand for good 1 c.p. increases labour demand and thereby lowers the unemployment rate \( u_1 \).

Labour force mobility now means that the labour force \( N_1 \) by itself is a function of \( w_1 \), \( w_2 \) and \( u_1 \) and therefore changes simultaneously with \( u_1 \) by migration. For a given sectoral allocation of labour force (i.e. a given value of \( N_1 \)) \( u_1 \) in this setting becomes a negative function of the demand shift parameter \( \eta_1 \) \( (\frac{\delta u_1}{\delta \eta_1} < 0) \), what simply expresses that a favourable demand shift for sector 1 will lower the unemployment rate in sector 1 because of higher ceteris paribus employment in sector 1 (s.: Appendix). Things will become more complicated, when we take into account that the sectoral labour force might change according to

\[
N_1 = \mu_1 (1 + \gamma [w_1 (1 - u_1) - w_2 (1 - u_2)]) \\
\]

(24)

Inserting that into (23) would yield an expression with the unemployment rate \( u_1 \) on the left and on the right hand side. Transforming to get an explicit treatment of \( u_1 \)
then gives a quadratic equation. The solution of this equation (s.: Appendix) reveals that the demand shift parameter \( \eta_1 \) now enters the root term of the solution. Assuming the 'normal' case of a positive root the impact of \( \eta_1 \) on \( u_1 \) would still be negative (s.: Appendix).

Anyway the partial derivatives for \( u_1 \) are only part of the adjustment mechanism taking place, because the other variables will change simultaneously. Again the system must be complemented by the wage equations and the income equation, which for \( w_1 \) and for \( Y \) have a slightly different form now:

\[
w_1 = \frac{(\sigma_1 + \varphi_1 (1/u_1) + \phi_1(1/\lambda_{1}\{w_1\}))}{(1 + \phi_1 \rho)} \tag{25}
\]

\[
w_2 = \left[ \frac{(1 - u_2^*) (N - N_1))}{(a_2 d_2 \{\eta_2, w_1, w_2\} Y)} \right]^{-1/\alpha_2} \tag{21}
\]

\[
Y = \frac{[w_1\{u_1\}\lambda_{1}\{w_1\}d_1\{\eta_1, w_1, w_2\} Y + w_2 \{d_2 \{\eta_2, w_1 , w_2 \} Y \} L_2]}{\{d_1(\eta_1, w_1 ,w_2)p_1(w_1) + d_2 (\eta_2, w_1 , w_2) p_2 (w_2)\}} \tag{26}
\]
The wage equation for $w_1$ could now be written as a function of $u_1$ only, whereas $w_2$ depends on $w_1$ and $Y$. As above $Y$ as the budget balance cannot be expressed explicitly, so again we end up with a small CGE model without explicit solution and where numerical solution can be found.

A demand shift in favour of sector 1 now has a first negative impact on $u_1$ which induces the wage rate $w_1$ to rise. Part of the adjustment mechanism in reaction to a demand shift now works through labour force mobility and migration. The system can be solved after a demand shift for a new equilibrium with an equilibrium migration rate following from sectoral wages and the unemployment rate $u_1$.

4. Demand Shift Effects

In this section I present a small numerical example for the model with equilibrium migration set up above, which mostly relies on stylized facts for Austria. The model is calibrated in a way to get an initial equilibrium with zero net migration, but the functions allow migration to achieve a new equilibrium after a shock.
The first column in Table 1 describes the data set for the initial equilibrium, where we have
\( w_1 (1 - u_1) - w_2 (1 - u_2^*) = 0 \), i.e. zero net migration. It is clear then that even small
changes in simulations affecting the wage differential \( w_1/w_2 \) and the unemployment rate \( u_1 \)
will induce migration in both directions. The total labour force \( N \) is given, the wage in the
primary sector (1,1) is about 1.4 times the wage in the secondary sector (0,8), which is
accompanied by a productivity in sector 1 which is 1.3 times the productivity in sector 2.
This is in line with stylized facts on wage differentials, which cannot be totally explained by
productivity. The baseline data on output and employment level reproduce the Austrian data
for manufacturing (sector 1) and the rest of the economy (sector 2).

As mentioned above the primary high wage sector in most European economies usually has a
low turnover rate and therefore a low unemployment rate, whereas the opposite is true for the
secondary sector. This fact represented in the data is consistent with the statistical concept we
have in measuring sectoral labour force by attaching unemployed persons to the segment of
the labour force, where they come from and in unemployment still belong to. Obviously this
concept is justified in the skilled/unskilled case, where we also have limited mobility. It is not
so clear in the case of industries and occupations, as Nickell (1997) also points out. In the
numerical example I impose the zero net migration condition in the initial equilibrium
situation. Fixing the full employment unemployment rate in sector 2 at 2% (which is below
the actual statistical value) and with all the other data given the unemployment rate in sector 1
is determined at 28.7%. Clearly this values for the sectoral unemployment rates are a major
deviation from stylized facts and follow from the theoretical framework developed in this
study. The unemployment rates must be interpreted as following from a concept that attaches
the unemployed in the labour force segment, where they search for jobs. As wages in sector 1 are high, persons are queuing for ‘good jobs’ expressed by high unemployment, which deters further people from searching for work in sector 1 (zero migration). The total rate of unemployment, which now must be interpreted also as a measure of people in search for work is 12.9%. The parameter values of the functions are as follows:

\[ a_1 = 0.79 \quad a_2 = 0.94 \quad \alpha_1 = 0.3 \quad \alpha_2 = 0.5 \quad \eta_1 = 0.408 \quad \eta_2 = 0.592 \quad \varepsilon_{11} = \varepsilon_{22} = -0.4 = -\varepsilon_{12} \]

\[ \gamma = 0.1 \quad \varphi_1 = 0.056 \quad \phi_1 = 0.19 \quad \rho = 0.6. \]

The two simulation experiments carried out are ex ante aggregate neutral but considerable demand shifts, where \( \delta \eta_1 = -\delta \eta_1 \). In the first case I assume, that \( \delta \eta_1 = +0.04 \), which gives an ex ante reallocation of 130 units (~10% of \( Y_1 \)) of income and in the second case we have \( \delta \eta_2 = +0.06 \), which means that 190 units (~10% of \( Y_2 \)) change from sector 1 to sector 2. The demand shift is the same in both directions in percentage of the output of the favoured sector. We would expect, that as the zero net migration condition is not imposed both shocks will induce migration in different directions. As the unemployment rate in sector 2 is fixed at full equilibrium and total labour force is given, the migration flow is always equal to the change in sector 2 employment. The results for the two cases are also listed in Table 1.

In the case of the demand shift into sector 1 \( (\delta \eta_1 = -\delta \eta_2 = 0.04) \) one observes the standard results, which we would expect according to the Layard, Nickell, Jackman (1991) model. The wage differential \( w_1/w_2 \) increases and the unemployment rate \( u_1 \) decreases. In contrast to the studies treating with the skilled/unskilled case this demand shift decreases the total unemployment rate. That result prevails, although considerable wage effects are induced
by the decrease in \( u_1 \) and the employment effects are accompanied by migration into sector 1. This migration flow of 15 into sector 1 determines the employment change in sector 2 ( - 15) and is consistent with a positive expected income differential (+ 0.107) in equilibrium. The feedback effects of the demand function brought about by a higher relative price of good 1 can be seen in the result, that the ex post change \( \delta d_1 \) is about + 0.027 compared to the initial change in the shift parameter of + 0.04. So the ex post absolute change in \( Y_1 \) (+ 122 units) is less than the ex ante (+130 units) change, although total income has increased. This increase in total income is due to the wage and employment shifts, the change in the aggregate price index \( (d_1 p_1 + d_2 p_2) \) is small and positive ( +0.3 %), i.e. real income reducing.

For the demand shift into sector 2 \( (\delta \eta_2 = - \delta \eta_1 = 0.06) \) we see that unemployment in sector 1 rises considerably accompanied by lower total output and a decreasing wage differential \( w_1/w_2 \). This result is not trivial, as migration into sector 2 also occurs, so the migration flow is reversed. As employment in sector 2 \( (L_2) \) is given with the full employment unemployment rate as \( (1 - u_2)(N - N_1) \), it becomes clear, that employment in sector 2 can only increase, if there is net migration into sector 2. Until this point is reached, the wage rate \( w_2 \) rises, thereby raising the output price \( p_2 \) with the corresponding negative feedback on demand for sector 2 goods, so that part of the demand shift to sector 2 will be reversed. This feedback impact on demand can be seen very clearly from the results for sectoral and total output. The \textit{ex ante} impact on \( Y_2 \) of 190 units is transformed into 58 units in the model solution, part of which comes about by the decrease in total income, \( Y \). Looking at the shares we see, that an \textit{ex ante} increase in the share \( d_2 \) (+ 0.06) to 0.6538 is dampened considerably to 0.6378 ( + 0.044). With a demand shift into sector 2 therefore there is a contradictory claim on
w₂ in order not to increase unemployment. On the one hand w₂ should become high enough to attract workers as only by that unemployment might not rise and on the other hand it should not rise too much in order not to reverse the demand shift. The economic intuition behind that is, that attracting workers for 'bad' jobs in a competitive sector (e.g.: personal services) is only possible if a rising unemployment rate in the other sector (e.g.: chemical industry) is combined with rising wages for 'bad' jobs. This wage rise on the other hand depresses the demand for products with a high 'bad' jobs content. The assumption of a given total labour force is crucial for this result. If we would introduce a flexible participation rate, 'bad' jobs possibly would be matched with new entrants in the labour market without the demand depressing feedback effect on the goods market.

Conclusions

The two sector general equilibrium model for the case of equilibrium migration developed in this study can be applied to analyse the different links between labour and goods markets in the presence of demand shifts. The feedbacks of the goods demand on the labour market are shown to be important in a numerical simulation exercise. It is worth noting, that in this setting a demand shift from the sector with 'good' jobs to the sector with 'bad' jobs has a negative labour market impact and the opposite is true for the demand shift in the other
direction. This is in slight contradiction to the studies dealing with demand shifts from unskilled to skilled labour, where this demand shift raises unemployment. In this study mobility between the sectors is possible and is an important adjustment mechanism to reach a new equilibrium. This kind of mobility is ruled out in most studies dealing with the skilled/unskilled case.

The model presented in this study may also contribute to an explanation of persistent unemployment over the business cycle. As the simulation exercises show in this model persistent unemployment would arise, if a negative demand shock for sector 1 is followed by a positive demand shock for sector 2 of the same magnitude.
Table 1: Simulation Results of an exogenous demand shift ($\delta \eta_i = - \delta \eta_j$)

<table>
<thead>
<tr>
<th>Demand Shifts</th>
<th>Baseline</th>
<th>$+ \delta \eta_1$ = $- \delta \eta_2$</th>
<th>$+ \delta \eta_2$ = $- \delta \eta_1$</th>
</tr>
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<tbody>
<tr>
<td>$\lambda_1$</td>
<td>0.7692</td>
<td>0.7612</td>
<td>0.7785</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>1.0527</td>
<td>1.0674</td>
<td>1.0323</td>
</tr>
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<td>W1</td>
<td>1.1</td>
<td>1.1394</td>
<td>1.0571</td>
</tr>
<tr>
<td>W2</td>
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<td>0.7781</td>
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</tr>
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<td>P1</td>
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<td>0.8672</td>
<td>0.8229</td>
</tr>
<tr>
<td>P2</td>
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<td>0.8305</td>
<td>0.8586</td>
</tr>
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<td>0.4332</td>
<td>0.3622</td>
</tr>
<tr>
<td>D2</td>
<td>0.5938</td>
<td>0.5668</td>
<td>0.6378</td>
</tr>
<tr>
<td>Y</td>
<td>3200</td>
<td>3282</td>
<td>3070</td>
</tr>
<tr>
<td>Y1</td>
<td>1300</td>
<td>1421</td>
<td>1112</td>
</tr>
<tr>
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<td>1860</td>
<td>1958</td>
</tr>
<tr>
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<td>1418</td>
<td>1382</td>
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<tr>
<td>N2</td>
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</tr>
<tr>
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<td>865</td>
</tr>
<tr>
<td>L2</td>
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<td>1985</td>
<td>2021</td>
</tr>
<tr>
<td>L</td>
<td>3000</td>
<td>3067</td>
<td>2887</td>
</tr>
</tbody>
</table>
Appendix

Case 1: zero net migration

Taking into account that \( \lambda_1 = \frac{L_1}{Y_1} \) and rearranging (20) one derives:

\[
(1A) \quad w_1 + w_1 \phi_1 \rho - \phi_1 [(\varepsilon_{11} a_1 w_1^{1-\alpha_1} Y) / ((1 - u_1) N_1)] = \\
\omega_1 + \phi_1 (1/u_1) + \phi_1 [(\eta_1 + \varepsilon_{12} a_2 w_2^{1-\alpha_2} Y) / ((1 - u_1) N_1)]
\]

Obviously an explicit formulation for \( w_1 \) is not feasible. The expression for \( w_1 \) can serve to derive the c.p. partial derivatives:

\[
(2A) \quad w_1 = \frac{(\omega_1 + \phi_1 (1/u_1) + \phi_1 [(\eta_1 + \varepsilon_{11} a_1 w_1^{1-\alpha_1} + \varepsilon_{12} a_2 w_2^{1-\alpha_2} Y) / ((1 - u_1) N_1)]}{(1 + \phi_1 \rho)}
\]

From (2A) we get:

\[
(3A) \quad \delta w_1 / \delta \eta_1 = \frac{\phi_1 Y / ((1 - u_1) N_1)}{(1 + \phi_1 \rho)} > 0
\]

Writing (21) as

\[
(4A) \quad w_2 = \left[ \frac{(1-u_2^a)N_2}{a2(\eta 2 + \varepsilon 12w1^{1-\alpha_1} + \varepsilon 22a2w2^{1-\alpha_2})Y} \right]^{-1/\alpha_2}
\]
As in (2A) one can observe, that an explicit treatment of \( w_2 \) is impossible. But again we can use this expression to derive the c.p. partial derivatives: \( \delta w_2 / \delta \eta_2 = \delta w_2 / \delta \eta_1 \).

\[
(3A) \quad \delta w_2 / \delta \eta_2 =
\]
\[
\left[ \frac{(-1/\alpha 2)(1-u^2*)N2}{a2(\eta 2 + \varepsilon 12 w_1^{1-\alpha 1} + \varepsilon 22 a2 w_2^{1-\alpha 2})Y} \right]^{-(1-\alpha 2)/\alpha 2}
- \frac{-a2Y(1-u^2*)N2}{(a2(\eta 2 + \varepsilon 12 w_1^{1-\alpha 1} + \varepsilon 22 a2 w_2^{1-\alpha 2})Y)^2}
\]

As \( \left[ \frac{(1-u^2*)N2}{a2(\eta 2 + \varepsilon 12 w_1^{1-\alpha 1} + \varepsilon 22 a2 w_2^{1-\alpha 2})Y} \right] \) is positive, we get that \( \delta w_2 / \delta \eta_2 = \delta w_2 / \delta \eta_1 > 0. \)

**Case 2: equilibrium migration**

Combining (2), (3), (5a) and (9) in the text and ignoring migration, so that \( N_1 \) is given we get the following explicit expression for \( u_1 \):

\[
(1A) \quad u_1 = 1 - \frac{a_1 (1/w_1)^{\alpha 1} [\eta_1 + \varepsilon_{11} a_1 w_1^{1-\alpha 1} + \varepsilon_{12} a_2 w_2^{1-\alpha 2}] Y}{N_1}
\]

The impact of \( \eta_1 \) on \( u_1 \) is given as:
\[
\begin{align*}
\frac{\partial u_1}{\partial \eta_1} = - \frac{a_1 (1/w_1)^{\alpha_1}}{N_1} < 0
\end{align*}
\]

Introducing migration as in (21) we get for \( u_1 \) the following expression:

\[
\begin{align*}
a_1 (1/w_1)^{\alpha_1} [\eta_1 + \varepsilon_{11} a_1 w_1^{1-\alpha_1} + \varepsilon_{12} a_2 w_2^{1-\alpha_2}] Y \\
\mu_1 (1 + \gamma [w_1 (1 - u_1) - w_2 (1 - u_2)])
\end{align*}
\]

An explicit treatment of \( u_1 \) yields the quadratic equation:

\[
\begin{align*}
\lambda_1 (w_1) d_1 (\eta_1, w_1, w_2) Y - 1 \\
u_1^2 + u_1 \left[ (w_2/w_1) (1 - u_2) - (1/\gamma w_1) - 1 \right] + \frac{\lambda_1 (w_1) d_1 (\eta_1, w_1, w_2)}{\gamma w_1 \mu_1} = 0
\end{align*}
\]

Here the term \( a_1 (1/w_1)^{\alpha_1} [\eta_1 + \varepsilon_{11} a_1 w_1^{1-\alpha_1} + \varepsilon_{12} a_2 w_2^{1-\alpha_2}] \) has been substituted again with \( \lambda_1 (w_1) d_1 (\eta_1, w_1, w_2) \).

The solution to this quadratic equation is:

\[
\begin{align*}
(5A) \quad u_1 (1,2) = - \left[ (w_2/w_1) (1 - u_2) - (1/\gamma w_1) - 1 \right]/2 & \pm \sqrt{\frac{\lambda(w_1) d_1 (\eta_1, w_1, w_2) Y - 1}{\gamma w_1 \mu_1}}
\end{align*}
\]
A negative root term would mean cyclical variations, which does not seem to make sense in a comparative static framework. For the 'normal' and economically feasible case, that the root term is positive, we get again that $\delta u_1 / \delta \eta_1 < 0$. 
References


Gundlach, E., (1994), Demand bias as an explanation for structural change, Kyklos, 47, 1994, 249 - 267


