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Marcel Bluhm

# Interbank Funding as Insurance Mechanism for (Persistent) Liquidity Shocks

SAFE Working Paper No. 117

**SAFE | Sustainable Architecture for Finance in Europe**

A cooperation of the Center for Financial Studies and Goethe University Frankfurt

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## Non-Technical Summary

The financial system serves to allocate funds from savers to investors. In this framework banks assume a crucial function via the provision of credit and liquidity to the real economy. To efficiently fulfill this function, the interbank market allows for the allocation of funds among banks. In particular, banks use the interbank market to hedge idiosyncratic liquidity shocks, that is, in- or outflow of funds. The recent financial crisis however has demonstrated that the interbank market can be prone to freezes and become a source of systemic risk and adverse spill-over effects to the banking system and the wider economy.

This paper extends the literature on interbank markets via taking into account persistence of liquidity shocks. In particular, a theory of short and long term interbank funding is developed and embedded in a micro-founded agent based network model. The model combines analytical rigour with a flexible simulation framework in which effects driven by individually motivated heterogeneous agents on financial system and real economy can be assessed. Besides numerous innovations it features interbank funding as an over-the-counter phenomenon and realistically replicates financial system phenomena of network formation, monetary policy transmission and endogenous money creation. After investigating the model's network properties, it is used for analyses to shed light on the purpose of the interbank market and its role for allocation and stability in the financial system.

Analyses in the modeling framework provide evidence that the interbank market renders the financial system more efficient relative to a setting without mutual insurance against persistent liquidity shocks. However, given that banks in the model can maximize profit via emitting credit and creating money while only facing limited liability, an unleashed financial system without a policymaker's regulation can well lead to economic outcomes below those resulting from a financial system with less efficient allocation. In a realistically calibrated simulation exercise featuring a policymaker who puts a lid on credit and money creation with a view on real economic activity while keeping financial fragility in check allows for significant output gains. Therefore the model provides evidence that interbank funding indeed plays a crucial role for welfare.

Furthermore, and related to those findings, the framework is used to carry out an optimal policy analysis in which the policymaker maximizes real activity via choosing the optimal interest rate in a trade-off between credit provision and financial fragility. This real world issue currently confronts central bank policymakers who have been criticized for not taking into account this trade-off sufficiently, therefore eventually sowing the seeds of 'tomorrow's' financial crisis with today's lax monetary policy in order to support current (short term) economic activity. In particular, it has been argued that central banks should raise interest rates from current abnormally low levels because there is a risk that long-run growth is adversely affected from low interest rates because of emerging financial fragility. The developed modeling framework and analyses indeed provide evidence in favor of these arguments.

# Interbank Funding as Insurance Mechanism for (Persistent) Liquidity Shocks \*

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## Abstract

The interbank market is important for the efficient functioning of the financial system, transmission of monetary policy and therefore ultimately the real economy. In particular, it facilitates banks' liquidity management. This paper aims at extending the literature which views interbank markets as mutual liquidity insurance mechanism by taking into account persistence of liquidity shocks. Following a theory of long-term interbank funding a financial system which is modeled as a micro-founded agent based complex network interacting with a real economic sector is developed. The model features interbank funding as an over-the-counter phenomenon and realistically replicates financial system phenomena of network formation, monetary policy transmission and endogenous money creation. The framework is used to carry out an optimal policy analysis in which the policymaker maximizes real activity via choosing the optimal interest rate in a trade-off between loan supply and financial fragility. It is shown that the interbank market renders the financial system more efficient relative to a setting without mutual insurance against persistent liquidity shocks and therefore plays a crucial role for welfare.

*Keywords:* Financial fragility, interbank market, liquidity, maturity, network model

*JEL Classification:* E44, E51, G01, G21, G28

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## 1. Introduction

The financial system serves to efficiently allocate funds from savers to investors. Banks assume a crucial function in the financial system via the provision of credit and liquidity to the real economy.<sup>1</sup> To efficiently fulfill this function, the interbank market allows for the allocation of funds among banks. In particular, banks use the interbank market to hedge idiosyncratic liquidity shocks which can be considered as an in- or outflow of

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<sup>1</sup>For the important role of banks in liquidity provision see, for example, Diamond and Dybvig (1983).

funds.<sup>2</sup> For example, customers withdrawals of bank deposits lead to an outflow of cash or reserves at the depositor's bank, yet at the same time to an inflow of liquidity at other banks.<sup>3</sup> Confronted with such liquidity shocks, banks can insure each other such that those experiencing liquidity outflows can borrow from those with liquidity inflows. However, besides facilitating banks' liquidity management and the financial system's efficiency, the interbank market can be prone to systemic risk and adverse spill-over effects.<sup>4</sup> In this paper a theory of short and long term segments of the interbank market is developed and embedded in a micro-founded agent based network model which is used for analyses to shed light on the purpose of the interbank market and its role for allocation and stability in the financial system.

While the extant literature investigates the interbank market mainly as a short term phenomenon,<sup>5</sup> Bluhm, Georg and Krahn (2015) provide evidence which points towards an important role of the term segment for interbank funding. Using German interbank data, they show that banks actually hold *simultaneously high proportions* of interbank assets and liabilities in a *highly stable* network structure<sup>6</sup> and argue that this cannot be easily reconciled with extent theories modeling the interbank market as a short term phenomenon only. Furthermore, Bluhm, Georg and Krahn (2015) point out that these features would almost automatically arise in an interbank market in which the long term segment plays an important role. Longer maturities automatically lead to a stable network structure because the links by definition persist for an extended time. Furthermore, in case of frequent (persistent) liquidity shocks banks can instead of netting interbank positions assume offsetting positions, building up relatively larger interbank positions both in terms of assets and liabilities. The important role of the long term segment on the interbank market is also pointed out by Upper and Worms (2004) who show that half of the interbank assets and liabilities on the German interbank market have a maturity of at least 4 years. While Bluhm, Georg and Krahn (2015) and Upper and Worms (2004) use German data sets for their analyses, interbank assets and liabilities similarly play an important role in other countries. For example, in France the ratio of interbank assets to total assets is about 0.23, for the Euroarea this metric amounts to 0.17. Furthermore, Georg and Gabrieli (2014), using a Euroarea-wide database of interbank loans, show that the long-term segment accounts for the major part of actual exposure between banks.

One reason for the importance of the long term segment in interbank markets can be motivated by the persistence of liquidity shocks. For example, if a bank provides a new loan to the real economy, it will subsequently face a persistent outflow of liquidity which it can refinance (partly) on the interbank market, matching the expected duration of

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<sup>2</sup>See Allen, Carletti and Gale (2009). Also see Allen and Gale (2000) who show theoretically that the interbank market can overcome a maldistribution of liquidity in a setting where banks face liquidity shocks.

<sup>3</sup>It is also possible that the withdrawn deposits are held in cash outside the banking system or flow back to the same bank.

<sup>4</sup>See Gorton and Metrick (2012) for an account of the role of the interbank market in the recent global financial crisis. For theoretical analyses see, for example, Bluhm and Krahn (2014).

<sup>5</sup>See, for example, Allen and Gale (2000).

<sup>6</sup>Note that their results hold irrespective of the subset of banks investigated. In particular, their findings also hold when using a subset of all commercial banks (excluding the arguably more stable sectors of savings and cooperative banks because of their institutional framework). Further evidence for stability features and important share of the German interbank market is provided in Battiston, Roukny and Georg (2014).

in- and outflows.<sup>7</sup> As a related example for persistent liquidity shocks consider liquidity surpluses or shortages driven by deposit fluctuations which gradually vanish over time. In line with the persistence of a large part of the deposit base, extant liquidity regulation allows banks to invest up to 90% of short term liquidity inflows at long term maturities. Both examples are developed and explored in more detail later in the analysis.

In sum, many aspects speak in favor of an important role of the entire maturity spectrum –as opposed to only the overnight segment– on the interbank market, in particular for banks’ short- and long-term liquidity management. Furthermore, the term segment can be an important building block to understand interbank (network) features pointed out in Battiston, Roukny and Georg (2014) and Bluhm, Georg and Krahen (2015). In line with the extant literature, this paper approaches the interbank market as a mutual liquidity insurance scheme. However, unlike previous theoretical and empirical analyses which mainly take into account the interbank market’s short term segment, the focus is put on the maturity structure on the interbank market.

Given the interbank market and its nature as a network of banks, tools developed for complex system analysis are particularly well suited for an analysis of its role as a buffer for (persistent) liquidity shocks.<sup>8</sup> From a network perspective, each bank is a node and connected to other banks via lending and borrowing on the interbank market (so-called edges in network theory). However, unlike physical networks such as an electricity grid, the nodes on the interbank market are heterogeneous and dynamically react to the system’s evolution to achieve their objective which is profit maximization. Therefore, to develop a theory on the maturity structure of the interbank market, a micro founded agent based complex network model which interacts with a real economic sector is developed.<sup>9</sup> This theoretical approach combines several realistic advantages. First, it allows for modeling heterogeneity in the financial system: banks’ assets and liabilities are determined endogenously based on an analytically derived profit maximization objective. Second, the interbank market is treated as an over-the-counter market with different term segments in which banks interact and bargain for funds: network formation is not random but depends on micro foundations and reflects real world interbank phenomena such as relationship lending, intermediation chains and lending and borrowing at the same time between the same counterparties. Third, a central bank that acts as lender of last resort, and whose policy decisions are transmitted via the interbank market is included to allow for policy driven welfare analysis. Besides developing a theory consistent with several observed interbank phenomena, the model is used for an optimal monetary policy experiment in which the policymaker faces a trade-off between short run stabilization of real activity and fragility of the financial system over the long run.

Previous research related to interbank markets can be subdivided into theoretical and empirical analyses. The extant theoretical literature motivates interbank lending mainly as insurance against (short term) liquidity shocks. Bhattacharya and Gale (1987) investigate the role of the central bank as a mechanism designer for (liquidity) risk-

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<sup>7</sup>In reality banks expect for each shock a specific cash-flow maturity profile over the horizon until it will disappear. In view of their liquidity management, banks can match their cash-flows at different maturity brackets on the interbank market. See European Central Bank (2002).

<sup>8</sup>For an overview on network and complex systems analysis see, for example, Lewis (2009).

<sup>9</sup>See Georg (2013), Bluhm and Krahen (2014), Bluhm, Faia and Krahen (2014a), Bluhm, Faia and Krahen (2014b) and Aldasoro, Delli Gatti and Faia (2015) for analyses of the financial system using agent based complex systems analysis.

sharing among banks. Allen and Gale (2000) model financial contagion in an interbank network as an equilibrium phenomenon driven by liquidity shocks and show that the possibility of contagion depends strongly on the completeness of the network. Freixas, Parigi and Rochet (2000) model systemic risk in an interbank market where banks face liquidity needs as consumers are uncertain about where they need to consume. Interbank credit lines allow coping with these liquidity shocks. The authors use this framework to investigate the justification for the 'Too-big-to-fail' policy. Finally, Allen, Carletti and Gale (2009) explicitly state that banks use the interbank market to hedge themselves against liquidity shocks. The basic role of interbank markets in their model is to allow re-allocation of liquidity from banks with an excess to banks with a deficit. They develop a simple model of the interbank market where banks hedge idiosyncratic and aggregate liquidity shocks. The authors show that the interbank market can be characterized by excessive price volatility. Furthermore they show that in such a situation an efficient allocation can be achieved via central bank's open market operations to fix the short term interest rate.

Related empirical analyses are mainly focused on the topology and stability features of (short term) interbank markets. Boss, Elsinger, Summer and Thurner (2004) investigate the Australian interbank market and find that it features a low clustering coefficient and relatively short average network path length. Upper and Worms (2004) use bank balance sheet information to estimate bilateral credit relationships for the German banking system. The authors find that the failure of a single bank could result in the breakdown of up to 15% of the banking system. Iori, Masi, Precup, Gabbi and Caldarelli (2008) explore the network topology of the Italian interbank market. The authors find that the system is characterized by a small group of large banks borrowing from a large number of small banks a structure which features high systemic risk. Cocco, Gomes and Martins (2009) use a dataset on the Portuguese interbank market and show that relationships between banks are an important determinant of their access to interbank market liquidity and that these relationships therefore allow banks to insure against liquidity shocks. Bech and Atalay (2010) investigate the federal funds market and find that it features a sparse network as well as small-world phenomenon.<sup>10</sup> Martinez-Jaramillo, Alexandrova-Kabadjova, Bravo-Benitez and Solorzano-Margain (2014) investigate the network of the Mexican interbank market and find that the interconnectedness of a bank is not necessarily related to its asset size but can rather be linked to the amount of systemic risk it causes. Raddant (2014) investigates volume and interest dynamics on the Italian interbank market. The author finds that before the Lehman default large net-borrowers obtained funds at a discount while post-Lehman borrowers with large net-exposures paid a premium. The main contribution of the proposed research project with respect to the outlined theoretical and empirical literature is to shed light on the role of the long-term segment of the interbank market and its role as a hedge for persistent liquidity shocks.

As outlined above, the theory on persistent liquidity shocks is embedded in a multi-agent network model. Related analyses investigating the interbank market and its stability properties, can be found in Bluhm and Krahnén (2014) who investigate macro prudential capital surcharges as a risk management tool, Bluhm, Faia and Krahnén (2014a) who investigate spillover effects between monetary and macroprudential policymakers,

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<sup>10</sup>In a mathematical graph, the small world phenomenon describes a situation in which most nodes are not directly connected to each other but can be reached with a small number of steps.

as well as Georg (2013) who analyzes optimal policy responses to different sources of systemic risk.<sup>11</sup> The main innovation of the proposal relative to that strand of the literature lies in extending those models along several dimensions including modeling the interbank market as an over-the-counter phenomenon featuring a range of maturities, endogenous money creation, and an adverse feedback loop between the real and financial sectors.

The remainder paper is structured as follows. Section 2 outlines theoretical arguments to explain the existence of long term segments on the interbank market. Based on these arguments a micro-founded agent based network model is developed. Section 4 investigates the model's (network) properties and carries out an optimal monetary policy experiment in which the central bank maximizes real activity via an interbank market transmission mechanism affecting liquidity and credit creation in the banking sector. Section 5 concludes.

## **2. A Theory of Interbank Funding as Insurance Mechanism for Persistent Liquidity Shocks**

### *2.1. The Interbank Market as Insurance Mechanism for Persistent Liquidity Shocks*

The case for the existence of a long term segment on the interbank market rests on the assumption of persistent liquidity shocks, that is, deposit in- or outflows which (in expectation) are beyond a short term horizon. In the following, persistence of liquidity shocks is motivated via building upon the 'socket-theory' introduced by Wagner (1857). His theory provides a rationale for banks' maturity transformation. In particular, Wagner (1857) points out that while the 'contractual' maturity of demand deposits is daily, many customers leave their deposits for much longer periods without withdrawing them. At the same time some of the outflows which actually take place are substituted by inflows of demand deposits. Since a bank has a large number of customers, and deposit in- and outflows are (in normal times) independent, the law of large numbers can be used to predict a stable base of deposits which the bank can invest for longer term investments, that is, carry out a maturity transformation. Note that with the emergence of central banks as lenders of last resort this maturity transformation is not too risky in view of a bank run risk. Existing liquidity regulations ultimately stem from the considerations laid out in that basic theory. For example, German liquidity regulations require 10% of demand deposits be held as reserves while 90% can be held at longer maturities.<sup>12</sup> In the following, theoretical extensions of Wagner (1857) are used to show how banks can use the interbank market as a buffer across a range of maturities.

Consider, a bank which experiences a sizable (net) inflow of deposits. Depending on its experience of past in- and outflows it forms an expectation about how long it can expect this increased stock of liquidity to remain. Based on that expectation it can invest (part of) the inflow in higher yielding (long term) assets instead of holding them as reserves. As a second example, consider a bank which provides a new loan to the real economy. Unlike assumed in fractional reserve banking theories,<sup>13</sup> a bank

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<sup>11</sup>Further related analyses are carried out in Bluhm, Faia and Krahen (2014b), and Aldasoro, Delli Gatti and Faia (2015).

<sup>12</sup>See Bundesbank (2014).

<sup>13</sup>See, for example, Mankiw (2012).



providing a loan is generally not lending out deposits it has previously acquired (or obtained as an inflow) but creates them 'ex-nihilo'.<sup>14</sup> That is, when the bank emits a new loan (asset side) it credits the deposit account (liability side) of the loan recipient with the loan. This leads to a balance sheet expansion by the assigned loan amount. Since the customer will however want to use the loan to finance spending (which was the reason for the loan application), the bank expects that a large fraction of these created deposits will flow out for the term of the loan, resulting in a long term liquidity outflow. Having an expectation about the maturity profile of the loan, the bank can refinance itself on the interbank market to fulfill liquidity regulations.<sup>15</sup> Note that the interbank market is particularly suited to buffer liquidity shocks because it allows for offsetting shocks with counterparties who experience the exact opposite shocks (the loan emitting bank's liquidity outflow becomes another bank's inflow). Indeed Cocco, Gomes and Martins (2009) find evidence that trading partners on the interbank market face negatively correlated liquidity shocks.

To depict the persistence of liquidity shocks in a simple framework, consider the following stylized modeling approach. For an individual bank, the amount of deposit outflows depends on *its* customers' payments for their economic transactions. For the same bank, the amount of deposit inflows depends on *all* banks' customer transactions and *that bank's* branch share. Banks with a bigger network of branches in the economy likely face higher deposit inflows than those with a smaller network of branches. Since deposit inflows and outflows are best separately modeled, in the following, deposit withdrawals and outflows are assumed to be uniformly distributed. Figure 1 displays deposit fluctuations for bank  $i$ .

$U$  denotes a draw from a uniform distribution,  $d^i$  designates bank  $i$ 's stock of deposits, and  $BS$  denotes a bank's branch share. Note that in expectation bank  $i$ 's stock of deposits is constant if its deposit inflows equal its outflows, that is,  $IF = OF$ . Assuming that  $BS^i = \frac{d_{init}^i}{\sum_j d_{init}^j}$ ,  $i \in j$ , in expectation each bank's deposits will remain stable as long as the amount of deposits in the economy remains stable and banks do not change their share of bank branches.<sup>16</sup>

Note that while in expectation the bank has a steady socket of deposits, random deposit fluctuations can lead to temporary disequilibria which will only gradually disappear. To fix ideas, consider the following example which is closely aligned to the outlined simple framework. A financial system consists of bank 1 and all remainder banks, denoted 'rest of financial system' (ROFS), which have a branch share of 0.3 and 0.7, respectively. Deposits are normed to 1 of which bank 1 currently has  $d_1^1 = 0.4$  and the ROFS  $d_1^2 = 0.6$ ,

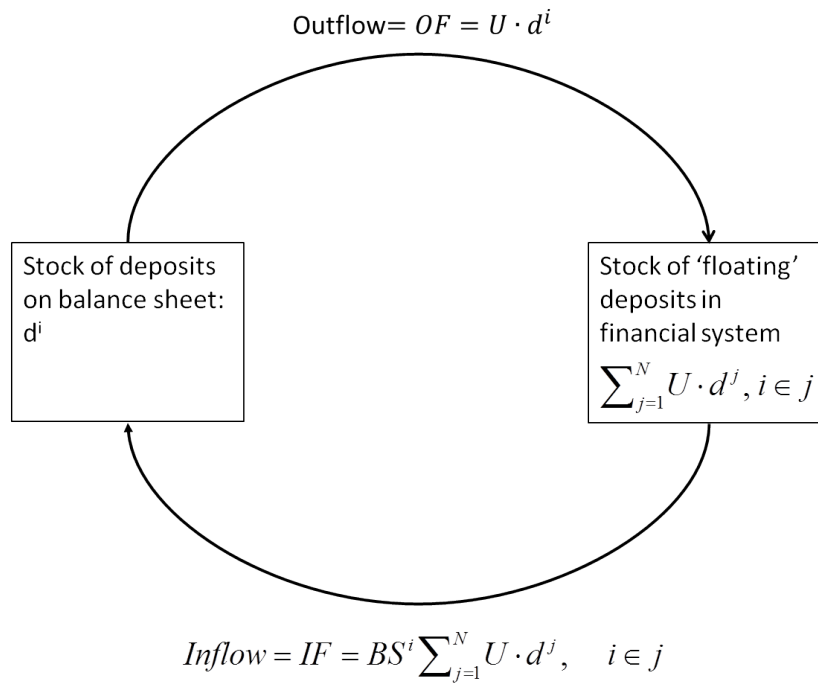
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<sup>14</sup>See McLeay, Radin and Thomas (2014).

<sup>15</sup>See European Central Bank (2002).

<sup>16</sup>On Figure 1, a steady state is achieved if  $E(OF^i) = E(IF^i)$ . Replacing inflows and outflows with the specific expressions on the figure and assuming bank  $i$  starts in period 1 with the deposits reflecting its share of total deposits as designated by its branch share, its expected outflows and inflows are given by  $OF_1 = U \cdot d_1^i$  and  $IF_1 = \frac{d_1^i}{\sum_j d_1^j} \cdot U \sum_j d_1^j = U d_1^i$ . The expected change of deposits is therefore  $\Delta d_1^i = IF_1 - OF_1 = 0$ . In the following period, the bank in expectation hence starts with  $d_2^i = d_1^i + \Delta d_1^i = d_1^i$  and so on. That is, in expectation the bank's deposits will remain unchanged in all future periods. Note that it is not necessary for banks to start with their equilibrium deposit distribution to obtain the result, however it simplifies exposition and allows for computationally less intensive simulations later on.

Figure 1: Deposit Flows between a Bank and Interbank Market



Note: The figure displays deposit flows between a bank  $i$  and the financial system. The financial system consists of  $j$  banks,  $j = 1 \dots N$ .  $U$  denotes a draw from a uniform distribution,  $d^i$  designates bank  $i$ 's stock of deposits, and  $BS$  denotes a bank's branch share. Note that in expectation bank  $i$ 's stock of deposits is constant if its deposit inflows equal its outflows, that is,  $IF = OF$ .

that is, their current deposits deviate from the expected equilibrium value by 0.1 (bank 1 should hold 0.3 and the ROFS 0.7, given branch shares). Still assuming that deposit inflows and outflows are uniformly distributed, in expectation bank 1's deposits after one period amount to

$$\begin{aligned}
E(d_2^1) &= d_1^1 + E(IF_1^1) - E(OF_1^1) \\
&= d_1^1 + BS(E(OF_1^1) + E(OF_1^{ROFS})) - E(OF_1^1) \\
&= d_1^1 + BS(0.5 \cdot d_1^1 + 0.5 \cdot d_1^{ROFS}) - 0.5 \cdot d_1^1 \\
&= 0.4 + 0.3(0.5 \cdot 0.4 + 0.5 \cdot 0.6) - 0.5 \cdot 0.4 \\
&= 0.35
\end{aligned} \tag{1}$$

where the third line uses the fact that the expected value of a draw from a uniform distribution equals 0.5. Similarly,  $d_2^{ROFS} = 0.65$ . After one more period,  $d_3^1 = 0.325$  and  $d_3^{ROFS} = 0.675$ . This process goes on and asymptotically bank 1 and the ROFS revert to their stable deposit bases of 0.3 and 0.7, respectively. Note that with each additional period the extant disequilibrium disappears by half, that is, after 5 periods most of the disequilibrium (about 97%) has disappeared.

So far, the example demonstrates a formal approach to model persistence of disequilibria arising from random liquidity shocks. Next, the example is extended to outline how the interbank market offers banks a convenient way to carry out their liquidity management to buffer these shocks. In aggregate, given the nature of deposit in- and outflows banks can mutually insure themselves, taking into account the maturity profile of shocks. Bank 1 and the ROFS' equilibrium balance sheets are given on Table 1.

Table 1: Initial equilibrium in financial system

(a) Bank 1 in initial equilibrium			(b) ROFS in initial equilibrium		
Bank 1			ROFS		
Assets		Liabilities	Assets		Liabilities
Cash	0.03	Deposits	0.3	Cash	0.07
Lending	0	Borrowing	0	Lending	0
Loans		Equity		Loans	

Note: The table displays stylized balance sheets of a financial system consisting of bank 1 and the rest of the financial system (ROFS). Lending and borrowing denote interbank lending and borrowing, respectively. Loans and equity are added for completeness.

Note that in the following it is assumed that banks have to maintain a liquidity ratio of 0.1, that is, they have to hold at least 10% of their deposits in cash. Now suppose that customer transactions lead to a net outflow of deposits of 0.05 from the ROFS to bank 1. The new situation is displayed on Table 2.

While the ROFS (Subtable 2b) has a shortfall of reserves (according to the liquidity requirement it should hold 0.065 units of reserves, however actually only holds 0.02), bank 1 (Subtable 2a) has a surplus. To fulfill the liquidity requirement, the ROFS needs to obtain 0.045 units of reserves while bank 1 overfulfills its liquidity ratio by the same amount. That is, absent other investment opportunities for bank 1 (in which case the ROFS would have to turn to the lender of last resort) it is mutually beneficial for banks 1

Table 2: Balance sheets after first liquidity shock

(a) Bank 1 after deposit withdrawal				(b) ROFS after deposit inflow			
Bank 1		Bank 1		ROFS		ROFS	
Assets		Liabilities		Assets		Liabilities	
Cash	0.08	Deposits	0.35	Cash	0.02	Deposits	0.65
Lending	0	Borrowing	0	Lending	0	Borrowing	0
Loans		Equity		Loans		Equity	

Note: The table displays stylized balance sheets of a financial system consisting of bank 1 and the rest of the financial system (ROFS). Lending and borrowing denote interbank lending and borrowing, respectively. Loans and equity are added for completeness.

and the ROFS to exchange funds on the interbank market. The new situation is displayed on Table 3.

Table 3: Banks' balance sheets after exchange of funds

(a) Bank 1 after borrowing from bank 2				(b) ROFS after lending to bank 1			
Bank 1		Bank 1		ROFS		ROFS	
Assets		Liabilities		Assets		Liabilities	
Cash	0.035	Deposits	0.35	Cash	0.065	Deposits	0.65
Lending	0.045	Borrowing	0	Lending	0	Borrowing	0.045
Loans		Equity		Loans		Equity	

Note: The table displays stylized balance sheets of a financial system consisting of bank 1 and the rest of the financial system (ROFS). Lending and borrowing denote interbank lending and borrowing, respectively. Loans and equity are added for completeness.

Note that given the previous outline on the persistence of (uniform) shocks, 50% of the funds exchanged have a maturity of more than one period. Both sides expecting the shock to have some persistence can use this information for their benefit: while bank 1 (Subtable 3a) can obtain a slightly higher return on the otherwise idle funds (the liquidity premium for lending at longer maturity), the ROFS (Subtable 3b) can lock-in the given interest rate at below the cost of turning to the lender of last resort<sup>17</sup> and therefore has funding security over the expected horizon of the liquidity shock. More precisely, banks' recourse to the long term segment instead of continuously rolling over debt on the short term segment can be motivated with four conditions: (i) Persistence of liquidity shocks, (ii) a liquidity premium  $\epsilon$ , (iii) an interest penalty  $\lambda > \epsilon$  for short term lending or borrowing with the lender of last resort, and, (iv) a positive probability that recourse to the lender of last resort becomes necessary in future periods. While conditions (i) to (iii) can motivate lenders' recourse to the long term segment, conditions (i), (iii), and (iv) can motivate borrowers to turn to the interbank market for long term. These conditions will be fulfilled in the network model developed in the next subsection. Note that banks' recourse to the short and long term segments can also be motivated via banks' desire not to go beyond a certain maturity mismatch, that is, the different maturity segments

<sup>17</sup>Normally the marginal refinancing facility of a central bank –if it provides long term funding at all– is above the interbank market rate.

of the interbank can be used to influence maturities on both sides of the balance sheets. For example, it can be shown that a bank's (long maturity) loan emission to the real economy resulting in a subsequent liquidity outflow at that bank increases its maturity mismatch while the maturity mismatch of the rest of the banking system decreases. In that situation both sides can use different maturity segments on the interbank market to influence the change in maturity mismatches.<sup>18</sup>

Next assume that in the following period banks face another deposit shock which leads to net outflows of 0.0575 from bank 1 to the ROFS. Note that following the above outline 50% of the existing interbank credit has been paid back at the beginning of the period (prior to the new deposit shock). The new balance sheets are displayed on Table 4.

Table 4: Banks' balance sheets after second deposit shock

(a) Bank 1 after deposit inflow				(b) ROFS after deposit outflow			
Bank 1		Bank 1		ROFS		ROFS	
Assets		Liabilities		Assets		Liabilities	
Cash	0	Deposits	0.2925	Cash	0.1	Deposits	0.7075
Lending	0.0225	Borrowing	0	Lending	0	Borrowing	0.0225
Loans		Equity		Loans		Equity	

Note: The table displays stylized balance sheets of a financial system consisting of bank 1 and the rest of the financial system (ROFS). Lending and borrowing denote interbank lending and borrowing, respectively. Loans and equity are added for completeness.

Bank 1 (Subtable 4a) now has a liquidity shortfall of 0.02925 while the ROFS (Subtable 4b) features this amount as a surplus. Exchanging the funds is again beneficial for both banks.<sup>19</sup> The new balance sheets after exchange of funds are displayed on Table 5.

Table 5: Bank's balance sheets after second exchange of funds

(a) Bank 1 after lending to bank 2				(b) ROFS after borrowing from bank 1			
Bank 1		Bank 1		ROFS		ROFS	
Assets		Liabilities		Assets		Liabilities	
Cash	0.02925	Deposits	0.2925	Cash	0.07075	Deposits	0.7075
Lending	0.0225	Borrowing	0.02925	Lending	0.02925	Borrowing	0.0225
Loans		Equity		Loans		Equity	

Note: The table displays stylized balance sheets of a financial system consisting of bank 1 and the rest of the financial system (ROFS). Lending and borrowing denote interbank lending and borrowing, respectively. Loans and equity are added for completeness.

At this point the example has established how a simple framework with persistent

<sup>18</sup>For further details on banks' asset-liability and liquidity management see European Central Bank (2002).

<sup>19</sup>Note that additionally, banks could net existing interbank exposure to some extent. However, the amounts and maturity structure resulting from both shocks do not match. Furthermore, canceling extent positions might involve some administrative cost and might rather be carried out in case of distress in the financial system. Finally, in a situation with multiple counterparties, it is not clear if counterparties are willing to net. In reality banks constant liquidity management on the interbank market might involve a combination of canceling or not rolling over extant positions, as well as assuming offsetting positions. See Bluhm, Georg and Krahen (2015).

liquidity shocks can rationalize banks featuring at the same time interbank assets and liabilities over different maturities.

To get an idea about the proportion of interbank lending and borrowing in the outlined stylized framework, a simulation is carried out, assuming that the interbank market has five maturity segments which allows for matching most of the maturity profiles of disequilibria emerging from uniform liquidity shocks. As outlined before, if a bank faces a net liquidity outflow in that framework, in expectation 50% of that disequilibrium will disappear after one period, 75% after two periods and so on until about 97% have vanished after 5 periods. Given a liquidity reserve requirement of 10%, a bank has to borrow  $B = 0.9 \cdot S$  on the interbank market for a given shock  $S = OF - IF$ . Using the interbank market to match the maturity profile, the bank should borrow  $B/2$  of the funds to fulfill the liquidity need for one period,  $B/4$  for two periods,  $B/8$  for three periods, and  $B/16$  for four and five periods (assuming the bank assigns any residual to the longest maturity bucket available). Considering five consecutive periods during each of which the bank experiences a new (uniform) deposit shock which it buffers at different maturities on the interbank market allows for gauging the proportion of interbank assets on the bank's balance sheet as given in Equation (2)

$$IBA = 0.9 \cdot [1/16 \cdot |S1| + 1/8 \cdot |S2| + 1/4 \cdot |S3| + 1/2 \cdot |S4| + |S5|] \quad (2)$$

Simulating 100000 such five period intervals in the subsequently developed network model results, on average, in a proportion of about 18% of interbank assets (or liabilities) to deposits (or about 12.5% to total assets) purely driven by deposit fluctuations. That is, in the outlined framework a sizeable fraction of banks' balance sheets results from buffering liquidity shocks. Furthermore, persistence of shocks automatically leads to a stable network structure, and in banks lending and borrowing at the same time.

The following section embeds the developed framework in a more sophisticated model which allows for stability and welfare analyses in a microfounded agent based network model.

## 2.2. A Microfounded Agent Based Network Model with Persistent Liquidity Shocks

To investigate the effect of persistent liquidity shocks on banks' balance sheets, network structure, systemic risk, and welfare, this sub-chapter develops a model consisting of a financial system with  $N$  banks, a central bank and a real sector. Whereas each bank chooses its optimal portfolio consisting of cash (reserves), loans to the real economy, interbank lending, deposits, and interbank borrowing to maximize expected profit,<sup>20</sup> the central bank steers interest rates on the interbank market to maximize economic activity. Banks use the interbank market which is modeled as a (partly collateralized) over-the-counter market featuring different maturities to fulfill regulatory requirements after liquidity shocks. The central bank transmits monetary policy decisions via an interest rate corridor on the interbank market, standing ready to provide unlimited amounts of reserves to banks who are willing to borrow at the marginal lending rate and taking on unlimited amounts of funds from banks who are willing to lend for the deposit

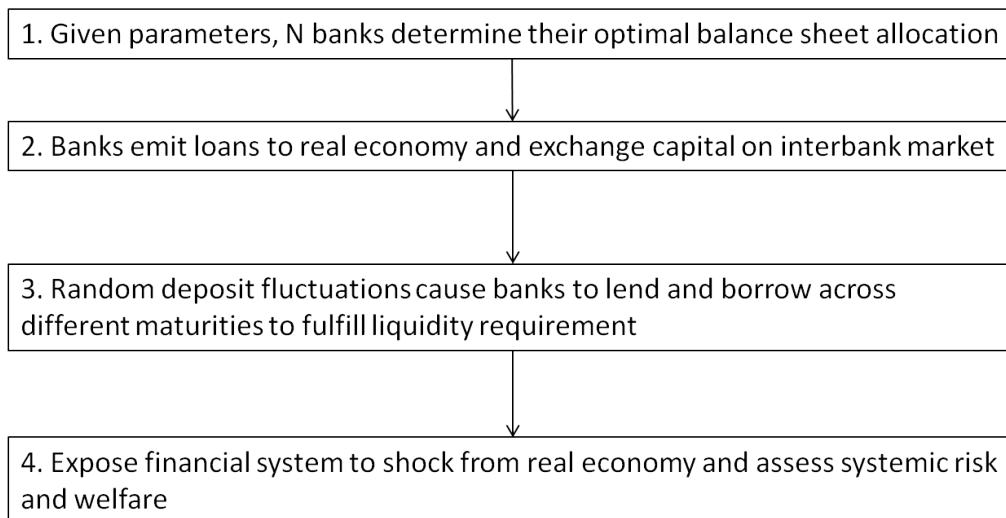
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<sup>20</sup>See Bluhm, Faia and Krahen (2014a) and Bluhm, Faia and Krahen (2014b) for modeling approaches in which banks determine portfolio positions via maximizing profit and Georg (2013) for a modeling approach in which banks maximize utility to determine their optimal portfolio.

rate. Furthermore, the model features a negative feedback loop between the financial and real sectors which serves as spillover and amplifying mechanism for shocks from the real economy and vice versa.

In the model banks are initially endowed with an exogenous amount of equity as well as a proportion of the branch network in the economy. Due to the different branch endowment, banks face different amounts of (i) credit demand from the real economy, and (ii) deposit shocks. The modeling approach consists of four main steps. First, the  $N$  banks choose their portfolio to maximize profit subject to regulatory constraints. In the second step, after emitting loans to the real economy which depends on banks' capacity to supply and demand from the real economy, banks lend and borrow on the interbank market to fulfill regulatory requirements. Third, a number of random deposit fluctuations takes place, causing banks to again turn to the interbank market to lend and borrow with a view to fulfill regulatory requirements. Fourth, the financial system is exposed to a shock from the real economy and after it has reached (a new) equilibrium is assessed in terms of systemic risk and welfare. Figure 2 gives an overview of the four steps which will be outlined in more detail in the following.

Figure 2: Model Steps



In step 1 (Figure 2) each bank determines its individual portfolio positions (except the exogenously given equity) which are outlined on Table 6.<sup>21</sup>

Table 6: Stylized bank balance sheet

Assets	Liabilities
Reserves (c)	Deposits (d)
Interbank lending (bl)	Interbank borrowing (bb)
Loans (l)	Equity (e)

To determine its optimal portfolio, each bank maximizes profit which is the revenue from providing loans to the real economy as well as lending on the interbank market minus the cost from borrowing on the interbank market and attracting deposits, subject to regulatory requirements and a number of further feasibility constraints. A detailed outline and derivation of the banks' optimization problem is given in Appendix A.

After each bank has determined its optimal portfolio, it creates the according amount of loans and deposits and (Step 2 on Figure 2) turns to the interbank market in case it has a regulatory shortfall or surplus of reserves.<sup>22</sup> The interbank market is modeled as an over-the-counter (OTC) market where counterparties are matched and bargain about funds. Furthermore, short term interest rates are steered by a central bank which stands ready to provide unlimited amounts of reserves at the marginal lending rate and borrow unlimited amounts of funds at the marginal deposit rate, both in the short term. The following setup draws upon Duffie, Gârleanu and Pedersen (2007) who investigate the valuation of assets in OTC markets in a continuous time setting. Assume that on the interbank market a lender's reservation price is  $r^{cbd_n}$ , the central bank's marginal borrowing rate and a borrower's reservation price is  $r^{cbup}$ , the central bank's marginal lending rate. The value of a trade between two counterparties,  $V$ , is given by the spread on the interbank market:  $V = r^{cbup} - r^{cbd_n}$ , and is shared between lender and borrower according to their bargaining power. In any possible lender-borrower match the assumption is that the bargaining power depends on the riskiness of the debtor, that is, riskier counterparties have weaker bargaining power in sharing the value of the trade. In case of a riskier borrower, the lender will be compensated with a fair risk premium. To derive this markup, first consider the profit from lending in absence of counterparty risk:  $\pi^l = [\frac{1}{2}(r^{cbup} - r^{cbd_n}) + r^{cbd_n}] \cdot bl^{ij} = r^{mid} \cdot bl^{ij}$  with  $bl^{ij}$  the interbank lending provided from bank  $i$  to bank  $j$ . In the absence of risk, lender and borrower have equal bargaining power and equally share the gains from trade. That is, trade takes place at  $r^{mid}$  which results in a gain of 50 basis points for the lender relative to lending its funds to the central bank at the marginal deposit facility and likewise for the borrower who gets the funds 50 basis points cheaper relative to borrowing at the central bank's marginal borrowing facility if the central bank puts a corridor of 100 basis points around its desired interest rate on the interbank market.

<sup>21</sup>A bank's equity parameter is drawn from a Weibull distribution which allows for strongly skewed distributions as is the case for most banking systems.

<sup>22</sup>At low capital requirement ratios it is possible that a bank's loan emission results in a deposit base in excess of 10 times their equity in which case the bank needs to borrow additional reserves to fulfill the liquidity requirement of 10% even if it holds all its equity in cash.



In case of a risky borrower, expected profit from lending becomes  $E(\pi^b l) = (1 - PD) \cdot (r^{mid} + r^r) \cdot bl^{ij} + PD \cdot (bl^{ij} - LGD \cdot l^{ij}) \cdot (r^{mid} + r^r)$  with  $r^r$  a risk premium for the borrower's default risk, and  $PD$  and  $LGD$  a bank's probability of default and loss given default, respectively. In a competitive market the lender charges a fair risk premium, that is, in expectation the profit from risky lending must equal the profit from safe lending. Therefore  $[\frac{1}{2}(r^{cbup} - r^{cbdn}) + r^{cbdn}] \cdot bl^{ij} = (1 - PD) \cdot (r^{mid} + r^r) \cdot bl^{ij} + PD \cdot (bl^{ij} - LGD \cdot l^{ij}) \cdot (r^{mid} + r^r)$ . Solving for  $r^r$  yields  $r^r = r^{mid} \cdot \frac{PD \cdot \xi}{1 - PD \cdot \xi}$ . That is, the interest rate on the interbank market between two counterparties is  $r^{ibb} = r^{mid} + r^r = r^{mid} + \frac{PD \cdot \xi}{1 - PD \cdot \xi} \cdot r^{mid} = r^{mid} \cdot (\frac{1}{1 - PD \cdot \xi})$ . Note that a possible trade only takes place if  $r^{bb} \leq r^{cbup}$ . If a bank faces a higher interest rate on the interbank market than charged by the central bank it borrows from the latter. Assuming market participants expect no interest rate changes by the central bank, an interbank yield curve can be computed as the geometric mean of the expected return of short term lending with a liquidity premium,  $\epsilon$ .<sup>23</sup> Note that interest rates on interbank loans and deposits are paid from banks' revenue from lending activity on the interbank market as well as loan provision to the real economy. Any remaining surplus, that is profit, is paid out to the equity holders.

On the interbank market, lenders and borrowers are score-matched where a lender-borrower pair's score is determined by criteria such as similarity of funds demanded and supplied as well as their lending-borrowing history. In particular, for each possible lender-borrower relation a weighted score is computed with relatively higher weights assigned if counterparties feature existing links to capture a relationship motive, and relatively lower weights assigned to the similarity of funds desired to lend and borrow to capture an efficiency motive.<sup>24</sup> A detailed outline of the matching algorithm is given in Appendix B. In this second step, funds among banks will only be lent at the longest maturity available because the lack/surplus of funds will persist over the entire period loans to the real economy exist. Note that loans are assumed to have a maturity of five periods which is in line with the longest maturity on the model's interbank market.

Next, in Step 3 on Figure 2, random deposit fluctuations take place. Similar to the example in Sub-section 2.1, each bank's outflows are modeled via drawing from a uniform distribution. Assuming a closed banking system and a cash-less economy (in the sense that all money is always circulating among deposits, for example via debit card payments) all deposit outflows end up as inflows at banks of the financial system. In particular, inflows are modeled via re-assigning the sum of all deposit outflows as a function of banks' branch network with larger branch networks resulting in larger deposit inflows. After a random deposit re-distribution some banks find themselves in (regulatory) excess of funds while others face a shortfall. As outlined before, it is mutually beneficial for these banks to trade on the interbank market (see Step 2 above), taking into account the expected persistence of the liquidity shock. Assuming as before that banks' initial amount of deposits reflects their branch share in the economy, banks have an idea about emerging disequilibria and their persistence (any emerging disequilibrium will disappear in expectation by about 97% after 5 random deposit fluctuations). Overall,

<sup>23</sup>For example, the return for lending for two periods,  $r_2$ , instead of one period,  $r_1$ , can be computed via  $(1 + r_2)^2 = \epsilon_2^2 + (1 + r_1)^2$ . In the model the liquidity premium  $\epsilon$  is generally set to  $1E - 7$ .

<sup>24</sup>For a related approach see Bluhm, Faia and Krahen (2014b) who use a tâtonnement and counterparty matching process to model the interbank market.

5 random deposit fluctuations are carried out, making sure that at the end of Step 3 any disequilibrium from the initial deposit fluctuation has almost completely disappeared in expectation therefore resulting in a realistically conceivable long-run interbank structure.

Finally, in Step 4 on Figure 2, the financial system is exposed to a shock in the real economy. In particular, an increase in non-performing loans leads to losses in the financial system, putting banks' balance sheets under pressure. In turn, banks aiming to fulfill regulatory requirements liquidate part of their loan-portfolio, thus reducing loan supply. This in turn lowers economic activity, further increasing the rate of non-performing loans which again increases pressure on banks' balance sheets and so on. This negative feedback loop goes on until eventually losses lead to the default of some banks in the financial system. Since the financial system is interconnected via interbank borrowing and lending further defaults can arise via direct contagion from counterparties not honoring their debt. Note that all borrowings with a maturity beyond one period are collateralized with banks' loan portfolios and subject to margin calls with the amount of collateralization influencing the severity of direct contagion in the model. Shock transmission is carried out similar to Bluhm and Krahen (2014) who use a modified version of Cifuentes, Ferrucci and Shin (2005) and Eisenberg and Noe (2001). Details on the shock transmission can be found in Appendix C. After a shock has been transmitted and the system has reached a new equilibrium, resulting real activity is computed as the aggregate value of loans which are not in distress.

Note that both banks' probabilities of default and loss given default for a given set of parameters are proxied via iterating over Steps 1 to 4 (see Figure 2) for a number of times. The emerged financial system after Step 3 on Figure 2 is based on  $N$  heterogenous banks' optimal portfolio decisions and score based counterparty matching on the interbank market. Step 4 on Figure 2 allows for investigating the model in terms of systemic risk and real economic activity.

The following section investigates the model with a focus on the interbank network and carries out a welfare analysis.

### 3. Model Analysis

In this chapter the model is first investigated with a view on interbank market phenomena and subsequently used to carry out an optimal monetary policy analysis. Throughout these analyses the following parameter values will be used in the model. Banks' (weighted) capital and liquidity requirements are set to 8% and 10%, respectively, with the weight on loans and interbank credits set to 1 and 0.2, respectively. The parameter governing the extent of negative feedback loops is set to a value under which the rate of non-performing loans increases to 25% if banks completely cease supplying loans to the real economy. Banks' equity is drawn from a Weibull distribution calibrated to match the first two moments in the empirical distribution of a sample of German banks. Finally, the parameter determining banks' branch network ( $C$ ), which determines the amount of individual credit demand and deposit inflows, is set such that on average banks' leverage ratio can reach 30 with a maximum possible value of about 60 if regulatory requirements are set to their minimum value (capital and liquidity requirements both set to 1%). These values are considered to be an upper band because they were among the most extreme observed before the financial crisis. Note that the following results hold qualitatively across a wide range of parameter values. The specific

Table 7: Model Parameters

Parameter	Designation in Model	Value	
Liquidity requirement ratio	$\alpha$	0.08	
Capital requirement ratio	$\gamma$	0.01	
Weight on loans	$\chi_1$	1	
Weight on interbank credits	$\chi_2$	0.2	
Branch network	$C$	$60 * U * \text{equity}$	25
Feedback parameter	$\iota$	$-\frac{\text{LOG}(0.75)}{\sum C}$	26
Equity distribution	$\varsigma$	$W(3,7)$	27
Shock to real economy	$\Psi$	$0.04 +  N * 0.01 $	

Note: The table displays the main model parameters. 'Weight on loans' and 'Weight on interbank credits' are the weight assigned to banks' loans and interbank assets in the capital requirement ratio, respectively. A bank's 'Branch network' determines its credit demand and deposit inflows. The 'Feedback parameter' determines the severity of negative feedback between shocks emerging from the real economy and the financial system. 'Equity distribution' governs the exogenously given amount of equity. 'U' and 'W' denote uniform distribution and Weibull distributions with scale parameter 3 and shape parameter 7, respectively. 'Shock to real economy' is an increase in the rate of non-performing loans sufficiently big to trigger defaults in the financial system with  $N$  a standard normal distribution.

values chosen here are regarded to reflect settings that can actually be found in financial systems and regulatory approaches. Table 7 summarizes these model parameters which are used in all following analyses.

To give a concise outline of a possible financial system emerging from the parameter values displayed on Table 7, a financial system with  $N = 10$  banks is generated. Note that here and in the following analyses the central bank chooses the interest rate which maximizes expected real activity.<sup>28</sup> The specific financial system is outlined on Table 8.

<sup>25</sup>Results in a maximum possible leverage ratio of 60 in the model.

<sup>26</sup>Results in a proportion of non-performing loans of 25% if banks cease supplying loans.

<sup>27</sup>Matches distribution of equity among sample of German banks.

<sup>28</sup>The optimal interest rate maximizing  $P \cdot \sum \text{loans}$  is obtained via a pattern search algorithm.

Table 8: Sample Banking System with N=10

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	CB	Loans	Reserves
B1	0	1	1	0	1	1	0	0	0	0	3	28	3
B2	0	0	1	0	0	1	0	0	0	0	8	14	1
B3	0	0	0	20	29	0	0	0	0	0	461	266	24
B4	0	0	14	0	14	14	2	0	0	0	12	926	87
B5	0	0	1	0	0	49	1	0	11	0	1250	17497	1817
B6	0	0	0	0	0	0	0	0	0	0	51	23055	1180
B7	0	2	0	113	492	491	0	0	417	472	10625	13732	2753
B8	0	0	4	0	6	4	0	0	0	0	13	192	21
B9	0	0	0	0	15	5	0	0	0	0	22	1181	60
B10	0	0	69	0	156	156	20	5	135	0	78	10885	1178
CB	0	0	0	0	63	9932	0	0	0	0	0	0	0
Deposits	31	13	222	825	16695	11141	27110	202	576	10961	0	0	0

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Table 8 displays a matrix representation of a randomly generated financial system based on the model outlined in Section 2.2 with  $N = 10$  banks and parameter values set as in Table 7.  $B1$  to  $B10$  identify banks 1 to 10, respectively,  $Cb$  designates the central bank,  $Deposits$ ,  $Loans$  and  $Reserves$  the respective amounts each bank holds in terms of those balance sheet positions. Banks' assets can be read row-wise while banks' liabilities can be read column-wise. For example, bank 1's assets are displayed in row 2 while its liabilities, excluding equity, are displayed in column 2. Equity for a specific bank is obtained via subtracting the bank's column sum from the respective row sum. Note that on the matrix representation row and column two to row and column 11 represent the interbank market. For example, bank 3 has lent 20 millions to bank 4 (row 4, col 5) while it has borrowed 14 millions from the same bank (row 5, col 4). Notice that banks' exposures to each other are not broken down by maturities on the table.

*B1* to *B10* identify banks 1 to 10, respectively, *Cb* designates the central bank, *Deposits*, *Loans* and *Reserves* the respective amounts each bank holds in terms of those balance sheet positions. Banks' assets can be read row-wise while banks' liabilities can be read column-wise. For example, bank 1's assets are displayed in row 2 while its liabilities, excluding equity, are displayed in column 2. Equity for a specific bank is obtained via subtracting the bank's column sum from the respective row sum. Note that on the matrix representation row and column two to row and column 11 represent the interbank market. For example, bank 3 has lent 20 millions to bank 4 (row 4, col 5) while it has borrowed 14 millions from the same bank (row 5, col 4). Notice that banks' exposures to each other are not broken down by maturities on the table. Finally notice that the ratio of bank lending and bank borrowing to total assets purely driven by deposit fluctuations is about 0.13.<sup>29</sup>

Figure 3 displays a visual outline of the interbank network from Table 8 (row and column 2 to row and column 11). Each bank is represented by a ball, with the respective identifier given by the balls' numbers. Lending and borrowing among banks is shown by an arrow emanating from creditor to debtor, with the thickness of the arrow indicating the relative amount lent (thicker lines indicate bigger amounts lent/borrowed). A ball's color expresses the according bank's total assets in relation to other banks with blue to beige to red indicating small to medium to large banks, respectively.<sup>30</sup> A ball's diameter gives an indication about the respective bank's degree (the number of banks it is connected to on the interbank market) with larger balls indicating a higher degree. Note that the network has been automatically arranged by a gravity algorithm which amounts to balls which are connected to each other to exert gravity on each other (weighted by the sums lent or borrowed) while balls that are unconnected repulse each other. This algorithm leads to banks which have a higher degree to be more central (so-called 'hubs').

Network characteristics of the financial network displayed on Figure 3 can be summarized by the following metrics: The network has an average degree<sup>31</sup> and density<sup>32</sup> of 3.6 and 0.4, respectively. Its average shortest path length<sup>33</sup> and average clustering coefficient<sup>34</sup> amount to 1.9 and 0.14, respectively. Overall, the visual representation and

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<sup>29</sup>Note that the ratios of interbank assets and liabilities to total assets including lending and borrowing from Stage 2 on Figure 2 are larger, resulting in 0.15 and 0.13, respectively. These metrics including interbank activity from Stage 2 differ from each other because the central bank –which is not included as an individual bank with its assets or liabilities– on balance drains funds from the banking system to achieve its optimal policy target. Furthermore note that bank assets to deposits and bank liabilities to deposits both purely driven by deposit fluctuations amount to 0.19 each.

<sup>30</sup>Note that because of the distributional assumptions (see Table 7) which automatically lead to a big number of small banks and a small number of big banks the diameter of a ball cannot be used to indicate banks' sizes. The figure becomes unreadable even when the logarithm of banks total assets is taken with large banks disproportionately larger than small banks.

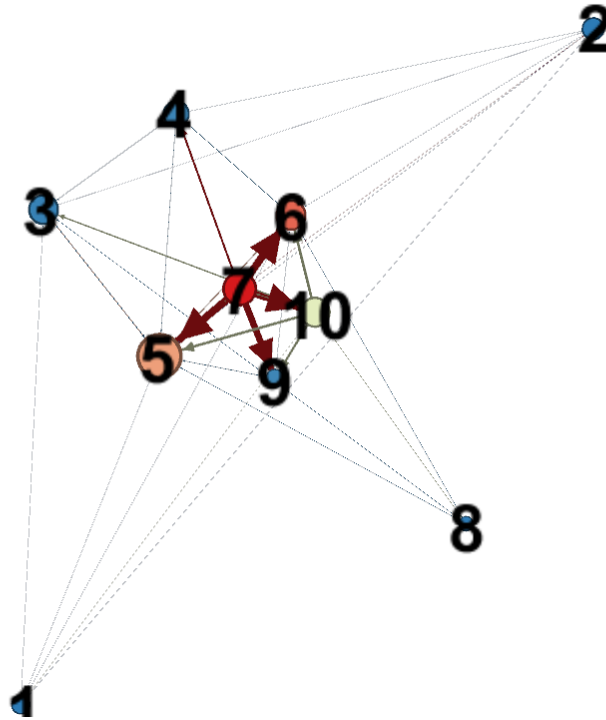
<sup>31</sup>The average degree is the average number of borrowing and lending counterparties a bank has in the network.

<sup>32</sup>The density of a network is the proportion of extant lender borrower relationships relative to all possible lender-borrower relationships.

<sup>33</sup>The average shortest path length is defined as the average number of steps along the shortest paths along connected nodes for all possible pairs of network nodes. It gives an indication about the efficiency of intermediation in a banking network.

<sup>34</sup>The clustering coefficient measures the degree to which nodes in a graph tend to cluster together. The local clustering coefficient of a node indicates how close its neighbors are to being a complete graph (all possible links exist). The average clustering coefficient is computed as the average of all local clustering coefficients.

Figure 3: Financial system draw for N=10 banks



Note: The figure displays a visual outline of the interbank network from Table 8 (row and column 2 to row and column 11). Each bank is represented by a ball, with the respective identifier given by the balls' numbers. Lending and borrowing among banks is shown by an arrow emanating from creditor to debtor, with the thickness of the arrow indicating the relative amount lent (thicker lines indicate bigger amounts lent/borrowed). A ball's color expresses the according bank's total assets in relation to other banks with blue to beige to red indicating small to medium to large banks, respectively.

the network metrics give the impression of a highly connected network with a money center bank in the core<sup>35</sup> and many (small) banks in the periphery. Notice that the density is high compared to actual financial systems because of the small number of banks chosen for the outline. A smaller number of banks *ceteris paribus* automatically leads to a higher density. For example, if a financial system consists of 2 banks only, it is highly likely that liquidity shocks lead to a density of 1.

To investigate model results for a more realistic financial system size, further simulations are carried out with  $N = 150$  banks. Table 9 displays the averages and standard deviations of the network metrics based on 100 financial systems generated along the parameters in Table 7.

<sup>35</sup>A money center bank is generally associated with large banks that dominate wholesale activity in money markets. For example, a bank in the core of a tiered interbank market can be regarded as a money center bank. See Craig and von Peter (2010).

Table 9: Network metrics

Variable	Average	Standard deviation
Degree	7.17	0.72
Density	0.05	0.00
Eigencentrality	0.07	0.00
Average shortest path	3.31	0.21
Clustering Coefficient	0.06	0.02
Interbank assets (liabilities) to total assets	0.12	0.01

Note: The table displays the averages and standard deviations of the network metrics based on 100 financial systems generated along parameters in Table 7. 'Interbank assets (liabilities) to total assets' refer to the shares purely driven by deposit fluctuations.

Given the larger number of banks, the outlined network metrics result in more realistic numbers close to those found for the German banking system.<sup>36</sup> On average banks have seven counterparties, about 5% of possible links in the financial system exist and the ratio of interbank assets to deposits purely driven by deposit fluctuations is about 0.12.<sup>37</sup>

Figure 4 displays one of the financial systems on which the average financial system metrics from Table 9 are based.

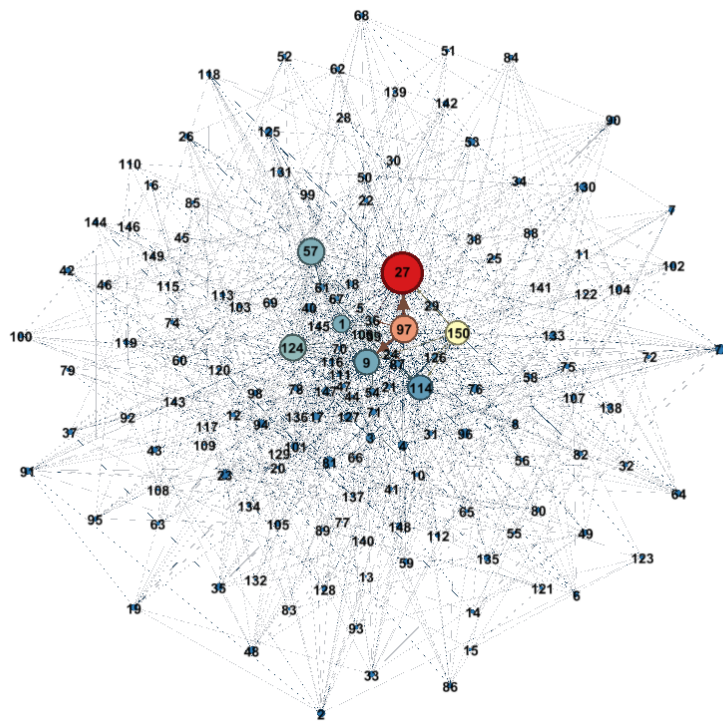
The main impression from the previously investigated financial system with  $N = 10$  banks remains: the model results in generating financial systems with a core-periphery structure with few money center banks at the core and many small peripheral banks connected to these hubs. Figure 5 displays a closer view on the core of the specific financial system with one of the main hubs (bank 97) and its connections highlighted. That bank is connected to the core and many peripheral banks at the same time and involved in lending (indicated by arrows emanating from the bank with the same color) as well as borrowing activities (indicating by arrows ending in the bank with the color of the creditor bank) at the same time.

Overall, the model developed in this paper reflects the stylized facts outlined in the introduction, namely that banks hold simultaneously high proportions of interbank assets and liabilities in a stable network structure. Furthermore, network metrics of the generated financial systems result in realistically conceivable values.

<sup>36</sup>See Battiston, Roukny and Georg (2014) and Bluhm, Georg and Krahen (2015).

<sup>37</sup>Note that the ratios of interbank assets and liabilities to total assets including lending and borrowing from Stage 2 on Figure 2 are larger, resulting in 0.19 and 0.13 on average, respectively. These metrics including interbank activity from Stage 2 differ from each other because the central bank –which is not included as an individual bank with its assets or liabilities– on balance drains funds from the banking system to achieve its optimal policy target. Furthermore note that bank assets to deposits and bank liabilities to deposits both purely driven by deposit fluctuations amount to 0.18 each on average.

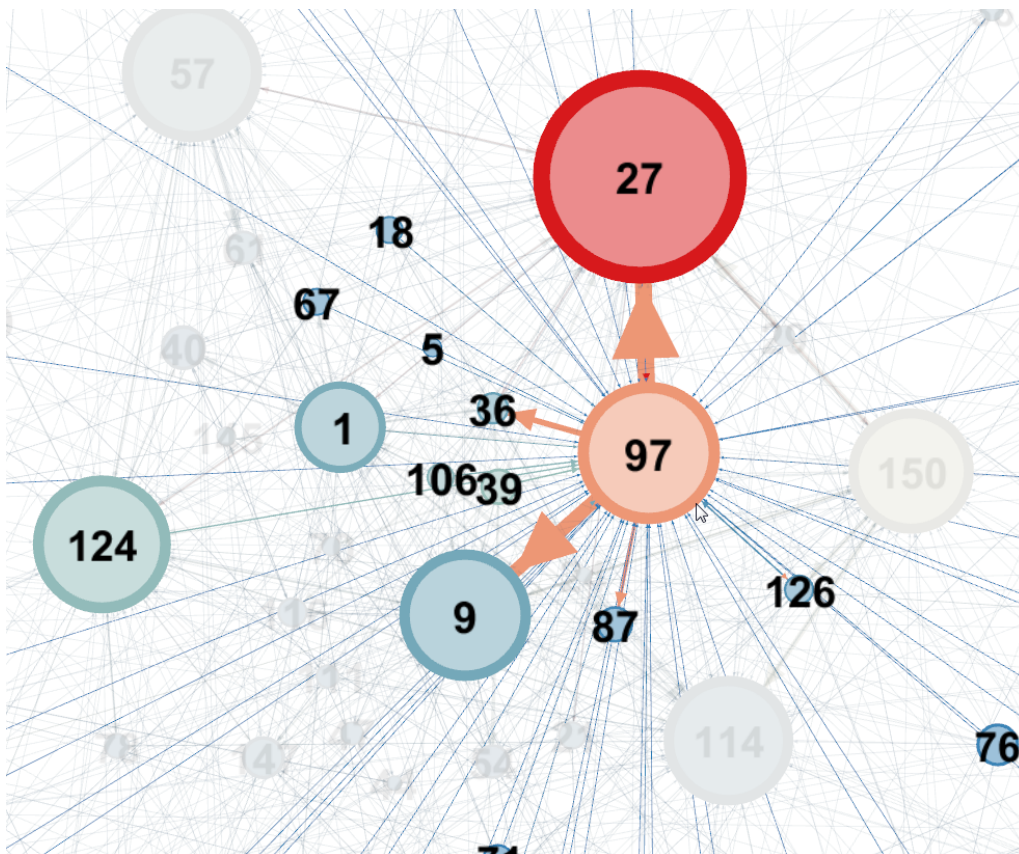
Figure 4: Financial system for  $N=150$  banks



Note: The figure displays an interbank network from the simulation of financial systems with  $N = 150$  banks. Each bank is represented by a ball, with the respective identifier given by the balls' numbers. Lending and borrowing among banks is shown by an arrow emanating from creditor to debtor, with the thickness of the arrow indicating the relative amount lent (thicker lines indicate bigger amounts lent/borrowed). A ball's color expresses the according bank's total assets in relation to other banks with blue to beige to red indicating small to medium to large banks, respectively.



Figure 5: Focus on Core of Interbank Market



Note: The figure displays the core of the interbank network displayed in Figure 4. Each bank is represented by a ball, with the respective identifier given by the balls' numbers. Lending and borrowing among banks is shown by an arrow emanating from creditor to debtor, with the thickness of the arrow indicating the relative amount lent (thicker lines indicate bigger amounts lent/borrowed). A ball's color expresses the according bank's total assets in relation to other banks with blue to beige to red indicating small to medium to large banks, respectively.

To investigate systemic risk and welfare properties within the model, an optimal monetary policy analysis is carried out in the following. Besides investigating real activity across a range of interest rates, outcomes are compared to a financial system without interbank market and central bank.<sup>38</sup> Note that such a reduced financial system is similar to early financial systems and can therefore give an indication about the efficiency gains emerging with a more sophisticated financial system framework. In particular, in case a bank cannot pay out the cash demanded by customers, a bank run takes place and the bank is liquidated. As a result, in the reduced financial system the lack of an interbank market to buffer liquidity shocks as well as the absence of a lender of last resort leads to banks being forced to hold a much higher amount of cash. In the model, in the absence of lending and borrowing possibilities, banks hold 85% of their deposits in liquid assets (lower liquidity ratios result in bank runs on major parts of the financial system, leading to very low economic activity measured as the aggregate sum of loans not in distress). Figure 6 displays the results of the optimal policy exercise for a specific financial system generated using the parameter values from Table 7.

The solid red and blue lines show real economic activity across a range of interest rates for the financial system with and without interbank market, respectively. The dashed lines are two standard deviation error bands.

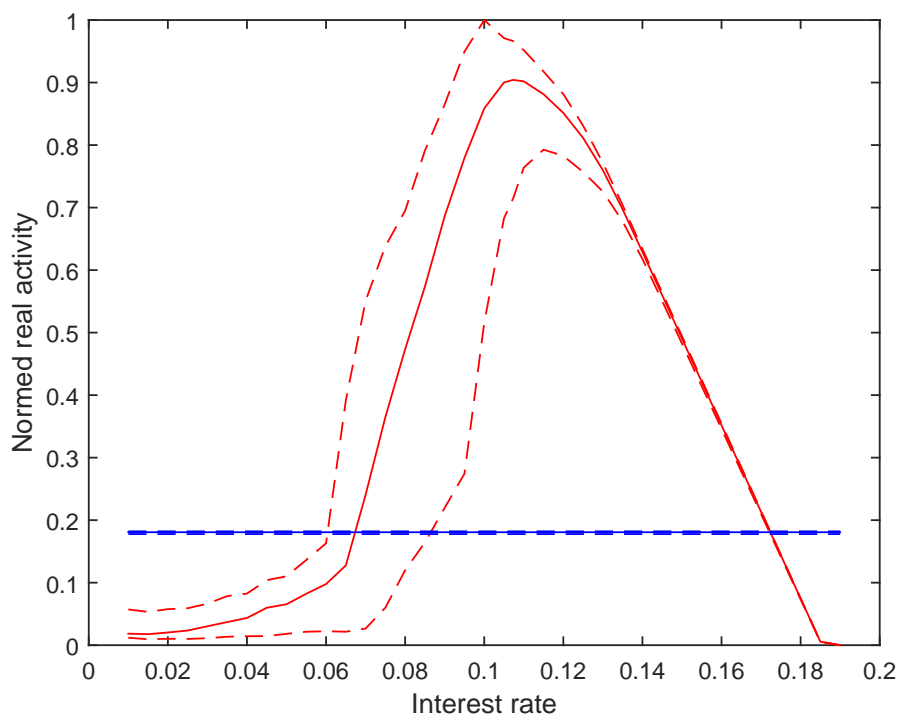
Two points are particularly worth noting. First, real activity in the financial system with interbank market (solid red line) is hump-shaped, with the maximum reached at the optimal interest rate. Across the range of possible interest rates there is a tension between higher credit supply at lower interest rates but higher risk of an adverse feedback loop arising when higher credit supply results in bad loans which in turn become distressed, leading to financial crises. The higher the interest rate, the lower the probability of adverse crisis events, but the less economic activity is supported by credit supply.

Second, a more sophisticated financial system results in higher real economic activity across most of the range of interest rates: real activity in the financial system with interbank market is significantly above the reduced financial system for most of the range. This is mainly driven by the ability of the central bank to steer the interest rate to the optimal value but also by the fact that being able to buffer liquidity shocks banks can *ceteris paribus* supply more credit relative to a situation in which banks' illiquidity can lead to insolvency. In a further simulation exercise, the optimal policy exercise is carried out 100 times based on financial systems generated from the parameter values on Table 7. On average, the more sophisticated financial system results in a (sustainable) increase of economic activity by a factor of 3.3 (with a  $p$ -value  $< 0.001$ ). Of course, to realize welfare gains in that magnitude, the policymaker needs to be aware of the optimal policy rate.

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<sup>38</sup>Interbank market and central bank can be shut down in the model via adding a set of further constraints, namely that banks' lending and borrowing on the interbank market, including to the central bank, is zero.

Figure 6: Optimal monetary policy exercise



#### 4. Conclusion

This paper has embedded a theoretical framework for the maturity structure of the interbank market in a micro founded agent based complex network model. Taking into account persistence of liquidity shocks and different maturity segments on the interbank market has important effects on banks' balance sheet composition, the interbank market and network formation. In particular, the theoretical model can explain as to why interbank funding is a very sizable component in the financial system and why banks are highly interconnected in a relatively stable network. Accounting for these phenomena is important because they affect efficiency of the financial system, systemic risk and monetary policy transmission.

The developed model can be called 'hybrid' because it embeds analytical micro-foundations in a flexible simulation framework. Using quadratic programming techniques for a concave optimization problem, each bank determines its globally optimal portfolio allocation. Heterogeneity of banks and an OTC modeling of the interbank market are then accommodated in a agent-based simulation setting. To provide for robustness of results, analyses are carried out with a large number of agents and across extensive simulations. Note that this type of model has only been used for financial system analysis since very recently. Despite many realistic elements included in the model and robustness of results, it is important to be aware of the model's capabilities and what it cannot (yet) achieve: for example, while a large number of draws allows for robustness of results on financial system and network properties (which can be gauged when observing the second moments of results) one cannot expect that employing the specific parameter values of an existing group of banks results in a model outcome which closely resembles their actual network. That is, individual results have to be taken with caution.

The developed model is used to carry out a stylized welfare analysis which underlines the importance of an efficient and stable financial system for real activity. In particular, this model does not only feature realistically microfounded financial system structures but shows a trade-off between credit provision (which allows for higher real activity) and financial fragility (which decreases real activity if financial and real crises developments amplify each other). This real world phenomenon currently confronts central banks which have been criticized for not taking into account this trade-off sufficiently, eventually sowing the seeds of tomorrow's financial collapse with today's lax monetary policy to support short-run economic activity. For example, in Bank for International Settlements (2015) it is argued that central banks should raise rates from an abnormally low level to support the real economy because there is a risk that long-run growth is adversely affected from low interest rates causing financial fragility and severe financial shocks. Furthermore, it is shown that while a financial system without interbank markets allows for stable but low output, an interbank market with lender of last resort renders the financial system more efficient and therefore can allow for higher welfare.

The analysis and developed model show that this (young) class of models offer a promising avenue for financial system and stability analysis. Extensions such as a central bank featuring besides its real activity and financial stability mandates also an inflation objective or a more realistically modeled real sector can allow for analyzing different policymakers' trade-offs among competing objectives. For example, given that endogenous money creation by the banking sector is already realistically included in the model, one could link real activity, price level and money supply via borrowing from

the quantity theory of money. Furthermore, the real sector could be enhanced via introducing additional classes of agents, a profit maximizing heterogeneous firm sector and utility maximizing heterogeneous household sector. Finally, in a dynamic setting, a Government could be introduced to smooth an emerging business cycle and stabilize the financial system in the face of large shocks from the real economy.

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## Appendix: Derivation of Model

The following Appendix derives model parts outlined in Sub-section 2.2 as depicted on Figure 2 and is subdivided into three parts. Part A outlines banks' maximization problem (Step 1 on Figure 2). Part B describes matching in the interbank market (Steps 2 and 3 on Figure 2) and Part C explains shock transmission in the model (Step 4 on Figure 2).

### A: Banks' maximization problem

A bank maximizes expected profit (Step 1 on Figure 2) given in Equation (3)

$$\begin{aligned}
 E(\pi^i) &= \text{revenue from interbank lending} + \text{revenue from emitting loans} \\
 &\quad - \text{cost of interbank borrowing} - \text{cost of deposits} \\
 &= bl^i \cdot r^{ibl} + loans^i \cdot r^{loans^i} \cdot P - bb^i \cdot r^{ibb} - d^i \cdot r^d,
 \end{aligned} \tag{3}$$

with  $r$  indicating different returns.

The individual elements in Equation (3) are expanded and explained in more detail in the following:

- $bl^i \cdot r^{ibl} = bl_{cb}^i \cdot r^{cbdn} + bl_b^i \cdot \tilde{r}^{mid}$  with  $r^{cbdn}$  the central bank's deposit facility and a tilde indicating that the interest rate captures the return at different maturities. That is, given persistence of shocks, 5 maturity segments in the model, and a liquidity premium of  $\epsilon$  for each period beyond a short term horizon, the return can be derived using the market expectations and liquidity premium hypotheses (See Sub-Section 2.2) as  $\tilde{r} = 1/2r_1 + 1/4(r_1 + \epsilon) + 1/8(r_1 + 2\epsilon) + 1/16(r_1 + 3\epsilon) + 1/16(r_1 + 4\epsilon) = r_1 + 15/16\epsilon$ . Overall, the equation gives a bank's periodical revenue from lending on the interbank market, where the first part represents the funds lent to the central bank ( $bl_{cb}^i$ ) and the second part the funds lent to other banks ( $bl_b^i$ ). Though banks generally get higher returns from lending to other banks, it is possible they lend to the central bank if (i) there is not sufficient demand on the interbank market for their funds, or (ii) if they are constrained by their capital requirement ratio such that they cannot invest in interbank lending (the risk weight for lending to the central bank is zero). Note that while banks' cost of *borrowing* ( $r^{ib}$ ) takes into account counterparty risk,<sup>39</sup> *lending* to banks uses the risk-free rate since charging a fair risk premium results in expectation in the distribution of the return on lending being centered at the risk-free rate (which is in the middle of the central bank's interest rate corridor,  $r^{mid}$ ).
- $loans^i \cdot P \cdot r^{loans^i} = P \cdot loans^i \cdot r^{loansmax} \cdot (1 - \frac{loans^i}{C^i})$  gives banks' periodical revenue from emitting loans to the economy with  $loans^i$  the amount of loans emitted,  $r^{loansmax}$  an upper threshold for the interest rate that can be obtained from emitting loans and  $C$  an indicator for a bank's share of branches in the economy.  $P$  is the fair market price of loans emitted, reflecting the amount of non-performing loans in the economy. Note that in that equation,  $r^{loansmax} \cdot (1 - \frac{loans^i}{C^i})$  ensures

<sup>39</sup>The cost of borrowing is derived in Sub-Section 2.2.



that banks have a declining return as they emit more loans (the yield approaches zero as  $loans^i$  approaches  $C^i$ ). The rationale is that banks with a larger branch network should face higher credit demand from the economy as a bigger pool of customers demands loans. The return from emitting loans is multiplied with  $P$ ,  $P \leq 1$ , to take into account that problems in the real economy can lead to non-performing loans which in turn lower banks' profitability. Note that prior to any shock in the economy  $P$  is set to 1 and only declines when a shock hits the economy in Step 4 on Figure 2.

- $bb^i \cdot r^{ibb} = bb^i \cdot \min(r^{mid} \cdot \frac{1}{1 - \xi^i PD^i}, r^{cbup})$ , with  $\xi^i$  and  $PD^i$  bank  $i$ 's loss given default and probability of default, respectively, and  $r^{cbup}$  the central bank's marginal refinancing rate. The equation gives a bank's periodical cost from borrowing funds on the interbank market (see Sub-section 2.2 for a derivation of the interbank interest rate). The minimum operator is used since a bank which faces an interbank rate higher than the marginal refinancing rate will turn to the central bank to borrow.
- $d^i \cdot r^d = loans^i \cdot r^{cbtn}$  is a bank's cost from holding deposits. Two points are important to note here. First, in the optimization the loans a bank emits result in a balance sheet expansion. That is, on its asset side it holds the loans and at the same time credits the debtor's deposits.<sup>40</sup> Note that in the model deposit shocks will change the amount of deposits on banks' balance sheets (see Steps 2 and 3 on Figure 2). Second, the interest rate on deposits is set equal to the marginal lending facility of the central bank. Assuming competition for deposits in the banking sector, the interest paid on deposits is closely aligned to the cost of obtaining funds from the central bank.

Banks' profit given in Equation (3) is maximized subject to a number of regulatory and feasibility constraints outlined in the following:

- $c^i \geq (\alpha + \tau^i) \cdot d^i = (\alpha + \tau^i) \cdot loans^i$ , with  $c$  banks' cash or reserves and  $\alpha$  the regulatory liquidity requirement ratio. The constraint makes sure that banks maintain a certain level of cash/reserves as a function of their deposits with  $\tau$  an individual parameter  $\tau^i = U \cdot 0.01$  assigning banks slightly different risk preferences as to how closely they desire to fulfill the regulatory requirements;
- $bl^i \geq \phi \cdot d^i = \phi \cdot loans^i$  is the expected amount of interbank lending given persistence of deposit shocks.  $\phi$  is the expected amount of interbank demand for excess funds bank  $i$  accrues from deposit fluctuations. (see Sub-chapter 2.1 which finds that in the model  $\phi$  equals 0.18) Banks can lend excess funds either to commercial banks (if their capital requirement ratio allows) or to the central bank. Note that interbank lending can be larger if banks lend part of their capital in Step 2 on Figure 2 to other banks.
- $bb^i \geq \phi di^i = \phi loans^i$  is banks expected amount of necessary interbank borrowing given deposit fluctuations Note that  $ib$  can be larger if banks borrow additional reserves in Step 2 on Figure 2 to fulfill the liquidity requirement ratio.

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<sup>40</sup>See McLeay, Radin and Thomas (2014) for a detailed outlined how banks create liquidity via emitting loans.

- $c^i + P \cdot loans^i + bl_b^i + bl_{cb}^i - d^i - bb^i \geq (\chi_1 \cdot P \cdot loans^i + \chi_2 \cdot bl_b^i) \cdot (\gamma + \tau^i)$   
 $\Leftrightarrow c^i + P \cdot loans^i + bl_b^i + bl_{cb}^i - loans^i - bb^i \geq (\chi_1 \cdot P \cdot loans^i + \chi_2 \cdot bl_b^i) \cdot \gamma + \tau^i$  is a regulatory requirement, namely that banks hold a minimum capital buffer as a function of their risk-weighted assets;
- $e^i = c^i + loans^i \cdot P + bl_b^i + bl_{cb}^i - d^i - bb^i = c^i + loans^i \cdot (P - 1) + bl_b^i + bl_{cb}^i - bb^i$  with  $e$  a bank's equity. The equation outlines a feasibility constraint which ensures that the balance sheet identity holds.

The above optimization problem features a quadratic objective function subject to linear constraints. Its optimal solution is found with quadratic programming techniques.

### B: Matching algorithm on the interbank market

In the following the interbank matching process used in Steps 2 and 3 on Figure 2 is outlined. To carry out matching on the interbank market, optimal lender and borrower combinations are determined via an iterative scoring algorithm. For a given adjacency matrix<sup>41</sup> and two vectors of length  $N \times 1$  containing banks desired amounts to lend or borrow, the following steps are carried out:

1. Set up a relationship matrix (*relatmatrix*) with weights for existing interbank links: a  $N \times N$  matrix filled with ones, zeros on the main diagonal and  $2(\sum bb + \sum bl)$  for extant lending-borrowing relations with  $\sum bb$  and  $\sum bl$  the sum of borrowing and lending vectors. Note that the weights chosen here make sure that banks always prefer lending/borrowing with existing counterparties in the *scorematrix* which is introduced in Step 4 below.
2. Set up a lender matrix (*lendermat*) with all rows filled with the amount desired to lend by banks; all rows with zeros are set to *+infinity*. For example, if bank 1's desired amount to lend is  $X > 0$ , and no other banks want to lend, then the lender matrix' first row consists of  $X$  in each element while all other (N-1) rows are set to *+infinity* in each element.
3. Set up a borrower matrix (*borrowermat*) with all columns filled with the amount desired to borrow by banks; all columns with zeros are set to *-infinity*. For example, if bank 1's desired amount to borrow is  $X > 0$ , and no other banks wants to borrow, then the borrower matrix' first column consists of  $X$  in each element while all other (N-1) columns are set to *-infinity* in each element.
4. Compute score matrix as  $scoremat = \frac{1}{|lendermat - borrowermat|} \cdot relatmatrix$ . In the score matrix, highest values are achieved by existing lender-borrower pairs who want to exchange relatively similar amounts, followed by lender-borrower pairs featuring an existing relationship but with larger differences in the desired amounts to exchange, followed by pairs having no existing relationship but small differences in desired amounts to exchange, followed by pairs having no existing relationship and larger differences in desired amounts to exchange. Note that further criteria for bank preferences can be easily added to this scoring approach.

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<sup>41</sup>An adjacency matrix indicates existing lender-borrower pairs in the interbank market. For example, a non-zero entry in matrix element (1,2) indicates that bank 1 has lent to bank 2, while a non-zero entry in matrix element (2,1) indicates that bank 1 has borrowed from bank 2.

5. Consecutively identify maxima in scoremat which assign the optimal borrower-lender matches  $(i, j)$  and set row  $i$  and column  $j$  to zero until scoremat is entirely filled with zeros.
6. Remove amounts exchanged (that is,  $\min(bl^i, bb^j)$ ) from the lender and borrower vectors and go to step 1 until at least one of the lender or borrower vectors consists only of zeros.
7. Any residual amount is given to/obtained from the central bank.

### C: Shock transmission

Shock transmission (Step 4 on Figure 2) in the financial system is modeled similarly to Bluhm and Krahen (2014) who draw on Cifuentes, Ferrucci and Shin (2005) as well as Eisenberg and Noe (2001). Note that Bluhm and Krahen (2014) and Cifuentes, Ferrucci and Shin (2005) carry out their analyses with an exogenously given financial system, while here the financial system which is exposed to a shock has been endogenously derived based on microfoundations. In the following shock transmission from Bluhm and Krahen (2014) is briefly reproduced.

In the model outlined in Sub-Section 2.2, an initial shock emanates from the real economy via an increase of non-performing loans. This increase reduces the return banks obtain for their loan portfolio which is reflected in the market price of this asset. For example, if the proportion of non-performing loans increases from 0% to 8% and the return on a bank's loan portfolio before the shock was 12%, then after the increase in distressed loans the price (which is set to 1 in the model during Steps 1 to 3 in Figure 2) declines to 0.92 and the return on the loan portfolio declines to  $0.92 \cdot 0.12 = 0.1104$ . Since banks have to mark their assets to market, this decline in the value of the loan portfolio has an effect on banks capital ratio given in Equation (4).

$$\frac{c^i + P \cdot loans^i + bl_b^i + bl_{cb}^i - d^i - bb^i}{\chi_1 \cdot P \cdot loans^i + \chi_2 \cdot bl_b^i} \geq \gamma. \quad (4)$$

In case of violation of the regulatory capital requirement,  $\gamma$ , banks have two possibilities to improve their capitalization, (a) netting extant interbank exposures, and (b) liquidating part of their loan portfolio. Netting interbank exposures is modeled similarly to Bluhm and Krahen (2014).

Consider the effect of netting interbank exposures. Equation (5) displays bank  $i$ 's capital ratio after netting (part of) its exposures with other banks by  $\theta$  units:

$$\gamma^i = \frac{(bl^i - \theta) + bl_{cb}^i + P \cdot loans^i + c^i - (bb^i - \theta) - d^i}{\chi_1 \cdot P \cdot loans^i + \chi_2 \cdot (bl_b^i - \theta)}. \quad (5)$$

Netting reduces the denominator by  $\theta$  units while the numerator remains unchanged.

If after netting interbank exposures banks still feature capital shortfalls, they can reduce its loan portfolio. This however eventually leads to a further deterioration in the economy which in turn increases the rate of non-performing loans and so on. A negative feedback loop between financial system and real economy starts. To model loan portfolio liquidation and the negative feedback loop between the real economy and the financial system we use a tâtonnement process. The following draws upon Bluhm and Krahen (2014) and Cifuentes, Ferrucci and Shin (2005). Note however, that while the

framework is similar, the market price here is efficient in the sense that it reflects the underlying proportion of non-performing loans while in the former it reflects firesales, eventually deviating from its fundamental value. In the tâtonnement process supply is driven by banks liquidating part of their distressed loan portfolio while demand comes from specialized institutions investing in distressed debt such as hedge funds, or a bad bank whose sole purpose is to liquidate the troubled assets.

Consider the effect of liquidating loans to improve a bank's capital ratio. Equation (6) shows the capital ratio bank  $i$  expects to obtain if it liquidates  $s_i$  units of its loans in exchange for  $p \cdot s_i$  units of liquid assets.

$$\gamma^* = \frac{bl^i + bl_{cb}^i + P \cdot (loans^i - s^i) + c^i + P \cdot s^i - bb^i - d^i}{\chi_1 \cdot P \cdot (loans^i - s^i) + \chi_2 \cdot bl_b^i}. \quad (6)$$

Similar to Cifuentes, Ferrucci and Shin (2005) the price of loans,  $P$ , is discovered by a function of supply and demand on the market. While aggregate supply is obtained via adding banks' loan-portfolio liquidations (via solving Equation (6) for  $s$  and summing up for all banks), the inverse demand function is assumed to follow Equation (7)

$$p = \exp(-\xi \sum_i s_i), \quad (7)$$

where  $\xi$  is a positive constant to scale the price responsiveness to an increase in the rate of non-performing loans in the real economy when banks liquidate their loan portfolios. In the model,  $\xi$  can be set such that the rate of non-performing loans increases to 25% if banks liquidate all their loans.<sup>42</sup>

A shock to the non-performing loans ratio in the real economy reduces the market price of loans from 1 to  $P_1$ . Eventually this decline in value puts banks' capital ratio under pressure, resulting in banks liquidating  $S(P_1(NPL))$  of their loans. Hedge funds (or a government set up 'bad bank') knowing that banks' reduction in supply leads to a further deterioration in the non-performing loan ratio offer  $P_2$  to take off the bad loans from banks' balance sheet and liquidate them. The decrease in market price which reflects the underlying increase in non-performing loans however puts further pressure on banks capital eventually resulting in further loan liquidations until a new equilibrium is reached at the intersection of demand and supply curves (Figure 7).

Using the above outlined building blocks, shock transmission is carried out by an iterative clearing algorithm after an initial exogenous shock (an increase in the rate of non-performing loans which decreases the market price of banks' loan portfolios) emerges in the real economy.<sup>43</sup> First, banks' net values (taking into account their direct interlinkages) and capital ratios are computed. Second, given banks' capitalization, counterparty netting is carried out and capital ratios are updated. Third, given the updated capital ratios, banks which do not fulfill the capital requirement ratio liquidate (parts of) their loan portfolio. Given the aggregate sum of loan liquidations, the rate of non-performing

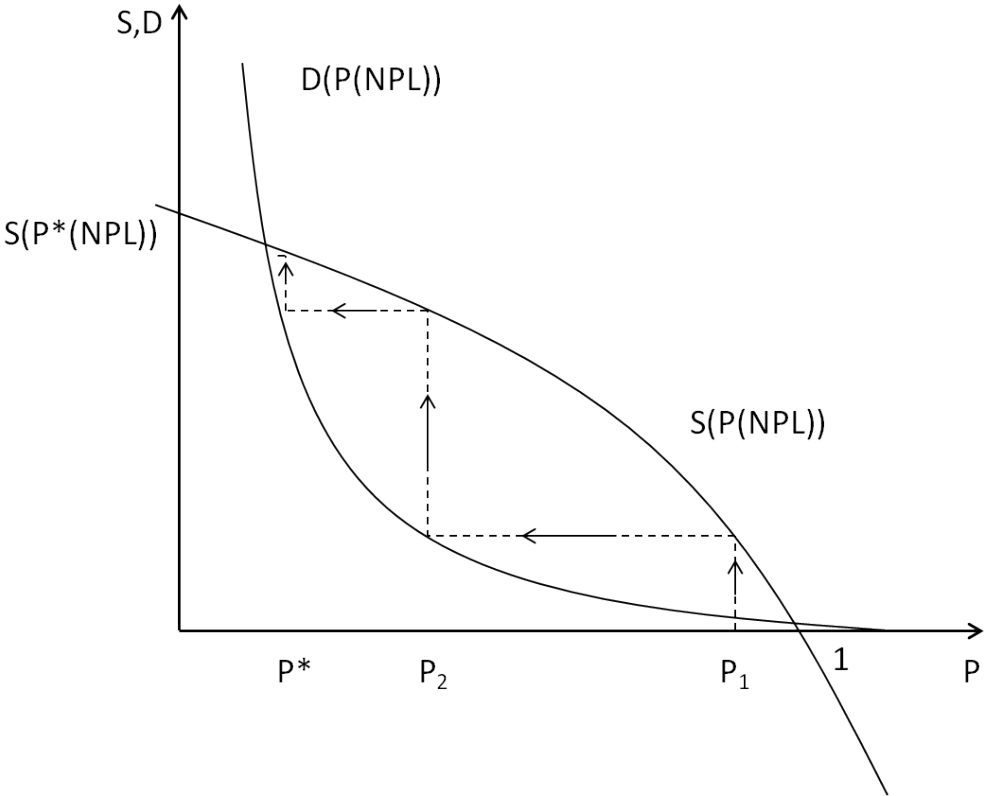
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<sup>42</sup>This value is also chosen in the model and considered in line with non-performing loan ratios found in countries with severe economic and financial crises. See The World Bank (2015) for an overview on non-performing loan ratios in a panel of countries.

<sup>43</sup>The clearing is described in detail in Bluhm and Krahen (2014) who build upon Cifuentes, Ferrucci and Shin (2005).

loans in the real economy increases and a new market price  $P$  is computed. The algorithm iterates over steps 1 to 3 until the financial system has reached a new equilibrium and does not change anymore.

Figure 7: Negative Feedback Loop Between Financial System and Real Economy



Note: The figure outlines the tâtonnement process used to discover market price,  $P$ , and rate of non-performing loans,  $NPL$ , in the real economy.  $D$  and  $S$  are demand and supply curves, respectively, for loans in liquidation.

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