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The Quest for Status and R&D-based Growth



by
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Abstract

We analyze the impact of status preferences on technological progress and long-run economic growth. For this purpose, we extend the standard relative wealth approach by allowing the two components of the representative household's wealth, physical capital and shares, to differ with respect to their status relevance. Relative wealth preferences imply that the effective rate of return of saving in the form of a particular asset is the sum of its market rate of return and its status-related extra return. It is shown that the status relevance of shares is of crucial importance: First, an increase in the intensity of the quest for status raises the steady-state economic growth rate only if the status-related extra return of shares is strictly positive. Second, for any given degree of status consciousness, the long-run economic growth rate depends positively on the relative status relevance of shares. Third, while in the standard model the decentralized long-run economic growth rate is less than its socially optimal counterpart, the wealth externalities in our model counterbalance this distortion to some extent provided that shares matter for status.

JEL classification: D31, D62, O10, O30.

Keywords: Status concerns, relative wealth, technological progress, long-run economic growth, social optimality.

1 Introduction

The idea that individuals derive utility not only from absolute consumption, leisure, or wealth but also from their social status is by now well established. It has long been recognized that individuals compare themselves with each other and that they derive extra felicity from outperforming their peers. For example, Adam Smith wrote in *The Wealth of Nations* that “With the greater part of rich people, the chief enjoyment of riches consists in the parade of riches” and John Stuart Mill stated in his *Essay on Social Freedom* that “Men do not desire merely to be rich, but to be richer than other men”. This all too human trait is also backed by empirical evidence: Clark and Oswald (1998) and Luttmer (2005) analyze 5,000 British workers and 8,000 US households, respectively, and find that self-reported happiness and life satisfaction are reduced if, ceteris paribus, neighbors an/or colleagues are better off. Their results are statistically and economically significant and are robust against various re-specifications of the regressions. Luttmer (2005) concludes that the most promising explanation for his result is the presence of a psychological externality that leads individuals to derive utility from their relative status as compared to others. For further empirical support of status concerns see McBride (2001) and Boyce et al. (2010).

In theoretical macroeconomic models with status preferences, the implications of the quest for status on private consumption, saving, work effort, the long-run economic growth rate, and the optimal design of distortionary taxation/subsidization are analyzed. The results of this literature with respect to long-run growth depend crucially upon whether i) social status is determined by relative consumption or relative wealth¹, ii) the production function of the representative firm is of the neoclassical type or exhibits externalities in accordance with Romer (1986) so that the aggregate production function is of the AK type, iii) labor supply is exogenously given or endogenously determined, and iv) economic agents are homogeneous in every respect or differ with respect to initial wealth and/or their subjective rate of time preference.

The *relative consumption* approach exhibits the following properties: i) If the production function is of the neoclassical type, then the quest for status affects only transitional dynamics, i.e., it cannot be an engine of long-run economic growth.² ii) If the production function is of the AK type, the consumption externalities resulting from status preferences affect the long-run economic growth rate. However, the sign of the qualitative effect is ambiguous. The *relative wealth* approach yields the following results: i) Even if the

¹ For the relative consumption specification or more general specifications of consumption externalities see for example Abel (1990, 2005), Galí (1994), Harbough (1996), Carroll et al. (1997), Rauscher (1997), Grossmann (1998), Fisher and Hof (2000), Ljungqvist and Uhlig (2000), Liu and Turnovsky (2005), Turnovsky and Monteiro (2007), Fisher and Heijdra (2009), and Strulik (2013). For the relative wealth specification see for example Corneo and Jeanne (1997, 2001a,b), Futagami and Shibata (1998), Fisher and Hof (2005, 2008), Van Long and Shimomura (2004), and Fisher (2010). For a framework that allows for both specifications see Tournemaine and Tsoukis (2008).

²In case that labor supply is endogenously determined, the steady state is characterized by excessive inputs of both real capital and labor as compared to the standard model without status preferences, while the resulting capital-labor ratio equals its standard counterpart.

production function is of the neoclassical type, the desire for status generates endogenous long-run growth provided that certain assumptions with respect to the intensity of the desire for status and/or the initial stock of physical capital are satisfied. However, a balanced growth path does not exist (Corneo and Jeanne, 2001b). ii) If the production function is of the *AK* type, then quite standard assumptions of relative wealth preferences ensure the existence of a balanced growth path. As long as households are homogeneous, the long-run growth rate depends positively on the intensity of the quest for status. However, if agents are heterogeneous, this result may be overturned (see Futagami and Shibata, 1998).

With respect to the analysis of long-run economic growth, this short overview has the following implications: The introduction of status preferences into models with a neoclassical production function is an unpromising avenue, regardless of whether status is determined by relative consumption or relative wealth. By contrast, at first glance it could be fruitful to combine a production function of the *AK* type with status preferences of the relative wealth type. However, this view has to be revised in light of the facts that i) many implications of the *AK* model are refuted by the available empirical evidence (cf. Aghion and Howitt, 2009, pp. 56–60); and that ii) *AK* models do not leave an explanatory role for what has been empirically and theoretically identified as the main driver of economic growth, namely technological progress (Acemoglu, 2009, pp. 402–403). Nowadays, multi-sector R&D-based growth models are used as state-of-the-art frameworks to analyze the driving forces behind technological change. We therefore aim to extend this literature to adequately analyze the effects of the quest for status on long-run economic growth.

The contribution of our paper is twofold: First, we close an important gap in the literature by introducing relative wealth preferences into the generic R&D-based growth model of the Romer (1990) type and by analyzing the implications of status concerns for technological progress. To the best of our knowledge this has not been attempted before.³ Second, we extend the standard relative wealth approach by allowing the two components of the representative household’s wealth, physical capital and shares, to differ with respect to their relative status relevance. The underlying idea is that saving in the form of tangible assets may be better suited for enhancing social status than saving in the form of intangible assets. As will become clear in our analysis, this specification is an important innovation because the long-run economic growth rate crucially depends on the relative status relevance of physical capital versus shares. Our generalization therefore yields additional insights that cannot be obtained in the standard relative wealth approach in which all assets are of equal importance with respect to status or in an *AK* type of growth model in which only one asset exists.

The introduction of generalized relative wealth preferences into the Romer (1990) model implies that the effective rate of return of saving in the form of a particular asset is the sum of its standard market rate of return and its status-related extra return. In both

³Using a semi-endogenous growth model of the Jones (1995) type instead of the Romer (1990) type would not change the basic mechanisms and channels that we describe below. Qualitatively, all our results would be present during the transition phase. However, this can only be shown numerically (see Trimborn et al., 2008; Prettnner and Trimborn, 2016), which is beyond the scope of our paper.

the Euler equation for consumption and in the no-arbitrage condition, the rental rate of physical capital and the market rate of return of shares are replaced by the corresponding effective rates of return. Hence, the status-seeking motive leads to a rise in the common steady-state effective rate of return of all assets as long as the positive effects of the status-related extra returns are not perfectly offset by a decrease in the market rates of return. The resulting stronger incentive to save causes the demand for both physical capital and shares to grow at a higher rate. The latter effect fosters technological progress, because the purchase of blueprints by intermediate firms is financed by the issuance of shares. Satisfying the faster-growing demand for blueprints is tantamount to the acceleration of technological progress. According to the underlying production technology in the R&D sector, this acceleration of technological progress is ultimately due to an increase in the employment of scientists.

The main implications of the introduction of generalized relative wealth preferences can be summarized as follows: First, an increase in the intensity of the quest for status raises the steady-state economic growth rate as long as the possession of shares matters for social status. If, however, solely the relative holdings of physical capital are status-relevant, then the status-augmented Romer (1990) model yields the same long-run balanced growth rate as the standard Romer (1990) model. Second, for any given degree of status consciousness both the share of wealth held in the form of stocks and the long-run economic growth rate depend positively on the relative status relevance of shares. This is consistent with the empirical findings of Levine and Zervos (1998) and Levine (2005) that initial stock market liquidity is a robust predictor for subsequent economic growth in cross-country growth regressions. Note that both of these results do not emerge in a standard relative wealth specification or in an *AK* type of growth model. Third, while in the standard Romer (1990) model the decentralized long-run economic growth rate is less than its socially optimal counterpart, in our status model the wealth externalities counterbalance this distortion to some extent provided that shares matter for status.

The paper is structured as follows. In Section 2 we introduce the basic assumptions with respect to status preferences and derive the status-augmented versions of i) the Euler equation for consumption, ii) the no-arbitrage condition with respect to the rates of return of physical capital and shares issued by intermediate goods producing firms, and iii) the transversality conditions of the representative household's optimization problem. In addition, we present the three sectors of the production side of the economy and derive the system of differential equations that governs the dynamic evolution of the economy. Special attention is given to the conditions for the existence of a balanced growth path and its determination. Section 3 contains the main results with respect to the impact of the quest for status on long-run economic growth. In Section 4 we discuss the results and conclude.

2 The model

2.1 Basic assumptions

Consider a modern knowledge-based economy with three sectors in the vein of Romer (1990)⁴: final goods production, intermediate goods production, and R&D. These sectors employ two production factors, physical capital and labor. Homogeneous labor is employed in the final goods sector and in the R&D sector (for simplicity we refer to labor employed in final goods production as “workers” and to labor employed in R&D as “scientists”). The final goods sector produces a single homogeneous commodity that is used either as consumption good or as physical capital. The firms of the intermediate goods sector rent the services of capital from the households that own the capital. The varieties produced by the intermediate goods sector are used as inputs in the production of the final good. The R&D sector develops patents for intermediate goods, which are sold to the new firms that enter intermediate goods production. To put it differently, an entrant into the intermediate goods sector has to purchase a new intermediate-specific patent from the R&D sector as a fixed up-front investment to be able to start the production process. These up-front investments are financed by issuing shares that are bought by the households in the economy, which, in turn, receive the associated dividend income and may experience valuation gains.

There exists a continuum of homogeneous households of mass one. The flow budget constraint of the representative household has the following form:

$$\dot{K} + p_Z \dot{Z} = rK + DZ + wL - C, \quad (1)$$

where K denotes physical capital, r is its rental rate, Z is the number of stocks, p_Z denotes the stock price, D refers to the dividend payments per stock, L is exogenously given supply of labor, w is the real wage rate, C refers to consumption, and a dot over a variable denotes the time derivative, i.e., for any variable x the derivative with respect to time t is given by $\dot{x} \equiv dx/dt$.⁵

Individuals earn labor income and asset income. The former is given by wL , while the latter consists of rental payments by the intermediate goods sector rK and dividend payments DZ . For simplicity and without loss of generality, we ignore the depreciation of physical capital. To summarize, the right-hand side of the flow budget constraint refers to total saving, while the left-hand side shows that total saving is either used for investments

⁴For the sake of simplicity, we follow the literature on horizontal innovations. Similar effects would, however, also be present in case of vertical innovations. For R&D-based growth models in general see Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Jones (1995), Kortum (1997), Segerström (1998), Peretto (1998), Young (1998), Howitt (1999), Dalgaard and Kreiner (2001), Strulik (2005), Bucci (2008), and Strulik et al. (2013). For extensive surveys see Gancia and Zilibotti (2005) and Aghion and Howitt (2005).

⁵Note that the derivative of total wealth $K + p_Z Z$ with respect to time t is obtained by adding the valuation gains of shares $\dot{p}_Z Z$ on both sides of the flow budget constraint (1) such that $\dot{K} + p_Z \dot{Z} + \dot{p}_Z Z = rK + DZ + \dot{p}_Z Z + wL - C$.

in physical capital or for purchasing the stocks that are issued by the intermediate goods sector to finance the outlays for the specific patents.⁶

In contrast to the standard framework, we employ status preferences to account for the fact that one's own felicity also depends on a comparison with others. More specifically, we assume that instantaneous utility u of the representative consumer depends not only on her consumption C , but also on her status S , i.e., the utility function has the form $u = u(C, S)$. With respect to the function u we assume the following:

$$\frac{\partial u}{\partial C} > 0, \quad \frac{\partial^2 u}{\partial C^2} < 0, \quad \frac{\partial u}{\partial S} > 0, \quad \frac{\partial^2 u}{\partial S^2} < 0, \quad \frac{\partial^2 u}{\partial C^2} \frac{\partial^2 u}{\partial S^2} - \left(\frac{\partial^2 u}{\partial C \partial S} \right)^2 \geq 0, \quad (2)$$

$$\frac{\partial \left(\frac{\partial u}{\partial S} / \frac{\partial u}{\partial C} \right)}{\partial C} > 0, \quad \frac{\partial \left(\frac{\partial u}{\partial S} / \frac{\partial u}{\partial C} \right)}{\partial S} < 0, \quad (3)$$

$$\lim_{C \rightarrow 0} \frac{\partial u(C, S)}{\partial C} = \infty, \quad \lim_{C \rightarrow \infty} \frac{\partial u(C, S)}{\partial C} = 0. \quad (4)$$

Assumption (2) signifies that the representative consumer derives positive but diminishing marginal utility from both consumption and status. Moreover, the utility function u is jointly concave in C and S . According to (3), the marginal rate of substitution of status for consumption $(\partial u / \partial S) / (\partial u / \partial C)$ depends positively on C and negatively on S . These properties ensure that both consumption and status are normal goods. Finally, (4) introduces standard Inada conditions with respect to the marginal utility of consumption.

With respect to status S , two alternative specifications are employed in the literature. In the relative consumption approach, status S is determined by a comparison of own consumption with average consumption of a reference group. In models with homogeneous agents, average consumption of the total household sector serves as the benchmark. In the relative wealth approach the determination of status rests on a comparison of own wealth with average wealth. We focus our attention on the latter approach, because, as discussed in the introduction, it has a more convincing interpretation and yields richer results.

A crucial and distinctive feature of our model is that the components of wealth (physical capital and shares) are allowed to differ with respect to their effect on social status. More specifically, we assume that

$$S = S(\Omega, \bar{\Omega}), \quad \frac{\partial S}{\partial \Omega} > 0, \quad \frac{\partial^2 S}{\partial \Omega^2} \leq 0, \quad \frac{\partial S}{\partial \bar{\Omega}} < 0, \quad (5)$$

where

$$\Omega \equiv \phi_K K + \phi_Z p_Z Z \quad \text{and} \quad \bar{\Omega} \equiv \phi_K \bar{K} + \phi_Z p_Z \bar{Z} \quad (6)$$

are the status-relevant measures of own wealth and average economy-wide wealth, respec-

⁶Note that we follow Romer (1990) and abstract from loans and bonds for the following reasons: i) we do not model a government such that there are no governmental bonds; ii) we abstract from risk by restricting attention to perfect foresight. Hence, we can assume, without loss of generality, that firms finance their outlays for patents solely by issuing stocks; iii) since individuals are by assumption identical in all respects, households neither lend to (nor borrow from) other households in equilibrium.

tively. The parameters $0 \leq \phi_K \leq 1$ and $0 \leq \phi_Z \leq 1$ are the status weights of physical capital and share capital, respectively. If $\phi_K = \phi_Z = 1$, then Ω is identical to the standard definition of wealth as given by $K + p_Z Z$. If $\phi_K = 1$ and $\phi_Z = 0$, only physical capital matters for status but not wealth held in the form of stocks. The converse holds for $\phi_K = 0$ and $\phi_Z = 1$. Note that the latter parameter specification is not very realistic because one would expect visible/tangible assets to be *at least* as status-relevant as intangible assets. We assume that status $S = S(\Omega, \bar{\Omega})$ increases in own wealth Ω , with marginal status being non-increasing, and decreases in average wealth $\bar{\Omega}$. The latter implies negative wealth externalities.

In the status literature it is common practice to restrict attention to symmetric equilibria in which identical agents make identical choices such that $\Omega = \bar{\Omega}$ holds along an equilibrium path. With respect to symmetric situations, we follow Fisher and Hof (2005) and assume that the following condition holds:

$$S(\Omega, \Omega) = \chi = \text{constant}, \quad \text{for } \Omega > 0. \quad (7)$$

Assumption (7) ensures that our approach corresponds to a pure *relative* wealth specification because the flow of utility is independent of the *level* of wealth Ω along any symmetric equilibrium path, i.e., $u[C, S(\Omega, \Omega)] = u(C, \chi)$.⁷ It is easily verified that two standard specifications of the status literature, the difference specification

$$S(\Omega, \bar{\Omega}) = \varphi(\Omega - \bar{\Omega}), \quad \varphi' > 0, \varphi'' \leq 0 \quad (8)$$

and the ratio specification

$$S(\Omega, \bar{\Omega}) = \varphi(\Omega/\bar{\Omega}), \quad \varphi' > 0, \varphi'' \leq 0 \quad (9)$$

satisfy both assumption (5) and assumption (7).

By optimally choosing the time paths of C , \dot{K} , and \dot{Z} , the representative household maximizes overall utility as given by

$$\int_0^\infty e^{-\rho t} u[C, S(\Omega, \bar{\Omega})] dt,$$

where $\rho > 0$ denotes the time-preference rate, subject to the flow budget constraint (1), the definitions of Ω and $\bar{\Omega}$ as given by (6), and the initial conditions $K(0) = K_0$ and $Z(0) = Z_0$. A crucial feature of this optimization problem is that the representative household takes the time paths of w , r , p_Z , D , and $\bar{\Omega}$ as given. This is due to the fact that in a continuum of households each single household has mass zero and its choices do not affect aggregate variables.

A detailed analysis of this optimization problem is provided in Appendix A. Here

⁷Note that absolute wealth Ω would play a role along symmetric equilibrium paths if the instantaneous utility function $u(C, S)$ was replaced by $u(C, \Omega, S)$.

we only mention the three aspects in which the symmetric equilibrium of the status-augmented model differs from the equilibrium of the standard Romer (1990) model.

The first modification is that the no-arbitrage condition between saving in terms of physical capital and saving in terms of shares that holds in the standard Romer (1990) model

$$r = \frac{D}{p_Z} + \frac{\dot{p}_Z}{p_Z} \quad (10)$$

has to be replaced by

$$r + \varepsilon^K(C, \Omega, \chi, \phi_K) = \frac{D}{p_Z} + \frac{\dot{p}_Z}{p_Z} + \varepsilon^Z(C, \Omega, \chi, \phi_Z) \quad (11)$$

where

$$\varepsilon^K(C, \Omega, \chi, \phi_K) \equiv MRS(C, \Omega, \chi) \times \phi_K, \quad (12)$$

$$\varepsilon^Z(C, \Omega, \chi, \phi_Z) \equiv MRS(C, \Omega, \chi) \times \phi_Z, \quad (13)$$

$$MRS(C, \Omega, \chi) \equiv \frac{\partial u(C, \chi)}{\partial S} \frac{\partial S(\Omega, \Omega)}{\partial \Omega} \left[\frac{\partial u(C, \chi)}{\partial C} \right]^{-1}. \quad (14)$$

On the left-hand side of (11), $r + \varepsilon^K$ is the *effective* rate of return of wealth accumulation in the form of physical capital, where r is the market rental rate of physical capital, while $\varepsilon^K(C, \Omega, \chi, \phi_K)$ as defined in (12) is the *status-related extra return* of physical capital. On the right-hand side of (11), $(D + \dot{p}_Z)/p_Z + \varepsilon^Z$ is the *effective* rate of return of wealth accumulation in the form of stocks, where the market return $(D + \dot{p}_Z)/p_Z$ results from dividend payments and valuation gains, while $\varepsilon^Z(C, \Omega, \chi, \phi_Z)$ as defined in (13) is the *status-related extra return* of stocks. The term $MRS(C, \Omega, \chi)$ defined in (14) is the marginal rate of substitution of status-relevant own wealth Ω for consumption C as perceived by the representative agent in a symmetric state in which $\Omega = \bar{\Omega}$ holds. Taking into account that ϕ_K is the weight of physical capital in the status-relevant measure of wealth, $\Omega = \phi_K K + \phi_Z p_Z Z$, it follows that $\varepsilon^K = MRS \times \phi_K$ is the symmetric MRS of own physical capital K for consumption C . Analogously, $\varepsilon^Z = MRS \times \phi_Z$ is the symmetric MRS of own stocks Z for consumption C . The economic interpretation of ε^K given above can also be verified as follows: From the perspective of the representative household, an increase in own physical capital K by a marginal unit causes the status-relevant measure of wealth Ω to increase by ϕ_K units. Since i) the household takes average wealth as given, and ii) $\Omega = \bar{\Omega}$ holds at the outset of our thought experiment, this increase in Ω by ϕ_K units causes status S to rise by $[\partial S(\Omega, \Omega)/\partial \Omega] \phi_K$ units and felicity u to increase by $[\partial u(C, \chi)/\partial S] [\partial S(\Omega, \Omega)/\partial \Omega] \phi_K$ units. Dividing the latter expression by the marginal utility of own consumption, $\partial u(C, \chi)/\partial C$, we obtain the amount of consumption C that the status-conscious household is willing to give up in exchange for an increase in K by a

marginal unit. Analogous considerations can be used for the interpretation of ε^Z .⁸

The second modification as compared to Romer (1990) refers to the Euler equation for consumption. In the standard framework, where $u = u(C)$, it holds that

$$\frac{\dot{C}}{C} = \sigma(C)(r - \rho), \quad \text{with} \quad \sigma(C) \equiv -\frac{u'(C)}{Cu''(C)}. \quad (15)$$

In our case this has to be replaced by

$$\frac{\dot{C}}{C} = \sigma^S(C, \chi) [r + \varepsilon^K(C, \Omega, \chi, \phi_K) - \rho], \quad (16)$$

where, according to assumption (7), $\chi = \text{constant} = S(\Omega, \Omega)$, for $\Omega > 0$. The term $\sigma^S(C, \chi)$ in which the superscript ‘‘S’’ refers to ‘‘status’’ is the *effective* elasticity of intertemporal substitution under relative wealth preferences in a symmetric equilibrium as given by

$$\sigma^S(C, \chi) \equiv -\frac{\partial u(C, \chi)}{\partial C} \left[C \frac{\partial^2 u(C, \chi)}{\partial C^2} \right]^{-1}. \quad (17)$$

For a given value of the rental rate of capital r , the modified Euler equation implies the following: the higher the status-related component of the effective rate of return, ε^K , the higher is the growth rate of consumption \dot{C}/C . In other words, the willingness to substitute future consumption for present consumption increases, implying that individuals save more. In a general macroeconomic equilibrium as analyzed below, r is determined endogenously. It is therefore possible that the positive impact of $\varepsilon^K > 0$ on the effective rate of return $r + \varepsilon^K$ is partially or even completely offset by a fall in the market rate of return r .

The third modification concerns the transversality conditions. In the standard model they are given by

$$\lim_{t \rightarrow \infty} \left\{ \exp \left[- \int_0^t r(v) dv \right] K \right\} = 0, \quad \lim_{t \rightarrow \infty} \left\{ \exp \left[- \int_0^t [r(v)] dv \right] p_Z Z \right\} = 0,$$

i.e., the present values of wealth held in the form of physical capital and shares, respectively, must converge to zero as time goes to infinity. In the model with relative wealth preferences, the market rate of return r is replaced by the effective rate of return $r + \varepsilon^K$ such that

$$\lim_{t \rightarrow \infty} \left\{ \exp \left[- \int_0^t [r(v) + \varepsilon^K(v)] dv \right] K \right\} = 0, \quad (18)$$

$$\lim_{t \rightarrow \infty} \left\{ \exp \left[- \int_0^t [r(v) + \varepsilon^K(v)] dv \right] p_Z Z \right\} = 0,$$

where $\varepsilon^K(v) = \varepsilon^K[C(v), \Omega(v), \chi, \phi_K]$.

⁸Note that in case of $\phi_K = \phi_Z$ the status-related rates of return are identical, i.e., $\varepsilon^K = \varepsilon^Z$, which, in turn, implies that the market rates of return are identical too. In this case $r = (D + \dot{p}_Z)/p_Z$ holds just as in the standard Romer (1990) model.

Remark 1. *If status preferences are of the pure relative consumption type, then $u = u(C, S)$ and*

$$S = S(C, \bar{C}), \quad \frac{\partial S}{\partial C} > 0, \quad \frac{\partial^2 S}{\partial C^2} \leq 0, \quad \frac{\partial S}{\partial \bar{C}} < 0,$$

$$S(C, C) = \zeta = \text{constant}, \quad \text{for } C > 0,$$

where \bar{C} denotes the economy-wide average of private consumption. This specification implies that the Euler equation for consumption of the standard model (15) is replaced by

$$\frac{\dot{C}}{C} = \sigma^{rc}(C)(r - \rho),$$

where

$$\sigma^{rc}(C) \equiv -\frac{\partial U(C, C)}{\partial C} \left\{ C \left[\frac{\partial^2 U(C, C)}{\partial C^2} + \frac{\partial^2 U(C, C)}{\partial C \partial \bar{C}} \right] \right\}^{-1}$$

is the effective elasticity of intertemporal substitution under relative consumption preferences and $U(C, \bar{C}) \equiv u[C, S(C, \bar{C})]$. The no-arbitrage equation and the transversality conditions remain unchanged. For a proof of this remark see Appendix C.

2.2 Production side

The production side of the economy follows the standard R&D-based growth literature so that our description will be short and focused on the main parts that we need in the subsequent analysis.

The final goods sector consists of a continuum of perfectly competitive firms of mass one, each of which produces the same single good by employing the same technology. The production function of the representative firm is given by

$$Y = L_Y^{1-\alpha} \int_0^A x_i^\alpha di, \quad (19)$$

where Y is output, L_Y denotes labor input, and x_i is the amount of the intermediate good of type $i \in [0, A]$ used in final goods production. In this context, A refers to the technological frontier, i.e., the spectrum of patents for specific varieties i that has already been discovered. For simplicity, the elasticities of output with respect to the various types of intermediate goods are identical and given by $\alpha \in (0, 1)$. Since, by assumption, the mass of firms equals one, output and labor input of the representative firm coincide with GDP and aggregate employment in the final goods sector, respectively. The perfectly competitive representative firm takes both the real wage w_Y in the final goods sector and the real prices of intermediate goods p_i as given and maximizes profits by choosing the inputs L_Y and x_i . The corresponding first-order conditions (FOCs) are

$$w_Y = (1 - \alpha) L_Y^{-\alpha} \int_0^A x_i^\alpha di = (1 - \alpha) \int_0^A \left(\frac{x_i}{L_Y} \right)^\alpha di, \quad (20)$$

$$p_i = \alpha L_Y^{1-\alpha} x_i^{-(1-\alpha)} = \alpha \left(\frac{x_i}{L_Y} \right)^{-(1-\alpha)}. \quad (21)$$

These conditions require that each input is utilized up to the point at which its marginal product equals its real price (i.e., its price in terms of the final good).⁹ From (20) and (21) it follows that – in a general equilibrium – the remuneration of workers equals $(1 - \alpha)$ percent of real revenue Y , while α percent are used to pay for the intermediate goods:

$$w_Y L_Y = (1 - \alpha) L_Y^{1-\alpha} \int_0^A x_i^\alpha di = (1 - \alpha) Y, \quad (22)$$

$$\int_0^A p_i x_i di = \alpha L_Y^{1-\alpha} \int_0^A x_i^\alpha di = \alpha Y. \quad (23)$$

Firms in the intermediate goods sector have to purchase a patent from the R&D sector as up-front investment before they can produce the patent-specific differentiated intermediate good $i \in [0, A]$. The intermediate goods producing firm $i \in [0, A]$ employs a single variable production factor, physical capital, which it rents from private households. The production function is assumed to be linear and, without loss of generality, the productivity of physical capital is normalized to one such that $x_i = k_i$. Taking into account this linear production function and, in addition, the first-order condition for the optimal input of x_i in the final goods sector [see (21)], operating profits of intermediate goods producers can be written as

$$\pi_i = p_i x_i - r x_i = \alpha L_Y^{1-\alpha} x_i^\alpha - r x_i.$$

Profit-maximization implies that prices are set according to the rule $p_i = \alpha^{-1} r$, where the rental rate r represents marginal cost and $\alpha^{-1} > 1$ is the gross markup we are familiar with from Dixit and Stiglitz (1977). Hence, firms have a certain degree of price setting power and operating profits will be positive. This price setting policy implies that production x_i and input of physical capital k_i in the intermediate goods sector depend negatively on the rental rate r and positively (in a linear way) on employment in the final goods sector L_Y :

$$x_i = k_i = \left(\frac{\alpha^2}{r} \right)^{1/(1-\alpha)} L_Y. \quad (24)$$

⁹Since the production function exhibits constant returns to scale, several well-known difficulties arise: i) to ensure that the unique solutions for the input ratios x_i/L_Y obtained from (21) also satisfy the FOC for the optimal choice of labor input L_Y as given by (20), factor prices have to satisfy the following condition (factor price frontier): $w_Y = (1 - \alpha) \alpha^{\alpha/(1-\alpha)} \int_0^A p_i^{-\alpha/(1-\alpha)} di$. While the real wage w_Y and the real prices of intermediate goods p_i are taken as given by the representative firm, these variables are endogenously determined at the market level and will adjust such that – in a general equilibrium – the factor price frontier condition holds; ii) while the profit-maximizing values of the input ratios x_i/L_Y can be uniquely determined provided that the factor price frontier condition holds, this is not true for the levels of inputs L_Y and x_i . However, again these levels can be determined uniquely in general equilibrium; iii) each profit-maximizing production plan yields zero profits, i.e., revenues are completely used up for the remuneration of the factors of production.

The same is true for operating profits:

$$\pi_i = (1 - \alpha) \alpha^{(1+\alpha)/(1-\alpha)} r^{-\alpha/(1-\alpha)} L_Y. \quad (25)$$

Since all firms $i \in [0, A]$ make identical choices, we can drop the index i in the subsequent analysis and use the notation p , x , k , and π instead.

The R&D sector employs scientists L_A to discover new blueprints A according to the production function

$$\dot{A} = \lambda A L_A, \quad (26)$$

where λ refers to the productivity of scientists. There is perfect competition in the research sector such that R&D firms take both the real price of blueprints p_A and the real wage of scientists w_A as given. Since the production function (26) is linear in L_A , the profit of the representative R&D firm $(p_A \lambda A - w_A) L_A$ is linear in L_A , too. Hence, the existence of profit-maximizing production plans with $L_A > 0$ requires that scientists are paid their marginal product, i.e.,¹⁰

$$p_A \lambda A = w_A. \quad (27)$$

2.3 Market clearing and equilibrium dynamics

We close the model by introducing the market clearing conditions for all markets. Afterwards we derive a system of differential equations that governs the dynamic evolution of the economy in a symmetric macroeconomic equilibrium. In such an equilibrium households maximize utility, firms maximize profits, and all market clearing conditions are satisfied. The word “symmetric” means that households – being identical in every respect – and firms – facing identical cost and demand functions – make identical choices.

Equilibrium in the labor market requires that the wage rates earned in the final goods sector and in the R&D sector are equal because labor is homogenous. In addition, the sum of labor inputs in these two sectors must equal the exogenously given labor supply of households:

$$w_Y = w_A = w \quad \text{and} \quad L_Y + L_A = L. \quad (28)$$

Equilibrium in the rental market for physical capital requires that the supply of capital by the households (K) is equal to the aggregate capital input of firms in the intermediate goods sector (Ak). Using (24), this condition can be written as

$$K = Ak = A \left(\frac{\alpha^2}{r} \right)^{1/(1-\alpha)} L_Y. \quad (29)$$

Equilibrium in the stock market prevails if all shares issued by the intermediate goods producers up to time t are held by households. The normalization of the number of shares that are issued by a single intermediate firm to 1 yields $Z = A$ and $\dot{Z} = \dot{A}$. Since all

¹⁰Just as in the final goods sector, labor input L_A and production \dot{A} cannot be determined uniquely at the level of the representative firm. However, a unique solution can be obtained in the general equilibrium.

firms in the intermediate goods sector earn identical profits, their shares will have the same price in equilibrium. From the no-arbitrage condition under status preferences (11) it follows that the common price of shares at time t is given by

$$p_Z(t) = \int_t^\infty \exp \left\{ - \int_t^\tau [r(v) + \Gamma(v)] dv \right\} D(\tau) d\tau, \quad (30)$$

where

$$\Gamma \equiv \varepsilon^K(C, \Omega, \chi, \phi_K) - \varepsilon^Z(C, \Omega, \chi, \phi_Z) = MRS(C, \Omega, \chi)(\phi_K - \phi_Z). \quad (31)$$

Future dividend payments are discounted by $r + \Gamma \equiv r + \varepsilon^K - \varepsilon^Z$, i.e., the sum of the market rental rate r and the difference between the status-related extra returns of wealth accumulation in the form of physical capital and stocks, $\varepsilon^K - \varepsilon^Z$. If $\phi_K = \phi_Z$, the extra returns ε^K and ε^Z are equal, such that the formula for the calculation of the fundamental value of a stock simplifies to the standard expression in Romer (1990). New entrants into the intermediate goods sector have to buy a patent at the price p_A the purchase of which is financed by issuing a new stock. Due to free entry, competition between new entrants will cause p_A to reach the highest possible level. Since the price of a stock attains its maximum if the operating profit is fully distributed in the form of dividends, we have that $D = \pi$ holds in equilibrium and the share price is given by

$$p_A(t) = p_Z(t) = \int_t^\infty \exp \left\{ - \int_t^\tau [r(v) + \Gamma(v)] dv \right\} \pi(\tau) d\tau. \quad (32)$$

Differentiating with respect to time t , we obtain the following differential equation for the evolution of the price of blueprints:

$$\frac{\dot{p}_A}{p_A} = r + MRS(C, \Omega, \chi)(\phi_K - \phi_Z) - \frac{\pi}{p_A}. \quad (33)$$

Substituting various results derived above into the flow budget constraint of the representative household (1), we show in Appendix B.1 that the market for final goods is also in equilibrium, i.e.,

$$Y = C + \dot{K}, \quad (34)$$

such that output of final goods is either consumed or used for investment in physical capital.

Putting all the information together, we show in Appendix B.2 that the dynamic evolution of the four variables A , K , L_A , and C is governed by the following system of differential equations:

$$\frac{\dot{A}}{A} = \lambda L_A, \quad (35)$$

$$\frac{\dot{K}}{K} = \left[\frac{K}{A(L - L_A)} \right]^{-(1-\alpha)} - \frac{C}{K}, \quad (36)$$

$$\frac{\dot{C}}{C} = \sigma^S(C, \chi) \left\{ \alpha^2 \left[\frac{K}{A(L - L_A)} \right]^{-(1-\alpha)} + \varepsilon^K(C, \Omega, \chi, \phi_K) - \rho \right\}, \quad (37)$$

$$\begin{aligned} \dot{L}_A = (L - L_A) \left\{ - (1 - \alpha) \left[\frac{K}{A(L - L_A)} \right]^{-(1-\alpha)} + \frac{C}{K} + \lambda L_A \right. \\ \left. - \lambda (L - L_A) + \frac{\varepsilon^K(C, \Omega, \chi, \phi_K) - \varepsilon^Z(C, \Omega, \chi, \phi_K)}{\alpha} \right\}, \quad (38) \end{aligned}$$

where

$$\Omega = \phi_K K + \phi_Z \frac{(1 - \alpha) A}{\lambda} \left[\frac{K}{A(L - L_A)} \right]^\alpha \quad (39)$$

holds in (37) and (38). Inspection of the system (35)–(39) reveals that, as in the standard framework of Romer (1990), we need to impose additional structure on the preferences to ensure the existence of a balanced growth path (BGP). The BGP is defined as a stationary equilibrium in which the variables A , K , C , and Ω grow at the same constant rate

$$g^* = \left(\dot{A}/A \right)^* = \left(\dot{K}/K \right)^* = \left(\dot{C}/C \right)^* = \left(\dot{\Omega}/\Omega \right)^* > 0,$$

while the variables L_A , $K/[A(L - L_A)]$, C/K , and

$$\frac{C}{\Omega} = \frac{\frac{C}{K}}{\phi_K + \phi_Z \frac{1 - \alpha}{\lambda(L - L_A)} \left[\frac{K}{A(L - L_A)} \right]^{-(1-\alpha)}} \quad (40)$$

remain unchanged at their steady-state levels L_A^* , $\{K/[A(L - L_A)]\}^*$, $(C/K)^*$, and $(C/\Omega)^*$. A crucial element of the derivation of the differential equations (36)–(38) is the fact that the variables r , p , π , w/A , p_A , and Y/K can be expressed as functions of $K/[A(L - L_A)]$ and $(L - L_A)$. Hence, along the BGP we are looking for, these variables are also constant, while aggregate output, per capita output, and wages grow at rate g^* .

In the rest of the paper we restrict our attention to specifications of the instantaneous utility function $u(C, S)$ and the status function $S(\Omega, \bar{\Omega})$ such that the symmetric effective elasticity of intertemporal substitution under relative wealth preferences does not depend on C , i.e.,

$$\frac{\partial \sigma^S(C, \chi)}{\partial C} = 0, \quad (41)$$

and the symmetric marginal rate of substitution of status-relevant own wealth Ω for consumption C , $MRS(C, \Omega, \chi)$, can be expressed as a function of C/Ω . Since we are also interested in analytical solutions, we employ the stronger assumption that $MRS(C, \Omega, \chi)$

depends linearly on C/Ω such that

$$MRS = \eta \times \frac{C}{\Omega} \quad \eta > 0, \quad (42)$$

where η represents the intensity of the quest for status of the representative consumer. Equation (42) implies that the status-related extra returns ε^K and ε^Z are linear functions of C/Ω :

$$\varepsilon^K = \phi_K \eta \times \frac{C}{\Omega}, \quad \varepsilon^Z = \phi_Z \eta \times \frac{C}{\Omega}. \quad (43)$$

The structure imposed by (41) and (42) on $u(C, S)$ and $S(\Omega, \bar{\Omega})$ is not very restrictive. In Appendix B.3 we show that the instantaneous utility function with the general form

$$u(C, S) = V[g(C)h(S)], \quad S = S(\Omega, \bar{\Omega}),$$

where

$$V' > 0, \quad V'' < 0, \quad g(C) > 0, \quad g'(C) > 0, \quad h(S) > 0, \quad h'(S) > 0,$$

$$V'' [g(C)h(S)] [g'(C)h(S)]^2 + V' [g(C)h(S)] g''(C) h(S) < 0,$$

satisfies the properties (41) and (42) if the instantaneous utility function has the form

$$u(C, S) = \frac{1}{1-\theta} \left\{ [C^\xi h(S)]^{1-\theta} - 1 \right\}, \quad \xi > 0, \quad \theta > 0, \quad 1 + \xi(\theta - 1) > 0, \quad (44)$$

and the status function exhibits the ratio specification

$$S(\Omega, \bar{\Omega}) = \varphi(\Omega/\bar{\Omega}), \quad \varphi' > 0, \quad \varphi'' \leq 0. \quad (45)$$

These specifications of $u(C, S)$ and $S(\Omega, \bar{\Omega})$ imply that $\chi = S(\Omega, \Omega) = \varphi(1)$ and

$$\sigma^S = \frac{1}{1 + \xi(\theta - 1)} > 0, \quad (46)$$

$$MRS = \eta \times \frac{C}{\Omega}, \quad \eta \equiv \frac{\beta}{\xi} > 0, \quad \beta \equiv \frac{h'[\varphi(1)] \varphi'(1)}{h[\varphi(1)]} = \frac{h'(\chi) \varphi'(1)}{h(\chi)} > 0. \quad (47)$$

For $\xi = 1$ these results simplify to $\sigma^S = 1/\theta$ and $\eta = \beta$.¹¹

¹¹It is easily verified that if the instantaneous utility function u is additively separable, $u(C, S) = g(C) + h(S)$, then the properties (41) and (42) are satisfied if $g(C) = \xi \ln C$ and $S(\Omega, \bar{\Omega}) = \varphi(\Omega/\bar{\Omega})$. This specification of status preferences implies that $\sigma^S = 1$ and $\eta \equiv \beta/\xi$, where $\beta \equiv h'(\varphi(1)) \varphi'(1) = h'(\chi) \varphi'(1)$. Since it rules out that $\sigma^S \neq 1$, it is obviously less general than the specification given by (44) and (45).

Substituting (43) into (37) and (38) yields

$$\frac{\dot{C}}{C} = \sigma^S \left\{ \alpha^2 \left[\frac{K}{A(L-L_A)} \right]^{-(1-\alpha)} + \phi_K \eta \times \frac{C}{\Omega} - \rho \right\}, \quad (48)$$

$$\begin{aligned} \dot{L}_A = (L-L_A) \left\{ - (1-\alpha) \left[\frac{K}{A(L-L_A)} \right]^{-(1-\alpha)} + \frac{C}{K} + \lambda L_A \right. \\ \left. - \lambda (L-L_A) + \frac{(\phi_K - \phi_Z) \eta}{\alpha} \times \frac{C}{\Omega} \right\}, \end{aligned} \quad (49)$$

where C/Ω is given by (40).

In the following we analyze the system that consists of the differential equations (35), (36), (48), and (49), where C/Ω is given by (40). In order to determine the BGP, we replace \dot{A}/A , \dot{K}/K , and \dot{C}/C by the common growth rate g and set $\dot{L}_A = 0$. From the differential equation (35) and from $L_A + L_Y = L$, it follows that

$$L_A^* = \frac{g^*}{\lambda}, \quad L_Y^* = L - L_A^* = \frac{\lambda L - g^*}{\lambda}. \quad (50)$$

The production function for new blueprints and the labor market equilibrium condition imply that any rise in the common growth rate g^* requires a reallocation of labor from the final goods sector to the R&D sector, i.e., an increase in L_A^* and a corresponding decrease in L_Y^* .

The differential equation (36), which results from the equilibrium condition of the market for final goods, implies that

$$\left(\frac{Y}{K} \right)^* = \left[\left(\frac{A}{K} \right)^* (L - L_A^*) \right]^{1-\alpha} = g^* + \left(\frac{C}{K} \right)^*. \quad (51)$$

Substituting (50) and (51) into the differential equations (48) and (49) and taking into account Equation (40), we derive a two-dimensional system of equations in the two variables g^* and $(C/K)^*$ (see Appendix B.4.1):

$$- (1 - \alpha^2 \sigma^S) g^* + \sigma^S \left\{ \alpha^2 + \frac{\phi_K \eta}{\phi_K + \frac{\phi_Z (1-\alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K} \right)^* \right]} \right\} \left(\frac{C}{K} \right)^* = \sigma^S \rho, \quad (52)$$

$$(1 + \alpha) g^* + \frac{1}{\alpha} \left\{ \alpha^2 + \frac{(\phi_K - \phi_Z) \eta}{\phi_K + \frac{\phi_Z (1-\alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K} \right)^* \right]} \right\} \left(\frac{C}{K} \right)^* = \lambda L. \quad (53)$$

Equation (52) is a representation of the steady-state version of the Euler equation for

consumption,

$$g^* = \sigma^S \left[r^* + (\varepsilon^K)^* - \rho \right], \quad (54)$$

which is obtained by expressing the rental rate r^* and the status-related extra return of wealth accumulation in the form of real capital $(\varepsilon^K)^*$ as functions of g^* and $(C/K)^*$ and taking into account that $(\dot{C}/C)^* = g^*$. Technically speaking, Equation (53) yields combinations of g^* and $(C/K)^*$ that exhibit the property that $\dot{L}_A = 0$. For the economic interpretations in the following chapter it will be of crucial importance that Equation (53) is equivalent to the steady-state version of the no-arbitrage condition

$$r^* + (\varepsilon^K)^* = (\pi/p_A)^* + (\varepsilon^Z)^*, \quad (55)$$

which is obtained by expressing r^* , $(\varepsilon^K)^*$, the dividend yield $(\pi/p_A)^*$, and the status-related extra return of wealth accumulation in the form of shares $(\varepsilon^Z)^*$ as functions of g^* and $(C/K)^*$ and taking into account that $(\dot{p}_A/p_A)^* = 0$.¹²

In Appendix B.4.2 we show that the transversality conditions given by (18) are satisfied along the balanced growth path if

$$-\frac{1 - \sigma^S}{\sigma^S} g^* - \rho < 0 \quad (56)$$

holds. Taking into account that $g^* > 0$, it is obvious that $\sigma^S < 1$ is sufficient for (56) to hold. If $\sigma^S > 1$, then (56) yields an upper bound for the growth rate, $g^* < \sigma^S \rho / (\sigma^S - 1)$.

3 The long-run economic effects of the quest for status

In the following we analyze the dependence of the BGP on the status parameter η . This parameter is an important determinant of the status-related extra returns $\varepsilon^K = \phi_K \eta \times (C/\Omega)$ and $\varepsilon^Z = \phi_Z \eta \times (C/\Omega)$. More precisely, we consider modifications in the specification of status preferences that cause variations in η , but leave the effective elasticity of intertemporal substitution σ^S unchanged. In the context of the general CIES preferences (44) and (45) and the resulting expressions for σ^S and η as given by (46) and (47), respectively, this implies that we consider variations in β , while leaving ξ unchanged. This thought experiment should be interpreted as the comparison between the balanced growth paths of two economies that differ only with respect to the status parameter η (high- η -economy or more status conscious economy versus low- η -economy or less status conscious economy).

One of the main results of this paper is that the impact of η on the growth rate g^* depends crucially upon the values of ϕ_K and ϕ_Z , i.e., on the status relevance of physical capital and stocks. In the rest of the paper we normalize $\phi_K = 1$ and consider alternative

¹²The equivalence of (53) and (55) is not obvious at first glance. In Appendix B.2.4 we show that i) the differential equation for p_A given by (33) is the starting point for the derivation of the differential equation for L_A given by (49) and ii) Equation (49) is equivalent to $\dot{L}_A = (L - L_A) \left\{ 1/\alpha [r + \varepsilon^K - (\pi/p_A + \varepsilon^Z)] - \dot{K}/K + \dot{A}/A \right\}$. Setting $\dot{L}_A = 0$ and taking into account that $(\dot{K}/K)^* = (\dot{A}/A)^* = g^*$, we obtain (55).

values of $\phi_Z \leq \phi_K$, i.e., we make alternative assumptions with respect to the *relative* status relevance of shares.¹³

To start with, we study the two particularly interesting polar cases $\phi_Z = 1$ and $\phi_Z = 0$, respectively, for which explicit solutions can be easily calculated. Subsequently, we also provide an assessment of the general intermediate case $0 < \phi_Z < 1$ in which the derivation of explicit solutions is tedious and we confine ourselves to the use of implicit differentiation.

Before we start with the analysis, we want to stress a crucial feature of our framework that is important for the interpretation of our results but which is absent in the standard Romer (1990) model. From the steady-state versions of the Euler equation for consumption, $g^* = \sigma^S [r^* + (\varepsilon^K)^* - \rho]$, and the no-arbitrage relation, $r^* + (\varepsilon^K)^* = (\pi/p_A)^* + (\varepsilon^Z)^*$, it is obvious that a rise in the steady-state growth rate g^* requires an increase in both the effective rate of return on physical capital, $r^* + (\varepsilon^K)^*$, and the effective rate of return on shares, $(\pi/p_A)^* + (\varepsilon^Z)^*$. The most important aspect of the model with status preferences is that the *market* rates of return r^* and $(\pi/p_A)^*$ and the *effective* rates of return $r^* + (\varepsilon^K)^*$ and $(\pi/p_A)^* + (\varepsilon^Z)^*$ may move *in opposite directions* in response to variations in the status parameter η . The production function of the R&D sector, $\dot{A} = \lambda AL_A$, implies that $g^* = \lambda L_A^*$. Hence, a higher growth rate requires more workers in the R&D sector. In any (stationary and non-stationary) equilibrium, the following positive relation between the dividend yield and employment in the final goods sector holds (for a proof see Appendix B.2.4):

$$\pi/p_A = \alpha \lambda L_Y = \alpha \lambda (L - L_A). \quad (57)$$

It follows that $(\pi/p_A)^* = \alpha (\lambda L - g^*)$, i.e., for given values of α , λ , and L , there is an inverse relation between the steady-state value of the dividend yield $(\pi/p_A)^*$ and the steady-state value of the common growth rate g^* . This result, together with the Euler equation for consumption and the no-arbitrage relation, implies the following: A stronger quest for status (higher η) can only be associated with a higher common growth rate g^* if there is an increase in the status-related extra return of shares $(\varepsilon^Z)^*$ that is only *partially* compensated by a decrease in the dividend yield $(\pi/p_A)^*$. In this case both effective rates of return (on physical capital and on shares) are higher and so is the growth rate.

First, we analyze the special case $\phi_K = \phi_Z = 1$ in which the status-relevant measure of wealth Ω equals the standard definition of wealth. In this case physical capital and shares are equally relevant for status such that status-related extra returns coincide at any time t . From the steady-state version of the no-arbitrage equation (55) it then follows that, along the BGP, the market rates of return of physical capital and shares are identical too, i.e., $r^* = (\pi/p_A)^*$. Altogether, we are able to state the following proposition for this case.

Proposition 1. *If $\phi_Z = \phi_K = 1$, the balanced growth path (BGP) exhibits the following properties:*

¹³As mentioned in the introduction, the restriction $\phi_Z \leq \phi_K$ is realistic, while $\phi_Z > \phi_K$ is very difficult to conceive because it would imply that intangible assets deliver more status gratification than tangible assets.

i) The solutions for g^* and $(C/K)^*$ are given by

$$g^* = \frac{\sigma^S [(\alpha + \eta) \lambda L - \rho]}{1 + \sigma^S [\alpha + \eta (1 + \alpha)]},$$

$$\left(\frac{C}{K}\right)^* = \frac{\lambda L - (1 + \alpha) g^*}{\alpha} = \frac{(1 - \alpha^2 \sigma^S) \lambda L + (1 + \alpha) \sigma^S \rho}{\alpha \{1 + \sigma^S [\alpha + \eta (1 + \alpha)]\}}.$$

The solution for $(C/K)^*$ is economically sensible if

$$\left(\frac{C}{K}\right)^* > 0 \Leftrightarrow g^* < \frac{\lambda L}{1 + \alpha} \Leftrightarrow (1 - \alpha^2 \sigma^S) \lambda L + (1 + \alpha) \sigma^S \rho > 0. \quad (58)$$

The steady-state growth rate g^* is strictly positive if and only if the representative household is sufficiently patient in the sense that

$$\rho < (\alpha + \eta) \lambda L. \quad (59)$$

ii) If (58) holds, then g^* depends positively on the status parameter η

$$\frac{\partial g^*}{\partial \eta} = \frac{\alpha \sigma^S}{1 + [\alpha + \eta (1 + \alpha)] \sigma^S} \left(\frac{C}{K}\right)^* > 0,$$

while the rest of the endogenous variables exhibit the following dependence on η :

$$\frac{\partial v^*}{\partial \eta} < 0, \text{ for } v = \frac{C}{K}, \frac{C}{Y}, \frac{C}{\Omega}, L_Y, \frac{Y}{K}, r, p, \frac{\pi}{p_A},$$

$$\frac{\partial v^*}{\partial \eta} > 0, \text{ for } v = L_A, \frac{x}{L_Y}, x, \varepsilon^K, \varepsilon^Z, r + \varepsilon^K, \frac{\pi}{p_A} + \varepsilon^Z, p_A, \frac{w}{A},$$

$$\text{sgn}\left(\frac{\partial \pi^*}{\partial \eta}\right) = \text{sgn}(2\alpha - 1).$$

iii) The composition of wealth does not depend on the status parameter η :

$$\left(\frac{K}{K + p_A A}\right)^* = \alpha, \quad \left(\frac{p_A A}{K + p_A A}\right)^* = 1 - \alpha.$$

Proof. See Appendix D.2.

In the following we provide the economic interpretation of the results described in this proposition. If $\phi_Z = \phi_K = 1$, any economically sensible BGP exhibits the property that the common growth rate g^* (of consumption, physical capital, the number of shares, the mass of intermediate goods, output of final goods, the representative household's wealth, and of real wages) depends positively on the status parameter η . A crucial feature of the case $\phi_Z = \phi_K = 1$ is that the identical effective rates of return $r^* + (\varepsilon^K)^*$ and

$(\pi/p_A)^* + (\varepsilon^Z)^*$ also depend positively on the status parameter η . As mentioned above, this results from the fact that a rise in η leads to an increase in the identical status-related components $(\varepsilon^K)^*$ and $(\varepsilon^Z)^*$ that is only partially offset by the fall in the identical market rates of return, r^* and $(\pi/p_A)^*$. According to the Euler equation for consumption, the rise in the effective rate of return $r^* + (\varepsilon^K)^*$ implies an increase in the growth rate of private consumption $(\dot{C}/C)^* = g^*$. In other words, the greater η , the steeper the consumption path chosen by the representative household, i.e., the higher the willingness to substitute future consumption for current consumption. The resulting changes in the saving behavior imply that the growth rate of the representative household's wealth increases. In contrast to its growth rate, the *composition* of wealth does not depend on η because α percent is held in the form of physical capital, while $1 - \alpha$ percent is held in the form of stocks.

Since the physical capital holdings of the households grow at a higher rate, the capital input in the intermediate goods sector also has to grow at a higher rate. In the high- η -economy, firms in the intermediate goods sector face a lower rental rate of physical capital r^* , i.e., a lower marginal cost, and hence they will charge a lower price for their products as determined by $p^* = (1/\alpha) \times r^*$. The lower price of intermediate goods p^* induces the representative firm of the final goods sector to produce with a higher intensity of intermediate goods. This increase in $(x/L_Y)^*$ originates from both an increase in the common input of each existing variety of intermediate goods x^* and a fall in labor input L_Y^* . The latter effect allows for the sectoral reallocation of labor from final goods production to R&D that is necessary to achieve a faster growth rate of patents.

As explained above, the number of shares that are held by individuals and issued by the firms to finance the purchase of blueprints grows faster in the high- η -economy. The price of shares and therefore also the price of blueprints depends positively on η , while the prices do not grow along the BGP. The reason for the level effect is the following. Since the status-related extra returns of both savings vehicles are equal in case of $\phi_K = \phi_Z = 1$, the dividends financed by operating profits are discounted with the rental rate of physical capital r^* . The rise in η implies a fall of the rental rate of physical capital, which guarantees a rise in the net present value of profits and therefore an increase in the price of shares and blueprints, irrespective of the fact that the dependence of operating profits π^* upon η is ambiguous.¹⁴ New entrants in the intermediate goods sector have to pay a higher price for the blueprints in the high- η -economy. However, since the effective rate of return on shares is higher in the high- η -economy, its inhabitants are more willing to acquire the associated issues of shares, in spite of the lower dividend yield.

Now we show why the R&D sector has to charge a higher price for the blueprints in the high- η economy. The rise in the intermediate goods intensity in the final goods sector implies an increase in the marginal product of labor and, hence, a rise in the ratio of the real wage to the mass of varieties $(w/A)^*$. Since the technology of the R&D sector is linear and we have perfect competition, the equilibrium has to be characterized by $p_A^* = \lambda^{-1} (w/A)^*$ and profits in the R&D sector are zero. In the high- η -economy, each

¹⁴Recall that an increase in η reduces p^* and raises x^* . Altogether we have: $\text{sgn}(\partial\pi^*/\partial\eta) = \text{sgn}(2\alpha - 1)$.

value of A is associated with a higher real wage. Moreover, the real wage also grows at a higher rate.

Finally, we show why the results of the proposition are consistent with the market clearing condition of the final goods sector, $Y = C + \dot{K}$, and the implied relation

$$g^* = (\dot{K}/K)^* = (Y/K)^* - (C/K)^* = [1 - (C/Y)^*](Y/K)^*.$$

An increase in η raises the growth rate of physical capital because the reduction of $(Y/K)^*$ is smaller than the reduction of $(C/K)^*$. In other words, the rise in the economy-wide savings rate $[1 - (C/Y)^*]$ is only partially compensated by the fall in $(Y/K)^*$. The reason for the fall of $(Y/K)^*$ is in turn given by the increase in the intermediate goods intensity of the final goods sector and the associated fall in the average product of the aggregate input of intermediate goods $[Y/(Ax)]^* = (Y/K)^*$.

Next we turn our attention to the special case $\phi_K = 1$ and $\phi_Z = 0$ in which wealth held in the form of shares is irrelevant for status, $\varepsilon^Z = 0$. The steady-state version of the no-arbitrage equation simplifies to $r^* + (\varepsilon^K)^* = (\pi/p_A)^*$. Hence, along the BGP, the rental rate of physical capital is less than the dividend yield, i.e., $r^* < (\pi/p_A)^*$. Moreover, to calculate the fundamental price of shares, future dividend payments are discounted by using the effective rate of return on physical capital, $r^* + (\varepsilon^K)^*$. Altogether, we are able to state the following proposition for this case.

Proposition 2. *If $\phi_Z = 0$ and $\phi_K = 1$, the balanced growth path (BGP) exhibits the following properties:*

i) The solutions for g^ and $(C/K)^*$ are given by*

$$g^* = \frac{\sigma^S (\alpha \lambda L - \rho)}{1 + \alpha \sigma^S},$$

$$\left(\frac{C}{K}\right)^* = \frac{\alpha [\lambda L - (1 + \alpha) g^*]}{\alpha^2 + \eta} = \frac{\alpha [(1 - \alpha^2 \sigma^S) \lambda L + (1 + \alpha) \sigma^S \rho]}{(1 + \alpha \sigma^S) (\alpha^2 + \eta)}.$$

The solution for $(C/K)^$ is economically sensible if*

$$\left(\frac{C}{K}\right)^* > 0 \Leftrightarrow g^* < \frac{\lambda L}{1 + \alpha} \Leftrightarrow (1 - \alpha^2 \sigma^S) \lambda L + (1 + \alpha) \sigma^S \rho > 0. \quad (60)$$

The steady-state growth rate g^ is strictly positive if and only if the representative household is sufficiently patient in the sense that*

$$\rho < \alpha \lambda L. \quad (61)$$

ii) The growth rate g^* is independent of the status parameter η ,

$$\frac{\partial g^*}{\partial \eta} = 0$$

and this independence is also true for the following variables:

$$\frac{\partial v^*}{\partial \eta} = 0, \quad \text{for } v = L_A, L_Y, r + \varepsilon^K, \frac{\pi}{p_A}, \varepsilon^Z.$$

The rest of the endogenous variables exhibit the following dependence on η :

$$\frac{\partial v^*}{\partial \eta} < 0, \quad \text{for } v = \frac{C}{K}, \frac{C}{\Omega}, \frac{C}{Y}, \frac{Y}{K}, r, p,$$

$$\frac{\partial v^*}{\partial \eta} > 0, \quad \text{for } v = \frac{x}{L_Y}, x, \pi, \varepsilon^K, p_A, \frac{w}{A}.$$

iii) The composition of wealth exhibits the following properties:

$$\frac{\partial [K/(K + p_A A)]^*}{\partial \eta} > 0, \quad \left(\frac{K}{K + p_A A} \right)^* \Big|_{\eta=0} = \alpha, \quad \lim_{\eta \rightarrow \infty} \left(\frac{K}{K + p_A A} \right)^* < 1.$$

Proof. See Appendix D.3.

The striking feature of the case $\phi_Z = 0$ and $\phi_K = 1$ is that the growth rate g^* is independent of the status parameter η . The technology used in the research sector and labor market clearing imply that employment in the R&D sector and in the final goods sector, L_A^* and L_Y^* , respectively, are also independent of η . A rise in η leads to an increase in the status-related component $(\varepsilon^K)^*$ that is perfectly offset by the fall in the rental rate r^* so that the effective rate of return of physical capital remains unchanged. From the Euler equation for consumption it follows that the growth rate of private consumption $(\dot{C}/C)^* = g^*$ remains unchanged, too. In other words, along the BGP, the willingness to substitute future consumption for current consumption is independent of the status parameter η . The no-arbitrage equation $r^* + (\varepsilon^K)^* = (\pi/p_A)^*$ implies that the dividend yield $(\pi/p_A)^*$ is also independent of η . Note that a rise in η leaves the growth rate of the representative household's wealth and the growth rate of its components unchanged, but alters the *composition* of wealth in favor of physical capital.¹⁵

In the high- η -economy, firms in the intermediate goods sector are confronted with a lower rental rate of physical capital (similar as in the case $\phi_Z = 1$). The resulting lower price of intermediate goods induces the representative firm of the final goods sector to choose a higher intermediate goods intensity. This increase of $(x/L_Y)^*$ originates from an increase of x^* , i.e., employment of each existing variety increases. In contrast to the case of $\phi_Z = 1$, employment in final goods production remains unchanged. For the aggregate

¹⁵In Appendix D.4 we provide an alternative proof (proof by contradiction) for the assertion that $\phi_Z = 0 \Rightarrow \partial g^*/\partial \eta = 0$ that does not rely on the explicit solution for g^* .

physical capital input $K = Ak^* = Ax^*$ we have that each value of A is associated with a higher value of K in the high- η -economy but that the growth rate of physical capital is the same as in the low- η -economy.

In contrast to the case $\phi_Z = 1$, operating profits of an intermediate goods producing firm, $\pi^* = (1 - \alpha)p^*x^*$, depend positively on η because the percentage change of x^* overcompensates the percentage change of p^* . For the fundamental price of shares we also have an unambiguous result: The assumptions $\phi_K = 1$ and $\phi_Z = 0$ imply that the stream of dividend payments is not discounted with the rental rate of capital r^* but with the effective rate of return $r^* + (\varepsilon^K)^*$. Since $\partial[r^* + (\varepsilon^K)^*]/\partial\eta = 0$, the higher dividends are discounted at the same rate such that the fundamental value of shares increases. New entrants into the intermediate goods sector are therefore able to pay for the patents by issuing more expensive shares. In contrast to the case of $\phi_Z = \phi_K = 1$, the number of shares that are held by the individuals grow with the same rate in the high- η -economy and in the low- η economy.

Again we have argued that the price for patents that new entrants in the intermediate goods sector pay depends positively on η . Analogous to the previous case, firms in the R&D sector in the high- η -economy have to charge higher prices because they are confronted with higher real wages. Regarding the growth rate of real wages, g^* , there is, however, no difference between the high- η -economy and the low- η -economy. Analogous to the case $\phi_Z = 1$ we have that the ratio of the real wage to the mass of varieties $(w/A)^*$ increases because the intermediate goods intensity in the final goods sector $(x/L_Y)^*$ rises and the corresponding increase in the marginal product of labor implies a higher economy-wide wage.

From the market clearing condition of the final goods market, $Y = C + \dot{K}$, and the associated condition

$$g^* = (\dot{K}/K)^* = (Y/K)^* - (C/K)^* = [1 - (C/Y)^*](Y/K)^*,$$

we get the following additional information: An increase of η does not affect the growth rate of physical capital because $(Y/K)^*$ and $(C/K)^*$ fall by the same amount. In other words, the increase in the economy-wide savings rate $[1 - (C/Y)^*]$ is fully compensated by a fall of $(Y/K)^*$. The reduction of $(Y/K)^*$ can be derived in analogy to the case $\phi_Z = 1$ because of the rise of the intermediate goods intensity in the final goods sector.

Next, we investigate what happens for $0 < \phi_Z < 1$ and $\phi_K = 1$.

Proposition 3. *If $0 < \phi_Z < 1$ and $\phi_K = 1$, the balanced growth path (BGP) exhibits the following properties:*

- i) *The stronger the quest for status, i.e., the higher the status parameter η , the higher the growth rate g^* :*

$$\frac{\partial g^*}{\partial \eta} > 0.$$

ii) The growth rate g^* depends positively on the relative importance of shares in the status-relevant measure of own wealth given by $\Omega = \phi_K K + \phi_Z p_Z Z$:

$$\frac{\partial g^*}{\partial \phi_Z} > 0.$$

Moreover, the following endogenous variables exhibit an unambiguous dependence on ϕ_Z , too:

$$\begin{aligned} \frac{\partial v^*}{\partial \phi_Z} > 0, & \quad \text{for } v = \frac{C}{K}, \frac{Y}{K}, p, r, L_A, r + \varepsilon^K, \frac{\pi}{p_A} + \varepsilon^Z, \varepsilon^Z, \frac{p_A A}{K + p_A A}, \\ \frac{\partial v^*}{\partial \phi_Z} < 0, & \quad \text{for } v = \frac{x}{L_Y}, L_Y, x, \pi, \frac{w}{A}, p_Z, p_A, \frac{\pi}{p_A}, \frac{K}{K + p_A A}. \end{aligned}$$

Proof. See Appendix D.5.

In Proposition 1 we showed that the growth rate g^* depends positively on the status parameter η if $\phi_Z = \phi_K = 1$ holds. Part i) of Proposition 3 shows that this property carries over to the case in which $0 < \phi_Z < 1$ and $\phi_K = 1$ holds. Since the analysis of the other endogenous variables does not lead to significant new insights, we did not include them in Proposition 3.

We now turn to the effects of ϕ_Z on the BGP. A *ceteris paribus* rise in ϕ_Z implies that private households re-adjust their portfolio by shifting wealth from physical capital holdings toward shares. The market rate of return on physical capital, r^* , rises, while the market rate of return on shares, $(\pi/p_A)^*$, falls. However, the rise in the status-related extra return of shares, $(\varepsilon^Z)^* = \phi_Z \eta \times (C/\Omega)^*$, more than offsets the fall in the dividend yield so that the *effective* rate of return on shares, $(\pi/p_A)^* + (\varepsilon^Z)^*$, depends positively on ϕ_Z . The no-arbitrage condition implies that the effective rate of return on physical capital also depends positively on ϕ_Z . According to the Euler equation for consumption, the rise in the effective rate of return implies an increase in the growth rate of private consumption, $(\dot{C}/C)^* = g^*$. In other words, the consumption path chosen by the representative household becomes steeper. The common growth rate of total wealth and its components, physical capital and shares, increases. Hence, while the proportion of physical capital decreases, its growth rate, $(\dot{K}/K)^* = g^*$, increases.

In the high- ϕ_Z -economy firms in the intermediate goods sector face a higher rental rate of physical capital, r^* , and hence they will charge a higher price for their products according to the mark-up pricing rule, $p^* = (1/\alpha) \times r^*$. The higher value of p^* causes the representative firm of the final goods sector to choose a lower intensity of intermediate goods, $(x/L_Y)^*$. Since the high- ϕ_Z -economy exhibits a higher common growth rate, g^* , employment in the R&D sector, L_A^* , is also higher, while the opposite is true for employment in the final goods sector, L_Y^* . Since both $(x/L_Y)^*$ and L_Y^* depend negatively on ϕ_Z , the identical input of the different varieties x^* depends negatively on ϕ_Z , too. To put it differently: Production and employment of each variety is lower but the stock of varieties

grows at a faster rate, $(\dot{A}/A)^* = g^*$. In the high- ϕ_Z -economy, firms in the intermediate goods sector face lower profits, $\pi^* = (1 - \alpha)p^*x^*$, because the increase in the price p^* is overcompensated by a decrease in the number of units sold, x^* . The fall in x^* implies that each firm in the intermediate goods sector uses less physical capital, $k^* = x^*$. However, since the stock of varieties grows faster, the aggregate stock of physical capital grows at a higher rate, too.

In the high- ϕ_Z -economy, the lower ratio of intermediate goods to labor, $(x/L_Y)^*$, implies a lower marginal product of labor. Consequently, the ratio of the real wage to the mass of varieties, $(w/A)^*$, is also lower, while, by contrast, the growth rate of the real wage, $(\dot{w}/w)^* = g^*$, is higher. The lower level of $(w/A)^*$ together with the linear technology in the research sector and perfect competition imply that the price of blueprints, p_A^* , is lower. Since it holds that $p_Z = p_A$ in equilibrium, shares are also cheaper. An increase in ϕ_Z leads to a decrease of the dividend yield, $(\pi/p_A)^*$, because π^* decreases by a higher percentage value than p_A^* . As explained above, the rise in ϕ_Z implies that the composition of the household's portfolio shifts in favor of shares. Furthermore, the growth rate of the stock of shares, $(\dot{Z}/Z)^* = (\dot{A}/A)^* = g^*$, rises.

Finally, inspired by Corneo and Jeanne (1997, 2001a) we end our detailed analysis with a remark on the social optimality of the decentralized long-run growth rate. The standard Romer (1990) model exhibits the well-known property that the decentralized long-run economic growth rate is less than its socially optimal counterpart due to several distortions. In our paper the quest for status acts so as to increase the decentralized long-run economic growth rate provided that shares are status relevant, i.e., $\phi_Z > 0$. However, we can show that the externality resulting from relative wealth preferences counterbalances the influence of the other distortions on the long-run growth rate *only partially* as long as our assumption that $0 < \phi_Z \leq \phi_K = 1$ holds. We can further show that there exists a pair of the exogenously given status parameters $(\tilde{\eta}, \tilde{\phi}_Z)$, where $\tilde{\phi}_Z > 1 = \phi_K$ such that the decentralized long-run growth rate equals its socially optimal counterpart in the absence of any government intervention. Hence, perfect replication of the socially optimal growth rate requires that, in contrast to our more realistic assumption, the status relevance of shares exceeds that of physical capital.¹⁶

4 Conclusions

In this paper we introduced status preferences into an R&D-based economic growth model with three sectors of production (final goods, intermediate goods, and blueprints) to analyze the impact of status concerns on technological progress and on long-run economic growth. We assumed that the instantaneous utility of the representative household depends not only on its consumption, but also on its relative wealth consisting of tangible assets in the form of physical capital and on intangible assets in the form of shares. In contrast to the standard relative wealth approach used in the status literature, we allow for

¹⁶The details are beyond the scope of this paper; they are provided upon request by the authors.

the possibility that these two components of the representative household's wealth differ with respect to their status relevance. The introduction of the generalized relative wealth preferences implies that the effective rate of return of saving in the form of a particular asset is the sum of its standard market rate of return and its status-related extra return. In both the Euler equation for consumption growth and in the no-arbitrage condition, the rental rate of physical capital and the market rate of return of shares are replaced by the corresponding effective rates of return.

First, we analyzed the effects of an increase in the intensity of the quest for status, i.e., a rise in the marginal rate of substitution (MRS) of status-relevant own wealth for consumption. Along the balanced growth path (BGP), variables such as the mass of intermediate goods, the aggregate input of physical capital, the mass of shares and the market capitalization, the representative household's wealth, aggregate output, private consumption, and real wages grow at the same rate. A rise in the MRS of status-relevant own wealth for consumption affects the economy by raising the extra returns of all assets that are status relevant. As long as these impact effects are not perfectly offset by a decrease in the market rates of return, the common steady-state effective rate of return of all assets rises. The resulting stronger incentive to save causes the demand for shares and hence for blueprints to grow at a higher rate, which fosters technological progress. According to the underlying production technology in the R&D sector, the acceleration of technological progress is ultimately due to an increase in the employment of scientists. Altogether, these effects induce the common long-run growth rate to rise.

One of our main results is that the effects of an increase in the intensity of the quest for status on the common growth rate depend crucially upon the relative status relevance of shares. We started with two special cases in which explicit solutions for all variables can be easily calculated: i) if physical capital and shares are equally status relevant, then the status-related extra returns of these two assets are identical. A rise in the intensity of the quest for status causes the common growth rate to rise unambiguously. This result is due to the fact that the rise in the status-related extra return of physical capital and shares is only partially compensated by the decrease in the rental rate and the dividend yield so that the common effective rate of return of the two assets increases; ii) if wealth held in the form of shares is irrelevant for status, then the status-related extra return of shares equals zero. A rise in the MRS of status-relevant own wealth for consumption causes the extra return of physical capital to increase. But since this rise is perfectly offset by a fall in the rental rate of capital, the effective rate of return of real capital and, hence, the common growth rate remain unchanged. While the growth rate of the representative household's wealth remains unchanged, its composition is altered in favor of physical capital. Finally, we considered the case in which wealth held in the form of shares is relevant for status, but to a smaller extent than physical capital. In this (realistic) intermediate case an increase in the intensity of the quest for status causes the common growth rate to rise similar to the special case of equal status relevance.

Second, we kept the intensity of the quest for status constant and analyzed the impli-

cations of an increase in the relative status relevance of shares. Private households adjust their portfolio by shifting wealth from physical capital holdings to shares. The rental rate of capital rises, while the dividend yield falls. However, this fall in the dividend yield is more than offset by the rise in the status-related extra return of shares so that the effective rate of return of shares rises. The no-arbitrage condition implies that the effective rate of return of physical capital rises, too. Consequently, the common growth rate along the BGP increases. The consumption path chosen by the representative household becomes steeper. While the proportion of shares increases, not only shares, but also physical capital are accumulated at a higher rate.

The findings of Levine and Zervos (1998) and Levine (2005) show that the initial stock market liquidity of a country is a robust predictor for subsequent economic growth in cross-country growth regressions. Our model with relative wealth preferences shares the importance of the stock market: the wealth proportion of stocks and the long-run economic growth rate depend positively on the relative status relevance of shares. The theoretical model is therefore consistent with the empirical evidence.

A final interesting feature of our framework is that, while the standard R&D-based economic growth model of Romer (1990) exhibits the property that the decentralized long-run growth rate is unambiguously smaller than its socially optimal counterpart, the externality resulting from relative wealth preferences counterbalances *to some extent* the influence of the other distortions that reduce savings incentives provided that shares matter for status: However, as long as the status relevance of shares does not significantly exceed that of physical capital, neither the perfect replication of the socially optimal growth rate nor excessive growth can occur.

With respect to further research we would like to mention three promising avenues: First, from a public economics point of view it would be interesting to analyze how the socially optimal taxation/subsidization is influenced by the introduction of relative wealth preferences and the possibility that the various assets differ with respect to their relative status relevance. Second, one could abandon the representative agent framework in order to allow for the heterogeneity of households. This could yield useful insights when analyzing the driving forces behind wealth disparities and assessing the conditions and policies under which the poor do not fall too far behind the rich. Third, in a thorough econometric analysis it could be tested whether the theoretical mechanism that we identified is indeed the relevant channel that explains the empirical results of Levine and Zervos (1998) and Levine (2005).

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Appendix

A The representative household's optimization problem

In the following we will analyze the optimal behavior of the representative household by employing *optimal control theory*. For this reason we dismantle the flow budget constraint (1), $\dot{K} + p_Z \dot{Z} = rK + DZ + wL - C$, into two differential equations for the state variables K and Z :

$$\dot{K} = rK + wL + DZ - p_Z Q - C, \quad (\text{A.1})$$

$$\dot{Z} = Q. \quad (\text{A.2})$$

The representative individual chooses time paths for C and Q so as to maximize lifetime utility given by

$$\int_0^{\infty} e^{-\rho t} u [C, S(\Omega, \bar{\Omega})] dt,$$

where

$$\Omega \equiv \phi_K K + \phi_Z p_Z Z \quad \text{and} \quad \bar{\Omega} \equiv \phi_K \bar{K} + \phi_Z p_Z \bar{Z}, \quad (\text{A.3})$$

subject to the differential equations (A.1) and (A.2) and the two initial conditions

$$K(0) = K_0, \quad Z(0) = Z_0, \quad (\text{A.4})$$

where K_0 and Z_0 are exogenously given. Since, by assumption, there is a continuum of homogeneous households, the representative household takes not only the time paths of r , w , p_Z , and D , but also the time paths of \bar{K} and \bar{Z} as given. Since individuals are identical in all respects, households neither lend to (nor borrow from) other households in equilibrium. For this reason we can ignore the no-Ponzi game condition.

The current value Hamiltonian is given by

$$\begin{aligned} H = & u [C, S(\phi_K K + \phi_Z p_Z Z, \phi_K \bar{K} + \phi_Z p_Z \bar{Z})] \\ & + \mu_K (rK + wL + DZ - p_Z Q - C) + \mu_Z Q, \end{aligned}$$

where the costate variables μ_K and μ_Z denote the shadow price of physical capital and shares, respectively. The necessary optimality conditions for an interior optimum, $\partial H / \partial C = 0$, $\partial H / \partial Q = 0$, $\dot{\mu}_K = \rho \mu_K - (\partial H / \partial K)$, and $\dot{\mu}_Z = \rho \mu_Z - (\partial H / \partial Z)$ can be written as

$$\mu_K = \frac{\partial u(C, S(\Omega, \bar{\Omega}))}{\partial C}, \quad (\text{A.5})$$

$$\mu_Z = \mu_K p_Z, \quad (\text{A.6})$$

$$\dot{\mu}_K = \rho\mu_K - \left[\frac{\partial u(C, S(\Omega, \bar{\Omega}))}{\partial S} \frac{\partial S(\Omega, \bar{\Omega})}{\partial \Omega} \phi_K + \mu_K r \right], \quad (\text{A.7})$$

$$\dot{\mu}_Z = \rho\mu_Z - \left[\frac{\partial u(C, S(\Omega, \bar{\Omega}))}{\partial S} \frac{\partial S(\Omega, \bar{\Omega})}{\partial \Omega} \phi_Z p_Z + \mu_K D \right], \quad (\text{A.8})$$

where Ω and $\bar{\Omega}$ are given by (A.3).

The transversality conditions are given by

$$\lim_{t \rightarrow \infty} e^{-\rho t} \mu_K K = 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} e^{-\rho t} \mu_Z Z = 0. \quad (\text{A.9})$$

Our assumptions imply that the Hamiltonian is jointly concave in C , Q , K , and Z . Hence, the transversality conditions ensure that the necessary optimality conditions are also sufficient.

Using (A.5) and (A.7) we obtain

$$\frac{\dot{\mu}_K}{\mu_K} = - \left[r + \frac{\frac{\partial U(C, S(\Omega, \bar{\Omega}))}{\partial S} \frac{\partial S(\Omega, \bar{\Omega})}{\partial \Omega} \phi_K}{\frac{\partial u(C, S(\Omega, \bar{\Omega}))}{\partial C}} - \rho \right]. \quad (\text{A.10})$$

From (A.6) it follows that

$$\dot{\mu}_Z = \dot{\mu}_K p_Z + \mu_K \dot{p}_Z. \quad (\text{A.11})$$

Substituting (A.5), (A.6), and (A.11) into (A.8) we can show that

$$\frac{\dot{\mu}_K}{\mu_K} = - \left[\frac{\dot{p}_Z}{p_Z} + \frac{D}{p_Z} + \frac{\frac{\partial u(C, S(\Omega, \bar{\Omega}))}{\partial S} \frac{\partial S(\Omega, \bar{\Omega})}{\partial \Omega} \phi_Z}{\frac{\partial u(C, S(\Omega, \bar{\Omega}))}{\partial C}} - \rho \right]. \quad (\text{A.12})$$

Equations (A.10) and (A.12) yield two alternative representations of $\dot{\mu}_K/\mu_K$. The required equality of the right-hand sides of (A.10) and (A.12) yields the no-arbitrage relation of the economy with relative wealth preferences:

$$r + \frac{\frac{\partial U(C, S(\Omega, \bar{\Omega}))}{\partial S} \frac{\partial S(\Omega, \bar{\Omega})}{\partial \Omega} \phi_K}{\frac{\partial u(C, S(\Omega, \bar{\Omega}))}{\partial C}} = \frac{\dot{p}_Z}{p_Z} + \frac{D}{p_Z} + \frac{\frac{\partial u(C, S(\Omega, \bar{\Omega}))}{\partial S} \frac{\partial S(\Omega, \bar{\Omega})}{\partial \Omega} \phi_Z}{\frac{\partial u(C, S(\Omega, \bar{\Omega}))}{\partial C}}. \quad (\text{A.13})$$

The expression on the left-hand side of (A.13) is the *effective* rate of return of wealth accumulation in the form of physical capital, where r is the market rental rate, while the second term is the *status-related extra return* of physical capital. The right-hand side gives the *effective* rate of return of wealth accumulation in the form of stocks, where the market return $(\dot{p}_Z/p_Z) + (D/p_Z)$ results from valuation gains and dividend payments, while the third term is the *status-related extra return* of stocks.

In the status literature it is common practice to restrict attention to symmetric equilibria in which agents that are identical in every respect make identical choices. In any symmetric situation, $\Omega = \bar{\Omega}$ holds. Moreover, due to assumption (7), we also have $S(\Omega, \Omega) = \chi$ for all $\Omega > 0$, where χ is an exogenously given constant. Equations (A.5), (A.10), and (A.13) simplify to

$$\mu_K = \frac{\partial u(C, \chi)}{\partial C}, \quad (\text{A.14})$$

$$\frac{\dot{\mu}_K}{\mu_K} = -[r + \varepsilon^K(C, \Omega, \chi, \phi_K) - \rho], \quad (\text{A.15})$$

$$r + \varepsilon^K(C, \Omega, \chi, \phi_K) = \frac{D}{p_Z} + \frac{\dot{p}_Z}{p_Z} + \varepsilon^Z(C, \Omega, \chi, \phi_Z), \quad (\text{A.16})$$

where

$$\varepsilon^K(C, \Omega, \chi, \phi_K) \equiv MRS(C, \Omega, \chi) \times \phi_K, \quad (\text{A.17})$$

$$\varepsilon^Z(C, \Omega, \chi, \phi_Z) \equiv MRS(C, \Omega, \chi) \times \phi_Z, \quad (\text{A.18})$$

$$MRS(C, \Omega, \chi) \equiv \frac{\partial u(C, \chi)}{\partial S} \frac{\partial S(\Omega, \Omega)}{\partial \Omega} \left[\frac{\partial u(C, \chi)}{\partial C} \right]^{-1}. \quad (\text{A.19})$$

Note that (A.16) is equal to the no-arbitrage relation (11) as given in the main text, while the definitions (A.17)-(A.19) coincide with the definitions ε^K , ε^Z , and MRS [see Equations (12)-(14)]. For an economic interpretation, we refer to reader to the main text.

From (A.14) it follows that

$$\frac{\dot{\mu}_K}{\mu_K} = C \frac{\partial^2 u(C, \chi)}{\partial C^2} \left[\frac{\partial u(C, \chi)}{\partial C} \right]^{-1} \times \frac{\dot{C}}{C}. \quad (\text{A.20})$$

Using (A.15) and (A.20), we obtain the Euler equation for consumption of a decentralized economy populated by households with relative wealth preferences:

$$\frac{\dot{C}}{C} = \sigma^S(C, \chi) [r + \varepsilon^K(C, \Omega, \chi, \phi_K) - \rho], \quad (\text{A.21})$$

where

$$\sigma^S(C, \chi) \equiv -\frac{\partial u(C, \chi)}{\partial C} \left[C \frac{\partial^2 u(C, \chi)}{\partial C^2} \right]^{-1}. \quad (\text{A.22})$$

Note that (A.21) and (A.22) are equivalent to (16) and (17) as given in the main text.

Using (A.6), the transversality conditions (A.9) can be written as

$$\lim_{t \rightarrow \infty} e^{-\rho t} \mu_K K = 0, \quad \lim_{t \rightarrow \infty} e^{-\rho t} \mu_K p_Z Z = 0. \quad (\text{A.23})$$

Integration of (A.15) yields

$$\mu_K(t) = \mu_K(0) e^{\rho t} \exp \left[- \int_0^t [r(v) + \varepsilon^K(C(v), \Omega(v), \chi, \phi_K)] dv \right].$$

The assumption that $\partial u(C, \chi) / \partial C > 0$ together with (A.14) implies that $\mu_K(t) > 0$ for $t \geq 0$. Hence, the transversality conditions (A.23) are equivalent to

$$\begin{aligned} \lim_{t \rightarrow \infty} \left\{ \exp \left[- \int_0^t [r(v) + \varepsilon^K(v)] dv \right] K \right\} &= 0, \\ \lim_{t \rightarrow \infty} \left\{ \exp \left[- \int_0^t [r(v) + \varepsilon^K(v)] dv \right] p_Z Z \right\} &= 0, \end{aligned}$$

where $\varepsilon^K(v) = \varepsilon^K[C(v), \Omega(v), \chi, \phi_K]$. Note that these conditions are identical to the conditions (18) as given in the main text.

B The decentralized equilibrium

B.1 Equilibrium in the market for final goods

From the flow budget constraint of the representative household (1) it follows that

$$\dot{K} = rK + wL + DZ - C - p_Z \dot{Z}.$$

Using i) the labor market equilibrium condition, $L = L_Y + L_A$, ii) the normalization of the number of shares, $Z = A$ ($\Rightarrow \dot{Z} = \dot{A}$), iii) the equilibrium condition of the market for blueprints, $p_A = p_Z$, and iv) the assumption that the operating profit of firms in the intermediate goods sector is fully distributed in the form of dividends at any time t , $D(t) = \pi(t)$, we obtain

$$\dot{K} = rK + w(L_Y + L_A) - C + \pi A - p_A \dot{A}.$$

Employing i) the equilibrium condition of the rental market of real capital, $K = Ak$, ii) the fact that the identical operating profit of the firms in the intermediate goods sector is equal to $\pi = px - rk$, and iii) the production function for blueprints of the representative firm in the R&D sector, $\dot{A} = \lambda AL_A$, we get

$$\begin{aligned} \dot{K} &= rAk + wL_Y + wL_A - C + (px - rk)A - p_A \lambda AL_A \\ &= (wL_Y + Apx) - C - (p_A \lambda AL_A - wL_A). \end{aligned}$$

Perfect competition in the R&D sector implies that the equilibrium value of profits of the representative firm is equal to zero: $p_A \dot{A} - wL_A = p_A \lambda AL_A - wL_A = 0$. Moreover, perfect competition in the final goods sector implies that the total real outlays of the representative firm for the two factors of production, labor and intermediate goods, is equal to total real revenue, $wL_Y + Ap_x = Y$, so that profits are equal to zero. Using these facts, we finally obtain

$$\dot{K} = Y - C. \quad (\text{B.1})$$

Obviously, (B.1) is equivalent to the representation of the equilibrium condition of the final goods market as given by (34). ■

B.2 Derivation of Equations (36)–(39)

B.2.1 Derivation of Equation (36)

In equilibrium, firms operating in the intermediate goods sector produce identical quantities, i.e., $x_i = x$ for $i \in [0, A]$. Substituting this into the production function of the representative firm in the final goods sector as given by (19) implies that

$$Y = L_Y^{1-\alpha} \int_0^A x^\alpha di = L_Y^{1-\alpha} Ax^\alpha = (AL_Y)^{1-\alpha} (Ax)^\alpha. \quad (\text{B.2})$$

Taking into account that the production functions of the firms in the intermediate goods sector are given by $x_i = k_i$ for $i \in [0, A]$, we obtain $k_i = k$ for $i \in [0, A]$, where $k = x$. From $k = x$ and the equilibrium condition for the rental market for physical capital, $K = Ak$, it then follows that $Ax = Ak = K$. Substituting the last result into (B.2) and using the labor market equilibrium condition, $L = L_Y + L_A$, we obtain

$$Y = K^\alpha (AL_Y)^{1-\alpha} = K^\alpha [A(L - L_A)]^{1-\alpha}. \quad (\text{B.3})$$

Using (B.3) and the equilibrium condition for the final goods market, $Y = \dot{K} + C$, we finally get

$$\frac{\dot{K}}{K} = \frac{Y}{K} - \frac{C}{K} = \left[\frac{K}{A(L - L_A)} \right]^{-(1-\alpha)} - \frac{C}{K}. \quad (\text{B.4})$$

Obviously, (B.4) is equivalent to the differential equation (36) as given in the main text. ■

B.2.2 Derivation of Equation (37)

Solving the equilibrium condition of the rental market for physical capital (29),

$$K = Ak = A \left(\frac{\alpha^2}{r} \right)^{1/(1-\alpha)} L_Y,$$

for r and taking into account the labor market equilibrium condition, $L = L_Y + L_A$, we obtain

$$r = \alpha^2 \left(\frac{K}{AL_Y} \right)^{-(1-\alpha)} = \alpha^2 \left[\frac{K}{A(L - L_A)} \right]^{-(1-\alpha)}. \quad (\text{B.5})$$

Substituting (B.5) into the general representation of the Euler equation for consumption under relative wealth preferences (16) yields

$$\frac{\dot{C}}{C} = \sigma^S(C, \chi) \left\{ \alpha^2 \left[\frac{K}{A(L - L_A)} \right]^{-(1-\alpha)} + \varepsilon^K(C, \Omega, \chi, \phi_K) - \rho \right\}. \quad (\text{B.6})$$

This is identical to the differential equation (37). ■

B.2.3 Derivation of Equation (39)

Taking into account that i) $Z = A$ holds due to the normalization of the number of shares and ii) the equilibrium price of blueprints equals the equilibrium price of shares, $p_A = p_Z$, it follows from (6) that

$$\Omega = \phi_K K + \phi_Z p_Z Z = \phi_K K + \phi_Z p_A A. \quad (\text{B.7})$$

The first order condition of the representative R&D firm with respect to the choice of employment as given by (27), $p_A \lambda A = w_A$, and the labor market equilibrium condition imply that

$$p_A = \frac{1}{\lambda} \frac{w}{A}.$$

The first order condition of the representative firm in the final goods sector with respect to the choice of labor input together with the labor market equilibrium condition and the fact that $x_i = x$ for $i \in [0, A]$ holds, implies that

$$w = (1 - \alpha) \int_0^A \left(\frac{x_i}{L_Y} \right)^\alpha di = (1 - \alpha) A \left(\frac{x}{L_Y} \right)^\alpha.$$

Using $x = k$ and the equilibrium condition for the rental market of physical capital, $K = Ak$, we obtain

$$\frac{x}{L_Y} = \frac{k}{L_Y} = \frac{Ak}{AL_Y} = \frac{K}{AL_Y} = \frac{K}{A(L - L_A)}.$$

The last three results imply

$$\frac{w}{A} = (1 - \alpha) \left[\frac{K}{A(L - L_A)} \right]^\alpha, \quad (\text{B.8})$$

and

$$p_A = \frac{1 - \alpha}{\lambda} \left[\frac{K}{A(L - L_A)} \right]^\alpha. \quad (\text{B.9})$$

Substituting (B.9) into (B.7) yields

$$\Omega = \phi_K K + \phi_Z \frac{(1-\alpha)A}{\lambda} \left[\frac{K}{A(L-L_A)} \right]^\alpha, \quad (\text{B.10})$$

which is identical to (39). ■

B.2.4 Derivation of Equation (38)

The starting point for the derivation of the differential equation for L_A is the no-arbitrage condition corresponding to relative wealth preferences (11). Taking into account that i) in equilibrium the price of blueprints equals the price of shares, $p_A = p_Z$, and ii) the operating profit of firms in the intermediate goods sector is fully distributed in the form of dividends at any point in time t , $D(t) = \pi(t)$, we obtain

$$r + \varepsilon^K(C, \Omega, \chi, \phi_K) = \frac{\pi}{p_A} + \frac{\dot{p}_A}{p_A} + \varepsilon^Z(C, \Omega, \chi, \phi_Z). \quad (\text{B.11})$$

From (25) and (B.5) it follows that

$$\pi = (1-\alpha) \alpha^{(1+\alpha)/(1-\alpha)} r^{-\alpha/(1-\alpha)} L_Y = (1-\alpha) \alpha \left[\frac{K}{A(L-L_A)} \right]^\alpha L_Y. \quad (\text{B.12})$$

Using (B.12), (B.9), and $L = L_Y + L_A$, we get

$$\frac{\pi}{p_A} = \alpha \lambda L_Y = \alpha \lambda (L - L_A). \quad (\text{B.13})$$

Note that (B.13) equals (57). From (B.9) it follows that

$$\frac{\dot{p}_A}{p_A} = \alpha \frac{\dot{K}}{K} - \alpha \frac{\dot{A}}{A} + \alpha \frac{\dot{L}_A}{L - L_A}. \quad (\text{B.14})$$

Substituting (B.14) into (B.11) yields

$$\dot{L}_A = (L - L_A) \left\{ \frac{1}{\alpha} \left[r + \varepsilon^K - \left(\frac{\pi}{p_A} + \varepsilon^Z \right) \right] - \frac{\dot{K}}{K} + \frac{\dot{A}}{A} \right\}, \quad (\text{B.15})$$

where $\varepsilon^K = \varepsilon^K(C, \Omega, \chi, \phi_K)$ and $\varepsilon^Z = \varepsilon^Z(C, \Omega, \chi, \phi_Z)$. Substituting (B.5), (B.13), (36) [= (B.4)], and (35) into (B.15), we obtain

$$\begin{aligned} \dot{L}_A = (L - L_A) \left\{ - (1-\alpha) \left[\frac{K}{A(L-L_A)} \right]^{-(1-\alpha)} + \frac{C}{K} + \lambda L_A \right. \\ \left. - \lambda (L - L_A) + \frac{\varepsilon^K(C, \Omega, \chi, \phi_K) - \varepsilon^Z(C, \Omega, \chi, \phi_Z)}{\alpha} \right\}. \end{aligned} \quad (\text{B.16})$$

This expression is identical to (38). ■

B.3 The status preferences given by (44) and (45)

The instantaneous utility function as given by (44),

$$u(C, S) = \frac{1}{1-\theta} \left\{ [C^\xi h(S)]^{1-\theta} - 1 \right\},$$

$$\text{where } \xi > 0, \quad \theta > 0, \quad 1 + \xi(\theta - 1) > 0, \quad h(S) > 0, \quad h'(S) > 0,$$

and the the ratio specification of the status function as given by (45),

$$S(\Omega, \bar{\Omega}) = \varphi(\Omega/\bar{\Omega}), \quad \varphi' > 0, \quad \varphi'' \leq 0,$$

exhibit the following properties:

$$\frac{\partial u(C, S)}{\partial C} = \xi C^{\xi(1-\theta)-1} [h(S)]^{1-\theta} > 0, \quad \frac{\partial u(C, S)}{\partial S} = C^{\xi(1-\theta)} [h(S)]^{-\theta} h'(S) > 0,$$

$$\frac{\partial^2 u(C, S)}{\partial C^2} = -[1 + \xi(\theta - 1)] \xi C^{\xi(1-\theta)-2} [h(S)]^{1-\theta} < 0,$$

$$S(\Omega, \Omega) = \varphi(\Omega/\Omega) = \varphi(1) \equiv \chi,$$

$$\frac{\partial S(\Omega, \bar{\Omega})}{\partial \Omega} = \varphi'(\Omega/\bar{\Omega}) \times (1/\bar{\Omega}), \quad \frac{\partial S(\Omega, \Omega)}{\partial \Omega} = \varphi'(1) (1/\Omega).$$

Evaluating the partial derivatives of the instantaneous utility function U at $(C, S) = (C, \chi) = (C, \varphi(1))$ and substituting the resulting expressions as well as the result for $\partial S(\Omega, \Omega)/\partial \Omega$ into the definitions of $\sigma^S(C, \chi)$ and $MRS(C, \Omega, \chi)$ as given by (14) and (17), respectively, we obtain:

$$\sigma^S(C, \chi) \equiv -\frac{\partial u(C, \chi)}{\partial C} \left[C \frac{\partial^2 u(C, \chi)}{\partial C^2} \right]^{-1} = \frac{1}{1 + \xi(\theta - 1)}, \quad (\text{B.17})$$

$$MRS(C, \Omega, \chi) \equiv \frac{\partial u(C, \chi)}{\partial S} \frac{\partial S(\Omega, \Omega)}{\partial \Omega} \left[\frac{\partial u(C, \chi)}{\partial C} \right]^{-1} = \frac{1}{\xi} \frac{h'(\varphi(1)) \varphi'(1)}{h(\varphi(1))} \times \frac{C}{\Omega}. \quad (\text{B.18})$$

Equation (B.17) implies that the symmetric effective elasticity of intertemporal substitution under relative wealth preferences does not depend on C , i.e., $\partial \sigma^S(C, \chi)/\partial C = 0$. From (B.18) it follows that the symmetric marginal rate of substitution of status-relevant own wealth Ω for consumption C , $MRS(C, \Omega, \chi)$, depends linearly on (C/Ω) :

$$MRS(C, \Omega, \chi) = \eta \times \frac{C}{\Omega}, \quad \text{where } \eta \equiv \frac{\beta}{\xi} > 0, \quad \beta \equiv \frac{h'[\varphi(1)] \varphi'(1)}{h[\varphi(1)]} = \frac{h'(\chi) \varphi'(1)}{h(\chi)} > 0.$$

Consequently, ε^K and ε^Z depend linearly on (C/Ω) , too:

$$\varepsilon^K = \phi_K \eta \times \frac{C}{\Omega}, \quad \text{and} \quad \varepsilon^Z = \phi_Z \eta \times \frac{C}{\Omega}.$$

The results given above prove the validity of (46) and (47). ■

B.4 Balanced growth path

B.4.1 The derivation of (54) and (55)

In this paper we focus our attention to specifications of the instantaneous utility function and the status function in which the *MRS* depends linearly on C/Ω such that $MRS = \eta \times (C/\Omega)$ holds [see (42)]. Hence, the dynamic evolution of the variables K , C , A , and L_A is governed by the four differential equations (35), (36), (48), (49), where C/Ω is given by (40). It is easily verified from these five equations that a balanced growth path (BGP) exhibits the following properties:

$$L_A = \text{constant}, \quad \frac{A}{K} = \text{constant}, \quad \frac{C}{K} = \text{constant}, \quad \frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \text{constant}.$$

Denote the steady-state value of a variable x by x^* and the common steady-state growth rate of C , K , and A by

$$g^* = \left(\frac{\dot{A}}{A}\right)^* = \left(\frac{\dot{K}}{K}\right)^* = \left(\frac{\dot{C}}{C}\right)^*.$$

Using (35), (36), (48), (49), and (40), we can show that g^* , L_A^* , $(A/K)^*$, $(C/K)^*$, and $(C/\Omega)^*$ are determined by the following system of equations:

$$g^* = \lambda L_A^*, \tag{B.19}$$

$$g^* = \left[\left(\frac{A}{K}\right)^* (L - L_A^*) \right]^{1-\alpha} - \left(\frac{C}{K}\right)^*, \tag{B.20}$$

$$g^* = \sigma^S \left\{ \alpha^2 \left[\left(\frac{A}{K}\right)^* (L - L_A^*) \right]^{1-\alpha} + \phi_K \eta \left(\frac{C}{\Omega}\right)^* - \rho \right\}, \tag{B.21}$$

$$\begin{aligned} 0 = & -(1-\alpha) \left[\left(\frac{A}{K}\right)^* (L - L_A^*) \right]^{1-\alpha} + \left(\frac{C}{K}\right)^* + \lambda L_A^* \\ & - \lambda (L - L_A^*) + \frac{(\phi_K - \phi_Z) \eta}{\alpha} \times \left(\frac{C}{\Omega}\right)^*, \end{aligned} \tag{B.22}$$

$$\left(\frac{C}{\Omega}\right)^* = \frac{\left(\frac{C}{K}\right)^*}{\phi_K + \phi_Z \frac{1-\alpha}{\lambda (L - L_A^*)} \left[\left(\frac{A}{K}\right)^* (L - L_A^*) \right]^{1-\alpha}}. \tag{B.23}$$

From (B.19) and (B.20) it follows that

$$L_A^* = \frac{g^*}{\lambda}, \quad L - L_A^* = \frac{\lambda L - g^*}{\lambda}, \tag{B.24}$$

$$\left[\left(\frac{A}{K}\right)^* (L - L_A^*) \right]^{1-\alpha} = \left(\frac{Y}{K}\right)^* = g^* + \left(\frac{C}{K}\right)^*. \tag{B.25}$$

Substituting (B.24) and (B.25) into (B.23) yields

$$\left(\frac{C}{\Omega}\right)^* = \frac{\left(\frac{C}{K}\right)^*}{\phi_K + \phi_Z \frac{1-\alpha}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^*\right]}. \quad (\text{B.26})$$

Substituting (B.24)-(B.26) into (B.21) and (B.22) and applying simple transformations, we obtain the following system of equations that determines g^* and $(C/K)^*$:

$$-(1 - \alpha^2 \sigma^S) g^* + \sigma^S \left\{ \alpha^2 + \frac{\phi_K \eta}{\phi_K + \frac{\phi_Z (1-\alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^*\right]} \right\} \left(\frac{C}{K}\right)^* = \sigma^S \rho,$$

$$(1 + \alpha) g^* + \frac{1}{\alpha} \left\{ \alpha^2 + \frac{(\phi_K - \phi_Z) \eta}{\phi_K + \frac{\phi_Z (1-\alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^*\right]} \right\} \left(\frac{C}{K}\right)^* = \lambda L.$$

These two equations are identical to (52) and (53). ■

B.4.2 The derivation of (56)

The transversality conditions are given by (18):

$$\lim_{t \rightarrow \infty} \left\{ \exp \left[- \int_0^t [r(v) + \varepsilon^K(v)] dv \right] K \right\} = 0,$$

$$\lim_{t \rightarrow \infty} \left\{ \exp \left[- \int_0^t [r(v) + \varepsilon^K(v)] dv \right] p_Z Z \right\} = 0.$$

The expressions in curly brackets can be interpreted as *modified* present values of the holdings of physical capital and shares, respectively. In contrast to the standard present values, the *modified* present values are calculated by using the effective rate of return instead of the pure market rate of return. Along the BGP the common constant growth rate of the *modified* present values is given by $-[r^* + (\varepsilon^K)^*] + g^*$. This results from the facts that i) $r^* + (\varepsilon^K)^* = \text{constant}$, ii) $p_Z^* = p_A^* = \text{constant}$, and iii) $(\dot{Z}/Z)^* = (\dot{A}/A)^* = (\dot{K}/K)^* = g^*$. From the Euler equation for consumption it follows that, along the BGP, $g^* = \sigma^S [r^* + (\varepsilon^K)^* - \rho]$ holds. This, in turn, implies that $r^* + (\varepsilon^K)^* = (1/\sigma^S) g^* + \rho$. The latter equation implies that the common constant growth rate of the modified present values can be expressed as

$$-[r^* + (\varepsilon^K)^*] + g^* = -\frac{1 - \sigma^S}{\sigma^S} g^* - \rho.$$

Obviously, the *modified* present values are asymptotically zero if the common constant growth rate of the modified present values is strictly negative. This requirement yields

condition (56) as given in the main text:

$$-\frac{1 - \sigma^S}{\sigma^S} g^* - \rho < 0. \quad \blacksquare$$

C Proof of Remark 1

The representative individual chooses the time paths of C and Q so as to maximize lifetime utility given by

$$\int_0^\infty e^{-\rho t} U(C, \bar{C}) dt, \quad U(C, \bar{C}) \equiv u(C, S(C, \bar{C})),$$

where the assumptions with respect to the status function $S = S(C, \bar{C})$ are given by

$$\frac{\partial S}{\partial C} > 0, \quad \frac{\partial^2 S}{\partial C^2} < 0, \quad \frac{\partial S}{\partial \bar{C}} < 0, \quad (\text{C.1})$$

$$S(C, C) = \zeta = \text{constant}, \quad \text{for } C > 0, \quad (\text{C.2})$$

subject to the differential equations (A.1) and (A.2) and the two initial conditions (A.4). Since, by assumption, there is a continuum of homogeneous households, the representative household takes not only the time paths of r , w , p_Z , and D , but also the time path of \bar{C} as given. For the reasons already given in the relative wealth case we can ignore the no-Ponzi game condition.

The current value Hamiltonian is given by

$$H = U(C, \bar{C}) + \mu_K (rK + wL + DZ - p_Z Q - C) + \mu_Z Q,$$

where the costate variables μ_K and μ_Z denote the shadow price of physical capital and shares, respectively. The necessary optimality conditions for an interior optimum, $\partial H / \partial C = 0$, $\partial H / \partial Q = 0$, $\dot{\mu}_K = \rho \mu_K - (\partial H / \partial K)$, and $\dot{\mu}_Z = \rho \mu_Z - (\partial H / \partial Z)$ can be written as

$$\mu_K = \frac{\partial U(C, \bar{C})}{\partial C}, \quad (\text{C.3})$$

$$\mu_Z = \mu_K p_Z, \quad (\text{C.4})$$

$$\dot{\mu}_K = - (r - \rho) \mu_K, \quad (\text{C.5})$$

$$\dot{\mu}_Z = \rho \mu_Z - \mu_K D. \quad (\text{C.6})$$

The transversality conditions are given by

$$\lim_{t \rightarrow \infty} e^{-\rho t} \mu_K K = 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} e^{-\rho t} \mu_Z Z = 0. \quad (\text{C.7})$$

Our assumptions with respect to $u(C, S)$ and $S(C, \bar{C})$ imply that the Hamiltonian is jointly concave in C , Q , K , and Z . Hence, the transversality conditions ensure that the

necessary optimality conditions are also sufficient.

From (C.4) it follows that

$$\dot{\mu}_Z = \dot{\mu}_K p_Z + \mu_K \dot{p}_Z. \quad (\text{C.8})$$

Using (C.4), (C.6), and (C.8) we obtain

$$\frac{\dot{\mu}_K}{\mu_K} = - \left(\frac{D}{p_Z} + \frac{\dot{p}_Z}{p_Z} - \rho \right). \quad (\text{C.9})$$

From (C.5) it is obvious that

$$\frac{\dot{\mu}_K}{\mu_K} = - (r - \rho) \quad (\text{C.10})$$

Equations (C.9) and (C.10) yield two alternative representations of $\dot{\mu}_K/\mu_K$. The required equality of the right-hand sides of (C.9) and (C.10) implies that the no-arbitrage relation of the economy with relative consumption preferences equals that of the standard model:

$$r = \frac{D}{p_Z} + \frac{\dot{p}_Z}{p_Z}. \quad (\text{C.11})$$

In the status literature it is common practice to restrict attention to symmetric equilibria in which agents that are identical in every respect make identical choices. In any symmetric situation, $C = \bar{C}$ holds. Hence, (C.3) implies that

$$\begin{aligned} \mu_K &= \frac{\partial U(C, C)}{\partial C}, \\ \dot{\mu}_K &= \left[\frac{\partial^2 U(C, C)}{\partial C^2} + \frac{\partial^2 U(C, C)}{\partial C \partial \bar{C}} \right] \dot{C}. \end{aligned}$$

Using the last two equations we get

$$\frac{\dot{\mu}_K}{\mu_K} = C \left[\frac{\partial^2 U(C, C)}{\partial C^2} + \frac{\partial^2 U(C, C)}{\partial C \partial \bar{C}} \right] \left[\frac{\partial U(C, C)}{\partial C} \right]^{-1} \times \frac{\dot{C}}{C}. \quad (\text{C.12})$$

Using (C.10) and (C.12), we obtain the Euler equation for consumption of a decentralized economy populated by households with relative consumption preferences:

$$\frac{\dot{C}}{C} = \sigma^{rc}(C) (r - \rho),$$

where

$$\sigma^{rc}(C) \equiv - \frac{\partial U(C, C)}{\partial C} \left\{ C \left[\frac{\partial^2 U(C, C)}{\partial C^2} + \frac{\partial^2 U(C, C)}{\partial C \partial \bar{C}} \right] \right\}^{-1}.$$

Using (C.4), the transversality conditions (C.7) can be written as

$$\lim_{t \rightarrow \infty} e^{-\rho t} \mu_K K = 0, \quad \lim_{t \rightarrow \infty} e^{-\rho t} \mu_K p_Z Z = 0. \quad (\text{C.13})$$

Integration of (C.5) yields

$$\mu_K(t) = \mu_K(0) e^{\rho t} \exp \left[- \int_0^t r(v) dv \right].$$

The assumption that $\partial U / \partial C > 0$, together with (C.3), implies that $\mu_K(t) > 0$ for $t \geq 0$. Hence, the transversality conditions (C.13) are equivalent to

$$\lim_{t \rightarrow \infty} \left\{ \exp \left[- \int_0^t r(v) dv \right] K \right\} = 0, \quad \lim_{t \rightarrow \infty} \left\{ \exp \left[- \int_0^t [r(v)] dv \right] p_Z Z \right\} = 0.$$

These conditions are identical to the conditions obtained in the standard model.

D Proofs of Propositions 1 – 3

D.1 Preliminaries

In the following proofs we restrict our attention to the mathematical aspects. Please note that detailed economic interpretations have already been given in the main text. The proofs given below for the three cases exhibit a common pattern. First, we use Equations (52) and (53) to analyze the properties of the steady-state values of the common growth rate and the consumption-capital ratio, i.e., g^* and $(C/K)^*$. In this context we derive the partial derivatives of g^* and $(C/K)^*$ with respect to η . In the two special cases “ $\phi_K = \phi_Z = 1$ ” (Proposition 1) and “ $\phi_K = 1$ and $\phi_Z = 0$ ” (Proposition 2) this task can be carried out easily because there exist quite simple explicit solutions for g^* and $(C/K)^*$. In the general case “ $\phi_K = 1$ and $0 < \phi_Z < 1$ ” (Proposition 3), however, we must resort to implicit differentiation. Moreover, we discuss the partial derivatives of g^* and $(C/K)^*$ with respect to ϕ_Z . Second, we analyze the properties of the rest of the endogenous variables by using the fact that the steady-state values of these variables can be expressed as functions of g^* and $(C/K)^*$. For the convenience of the readers, these representations are summarized in a compact way.

$$L_A^* = \frac{g^*}{\lambda}, \tag{D.1}$$

$$L_Y^* = L - L_A^* = \frac{\lambda L - g^*}{\lambda}, \tag{D.2}$$

$$\left(\frac{Y}{K} \right)^* = g^* + \left(\frac{C}{K} \right)^*, \tag{D.3}$$

$$\left(\frac{K}{A} \right)^* \frac{1}{L_Y^*} = \left(\frac{K}{A} \right)^* \frac{1}{(L - L_A^*)} = \left[\left(\frac{Y}{K} \right)^* \right]^{-1/(1-\alpha)}, \tag{D.4}$$

$$r^* = \alpha^2 \left[\left(\frac{K}{A} \right)^* \frac{1}{(L - L_A^*)} \right]^{-(1-\alpha)} = \alpha^2 \left(\frac{Y}{K} \right)^*, \tag{D.5}$$

$$p^* = \frac{1}{\alpha} \times r^* = \alpha \left(\frac{Y}{K} \right)^*, \quad (\text{D.6})$$

$$x^* = k^* = \left(\frac{\alpha^2}{r^*} \right)^{1/(1-\alpha)} L_Y^* = \frac{\lambda L - g^*}{\lambda} \left[\left(\frac{Y}{K} \right)^* \right]^{-1/(1-\alpha)}, \quad (\text{D.7})$$

$$\left(\frac{x}{L_Y} \right)^* = \left[\left(\frac{Y}{K} \right)^* \right]^{-1/(1-\alpha)}, \quad (\text{D.8})$$

$$\pi^* = (1-\alpha) \alpha L_Y^* \left[\left(\frac{Y}{K} \right)^* \right]^{-\alpha/(1-\alpha)} = (1-\alpha) \alpha \frac{\lambda L - g^*}{\lambda} \left[\left(\frac{Y}{K} \right)^* \right]^{-\alpha/(1-\alpha)}, \quad (\text{D.9})$$

$$p_A^* = \frac{(1-\alpha)}{\lambda} \left(\frac{\alpha^2}{r^*} \right)^{\alpha/(1-\alpha)} = \frac{1-\alpha}{\lambda} \left[\left(\frac{Y}{K} \right)^* \right]^{-\alpha/(1-\alpha)}, \quad (\text{D.10})$$

$$\left(\frac{\pi}{p_A} \right)^* = \alpha \lambda L_Y^* = \alpha (\lambda L - g^*), \quad (\text{D.11})$$

$$\left(\frac{w}{A} \right)^* = \lambda p_A^* = (1-\alpha) \left[\left(\frac{Y}{K} \right)^* \right]^{-\alpha/(1-\alpha)}, \quad (\text{D.12})$$

$$\begin{aligned} \left(\frac{C}{\Omega} \right)^* &= \frac{\left(\frac{C}{K} \right)^*}{\phi_K + \phi_Z \frac{1-\alpha}{\lambda(L-L_A^*)} \left[\left(\frac{K}{A} \right)^* \frac{1}{(L-L_A^*)} \right]^{-1(1-\alpha)}} \\ &= \frac{\left(\frac{C}{K} \right)^*}{\phi_K + \phi_Z \frac{1-\alpha}{\lambda L - g^*} \left(\frac{Y}{K} \right)^*}, \end{aligned} \quad (\text{D.13})$$

$$\left(\frac{C}{Y} \right)^* = \frac{(C/K)^*}{(Y/K)^*} = \frac{\left(\frac{C}{K} \right)^*}{g^* + \left(\frac{C}{K} \right)^*}, \quad (\text{D.14})$$

$$r^* + (\varepsilon^K)^* = \alpha^2 \left(\frac{Y}{K} \right)^* + \frac{\phi_K \eta \left(\frac{C}{K} \right)^*}{\phi_K + \phi_Z \frac{1-\alpha}{\lambda L - g^*} \left(\frac{Y}{K} \right)^*} = \frac{1}{\sigma_S} g^* + \rho, \quad (\text{D.15})$$

$$\left(\frac{\pi}{p_A} \right)^* + (\varepsilon^Z)^* = \alpha (\lambda L - g^*) + \frac{\phi_Z \eta \left(\frac{C}{K} \right)^*}{\phi_K + \phi_Z \frac{1-\alpha}{\lambda L - g^*} \left(\frac{Y}{K} \right)^*} = \frac{1}{\sigma_S} g^* + \rho, \quad (\text{D.16})$$

$$(\varepsilon^K)^* = \frac{\phi_K \eta \left(\frac{C}{K} \right)^*}{\phi_K + \phi_Z \frac{1-\alpha}{\lambda L - g^*} \left(\frac{Y}{K} \right)^*} = \frac{1}{\sigma_S} g^* + \rho - \alpha^2 \left(\frac{Y}{K} \right)^*, \quad (\text{D.17})$$

$$(\varepsilon^Z)^* = \frac{\phi_Z \eta \left(\frac{C}{K}\right)^*}{\phi_K + \phi_Z \frac{1-\alpha}{\lambda L - g^*} \left(\frac{Y}{K}\right)^*} = \frac{1 + \alpha \sigma^S}{\sigma^S} g^* + \rho - \alpha \lambda L, \quad (\text{D.18})$$

$$\left(\frac{K}{K + p_{AA}}\right)^* = \frac{1}{1 + \frac{1-\alpha}{\lambda L_Y^*} \left(\frac{Y}{K}\right)^*} = \frac{1}{1 + \frac{1-\alpha}{\lambda L - g^*} \left(\frac{Y}{K}\right)^*}. \quad (\text{D.19})$$

D.2 Proof of Proposition 1

Proof of i):

Setting $\phi_Z = \phi_K = 1$ in (52) and (53) yields

$$-(1 - \alpha^2 \sigma^S) g^* + \sigma^S \left\{ \alpha^2 + \frac{\eta}{1 + \frac{(1-\alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^* \right]} \right\} \left(\frac{C}{K}\right)^* = \sigma^S \rho,$$

$$(1 + \alpha) g^* + \alpha \left(\frac{C}{K}\right)^* = \lambda L.$$

Solving this system of two equations for the common growth rate g^* and the consumption-capital ratio $(C/K)^*$, we obtain:

$$g^* = \frac{\sigma^S [(\alpha + \eta) \lambda L - \rho]}{1 + \sigma^S [\alpha + \eta (1 + \alpha)]}, \quad (\text{D.20})$$

$$\left(\frac{C}{K}\right)^* = \frac{\lambda L - (1 + \alpha) g^*}{\alpha} = \frac{(1 - \alpha^2 \sigma^S) \lambda L + (1 + \alpha) \sigma^S \rho}{\alpha \{1 + \sigma^S [\alpha + \eta (1 + \alpha)]\}}. \quad (\text{D.21})$$

It is obvious from (D.20) and (D.21) that

$$\left(\frac{C}{K}\right)^* > 0 \Leftrightarrow g^* < \frac{\lambda L}{1 + \alpha} \Leftrightarrow (1 - \alpha^2 \sigma^S) \lambda L + (1 + \alpha) \sigma^S \rho > 0,$$

$$g^* > 0 \Leftrightarrow \rho < (\alpha + \eta) \lambda L.$$

The last two results prove the validity of the conditions (58) and (59). ■

Proof of ii)

The partial derivative of g^* [defined by (D.20)] with respect to the status parameter η is given by

$$\begin{aligned} \frac{\partial g^*}{\partial \eta} &= \frac{\sigma^S \{ (1 - \alpha^2 \sigma^S) \lambda L + (1 + \alpha) \sigma^S \rho \}}{\{1 + \sigma^S [\alpha + \eta (1 + \alpha)]\}^2} \\ &= \frac{\alpha \sigma^S}{1 + [\alpha + \eta (1 + \alpha)] \sigma^S} \left(\frac{C}{K}\right)^*, \end{aligned} \quad (\text{D.22})$$

where the second line is obtained by using (D.21). If (58) holds, then $(C/K)^* > 0$ and

$$\frac{\partial g^*}{\partial \eta} > 0, \quad (\text{D.23})$$

i.e., the common growth rate g^* depends positively on the status parameter η . From (D.21) it then follows that

$$\frac{\partial (C/K)^*}{\partial \eta} = -\frac{1 + \alpha}{\alpha} \frac{\partial g^*}{\partial \eta} < 0.$$

Using (D.21) and (D.1)–(D.3) and taking into account that $\partial g^*/\partial \eta > 0$, we obtain

$$\begin{aligned} L_A^* &= \frac{g^*}{\lambda} \Rightarrow \frac{\partial L_A^*}{\partial \eta} = \frac{1}{\lambda} \frac{\partial g^*}{\partial \eta} > 0, \\ L_Y^* &= \frac{\lambda L - g^*}{\lambda} \Rightarrow \frac{\partial L_Y^*}{\partial \eta} = -\frac{1}{\lambda} \frac{\partial g^*}{\partial \eta} < 0, \\ \left(\frac{Y}{K}\right)^* &= \frac{\lambda L - g^*}{\alpha} \Rightarrow \frac{\partial (Y/K)^*}{\partial \eta} = -\frac{1}{\alpha} \frac{\partial g^*}{\partial \eta} < 0. \end{aligned} \quad (\text{D.24})$$

Substituting (D.24) into (D.5)–(D.18) and taking into account that $\partial g^*/\partial \eta > 0$, we obtain

$$\begin{aligned} r^* &= \alpha(\lambda L - g^*) \Rightarrow \frac{\partial r^*}{\partial \eta} = -\alpha \frac{\partial g^*}{\partial \eta} < 0, \\ p^* &= \lambda L - g^* \Rightarrow \frac{\partial p^*}{\partial \eta} = -\frac{\partial g^*}{\partial \eta} < 0, \\ x^* &= \frac{\alpha^{1/(1-\alpha)}}{\lambda} (\lambda L - g^*)^{-\alpha/(1-\alpha)} \Rightarrow \frac{\partial x^*}{\partial \eta} = \frac{\alpha^{(2-\alpha)/(1-\alpha)}}{(1-\alpha)\lambda} (\lambda L - g^*)^{-1/(1-\alpha)} \frac{\partial g^*}{\partial \eta} > 0, \\ \left(\frac{x}{L_Y}\right)^* &= \left(\frac{\lambda L - g^*}{\alpha}\right)^{-1/(1-\alpha)} \Rightarrow \frac{\partial (x/L_Y)^*}{\partial \eta} = \frac{\alpha^{1/(1-\alpha)}}{1-\alpha} (\lambda L - g^*)^{-(2-\alpha)/(1-\alpha)} \frac{\partial g^*}{\partial \eta} > 0, \\ \pi^* &= \frac{(1-\alpha)\alpha^{1/(1-\alpha)}}{\lambda} (\lambda L - g^*)^{-(2\alpha-1)/(1-\alpha)} \Rightarrow \frac{\partial \pi^*}{\partial \eta} = \frac{(2\alpha-1)\alpha^{1/(1-\alpha)}}{\lambda} (\lambda L - g^*)^{-\alpha/(1-\alpha)}, \\ p_A^* &= \frac{1-\alpha}{\lambda} \left(\frac{\lambda L - g^*}{\alpha}\right)^{-\alpha/(1-\alpha)} \Rightarrow \frac{\partial p_A^*}{\partial \eta} = \frac{1}{\lambda} \left(\frac{\lambda L - g^*}{\alpha}\right)^{-1/(1-\alpha)} \frac{\partial g^*}{\partial \eta} > 0, \\ \left(\frac{\pi}{p_A}\right)^* &= \alpha(\lambda L - g^*) \Rightarrow \frac{\partial (\pi/p_A)^*}{\partial \eta} = -\alpha \frac{\partial g^*}{\partial \eta} < 0, \\ \left(\frac{w}{A}\right)^* &= (1-\alpha) \left(\frac{\lambda L - g^*}{\alpha}\right)^{-\alpha/(1-\alpha)} \Rightarrow \frac{\partial (w/A)^*}{\partial \eta} = \left(\frac{\lambda L - g^*}{\alpha}\right)^{-1/(1-\alpha)} \frac{\partial g^*}{\partial \eta} > 0, \\ \left(\frac{C}{\Omega}\right)^* &= \lambda L - (1+\alpha)g^* \Rightarrow \frac{\partial (C/\Omega)^*}{\partial \eta} = -(1+\alpha) \frac{\partial g^*}{\partial \eta} < 0, \\ \left(\frac{C}{\bar{Y}}\right)^* &= \frac{\lambda L - (1+\alpha)g^*}{\lambda L - g^*} \Rightarrow \frac{\partial (C/\bar{Y})^*}{\partial \eta} = -\frac{\alpha\lambda L}{(\lambda L - g^*)^2} \frac{\partial g^*}{\partial \eta} < 0, \end{aligned}$$

$$r^* + (\varepsilon^K)^* = \left(\frac{\pi}{p_A}\right)^* + (\varepsilon^Z)^* = (\alpha + \eta) \lambda L - [\alpha + \eta(1 + \alpha)] g^* = \frac{1}{\sigma^S} g^* + \rho$$

$$\Rightarrow \frac{\partial [r^* + (\varepsilon^K)^*]}{\partial \eta} = \frac{\partial [(\pi/p_A)^* + (\varepsilon^Z)^*]}{\partial \eta} = \frac{1}{\sigma^S} \frac{\partial g^*}{\partial \eta} > 0,$$

$$(\varepsilon^K)^* = (\varepsilon^Z)^* = \eta [\lambda L - (1 + \alpha) g^*] = \frac{1 + \alpha \sigma^S}{\sigma^S} g^* - \alpha \lambda L + \rho$$

$$\Rightarrow \frac{\partial (\varepsilon^K)^*}{\partial \eta} = \frac{\partial (\varepsilon^Z)^*}{\partial \eta} = \frac{1 + \alpha \sigma^S}{\sigma^S} \frac{\partial g^*}{\partial \eta} > 0.$$

The results given above prove the validity of the following assertions made in ii) of proposition 1:

$$\begin{aligned} \frac{\partial v^*}{\partial \eta} < 0, \text{ for } v &= \frac{C}{K}, \frac{C}{Y}, \frac{C}{\Omega}, L_Y, \frac{Y}{K}, r, p, \frac{\pi}{p_A}, \\ \frac{\partial v^*}{\partial \eta} > 0, \text{ for } v &= L_A, \frac{x}{L_Y}, x, \varepsilon^K, \varepsilon^Z, r + \varepsilon^K, \frac{\pi}{p_A} + \varepsilon^Z, p_A, \frac{w}{A}, \\ \operatorname{sgn} \left(\frac{\partial \pi^*}{\partial \eta} \right) &= \operatorname{sgn} (2\alpha - 1). \quad \blacksquare \end{aligned}$$

Proof of iii):

Using (D.24) and (D.19), we finally obtain

$$\left(\frac{K}{K + p_{AA}}\right)^* = \alpha \Rightarrow \left(\frac{p_{AA}}{K + p_{AA}}\right)^* = 1 - \alpha. \quad \blacksquare$$

D.3 Proof of Proposition 2

Proof of i):

Setting $\phi_Z = 0$ in (52) and (53) yields

$$-(1 - \alpha^2 \sigma^S) g^* + \sigma^S (\alpha^2 + \eta) \left(\frac{C}{K}\right)^* = \sigma^S \rho,$$

$$(1 + \alpha) g^* + \frac{1}{\alpha} (\alpha^2 + \eta) \left(\frac{C}{K}\right)^* = \lambda L.$$

Solving this system of two equations for g^* and $(C/K)^*$, we obtain

$$g^* = \frac{\sigma^S (\alpha \lambda L - \rho)}{1 + \alpha \sigma^S}, \quad (\text{D.25})$$

$$\left(\frac{C}{K}\right)^* = \frac{\alpha [(1 - \alpha^2 \sigma^S) \lambda L + (1 + \alpha) \sigma^S \rho]}{(1 + \alpha \sigma^S) (\alpha^2 + \eta)}. \quad (\text{D.26})$$

It is obvious from (D.25) and (D.26) that

$$\left(\frac{C}{K}\right)^* > 0 \Leftrightarrow g^* < \frac{\lambda L}{1+\alpha} \Leftrightarrow (1-\alpha^2\sigma^S)\lambda L + (1+\alpha)\sigma^S\rho > 0,$$

$$g^* > 0 \Leftrightarrow \rho < \alpha\lambda L.$$

These two results prove the validity of the conditions (60) and (61). ■

Proof of ii):

From (D.25) it is obvious that the common growth rate g^* is independent of the status parameter η ,

$$\frac{\partial g^*}{\partial \eta} = 0. \quad (\text{D.27})$$

Using (D.26) it can be shown that $(C/K)^*$ depends negatively on η :

$$\frac{\partial (C/K)^*}{\partial \eta} = -\frac{\alpha [(1-\alpha^2\sigma^S)\lambda L + (1+\alpha)\sigma^S\rho]}{(1+\alpha\sigma^S)(\alpha^2+\eta)^2} = -\frac{(C/K)^*}{\alpha^2+\eta} < 0. \quad (\text{D.28})$$

From $\partial g^*/\partial \eta = 0$, $\partial (C/K)^*/\partial \eta < 0$, and (D.1)–(D.3) it follows that

$$L_A^* = \frac{g^*}{\lambda} \quad \text{and} \quad L_Y^* = \frac{\lambda L - g^*}{\lambda} \Rightarrow \frac{\partial L_A^*}{\partial \eta} = \frac{\partial L_Y^*}{\partial \eta} = 0, \quad (\text{D.29})$$

$$\left(\frac{Y}{K}\right)^* = g^* + \left(\frac{C}{K}\right)^* \Rightarrow \frac{\partial (Y/K)^*}{\partial \eta} = \frac{\partial (C/K)^*}{\partial \eta} < 0. \quad (\text{D.30})$$

Using $\partial g^*/\partial \eta = 0$, $\partial L_Y^*/\partial \eta = 0$, $\partial (Y/K)^*/\partial \eta = \partial (C/K)^*/\partial \eta < 0$, and (D.5)–(D.18), we then obtain

$$r^* = \alpha^2 \left(\frac{Y}{K}\right)^* \Rightarrow \frac{\partial r^*}{\partial \eta} = \alpha^2 \frac{\partial (Y/K)^*}{\partial \eta} < 0,$$

$$p^* = \alpha \left(\frac{Y}{K}\right)^* \Rightarrow \frac{\partial p^*}{\partial \eta} = \alpha \frac{\partial (Y/K)^*}{\partial \eta} < 0,$$

$$x^* = L_Y^* \left[\left(\frac{Y}{K}\right)^*\right]^{-1/(1-\alpha)} \Rightarrow \frac{\partial x^*}{\partial \eta} = -\frac{L_Y^*}{1-\alpha} \left[\left(\frac{Y}{K}\right)^*\right]^{-(2-\alpha)/(1-\alpha)} \frac{\partial (Y/K)^*}{\partial \eta} > 0,$$

$$\left(\frac{x}{L_Y}\right)^* = \left[\left(\frac{Y}{K}\right)^*\right]^{-1/(1-\alpha)}$$

$$\Rightarrow \frac{\partial (x/L_Y)^*}{\partial \eta} = -\frac{1}{1-\alpha} \left[\left(\frac{Y}{K}\right)^*\right]^{-(2-\alpha)/(1-\alpha)} \frac{\partial (Y/K)^*}{\partial \eta} > 0,$$

$$\pi^* = (1-\alpha)\alpha L_Y^* \left[\left(\frac{Y}{K}\right)^*\right]^{-\alpha/(1-\alpha)} \Rightarrow \frac{\partial \pi^*}{\partial \eta} = -\alpha^2 L_Y^* \left[\left(\frac{Y}{K}\right)^*\right]^{-1/(1-\alpha)} \frac{\partial (Y/K)^*}{\partial \eta} > 0,$$

$$p_A^* = \frac{1-\alpha}{\lambda} \left[\left(\frac{Y}{K}\right)^*\right]^{-\alpha/(1-\alpha)} \Rightarrow \frac{\partial p_A^*}{\partial \eta} = -\frac{\alpha}{\lambda} \left[\left(\frac{Y}{K}\right)^*\right]^{-1/(1-\alpha)} \frac{\partial (Y/K)^*}{\partial \eta} > 0,$$

$$\begin{aligned}
\left(\frac{\pi}{p_A}\right)^* &= \alpha \lambda L_Y^* = \alpha (\lambda L - g^*) \Rightarrow \frac{\partial (\pi/p_A)^*}{\partial \eta} = 0, \\
\left(\frac{w}{A}\right)^* &= (1 - \alpha) \left[\left(\frac{Y}{K}\right)^*\right]^{-\alpha/(1-\alpha)} \Rightarrow \frac{\partial (w/A)^*}{\partial \eta} = -\alpha \left[\left(\frac{Y}{K}\right)^*\right]^{-1/(1-\alpha)} \frac{\partial (Y/K)^*}{\partial \eta} > 0, \\
\left(\frac{C}{\Omega}\right)^* &= \left(\frac{C}{K}\right)^* \Rightarrow \frac{\partial (C/\Omega)^*}{\partial \eta} = \frac{\partial (C/K)^*}{\partial \eta} < 0, \\
\left(\frac{C}{Y}\right)^* &= \frac{\left(\frac{C}{K}\right)^*}{g^* + \left(\frac{C}{K}\right)^*} \Rightarrow \frac{\partial (C/Y)^*}{\partial \eta} = \frac{g^*}{\left[g^* + \left(\frac{C}{K}\right)^*\right]^2} \frac{\partial (C/K)^*}{\partial \eta} < 0, \\
(\varepsilon^K)^* &= \frac{1}{\sigma^S} g^* + \rho - \alpha^2 \left(\frac{Y}{K}\right)^* \Rightarrow \frac{\partial (\varepsilon^K)^*}{\partial \eta} = -\alpha^2 \frac{\partial (Y/K)^*}{\partial \eta} > 0, \\
(\varepsilon^Z)^* &= 0 \Rightarrow \frac{\partial (\varepsilon^Z)^*}{\partial \eta} = 0, \\
r^* + (\varepsilon^K)^* &= \left(\frac{\pi}{p_A}\right)^* + (\varepsilon^Z)^* = \frac{1}{\sigma^S} g^* + \rho \\
&\Rightarrow \frac{\partial [r^* + (\varepsilon^K)^*]}{\partial \eta} = \frac{\partial [(\pi/p_A)^* + (\varepsilon^Z)^*]}{\partial \eta} = 0.
\end{aligned}$$

The results given above prove the validity of the following assertions made in ii) of proposition 2:

$$\begin{aligned}
\frac{\partial v^*}{\partial \eta} &= 0, & \text{for } v &= L_A, L_Y, r + \varepsilon^K, \frac{\pi}{p_A}, \varepsilon^Z, \\
\frac{\partial v^*}{\partial \eta} &< 0, & \text{for } v &= \frac{C}{K}, \frac{C}{\Omega}, \frac{C}{Y}, \frac{Y}{K}, r, p, \\
\frac{\partial v^*}{\partial \eta} &> 0, & \text{for } v &= \frac{x}{L_Y}, x, \pi, \varepsilon^K, p_A, \frac{w}{A}. \quad \blacksquare
\end{aligned}$$

Proof of iii)

Using (D.25), (D.26), (D.29), and (D.30), we obtain

$$\begin{aligned}
\left(\frac{Y}{K}\right)^* &= \frac{(1 + \eta \sigma^S) \alpha \lambda L + (\alpha - \eta) \sigma^S \rho}{(1 + \alpha \sigma^S) (\alpha^2 + \eta)}, \\
L_Y^* &= \frac{\lambda L + \sigma^S \rho}{\lambda (1 + \alpha \sigma^S)}.
\end{aligned}$$

Substituting these results into (D.19) yields

$$\left(\frac{K}{K+p_A A}\right)^* = \left[1 + \frac{1-\alpha}{\lambda L_Y^*} \left(\frac{Y}{K}\right)^*\right]^{-1} \quad (\text{D.31})$$

$$= \frac{(\alpha^2 + \eta)(\lambda L + \sigma^S \rho)}{\alpha(\lambda L + \sigma^S \rho) + \eta\{[1 + \alpha\sigma^S(1-\alpha)]\lambda L + \alpha\sigma^S \rho\}}. \quad (\text{D.32})$$

Differentiating (D.31) with respect to η and taking into account that $\partial L_Y^*/\partial\eta = 0$ and $\partial(Y/K)^*/\partial\eta < 0$, we obtain

$$\frac{\partial [K/(K+p_A A)]^*}{\partial\eta} = - \left[1 + \frac{1-\alpha}{\lambda L_Y^*} \left(\frac{Y}{K}\right)^*\right]^{-2} \frac{1-\alpha}{\lambda L_Y^*} \times \frac{\partial(Y/K)^*}{\partial\eta} > 0.$$

It is obvious from (D.32) that

$$\left(\frac{K}{K+p_A A}\right)^* \Big|_{\eta=0} = \alpha.$$

Moreover, recalling that g^* is strictly positive if and only if assumption (61), i.e., $\rho < \alpha\lambda L$, holds, we obtain

$$\lim_{\eta \rightarrow \infty} \left(\frac{K}{K+p_A A}\right)^* = \frac{\lambda L + \sigma^S \rho}{\lambda L + \sigma^S \rho + (1-\alpha)\sigma^S(\alpha\lambda L - \rho)} < 1.$$

These results prove the validity of all assertions made in iii) of Proposition 2. ■

D.4 Alternative proof for $\phi_Z = 0 \Rightarrow \partial g^*/\partial\eta = 0$

In the following we assume that $\phi_Z = 0$. We show that, under this assumption, $\partial g^*/\partial\eta = 0$ holds by demonstrating the impossibility of both $\partial g^*/\partial\eta > 0$ and $\partial g^*/\partial\eta < 0$.

Part 1: We show by contradiction that $\partial g^*/\partial\eta > 0$ cannot hold. Let us assume that the growth rate g^* depends positively on the status parameter η , $\partial g^*/\partial\eta > 0$, just as in the special case $\phi_Z = \phi_K = 1$ discussed above. The production function of the R&D sector, $\dot{A} = \lambda A L_A$, then implies that the employment of researchers also depends positively on η , $\partial L_A^*/\partial\eta = \lambda^{-1}(\partial g^*/\partial\eta) > 0$. Using the equilibrium condition for the labor market, $L_Y + L_A = L$, we obtain $\partial L_Y^*/\partial\eta = -\partial L_A^*/\partial\eta < 0$, i.e., employment in the final goods sector depends negatively on η . Using the fact that in any equilibrium $\pi/p_A = \alpha\lambda(L - L_A) = \alpha\lambda L_Y$ holds [see (57)], we obtain $\partial(\pi/p_A)^*/\partial\eta = \alpha\lambda(\partial L_Y^*/\partial\eta) < 0$, i.e., the dividend yield depends negatively on η . The assumption that $\phi_Z = 0$ implies that the steady-state version of the no-arbitrage equation (55) simplifies to $r^* + (\varepsilon^K)^* = (\pi/p_A)^*$. Consequently, the effective rate of return of physical capital also depends negatively on η , $\partial[r^* + (\varepsilon^K)^*]/\partial\eta = \partial(\pi/p_A)^*/\partial\eta < 0$. From the latter result and the steady-state version of the Euler equation for consumption (54), $g^* = \sigma^S[r^* + (\varepsilon^K)^* - \rho]$, it finally follows that $\partial g^*/\partial\eta = \sigma^S(\partial[r^* + (\varepsilon^K)^*]/\partial\eta) < 0$. Consequently, the growth rate g^*

depends negatively on the status parameter η , which contradicts the assumption $\partial g^*/\partial \eta > 0$ that we made at the beginning of part 1 of the proof.

Part 2: We show by contradiction that $\partial g^*/\partial \eta < 0$ cannot hold. Now let us assume that the growth rate g^* depends negatively on the status parameter η , $\partial g^*/\partial \eta < 0$. From considerations that are analogous to those made in part 1, we finally obtain $\partial g^*/\partial \eta > 0$, which contradicts the initial assumption $\partial g^*/\partial \eta < 0$. ■

D.5 Proof of Proposition 3

Proof of i)

Equations (52) and (53) can be written as

$$M_1 \left(g^*, \left(\frac{C}{K} \right)^*, \eta, \phi_K, \phi_Z, \sigma^S, \rho \right) = 0,$$

$$M_2 \left(g^*, \left(\frac{C}{K} \right)^*, \eta, \phi_K, \phi_Z, \sigma^S, \rho \right) = 0,$$

where

$$M_1 \equiv - (1 - \alpha^2 \sigma^S) g^* + \sigma^S \left[\alpha^2 + \frac{\phi_K \eta}{\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K} \right)^* \right]} \right] \left(\frac{C}{K} \right)^* - \sigma^S \rho,$$

$$M_2 \equiv (1 + \alpha) g^* + \frac{1}{\alpha} \left[\alpha^2 + \frac{(\phi_K - \phi_Z) \eta}{\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K} \right)^* \right]} \right] \left(\frac{C}{K} \right)^* - \lambda L.$$

In the following, we restrict our attention to cases in which the Jacobian matrix of this system with respect to the endogenous variables g^* and $(C/K)^*$ is nonsingular. The partial derivatives of the implicitly defined functions

$$g^* = g^*(\eta, \phi_K, \phi_Z, \sigma^S, \rho) \quad \text{and} \quad \left(\frac{C}{K} \right)^* = \left(\frac{C}{K} \right)^*(\eta, \phi_K, \phi_Z, \sigma^S, \rho)$$

are determined by the following system of equations, where $var = \eta, \phi_K, \phi_Z, \sigma^S$, and ρ :

$$\begin{pmatrix} \frac{\partial M_1}{\partial g^*} & \frac{\partial M_1}{\partial (C/K)^*} \\ \frac{\partial M_2}{\partial g^*} & \frac{\partial M_2}{\partial (C/K)^*} \end{pmatrix} \begin{pmatrix} \frac{\partial g^*}{\partial var} \\ \frac{\partial (C/K)^*}{\partial var} \end{pmatrix} = \begin{pmatrix} -\frac{\partial M_1}{\partial var} \\ -\frac{\partial M_2}{\partial var} \end{pmatrix}. \quad (\text{D.33})$$

Please note that all partial derivatives are evaluated at $g^* = g^*(\eta, \phi_K, \phi_Z, \sigma^S, \rho)$ and $(C/K)^* = (C/K)^*(\eta, \phi_K, \phi_Z, \sigma^S, \rho)$. The solutions of (D.33) can be written as

$$\frac{\partial g^*}{\partial var} = \frac{1}{\Psi} \left[\frac{\partial M_2}{\partial (C/K)^*} \frac{\partial M_1}{\partial var} - \frac{\partial M_1}{\partial (C/K)^*} \frac{\partial M_2}{\partial var} \right],$$

$$\frac{\partial (C/K)^*}{\partial var} = \frac{1}{\Psi} \left[-\frac{\partial M_2}{\partial g^*} \frac{\partial M_1}{\partial var} + \frac{\partial M_1}{\partial g^*} \frac{\partial M_2}{\partial var} \right],$$

where

$$\Psi \equiv - \left[\frac{\partial M_1}{\partial g^*} \frac{\partial M_2}{\partial (C/K)^*} - \frac{\partial M_1}{\partial (C/K)^*} \frac{\partial M_2}{\partial g^*} \right]$$

and $(-\Psi)$ equals the determinant of the Jacobian matrix. It can be shown that

$$\begin{aligned} \frac{\partial M_1}{\partial g^*} &= - \left((1 - \alpha^2 \sigma^S) + \frac{\eta \phi_K \phi_Z (1 - \alpha) \sigma^S \left(\frac{C}{K} \right)^* \left(\lambda L + \left(\frac{C}{K} \right)^* \right)}{(\lambda L - g^*)^2 \left(\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K} \right)^* \right] \right)^2} \right), \\ \frac{\partial M_1}{\partial (C/K)^*} &= \sigma^S \alpha^2 + \frac{\sigma^S \phi_K \eta \left(\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} g^* \right)}{\left(\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K} \right)^* \right] \right)^2}, \\ \frac{\partial M_2}{\partial g^*} &= (1 + \alpha) - \frac{(\phi_K - \phi_Z) \phi_Z (1 - \alpha) \eta \left[\lambda L + \left(\frac{C}{K} \right)^* \right] \left(\frac{C}{K} \right)^*}{\alpha \left(\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K} \right)^* \right] \right)^2 (\lambda L - g^*)^2}, \\ \frac{\partial M_2}{\partial (C/K)^*} &= \alpha + \frac{(\phi_K - \phi_Z) \eta \left(\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} g^* \right)}{\alpha \left(\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K} \right)^* \right] \right)^2}. \end{aligned}$$

Using these results, we obtain

$$\begin{aligned} \Psi &= \alpha (1 + \sigma^S \alpha) + \frac{\eta [\phi_K (1 + \sigma^S \alpha) - (1 - \alpha^2 \sigma^S) \phi_Z] \left[\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} g^* \right]}{\alpha \left(\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K} \right)^* \right] \right)^2} \\ &\quad + \frac{\eta (1 - \alpha) \phi_Z^2 \alpha^2 \sigma^S \left[\lambda L + \left(\frac{C}{K} \right)^* \right] \left(\frac{C}{K} \right)^*}{\alpha (\lambda L - g^*)^2 \left(\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K} \right)^* \right] \right)^2}. \end{aligned}$$

In Proposition 3, we restrict our attention to the case in which $\phi_K = 1$ and $0 < \phi_Z < 1$. Under this assumption, we have

$$\phi_K (1 + \sigma^S \alpha) - (1 - \alpha^2 \sigma^S) \phi_Z = (1 - \phi_Z) + \sigma^S \alpha (1 + \alpha \phi_Z) > 0.$$

Obviously, this inequality is sufficient (but not necessary) for $\Psi > 0$ to hold.

In order to analyze the effects of changes in η we substitute

$$\frac{\partial M_1}{\partial \eta} = \frac{\phi_K \sigma^S \left(\frac{C}{K}\right)^*}{\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^* \right]},$$

$$\frac{\partial M_2}{\partial \eta} = \frac{(\phi_K - \phi_Z) \left(\frac{C}{K}\right)^*}{\alpha \left(\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^* \right] \right)}$$

into the general representation of the solutions. Taking into account that, by assumption, $\phi_K = 1$ and $0 < \phi_Z < 1$, we obtain

$$\frac{\partial g^*}{\partial \eta} = \frac{1}{\Psi} \frac{\phi_Z \alpha \sigma^S \left(\frac{C}{K}\right)^*}{\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^* \right]} > 0.$$

According to Proposition 1, the growth rate g^* depends positively on the status parameter η if $\phi_Z = \phi_K = 1$ holds. Now we have shown by implicit differentiation that this property carries over to the more general case in which $0 < \phi_Z < 1$ and $\phi_K = 1$. This completes the proof of part i) of Proposition 3. ■

Remark 2. Many other results given in Proposition 1 for the special case $\phi_Z = \phi_K = 1$ remain valid in the more general case $0 < \phi_Z < 1$ and $\phi_K = 1$. For instance, we obtain

$$\frac{\partial (C/K)^*}{\partial \eta} = -\frac{1}{\Psi} \frac{[\phi_K (1 + \sigma^S \alpha) - (1 - \alpha^2 \sigma^S) \phi_Z] \left(\frac{C}{K}\right)^*}{\alpha \left(\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^* \right] \right)} < 0,$$

$$\frac{\partial (Y/K)^*}{\partial \eta} = \frac{\partial g^*}{\partial \eta} + \frac{\partial (C/K)^*}{\partial \eta} = -\frac{1}{\Psi} \frac{[\phi_K (1 + \sigma^S \alpha) - \phi_Z] \left(\frac{C}{K}\right)^*}{\alpha \left(\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^* \right] \right)} < 0.$$

The last results together with (D.1)–(D.18) imply that

$$\frac{\partial v^*}{\partial \eta} < 0, \text{ for } v = \frac{C}{K}, L_Y, \frac{Y}{K}, r, p, \frac{\pi}{p_A},$$

$$\frac{\partial v^*}{\partial \eta} > 0, \text{ for } v = L_A, \frac{x}{L_Y}, \varepsilon^K, \varepsilon^Z, r + \varepsilon^K, \frac{\pi}{p_A} + \varepsilon^Z, p_A, \frac{w}{A}.$$

Proof of ii)

In order to analyze the effects of changes in ϕ_Z , we substitute

$$\frac{\partial M_1}{\partial \phi_Z} = - \frac{\phi_K \eta (1 - \alpha) \sigma^S \left(\frac{C}{K}\right)^* \left(g^* + \left(\frac{C}{K}\right)^*\right)}{(\lambda L - g^*) \left(\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^*\right]\right)^2},$$

$$\frac{\partial M_2}{\partial \phi_Z} = - \frac{\eta \phi_K \left(1 + \frac{1 - \alpha}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^*\right]\right) \left(\frac{C}{K}\right)^*}{\alpha \left(\phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^*\right]\right)^2}$$

into the general representation of the solutions. This yields

$$\frac{\partial g^*}{\partial \phi_Z} = \frac{1}{\Psi} \frac{\phi_K \eta \sigma^S \left(\frac{C}{K}\right)^* \left\{ \phi_K (\alpha^2 + \eta) + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[(\alpha^2 + \eta) g^* + \alpha^2 \left(\frac{C}{K}\right)^* \right] \right\}}{\alpha \left\{ \phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^*\right] \right\}^3},$$

$$\begin{aligned} \frac{\partial (C/K)^*}{\partial \phi_Z} &= \frac{1}{\Psi} \frac{\phi_K \eta \left(\frac{C}{K}\right)^* \left\{ (1 + \sigma^S \alpha) \frac{1 - \alpha}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^*\right] + 1 - \alpha^2 \sigma^S \right\}}{\alpha \left\{ \phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^*\right] \right\}^2} \\ &+ \frac{1}{\Psi} \frac{\phi_K \phi_Z \eta^2 (1 - \alpha) \sigma^S \left[\left(\frac{C}{K}\right)^*\right]^2 \left[\lambda L + \left(\frac{C}{K}\right)^*\right]}{\alpha (\lambda L - g^*)^2 \left\{ \phi_K + \frac{\phi_Z (1 - \alpha)}{\lambda L - g^*} \left[g^* + \left(\frac{C}{K}\right)^*\right] \right\}^3}. \end{aligned}$$

These solutions imply that

$$\frac{\partial g^*}{\partial \phi_Z} > 0 \quad \text{and} \quad \frac{\partial (C/K)^*}{\partial \phi_Z} > 0.$$

These two results together with (D.1)–(D.3) yield

$$L_A^* = \frac{g^*}{\lambda} \Rightarrow \frac{\partial L_A^*}{\partial \phi_Z} = \frac{1}{\lambda} \frac{\partial g^*}{\partial \phi_Z} > 0,$$

$$L_Y^* = \frac{\lambda L - g^*}{\lambda} \Rightarrow \frac{\partial L_Y^*}{\partial \phi_Z} = -\frac{1}{\lambda} \frac{\partial g^*}{\partial \phi_Z} < 0, \quad (\text{D.34})$$

$$\left(\frac{Y}{K}\right)^* = g^* + \left(\frac{C}{K}\right)^* \Rightarrow \frac{\partial (Y/K)^*}{\partial \phi_Z} = \frac{\partial g^*}{\partial \phi_Z} + \frac{\partial (C/K)^*}{\partial \phi_Z} > 0. \quad (\text{D.35})$$

Using (D.5)–(D.15) and taking into account that $\partial g^*/\partial \phi_Z > 0$, $\partial L_Y^*/\partial \phi_Z < 0$, and

$\partial (Y/K)^* / \partial \phi_Z > 0$, we obtain

$$r^* = \alpha^2 \left(\frac{Y}{K} \right)^* \Rightarrow \frac{\partial r^*}{\partial \phi_Z} = \alpha^2 \frac{\partial (Y/K)^*}{\partial \phi_Z} > 0,$$

$$p^* = \alpha \left(\frac{Y}{K} \right)^* \Rightarrow \frac{\partial p^*}{\partial \phi_Z} = \alpha \frac{\partial (Y/K)^*}{\partial \phi_Z} > 0,$$

$$x^* = L_Y^* \left[\left(\frac{Y}{K} \right)^* \right]^{-1/(1-\alpha)}$$

$$\Rightarrow \frac{\partial x^*}{\partial \phi_Z} = \left[\left(\frac{Y}{K} \right)^* \right]^{-1/(1-\alpha)} \frac{\partial L_Y^*}{\partial \phi_Z} - \frac{L_Y^*}{1-\alpha} \left[\left(\frac{Y}{K} \right)^* \right]^{-(2-\alpha)/(1-\alpha)} \frac{\partial (Y/K)^*}{\partial \phi_Z} < 0,$$

$$\left(\frac{x}{L_Y} \right)^* = \left[\left(\frac{Y}{K} \right)^* \right]^{-1/(1-\alpha)}$$

$$\Rightarrow \frac{\partial (x/L_Y)^*}{\partial \phi_Z} = -\frac{1}{1-\alpha} \left[\left(\frac{Y}{K} \right)^* \right]^{-(2-\alpha)/(1-\alpha)} \frac{\partial (Y/K)^*}{\partial \phi_Z} < 0,$$

$$\pi^* = (1-\alpha) \alpha L_Y^* \left[\left(\frac{Y}{K} \right)^* \right]^{-\alpha/(1-\alpha)}$$

$$\Rightarrow \frac{\partial \pi^*}{\partial \phi_Z} = (1-\alpha) \alpha \left\{ \left[\left(\frac{Y}{K} \right)^* \right]^{-\alpha/(1-\alpha)} \frac{\partial L_Y^*}{\partial \phi_Z} - \frac{\alpha L_Y^*}{1-\alpha} \left[\left(\frac{Y}{K} \right)^* \right]^{-1/(1-\alpha)} \frac{\partial (Y/K)^*}{\partial \phi_Z} \right\} < 0,$$

$$p_A^* = \frac{1-\alpha}{\lambda} \left[\left(\frac{Y}{K} \right)^* \right]^{-\alpha/(1-\alpha)} \Rightarrow \frac{\partial p_A^*}{\partial \phi_Z} = -\frac{\alpha}{\lambda} \left[\left(\frac{Y}{K} \right)^* \right]^{-1/(1-\alpha)} \frac{\partial (Y/K)^*}{\partial \phi_Z} < 0,$$

$$\left(\frac{\pi}{p_A} \right)^* = \alpha \lambda L_Y^* \Rightarrow \frac{\partial (\pi/p_A)^*}{\partial \phi_Z} = \alpha \lambda \frac{\partial L_Y^*}{\partial \phi_Z} < 0,$$

$$\left(\frac{w}{A} \right)^* = (1-\alpha) \left[\left(\frac{Y}{K} \right)^* \right]^{-\alpha/(1-\alpha)} \Rightarrow \frac{\partial (w/A)^*}{\partial \phi_Z} = -\alpha \left[\left(\frac{Y}{K} \right)^* \right]^{-1/(1-\alpha)} \frac{\partial (Y/K)^*}{\partial \phi_Z} < 0,$$

$$r^* + (\varepsilon^K)^* = \left(\frac{\pi}{p_A} \right)^* + (\varepsilon^Z)^* = \frac{1}{\sigma^S} g^* + \rho$$

$$\Rightarrow \frac{\partial [r^* + (\varepsilon^K)^*]}{\partial \phi_Z} = \frac{\partial [(\pi/p_A)^* + (\varepsilon^Z)^*]}{\partial \phi_Z} = \frac{1}{\sigma^S} \frac{\partial g^*}{\partial \phi_Z} > 0,$$

$$(\varepsilon^Z)^* = \frac{1 + \alpha \sigma^S}{\sigma^S} g^* + \rho - \alpha \lambda L \Rightarrow \frac{\partial (\varepsilon^Z)^*}{\partial \phi_Z} = \frac{1 + \alpha \sigma^S}{\sigma^S} \frac{\partial g^*}{\partial \phi_Z} > 0,$$

$$\left(\frac{K}{K+p_{AA}}\right)^* = \frac{1}{1 + \frac{1-\alpha}{\lambda L_Y^*} \left(\frac{Y}{K}\right)^*}$$

$$\Rightarrow \frac{\partial [K/(K+p_{AA})]^*}{\partial \phi_Z} = -\frac{(1-\alpha) \left[L_Y^* \frac{\partial (Y/K)^*}{\partial \phi_Z} - \left(\frac{Y}{K}\right)^* \frac{\partial L_Y^*}{\partial \phi_Z} \right]}{\lambda (L_Y^*)^2 \left[1 + \frac{1-\alpha}{\lambda L_Y^*} \left(\frac{Y}{K}\right)^* \right]^2} < 0.$$

Obviously, the last result implies that

$$\frac{\partial [(p_{AA}) / (K + p_{AA})]^*}{\partial \phi_Z} > 0.$$

The results given above prove the validity of the following assertions made in ii) of Proposition 3:

$$\frac{\partial v^*}{\partial \phi_Z} > 0, \quad \text{for } v = \frac{C}{K}, \frac{Y}{K}, p, r, L_A, r + \varepsilon^K, \frac{\pi}{p_A} + \varepsilon^Z, \varepsilon^Z, \frac{p_{AA}}{K + p_{AA}},$$

$$\frac{\partial v^*}{\partial \phi_Z} < 0, \quad \text{for } v = \frac{x}{L_Y}, L_Y, x, \pi, \frac{w}{A}, p_A, \frac{\pi}{p_A}, \frac{K}{K + p_{AA}}. \quad \blacksquare$$

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