Auer, Raphael A.

Working Paper
Human Capital and the Dynamic Effects of Trade

Working Paper, Study Center Gerzensee, No. 15.03

Provided in Cooperation with:
Study Center Gerzensee, Swiss National Bank

Suggested Citation: Auer, Raphael A. (2015) : Human Capital and the Dynamic Effects of Trade, Working Paper, Study Center Gerzensee, No. 15.03

This Version is available at:
http://hdl.handle.net/10419/128092

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Human Capital and the Dynamic Effects of Trade

Raphael A. Auer

Working Paper 15.03

This discussion paper series represents research work-in-progress and is distributed with the intention to foster discussion. The views herein solely represent those of the authors. No research paper in this series implies agreement by the Study Center Gerzensee and the Swiss National Bank, nor does it imply the policy views, nor potential policy of those institutions.
Human Capital and the Dynamic Effects of Trade

Raphael A. Auer†

July 7, 2015

Abstract

This paper examines the cross-country income and welfare consequences of trade-induced human capital (dis-)accumulation. The model is based on heterogeneous workers who make educational decisions in the presence of complete markets. When such heterogeneous workers invest in schooling, high type agents earn a surplus from their investment. In the presence of cross-country differences in skill-augmenting technology, trade shifts this surplus to rich countries that can use skills more efficiently. Thus, while the static gains from trade may lead to convergence, the dynamic gains from trade occur to initially rich countries, thus leading to cross-country divergence of income and welfare. The second part of the paper endogenizes world prices, documenting that as trade liberalization concentrates skills in countries with a high level of skill augmenting technology, it thereby increases the effective global supply of skilled labor. Despite the resulting decline in the price of skill-intensive goods, trade is shown to be skill-biased.

Keywords: Factor Content of Trade, Employment, Human Capital, Economic Growth

JEL: F11, F14, and F16

---

*I thank Sylvain Chassang, Emmanuel Farhi, Guido Lorenzoni, Dan Trefler, Ivan Werning, and especially Daron Acemoglu and Xavier Gabaix for helpful discussions; seminar participants at the Institute for International Economic Studies, ETH Zurich, KU Leuven, MIT, Singapore Management University, the Swiss National Bank, and at the EEA-ESSEM meetings for comments; and Gian Humm for excellent research assistance. The views expressed in this document do not necessarily reflect those of the Swiss National Bank.

† Swiss National Bank and CEPR. Email: raphael.auer@snb.ch.
1 Introduction

Among economists and policymakers alike, there is agreement that import competition from low-wage countries has caused a decline in the relative wage of unskilled workers in rich nations. For example, building on Bernard et al. (2006) and Auer and Fischer (2010), Autor et al. (2013) document that over the period 1990 to 2007, import competition from China can account for around a third of the decline in US manufacturing employment.\footnote{Also see the discussion in Krugman (2008), who reconsiders his earlier verdicts that such effects are negligible.} Much less discussed is the flipside of this argument, namely that trade with richer nations tends to depress the relative wage of skilled workers in less developed countries and, therefore, the incentives to invest in education.\footnote{Atkin (2013) documents that the latter consideration is indeed empirically relevant: school dropout rates in Mexico increased substantially following the establishment of the North American Free Trade Area. His findings suggest that for every 20 new jobs created in the Mexican export industry, one student dropped out of high school.}

This paper examines the welfare consequences of trade-induced (dis-)accumulation of human capital in a model featuring within-country worker heterogeneity and cross-country differences in the relative productivity of human capital. The first assumption of the model is that workers are heterogeneous in their relative ability to provide skilled versus unskilled labor as in the seminal contribution of Findlay and Kierzkowski (1983) and its extension to a continuous distribution of abilities in Borsook (1987).\footnote{The mechanism at work is similar to the one in Galor and Mountford (2006 and 2008), who analyze how trade can influence population growth in a Malthusian setting and result in a less-educated workforce in initially poor countries.} The second assumption of the model is that countries differ in their level of human capital-augmenting technology. This assumption is based on Caselli and Coleman (2006), who show that – while the productivity of unskilled labor is similar across the world – there are substantial differences in the efficiency with which different nations use skilled labor.

Under these two modeling assumptions, liberalization is associated with divergence of the world distribution of income because it shifts educational investment to rich nations. Due to the underlying worker heterogeneity, also the gains from trade may favor rich nations: when heterogeneous workers invest in schooling, there exists a cutoff worker that is indifferent between choosing schooling or not, while all higher type workers receive a surplus from their investment. A trade-induced increase in the skill premium increases the expected lifetime income for the workers who already would have chosen schooling in the autarky economy. In addition, an increase in the relative wage induces more entry into the skilled labor force. In total, the net return from education – taking into consideration the opportunity cost of forgone unskilled labor – responds more than proportionally to changes in the relative wage. Skill scarce nations, in contrast, have their comparative advantage in unskilled labor, a factor that is in fixed supply and cannot be accumulated. Thus, trade may create divergence of welfare since richer nations gain proportionally the most from liberalization.\footnote{It is noteworthy that the model does not feature any externalities such as monopolistic competition or matching imperfections in the labor market. Rather, it is the nature of the worker heterogeneity itself that causes the gains from trade to be distributed asymmetrically across rich and poor countries.}
The second part of the paper evaluates the general equilibrium response of simultaneously opening many countries to trade. Since trade equates goods prices across the world, liberalization concentrates human capital in countries that are efficient at using skills. With open markets, the typical skilled worker is thus located in a country with a higher level of human capital-augmenting technology than in the closed economy. Thus, trade increases the relative output of skill-intensive goods, i.e., trade liberalization creates a skill bias. This result is closely related to the skill bias mechanism of Burstein and Vogel (2012), except that it derives from the trade-induced reallocation of human capital towards countries that are efficient at using skills rather than the within industry shift towards more skill intensive firms.

In emphasizing the impact of trade on factor accumulation, the model is related to the literature examining the dynamic implications of factor price equalization. This literature on dynamic Heckscher-Ohlin models dates back to studies analyzing long-run specialization patterns when saving rates are determined exogenously (see, among many examples, Bardhan (1966) or Stiglitz (1970)), while successive contributions derive the savings rate from microeconomic foundations. Contributions to this literature include, but are not limited to, frameworks showing that already small differences in technologies lead to complete specialization (see Baxter (1992)), and that differences in initial conditions can lead to hysteresis (Cuñat and Maffezoli (2004), Chen (1992), and Atkeson and Kehoe (2010)) and to multiple equilibria (Bond et al. (2003)). Mountford (1998) documents that even income reversals are possible in a dynamic Heckscher Ohlin model nested in the overlapping generations model of Galor (1992) (see also Mountford (1999)). Caliendo (2011) presents a general characterization of the conditions under which countries specialize and incomes converge; he also examines how non-monotonic changes in specialization throughout different phases of development can arise.

Compared to the literature on dynamic Heckscher-Ohlin models, the paper’s main contribution is to establish the cross-country welfare implications of trade liberalization in a model that takes into account the nature of human capital accumulation. In an important contribution, Baldwin (1992) establishes that trade-induced accumulation of physical capital has no welfare consequences in the absence of externalities. His main insight is simple: since the marginal cost of accumulating capital is constant and, in equilibrium, the return to capital equals precisely the constant cost of accumulation, investors do not record a net gain from the additional accumulated capital (as the envelope theorem would predict).

\[\text{Relatedly, Ventura (1997) examines the catchup process in a Ramsey model with trade in intermediate goods that embody accumulated capital, and demonstrates how convergence is achieved via price effects. A similar mechanism is present in Acemoglu and Ventura (2002), were price effects imply a stable world income distribution also for heterogeneous savings rates. In contrast, Cuñat and Maffezoli (2007) document how the dynamic response of factor accumulation amplifies the initial growth effect of a marginal trade liberalization.}\]

\[\text{Of course, it is likely that dynamic externalities matter a great deal for the accumulation of human capital. Baldwin’s (1992) main contribution is hence to establish a clear benchmark under which physical capital accumulation has no welfare implications and to then quantify departures from this benchmark under alternative assumptions on the size of externalities.}\]
The latter is different for the case of human capital accumulation, as the upward-sloping supply curve introduced by the presence of heterogeneous abilities implies that some workers earn a net surplus from their educational investment. The analysis of this paper demonstrates that owing to how trade shifts this net surplus from poor to rich countries, liberalization, while benefiting all nations, may also lead to divergence in welfare.\(^7\)

In modeling how human capital is accumulated, the present paper extends the work of Findlay and Kierzowski (1983) to the case of a continuous distribution of abilities. This continuous distribution of abilities is necessary, since it gives rise to the above-mentioned net surplus from education: the skill premium is determined by the cutoff worker who is indifferent between choosing schooling or not, while higher type workers earn a surplus from their educational decision.\(^8\) This feature of the model is also shared with Borsook (1987), who examines the effect of trade liberalization on within-country income dispersion in the presence of heterogeneous abilities, a complementarity between schooling and ability, and educational capital that can be used for schooling.

Compared to Borsook (1987), the contribution of this paper is to solve for the general equilibrium effects of trade liberalization in a model featuring a continuous distribution of worker abilities. In Borsook (1987), the cost of educational capital is only solved for graphically; in the context of his focus on within-country differences, this seems appropriate. However, there are also important cross-country differences that Borsook (1987) cannot analyze because they depend on the exact cost of educational capital in the open economy. This paper develops a model with heterogeneous abilities that can be solved for analytically, thus allowing to examine also the cross-country welfare implications of liberalization.

It is also noteworthy that the model developed in this paper results in fundamentally different cross-country effects of liberalization due to a difference in the modeling strategy: Borsook (1987) introduces a country-specific fixed amount of “educational capital.” With this assumption, a higher skill premium makes more people choose to become educated, thereby reducing the average educational capital per worker for those abilities that would have chosen to become educated also with a low skill premium. This effect is not present in the model at hand, in which the aggregate amount of investment in human capital is allowed to adjust freely.\(^9\)

\(^7\) This effect also discerns the model from the economy of Bond et al. (2003), who do not model the private nature of human capital accumulation. More generally, this paper is not the first to model accumulated human capital in a dynamic Heckscher Ohlin setting. Rather, the novelty is to take into account how human capital is accumulated differently than is physical capital, and to then show that this difference matters for the cross-country welfare implications of a trade liberalization in the presence of complete markets.

\(^8\) The fact that factor supply does respond to trade, yet that countries do not specialize completely due to the upward sloping supply curve of skilled labor introduced by the continuous distribution of worker types is of interest to the discussion on the conditions under which countries specialize completely in dynamic Heckscher Ohlin models of trade. For example, Caliendo (2011) examines the importance of country size and the dynamics of factor accumulation and income outside the cone of specialization; the latter is also examined in Cuñat and Maffezzoli (2004) and Banjona and Kehoe (2006).

\(^9\) Note that there does not exist any limit case of parameters that would map the model of Borsook (1987) into the one developed in this paper. The reason for this is that both the assumption of decreasing returns to education at the worker level and the assumption of a fixed amount of educational capital in the aggregate are necessary for
In emphasizing the general equilibrium response of human capital, the skill premium, and the price of skill intensive goods to trade liberalization, the model also relates to the literature documenting and rationalizing the skill bias of world trade, i.e., the fact that trade liberalization has increased the skill premium, yet decreased the relative price of skill-intensive goods.

The existing literature rationalizes this skill bias of global trade with what Burstein and Vogel (2012) term "skill bias mechanisms": the increased exposure to international trade has resulted in a higher relative productivity for skilled labor within all sectors and all countries alike. This includes Dinopolous and Segerstrom (1999) and Gancia and Epifani (2008), who argue that the skill-intensive sector is more sensitive to scale. A second class of models builds on the directed technical change literature, with contributions including Acemoglu and Zilibotti (2001) and Acemoglu (2003). Here, it is a combination of unequal protection of intellectual property rights and differential factor endowments that creates technical change biased towards skilled workers. By increasing the market size for skill-complementary technologies in those countries that have good intellectual property rights protection, trade increases the skill bias of global technology (see Gancia (2012) for an overview of this literature).

In contrast to these papers arguing that trade liberalization affects the incentive to innovate and thus technology itself, Burstein and Vogel (2012) offer an interpretation that is centered on the trade-induced shift towards large firms in the presence of export selection and the fact that large firms generally use more skill-intensive technologies. Trade thus does not change technology itself; rather, it shifts economic activity towards skill-intensive firms, which implies that the effective (weighted) level of human capital-augmenting technology increases in all sectors and countries.

Compared to this literature, the contribution of this paper is to highlight that trade can be skill-biased even in the absence of a skill bias mechanism. Since human capital is a factor that can be accumulated and is thus in essence mobile, its reallocation towards countries with higher human capital-augmenting technology creates a skill bias at the global level even if technology is unchanged at the country level. As in Burstein and Vogel (2012), the mechanism does not rely on changing technology, but on how trade liberalization shifts economic activity in the presence of static technological differences. In the model at hand, human capital reallocates towards countries with a high level of human capital augmenting technology, thus influencing the effective average technology at a global scale. In contrast to Burstein and Vogel (2012), however, this process rests exclusively on the basic forces of the Heckscher Ohlin model (also see Burstein and Vogel (2011)). Since the reallocation of human capital is driving effective technology at a global level, this effect could be termed the human capital reallocation mechanism.

The results regarding cross-country welfare differences and the skill bias of international trade derive from static cross-country differences in technology. In the main part of the analysis, the

---

10 This difference is important in terms of economic observables: whereas models emphasizing skill bias mechanisms predict trade to be associated with increases in the relative level of human capital augmenting technology in all sectors and countries alike, in the model at hand, the skill bias can only be observed at the global level.
latter are assumed to exist for exogenous reasons. In two extensions of the model, I show that the divergence of welfare and the global skill bias are amplified if technology is endogenized or if trade is associated with a skill bias mechanism. Most importantly, the second extension of the model documents that the human capital reallocation mechanism does not counteract the skill bias mechanism. In fact, these two mechanisms tend to reinforce each other.

The structure of the paper is as follows. Section 2 sets up an economy featuring heterogeneous and finitely lived workers who invest in human capital and characterizes the resulting autarky equilibrium. Section 3 establishes the income and welfare effects of opening a small economy to trade. Section 4 endogenizes world prices and establishes the skill bias of world trade. Section 5 presents extensions of the model endogenizing technology and allowing for additional skill bias mechanisms. Section 6 concludes.

2 A Model of Heterogeneous Workers and Cross-Country Differences in Technology

The framework of this paper builds upon Findlay and Kierzkowski (1983), who propose a general equilibrium model of human capital accumulation in the presence of international trade. I depart from their model with two key assumptions: countries are characterized by exogenously given differences in the efficiency of human capital (see Caselli and Coleman (2006) and also Trefler (1995)). Second, building on Borsook (1987), I assume that, while workers are homogeneous in how well they can provide unskilled labor, there is a continuous distribution of abilities determining how effective workers can supply skilled labor if they chose to get an education.

2.1 Preferences, Production Relations and Demography

The model is formulated in continuous time, which is indexed by $t$ ($t \geq 0$). The world economy consists of many small countries that are indexed by $i$. Each country $i$ has mass 1 of identical and infinitely lived households. Each household is composed of a mass of heterogeneous and finitely lived workers. Households take education decisions for workers and have preferences over consumption that are additive, time separable, and exhibit a constant rate of time preference so that

$$V(t,i) = \int_t^\infty U(C_{t,i}) e^{-\delta(\tau-t)} d\tau.$$  

I assume that $U$ is strictly increasing, strictly concave and twice continuously differentiable, with $U'(0) = \infty$. Infinite marginal utility at $C_{\tau,i} = 0$ is assumed for convenience so that the economy is never on a path where investment is equal to zero for all times. A standard budget constraint applies, which restricts the net present cost of the path of consumption being at most as large as the net present value of future income. Let $Y_{i,\tau}$ denote a country’s production. The budget
constraint of the representative household is given by

\[
\int_{t}^{\infty} C_{t} \sigma e^{-\int_{t}^{\infty} r_{e} \sigma - \sigma_{t} \rho_{t}} d\tau \leq \int_{t}^{\infty} Y_{i,t} e^{-\int_{t}^{\infty} r_{e} \sigma - \sigma_{t} \rho_{t}} d\tau + B_{t,i}.
\]

(2)

The interest rate \( r_{t} \) is not country specific, i.e. global capital markets exist. \( B_{t,i} \) denotes the net asset position of country \( i \).\(^{11}\)

Omitting time subscripts, Final output \( Y_{i} \) is defined over a constant elasticity of substitution (CES) aggregate of a skill-intensive and a labor-intensive good. Denoting the amount of the labor-intensive intermediate goods used in production by \( X_{l,i} \) and the amount of the human capital-intensive good by \( X_{h,i} \), final output in country \( i \) is given by

\[
Y_{i} = \left( X_{l,i}^\beta + X_{h,i}^\beta \right)^{1/\beta}.
\]

(3)

The final good is produced competitively. The elasticity of substitution between the two intermediate goods is constant and equal to \((1 - \beta)^{-1}\). Throughout the analysis, I assume that the intermediate goods are gross substitutes, i.e. that \( 0 < \beta < 1 \). This assumption implies that price effects are not extreme so that in equilibrium, a human capital abundant economy is characterized by a low price of skill-intensive goods, yet still large total expenditures on skill-intensive goods. Autor et al. (1998) have estimated the elasticity between skilled and unskilled labor directly. They conclude that it is unlikely to fall outside the interval \([1, 2]\), which in this model corresponds to \( 0 < \beta < 0.5 \).

I denote the prices of the two intermediate goods in country \( i \) by \( p_{l,i} \) and \( p_{h,i} \). Normalizing the price of the final good to unity implies \( p_{h,i}^{-\frac{1}{1-\beta}} + p_{l,i}^{-\frac{1}{1-\beta}} = 1 \).

The two intermediate goods are produced from two factors, human capital and "raw" unskilled labor. Human capital \( H_{i} \) can be used to produce the skill-intensive good using a linear transformation technology. Labor \( L_{i} \) can be used to produce the labor-intensive good using a linear transformation technology. In the remainder of the analysis, I sometimes refer to these two goods as the skill-intensive sector and the labor-intensive sector respectively. While raw labor can be used equally efficient in all countries, I assume that the effectiveness of human capital depends on some exogenously given country specific parameter \( A_{i} \).\(^{12}\)

Denoting the output of the skill-intensive good in country \( i \) by \( Y_{h,i} \) and the output of the labor-intensive good by \( Y_{l,i} \), it holds that

\[
Y_{l,i} = L_{i} \quad \text{and} \quad Y_{h,i} = A_{i} H_{i}.
\]

(4)

\(^{11}\)This implies that final output can always be traded so that countries can borrow, lend, and repay to each other.

\(^{12}\)These cross-country differences in \( A_{i} \) can be seen as stemming from differences in the institutional setup of a country, see Caselli and Coleman (2006).
The two intermediate goods are produced competitively. There are no factors of production other than human capital and labor. Equation (4) incorporates the simplification that production in each sector requires either only unskilled labor or only human capital.\textsuperscript{13}

Regarding the supply of skilled and unskilled labor, each household consists of a mass of heterogeneous and finitely lived workers. Per household and unit of time, a mass of $\delta$ workers is born. Young workers are of type $\theta$ and can spend time educating themselves. If they choose to get an education, they enter the labor force after a fixed period of time $T$ and start supplying one unit of unskilled labor and $\theta$ units of skilled labor. Workers that do not get an education supply one unit of unskilled labor from their moment of birth. After entering the labor force, all agents face a constant and age-independent rate of death $\delta$.

Types are distributed equally in all households and countries following a Pareto density function with shape parameter $\left(1 - \eta\right)^{-1}$ and scale parameter $\eta c$:

$$F (\theta) = 1 - \left(\frac{\eta c}{\theta}\right)^{\frac{1}{\eta}}.$$ \tag{5}

The parameter restrictions $0 < \eta < 1$ and $0 < c$, as well as the lower bound of $\theta \geq \eta c$, apply. A lower $\eta$ is associated with more heterogeneous workers. The scale parameter in (5) is chosen such that $\eta$ does not affect the average type and it is always true that $E (\theta) = c$. With this formulation, a decrease of $\eta$ is a mean-preserving spread of the distribution of types.

For each type $\theta$ and each cohort born at time $\tau$, the households decide whether the worker does get an education or not. Let $h (\tau, i, \theta)$ denote the education decision for a worker of type $\theta$ in country $i$ born at time $\tau$. $h (\tau, i, \theta)$ equals 1 if this worker gets an education and 0 otherwise. There is no cost of education other than time spent in school.

Since workers are perfectly substitutable, the total supply of human capital is given by the sum over past education decisions adjusted for types, the probability of survival, and whether a worker is currently schooling or working. It thus holds that

$$H_{t,i} = \delta \int \int e^{-(t-(T+\tau))\delta} \int f (\theta) \Upsilon_{t,i,\tau} h (t, i, \theta) \theta d\theta d\tau,$$ \tag{6}

where $\Upsilon_{t,i,\tau} = 1$ if $t \geq T + \tau$ and 0 otherwise, i.e. $\Upsilon_{t,i,\tau}$ denotes the indicator function that equals 1 if a worker has left school and is equal to 0 otherwise. Since education is restricted to take place at the beginning of an individual’s life, $\Upsilon_{t,i,\tau}$ takes the value 1 whenever $\tau \leq t - T$. Similarly, the aggregate supply of labor takes into consideration that some agents are currently in school and is

\textsuperscript{13}As is well known from the classical theory of trade, a generalization of the model at hand with both goods requiring both factors of production at different intensities would not change the results as long as countries have similar enough factor supplies such that factor price equalization holds in the open economy equilibrium.
given by

\[ L_{i,t} = \delta \int_{-\infty}^{t} e^{-\delta(t-\tau)} \int_{\theta} f(\theta) (1 - h(t, i, \theta)) d\theta d\tau + \delta \int_{-\infty}^{t} e^{-(t-(T+\tau))\delta} \int_{\theta} f(\theta) \Upsilon_{t,i,\tau} h(t, i, \theta) d\theta d\tau. \]  

(7)

Supply of services from labor \( L \) comes from two groups: unskilled workers and skilled workers who have finished their education.

### 2.2 Autarky Wage Patterns

This subsection establishes the equilibrium in a closed economy. Before solving for the stationary equilibrium path of the economy in autarky, I establish the instantaneous competitive equilibrium. Thereafter, I establish the stationary equilibrium and highlight the origin of income and consumption differences in autarky.

**Definition 1** A feasible autarky allocation in country \( i \) given the supply of labor (7) and the supply of human capital (6), consists of functions \([h(t, i, \theta), Y_{i,t}, C_{t,i}]\) that satisfy (4) and (2) such that the integral over (1) is finite and well defined. A resource constraint restricting input use in (3) to \( X_{l,i} \leq Y_{l,i} \) and \( X_{h,i} \leq Y_{h,i} \) applies.

At each point in time \( t \), there are perfectly competitive spot markets for the two intermediates and the final good. Non-satiation of the instantaneous utility together with the strictly positive marginal product of inputs in (3) ensures that all inequalities hold. I first establish the instantaneous equilibrium given factor supplies. For simplicity, I drop time subscripts \( t \) unless there is danger of confusion. I denote the wage of raw labor by \( w_{l,i} \), the factor return of one unit of human capital by \( w_{h,i} \) and the relative wage by \( w_i = \frac{w_{h,i}}{w_{l,i}} \). Profit maximization by competitive final goods producers (3) relates the relative price of intermediate goods to relative input use. Also, I denote the relative prices of the skill-intensive good in country \( i \) by \( p_i \), implying

\[ p_i \equiv \frac{p_{h,i}}{p_{l,i}} = \left( \frac{Y_{h,i}}{Y_{l,i}} \right)^{-(1-\beta)}. \]  

(8)

Intermediate goods are produced using a linear transformation technology and (8) also determines the relative wage

\[ w_i = A_i^\beta \left( \frac{H_i}{L_i} \right)^{-\beta}. \]  

(9)

The relative wage is increasing in the efficiency of technology but decreasing in the relative abundance of human capital. Since the price of the final good is normalized to 1, the relative price \( p_i \)
alone pins down \( p_{t,i} \) and \( p_{h,i} \), and consequently also wages.

Each household chooses the education taking the actions of other households in the economy as given. A strategy for a household is a subset of each cohort of workers that are sent to the educational sector and the intertemporal consumption decision. I evaluate first the education decision \( h(t,i) \) of each household. Since there exist perfect capital markets, each household maximizes the net present flow of labor income from each worker. Denote by \( N(t,i,\theta,h) \) the net present value of the lifetime income that a worker of type \( \theta \) born at \( t \) in country \( i \) receives when the education decision is \( h(t,i,\theta) \). Income is discounted to the point of birth \( t \) of the respective worker and equal to

\[
N(t,i,\theta,h) = \begin{cases} 
\int_t^\infty w_{t,\tau,i} e^{-\int_t^\tau \delta + r(\nu) d\tau} d\tau & \text{if } h(t,i,\theta) = 0 \text{, or} \\
\int_{t+T}^\infty (\theta w_{h,\tau,i} + w_{t,\tau,i}) e^{\delta T - \int_t^\tau \delta + r(\nu) d\tau} d\tau & \text{if } h(t,i,\theta) = 1.
\end{cases}
\] (10)

The effective cost of education is giving up the unskilled wage from time \( t \) to \( t + T \). The benefit is the additional income equal to \( \theta \) times the skilled wage from time \( t + T \) onwards. Along any path of the economy, (10) leads to a threshold for the worker type and the education decision of a household: if it is optimal for a household to choose \( h(t,i,\theta) = 1 \), then the same is true for any other type \( \theta' > \theta \). Therefore, there exists a cutoff level \( \theta_{i,t} \) such that all types \( \theta \geq \theta_{i,t} \) get an education and all other types do not.

The main results of this paper concern cross-country comparisons of aggregate gains from trade. I therefore define the aggregate net present income from the current cohort of workers \( I_{t,i} \). Total income is equal to the integration of the maximal income (10) over types. This defines the discounted flow of income from the current generation of workers, which is given by

\[
I_{t,i} = \int_0^\infty f(\theta) \max_{h(t,i,\theta)} N_{t,i}(\theta, h) d\theta.
\] (11)

There is no aggregate uncertainty in this economy. Given (11) for past, present and future generations, the household has a separate consumption decision. Optimization of intertemporal utility (1) subject to (2) yields a familiar result for the slope of the consumption process.

**Definition 2** A competitive static equilibrium, given by the initial stock of human capital (6), labor (7) and \( A_i \) consists of a feasible allocation of functions for \( [c(\tau,i), T(t,i), r(t), p(x_i)] \) such that (8) and (9) hold, \( h(t,i,\theta) \) maximizes lifetime income for all cohorts (11) and the path of consumption maximizes (1) subject to (2).

I next consider the existence and uniqueness of the autarky stationary equilibrium. Let an "A" superscript denote expressions along such a stationary equilibrium, in which the relative price is constant and equal to \( p^A_{t} \), the relative wage is a function of \( A_i \) and \( p_i \) and the interest rate is
stable. Households choose a cutoff level $\theta^A_i$ and, since there is no technological progress, output and consumption are constant. Convergence to a stationary equilibrium is established easily because investment and intertemporal consumption decisions are independent. First, evaluate the cutoff condition (12) along any path of development. A single household has no influence on wages or interest rates. Even if it is optimal to school all types of workers, there is still a well-defined and finite supply of unskilled and skilled labor for any path of wages and interest rates that leads to a finite net discounted value of income. Arbitrage considerations ensure a non-negative rate of interest at any point in time. A nonzero interest rate combined with a positive rate of death $\delta$ implies that the discounted value of income is finite for any worker. Hence, intertemporal income of a household is always defined. By standard arguments, time separable and concave preferences combined with a constant rate of time preference lead to a constant interest rate of $r = \rho$ along any path with stable income. If $\rho > 0$, a unique and stable stationary equilibrium exists in which the choice of the cutoff point is a constant function of the interest rate and the autarky wages $w^A_{h,i}$ and $w^A_{l,i}$. Evaluating the entry condition (10) at the worker of type $\theta = \theta^A_i$ who is indifferent between going to school or not, this cutoff level solves
\begin{equation}
    w^A_{i,i} = e^{-\rho T} \left( \theta^A_i w^A_{h,i} + w^A_{l,i} \right).
\end{equation}

Given the optimal choice of $\theta^A_i$, one can solve for the maximal net present value of income from the present cohort of workers, which is given by (11) in autarky. Along any path of the economy with constant wages and cutoff level $\theta_i$, I denote the net present value of income from the current cohort of workers by $I(\theta_i, w_{l,i}, w_{h,i})$. Without assuming any specific distribution of types, it is always possible to express the net present income of a cohort of workers depending exclusively on the two wages. Evaluated at $\theta^A_i$, the total income discounted to the point of birth of a generation of workers is equal to
\begin{equation}
    I(\theta^A_i, w_{l,i}, w_{h,i}) = \frac{\delta}{\rho + \delta} \left( 1 + e^{-\rho T w_{h,i}} \int_{\theta^A_i}^{\infty} f(\theta) \left( \theta - \theta^A_i \right) d\theta \right) w_{l,i}.
\end{equation}

For any relative wage $w_i = \frac{w_{h,i}}{w_{l,i}}$, income is at least equal to $\frac{\delta}{\rho + \delta} w_{l,i}$. There are $\delta$ young workers who could start working right away and earn the unskilled wage forever. The future is discounted at rate $\rho + \delta$ to account for the probability of death. Secondly, for any $w_{h,i} > 0$, there may exist high type agents that find it worthy to get an education. The marginal worker of type $\theta = \theta^A_i$ just breaks even on his educational investment, but for all workers of higher type $\theta$, the possibility to get educated increases their lifetime income.

I solve for general equilibrium prices, wages, and incomes (13) in the case of the Pareto distribution of types (5). In the autarky stationary equilibrium the only source of cross-country variation is $A_i$. Solving the supply of labor (7) and human capital (6) for the constant cutoff $\theta^A_i$,
factor supply is given by \( L^A_i = 1 \) and
\[
H^A_i = \lambda^\frac{1}{\eta} A^\frac{\eta}{1-\eta},
\]
where \( \lambda \equiv \eta^\frac{\eta}{1-\eta} \left( e^{\rho T} - 1 \right)^{-\frac{\eta}{1-\eta}} e^{\frac{\eta}{1-\eta}} \). In equilibrium, the higher a country’s relative efficiency of human capital \( A_i \), the more skill abundant is this country. With the supply of factors given, prices (8) and consequently wages (9) are determined uniquely. In autarky, skill abundant countries have a lower relative price of the skill-intensive good, but still a higher relative wage.

The relative abundance of factors, technology and the normalization of the price of the final good to 1 relate the equilibrium unskilled wage \( w^A_{l,i} \) to the level of domestic skill-augmenting technology \( A_i \) as
\[
w^A_{l,i} = \left( 1 + \lambda A^{\frac{1}{1-\eta}}_i \right)^{\frac{1-\eta}{\eta}}.
\]
A country that is characterized by a high \( A_i \) has a low autarky price of the skill-intensive good. Because the normalization of the final good relates relative and absolute prices one to one, the price of the labor-intensive good is high in these countries. Since each unit of raw labor can produce one unit of the unskilled good it thus receives a high wage. I denote stationary output by \( Y(\bar{\theta}_i, w^A_{l,i}, w^A_{h,i}) \), which in autarky is equal to
\[
Y(\bar{\theta}_i, w^A_{l,i}, w^A_{h,i}) = \left( 1 + \lambda A^{\frac{1}{1-\eta}}_i \right) w^A_{l,i}.
\]
In equilibrium, a country that is characterized by a high efficiency of human capital has a high level of net income (16), i.e. it is "rich." The stationary net present income (11) of each young cohort of workers is equal to the total income from skilled labor plus the net income from human capital
\[
I(\bar{\theta}_i, w^A_{l,i}, w^A_{h,i}) = \frac{\delta}{\rho + \delta} \left( 1 + e^{-\rho T} (1 - \eta) \lambda A^{\frac{1}{1-\eta}}_i \right) w^A_{l,i}.
\]
High \( A_i \) countries have a high level of net income and are rich. Because of the convenient Pareto distribution of types, the net income from human capital is equal to a fraction \( e^{-\rho T} (1 - \eta) \) of the total income from skilled labor services.

How does the heterogeneity of workers influence the lifetime income of a cohort of workers? Consider first the case of homogeneous types (\( \eta \to 1 \)), in which all workers earn \( w^A_{l,i} \). The model then becomes very similar to that of Findlay and Kierzkowski (1983). All workers earn the unskilled wage (15) and technology differences matter only through relative supply and price effects: a country with high \( A_i \) is characterized by a high supply of human capital and hence a lower price of the skill-intensive good. A low price of the skill-intensive good implies a high price of the labor-intensive good and consequently a high unskilled wage. Consider now the case of a decrease in \( \eta \), i.e. a mean preserving spread of the distribution of types. In autarky equilibrium a
low \( \eta \) is associated with a large share of surplus as a fraction of total revenue of the skill-intensive sector.\(^{14}\)

More important than the impact of \( \eta \) on absolute levels of income and output is the impact it has on relative cross-country differences. Nations intrinsically only differ with respect to their level of human capital-augmenting technology \( A_i \). The heterogeneity of workers guides how differences in technology translate into differences of income and factor abundance. If types are similar, small differences in human capital efficiency translate into large differences of relative factor abundance and income. If the degree of worker heterogeneity is large, differences in \( A_i \) translate into only moderate differences in factor endowments: the more spread the distribution of types is, the lower is the density of workers at any point along the distribution \( F(\theta) \). For a given intrinsic difference in \( A_i \), the resulting international dispersion of relative factor supply is larger if the distribution of workers is more homogeneous.

Cross-country differences are influenced by the elasticity of substitution between skill- and labor-intensive intermediate goods. Consider first the case of \( \beta \) larger than, but close to 0. In this case, price effects in (3) are offsetting differences in technology and countries have nearly identical factor supplies. Countries thus only differ in their level of technology and hence output. A higher beta is associated with weaker price effects and thus increasingly pronounced cross-country differences in autarky factor supply. In the case of \( \beta = 1 \), the production of the final good (3) is linear in inputs used, relative input prices are fixed and therefore international factor abundance levels are very different.

\section{3 Income and Welfare Effects of Liberalization}

The notion that exchange – if it happens in the absence of externalities – must benefit all involved parties is an axiomatic insight of economic theory, and the same is true for exchange between countries. But how are the gains from trade split up between nations at different stages of their economic development? This section establishes the gains from liberalization in a partial equilibrium setting taking as given world prices. The global prices derived in the next section.

The structure of the present section is the following. First, as a benchmark model, I establish the gains from trade that would prevail in a world where education decisions are fixed at autarky levels. This exercise is a relevant benchmark, as the gains from trade in this static setting are equivalent to the welfare effects in a standard Heckscher Ohlin model of trade with factors of production being in fixed supply. In this static setting, a country gains from trade because it is different from the rest of the world. I show that the initial gains from trade are likely to lead to neutral gains from trade that favor neither rich nor poorer countries, thus leaving the relative

\(^{14}\)For given wages and therefore cutoff level \( \bar{\theta}_i \), the supply of skilled workers (6) is lower if types are more heterogeneous. Although the expected value of the distribution of types is unaffected by \( \eta \), the truncated expected value (that is the expected value given that the type is higher than \( \bar{\theta}_i \)) actually increases with \( \eta \). This effect is captured in the value of \( \lambda \).
dispersion of income and welfare unchanged.

Second, I establish the long run impact of liberalization after human capital has adjusted to trade. Because factor accumulation is costly, one has to distinguish between income divergence and divergence of welfare. It always holds that the steady state of an open world is characterized by larger differences in human capital abundance and also larger output differences than under autarky. Regarding welfare, there are always additional efficiency gains that occur to countries because the educational decisions can adjust to international prices. However, because of the way in which trade affects the surplus from education, welfare diverges when human capital adjusts to trade. Finally, I develop conditions for when trade leads to absolute divergence of welfare compared to autarky and argue that these conditions are likely to hold in reality.

3.1 The Static Impact of Liberalization

The focus of this paper is to investigate how the adjustment of human capital impacts the gains from trade. To highlight the mechanism at work requires comparing the results with what would happen in a world without such adjustment. This subsection thus establishes the static impact of an unanticipated liberalization, which is equivalent to the gains from trade that would prevail in a static Heckscher Ohlin model without factor accumulation.

Assume a small country $i$ has a level of human capital efficiency of $A_i$ and is in its autarky stationary equilibrium. At point in time $t^*$, markets are unexpectedly opened to trade with a large world that is characterized by $A_w$ and a resulting relative price of the skill-intensive good $p_w = \frac{1}{\lambda A_w (1 - \eta \beta)}$. $A_w$ is endogenized in the next section. Instantaneously after opening to trade, output of country $i$ is given by autarky factor supplies (6) and (7), but valued at international prices

$$Y \left( \tilde{\tilde{y}}, w_i, A_i p_w \right) = \left( 1 + \lambda \left( \frac{A_i}{A_w} \right)^{1 - \eta \beta} A^{\frac{\eta \beta}{1 - \eta \beta}} \right) \left( 1 + \lambda A_w^{\frac{\eta \beta}{1 - \eta \beta}} \right)^{1 - \beta \lambda}.$$  

Opening to trade has two effects on income: it influences both relative wage $w_i$ and the unskilled wage $w_l$. These two effects always work in opposite directions. If a country is more skill abundant than the rest of the world ($\frac{A_i}{A_w} > 1$), it benefits from trade because the relative wage $w_i$ increases, but at the same time it loses from trade because the unskilled wage decreases. The opposite is true for a country $j$ that is less skill abundant than the rest of the world.

Trade benefits all economies and the gains from trade are the larger, the more different the economy is from the rest of the world. A country that happens to have autarky prices that are equal to the prices in the rest of the world is not affected by trade; all other countries strictly gain from trade and these gains are increasing in $|A_i - A_w|$. The intuition for this result follows from Falvey et al. (2010) analyze the impact of anticipated and unanticipated trade liberalizations, focusing on the impact of skill upgrading of unskilled workers (and how it differs across workers of different age and ability) in response to trade liberalization.
standard trade theory. Each country faces a concave frontier of how much it can supply of the two factors and because there are no market failures, the current supply is on and not inside this frontier. Statically, factor supply is fixed, but trade can change the relative price. At any relative price, the input constraint of final goods producers under trade passes through the current factor supply (4), is tangent to the concave factor supply frontier and hence encompasses the latter. Trade enables producers to a strictly larger set of input bundles, and since production isoquants are convex, output increases.\(^\text{16}\)

At the moment of liberalization, do poor or rich nations benefit relatively more from liberalization? The answer to this question on convergence or divergence involves comparing income differences before and after opening to trade, i.e. four different levels of income. To establish the direction of relative gains from trade, I evaluate income differences for two small economies, a country form the North \((n)\) and a country from the South \((s)\). I assume that the North is skill abundant compared to the rest of the world, so that that \(A_n = (1 + \gamma)A_w\), where \(\gamma > 0\). South is skill scarce and I assume that \(A_s = (1 + \gamma)^{-1}A_w\). \(n\) and \(s\) are hence symmetrically different from the rest of the world. If for every pair of countries defined in this way there is divergence of output, I speak about uniform relative divergence.

**Definition 3 (Uniform relative Di- and Convergence)** Let \(n\) and \(s\) be two small countries with \(A_n = (1 + \gamma)A_w = (1 + \gamma)^2A_s\). There is uniform relative divergence (convergence) of output if trade results in an increase (decrease) of relative income differentials for every \(\gamma > 0\) and for every \(A_w\).

In this section, statements of convergence or divergence will be made for country pairs. If, for all possible pairs, opening to trade differences in output and net present income are increased, one can make statements regarding the world distribution of income.\(^\text{17}\)

The appealing feature of the definition at hand is that it helps to establish for which range of world prices there will be divergence when opening to trade. The following lemma establishes instantaneous effects from trade.

**Lemma 1 (Static Output Effects of Trade)** Consider the moment of opening to trade \(\tau^*\). There is uniform relative convergence (divergence) of output if the global size of the labor-intensive sector is smaller (larger) than the human capital-intensive one.

\(^{16}\)This can formally be shown by evaluating the first order condition of the ratio of (18) divided by (16) with respect to \(A_i\). The minimum level of this ratio is equal to 1 and occurs at \(A_i = A_w\). Evaluating the second order condition of this ratio establishes that countries that are more different from the rest of the world gain relatively more from trade.

\(^{17}\)A caveat with the adopted definition of uniform relative di- or convergence concerns the role of relative country size. In this section, \(A_w\) is given exogenously, and the analysis implicitly assumes that \(n\) and \(s\) are both small economies. Obviously, the results established in this section would not go through for example if a very large rich country and a very small poor country liberalize (in that case only the small country gains from trade). Note, however, that the next section endogenizes \(A_w\), which is possible for the case of non-negligible countries.
Proof. The appendix establishes that

\[
\frac{Y(\tilde{\theta}_n^A, w_{l,w}, A_n P_w)}{Y(\tilde{\theta}_s^A, w_{l,w}, A_s P_w)} \begin{cases} 
\geq 1 & \text{if } \lambda A_w^{\frac{\beta}{1-\sigma}} \leq 1 \text{ and } \\
< 1 & \text{if } \lambda A_w^{\frac{\beta}{1-\sigma}} > 1.
\end{cases}
\]

It is also true that if \( \lambda A_w^{\frac{\beta}{1-\sigma}} > 1 \), the skill-intensive sector is larger in terms of output and revenue than the labor-intensive sector. ■

If the skill-intensive sector is large there is convergence. This result seems striking at first sight, but thinking in terms of wages offers a good intuition. If the skill-intensive sector is large, the gains from trade for unskilled labor are relatively large because unskilled labor is a globally scarce factor. Poor countries that export labor then benefit more from trade than do rich countries.

Under realistic parameterizations, the static gains from trade occur roughly equiproportional to rich and poor nations. Mankiw et al. (1992) estimate that the global income share of human capital is about as large as the one of unskilled labor. A similar comparison can be made based on the calculations of Hall and Jones (1999): estimates suggest that they are of about the same size. Hence, trade is in a static sense neither likely to favor poor nor rich nations.

3.2 The Open Economy Steady State

Throughout the following analysis, I denote open economy expressions with an "O" superscript. The new optimal cutoff level for the education decision is hence denoted by \( \tilde{\theta}_i^O \) and solves

\[
\frac{\partial u}{\partial \theta} = \frac{e^{-\rho T} \left( A_i^O A_i^w p_h + w_l \right)}{Y(\tilde{\theta}_i^O, w_{l,w}, A_i P_w)} \frac{Y(\tilde{\theta}_i^O, w_{l,w}, A_i P_w)}{Y(\tilde{\theta}_i^O, w_{l,w}, A_i P_w)} 
\]

The resulting long term level of output is given by

\[
Y(\tilde{\theta}_i^O, w_{l,w}, A_i P_w) = \left( 1 + \lambda \left( \frac{A_i}{A_w} \right)^{\frac{1}{1-\sigma}} A_i^w \right) \left( 1 + \lambda A_i^w \right)^{\frac{1-\beta}{\rho}}. \tag{20}
\]

For any country \( n \) with \( \frac{A_n}{A_w} > 1 \), the long run level of output is necessarily larger than the one prevailing at the moment of opening to trade. This reflects the increased investment activity compared to autarky. Similarly, the long term level of output under trade for any country \( s \) with \( \frac{A_n}{A_w} < 1 \) is necessarily smaller than the one prevailing just after autarky. The following proposition summarizes trade-induced changes of output after opening to trade.

Proposition 2 (Trade and the Dynamics of Income) Let \( n \) and \( s \) be two small countries with \( A_n = (1 + \gamma) A_w = (1 + \gamma)^2 A_s \). There is uniform relative divergence of output comparing the output just after opening to trade (18) to the one in the stationary equilibrium under free trade (20). There is also uniform relative divergence of output comparing the output in autarky stationary equilibrium (16) to the stationary equilibrium under free trade (20).
Proof. see appendix

Output must diverge after opening to trade as trade increases investment rates in rich countries while it decreases them in poor countries. Naturally, this leads to a more disperse distribution of incomes.

Regarding welfare, there are two questions of interest. The first is whether countries gain from trade and the second is whether countries gain more from trade than they would have if the education choice had not adjusted. First evaluate the net present value of income for cohorts of workers born at or after \( \tau^* \) if the cutoff point had not changed from its autarky level \( \sigma_i^A \), so that

\[
I \left( \sigma_i^A, w_{l,w}, A_ip_w \right) = \frac{\delta}{\rho + \delta} \left( 1 + \left( \frac{A_i}{A_w} \right)^{\frac{1}{1-\eta}} - \eta \left( \frac{A_i}{A_w} \right)^{\frac{\beta}{1-\eta}} \right) \lambda e^{-\rho T} A_w^{\frac{\beta}{1-\eta^2}} w_{l,w}. \tag{21}
\]

Compare this to the level of net present income that the same cohort of workers receives from adjusting to the new optimal cutoff level \( \sigma_i^O \):

\[
I \left( \sigma_i^O, w_{l,w}, A_ip_w \right) = \frac{\delta}{\rho + \delta} \left( 1 + (1-\eta) \left( \frac{A_i}{A_w} \right)^{\frac{1}{1-\eta}} \lambda e^{-\rho T} A_w^{\frac{\beta}{1-\eta^2}} \right) w_{l,w}. \tag{22}
\]

(21) is the net present value that a worker born just before \( \tau^* \) receives. For all \( A_i \) and any \( A_w \), there are gains from trade also when the cutoff remains at \( \sigma_i^A \). There are additional gains from trade when \( \sigma_i \) adjusts optimally (i.e. \( I \left( \sigma_i^O, w_{l,w}, A_ip_w \right) \geq I \left( \sigma_i^A, w_{l,w}, A_ip_w \right) \geq I \left( \sigma_i^1, w_{l,1}, A_i^1, w_{h,1}^1 \right) \)).

Consider again the impact of worker heterogeneity \( \eta \) on relative cross-country differences in welfare in the open economy in (22) and compare it to such differences in autarky (17). In both the open and the closed economy, if types are similar, small differences in human capital efficiency translate into large differences of relative factor abundance and income. However, in the autarky economy, differences in factor supply are dampened by an adjusting skill premium, a channel absent in the open economy. What happens in relative terms? The following proposition establishes whether there is divergence of net present income.

**Proposition 3 (Post Opening Divergence)** Let \( n \) and \( s \) be two small countries with \( A_n = (1+\gamma) A_w = (1+\gamma)^2 A_s \). It is always the case that comparing \( I \left( \sigma_i^O, w_{l,w}, A_ip_w \right) \) to \( I \left( \sigma_i^A, w_{l,w}, A_ip_w \right) \), there is uniform relative divergence. There is uniform relative divergence of \( I \left( \sigma_i^O, w_{l,w}, A_ip_w \right) \) and \( I \left( \sigma_i^A, w_{l,i}^A, w_{h,i}^A \right) \) iff

\[
e^{-\rho T} \left( 1 + \eta \lambda A_w^{\frac{\beta}{1-\eta^2}} \right) > 1. \tag{23}
\]

Proof. see Appendix. □

In the presence of a continuous distribution of worker types, the dynamic effects of trade lead to divergence if the cost of an education (captured by \( e^{-\rho T} \)) is not too high. The condition for
total divergence of welfare is similar to the one for post opening divergence. Different countries are more likely to diverge if the time of schooling is short, the human capital-intensive sector is relatively important and if workers are more homogeneous (of course, there is a discontinuity to the latter statement in the limit of $\eta \rightarrow 1$).

Under which parameter values does liberalization lead to divergence? Consider first the conditions for post opening divergence of net present income (23). If the duration of education is sufficiently short or $\rho$ approaches 0, there is always divergence. This result is straightforward: as $e^{-\rho T}$ goes to 1, workers do not have to invest much in order to become skilled. Any human capital accumulation that is induced by trade hence leads to large net gains for human capital abundant countries. If $e^{-\rho T}$ is substantially below one, there is a significant cost of education. In this case, rich countries are likely to gain more from trade than poor nations if the global skill-intensive sector is large compared to the labor-intensive sector and if the heterogeneity of workers is small.

It is important to clarify the welfare statements embodied in Proposition 2 and 3: in the model there are neither static externalities (such as monopolistic competition in Gancia and Epifani (2008)) nor dynamic externalities (arising for example in the presence of overlapping generations as in Mountford (1998), or in models of external learning by doing such as Young (1991)) and trade opening results in positive gains from trade that occur to both countries. Divergence, as established in Proposition 3, does not mean that country $s$ is made worse off; it merely means that there are conditions under which country $s$ gains comparatively less than does country $n$. Even if income can be shown to be lower for country $s$ in the open economy, this reflects the optimal decision to dis-save amidst a skill premium that is lower than in autarky.\footnote{Further, note that Proposition 3 establishes whether the net present income of young workers diverges when opening to trade. However, the household receives additional income from old cohorts of workers that were born before $\tau^*$. To establish whether household's net present income diverges, one has to evaluate the total relative increase in consumption, which is a combination of contributions from generations born before $\tau^*$ and from younger cohorts born thereafter. The working paper version of this paper establishes the conditions under which opening to trade results in uniform divergence of welfare for households.}

4 General Equilibrium and the Skill Bias of Trade

This section evaluates the general equilibrium response of simultaneously opening many countries to trade, highlighting a novel mechanism how trade can be globally skill biased.

This novel mechanism does not rely on how trade influences technology, but on how trade influences the international location of human capital. Trade equates goods prices across the world, and the dynamic response of education decisions tends to concentrate human capital in countries that can use skills efficiently. With the average skilled worker working in a country with a higher level of human capital augmenting technology, the output of skill intensive goods increases. This results in a decrease in the price of skill intensive goods. The expansion of the skill intensive sector takes place slowly as new cohorts enter the labor force.
Despite the decrease in the price of the skill-intensive good, I show that an open economy is skill biased. The supply of human capital decreases in countries that are skill scarce and increases elsewhere, resulting in a further increase in the arithmetic average of the skill premium. The results of the model in general equilibrium hence explain why a globalizing world is characterized by both a decreasing price of the skill-intensive good while at the same time resulting in a pervasive increase in the skill premium.

I order all countries \( i \) by their relative human capital effectiveness \( A_i \). I assume that this measure is distributed with probability density function \( g(A_i) \).

I assume that countries are not too different, so that there exists no country that would only have skilled workers in equilibrium.

The global resource constraint restricts total input use to be at most as large as global output of the two intermediate goods

\[
\int_i X_{l,i} di \leq \int_i Y_{l,i} di \quad \text{and} \quad \int_i X_{h,i} di \leq \int_i Y_{h,i} di .
\]

At the moment of opening to trade, global relative supply of factors is given by steady state autarky levels

\[
\frac{Y_{h,w}}{Y_{l,w}} \big|_{t=\tau^*} = \lambda^{\frac{1}{\beta}} \int_{A_i} g(A_i) (A_i)^{\frac{1}{1-\eta}} dA_i. \tag{24}
\]

I denote the average world level of human capital efficiency at the moment of opening to trade by \( A^A_w \)

\[
A^A_w \equiv \left( \int_{A_i} g(A_i) (A_i)^{\frac{1}{1-\eta}} dA_i \right)^{1-\eta^\beta} . \tag{25}
\]

Instantaneously after \( \tau^* \), countries with \( A_i > A^A_w \) accumulate further human capital, while other nations disaccumulate. In a stationary equilibrium, each country chooses a level of human capital dependent on its level of \( A_i \) and on global prices \( H^O_i = \lambda^{2-\beta} A_i^{\frac{\eta}{1-\eta}} p^O_w \), resulting in a total level of global output of

\[
\frac{Y^O_{h,w}}{Y^O_{l,w}} = \lambda^{\frac{1}{\beta}} \left( \int_{A_i} g(A_i) (A_i)^{\frac{1}{1-\eta}} dA_i \right)^{1-\eta^\beta} . \tag{26}
\]

Similarly to the definition of the average world level of human capital efficiency at the moment of opening to trade \( A^A_w \), I denote the average long run global level of human capital efficiency in an

\[\text{If a country of non-negligible size liberalizes, this affects the world’s average level of human capital augmenting technology. Denote the employment share of each country by } l_i = L_i / (\sum_{i \in C} L_i) \text{ and denote the effective average of human capital augmenting technology in the world without country } i \text{ by } A_{w,-i} \equiv \left( \sum_{i \neq i} (A_i)^{\frac{1}{1-\eta}} \right)^{1-\eta^\beta}. \]

It holds that after liberalization with a large country \( i \), \( A^A_w = l_i (A_i)^{\frac{1}{1-\eta}} + (1-l_i) \left( A^A_{w,-i} \right)^{\frac{1}{1-\eta}} \). Country size is thus important in that a large country impacts the global level of \( A^A_w \), thus mitigating differences between autarky and the open economy.

\[\text{In the long run equilibrium, this restriction is equivalent to } A_{MAX} < \eta c (e^{\eta^\beta} - 1) p_w^{-1}.\]
open world by $A_w^Q$, given by

$$A_w^Q = \left( \int_{A_i} g(A_i) (A_i)^{\frac{1}{1-\eta}} dA_i \right)^{1-\eta}. \tag{27}$$

What is the difference between the relative global supply of skilled labor in autarky (25) and in the open economy (27)? In autarky, the general equilibrium response of prices dampens differences in the supply of human capital: a nation that is characterized by a low $A_i$ has a high price of the skill-intensive good, thereby increasing demand. In an open economy, all countries face the same price and cross-country differences in the supply of human capital are thus more pronounced. The next proposition establishes that this concentration of skills also affects the absolute size of the skill-intensive sector.

**Proposition 4 (Expansion of the Skill Intensive Sector)** The stationary equilibrium under trade is characterized by a larger world production of skill-intensive goods than in autarky.

**Proof.** The dynamic relative supply of skill-intensive goods (26) is larger than the static one (24) if the following inequality holds

$$\left( \int_{A_i} g(A_i) (A_i)^{\frac{1}{1-\eta}} dA_i \right)^{1-\eta} > \left( \int_{A_i} g(A_i) (A_i)^{\frac{1}{1-\eta\beta}} dA_i \right)^{1-\eta\beta}. \tag{27}$$

As $\beta < 1$ and due to the general means inequality, this is always true.

Trade, to a first order, "shifts" skilled workers from low $A$ to high $A$ countries. While trade also reduces skill abundance and the supply of the skill-intensive good in some nations, it raises the supply in exactly those countries that can use them very efficiently. This concentration results in an expansion of the skill-intensive sector.

A higher relative output implies a lower relative price of the skill-intensive good, and one might suspect that trade therefore lowers the average skill premium. Interestingly, the opposite is the case. In the context of the present model, a skill bias is not easily established, since some countries may see their skill premium increase with trade, but in other countries there may be a decrease of the relative skilled wage. I therefore define "skill bias" in an average sense.

**Definition 4 (Pervasive Skill Bias)** Trade is pervasively skill biased if the arithmetic average of the relative wage of human capital increases with trade.

There are two questions of interest. First, is there pervasive skill bias at the moment of opening to trade? Second, is there additional skill bias along the dynamic path of the global economy? The following proposition answers both of these questions.

**Proposition 5 (The Skill Bias of Trade)** There is pervasive skill bias at $\tau^*$. The dynamic response of educational investment results in further skill bias.
Proof. Compare the arithmetic average of the skill premium before, at the moment of, and in the long run after trade liberalization. It is both true that

\[ \int_{A_i} g(A_i) H_i^A A_i p_w \left( \frac{Y_{h,w}}{Y_{l,w}} \bigg| t = t^* \right) dA_i > \int_{A_i} g(A_i) H_i^A w_i A_i dA_i \]

and

\[ \int_{A_i} g(A_i) H_i^O A_i p_w^O dA_i > \int_{A_i} g(A_i) H_i^A A_i p_w \left( \frac{Y_{h,w}}{Y_{l,w}} \bigg| t = t^* \right) dA_i. \]

These two inequalities are satisfied by the general means inequality.

The same mechanism that is responsible for the output increase of the skill-intensive sector is responsible for the skill bias of trade. At the moment of opening to trade, the skill premium increases in skill abundant countries, while it decreases in skill scarce countries. Since all countries have an equal endowment of unskilled labor, this channel is not present for unskilled sector. Trade then induces skill accumulation in high wage countries and dis-accumulation elsewhere. The arithmetic average of the wage hence increases further.

This result is related to Dinopolous and Segerstrom (1999), Gancia and Epifani (2008), Acemoglu and Zilibotti (2001), and Acemoglu (2003), who argue that trade changes technology, as well as to Burstein and Vogel (2012), who argue that trade shifts economic activity towards large firms that use more skill-intensive technologies. In contrast to such arguments predicting that trade will increase the relative level of human capital-augmenting technology across all nations and sectors alike, in the model at hand this does not hold: technology is entirely unchanged, but the reallocation of human capital towards countries with higher human capital-augmenting technology creates a skill bias at the global level.

5 Extensions to Endogenous Technology and a Skill Bias Mechanism

In the above analysis, the mechanisms of the model derive from static cross-country differences in technology that are assumed to exist for exogenous reasons. In this section, I show that the main results documented above are re-enforced if technology differences are determined endogenously or if trade is associated with a skill bias mechanism.

5.1 Endogenous Technology

Trade affects market size, and in the presence of increasing returns, this also affects the incentives to innovate. This subsection thus endogenizes technology in a model of endogenous growth. Assume that the production of intermediate goods is modified in the spirit of Romer (1990) in the two sector version of Acemoglu (1998). Technology is local and in each country, the production function combines factor-specific differentiated input goods and the respective factor. Each of
these input goods is produced using a linear transformation of the respective intermediate good.\footnote{To be concise, in this subsection, there are two sorts of input goods that are used to produce two distinct intermediate goods.}

I denote the amount of each input good used in the labor-intensive sector by $i_{L,i}$ and the one used for production of the skill-intensive good by $i_{H,i}$. The net output of each intermediate (denoted by $\bar{Y}_{j,i}$ for $j \in [L, H]$) is given by

\begin{align}
\bar{Y}_{L,i} &= \left( \int_0^{N_L,i} i_{L,i}^\gamma di \right) L_i^{1-\gamma} - \int_0^{N_L,i} i_{L,i} di - R_{L,i} \\
\bar{Y}_{H,i} &= \left( \int_0^{N_H,i} i_{H,i}^\gamma di \right) H_i^{1-\gamma} - \int_0^{N_H,i} i_{H,i} di - R_{H,i}.
\end{align}

(28) and (29)

(where $1 < \gamma < 1$). $R_{L,i}$ and $R_{H,i}$ are the flows of R&D expenditures that are used to invent new blueprints in each sector.\footnote{Epifani and Gancia (2005) show how trade can lead to skill bias when the elasticity of substitution $\frac{1}{1-\gamma}$ between varieties is larger in the skill intensive sector than in the labor intensive one. Stokey (1996) examines the effects of trade liberalization on human capital accumulation if capital tends to complement skilled labor, but substitutes unskilled labor. Since technology in (28) and (29) requires costly innovation and thus can be interpreted as sector-specific capital, also her approach would be equivalent to letting $\gamma$ differ across the two sectors.}

I assume that innovation in sector $j$ uses only the respective intermediate good as input to produce new innovations. Furthermore, as in Jones (1995), innovation becomes the more difficult the higher the current level of innovation. Denoting the flow-cost of innovation in terms of the respective intermediate goods in sector $j$ and country $i$ by $\psi(N_{j,i})$, I assume that

$$\psi(N_{j,i}) = N_{j,i}^\mu.$$ 

With $\mu > 0$, innovation in each country runs into decreasing returns. I now characterize countries not by their intrinsic difference in technology, but by their difference in their educational sector. That is, some countries are essentially better at educating their workforce. The demographic structure is unchanged, except that in country $i$, a skilled worker of type $\theta$ now supplies $s_i\theta$ units of skilled labor if she chooses to get educated. $s_i$ is the country-specific efficiency of the educational system that is given exogenously.

Models of endogenous investments in technology that are financed with profits from monopolistic competition feature two related market failures: because each input good monopolist cannot price discriminate, it charges a constant markup, hence producing a suboptimal amount. For given levels of technology, the production of a country is thus suboptimal. More importantly, the same lack of ability to price discriminate also leads to the monopolist not capturing the full social surplus from her invention. There is thus also suboptimal entry into the input producing sector, with important dynamic consequences for technology, output and welfare. Because innovators face a constant demand elasticity, they charge a price of $1/\gamma$ times their marginal costs. Each innovator in the L sector hence sells $i_{L,i} = \gamma^2 L_i$ units while a firm in the H sector sells $i_{H,i} = \gamma^2 H_i$. 

---

21To be concise, in this subsection, there are two sorts of input goods that are used to produce two distinct intermediate goods.

22Epifani and Gancia (2005) show how trade can lead to skill bias when the elasticity of substitution $\frac{1}{1-\gamma}$ between varieties is larger in the skill intensive sector than in the labor intensive one. Stokey (1996) examines the effects of trade liberalization on human capital accumulation if capital tends to complement skilled labor, but substitutes unskilled labor. Since technology in (28) and (29) requires costly innovation and thus can be interpreted as sector-specific capital, also her approach would be equivalent to letting $\gamma$ differ across the two sectors.
In turn, free entry into the market for input goods implies that

\[ N_{L,i} = ((1 - \gamma) \gamma^2 L_i)^\frac{1}{\hat{\mu}} \quad \text{and} \quad N_{H,i} = ((1 - \gamma) \gamma^2 H_i)^\frac{1}{\hat{\mu}}. \]

The net output in each sector is given by

\[
\tilde{Y}_{L,i} = ((1 - \gamma) \gamma^2)^\frac{1}{\hat{\mu}} \gamma^2 \gamma^2 L_i^\frac{1+\mu}{\mu} - \int_0^{N_{L,i}} i_{L,i} di - R_{L,i} \quad \text{and}
\]

\[
\tilde{Y}_{H,i} = \left( \int_0^{N_{H,i}} \gamma_i H_i di \right) H_i^{1-\gamma} - \int_0^{N_{H,i}} i_{H,i} di - R_{H,i}. \]

and the relative wage is given by

\[
w_i = \left( \frac{N_{H,i}}{N_{L,i}} \right)^\beta \left( \frac{H_i}{L_i} \right)^{(1-\beta)} = \left( \frac{H_i}{L_i} \right)^{\frac{\beta}{\hat{\mu}}(1-\beta)}. \]

Because relative technology is increasing in factor abundance, the relative wage may now be increasing in the supply of skilled labor. The steady state education supply in each country is a function of the wage and the domestic schooling technology

\[
\frac{H_i}{L_i} = (\eta c)^{\frac{1}{1-\eta}} \eta (e^{\eta T} - 1)^{-\frac{1}{1-\eta}} \frac{1}{n \pi^\eta \sigma_i^{\frac{1}{1-\eta}}}. \]

\( s_i \) matters more than proportional, because it influences both the cutoff and the average level of education per skilled worker

\[
\left( \frac{H_i}{L_i} \right)^{1-\beta \eta \frac{1+\mu}{\mu}} = (\eta c)^{\eta} \eta^{1-\eta} (e^{\eta T} - 1)^{-1} s_i. \]

An non-explosive equilibrium in the autarky economy requires that \( \beta \eta \frac{1+\mu}{\mu} < 1 \), and the condition \( \eta \frac{1+\mu}{\hat{\mu}} < 1 \) is required for a non-explosive open equilibrium. Because also technology adjusts, countries tend to be more dissimilar than under exogenous cross-country technology differences. Otherwise, the results of the model with endogenous education and an intrinsic difference in the efficiency in the educational system is equivalent to the model with fixed technology.

### 5.2 Skill Bias Mechanism

It is well established that trade liberalization is associated with what Burstein and Vogel (2012) refer to as a "skill bias mechanism," that is, trade liberalization results in a global increase in the relative level of human capital-augmenting technology across all sectors and countries equally. This raises the question as to whether the above-documented results concerning the divergence of welfare continue to hold if one allows for the presence of a skill bias mechanism.
This subsection establishes that the existence of a skill bias mechanism actually amplifies the above-established results: a uniform increase in the level of factor augmenting technology $A_i \rightarrow \varphi_i A_i$ with $\varphi > 1$ tends, in welfare terms, to favor rich countries. The reason is that a global skill bias mechanism results in a larger market size for skill-intensive goods, thus amplifying the reallocation of educational investments towards initially richer countries and the associated welfare effects.

Assume that trade creates a country-specific skill bias mechanism $\varphi_i$ that increases the level of factor augmenting technology proportionally: $A_i \rightarrow \varphi_i A_i$, and that consequently leads to $A_w \rightarrow \varphi_w A_w$ with $\varphi_w \equiv \left( \frac{1}{\int_{A_i} g(A_i)(\varphi_i A_i) \frac{1}{1-\eta} dA_i} \right)^{1-\eta}$ being a weighted average of the individual $\varphi_i$. It holds that the relative welfare of a northern country $n$ compared to that of a southern country $s$ is equal to

$$I \left( \tilde{p}^O_n, w_{l,w}, A_n p_w, \varphi_n \right) = 1 + (1 - \eta) \left( \frac{\varphi_n}{\varphi_w} \frac{A_n}{A_w} \right)^{1-\eta} \lambda e^{-\rho T} (\varphi_w A_w)^{\frac{\beta}{1-\eta}}.$$

Divergence of welfare is more likely if $\frac{\varphi_n}{\varphi_w}$ is large and $\frac{A_n}{A_w}$ is small (i.e.: if the skill bias mechanism is relatively stronger for richer countries, there is divergence).\(^{23}\)

If the skill bias mechanism is uniform (as is predicted, for example, by the frameworks of Acemoglu and Zilibotti (2001) and Acemoglu (2003)), and $A_i \rightarrow \varphi A_i$ with $\varphi > 1$, it holds that

$$\frac{\partial I \left( \tilde{p}^O_n, w_{l,w}, A_n p_w, \varphi_n \right)}{\partial \varphi} / I \left( \tilde{p}^O_s, w_{l,w}, A_s p_w, \varphi_s \right) > 0:$$

a uniform shift towards human capital-augmenting technology increases the total global market for skill-intensive goods, which benefits the nations specializing in this sector.

A second topic of interest concerns the response of human capital to trade liberalization, and how it amplifies any intrinsic skill bias mechanism. If there exists a skill bias mechanism, the formation of human capital responds to the increase of the relative productivity of human capital. This, in turn, multiplies the intrinsic skill bias mechanism: consider $Y_{h,w}^O / Y_{l,w}^O$, the long run relative supply of skill-intensive goods (26) and how it responds to an uniform upward shift in human capital-augmenting technology of the form $A_i \rightarrow \varphi A_i$. It holds that the elasticity of $Y_{h,w}^O / Y_{l,w}^O$ with respect to $\varphi$ is equal to $(1 - \eta/\beta)^{-1}$ and thus larger than one.\(^{24}\)

\(^{23}\)In the general case with country-specific $\varphi_i$, trade leads to uniform divergence of welfare if

$$\left( \gamma \right)^{\frac{1-\beta}{1-\eta}} - \left( \gamma \right)^{\frac{1-\beta}{1-\eta}} +$$

$$\left( e^{-\rho T} (1 - \eta) \lambda \left( \varphi_w A_w \right)^{\frac{\beta}{1-\eta}} \right) \left( \gamma \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\varphi_n}{\varphi_w} \right)^{\frac{1}{1-\eta}} - \left( \gamma \right)^{\frac{1-\beta}{1-\eta}} \left( \frac{\varphi_n}{\varphi_w} \right)^{\frac{1}{1-\eta}}$$

$$< \left( \frac{\varphi_n}{\varphi_w} \right)^{\frac{1}{1-\eta}} - \left( \frac{\varphi_n}{\varphi_w} \right)^{\frac{1}{1-\eta}} \gamma^\frac{1}{1-\eta} \frac{\varphi_n}{\varphi_w}.$$

\(^{24}\)More generally, if $\varphi_i$ is country-specific, it holds that also the distribution of $\varphi_i$ and how it co-varies with $A_i$ matters for the amplification of the skill bias: $Y_{h,w}^O / Y_{l,w}^O = \lambda^\frac{1}{\beta} \left( \int_{A_i} g(A_i) (\varphi_i A_i) \frac{1}{1-\eta} dA_i \right)^{\frac{1}{1-\eta}}$
The results of this subsection thus demonstrate that the effects established in the main section of the paper do not only hold in the presence of an additional skill bias, but also, that these two mechanisms amplify each other.

6 Conclusion

The literature on economic growth argues that factor accumulation is a key ingredient for long term economic success. Countries that have sustained high rates of growth did so because of their high levels of savings and investment in human capital (see Young (1995) and Mankiw et al. (1992)). Other nations stagnated precisely because their institutional setups hindered private savings and investment (see for example Hall and Jones (1999)). To evaluate whether trade has sizeable and first order effects on economic performance, it is therefore essential to show how exposure to international prices influences factor accumulation.

This paper establishes the cross-country welfare implications of trade in a model that takes into account the private nature of human capital accumulation. Being a net supplier of human capital that can be accumulated increases the growth potential of the economy, while specialization in labor-intensive sectors means specialization in a factor that is in fixed supply. In addition to showing how trade can result in divergence of income, this paper documents that there are important cross-country welfare implications. The dynamic responses of education decisions introduces an asymmetry between economies to the model. This stems from the two margins in which the relative wage influences the surplus from education: a higher relative wage increases the income for all workers that already would have chosen schooling at lower wages. In addition, an increase in the relative wage induces more entry into the skilled labor force. In total, the surplus from education responds more than proportionally to changes in the relative wage. Skill scarce nations, in contrast, have their comparative advantage in a factor that is in fixed supply and cannot be accumulated. The key insight of the mechanism at work is that while all countries gain from trade, it is the group of already developed nations that gain proportionally the most from trade; trade liberalization hence can result in a divergence of welfare.
References


Lemma 1 (remined) Consider the moment of opening to trade $\tau^*$. There is uniform relative convergence (divergence) of output if the global size of the labor-intensive sector is smaller (larger) than the human capital-intensive one.

Proof. Evaluate autarky output (16) to output at $\tau^*$ (18) for two countries $A_n = (1 + \gamma)A_w = (1 + \gamma)^2A_s$. The pre-opening ratio of output is equal to

$$
Y \left( \tilde{\theta}_n, w_{l,w}, A_n p_w \right) / Y \left( \tilde{\theta}_s, w_{l,w}, A_s p_w \right) = \frac{1 + \lambda \left( A_n / A_w \right)^{\frac{1}{1-\gamma \beta}} \frac{1}{A_w^\frac{1}{1-\gamma \beta}} \frac{A_s}{A_n}}{1 + \lambda \left( A_s / A_w \right)^{\frac{1}{1-\gamma \beta}} \frac{1}{A_w^\frac{1}{1-\gamma \beta}} \frac{A_n}{A_s}}.
$$

The post opening ratio of output is equal to

$$
Y \left( \tilde{\theta}_n, w_{l,w}, A_n p_w \right) / Y \left( \tilde{\theta}_s, w_{l,w}, A_s p_w \right) = \left( \frac{1 + \lambda A_n^{\frac{1}{1-\gamma \beta}}}{1 + \lambda A_s^{\frac{1}{1-\gamma \beta}}} \right)^{1/\beta}.
$$

When is there absolute divergence? The relative level of output has a form of $\left( \frac{z^{k,x+1}}{z^{-k,x+1}} \right)^{\frac{1}{z}}$, where $z = 1 + \gamma$. The autarky level of $k$ is higher than under autarky. If

$$
\frac{\partial}{\partial k} \left( \frac{z^{k,x+1}}{z^{-k,x+1}} \right)^{\frac{1}{z}} > 0
$$

holds, there is divergence. First note that when $x = 1$, $\left( \frac{z^{k,x+1}}{z^{-k,x+1}} \right)^{\frac{1}{z}} = z$. If $x < 1$, $\left( \frac{z^{k,x+1}}{z^{-k,x+1}} \right)^{\frac{1}{z}} < z$. Rewriting this expression

$$
\frac{\partial}{\partial k} \exp \left[ \frac{1}{k} \log \left( z^{k,x+1} \right) - \frac{1}{k} \log \left( z^{-k,x+1} \right) \right],
$$

where the inner exponents need to be rewritten in log-exponential form, too. This can be shown to be

$$
\left( \frac{z^{k,x+1}}{z^{-k,x+1}} \right)^{\frac{1}{z}} \left( \frac{\partial}{\partial k} \left( \frac{1}{k} \log \left( z^{k,x+1} \right) \right) - \frac{\partial}{\partial k} \left( \frac{1}{k} \log \left( z^{-k,x+1} \right) \right) \right).
$$

Omitting terms that are positive it remains to be shown that

$$
\frac{\partial}{\partial k} \left( \frac{1}{k} \log \left( z^{k,x+1} \right) \right) - \frac{\partial}{\partial k} \left( \frac{1}{k} \log \left( z^{-k,x+1} \right) \right) > 0.
$$
multiplying by \( k^2 \) this is true, if
\[
\log(z) \left[ \frac{z^k x}{(z^k x + 1)} + \frac{z^{-k} x}{(z^{-k} x + 1)} \right] - \log \left( \frac{z^k x + 1}{z^{-k} x + 1} \right)^{\frac{1}{k}} > 0. \tag{30}
\]

Recalling that when \( x = 1 \), \( \log \left( \frac{z^k x + 1}{z^{-k} x + 1} \right)^{\frac{1}{k}} = \log(z) \) so (30) is equal to 0. In addition, (30) takes the value 0 if \( x \to \infty \) and if \( x = 0 \). Evaluating the slope of (30) with respect to \( x \), it can be shown that at levels of \( x \) just below 1, (30) is decreasing, implying that (30) is larger 0 for any \( 1 > x > 0 \).

In addition, it can be shown that there exist at most 3 levels of \( x \) where (30) equals 0. Hence, (30) is increasing in \( k \) whenever \( 1 > x > 0 \).

**Proposition 2 (reminded).** Let \( n \) and \( s \) be two small countries with \( A_n = (1 + \gamma) A_w = (1 + \gamma)^2 A_s \). There is uniform relative divergence of output comparing the output just after opening to trade (18) to the one in the stationary equilibrium under free trade (20). There is also uniform relative divergence of output comparing the output in autarky stationary equilibrium (16) to the stationary equilibrium under free trade (20).

**Proof.** of Proposition 2 (Trade and The Dynamics of Income)

To establish the two claims of the proposition, compare the relative ratio of output for two countries \( N \) and \( S \) in autarky (16), just after opening to trade (18), and in the stationary equilibrium with trade (20). It is both true that for any \( \gamma > 0 \), the following inequalities hold
\[
\frac{Y \left( \overline{\theta}_n^O, w_{l,w}, A_{n} p_{w} \right)}{Y \left( \overline{\theta}_s^O, w_{l,w}, A_{s} p_{w} \right)} > \frac{Y \left( \overline{\theta}_n^A, w_{l,w}, A_{n} p_{w} \right)}{Y \left( \overline{\theta}_s^A, w_{l,w}, A_{s} p_{w} \right)}. \tag{31}
\]

First note that if \( \gamma = 0 \), this ratio is equal to 1. Now evaluate the first and second derivative of (31) with respect to \( \gamma \). The first derivative is positive at \( \gamma = 0 \), while the second derivative is positive for any \( \gamma \). Hence, for any \( \gamma > 0 \) (31) takes a value larger than 1. A proof along the same lines establishes that for any \( \gamma > 0 \) comparing (16) to (20) results in dynamic divergence, i.e.
\[
\frac{Y \left( \overline{\theta}_n^O, w_{l,w}, A_{n} p_{w} \right)}{Y \left( \overline{\theta}_s^O, w_{l,w}, A_{s} p_{w} \right)} > \frac{Y \left( \overline{\theta}_n^A, w_{l,w}, A_{n} p_{w} \right)}{Y \left( \overline{\theta}_s^A, w_{l,w}, A_{s} p_{w} \right)}. \]

**Proposition 3 (reminded).** Let \( n \) and \( s \) be two small countries with \( A_n = (1 + \gamma) A_w = (1 + \gamma)^2 A_s \). It is always the case that comparing \( I \left( \overline{\theta}_i^O, w_{i,w}, A_{i} p_{w} \right) \) to \( I \left( \overline{\theta}_i^A, w_{i,w}, A_{i} p_{w} \right) \), there is uniform relative divergence. There is uniform relative divergence of \( I \left( \overline{\theta}_i^O, w_{i,w}, A_{i} p_{w} \right) \) and \( I \left( \overline{\theta}_i^A, w_{i,w}, A_{i} p_{w} \right) \) iff
\[
e^{-\rho T} \left( 1 + \eta \lambda \frac{A_{i} p_{w}}{w_{i,w}} \right) > 1.\tag{30}
\]
Proof. Evaluate the ratio of (22) to (21) for two countries $N$ and $S$.

\[
\frac{I \left( \bar{\sigma}_n^O, w_{l,w}, A_n p_w \right)}{I \left( \bar{\sigma}_n^A, w_{l,w}, A_n p_w \right)} / \frac{I \left( \bar{\sigma}_s^O, w_{l,w}, A_s p_w \right)}{I \left( \bar{\sigma}_s^A, w_{l,w}, A_s p_w \right)}.
\]

If $\gamma = 0$, this ratio equal to 1. For any $\gamma > 0$, this ratio can be shown to be larger than 1 if (23) holds. The second claim involves a similar comparison of (22) to (17). The equivalent ratio is larger than 1 for any $\gamma > 0$ if (23) holds. ■

31