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## **Austerity**

Harris Dellas and Dirk Niepelt

Working Paper 14.07

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# Austerity\*

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## Abstract

We shed light on the function, properties and optimal size of austerity using the standard sovereign debt model augmented to include incomplete information about credit risk. Austerity is defined as the shortfall of consumption from the level desired by a country and supported by its repayment capacity. We find that austerity serves as a tool for securing a more favourable loan package; that it is associated with over-investment even when investment does *not* create collateral; and that low risk borrowers may favour more to less severe austerity. These findings imply that the amount of fresh funds obtained by a sovereign is not a reliable measure of austerity suffered; and that austerity may actually be associated with higher growth. Our analysis accommodates costly signalling for gaining credibility and also assigns a novel role to spending multipliers in the determination of optimal austerity.

**JEL class:** F34, H63

**Keywords:** Austerity; credit rationing; default; incomplete information; investment; growth; pooling equilibrium; separating equilibrium.

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# 1 Introduction

The ongoing European debt crisis has brought “austerity” to center stage. The public debate contains references to “austerity” as a means of gaining credibility; a self-defeating scheme; or, excessive retrenchment.<sup>1</sup> Yet, a clear, operational, model based definition of austerity as well as a coherent analysis of its properties and consequences for macroeconomic activity and welfare are missing. This paper aims at filling this gap.

In the context of sovereign debt, the term austerity is typically used to describe borrowing constraints faced by governments as manifested by restrictions on the size of their budget deficits. A problem with this description is that it confounds two different sources of debt limits. On the one hand, a debt ceiling could reflect creditors’ beliefs about a country’s inability or unwillingness to honor obligations beyond that ceiling. Using the term austerity to describe this situation seems meaningless. On the other hand, a debt ceiling could represent a creditor imposed bound that falls short of the country’s repayment capacity, giving rise to a gap between the actual debt and the ceiling that reflects the country’s fundamental ability or willingness to repay. Referring to the presence of such a gap as austerity seems useful and can also make sense of many of the arguments made in the current debates between opponents and proponents of austerity.

Based on this consideration, we propose a definition of austerity that relates to the second source of debt limits described above but is stated in terms of a consumption rather than a debt gap.<sup>2</sup> We define austerity as the difference between the consumption level supported by the actual debt issued and the level of consumption that the country would like to and could afford to enjoy given its fundamentals (ability/willingness to repay).<sup>3</sup> In other words, austerity refers to a situation where a country both wishes to consume more than she actually does and this would be feasible given the country’s debt fundamentals.

In light of this definition, we know of no theoretical framework in the sovereign debt literature that can be used to rationalize austerity and determine its optimal size. The standard sovereign debt model (Eaton and Gersovitz (1981), Obstfeld and Rogoff (1996, ch. 6)) implies an endogenous debt and consumption ceiling that reflects the borrower’s willingness to repay but it predicts austerity to equal zero because there is no reason in this model to justify restricting funds below that ceiling. Moreover, in the standard model, the relationship between austerity and growth is unambiguous: Austerity lowers investment and impacts negatively on growth.

We extend the standard sovereign debt model to render it applicable for an analysis of austerity. In our model, debt and consumption gaps arise due to the presence of incomplete information in credit markets. Our approach exploits the similarity between austerity and credit rationing in markets with adverse selection.<sup>4</sup> As in the credit rationing literature,

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<sup>1</sup>See, for example, Giancarlo Corsetti, “Has austerity gone too far?,” *Vox.eu*, 2 April 2012.

<sup>2</sup>As we elaborate later, the two definitions often have the same implications. But with endogenous investment, the consumption based definition is more general and more accurate than the debt based one.

<sup>3</sup>The time of the appearance of the consumption gap—austerity—does not have to coincide with a debt crisis period. A country may opt for “preemptive” austerity rather than risk financial markets imposing it in a harsher way in the future.

<sup>4</sup>While the two are not the same (austerity may arise even in the absence of conventionally defined

we assume that debtors differ with regard to their—unobserved to creditors—willingness to honour debt commitments. The difference arises from the existence of type specific default costs, defined as the output a defaulting country forfeits when not repaying in full:<sup>5</sup> Highly creditworthy governments face high default costs while less creditworthy types face low costs.

The model has two periods and in the benchmark case there is no investment decision. In the first period, a government inherits an amount of debt and decides whether to repay or not. If the government defaults, it suffers the type specific cost. Following the default decision, the government may borrow fresh funds in the form of non-state contingent debt that is due for repayment in the second period. The amount and price of these fresh funds depend on the perceptions of creditors about the type of the debtor government they face. In turn, these perceptions may be affected by the government's default decision in the first period. Creditors are risk neutral and operate under perfect competition.

We focus on the equilibrium that generates the highest level of welfare for the borrower. Depending on parameter values, the optimal equilibrium may be a pooling one where both types take the same action in the first period and a single amount and price of fresh funds is offered. Or, the optimal equilibrium may be a separating one in which the government's type is revealed by its default decision in the first period, and the loan contract is type specific. In general, a large (small) probability of facing a high type government makes the pooling (separating) equilibrium more likely to emerge.

In the pooling equilibrium, the high type country generally faces austerity. More interestingly, it also faces austerity in the *separating* equilibrium.<sup>6</sup> The culprit of austerity is the self-selection constraint of a low type government. The loan to the high type must be capped at a level that makes it unprofitable for the low type to mimic the high type (by honoring debt in the first period). As long as governments face different costs of repudiating debt, and these costs are private information, the most committed governments will invariably have to face austerity independently of whether a government reveals its true type or not.

The credit rationing literature has established that the properties of equilibrium critically depend on the menu of contracts and the set of financial instruments available. For instance, the availability of equity along side debt financing or the existence of co-investment may make it easier for creditors to induce a separating equilibrium (Meza and Webb (1987), Brennan and Kraus (1987)). In this spirit, we introduce endogenous investment and allow for the possibility that creditors may require a specific level of investment as part of the loan contract. We find that adding investment requirements

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credit rationing) they are closely related.

<sup>5</sup>The actual costs of defaulting in terms of an aggregate measurable quantity such as GDP losses may well be the same across types. Nonetheless, the trade-off involved in the default decision may still differ across government types if the incidence of the default costs is asymmetric across groups and different types weigh the welfare of these groups differently. We do not model incidence but assume instead type specific aggregate default costs.

<sup>6</sup>The standard result in the credit rationing literature is that the availability of a menu of contracts conditioned on observed collateral is sufficient to induce sorting and eliminate credit rationing (Bester (1985)). In our case, there is incomplete information on collateral (the default cost) and rationing also obtains in the optimal separating equilibrium.

to the loan package indeed makes separation easier and also increases the welfare of a creditworthy government even when the proceeds from the investment *cannot* serve as collateral. The optimal package requires *over-investment* relative to the case where the government can freely choose the investment level. Moreover, this over-investment takes a special form. For any fresh funds offered above a certain level, not only must all these funds be invested but investment must also be co-financed by the borrower. That is, investment increases by more than one-to-one with such funds. While the availability of a costly action—over-investment in our case—is known to promote separation in signalling games, these results are both novel and unexpected from the point of view of the extant sovereign debt literature. In this literature, the only reason for over-investment is to provide more collateral and thus make it possible to obtain a larger loan. Moreover, the extra funds received through (over-)investment’s enhancement of the collateral are split between investment and consumption. In our model, by contrast, over-investment need not contribute to higher collateral and the effect on consumption of the marginal unit of debt made possible by over-investment is negative. That is, beyond some level, more debt implies greater austerity. The fact that the optimal level of debt is found in the greater austerity region implies that a low credit risk borrower is better off with more rather than with less austerity.

The key to understanding this result lies in the fact that low credit risk borrowers have a higher propensity to invest because they need more funds to repay debt in the second period. Consequently, increasing investment beyond the conditionally optimal level hurts more a less creditworthy type who tries to mimic than a high type. The over-investment requirement then represents a costly signal that the high type can employ in order to distinguish himself from a mimicking low type, paving the way for obtaining more funds. While these additional funds cannot be used to increase consumption and close the consumption gap, they are still valuable because they help close the investment gap (which is due to the fact that a debt constrained sovereign also under-invests relative to the first best).

The role of investment as a sorting device has several implications. First, it makes austerity a non-monotone function of the quantity of new loans. As the amount of new loans increases from some low level, austerity initially decreases. But beyond a certain level of new debt, it starts to increase. As mentioned above, the optimal level of austerity is found in the increasing portion of this function and consequently, more austerity is associated with higher welfare for creditworthy borrowers. Second, it gives rise to an ambiguous relationship between the severity of austerity and economic growth. The same level of austerity may be associated with different rates of growth. At the optimum, austerity is more severe but investment and growth are higher than in the equilibrium in which the investment instrument cannot be used in the loan package. And third, it drives a discrepancy between debt based (credit rationing) and consumption based (austerity) gaps. In particular, with forced investment, credit rationing—the distance between actual debt and the level under complete information (the natural borrowing ceiling)—decreases with the amount of fresh funds while austerity becomes harsher. That is, the debt gap could be indicating an amelioration of credit rationing while the consumption gap would at the same time be indicating more severe austerity.

The preceding discussion has not explicitly referred to any of the standard arguments present in the current debates on austerity. The standard view appears to be that the main function of “austerity” is to help establish—signal—a government’s level of creditworthiness and thus, suppress sovereign debt default premia and increase the flow of fresh funds. Our model with over-investment offers an example of this mechanism. The opponents of “austerity,” while recognizing its direct contribution to credibility argue that this effect may be overwhelmed by negative macroeconomic implications. “Austerity” is thought to depress economic activity through standard spending (Keynesian) multiplier effects and thus to lower a country’s debt repayment *ability*. Consequently, severe “austerity” could actually further reduce the flow of fresh funds by making default more rather than less likely.

The current debates thus mix *willingness* (the credibility side) with *ability* (the multipliers side) to repay considerations. Assessing the merits of these considerations requires that both mechanisms be embedded in a common framework.<sup>7</sup> Our model makes this possible and in the process establishes a new role for spending multipliers. We show that the size of the multiplier may matter for the terms of financing and the default decision if it matters for the severity of the agency problem (the identification of credit risks). This function does not require that larger multipliers enhance a country’s ability to repay debt.

We also consider extensions of the model that help shed light on costly signalling other than through over-investment and on the inclusion of reform requirements in loan packages. We argue that having the borrower undertake costly—in the short term—reforms can increase the flow of funds. But unlike popular thinking, reforms accompanied by the relaxation of fiscal stance do not necessarily prevent the loss of current consumption. There is simply no clear relationship between the size of new funding and austerity.

**Related Literature** Our paper combines the sovereign debt literature with the literature on credit rationing in models with heterogeneous borrowers and incomplete information. Two implications of the standard sovereign debt model are of relevance for our analysis. First, that the maximum level of debt that can be issued is suboptimally low, constrained by the country’s willingness to repay. And second, that the level of investment plays an important role for that debt ceiling. In particular, investment relaxes the debt ceiling due to its ability to create collateral and thus increase the cost of default ((Obstfeld and Rogoff, 1996, ch. 6)). In contrast to this line of work, our analysis attributes benefits to investment that go beyond those operating via collateral creation.

Concerning the use of incomplete information in the sovereign debt literature, the closest precursor to our work is Cole, Dow and English (1995). These authors develop a model in which governments come in two types, a high and a low discount, that alternate stochastically and with the type being private information. As in our model, the high types find it beneficial to costly signal their type (and hence their greater willingness to repay future loans). In Cole et al. (1995) they do so by making payment on debt defaulted upon by previous, low type governments (that is, by settling old debts).

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<sup>7</sup>As we argued above, the concept of austerity may lack content in the absence of incomplete information about the level of credit risk.

The credit literature with incomplete information has pre-occupied itself primarily with the existence of rationing, a concept closely related to our definition of austerity. While the seminal paper of Stiglitz and Weiss (1981) exhibits credit rationing in equilibrium, subsequent work has demonstrated that the rationing problem is not present under alternative assumptions about the incidence of informational asymmetry (that is, risk versus return, see Meza and Webb (1987)) or, that it can be solved with a rich enough menu of financial contracts<sup>8</sup> (see Bester (1985), Milde and Riley (1988), and Brennan and Kraus (1987)). Such contracts can induce self-selection and support a separating equilibrium in which the asymmetric information is revealed and there is *no* credit rationing. Unlike the results in this literature, “credit rationing” remains a feature of the separating equilibria in our model. It is required in order to deter the less creditworthy type from mimicking the high type. The use of this sanction is akin to that employed by Green and Porter (1984), to deter cheating. In Green and Porter (1984), punishment is imposed following certain events in spite of the fact that there is no cheating in equilibrium. In our model, the sanction (credit rationing) is essential in order to support the truthful revelation of type.

Our analysis of sovereign debt under asymmetric information bears resemblance to that in a branch of the literature on monetary policy credibility that developed during the 1980s and 1990s following Kydland and Prescott’s (1977) contribution on rules vs. discretion. Canzoneri (1985), Vickers (1986) and Backus and Driffill (1985) represent prominent examples of this body of work. As in our paper, there are two types of policymakers (a “hard nosed” and a “wet” one), each with its own welfare function;<sup>9</sup> type is unobserved but may be revealed through the action taken. The objective of the public is to guess which type they face in order to form expectations about inflation accordingly. Canzoneri (1985) relies on the model of Green and Porter (1984) to argue that some punishment is always present in order to discourage opportunistic behavior even if it is known that no opportunistic actions are ever taken in equilibrium. The punishment takes the form of expectations of inflation by the public that are too high given the absence of opportunistic behavior. Vickers (1986) applies the model of Cho and Kreps (1987). His analysis of costly signalling and the characterization of pooling and separating equilibria is related to the versions of our model with costly signalling.

The rest of the paper is organized as follows: Section 2 lays out the basic model and characterizes the pooling and separating equilibria. In sections 3 and 4, we analyze the consequences of contractible and non-contractible investment, respectively. Section 5 contains extensions and additional discussions, including on multipliers, costly signalling and structural reform, and section 6 concludes.

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<sup>8</sup>For instance, contracts that impose restrictions on capital structure, require co-investment and so on.

<sup>9</sup>The two types are formally modelled in Vickers (1986). In Canzoneri (1985) the version with the conservative policymaker can be interpreted as a two type game.



## 2 Basic Model

### 2.1 Environment

The economy lasts for two periods,  $t = 1, 2$ . It is inhabited by a representative taxpayer, a government and foreign investors. Taxpayers neither save nor borrow. Their lifetime utility is given by

$$\mathbb{E} \left[ \sum_{j \geq t} \delta^{j-t} u(\bar{y}_j - \tau_j) | \mathcal{I}_t \right],$$

where  $\bar{y}_t$  denotes pre-tax income,  $\tau_t$  taxes and  $\mathcal{I}_t$  the information set (to be specified below).

Foreign investors are competitive and risk neutral, require a risk free gross interest rate  $\beta^{-1} > 1$  and hold all government debt (since taxpayers do not save).<sup>10</sup> To guarantee positive debt positions, we assume  $\delta \ll \beta$  as is standard in the sovereign debt literature.<sup>11</sup>

The government maximizes the welfare of taxpayers. In period  $t$ , it chooses the repayment rate on maturing debt,  $r_t$ , issues zero-coupon, one period debt,  $b_{t+1}$ , and (residually) levies taxes. Without loss of generality, public spending other than debt repayment is set to zero. The government cannot commit its successors (or future selves). Short-sales are ruled out.

A sovereign default—a situation where the repayment rate falls short of unity—triggers a contemporaneous, temporary income loss for taxpayers (see Eaton and Gersovitz, 1981; Cole and Kehoe, 2000; Aguiar and Gopinath, 2006; Arellano, 2008). More specifically, a default in period  $t$  reduces the exogenous income  $y_t$  by the fraction  $\lambda \geq 0$  so that  $\bar{y}_t = y_t$  when there is no default and  $\bar{y}_t = y_t(1 - \lambda)$  when there is default. For simplicity, we treat  $y_t$  as deterministic. There is no exclusion from credit markets following default.

The default cost parameter  $\lambda$  takes one of two values,  $\lambda^h$  or  $\lambda^l$ , with  $0 \leq \lambda^l < \lambda^h$ . We refer to a government facing  $\lambda^h$  ( $\lambda^l$ ) as a government with high (low) creditworthiness or simply as a “high (low) type.” The values of  $\lambda^h$  and  $\lambda^l$  are common knowledge but the type of government is private information. The prior probability that a given country has a high type government equals  $\theta \in (0, 1]$ .

Events unfold as follows. In the beginning of the first period, the government chooses the repayment rate  $r_1 \in \mathcal{R} \subseteq [0, 1]$  on maturing debt  $b_1$ . Lenders observe this choice, form the posterior belief  $\theta_1$  that they face a high type, and buy new debt  $b_2 \in \mathcal{B} \equiv [0, \infty)$  at price  $q_1 \in [0, \beta]$ . For brevity, we let  $\mathcal{F}_1 \equiv (q_1, b_2)$  denote this financing arrangement. Finally, taxes  $\tau_1 = b_1 r_1 - q_1 b_2$  are levied. In the second period, the government chooses the repayment rate  $r_2 \in \mathcal{R}$  on debt  $b_2$  and levies taxes  $\tau_2 = b_2 r_2$ .

The indirect utility function of taxpayers in a country of type  $i = h, l$  (or “of type  $i$ ” for short) in period  $t = 2$  can be expressed as

$$U_2^i(\mathcal{F}_1, r_2) = u \left( y_2(1 - \lambda^i \mathbf{1}_{\{r_2 < 1\}}) - b_2 r_2 \right)$$

<sup>10</sup>The assumption that the sets of taxpayers and investors do not “overlap” simplifies the analysis and does not matter for the main results.

<sup>11</sup>For recent examples, see Aguiar and Gopinath (2006) or Arellano (2008).

where  $\mathbf{1}_{\{x\}}$  denotes the indicator function for event  $x$ . Welfare of type  $i = h, l$  is given by

$$U_1^i(r_1, \mathcal{F}_1) = u(y_1(1 - \lambda^i \mathbf{1}_{\{r_1 < 1\}}) - b_1 r_1 + q_1 b_2) + \delta \max_{r_2 \in \mathcal{R}} U_2^i(\mathcal{F}_1, r_2).$$

We define *austerity* as the difference between the actual level of consumption and the level of consumption that would have been achieved in the economy without incomplete information.<sup>12</sup> Let  $b_2^{i\text{sb}}$  denote the –second best– level of debt under complete information. It is given by

$$b_2^{i\text{sb}} = \arg \max_{b_2^i \in \mathcal{B}} u(y_1 - \min[\lambda^i y_1, b_1] + \beta b_2^i) + \delta u(y_2 - b_2^i) \quad \text{s.t.} \quad b_2^i \leq \lambda^i y_2$$

The corresponding level of consumption,  $c_t^{i\text{sb}}$ , is  $c_1^{i\text{sb}} \equiv y_1 - \min[\lambda^i y_1, b_1] + \beta b_2^{i\text{sb}}$  and  $c_2^{i\text{sb}} \equiv y_2 - b_2^{i\text{sb}}$  for  $i = h, l$ . Austerity  $a_t^i$  for type  $i = h, l$  in period  $t$  is then given by

$$a_t^i \equiv c_t^{i\text{sb}} - c_t^i.$$

When referring to austerity without specifying a particular period, we mean austerity in the first period.

## 2.2 Equilibrium

An *equilibrium* is a repayment rate for each type in the first period,  $r_1^i, i = h, l$ ; a posterior belief and a financing arrangement that depend on the repayment rate in the first period,  $\theta_1(\cdot) : \mathcal{R} \rightarrow [0, 1]$  and  $\mathcal{F}_1(\cdot) : \mathcal{R} \rightarrow \mathbf{R}_+^2$ , respectively; and a repayment rate for each type in the second period that depends on the financing arrangement,  $r_2^i(\cdot) : \mathbf{R}_+^2 \rightarrow \mathcal{R}, i = h, l$ , such that the following conditions are satisfied:<sup>13</sup>

- i. For each  $\mathcal{F}_1$  and each type, the repayment rate in the second period is optimal,

$$r_2^i(\mathcal{F}_1) = \arg \max_{r_2 \in \mathcal{R}} U_2^i(\mathcal{F}_1, r_2);$$

- ii. for each type, the repayment rate in the first period is optimal conditional on  $\mathcal{F}_1(\cdot)$ ,

$$r_1^i = \arg \max_{r_1 \in \mathcal{R}} U_1^i(r_1, \mathcal{F}_1(r_1));$$

- iii. the posterior belief satisfies Bayes' law where applicable,

$$\theta_1(r_1) = \text{prob}(i = h | r_1, \mathcal{F}_1(\cdot));$$

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<sup>12</sup>Note that the latter level of consumption falls short of the first best level due to the absence of repayment commitment.

<sup>13</sup>We specify  $r_2^i(\cdot)$  to be a function of  $\mathcal{F}_1$  rather than only  $b_2$  to render the notation consistent across the different sections of the paper. In a subsequent section, the repayment rate will depend on an additional argument that is also part of  $\mathcal{F}_1$ .

- iv. for each  $r_1$ , the financing arrangement  $\mathcal{F}_1(\cdot)$  satisfies the break even condition of lenders given their posterior,

$$q_1(r_1) = \beta\{\theta_1(r_1)r_2^h(\mathcal{F}_1(r_1)) + (1 - \theta_1(r_1))r_2^l(\mathcal{F}_1(r_1))\}.$$

Since Bayes' law constrains lenders' beliefs only along the equilibrium path, there exists (as usual) a multiplicity of equilibria. We distinguish between *pooling* and *separating equilibria*. In a pooling equilibrium, both types choose the same repayment rate in the first period and lenders therefore do not update their beliefs. In a separating equilibrium, first-period repayment rates differ across types and the posterior beliefs of lenders either equal zero or unity. In both types of equilibrium, the repayment rate in the second period may differ across types.<sup>14</sup>

The number of equilibria can be reduced via specific refinements (see, for example, Cho and Kreps, 1987). We focus on the *optimal equilibrium*, that is, the equilibrium that maximizes the social welfare function  $W(\cdot)$  defined as

$$W(r_1^h, r_1^l, \mathcal{F}_1(\cdot)) \equiv \theta U_1^h(r_1^h, \mathcal{F}_1(r_1^h)) + (1 - \theta)\omega U_1^l(r_1^l, \mathcal{F}_1(r_1^l)).$$

The parameter  $\omega$  in the social welfare function  $W(\cdot)$  denotes the relative weight of low types; for  $\omega = 0$ , the equilibrium is (constrained) optimal for high types.

Since the cost of default is independent of whether default is full,  $r_2 = 0$ , or partial,  $0 < r_2 < 1$ , the optimal repayment rate in the second period equals either zero or unity. In particular, equilibrium requirement (i) implies

$$r_2^i(\mathcal{F}_1) = \begin{cases} 1 & \text{if } \lambda^i y_2 \geq b_2 \\ 0 & \text{if } \lambda^i y_2 < b_2 \end{cases}, i = h, l. \quad (1)$$

We refer to conditions (1) as the repayment constraints. Consistent with (1), we restrict the choice set of borrowers in the first and second period and thus, the domain of  $\mathcal{F}_1(\cdot)$  to  $\mathcal{R} \equiv \{0, 1\}$ .

Equilibrium requirement (ii) implies

$$U_1^i(r_1^i, \mathcal{F}_1(r_1^i)) \geq U_1^i(r_1, \mathcal{F}_1(r_1)), \forall r_1 \in \mathcal{R}, i = h, l. \quad (2)$$

We refer to conditions (2) as the (self-)selection constraints. We assume that

$$\lambda^l < b_1/y_1 \leq \lambda^h \quad (L)$$

that is, the immediate cost of defaulting is lower than the cost of repaying the initial debt for a low type, but higher for a high type. Repayment of debt due in the first period generates a net immediate loss for the low type but a net immediate gain for the high type. Consequently, if we were to think of repayment as serving as a signal, this signal

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<sup>14</sup>While we describe the equilibrium in terms of a signalling equilibrium we are not tied to this type of equilibrium. With some minor modifications, our analysis can alternatively be conducted in the context of a model of screening. See Bolton and Dewatripont (2005, ch. 2, 3) for a discussion of signalling and screening equilibria.

would be costly for the low and costless for the high type. We examine later the case where it is also costly for the high type to signal. Our key result that the separating equilibrium involves austerity turns out to be independent of this consideration.

In addition, we assume that the following condition holds:

$$b_2^{lfb} \equiv \arg \max_{b_2^l} u(y_1(1 - \lambda^l) + \beta b_2^l) + \delta u(y_2 - b_2^l) > \lambda^l y_2. \quad (\text{B})$$

Condition (B) implies that the low type is borrowing constrained independent of whether he defaults in the first period or not. We make this assumption to guarantee that the economy would be borrowing constrained under complete information, so that that economy represents the relevant reference point. The condition is satisfied if  $\beta \gg \delta$  or  $y_2 \gg y_1$  and if  $\lambda^l$  is small. Since the first-best financing arrangement for the high type involves a loan size that exceeds  $b_2^{lfb}$ ,  $b_2^{hfb}$  say, condition (B) also implies that  $b_2^{lfb} > \lambda^l y_2$ .

The break even requirement (iv) and the repayment constraints (1) imply that the price satisfies

$$q_1(r_1) = \begin{cases} \beta & \text{if } b_2(r_1) \leq \lambda^l y_2 \\ \beta \theta_1(r_1) & \text{if } \lambda^l y_2 < b_2(r_1) \leq \lambda^h y_2 \\ 0 & \text{otherwise} \end{cases}. \quad (3)$$

In conclusion, an equilibrium is given by the tuple  $(r_1^h, r_1^l, \theta_1(\cdot), \mathcal{F}_1(\cdot), r_2^h(\cdot), r_2^l(\cdot))$  that satisfies conditions (1), (2), (3) as well as Bayes' law (where applicable).

## 2.3 Pooling Equilibrium

In pooling equilibrium, both types select the same first-period repayment rate,  $r_1^h = r_1^l = r_1^p$ . Conditional on observing this repayment rate, lenders form the posterior belief  $\theta_1(r_1^p) = \theta$  and extend the loan  $\mathcal{F}_1(r_1^p) = (q_1(r_1^p), b_2(r_1^p))$ . Off the equilibrium path, a choice of  $r_1 = 1 - r_1^p$  induces the posterior belief  $\theta_1(1 - r_1^p)$  and lenders extend the loan  $\mathcal{F}_1(1 - r_1^p) = (q_1(1 - r_1^p), b_2(1 - r_1^p))$ . In both cases, condition (3) must hold. The selection constraints (2) take the form

$$U_1^i(r_1^p, \mathcal{F}_1(r_1^p)) \geq U_1^i(1 - r_1^p, \mathcal{F}_1(1 - r_1^p)), i = h, l, \quad (4)$$

where  $q_1(r_1)$  satisfies (3) subject to the specified posterior beliefs.

A pooling equilibrium is fully characterized by  $\kappa^p \equiv (r_1^p, q_1(r_1^p), b_2(r_1^p), \theta_1(1 - r_1^p), q_1(1 - r_1^p), b_2(1 - r_1^p))$ . The set of pooling equilibria,  $K^p \subseteq \mathcal{R} \times [0, 1] \times \mathcal{B} \times [0, 1]^2 \times \mathcal{B}$ , is composed of all  $\kappa^p$  satisfying both (3) and (4) subject to  $\theta_1(r_1^p) = \theta$ . Accordingly, the optimal pooling equilibrium  $\kappa^{p*}$  solves

$$\kappa^{p*} = \arg \max_{\kappa^p \in K^p} W(r_1^p, r_1^p, \mathcal{F}_1(\cdot)).$$

Note that while Bayes' law pins down the posterior belief along the equilibrium path,  $\theta_1(r_1^p) = \theta$ , it does not pin down the posterior belief after a deviation,  $\theta_1(1 - r_1^p)$ . Similarly, the break even condition (3) does not pin down the loan size after a deviation,  $b_2(1 - r_1^p)$ . Both these instruments can be chosen to relax the selection constraints.

To avoid unnecessary complications that distract from the central questions of interest we assume that  $\lambda^h = \infty$ .<sup>15</sup> This implies that high types never default and their selection constraint does not bind. Low types therefore do not default in any period. When the appropriate choice of  $\theta_1(0)$  and  $b_2(0)$  can deter a default by the low type and thus deliver a pooling equilibrium,<sup>16</sup>  $b_2(1)$  maximizes  $W(1, 1, \mathcal{F}_1(1))$  subject to (3) with  $\theta_1(1) = \theta$ . There are two possibilities, either  $b_2(1) = \lambda^l y_2$  (a smaller value for  $b_2(1)$  is ruled out by condition (B)) and  $q_1(1) = \beta$  or  $b_2(1) > \lambda^l y_2$  and  $q_1(1) = \beta\theta$ . In the former case, the objective function takes the value

$$\underline{W}^p = (\theta + (1 - \theta)\omega)\{u(y_1 - b_1 + \beta\lambda^l y_2) + \delta u(y_2(1 - \lambda^l))\}.$$

In the latter, it equals

$$\overline{W}^p = (\theta + (1 - \theta)\omega)u(y_1 - b_1 + \beta\theta b_2(1)) + \delta\theta u(y_2 - b_2(1)) + \delta(1 - \theta)\omega u(y_2(1 - \lambda^l)),$$

where  $b_2(1) > \lambda^l y_2$  satisfies the first-order condition

$$(\theta + (1 - \theta)\omega)u'(y_1 - b_1 + \beta\theta b_2(1))\beta\theta = \delta\theta u'(y_2 - b_2(1)).$$

In the former case, the high type clearly suffers austerity. In the latter case, the same holds true if the high type would be borrowing constrained under perfect information because in this case the equilibrium loan size is weakly smaller than it would be under perfect information and at the same time, the price of the loan is smaller.<sup>17</sup> If the high type would not be borrowing constrained under complete information and if  $b_2(1) > \lambda^l y_2$  then again, the high type suffers austerity unless  $\omega$  is very large.<sup>18</sup>

While a large loan size  $b_2(1) > \lambda^l y_2$  may be in the interest of the high type because it improves consumption smoothing it always lies in the interest of the low type because the latter does not repay such a loan. In effect, any loan  $b_2(1) > \lambda^l y_2$  amounts to a transfer from high to low types; holding the loan size fixed, this transfer per capita of a high type increases in the fraction of low types,  $1 - \theta$ . Accordingly, a pooling equilibrium becomes less attractive for high types as their share in the population decreases.

## 2.4 Separating Equilibrium

In a separating equilibrium, the high and low type choose different repayment rates in the first period,  $r_1^h \neq r_1^l$ . Lenders form the posterior belief  $\theta_1(r_1^h) = 1$  and  $\theta_1(r_1^l) = 0$

<sup>15</sup>Throughout the analysis, we state the problem for general  $\lambda^h$  and  $\lambda^l$  but assume  $\lambda^h = \infty$  when characterizing equilibrium.

<sup>16</sup>Setting  $\theta_1(0) = 0$  and  $b_2(0) > \lambda^l y_2$  implies  $q_1(0) = 0$  (from (3)) so that were the low type to default his welfare  $U_1^l(0, \mathcal{F}_1(0))$  would equal the level obtained under autarky.

<sup>17</sup>Strictly speaking, the high type could not be borrowing constrained under perfect information because of our assumption that  $\lambda^h = \infty$ . Under the assumption that  $\lambda^h$  is finite but sufficiently high to render the high type's repayment and selection constraints non binding, the reasoning in the text applies.

<sup>18</sup>For  $\omega = 1$  the condition determining  $b_2(1)$  reduces to  $u'(y_1 - b_1 + \beta\theta b_2(1))\beta = \delta u'(y_2 - b_2(1))$  while in the perfect information case it reads  $u'(y_1 - b_1 + \beta b_2^{h\text{sb}})\beta = \delta u'(y_2 - b_2^{h\text{sb}})$ . This implies  $b_2(1) > b_2^{h\text{sb}}$  but  $\beta\theta b_2(1) < \beta b_2^{h\text{sb}}$  and thus, austerity. Lower values for  $\omega$  further aggravate austerity but very high values may change the result. We do not consider this last possibility to be of much interest.

and, based on this belief, they extend financing  $\mathcal{F}_1(r_1^h) = (q_1(r_1^h), b_2(r_1^h))$  or  $\mathcal{F}_1(r_1^l) = (q_1(r_1^l), b_2(r_1^l))$  subject to (3). The selection constraints (2) take the form

$$U_1^i(r_1^i, \mathcal{F}_1(r_1^i)) \geq U_1^i(r_1^j, \mathcal{F}_1(r_1^j)), i = h, l; i \neq j, \quad (5)$$

subject to (3) and the specified posteriors.

A separating equilibrium is fully characterized by  $\kappa^s \equiv (r_1^h, r_1^l, q_1(r_1^h), b_2(r_1^h), q_1(r_1^l), b_2(r_1^l))$ . The set of separating equilibria,  $K^s \subseteq \mathcal{R}^2 \times [0, 1] \times \mathcal{B} \times [0, 1] \times \mathcal{B}$ , is composed of all  $\kappa^s$  satisfying both (3) and (5) subject to  $\theta_1(r_1^h) = 1$  and  $\theta_1(r_1^l) = 0$ . Accordingly, the optimal separating equilibrium  $\kappa^{s*}$  solves

$$\kappa^{s*} = \arg \max_{\kappa^s \in K^s} W(r_1^h, r_1^l, \mathcal{F}_1(\cdot)).$$

The only feasible separating equilibrium is one where the high type chooses  $r_1^h = 1$  and the low type  $r_1^l = 0$ .<sup>19</sup> The loans in the optimal separating equilibrium therefore satisfy

$$\begin{aligned} (b_2(1), b_2(0)) &= \arg \max_{(b_2^h, b_2^l) \in \mathcal{B}^2} W(1, 0, ((\beta, b_2^h), (\beta, b_2^l))) \\ \text{s.t.} \quad &U_1^h(1, (\beta, b_2^h)) \geq U_1^h(0, (\beta, b_2^h)), \\ &U_1^l(0, (\beta, b_2^l)) \geq U_1^l(1, (\beta, b_2^h)), \\ &b_2^h \leq \lambda^h y_2, \\ &b_2^l \leq \lambda^l y_2. \end{aligned}$$

As before, we let  $\lambda^h = \infty$ . The selection and repayment constraints of the high type then do not bind and can be ignored. There are two possibilities. Either the low type receives the loan  $b_2(0) = \lambda^l y_2$  and the high type a loan that is equal to the amount he would have received under complete information,  $b_2^{h, \text{sb}}$ . This can happen only if the selection constraint of the low type does not bind at this loan level. Or, the low type receives the loan  $b_2(0) = \lambda^l y_2$  and the high type receives less than  $b_2^{h, \text{sb}}$  because the selection constraint of the low type binds.<sup>20</sup>

In either case,  $b_2(0) = \lambda^l y_2$  and  $b_2(1) \geq b_2(0)$ .<sup>21</sup> The latter inequality implies  $U_2^l((\beta, b_2(0)), 1) = U_2^l((\beta, b_2(1)), 0)$ . Accordingly, the selection constraint of the low type reduces to the requirement that first period consumption of the low type when defaulting and receiving  $\mathcal{F}_1(0)$  must be greater or equal to consumption when repaying and receiving  $\mathcal{F}_1(1)$ . Formally, the constraint reduces to  $y_1(1 - \lambda^l) + \beta b_2(0) \geq y_1 - b_1 + \beta b_2(1)$  or

$$b_2(1) \leq b_2(0) + \frac{b_1 - y_1 \lambda^l}{\beta}. \quad (6)$$

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<sup>19</sup>A separating equilibrium with  $r_1^h = 0$  and  $r_1^l = 1$  is not feasible because of the selection constraints. Making the high type better off when he defaults requires a larger loan after default than after no default (from condition (L)); making the low type better off when he does not default requires a larger loan after no default than after default (from condition (L)).

<sup>20</sup>If the selection constraint binds, the repayment constraint of the low type must bind as well. Otherwise, one could increase  $b_2(0)$  and, from the relaxed selection constraint of the low type,  $b_2(1)$  too.

<sup>21</sup>When the selection constraint binds, this follows from condition (L).

Condition (6) caps the loan that can be extended to the high type without encouraging mimicking by the low type; if the condition were violated, mimicking would generate more funds to the low type in the first period at no cost in the second period (since the low type defaults in the second period if the loan exceeds  $\lambda^l y_2$ ). The constraint is tighter and the maximal loan that can be extended to the high type is smaller for lower values of initial debt,  $b_1$ , and for lower growth rates,  $y_2/y_1$  (recall that  $b_2(0) = \lambda^l y_2$ ). Interestingly, it is also tighter, the larger the current level of output, that is, austerity is procyclical. This is due to the fact that the incentive of the low type to mimic is procyclical because the cost of default is an increasing function of output. In the special case where  $\lambda^l = 0$  the constraint reduces to  $b_2(1) \leq b_1/\beta$ . That is, the high type country must produce a budget surplus or equivalently, a *current account surplus*.

In conclusion, the separating equilibrium satisfies  $b_2(0) = y_2 \lambda^l$  and either  $b_2(1) = b_2^{h\text{sb}}$  with (6) not binding, or  $b_2(1) = (\lambda^l(\beta y_2 - y_1) + b_1)/\beta$  with (6) binding. In the relevant case with a binding selection constraint the high type suffers austerity because the loan size is smaller than it would have been under complete information. The low type, in contrast, does not suffer austerity.

The objective function takes the value

$$W^s = (\theta + (1 - \theta)\omega)u(y_1(1 - \lambda^l) + \beta\lambda^l y_2) + \delta \left\{ \theta u \left( y_2(1 - \lambda^l) - \frac{b_1 - \lambda^l y_1}{\beta} \right) + (1 - \theta)\omega u(y_2(1 - \lambda^l)) \right\}.$$

This can be compared to the value that obtains in the pooling equilibrium.

For  $\theta \rightarrow 1$ , the optimal pooling equilibrium is associated with a loan size and price that converge to the financing arrangement extended to a high type under complete information. Hence, both types prefer the optimal pooling equilibrium over the optimal separating equilibrium in this limiting case. For  $\theta \rightarrow 0$ , in contrast, the optimal pooling equilibrium fares worse than the optimal separating equilibrium. It can be verified that there exists a critical value of  $\theta = \theta^*$ , above (below) which pooling gives higher (lower) welfare than separation.

**Costly Signalling** In the analysis so far, creditworthy borrowers find it in their interest to repay outstanding debt in the first period even abstracting from signalling considerations, because the immediate cost of default exceeds their debt obligation (assumption (L)). Hence, such borrowers do not face a meaningful choice between default and repayment. One could think of an alternative environment, though, in which the level of outstanding debt is high enough as to make the short run gains from default exceed the short term losses. Under what conditions would a high creditworthiness type choose to suffer a net short term loss in consumption (suffer austerity) in order to signal his type and secure a better loan deal? Popular arguments in the policy debate suggest that austerity may indeed serve as a costly prerequisite for establishing "credibility" and thus securing a better loan package. We derive such conditions in a simple variant of our endowment model.

We modify condition (L) to

$$\lambda^l < \lambda^h < b_1/y_1, \tag{L'}$$

so that the direct cost of debt repayment exceeds the default losses in the first period, independently of the type of government. A separating equilibrium with  $r_1^h = 1$ ,  $r_1^l = 0$  and  $b_2(1) \geq b_2(0) = y_2\lambda^l$ , is feasible and it dominates the pooling equilibrium if

$$\begin{aligned} u(y_1 - b_1 + \beta b_2(1)) + \delta u(y_2 - b_2(1)) &\geq u(y_1(1 - \lambda^h) + \beta y_2\lambda^l) + \delta u(y_2(1 - \lambda^l)), \\ b_2(1) &\leq y_2\lambda^h, \\ b_2(1) &\leq y_2\lambda^l + \frac{b_1 - y_1\lambda^l}{\beta}, \end{aligned}$$

where the first and second constraint represent the selection and repayment constraint of the high type, respectively—which can no longer be ignored under condition (L') where  $\lambda^h < \infty$ —and the last constraint represents the selection constraint of the low type which is unchanged relative to section 2. The new element here is that the first equation generates a lower bound on the amount of fresh loans that is needed in order to induce the high type to not default in the first period. Consequently, the austerity level required to support a separating equilibrium can be neither too light (because the low type would then mimic) nor too severe (because the high type would default in the first period).

In order to produce a more concrete example we set  $\lambda^l = 0$ ,  $\omega = 0$ . We saw earlier that in this case, the best separating equilibrium involved a fresh loan  $\beta b_2 = b_1$  at the price  $\beta$  if there were no default and a loan of zero if there were default. Can this contract still support separation? That is, does the high type prefer  $b_1, \beta$  to default? The condition for an affirmative answer is

$$U_1^h(0) \geq U_1^h(1, (\beta, b_2)) \Rightarrow u(Y_1) + \delta u(Y_2 - b_2) \geq u(Y_1(1 - \lambda^h)) + \delta u(Y_2). \quad (7)$$

A sufficiently high  $Y_2/Y_1$  ratio or/and a low  $b_1/\lambda^h$  will make this condition satisfied and deliver the best separating equilibrium. Consequently, the requirements for a separating equilibrium now become *more* stringent as the selection constraint of the high type must also be satisfied. But the properties of the optimal separating equilibrium remain the same.

## 3 Contractible Investment

### 3.1 Environment and Equilibrium

We now introduce a decreasing returns to scale technology  $f(\cdot)$  that transforms investment  $I_1 \in \mathcal{I} \equiv [0, \infty)$  in the first period into output  $f(I_1)$  in the second. We interpret investment broadly: It might represent physical investment in productive capacity or investments in institutions that increase future productivity. In line with either interpretation, we allow for the possibility that investment might make it costlier to default in the second period, by triggering default costs  $\tilde{\lambda}^i f(I_2)$  in addition to the income losses  $\lambda^i y_2$ . Clearly, for  $\tilde{\lambda}^i > 0$ , investment increases the collateral of a borrowing country and this alleviates the borrowing constraint, as is well known. But in order to highlight the fact that the main mechanism at work in our model concerns the role of investment as a signalling device



rather than as collateral enhancer we will study both the case of  $\tilde{\lambda}^i = 0$  and of  $\tilde{\lambda}^i = \lambda^i$  when this distinction is relevant.

We first consider the case of contractible investment before turning to the case of non-contractible investment in the subsequent section. With contractible investment, a financing arrangement specifies a level of investment in addition to the price and quantity of debt,  $\mathcal{F}_1 = (q_1, b_2, I_1)$ . Utility of type  $i = h, l$  in period  $t = 2$  now is given by

$$U_2^i(\mathcal{F}_1, r_2) = u\left(y_2(1 - \lambda^i \mathbf{1}_{\{r_2 < 1\}}) + f(I_1)(1 - \tilde{\lambda}^i \mathbf{1}_{\{r_2 < 1\}}) - b_2 r_2\right)$$

and welfare of type  $i = h, l$  equals

$$U_1^i(r_1, \mathcal{F}_1) = u\left(y_1(1 - \lambda^i \mathbf{1}_{\{r_1 < 1\}}) - b_1 r_1 + q_1 b_2 - I_1\right) + \delta \max_{r_2 \in \mathcal{R}} U_2^i(\mathcal{F}_1, r_2).$$

The definition of *equilibrium* is the same as in the basic model. The repayment constraints (1) are modified to

$$r_2^i(\mathcal{F}_1) = \begin{cases} 1 & \text{if } \lambda^i y_2 + \tilde{\lambda}^i f(I_1) \geq b_2 \\ 0 & \text{if } \lambda^i y_2 + \tilde{\lambda}^i f(I_1) < b_2 \end{cases}, i = h, l, \quad (8)$$

while the selection constraints (2) (which take the form (4) in pooling equilibrium and (5) in separating equilibrium) remain unchanged. The price therefore satisfies

$$q_1(r_1) = \begin{cases} \beta & \text{if } b_2(r_1) \leq \lambda^l y_2 + \tilde{\lambda}^l f(I_1) \\ \beta \theta_1(r_1) & \text{if } \lambda^l y_2 + \tilde{\lambda}^l f(I_1) < b_2(r_1) \leq \lambda^h y_2 + \tilde{\lambda}^h f(I_1) \\ 0 & \text{otherwise} \end{cases}. \quad (9)$$

### 3.2 Pooling Equilibrium

In addition to the objects introduced in the previous section, a pooling equilibrium now also involves the levels of investment,  $I_1(r_1^p)$  and  $I_1(1 - r_1^p)$ . The selection constraints are still given by (4). A pooling equilibrium is characterized by  $\kappa^p \equiv (r_1^p, q_1(r_1^p), b_2(r_1^p), I_1(r_1^p), \theta_1(1 - r_1^p), q_1(1 - r_1^p), b_2(1 - r_1^p), I_1(1 - r_1^p))$  and the set of pooling equilibria,  $K^p \subseteq \mathcal{R} \times [0, 1] \times \mathcal{B} \times \mathcal{I} \times [0, 1]^2 \times \mathcal{B} \times \mathcal{I}$ , is composed of all  $\kappa^p$  satisfying both (4) and (9) subject to  $\theta_1(r_1^p) = \theta$ . Accordingly, the optimal pooling equilibrium  $\kappa^{p*}$  solves

$$\kappa^{p*} = \arg \max_{\kappa^p \in K^p} W(r_1^p, r_1^p, \mathcal{F}_1(\cdot)).$$

Analogously to the situation without investment, the off-equilibrium objects  $\theta_1(1 - r_1^p)$  and  $b_2(1 - r_1^p)$  (and  $I_1(1 - r_1^p)$ ) can be chosen to make the selection constraints non binding.

We again assume that  $\lambda^h = \infty$ , implying that  $r_1^p = 1$ . When the selection constraints do not bind, the quantities  $(b_2(1), I_1(1))$  maximize  $W(1, 1, \mathcal{F}_1(1))$  subject to (9) with  $\theta_1(1) = \theta$ . As before, two cases can be distinguished: Either  $b_2(1)$  equals  $\lambda^l y_2 + \tilde{\lambda}^l f(I_1(1))$  with  $q_1(1) = \beta$ ; or, it exceeds that value and  $q_1(1) = \beta \theta$ .

If  $b_2(1) \leq \lambda^l y_2 + \tilde{\lambda}^l f(I_1(1))$ , the objective function takes the value

$$W^p = (\theta + (1 - \theta)\omega)\{u(c_1) + \delta u(c_2)\}$$

with  $c_1 \equiv y_1 - b_1 + \beta b_2(1) - I_1(1)$  and  $c_2 \equiv y_2 - b_2(1) + f(I_1(1))$  where  $I_1$  solves

$$u'(c_1) = \delta f'(I_1(1))u'(c_2) + \tilde{\lambda}^l f'(I_1(1))[u'(c_1)\beta - \delta u'(c_2)].$$

If investment contributes collateral ( $\tilde{\lambda}^l > 0$ ) and the repayment constraint of the low type binds (as reflected in the wedge  $[u'(c_1)\beta - \delta u'(c_2)]$ ) investment is distorted upwards in order to increase collateral.<sup>22</sup> Otherwise, the investment decision is optimal given the loan size. But in either case, the high type suffers austerity because as the high type's loan size falls below the level that would have been extended under complete information, the investment level drops too but by less than one to one (due to the normality of consumption).

If  $b_2(1) > \lambda^l y_2 + \tilde{\lambda}^l f(I_1(1))$ , the objective function takes the value

$$W^p = (\theta + (1 - \theta)\omega)u(c_1) + \delta\{\theta u(c_2^h) + (1 - \theta)\omega u(c_2^l)\}$$

with  $c_1 \equiv y_1 - b_1 + \beta\theta b_2(1) - I_1(1)$ ,  $c_2^h \equiv y_2 - b_2(1) + f(I_1(1))$  and  $c_2^l \equiv y_2(1 - \lambda^l) + f(I_1(1))(1 - \tilde{\lambda}^l)$  where  $(b_2(1), I_1(1))$  solves

$$\begin{aligned} (\theta + (1 - \theta)\omega)u'(c_1)\beta\theta &= \delta\theta u'(c_2^h), \\ (\theta + (1 - \theta)\omega)u'(c_1) &= \delta f'(I_1(1))\{\theta u'(c_2^h) + (1 - \theta)\omega u'(c_2^l)(1 - \tilde{\lambda}^l)\}. \end{aligned}$$

Note that strictly positive values for  $\omega$  imply that the investment level conditional on loan size and price is smaller than the conditional investment level of the high type in the complete information case. This is due to two factors. First, the low type's preferred conditional investment level is lower than the one of the high type because the former defaults in the second period whereas the latter does not. Second, if  $\tilde{\lambda}^l > 0$ , the return to investment is lower for the low type because he defaults.

The implications for austerity for the high type are similar to those discussed previously, in the model without investment. But high values for  $\omega$  make austerity lighter not only for the reasons discussed there but also because investment is lower (conditional on loan size and price) and thus consumption higher if  $\omega$  is large.

### 3.3 Separating Equilibrium

A separating equilibrium is fully characterized by  $\kappa^s \equiv (r_1^h, r_1^l, q_1(r_1^h), b_2(r_1^h), I_1(r_1^h), q_1(r_1^l), b_2(r_1^l), I_1(r_1^l))$  and the selection constraints are given by (5) subject to (9) as well as the posterior beliefs  $\theta_1(r_1^h) = 1$  and  $\theta_1(r_1^l) = 0$ . The set of pooling equilibria,  $K^s \subseteq \mathcal{R}^2 \times [0, 1] \times \mathcal{B} \times \mathcal{I} \times [0, 1] \times \mathcal{B} \times \mathcal{I}$ , is composed of all  $\kappa^s$  satisfying both (5) and (9) subject to the stated posteriors. The optimal separating equilibrium  $\kappa^{s*}$  solves

$$\kappa^{s*} = \arg \max_{\kappa^s \in K^s} W(r_1^h, r_1^l, \mathcal{F}_1(\cdot)).$$

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<sup>22</sup>This case corresponds to the situation with a single type that has been studied in the literature, see Obstfeld and Rogoff (1996, 6.2.1.3).

As before, the only separating equilibrium is one where the high type chooses  $r_1^h = 1$  and the low type  $r_1^l = 0$ . The loan sizes and investment levels in the optimal separating equilibrium therefore solve

$$\begin{aligned} (b_2(1), b_2(0), I_1(1), I_1(0)) &= \arg \max_{(b_2^h, b_2^l, I_1^h, I_1^l) \in \mathcal{B}^2 \times \mathcal{I}^2} W(1, 0, ((\beta, b_2^h, I_1^h), (\beta, b_2^l, I_1^l))) \\ \text{s.t.} \quad &U_1^h(1, (\beta, b_2^h, I_1^h)) \geq U_1^h(0, (\beta, b_2^l, I_1^l)), \\ &U_1^l(0, (\beta, b_2^l, I_1^l)) \geq U_1^l(1, (\beta, b_2^h, I_1^h)), \\ &b_2^h \leq \lambda^h y_2 + \tilde{\lambda}^h f(I_1^h), \\ &b_2^l \leq \lambda^l y_2 + \tilde{\lambda}^l f(I_1^l). \end{aligned}$$

As before, we assume that  $\lambda^h = \infty$ . Accordingly, the selection and repayment constraints of the high type do not bind and can be ignored.

**Investment Does Not Enhance Collateral** We establish two important results. First, conditional on loan size there is over-investment even if investment does not increase collateral ( $\tilde{\lambda}^i = 0$ ), because investment serves as a means of mitigating the adverse selection friction. And second, this over-investment is so severe as to make the high type's consumption lower than it would have been were it not possible to use investment as a device for that purpose. Stated differently, investment helps the high type to partly overcome the adverse selection friction, but it does so at the cost of even more severe austerity.

In general, distorted investment creates a welfare loss. But in the presence of adverse selection the high type benefits from distorted investment because this slackens the selection constraint of the low type and thus, makes it possible for the high type to obtain a larger loan. Although at the margin the increased loan size is more than fully absorbed by higher investment, the high type still enjoys a net benefit.

These results differ from the standard result in the sovereign debt literature that over-investment is useful because it relaxes the repayment constraint (see Obstfeld and Rogoff (1996, 6.2.1.3) and the discussion in the preceding subsection of pooling equilibrium when the loan size is small). The latter, well-known result requires the assumption that investment serves to increase collateral ( $\tilde{\lambda}^i > 0$ ). Our result has a different source (the existence of adverse selection) and role (the mitigation of the resulting friction) and holds independently of whether  $\tilde{\lambda}^i = 0$  or not.

Consider the program above (with  $\lambda^h = \infty$ ) that characterizes the optimal separating equilibrium. Let  $\mu$  and  $\nu$  denote the multipliers on the low type's selection and repayment constraints, respectively, and let  $c_1^h \equiv y_1 - b_1 + \beta b_2(1) - I_1(1)$ ,  $c_2^h \equiv y_2 - b_2(1) + f(I_1(1))$ ,  $c_1^l \equiv y_1(1 - \lambda^l) + \beta b_2(0) - I_1(0)$  and  $c_2^l \equiv y_2 - b_2(0) + f(I_1(0))$  denote the first- and second period consumption levels of the high and low type in equilibrium. The Lagrangian is

$$\begin{aligned} \mathcal{L} &= \theta \{u(c_1^h) + \delta u(c_2^h)\} + (1 - \theta) \omega \{u(c_1^l) + \delta u(c_2^l)\} + \nu \{\lambda^l y_2 - b_2(0)\} \\ &\quad + \mu \{u(c_1^l) + \delta u(c_2^l) - u(c_1^h) - \delta u(y_2(1 - \lambda^l) + f(I_1(1)))\}. \end{aligned}$$

In addition to the complementary slackness conditions, we have the following first-order

conditions:

$$\begin{aligned}
b_2(1) : & \quad \theta\{u'(c_1^h)\beta - \delta u'(c_2^h)\} = \mu\beta u'(c_1^h), \\
b_2(0) : & \quad ((1 - \theta)\omega + \mu)\{u'(c_1^l)\beta - \delta u'(c_2^l)\} = \nu, \\
I_1(1) : & \quad \theta\{-u'(c_1^h) + \delta f'(I_1(1))u'(c_2^h)\} = \mu\{-u'(c_1^h) + \delta f'(I_1(1))u'(y_2(1 - \lambda^l) + f(I_1(1)))\}, \\
I_1(0) : & \quad ((1 - \theta)\omega + \mu)\{-u'(c_1^l) + \delta f'(I_1(0))u'(c_2^l)\} = 0.
\end{aligned}$$

The first condition states that the high type's consumption profile is distorted ( $u'(c_1^h)\beta \neq \delta u'(c_2^h)$ ) whenever the selection constraint of the low type binds.<sup>23</sup> The second condition indicates that the shadow cost of the low type's repayment constraint,  $\nu$ , is non-zero if his consumption profile is distorted ( $u'(c_1^l)\beta \neq \delta u'(c_2^l)$ ) and either  $\omega > 0$  or  $\mu > 0$  (the selection constraint binds). The third condition states that investment of the high type is *distorted*—conditional on loan size—if the selection constraint binds and if a low type mimicking a high type is forced to over- or under-invest. Intuitively, the cost of distorting investment for the high type upwards is balanced by the benefit from relaxing the selection constraint of the low type, and such relaxation results by distorting the mimicking low type's investment. Finally, the last constraint states that along the equilibrium path, the investment of the low type cannot be distorted if  $\omega > 0$  or if the selection constraint binds. Intuitively, allowing the low type to invest optimally when he does not mimic increases his utility and thus helps relax his selection constraint.

Consider first the case where the selection constraint does not bind, that is  $\mu = 0$ . The first and third first-order conditions then imply that the presence of asymmetric information is of no consequence for the high type. For this outcome to obtain it must be that  $U_1^l(0, (\beta, \lambda^l y_2, I_1(0))) \geq U_1^l(1, (\beta, b_2^{h\text{sb}}, I_1^{h\text{sb}}))$  where  $I_1(0)$  is the low type's optimal investment conditional on  $r_1^l = 0$  and  $b_2(0) = \lambda^l y_2$ .

If, in contrast, the latter inequality is violated, then the selection constraint binds and  $\mu > 0$ . In this case, the equilibrium involves conditionally undistorted investment for the low type along the equilibrium path but distorted investment when there is mimicking. Combining the first and third first-order conditions gives

$$\frac{u'(c_1^h)\beta - \delta u'(c_2^h)}{\beta u'(c_1^h)} = \frac{u'(c_1^h) - \delta f'(I_1(1))u'(c_2^h)}{u'(c_1^h) - \delta f'(I_1(1))u'(y_2(1 - \lambda^l) + f(I_1(1)))} > 0. \quad (10)$$

The numerator on the right-hand side of condition (10) measures the investment distortion for the high type and the denominator measures the investment distortion for the low type when mimicking. The two wedges have the same sign (because  $u'(c_1^h)\beta > \delta u'(c_2^h)$  from the first-order condition with respect to  $b_2(1)$ .) Moreover, as established in the following proposition, their sign is positive. A binding selection constraint therefore implies *over-investment*—conditional on loan size—for the high type.

Perhaps more surprisingly, a binding selection constraint implies an extreme form of over-investment: At the margin, the marginal propensity to invest borrowed funds exceeds unity. In other words, an increase in first-period funding is associated with a more than

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<sup>23</sup>The consumption profile would further be distorted if the repayment constraint limited borrowing by the high type. Our assumption that  $\lambda^h = \infty$  rules this out.

one-to-one increase in investment and thus, *lower* consumption and *higher* austerity for the high type. This is a direct consequence of the need to discourage mimicking by low types. Although the high type's investment is too high—conditional on loan size,—the arrangement is still better than other incentive compatible arrangements with a smaller loan size.

Figure 1 shows the selection constraint of the low type as well as the indifference curves of the high type in  $(b_2(1), I_1(1))$ -space. The demarcation line between the colored region on the left-hand side of the contour plot and the white region on the right-hand side represents the selection constraint of the low type (with equality) where the low type's equilibrium loan size  $b_2(0)$  is fixed at the maximal value compatible with her repayment constraint,  $b_2(0) = \lambda^l y_2$ , and  $I_1(0)$  is the optimal investment level of a low type (conditional on  $r_1 = 0$  and  $b_2 = \lambda^l y_2$ ). The demarcation line indicates that given  $I_1(1)$ , mimicking by the low type can only be prevented for sufficiently low values of  $b_2(1)$ .

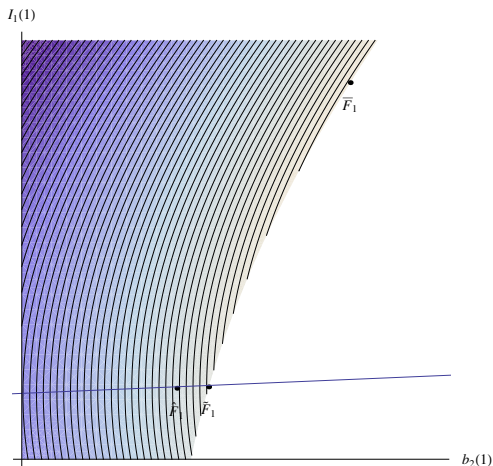


Figure 1: Separating equilibrium with contractible investment.

The colored region on the left-hand side of the contour plot contains the  $(b_2(1), I_1(1))$ -combinations that are associated with financing arrangements  $\mathcal{F}_1 = (\beta, b_2(1), I_1(1))$  compatible with a separating equilibrium. The financing arrangement in the optimal separating equilibrium is indicated by  $\bar{\mathcal{F}}_1 = (\bar{b}_2^h, \bar{I}_1^h)$  (we leave the price  $q_1(1) = \beta$  implicit) in the upper right part of the figure. It represents the point on the right-most indifference curve of the high type that lies in the set of incentive compatible arrangements. The figure also displays two other incentive compatible financing arrangements, indicated by  $\hat{\mathcal{F}}_1$  and  $\tilde{\mathcal{F}}_1$  in the lower part of the figure; they are useful for building intuition about the properties of the optimal equilibrium.

Consider first the arrangement  $\hat{\mathcal{F}}_1 = (\hat{b}_2^h, \hat{I}_1^h)$ , with  $\hat{b}_2^h = (\lambda^l(\beta y_2 - y_1) + b_1)/\beta = b_2(0) + (b_1 - \lambda^l y_1)/\beta$  and  $\hat{I}_1^h$  the conditionally optimal investment level of a high type that receives a loan of size  $\hat{b}_2^h$ . This financing arrangement represents the optimal separating equilibrium in the model without investment, augmented with the conditionally optimal investment level. Note that the selection constraint of the low type is guaranteed to not bind at  $\hat{\mathcal{F}}_1$  because the optimal investment level of a high type with loan size  $\hat{b}_2^h$

differs from the optimal investment level of a mimicking low type that receives a loan of that size.<sup>24</sup> In contrast, the arrangement  $\tilde{\mathcal{F}}_1 = (\tilde{b}_2^h, \tilde{I}_1^h)$  is just incentive compatible. It represents the loan size  $\tilde{b}_2^h$  and conditionally optimal investment level  $\tilde{I}_1^h$  of the high type such that the selection constraint of the low type holds with equality. The line through points  $\hat{\mathcal{F}}_1$  and  $\tilde{\mathcal{F}}_1$  indicates the conditionally optimal investment level of the high type for every loan size.

With the arrangement  $\tilde{\mathcal{F}}_1$ , the high type invests his preferred level (given  $\tilde{b}_2^h$ ). The indifference curve of the high type in  $(b_2(1), I_1(1))$  space is therefore vertical at the point  $\tilde{\mathcal{F}}_1$  while the indifference curve of a mimicking low type has slope

$$S = -\frac{u'(y_1 - b_1 + \beta\tilde{b}_2^h - \tilde{I}_1^h)\beta}{-u'(y_1 - b_1 + \beta\tilde{b}_2^h - \tilde{I}_1^h) + \delta f'(\tilde{I}_1^h)u'(y_2(1 - \lambda^l) + f(\tilde{I}_1^h))} > 0.$$

The numerator of  $S$  captures the fact that a larger loan  $\tilde{b}_2^h$  increases first-period consumption of a mimicking low type but does not affect second-period consumption because the mimicking low type defaults in the second period. The denominator shows that larger investment reduces (increases) first- (second-) period consumption of a mimicking low type. The denominator is negative because  $\tilde{I}_1^h$  is higher than the preferred investment level of a mimicking low type that repays in the first period, receives a loan of size  $\tilde{b}_2^h$  and defaults in the second period.<sup>25</sup>

Since the indifference curve of the high type at the point  $\tilde{\mathcal{F}}_1$  is vertical and the indifference curve of a mimicking low type has a finite positive slope there exist other arrangements for the high type with  $(b_2(1), I_1(1)) > (\tilde{b}_2^h, \tilde{I}_1^h)$  that make the high type strictly better off and still satisfy the selection constraint of the low type. The best among those arrangements is at point  $\bar{\mathcal{F}}_1$  where the indifference curves of the high type and the mimicking low type are tangent, that is, where both the selection constraint of the low type (with equality) and condition (10) hold.

The following proposition summarizes this discussion and establishes further results concerning the slope  $S$  as well as the loan size and investment level of the high type.

**Proposition 1.** *Consider the separating equilibrium in the model with contractible investment, no collateral contributing role for investment and  $\lambda^h = \infty$ . Suppose that the low type's selection constraint binds. Then:*

(a) *The high type is borrowing constrained,  $u'(c_1^h)\beta > \delta u'(c_2^h)$ , even if his repayment constraint does not bind.*

<sup>24</sup>That the selection constraint is slack follows from

$$\begin{aligned} & u(y_1(1 - \lambda^l) + \beta b_2(0) - I_1(0)) + \delta u(y_2 + f(I_1(0)) - b_2(0)) \\ &= u(y_1 - b_1 + \beta\hat{b}_2^h - I_1(0)) + \delta u(y_2 + f(I_1(0)) - b_2(0)) \\ &= u(y_1 - b_1 + \beta\hat{b}_2^h - I_1(0)) + \delta u(y_2(1 - \lambda^l) + f(I_1(0))) \\ &> u(y_1 - b_1 + \beta\hat{b}_2^h - \hat{I}_1^h) + \delta u(y_2(1 - \lambda^l) + f(\hat{I}_1^h)). \end{aligned}$$

The last inequality holds because  $\hat{I}_1^h \neq I_1(0)$  since the second period endowments after default cost and debt repayment differ between the high type and the mimicking low type.

<sup>25</sup>This follows from  $-u'(y_1 - b_1 + \beta\tilde{b}_2^h - \tilde{I}_1^h) + \delta f'(\tilde{I}_1^h)u'(y_2 - \tilde{b}_2^h + f(\tilde{I}_1^h)) = 0$  and  $\tilde{b}_2^h > \hat{b}_2^h > \hat{b}_2^l = \lambda^l y_2$ .

(b) Conditional on the constrained loan size, investment of the high type is distorted upwards,  $u'(c_1^h) > \delta f'(I_1(1))u'(c_2^h)$ .

(c) At the margin, the investment level of the high type increases by more than one-to-one with the loan size. That is, at the margin, austerity increases with the loan size.

(d) The investment level of the high type is strictly smaller than the first-best investment level  $I_1^{\text{fb}}$  which satisfies  $\beta f'(I_1^{\text{fb}}) = 1$ . Moreover, the loan size for the high type is strictly smaller than the first best.

*Proof.* Part (a): Follows directly from the first-order condition.

Parts (b) and (c): The indifference curve of a mimicking low type at the point  $\tilde{\mathcal{F}}_1$  has slope  $S$  which can be expressed as

$$S = \frac{\beta}{1 - \delta f'(\tilde{I}_1^h) \frac{u'(y_2(1-\lambda^l) + f(\tilde{I}_1^h))}{u'(y_1 - b_1 + \beta \tilde{b}_2^h - \tilde{I}_1^h)}} > \beta.$$

At point  $\tilde{\mathcal{F}}_1$ , a marginal increase of  $b_2(1)$  by  $\Delta$  (which means that the funds raised increase by  $\beta\Delta$ ) must be accompanied by a marginal increase of  $I_1(1)$  by more than  $\beta\Delta$  in order to satisfy the low type's selection constraint. That is, the marginal increase of  $b_2(1)$  by  $\Delta$  goes hand in hand with a reduction in the first-period equilibrium consumption of the high type. The slope falls as we move further along the indifference curve of the mimicking low type but it remains bounded below by  $\beta$ . This follows from the fact that conditional on  $b_2(1)$ ,  $I_1(1)$  continues to be excessively high from the perspective of a mimicking low type,  $u'(y_1 - b_1 + \beta b_2(1) - I_1(1)) > \delta f'(I_1(1))u'(y_2(1 - \lambda^l) + f(I_1(1)))$ .

(d) Let  $\alpha \equiv 1/[\beta f'(I_1(1))]$  where  $\alpha < 1$  indicates under-investment relative to first best and let  $M^h$  and  $M^l$ , respectively, denote the normalized marginal rates of substitution between first- and second-period consumption,  $\delta u'(c_2)/[\beta u'(c_1)]$ , of the high type and the mimicking low type. The left-hand side of condition (10) can then be expressed as  $1 - M^h$  and the right-hand side as

$$\frac{u'(c_1^h) - \delta u'(c_2^h)/(\alpha\beta)}{u'(c_1^h) - \delta u'(y_2(1 - \lambda^l) + f(I_1^h))/(\alpha\beta)} \quad \text{or} \quad \frac{\alpha - M^h}{\alpha - M^l}.$$

Condition (10) therefore reduces to

$$1 - M^h = \frac{\alpha - M^h}{\alpha - M^l} \quad \text{or} \quad \alpha = 1 + M^l \left( 1 - \frac{1}{M^h} \right).$$

Since the high type is borrowing constrained,  $M^h < 1$ . This implies  $\alpha < 1$  and thus, under-investment relative to first best.

As shown earlier,  $\tilde{I}_1^h$  is optimal (conditional on  $\tilde{b}_2^h$ ) and the increase  $\bar{b}_2^h - \tilde{b}_2^h$  is associated with a change  $\bar{I}_1^h - \tilde{I}_1^h > \beta(\bar{b}_2^h - \tilde{b}_2^h)$ . This implies  $\bar{b}_2^h < \tilde{b}_2^h + (\bar{I}_1^h - \tilde{I}_1^h)/\beta$ . Since first-best, first-period consumption is higher than first-period consumption under  $(\tilde{b}_2^h, \tilde{I}_1^h)$  we have  $\beta \tilde{b}_2^{\text{fb}} - \tilde{I}_1^{\text{fb}} > \beta \tilde{b}_2^h - \tilde{I}_1^h$  or  $\tilde{b}_2^{\text{fb}} > \tilde{b}_2^h + (\tilde{I}_1^{\text{fb}} - \tilde{I}_1^h)/\beta$ . Combining these inequalities and using the fact that  $\bar{I}_1^h < \tilde{I}_1^{\text{fb}}$  implies  $\bar{b}_2^h < \tilde{b}_2^{\text{fb}}$ .  $\square$

**Investment Enhances Collateral** Suppose now that investment proceeds also serve as collateral,  $\tilde{\lambda}^l = \lambda^l$ . This has two implications for the Lagrangian. First, the repayment constraint of the low type changes from  $\lambda^l y_2 \geq b_2(0)$  to

$$\lambda^l (y_2 + f(I_1(0))) \geq b_2(0).$$

And second, the low type's selection constraint becomes

$$u(c_1^l) + \delta u(c_2^l) \geq u(c_1^h) + \delta u((y_2 + f(I_1(1)))(1 - \lambda^l))$$

instead of  $u(c_1^l) + \delta u(c_2^l) \geq u(c_1^h) + \delta u(y_2(1 - \lambda^l) + f(I_1(1)))$ , reflecting the fact that a mimicking low type that defaults in the second period suffers losses on the return on investment in addition to those on the exogenous income.

The first-order conditions with respect to  $b_2(1)$  and  $b_2(0)$  are not affected by these changes. In contrast, the first-order conditions with respect to the investment levels change to

$$\begin{aligned} I_1(1) : \quad & \theta \{-u'(c_1^h) + \delta f'(I_1(1))u'(c_2^h)\} \\ & = \mu \{-u'(c_1^h) + \delta f'(I_1(1))(1 - \lambda^l)u'((y_2 + f(I_1(1)))(1 - \lambda^l))\}, \\ I_1(0) : \quad & ((1 - \theta)\omega + \mu) \{-u'(c_1^l) + \delta f'(I_1(0))u'(c_2^l)\} = -\nu \lambda^l f'(I_1(0)). \end{aligned}$$

The first condition indicates that the marginal return on investment for a mimicking low type who defaults in the second period equals  $f'(I_1(1))(1 - \lambda^l)$  rather than  $f'(I_1(1))$  as was the case before. The term on the right-hand side of the second first-order condition reflects the fact that investment of the low type slackens his repayment constraint.

The central results established in the setting without a collateral contributing role for investment continue to hold. In particular, when the selection constraint does not bind,  $\mu = 0$ , the presence of asymmetric information does not affect the financing arrangement for the high type. When it binds ( $\mu > 0$ ), investment of the high type is conditionally distorted as in the case where investment proceeds do not serve as collateral. In this case, combining the first-order conditions with respect to  $b_2(1)$  and  $I_1(1)$  yields a version of equation (10) that only differs insofar as the denominator on the right-hand side reflects the modified marginal return on  $I_1(1)$  for a mimicking low type.

When the selection constraint binds, investment of the high type is conditionally distorted upwards, exactly for the same reasons as before. Moreover, at the margin, a larger loan size for the high type continues to go hand in hand with a rise of investment by more than one-to-one. At the margin, a larger loan size therefore continues to imply harsher austerity for the high type. Finally, relative to first best, the investment level and loan size of the high type continue to be depressed. All the results derived in the setting without a collateral contributing role for investment thus continue to hold. These results essentially hinge on the presence and properties of the low type's selection constraint, in particular with respect to the interaction between  $b_2(1)$  and  $I_1(1)$ . But this interaction is not altered in important ways when investment enhances collateral.



## 4 Non-Contractible Investment

If investment is not contractible, the financing arrangement only specifies the price and quantity of debt,  $\mathcal{F}_1 = (q_1, b_2)$ , as it was the case in the baseline model. Investment is chosen by the sovereign after the lenders have extended the loan. It is therefore a function of  $\mathcal{F}_1$  and  $r_1$  rather than of  $r_1$  only as in the case with contractible investment. The repayment rate in the second period is still a function of  $(q_1, b_2, I_1)$ .

This change in timing protocol implies an additional equilibrium condition: Investment must be optimal for the sovereign conditional on the loan size and price. Referring back to figure 1, this restriction adds the requirement that not only must the financing arrangement have to lie in the colored region in order to satisfy the low type's selection constraint but it must also lie on the line through points  $\hat{\mathcal{F}}_1$  and  $\tilde{\mathcal{F}}_1$  in order to satisfy—conditional—optimality of investment by the high type. The optimal separating equilibrium with non-contractible investment therefore involves the arrangement  $\tilde{\mathcal{F}}_1$ .

If investment is not contractible, then its level cannot be used to assist separation. Consequently, the model with non-contractible investment therefore does not generate any new insights relative to the model without investment.

## 5 Extensions and Further Implications

We now extend the model in order to address the role of spending multipliers in the determination of the optimal degree of austerity. We derive the main implications in the context of a simple version of the endowment economy of section 2, but the main insights carry over to the more general case. In another extension, we allow for costly signalling in the version of the model with endowments.

**Multipliers** Rather than embedding the mechanism outlined so far into a standard DSGE model of the New Keynesian variety—an approach that seems both daunting and unnecessary—we introduce a simple modification that allows our model to shed light on the relationship between the size of the multipliers and the optimal degree of austerity.

The essence of the concept of multiplier is that an autonomous change in spending in the private or public sector can have an amplified effect on spending and income in the economy at large. We capture this by assuming that disposable income and consumption in the first period are given by  $y_1(1 - \lambda^i \mathbf{1}_{\{r_1 < 1\}}) + m(q_1 b_2 - b_1 r_1)$  with  $m \geq 1$ ; in the baseline model, we posited  $m = 1$ .<sup>26</sup> Consequently, in the presence of the multiplier, the selection constraint of the low type, condition (6), generalizes to  $y_1(1 - \lambda^l) + m\beta b_2(0) \geq y_1 + m(\beta b_2(1) - b_1)$  or

$$b_2(1) \leq b_2(0) + \frac{b_1 - y_1 \lambda^l / m}{\beta}.$$

Two points are worth making. First, allowing for multiplier effects does not alter the role of austerity as a device for inducing separation. That is, the amount of debt issued in the absence of default is constrained to be below the level under complete information.

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<sup>26</sup>The assumption that  $m > 1$  may also be motivated by distorting taxation.

Second, the required amount of austerity is decreasing in the size of the multiplier (the optimal loan size  $b_2(1)$  is increasing in  $m$ ). That is, less austerity should be applied when austerity has large negative effects on output. While this property seems to corroborate conventional thinking on this subject, it is important to note that it does not represent a robust implication of the model and arises from our assumption that the multiplier effects only apply to a specific subset of disposable income. Suppose, instead that we make the assumption that the multiplier effects apply also to the income losses arising from default, that is,  $y_1(1 - m_1\lambda^i\mathbf{1}_{\{r_1 < 1\}}) + m_2(q_1b_2 - b_1r_1)$  where  $m_1$  and  $m_2$  are multipliers associated with different sources of disposable income. Under this specification, if  $m_1 = m_2$  the optimal level of austerity (6) is invariant to the size of the multiplier.

**Structural Reform and Austerity** The management of the Greek sovereign debt crisis has recently witnessed a shift of emphasis away from fiscal towards structural reform measures. Greece’s official creditors have offered her a relaxation of the fiscal requirements that were already agreed previously, essentially allowing the country to run above target budget deficits financed by the official creditors, in exchange for the implementation of a package of reforms drawn up by the task force and the OECD. This development has been greeted as a relaxation of austerity. But is it? As we noted when discussing the model with investment, structural reform could be an example of some sort of investment, requiring resources in the short run but generating returns in the long run. In light of these results, and for the same reasons, it is clear that a high type government would be more willing to engage in structural reform than a low type, and that a high type could be best off exhibiting ”excessive” reform zeal in order to reveal her type and get more funds. What supports such action is that high crediworthiness governments value resources in the future more than the low types do.

In light of this discussion, the extension of more financing in combination with stricter requirements for structural reform (as currently being discussed for Greece) should then not be mis-interpreted as leniency; it rather constitutes harsh austerity.<sup>27</sup>

## 6 Conclusions

The debate on the role and implications of austerity in the context of sovereign debt seems to be conducted in a haphazard manner due to the lack of a suitable theoretical framework. There exists no model based definition of austerity that can accommodate the various functions it allegedly has. The present paper aims at filling this gap by providing a unified approach that combines the standard sovereign debt model with that on credit rationing under incomplete information about credit risk. We have offered a

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<sup>27</sup>An alternative theory of structural reform as selection device could assume that reform imposes other cost on government than reduced first-period consumption, for example a reduction of policy makers’ rents due to reduced voter support. In such an alternative theory with non-benevolent policy makers, structural reforms may not be associated with austerity as defined in this paper. But it remains unclear how such theory with non-benevolent policy makers could explain that the latter engage in structural reform although it does not benefit them.

coherent definition of austerity, namely, the drop in consumption due to the incomplete information friction.

The fusion of the two literatures gives rise to properties that are different from those obtained in the constituent parts. For instance, unlike the sovereign debt literature where the optimal degree of austerity is zero and investment supports larger loans exclusively through its capacity to create collateral, in our model the optimal degree of austerity is non-zero and investment supports larger loans even without collateral creation. Unlike the credit rationing literature, in our model there is credit rationing and austerity even in separating equilibria where the sovereign's credit risk is revealed through its action to honor (or not) current debt obligations. Austerity is necessary in order to deter the misrepresentation of credit risks and support separation.

Our analysis has a number of novel, useful implications. First, it demonstrates that low credit risk sovereigns may prefer more severe austerity —brought about by a commitment to over-invest fresh funds obtained— to the lighter austerity they would have suffered if they forewent such a commitment. Second, the same property is present when governments can use reforms instead of investment. Committing to financially costly —in the short term—reforms can increase the flow of funds but does not alleviate the loss of current consumption. Consequently, the model implies the absence of a clear relationship between the size of new funding and austerity. Nonetheless, the relationship is unambiguously negative when such costly signals of high creditworthiness are not available (as it is the case in the absence of commitment to invest). And third, it provides a novel perspective on the relationship between the size of spending multipliers and the severity of austerity suffered. In particular, it establishes that multipliers may be linked to austerity because their size matters for the *identification* of credit risk even when their alleged effects on economic growth and ability to pay are limited.

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