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Product Heterogeneity, Within-Industry Trade Patterns, and the Home Bias of Consumption*

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Abstract

Starting with Krugman (1980), much literature has analyzed how trade liberalization affects the economy based on the notion that trade is motivated by consumer’s love of variety. In this paper, I augment these preferences by the determinants of demand for heterogeneous products. The model features products with heterogeneous attributes and consumers with heterogeneous tastes for attributes. Allowing for international trade, the model predicts a within-industry home market effect, i.e., that high domestic demand for an attribute leads to entry of firms producing a fitting output and, consequently, net exports of products embodying the attribute. Second, the model rationalizes why consumption is home-biased in the short run. Each country’s industry is optimized for the preferences of domestic consumers and thus somewhat inappropriate for the export market. Third, in the long run, countries specialize further and the within-industry home market effect intensifies. Intriguingly, (as long as it is incomplete) this specialization implies that the home bias disappears completely, thus demonstrating that Linder’s (1961) conjecture describes a temporary phenomenon that does not prevail in general equilibrium.

Keywords: Intra-Industry Trade, Monopolistic Competition, Firm Dynamics, Product Heterogeneity, Industrial Structure

JEL: F12, F15, L15, L16

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1 Introduction

Starting with the seminal work of Krugman (1979, 1980, and 1981), increasingly returns that arise from consumer’s “love of variety” (or from their similar search for the “ideal variety” as in Lancaster (1980)) have been regarded as the major motive for international trade. Due to their intuitive appeal and analytical tractability, the preferences of Dixit and Stiglitz (1977) that Krugman’s analysis is based upon have become the workhorse of international trade theory.

While trade theorists continue to gain important insights from using these preferences, they also mask significant aspects of product demand. For example, the classical Armington (1969) assumption that consumption is differentiated by the location of production is often needed to match aggregate trade patterns (Trefler (1995)), yet it remains somewhat unclear why this is the case and why the expenditure share on foreign goods is generally rather small. Also more recent findings that there are pronounced systematic patterns in the quality composition of production, trade, and consumption (Schott (2004), Hummels and Klenow (2005), Hallak (2006 and forthcoming), Hallak and Schott (2009))¹ or that intra-industry trade volume can best be explained by factors that are specific to country-pairs (Hummels and Levinson (1995)) are not easily rationalized in frameworks based on the Dixit and Stiglitz preference framework.

It is worthwhile to examine which underlying preference structure of rational agents can explain these patterns of trade and to then analyze how trade liberalization affects the aggregate economy once these preference are properly modeled. For example, as famously conjectured by Linder (1961, p. 94), it is likely that “[t]he more similar is the demand structure of two countries, the more intensive, potentially, is the trade between these two countries.” Implicit in Linder’s hypothesis is the argument that domestic firms tend to produce goods that are optimized for the local taste and less for the taste of foreign consumers. Consequently, differences in the tastes and products impede trade and reduce the welfare gains from trade liberalization.

In this paper, I thus develop a model of the determinants of demand for heterogeneous products, in which consumers do display a love of variety as in Dixit and Stiglitz (1977), but they are also characterized by a “taste”¹²

¹Hummels and Skiba (2004) argue that per unit transportation costs induce firms to export high quality goods, while Hallak and Sivadasan (2009) argue that exporter quality constraints influence the composition of exports. In this paper, I do not take into account such supply-side explanations and analyze to what extent the preference structure alone affects within-industry trade patterns and aggregate trade volume.

²To some extent, the Dixit and Stiglitz (1977) utility formulation can be criticized on the grounds that the love-for-variety motive is an assumption rather than an equilibrium outcome. In what follows below, I thus return to work demonstrating how this motive can arise from consumer-firm specific taste shocks in a discrete-choice framework similar to McFadden (1981) and Anderson et al. (1987 and 1992), which I augment by the described
for good attributes as in Mussa and Rosen (1978). The model features products that are heterogeneous in their attributes and consumers that are heterogeneous in their taste for product attributes. The latter two-sided heterogeneity results in an equilibrium matching in which consumers with a high preference for a given attribute tend to buy from firms with a fitting high-attribute good, i.e., the two-sided heterogeneity leads to an assortative equilibrium matching of consumer valuations and good attributes.

Consumers are randomly assigned their taste draw. A key assumption of the model is that firms can decide with what kind of good to enter the market. Consequently, in the equilibrium of the closed economy, a high proportion of consumers with a high taste for the attribute leads to a high proportion of firms producing high-attribute goods.3

I nest these preferences in a model of the international economy featuring iceberg transportation costs and two countries that differ in the distribution of consumer tastes. The model comprises the economy described in Krugman (1980) as a special case without product or taste heterogeneity. Therefore, I can directly evaluate the effect of such heterogeneity on trade flows, industrial composition dynamics, and the welfare effects of trade. The model has three novel main predictions.

The first novel prediction is the within-industry extension of Krugman’s (1980) "home market effect". In Krugman’s model, a country with a larger home market has more entry of firms producing manufactured goods for the domestic market and, with open markets, is also the net exporter of these goods. This prediction has been extended to the many-industry case by Hanson and Xiang (2004), who predict that a relative home market effect can arise across industries with different transportation costs or demand structures. Moreover, as Fajgelbaum et al. (2009) demonstrate, a home market effect can arise along the dimension of good quality when consumer preferences are non-homothetic: since richer countries have a relatively larger domestic market for high quality goods, in equilibrium, they also tend to export of such goods.

The intuition of the within-industry home market effect of this paper is closely related to the relative notion in Hanson and Xiang (2004) and Fajgelbaum et al. (2009). Even if two countries, say Germany and France, are characterized by an equal domestic market size for cars in general, a home market effect can arise in the type of cars these countries produce.  

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3The fact that firms can choose with what type of good to enter the industry implies that although firm’s output is heterogeneous, in general equilibrium, firms are not heterogeneous in profitability. The model at hand, therefore, does neither feature trade-induced shifts in the distribution of firm-productivities as in Melitz (2003) nor does it display trade-induced within-firm shifts in the composition of products as in Bernard et al. (2009) and Melitz, Mayer, and Ottaviano (2009). I analyze the case where product attributes are randomly assigned to firms in Auer (2008).
For example, if the French consumers put relatively more emphasis on fuel efficiency, while the German consumers tend to emphasize top-speed, in general equilibrium, each country’s industry is adapted to the needs of the local population. Immediately after trade liberalization, France thus becomes a net exporter of fuel-efficient cars, while Germany becomes a net exporter of fast cars. Neither of the two economies, however, need to become a net exporter of cars.\(^4\)

The model’s second prediction is that in the short run after a trade liberalization, consumption is home-biased in the sense that the volume of trade is lower than what would be expected on the basis of transportation costs and the elasticity of demand.\(^5\) Consider the moment just after opening markets to trade were each country’s industry is optimized for the tastes of domestic consumers only. While the few German producers of fuel-efficient cars experience high demand in France, this is more than offset by the many producers of fast cars that experience low demand in France. Overall, the volume of trade is reduced by product heterogeneity since the German industry, which is optimized for the fast car-loving German consumer, is inappropriate for the average French consumer, who is characterized by a love for fuel efficiency. In the short run, the model thus supports Linder’s conjecture that taste differences across nations impede trade.\(^6\)

The third prediction is that after trade liberalization, the within-industry home market effect intensifies, while the home bias of consumption disappears. When markets are opened to trade, French firms experience relatively more import competition in the segment for fast cars than in the segment for fuel-efficient cars. Thus, domestic sales and profits are relatively higher for the makers of fuel-efficient cars. The latter effect is only partly offset by export possibilities being relatively better for the French makers of fast cars and the country thus specializes into the fuel-efficient market segment. In contrast, the German car industry specializes into producing fast cars, in turn further reinforcing the specialization in France. In the long run, countries end up with more specialized industrial structures and the within-

\(^4\)This example raises the question of whether fast cars and fuel-efficient cars should be seen as two separate industries. While – for reasons of expositional clarity – the model developed below features one attribute which can take two possible values, an extension featuring a continuous distributions of potential values for several attribute dimensions is straightforward (see Auer (2008)). Every industry is characterized by some degree of product heterogeneity, the effect of which is analyzed below.

\(^5\)There is ample empirical evidence that the volume of trade is far lower than what theory suggests. For example, trade Anderson and Van Wincoop (2003) estimate that conditional on distance, country size, and other observable factors, a border reduces trade flows in the order of 30-40%.

\(^6\)Foellmi et al. (2008) document how the non-divisibility of goods can impede the volume of trade. In their setup, richer individuals demand a larger variety of goods and richer nations thus offer a market for a large set of different varieties. Many of these varieties cannot be exported to poorer nations since poor consumers can afford only a narrow set of varieties.
industry home market effect intensifies.

An intriguing result of the model is that the home bias of consumption prevailing immediately after trade liberalization vanishes in the long run although (indeed: because) both economies specialize even more. Consider again the example of German and French cars. At the moment of opening to trade, the composition of French industry is tailored exactly to the demand of French consumers, while the additional import supply from Germany is concentrated in the fast-car segment. Immediately after trade liberalization, competition in France is tougher in precisely the segment where German exports are concentrated in, thus impeding trade.

Dynamically, however, German exporters crowd out French producers of fast cars. This raises the volume of trade since it makes competition in France weaker in the segment where Germans exports are concentrated in. Owing to this, the group of French consumers who do prefer fast cars are not well served by the domestic industry, leading to higher import demand for fast cars. As I demonstrate below, as long as specialization is incomplete, the latter effect is so strong that the home bias of consumption disappears completely and the long-run volume of trade is exactly equal to the one that would prevail in the absence of across-country taste differences.

The dynamic response of industrial composition also has stark results for the welfare effects of trade: under incomplete specialization, the long run welfare gains from trade occur to all consumers in the same proportion irrespective of the foreign distribution of valuations. Again, this somewhat counter-intuitive result can be explained by how trade affects the composition of the domestic industry. For example, given that the French produce a large variety of fuel efficient cars, German consumers with a preference for fuel efficiency do gain more at the moment of liberalization than do the German consumers with a preference for speed. Dynamically, however, trade induces German producers of fuel efficient cars to exit the industry, which favors the lovers of fast cars. In general equilibrium the these two effects exactly offset each other and all consumers benefit from trade in the same proportion irrespective of the distribution of tastes in the other nation.

These findings document that endogenizing how a nation’s industrial composition responds to trade liberalization is of first order importance for understanding trade patterns and the welfare gains from open markets. For example, Linder’s (1961) often-cited hypothesis hinges on the intuitive idea that a lower fraction of consumers who value a certain attribute is associated with a lower volume of imports embodying the attribute. While the latter statement is true for a given domestic industry structure, the reverse holds true in general equilibrium: under trade, lower domestic valuation for an attribute is associated with an over-proportional reduction in domestic production of goods embodying the attribute, and consequently, a higher import volume of such goods.

Modelling the dynamic response of industrial composition to trade liber-
alization and the subsequent increase in trade volumes can also contribute to our understanding of why trade grows very sluggish after liberalization (see Yi (2003), Ruhl (2008), and Hummels (2007)). After such liberalization, each country’s industrial composition has to adapt, which requires firm exit and entry and, thus, time. It is also noteworthy that the model predicts a substantial amount of new trade due to the extensive margin as documented by Kehoe and Ruhl (2008). In contrast to the existing literature (Arkolakis (2008), Baldwin and Harrigan (2007), Bernard et al. (2003), Chaney (2008), Kugler and Verhoogen (2008), Johnson (2007), Melitz (2003), and Verhoogen (2008)), this is not driven by the trade-induced shift towards ex-ante more profitable entities, but rather, by the adaptation of a country’s industrial composition to the taste structure of a globalized economy.\footnote{Cunat and Maffezzoli (forthcoming) model a similar structural transition process in which trade-induced factor accumulation slowly transforms a country’s industrial structure, leading to a sluggish response of trade volume to liberalization.}

The structure of this paper is the following. In section 2, I develop the a new set of preferences where consumers are characterized by love of variety and heterogeneity tastes. I analyze the steady state of the closed economy in section 3. In next open markets to trade and analyze the static impact of trade liberalization in section 4 and the long run impact in section 5. Section 6 concludes.

2 A Model of the Demand for Heterogeneous Products

In this section, I develop a preference model that combines two motives of consumption decisions: the love of variety motive from Dixit and Stiglitz (1977) and the two-sided heterogeneity of good attributes and consumer valuations developed in Mussa and Rosen (1978).

The world is populated by two countries named Home and Foreign, which are populated by a mass of $L$ and $L^*$ consumers respectively. Each consumer has preferences over a homogenous $O$ (outside) good and over a finite set of differentiated $M$ (manufacturing) varieties. Each $M$ firm produces exactly one differentiated variety that is characterized by its attribute $a$. Each consumer has a valuation $v$ for the attribute $a$ and is also characterized by an idiosyncratic and consumer-firm specific utility draw $x$.

Differences in attributes $a$ can be seen as differences in good quality, but may also reflect more trivial product characteristics such as the good’s color or the language used to label a product. Similarly, differences in valuations $v$ reflect differences in people’s tastes for the attribute. For example, some consumers might have a preference for cars painted in Ferrari Red, while others prefer British Racing Green. The two-sided heterogeneity results in an equilibrium matching in which consumers with a preference for green cars
tend to buy from firms producing green cars, i.e., the two-sided heterogeneity leads to an assortative equilibrium matching of consumer valuations and good attributes.\(^8\)

Consumers also value variety, i.e., they prefer an economy featuring many different varieties of cars painted in British Racing Green to an economy featuring only one such variety. This love for variety motive is derived from a discrete choice setting in the spirit of McFadden (1981), Anderson et al. (1987 and 1992), and in particular Gabaix et al. (2006). Each consumer is endowed with an idiosyncratic and consumer-firm specific utility draw \(x\), reflecting that a specific consumer, by chance, might like or dislike the output of a specific firm. Since having a larger number of such draws raises the expected maximum draw, consumer welfare rises with the number of available varieties. The love of variety motive partly blurs the assortative matching of consumer valuations and good attributes, since a high consumer-firm specific utility draw might lead to a consumer buying the good even if the good’s attributes do not fit the consumer’s valuations very well.

In equilibrium, the economy thus features expected assortative matching, i.e., consumer valuations and product attributes are on average matched assortative and a high attribute producer has relatively more consumers with a high preference for the attribute than a low attribute producer. The key implication of this matching, in turn, is that a firm’s sales decrease more when a new firm with a similar product enters the industry than when a firm with a dissimilar product enters. Therefore, with trade, the composition of the foreign industry matters for the composition of the domestic industry.

I next lay out the functional forms used in this paper to model these intuitions, derive a firm’s demand, and then describe the supply side of the economy.

2.1 Preferences

Throughout the analysis, let \(i \in I\) index consumers (individuals) and \(j \in J\) index manufacturing firms. Each of these consumers \(i\) is endowed with income \(\theta_i = \theta^0\) in terms of labor and a valuation draw \(v_i\). The consumer is also endowed with a consumer-firm specific draw \(x_{i,j}\) for each firm in \(j \in J\).

Consumers care about the valuation- and idiosyncratic draw- adjusted effective quantity of the manufacturing \(M\) good and the absolute quantity of the outside good \(O\). Denoting the quantity consumer \(i\) consumes of the

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\(^{8}\)For the case of good quality, differences in valuations mean that while all consumers do prefer higher quality goods, they do so at different rates. See Auer and Chaney (2007 and 2008) and Auer and Saurè (2009) for other applications of the Mussa and Rosen (1978) model to the open economy.

\(^{9}\)The preferences of the model developed below are homothetic so that the model’s predictions with \(L\) equal workers who supply \(\theta\) units of effective labor each are exactly equal to the predictions in a model with heterogeneous workers satisfying \(\theta L = \sum_{i \in I} \theta_i\).
O good by \( o_i \) and the quantity she consumes from manufacturing firm \( j \) by \( q_{i,j} \), consumer \( i \)'s utility \( U_i \) is given by

\[
U_i = (o_i)^{1-\alpha} \left( \sum_{j \in J} q_{i,j} x_{i,j}^{x_i,j + a_j v_i} \right)^{\alpha}.
\] (1)

Her consumption decision is subject to non-negativity for \( o_i \) and each pair \( i,j \) \( q_{i,j} \geq 0 \), as well as to her budget constraint

\[
o_i p_O + \sum_{i \in I} q_{i,j} p_j \leq \theta_i.
\] (2)

The utility function (1) implies that for all consumers, all manufacturing goods are perfectly substitutable. However, different consumers have different rates of substitution between different varieties; in equilibrium, therefore, certain types of consumers are more or less likely to buy certain types of goods.

Consider first only the term \( e^{a_j v_i} \) in (1).10 The key feature of this term in the preferences is that the rate at which consumers value (or dislike) the attribute differs between consumers with different \( v_i \). Assume that two consumers of valuations \( v_L \) and \( v_H > v_L \) are offered to buy a certain good \( a_L \) at price \( p_L \) or a good \( a_H \) at price \( p_H \) where \( a_H > a_L \). What is the maximum price difference between \( p_L \) and \( p_H \) at which each consumer would prefer the high \( a \) good? For the \( H \) – valuation consumer, this would be price ratio \( p_H/p_L = e^{v_H(a_H-a_L)} \), while it would be \( p_H/p_L = e^{v_L(a_H-a_L)} \) for the \( L \) – valuation consumer. Because higher valuation consumers value the attribute more, in equilibrium, they constitute the relatively larger group of consumers of \( H \) – attribute goods. For expositional clarity, a large part of the analysis below assumes that \( v_i \) can take only one of two possible values \( v_L, v_H \). However, in general, this assumption is not necessary and valuations can take any positive value, i.e.,

\[
v_i \sim F_v(v).
\] (3)

\[
f_v(v) \begin{cases} 
\geq 0 \text{ for } v \geq 0 \\
= 0 \text{ for } v < 0
\end{cases}
\]

Next, consider only the term \( e^{x_{i,j}} \) in (1). \( x_{i,j} \) is a consumer-firm specific shock, reflecting the fact that some consumers like or dislike the variety of a specific firm irrespective of the variety’s attribute. In (1), the idiosyncratic

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10 Both \( a_j \) and \( v_i \) are scalars. It is straightforward to extend the model at hand to the case of multiple attributes. For example, if each consumer is characterized by independent valuations over \( K \) attribute dimensions, the predictions developed below continue to hold exactly as long as specialization is incomplete in the \( K \) attribute dimensions.
taste shock introduces market power to the model: although firms cannot observe \( x_{i,j} \), they can engage in first degree price discrimination by charging a higher price and only attracting consumers with high \( x_{i,j} \) draws. Throughout the analysis, I assume that \( x_{i,j} \) is distributed (maximum) Gumbel with scale and shape parameters 0 and \( 1/\beta \) respectively.

\[
G_x(x_{i,j}) = \exp[-\exp(-x_{i,j}/\beta)]
\]  

(4)

The consumer-firm specific shocks are orthogonal to firm attribute or consumer valuation and are independent across firms and consumers: \( x_{i,j} \perp x_{i,n} \) for \( n \neq j \). Gabaix et al. (2006) demonstrate that these assumptions in combination with a utility function similar to (1) yield an ideal-variety microfoundation for the constant elasticity of substitution (CES) demand system of Dixit and Stiglitz (1977). It is noteworthy that the closed-form assumption on the consumer-firm specific taste shocks (4) is not very restrictive, since in equilibrium consumers buy only from the attribute-adjusted maximum realization of \( x_{i,j} \). Since the economy features a large number of firms, the distribution of this maxima converges to the Type I Extreme value function for a wide set of underlying distributions.11

2.2 Demand and Consumer Welfare

I next solve for a firm’s demand and consumer welfare using the general distribution of valuations \( F_v(v) \). Consumer \( i \) consumes the agricultural \( O \) good and the manufacturing composite \( M_i \equiv \sum \limits_{j \in J} q_{i,j} e^{x_{i,j} + a_i v_i} \). Before considering the choice among the single manufactured goods, consider first the decision of how much of the \( O \) good to consume. The first order conditions of the utility function (1) with respect to these two quantities and the budget constraint (2) imply that an agent with income 1 consumes

\[
M_i = (1 - \alpha)/p_{M,i} \quad \text{and} \quad O_i = \alpha/p_O,
\]

where \( p_{M,i} \) is the price of the manufacturing composite for consumer \( i \) (which is NOT the same for all \( i \)). Irrespective of this price, the consumer always spends a fraction \((1 - \alpha)\) of her income on manufactured goods.

Thus, the consumer spends the remainder fraction of \((1 - \alpha)\) on the manufacturing composite. Within the manufacturing composite, since all goods are perfect substitutes, each consumer then chooses the variety that yields the highest ratio of effective quantity per unit divided by the price of the variety. Since consumers with different valuation \( v_i \) differ in their average rate at which they substitute goods of different attributes \( a \), demand is of a different shape for each \( v \).

11 The preference structure at hand makes the model’s results highly comparable to the work of Bernard et al. (2003), who extend the Eaton and Kortum (2002) model of trade to allow for positive markups.
**Proposition 1 (Demand)** The demand function \( D(a_j, p_j) \) of a firm with attribute \( a_j \) and price \( p_j \) is equal to

\[
D(a_j, p_j) = (1 - \alpha) \theta L \Gamma(1 - \beta) p_j^{-(1+\beta)} \int_{v \in V} \frac{\exp[\beta x_i]}{P(v)} dv, \tag{5}
\]

where \( \Gamma(\ldots) \) is the beta function and \( P(v) \) denotes the ideal price index for all consumers with \( v_i = v \), which is given by

\[
\overline{P(v)} \equiv \left( \sum_{n \in J} \left( \frac{p_n}{\exp[\beta a_n]} \right)^{-\beta} \right)^{-1/\beta}. \tag{6}
\]

**Proof.** See Appendix.}

The proof of Proposition 1 follows previous research demonstrating how the love of variety motive can arise in a discrete choice setting: each consumer has a consumer-variety specific taste shock \( x_{i,j} \). For equal prizes and good attributes, the consumer chooses the maximum realization of the taste shock, i.e., she chooses \( j = \arg \max_{j \in J} x_{i,j} \). Owing to the functional form assumption that the idiosyncratic taste shocks are distributed Gumbel with shape parameter \( 1/\beta \), all firms face a constant elasticity of demand equal to \( -(1 + \beta) \).

Compared to the existing literature, the novel ingredient in the derivation of (5) is that the probability of consumer \( i \) with valuation \( v_i = v \) buying from firm \( j \) with attribute \( a_j = a \) depends on the match of \( v \) and \( a \), as well as on how well the other goods in the economy match with the consumer’s taste, i.e., the ideal price index of consumers with \( v_i = v \). First, sales are shifted by the match between the consumer’s taste and the firm’s attribute, i.e., in (5), demand is shifted by \( \exp[\beta v_i a_j] \). Second, it is not only the match between firm \( j \) and consumer \( i \) with \( v_i = v \) that determines sales, but also how well the competition’s output matches with the consumers preferences, i.e., the ideal price index of each consumer type is a function of the attribute composition of the economy. The latter average match is summarized in the ideal price index \( \overline{P(v)} \).

The latter two matchings generate the key difference between the preference structure of this paper and the existing literature. In existing frameworks, due to the constant elasticity demand structure, entry of new competitors hurts the sales of all existing firms in the same proportion. In the preferences at hand, the effect of such an increase in competition on a firm’s sales will be different for different types of firms.

Last, there is not one type of consumer, but a distribution of consumers with varying valuations. Total demand for a firm equals the sum of demand from all possible valuations, hence explaining the outer integral over all possible realizations of \( v \) in (5).
Since the expected maximum draw is increasing in the number of draws, consumers prefer having a larger number of varieties to choose from, i.e., they love variety. A key feature of the preferences developed here is that consumer welfare is highly comparable to the one in Dixit and Stiglitz (1977).

**Corollary 1 (Consumer Welfare)** Denote the expected welfare of consumer \( i \) with \( v_i = v \) and income \( \theta_i \) by \( E(U_i | v, \theta_i) \). If \( p_O = 1 \),

\[
E(U_i) = (1 - \alpha)^{1-\alpha} \alpha^\alpha \Gamma \left( 1 - \frac{\beta}{\alpha} \right) \left( \frac{\Gamma}{P(v)} \right)^{-\alpha} \theta_i
\]

where the ideal price index \( P(v) \) is as defined in (6) and \( \Gamma(\cdot) \) is the gamma function.

**Proof.** see Appendix □

Corollary 1 is very convenient: the developed preference structure allows to directly map changes in the toughness of competition for all consumers with \( v_i = v \) into welfare changes for this group of consumers. As I document below, with open markets, the interplay of the free entry conditions at Home and abroad pins down the ideal relative price indices for different \( v \)'s uniquely, hence leading to very sharp prediction regarding the welfare effects of trade.

Since both demand is CES-shaped and consumer welfare (up to some constants) coincides with the one in the Dixit and Stiglitz (1977) framework, one can directly relate the findings of this paper to the existing literature. In the case where all firms produce the same good \( a_i = a_j = a \), the valuation-attribute match in (5) cancels out and the demand curve is the same as in Dixit and Stiglitz (1977). The model at hand, therefore, includes the Krugman (1980) model as a special case without product heterogeneity, which is convenient since it allows clearly highlighting the impact of such heterogeneity.\(^{12}\)

### 2.3 Supply

In each country and at each moment in time, a large set of potential entrepreneurs can enter the \( M \) industry by paying a fixed cost of \( F \) labor units. When entering the industry, each entrepreneur can choose with what type of attribute to enter the industry. After paying the entry cost \( F \) and deciding

\(^{12}\)The welfare gains from trade in the model of this paper are also comparable to the one’s in the literature on firm heterogeneity. Arkolakis et al. (2009) evaluate the welfare effects of trade for several such models with heterogeneous firms, demonstrating that one can calculate the welfare gains from trade on the basis of knowing only the share of expenditure on domestic goods and the elasticity of demand. In this paper, the same holds true.
with what kind of good to enter the industry, the entrepreneur $j$ receives the blueprint to produce a new variety of the manufacturing good with attribute $a_j$. While $a_j$ can be chosen at the moment of entry, it cannot be changed thereafter. The entrepreneur has a perpetual monopoly over that specific variety from the moment of entry onwards and faces an exogenous probability of firm death of $\delta > 0$.

For expositional clarity, I restrict the universe of potential levels the attribute can take and assume that $a_j \in \{a_L, a_H\}$, where $0 < a_L < a_H$. I refer to the two attribute levels as the $H$-attribute or $L$-attribute "good", "firm", or "variety" in the remained of the paper.

While alive, each firm can produce any quantity of its good at constant marginal costs (in units of labor) equal to

$$c_j = e^{ca_j}$$  \hspace{1cm} (7)

$\frac{\partial c(a_j)}{\partial a_j}$ can be positive, zero, or negative. For example, if $a_j$ measures the wavelength of the good’s color, it may be the cheapest to produce high wavelength colors and $c < 0$. If the lowest possible valuation $v_{\min}$ is larger than 0, it is reasonable to assume that higher $a_j$ (higher quality) goods are more expensive and that $c > 0$.

The outside good $O$ is produced in a competitive sector at a marginal cost of one unit of labor. In total, the Home economy thus has to satisfy the resource constraint that domestic production of the $O$ and $M$ sector and entry into the $M$ sector do not use more than $\theta L$ units of Home labor.

If markets are opened to trade, manufacturing firms can sell abroad at a cost $c^*_j = \tau c_j$, where $\tau > 1$. In contrast, the outside $O$ good can freely be traded.\footnote{The economy does not feature a cost to access the export market as in Melitz (2003). The key reason for this modeling choice is that in the economy presented in this paper, firms are free to choose with what kind of attribute to enter the industry and in equilibrium, all firms are equally profitable. Therefore, a fixed cost to access export markets has no effect on the equilibrium industry composition.}

3 Equilibrium in the Closed Economy

I next solve the closed economy equilibrium. To better convey the model’s intuitions, I solve the two attribute - two valuation case and assume that $v_i \in \{\hat{v}_L, \hat{v}_H\}$ where $\hat{v}_L < \hat{v}_H$. Note that when $\hat{v}_L > 0$, all consumers value higher attribute goods and one can speak of good "quality" as in Auer and Chaney (2007 and 2008), Auer and Saurè (2009), and Fajgelbaum et al. (2009). In this paper, I however, do not necessarily assume that $\hat{v}_L > 0$, so that the analysis also extends to product characteristics that is not strictly preferred by all consumers. I denote the fraction of the population that has a valuation draw of $v_i = \hat{v}_H$ by $0 \leq \pi_H \leq 1$. 

$\pi_L = 1 - \pi_H$. 

$$\pi_L = 1 - \pi_H$$
Firms face a constant price elasticity of \( (1 + \beta) \) and thus charge a price of \( p_j = \frac{1+\beta}{\beta}c_j = \frac{1+\beta}{\beta}e^{c\alpha_j} \). For each type of consumer, (5) thus simplifies to \( e^{\beta(v_i-c)a_j}/\sum_{n \neq j} e^{\beta(v_i-c)a_n} \), i.e., valuations \( v_i \) can simply be adjusted by the fact that also costs vary with \( a \). Thus, in the remainder of the analysis, I will only evaluate the cost adjusted \( H - valuations \) and \( L - valuations \) \( v_H \) and \( v_L \) satisfying

\[
v_L \equiv (\bar{v}_L - c) \quad \text{and} \quad v_H \equiv (\bar{v}_H - c).
\]

Throughout the analysis, let \( N \) denote the total number of active firms in the industry at Home and let \( n_H \) denote the fraction these firms producing a good with \( a_j = a_H \). Normalizing \( \Gamma (1 - \beta) \theta (1 - \alpha) \equiv 1 \), revenue \( \Pi(a_j) \) in the home market economy is equal to

\[
\Pi(a_j) = L \pi_H e^{\beta v_H a_j} \frac{e^{\beta v_H a_j}}{N(n_H e^{\beta v_H a_H} + (1 - n_H) e^{\beta v_H a_L})} \\
+ L (1 - \pi_H) e^{\beta v_L a_j} \frac{e^{\beta v_L a_j}}{N(n_H e^{\beta v_L a_H} + (1 - n_H) e^{\beta v_L a_L})}
\]

Given the constant markup-pricing, firm profits are proportional to revenue. In the closed economy, this revenue depends on the distribution of consumer valuations. For any given attribute, a higher proportion of \( H - valuation \) consumers implies a larger market size for \( H - attribute \) firms.

Similarly, the revenue (8) of a firm reacts more to entry of firms producing a similar good than to entry of firms producing a dissimilar good, i.e., \( \frac{\partial \Pi(a_H)}{\partial N_H} > \frac{\partial \Pi(a_L)}{\partial N_H} \) and \( \frac{\partial \Pi(a_H)}{\partial N_L} < \frac{\partial \Pi(a_L)}{\partial N_L} \). The latter feature implies that industries are partially segmented: for example, the sales of BMW depend much more on the product strategy of Mercedes rather than the one of Toyota, which caters to a slightly different set of consumers. Similarly, Armani’s sales depend much more on the success of the latest collections by Prada than they do depend on the success of the collections Louis Vuitton or Hermes.

Since cost differences and attribute differences can be reduced to one dimension only the cost-adjusted valuations \( v_L = (\bar{v}_L - c) \) and \( v_H = (\bar{v}_H - c) \) enter the equilibrium demand function (8). A necessary condition for an equilibrium with positive entry of firms is that

\[
e^{\beta v_H a_H} > e^{\beta v_H a_L} \quad \text{and} \quad e^{\beta v_L a_L} > e^{\beta v_L a_H},
\]

i.e., that no type of good, when adjusted for its relative cost of production, is preferred by both types of consumers. Throughout the rest of this paper, I assume this to hold.

With demand being pinned down, it is straightforward to derive entry in the closed economy. Denoting the value that a variable takes in the autarky steady state by an \( A \) superscript, the following holds.
Proposition 2 (Autarky Equilibrium) Denote by $N^A$ the total number of firms in autarky equilibrium and by $n_H^A \in [0, 1]$ the autarky equilibrium fraction of entrepreneurs choosing to produce the $H$–attribute good. There exists a unique autarky equilibrium featuring $N = \frac{L}{\beta v_A}$ and

$$n_H^A = \begin{cases} 
0 & \text{if } \pi_H < e^{\beta v_L a_H} \frac{e^{\beta v_H a_H} - e^{\beta v_L a_L}}{e^{\beta v_L a_L} - e^{\beta v_H a_H}} \pi_H - (1 - \pi_H) \frac{e^{\beta v_L a_L} - e^{\beta v_H a_H}}{e^{\beta v_L a_L} - e^{\beta v_H a_H}} \), \\
\frac{e^{\beta v_H a_H} - e^{\beta v_L a_L}}{e^{\beta v_H a_H} e^{\beta v_H a_H} - e^{\beta v_L a_L} e^{\beta v_L a_L}} & \text{otherwise}.
\end{cases} \quad (9)$$

Proof. Since firms are free to enter with an $H$ or the $L$ good, an equilibrium with positive entry of both types of firms requires that the flow of revenues are equal for both types of firms, or that

$$L \left( \frac{\pi_H e^{\beta v_H a_H}}{P(v_H)^{-\beta}} + \frac{(1 - \pi_H) e^{\beta v_L a_L}}{P(v_L)^{-\beta}} \right) = L \left( \frac{\pi_H e^{\beta v_H a_L}}{P(v_H)^{-\beta}} + \frac{(1 - \pi_H) e^{\beta v_L a_H}}{P(v_L)^{-\beta}} \right),$$

where $P(v_H)$ and $P(v_L)$ are the ideal price indices of $H$– and $L$–valuation consumers, which are $P(v_H)^{-\beta} = N (n_H e^{\beta v_H a_H} + (1 - n_H) e^{\beta v_L a_L})$ and $P(v_L)^{-\beta} = N (n_H e^{\beta v_H a_L} + (1 - n_H) e^{\beta v_L a_H})$. Thus, reformulating (10) as the difference in sales to $H$–valuation and $L$–valuation consumers yields

$$\frac{\pi_H}{1 - \pi_H} \frac{e^{\beta v_H a_H} - e^{\beta v_L a_L}}{n_H e^{\beta v_H a_H} + (1 - n_H) e^{\beta v_L a_L}} = \frac{e^{\beta v_L a_L} - e^{\beta v_L a_H}}{n_H e^{\beta v_L a_H} + (1 - n_H) e^{\beta v_L a_L}}.$$  

(11)

Since $e^{\beta v_H a_H} > e^{\beta v_L a_L}$, the LHS of (11) is increasing in relative entry of $H$ firms $n_H$. Since $e^{\beta v_L a_L} > e^{\beta v_L a_H}$ the RHS is decreasing in $n_H$. Thus, $n_H$ is uniquely determined. $N^A$ depends on the flow of instantaneous profits which have to be discounted at rate $\delta$ and pin down the number of firms by the free entry condition $F = \frac{L}{\beta N^A}$. 

Since $e^{\beta v_H a_H} > e^{\beta v_L a_L}$, $H$–attribute firms sell more to $H$–valuation consumers than do $L$–valuation firms. Similarly, $e^{\beta v_L a_L} > e^{\beta v_L a_H}$ and $L$ firms sell more to $L$–valuation consumers. Sales to each group are proportional to the number of consumers (there are $L \pi_H$ $H$–valuation consumers) and increasing in the ideal price indices $P(v_H)$ and $P(v_L)$.

It is noteworthy that in general equilibrium, as long as $n_H^A \in [0, 1]$, $n_H^A$ is increasing in the number of $H$–valuation consumers ($\frac{\partial n_H^A}{\partial \pi_H} > 0$) and also that $n_H^A$ is increasing in both valuations $v_L$ and $v_H$ ($\frac{\partial n_H^A}{\partial v_L} > 0$, $\frac{\partial n_H^A}{\partial v_H} > 0$). Furthermore, denoting the autarky equilibrium ideal price indices by $P^A(v_j)$ it is true that if $n_H^A$ is interior
\[ P^A (v_H) = \left( \frac{\phi \pi_H N^A}{e^{\Delta v_{HL}} - e^{\Delta v_{H}}} \right)^{-1/\beta} \]
\[ P^A (v_L) = \left( \frac{\phi (1 - \pi_H) N^A}{e^{\Delta v_{HL}} - e^{\Delta v_{L}}} \right)^{-1/\beta} \]

where \( \phi \equiv e^{\beta v_{H} a_{H}} e^{\beta v_{L} a_{L}} - e^{\beta v_{H} a_{L}} e^{\beta v_{L} a_{H}} > 0 \). In the autarky equilibrium revenue is constant across firms \( \Pi (a_H) = \Pi (a_H) = \frac{1}{N^A} \) for any level of \( \pi_H \), i.e., in equilibrium, the sales of a firm do not depend on the relative distribution of consumer tastes. The later result is a direct consequence of the fact that firms can decide with what kind of product to enter the industry. Therefore, a higher \( \pi_H \) has to be offset exactly by an increase in \( n_H \) so that firms of with different attributes still make the same profits, i.e. in the closed economy, the level of competition for \( H \) and \( L \) valuation consumers is proportional to the number of customers \( \pi_H \) respectively \((1 - \pi_H)\).

Summarizing, the equilibrium in the closed economy has the following properties. First, a necessary condition for an equilibrium featuring both kind of firms is that \( e^{\beta v_{HL}} > e^{\beta v_{L}} \) and \( e^{\beta v_{H}} > e^{\beta v_{H} a_{L}} \), i.e., that there exists both a group of consumers that prefers \( L \) goods as well as a group that prefers \( H \) goods. Second, in an equilibrium featuring positive entry of both types of firms, the fraction of \( H \) attribute firms is increasing in the number of \( H \) valuation consumers. The fraction of such firms is also increasing in \( \pi_H \) and \( v_L \), since an increase in either valuation leads to higher relative expenditures on \( H \) attribute goods. Third, in equilibrium, owing to the free entry condition, all firms have the same revenue and profit flows.

4 The Static Impact of Trade Liberalization

I next examine the immediate impact of an unanticipated trade liberalization. In particular, I analyze how such liberalization impacts the economy if the two countries differ in the fraction of \( H \) and \( L \) valuation consumers (for expositional clarity I assume that \( \pi_H \geq \pi^*_H \)) and contrast this to the static effects of liberalization in the Krugman (1980) economy. As in the latter work, I also allow for the countries to differ in size \( L \) and \( L^* \).

At the instant after opening markets to trade, the number of firms is at its autarky level (9). Since accessing the export market is not subject to any fixed cost, all firms export and there are \( N^H n^A_H \) \( H \) attribute produc-

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14 The average tastes of consumers in Home and Foreign are assumed to be different for exogenous reasons. Atkin (2009) shows how such taste differences can be an equilibrium outcome of a model featuring habit formation and comparative advantage. In his work, since the prices of comparative advantage goods are cheaper at home than abroad, consumers develop a preference for domestically produced good.
ers exporting from Home to Foreign and \( N^* n^A_{H} \) \( H \) – attribute producers exporting from Home to Foreign.

The aggregate volume of manufacturing trade is equal to the sum of trade in \( H \) – attribute and \( L \) – attribute goods. To understand how product heterogeneity affects the aggregate volume of trade, it is thus expedient to evaluate trade volumes in these two segments of the industry. In the domestic economy, a larger share of \( H \) – valuation consumers is associated with a higher number of \( H \) – attribute entrants and since \( \pi_H > \pi_{H}^* \), it is also true that \( n^A_H > n^A_{H}^* \). Since all firms export, Home’s exports are more \( H \) – attribute intensive than is the domestic production in Foreign. Trade, therefore, intensifies competition more in the sector where the country has relatively fewer consumers. Denoting the values that variables take immediately at the moment after opening to trade by \( \pi \), \( \mu \), \( \delta \), the ideal price index of each group can be mapped one-to-one into well-distribution of tastes, it is the relatively smaller group of consumers that gains more at the moment of trade liberalization. This result if intuitive: if markets are opened to trade, a French consumer with a preference for fast cars suddenly has access to a large number of German fast car varieties. In contrast, a French consumer with a preference for fuel efficient cars gains relatively little, since Germany offers few of these varieties compared to the French industry.

\[ \text{Lemma 1 (Liberalization and Short Run Relative Competition): Assume that } \pi_H > \pi_{H}^* \text{ and } n^A_H, n^A_{H}^* \epsilon [0,1]. \text{ When opening markets to trade, competition in Home intensifies more in the } L \text{ – attribute segment of the industry than in the } H \text{ – attribute segment, while competition in Foreign intensifies more in the } H \text{ – attribute segment of the industry than the } L \text{ – attribute segment. I.e., it is true that} \]

\[
\frac{P^S(v_H)}{P^A(v_H)} < \frac{P^S(v_L)}{P^A(v_L)} \text{ and } \frac{P^{S*}(v_H)}{P^{A*}(v_H)} < \frac{P^{S*}(v_L)}{P^{A*}(v_L)}. \]

**Proof.** Since accessing the export market is free, all firms export. With entry given by the autarky equilibrium values \( P^S(v_H) = (N^A_H + n^A_H P^A(v_L) = (\beta \delta F)^{-1} \frac{e^{1/2} L_{H}^{1/2} a_{H}}{e^{1/2} L_{H}^{1/2} - e^{1/2} L_{H}^{1/2}} \), \( P^{S*}(v_H) = \left( \frac{P^S(1-H) + L_{H}^{*} \pi_{H}^{*}}{e^{1/2} L_{H}^{1/2} - e^{1/2} L_{H}^{1/2}} \right)^{-1/1} \), \( P^{S*}(v_L) = \left( \frac{P^S(1-H) + L_{H}^{*} \pi_{H}^{*}}{e^{1/2} L_{H}^{1/2} - e^{1/2} L_{H}^{1/2}} \right)^{-1/1} \), which satisfy the stated inequalities. \( \blacksquare \)

The above Lemma (1) has two implications. First, since by Corollary (1) the ideal price index of each group can be mapped one-to-one into welfare changes, the above result implies that when countries differ in their distributions of tastes, it is the relatively smaller group of consumers that gains more at the moment of trade liberalization. This result if intuitive: if markets are opened to trade, a French consumer with a preference for fast cars suddenly has access to a large number of German fast car varieties. In contrast, a French consumer with a preference for fuel efficient cars gains relatively little, since Germany offers few of these varieties compared to the French industry.
Second, Lemma (1) is also indicative of why differences in the distribution of tastes across Home and Foreign reduce the aggregate volume of trade. The aggregate volume of Home’s exports is equal to the number of $H$–attribute firms times exports per such firm plus the number of $L$–attribute firms times exports per such firms. Since trade intensifies competition in Foreign relatively more in the H sector, each Home $H$–attribute exporter sells a smaller amount that she would in an economy without product heterogeneity. In contrast, each Home $L$-exporter sells a larger amount that she would in an economy without product heterogeneity.

**Proposition 3 (Short Run Trade Volume)** Assume that parameters are such that $n^A_H, n^{A*}_H \in [0, 1]$. At the moment after trade liberalization, the following holds. If $\pi^*_H = \pi_H$, the volume of trade is the same as in the absence of consumer heterogeneity and home is a net exporter of the $M$ good iff $L > L^*$. If $\pi_H \neq \pi^*_H$, the volume of trade is lower than in the absence of consumer heterogeneity and decreasing in $|\pi_H - \pi^*_H|$. 

**Proof.** See Appendix.

In the model at hand, if there are no differences in the distributions of valuations between Home and Foreign ($\pi^*_H = \pi_H$), trade volume is the same as in Krugman (1980), i.e., Home’s export volume is proportional to $N_A \tau^{-\beta} L^*$, the latter volume is decreasing in trade costs, increasing in the size of the domestic labor force (because a larger domestic labor force is associated with more domestic firms) and also in the size of the foreign labor force (since a larger foreign labor force consumes more). The volume of trade is less than proportionally increasing in $\tau^{-\beta}$ since the global toughness of competition is increasing in the inverse of trade costs.

Next, consider the impact of taste heterogeneity and assume that $\pi_H > \pi^*_H$. With such preferences, each home firm faces relatively more demand from $L$–valuation consumers, and sales per firm such firm equal 

$$\frac{(1-\pi^*_H)L^*}{\tau^{-\beta} L(1-\pi_H)+L^*(1-\pi^*_H)} > \frac{L^*}{\tau^{-\beta} L^*}. \quad H$–valuation consumers face less demand and each have sales of 

$$\frac{\pi^*_H L^*}{\tau^{-\beta} L^*}. \quad\text{Compared to the benchmark economy without product heterogeneity, there is thus one sub-sector with larger export volume and one with smaller export volume per firm. The overall effect of such product heterogeneity on the volume of trade is still unambiguously negative on Home’s export volume, since the losses in the large $H$–attribute segment are not fully outweighed by gains in the comparatively small $L$–attribute segment.}

In the short run, the composition of the domestic industry is not appropriate for the average foreign consumer. This notion of the "appropriateness" of the domestic industry relates well to the notion of appropriate technology in the endogenous growth literature. For example, Acemoglu and Zilibotti (2001) show how even when technology is freely adoptable, the
poorer countries may be less productive because technologies are adapted to the skill endowment of rich nations and are can only be used sub-optimally in poor nations. In this paper, in autarky, each country develops an industry that is suited best to the tastes of the local consumer. In the short run after opening to trade, the country’s export bundle is inappropriate for the taste distribution of Foreign consumers.

The model includes Krugman’s classical aggregate home market effect as a special case when home and foreign share the distribution of consumer tastes. For the strength of the home bias, the share of $H$ – and $L$ – valuation consumers is not important, since the returns to scale are equally strong in the $L$ and $H$ segment of the market. The model also predicts that net exports can be nonzero even in the case of equal country sizes. In this case, the direction of net exports is the following: if $\pi_H + \pi_H^* > 1$, i.e., if the global market for the type of good that Home’s exports are concentrated in is large, Home is a net importer of manufacturing goods. If $\pi_H + \pi_H^* > 1$ there are in more $H$ – valuation consumers than $L$ – valuation consumers in the world (since $L = L^*$) and accordingly, there are also more $H$ – attribute firms in the world than $L$ – attribute good ones. Global competition is thus tougher in the $H$ segment of the industry, which happens to be the segment were home’s exports are concentrated in. Similarly, competition is less tough in the market segment were foreign exports are concentrated in. Thus, home’s overall exports are smaller than its imports from foreign if its export tend to be concentrated in the more competitive industry, which is the case if $\pi_H + \pi_H^* > 1$.

Both reasons for why countries can be net exporters of manufactured goods have to be interpreted with care, since these predictions rely on trade in the agricultural sector (the $O$ sector) being costless. As demonstrated by Davis (1998), the aggregate home market effect does not necessarily arise once one allows for the possibility that trade in the $O$ sector is also subject to trade costs. The model at hand, however, also predicts a home market effect that is not subject to Davis’s critique.

**Proposition 4 (The Within-Industry Home Market Effect)** Assume that parameters are such that $n_{HA}, n_{HA^*} \in [0, 1]$. At the moment after trade liberalization, if $L = L^*$, Home is a net exporter of $H$ – attribute goods iff $\pi_H > \pi_H^*$. If $L \neq L^*$ Home’s manufacturing exports contain a larger fraction of $H$ – attribute goods than do Foreign’s exports.

**Proof.** see Appendix

Proposition 4 presents the model’s second prediction, the within-industry home market effect. In models following Krugman (1980), a country with a larger home market for manufacturing goods has more entry of firms producing for the domestic market. Thus, with open markets, this nation is a net exporter of industrial output. In the model at hand, the intuition of the
within-industry home market effect is closely related to Hanson and Chen’s (2004) notion of the relative across-industry home market effect. The within-industry home market effect is also reminiscent of Fajgelbaum et al.’s (2009) prediction that richer countries tend to export high-quality goods and import low-quality one’s. Fajgelbaum et al.’s model features non-homogenous preferences which result in richer consumers tending to buy higher quality goods. Since a larger fraction of high income consumers is associated with a larger domestic market for high quality goods, richer nations have a larger number of high quality producers. When markets are opened to trade, richer nations thus become net exporters of high quality goods.

In the model at hand, the within-industry home market effect follows a closely related notion: even if two countries, say Germany and France, are characterized by an equal domestic market size for cars in general, a home market effect can arise in the type of cars these countries produce. If French consumers put relatively more emphasis on fuel efficiency, while German consumers tend to emphasize top-speed, in general equilibrium, each country’s industry is adapted to the needs of the local population. Immediately after trade liberalization, France thus becomes a net exporter of fuel-efficient cars, while Germany becomes a net exporter of fast cars. Neither of the two economies, however, need to become a net exporter of cars.

Summarizing, three major trade patterns arise immediately after opening markets to trade. First, if countries are of unequal size, the home market effect applies and the larger country becomes the net exporter of manufactured goods, while the other country becomes the net exporter of agricultural goods. Second, even if countries are of equal size, there can be net exports in each segment of the industry. Third, owing to the differences in countries’ average tastes, trade volume is lower than what one would observe in Krugman’s (1980) model. I next examine whether and to what extent these predictions hold in general equilibrium.

5 The Steady State In The Open Economy

In the long run equilibrium with open markets each countries’s industrial composition has adjusted to the distribution of tastes in both Home and Foreign. In the short run, trade intensified competition more in the sector where the trade partner has a relatively larger industry size, hence explaining why trade volume is rather low: most exporters are in the crowded market segment. To understand why the home bias of consumption disappears entirely in the long run, it is again expedient to evaluate how the relative level of competition in the $H-$ and $L-$ attribute segments of the market develop.

Denoting the value that a variable takes in the long run equilibrium by
Lemma 2 (Long Run Specialization and Relative Competition) Assume that $\pi_H > \pi_H^*$, $\pi_H > e^{\beta v_H a_H} e^{\beta v_L v_L} e^{\beta a_H H H} (1 - \tau) + \pi_H^* - \beta \frac{L^*}{T} \theta$, and $\pi_H < e^{\beta v_H a_H} e^{\beta v_L v_L} e^{\beta a_H H H} (1 - \tau) + \pi_H^* - \beta \frac{L^*}{T} \theta$, i.e., that parameters are such that in the long run equilibrium of the open economy $n_H^T, n_H^T \in [0, 1]$. After trade liberalization, the Home industry specializes into the $H$-attribute sector and the Foreign industry specializes into the $L$-attribute sector until $P^H(v_H) = \frac{P^T(v_H)}{P^T(v_L)}$ and $P^L(v_H) = \frac{P^T(v_H)}{P^T(v_L)}$.

Proof. Denote by $\Pi(a_j) \equiv L \pi_H \frac{e^{\beta a_H H^L} + L (1 - \pi_H)}{P(v_H)}$ the domestic revenue of a Home firm with good $a_j$ in the Home market and by $\Pi^*(a_j) \equiv L^* \pi_H \frac{e^{\beta a_H H^L} + L (1 - \pi_H)}{P^*(v_H)}$ the domestic revenue of a Foreign firm with good $a_j$ in the Foreign market. Since all firms export, face a constant elasticity of demand, and are subject to iceberg transportation costs, the export revenue of a Home firm is equal to $\tau^{-\beta} \Pi^*(a_j)$ so that total revenue of a home firm is equal to $\Pi(a_j) + \tau^{-\beta} \Pi^*(a_j)$. Similarly, the total revenue of a Foreign firm equals $\tau^{-\beta} \Pi(a_j) + \Pi^*(a_j)$. An equilibrium without complete specialization requires that the discounted sales of an $H$-attribute and an $L$-attribute firm are equal in Home and in Foreign:

$$\int_0^\infty e^{-\delta t} \frac{1}{\beta} \left( \Pi(a_H) + \tau^{-\beta} \Pi^*(a_H) \right) dt = \int_0^\infty e^{-\delta t} \frac{1}{\beta} \left( \Pi(a_L) + \tau^{-\beta} \Pi^*(a_L) \right) dt.$$  
$$\int_0^\infty e^{-\delta t} \frac{1}{\beta} \left( \tau^{-\beta} \Pi(a_H) + \Pi^*(a_H) \right) dt = \int_0^\infty e^{-\delta t} \frac{1}{\beta} \left( \tau^{-\beta} \Pi(a_L) + \Pi^*(a_L) \right) dt.$$  

Solving the integral and subtracting $\tau^{-\beta}$ times (13) from (12) yields $\Pi(a_H) = \Pi(a_L)$ and subtracting $\tau^{-\beta}$ times (12) from (13) yields $\Pi^*(a_H) = \Pi^*(a_L)$. Recalling the definition of $\Pi(a_j)$ and $\Pi^*(a_j)$ yields

$$\frac{P^T(v_H)}{P^T(v_L)} - \beta = e^{\beta a_H v_H} - e^{\beta a_L v_L} \frac{\pi_H}{1 - \pi_H}$$

and

$$\frac{P^T(v_H)}{P^T(v_L)} - \beta = e^{\beta a_H v_H} - e^{\beta a_L v_L} \frac{\pi_H^*}{1 - \pi_H^*},$$

which is the same as in autarky (see Proposition (2)).

Lemma (2) states a surprising and very stark result: regardless of the relative size of countries and the distribution of the preferences in the other nation (as long as there is incomplete specialization), the long-run relative toughness of competition in the $H$ and $L$ segments is the same as in autarky. The underlying intuition for this effect is indeed trivial and derives from the fact that the home market matters relatively more for Home firms than for Foreign firms, while, in contrast, the Foreign market is relatively
more important for Foreign firms than for Home firms: there can never be incomplete specialization unless the domestic revenue is equal for an $H$ and $L - attribute$ firms in both the Foreign and the Home market.

The intuition of this result is the following. If a Home $H - attribute$ firm has higher sales at Home than a Home $L - attribute$ firm, there must be some offsetting advantage for $L - attribute$ firms in the Foreign market for the free entry condition in Home to hold. Moreover, because the foreign market matters relatively less than the domestic market due to the existence of transportation costs, the offsetting advantage for $L - attribute$ firms in Foreign must be larger than the advantage for $H - attribute$ firms at Home. In contrast, for Foreign firms transportation costs apply to sales in the Home market, thus requiring that the Home advantage for $H - attribute$ firms is relatively stronger than the $L - attribute$ advantage in Foreign. Together, these two requirements are inconsistent unless there is not advantage for either type of firm in either market.\footnote{To formulize this insight, denote the difference in domestic sales at home between a $H - attribute$ good and a $L - attribute$ good firm by $Z \left( Z = \Pi \left( aH \right) - \Pi \left( aL \right) \right)$ and the same difference in foreign by $Z^* \left( Z^* = \Pi^* \left( aH \right) - \Pi^* \left( aL \right) \right)$. The free attribute-entry condition at home implies that $Z + \tau^{-\beta} Z^* = 0$, while the same condition in Foreign is $\tau^{-\beta} Z + Z^* = 0$. Next, note that if $Z > 0$, the free entry condition in home can indeed hold if $- Z^* = Z/\tau^{-\beta} > Z$. However, if the latter is the case, the symmetric condition in foreign would imply that $- Z^* = \tau^{-\beta} Z < Z$, i.e., only $Z = Z^* = 0$ can satisfy the free attribute-entry conditions both at Home and in Foreign.}

The result of Lemma (2) holds irrespective of country size, but only under the assumption that specialization is incomplete. It is straightforward to show that the total number of firms equals

$$N^T = \frac{L - \tau^{-\beta} L^*}{(1 - \tau^{-\beta}) F \beta \delta}$$

and

$$N^T_* = \frac{L^* - \tau^{-\beta} L^*}{(1 - \tau^{-\beta}) F \beta \delta},$$

while relative entry satisfies

$$n^T_H = n^A_H + \left( \pi_H - \pi_H^* \right) \frac{\tau^{-\beta} \left( \frac{N^T_0}{N^T} \right)}{1 - \tau^{-2\beta}} \Lambda$$

and

$$n^T_* = \frac{n^A_H - \left( \pi_H - \pi_H^* \right) \frac{\tau^{-\beta} \left( \frac{N^T_0}{N^T} \right)}{1 - \tau^{-2\beta}} \Lambda}{(14)}$$

where $\Lambda = \frac{e^{\beta \beta L - e^{\beta \beta L} \gamma L}}{e^{\beta \beta L} - e^{\beta \beta L} \gamma L} + \frac{e^{\beta \beta L \gamma L}}{e^{\beta \beta L} - e^{\beta \beta L} \gamma L}$. The conditions for incomplete specialization are thus that $\pi_H > \pi_H^* > e^{\beta \beta H H L} e^{\beta \beta L L - e^{\beta \beta H H L}} \left( 1 - \tau^{-\beta} \frac{L}{F} \right) + \pi_H \tau^{-\beta} L$(? \text{ and } \pi_H < e^{\beta \beta H H L} e^{\beta \beta L L - e^{\beta \beta H H L}} \left( 1 - \tau^{-\beta} \frac{L}{F} \right) + \pi_H \tau^{-\beta} \frac{L}{F} \). Complete specialization is thus more likely if countries are more unequal in tastes (higher $|\pi_H - \pi_H^*|$) or if countries are of more unequal size. Note that the condition that countries do not specialize in the $O$ sector is less restrictive than the condition that $n^T_H$ and $n^T_* \in [0, 1]$ if $\pi_H \neq \pi_H^*$. If $\pi_H = \pi_H^*$ one only needs the restriction that $\tau^{-\beta} < \frac{L}{F} \tau^{-\beta}$.\footnote{To formulize this insight, denote the difference in domestic sales at home between a $H - attribute$ good and a $L - attribute$ good firm by $Z \left( Z = \Pi \left( aH \right) - \Pi \left( aL \right) \right)$ and the same difference in foreign by $Z^* \left( Z^* = \Pi^* \left( aH \right) - \Pi^* \left( aL \right) \right)$. The free attribute-entry condition at home implies that $Z + \tau^{-\beta} Z^* = 0$, while the same condition in Foreign is $\tau^{-\beta} Z + Z^* = 0$. Next, note that if $Z > 0$, the free entry condition in home can indeed hold if $- Z^* = Z/\tau^{-\beta} > Z$. However, if the latter is the case, the symmetric condition in foreign would imply that $- Z^* = \tau^{-\beta} Z < Z$, i.e., only $Z = Z^* = 0$ can satisfy the free attribute-entry conditions both at Home and in Foreign.}
If $\pi_H^* > \pi_H$, it is true that

$$n_H^T > n_H^A > n_H^A > n_H^T$$

as long as $\tau^γ > 0$. I.e., after trade liberalization, the industrial structure of the two countries diverges. This result is a direct consequence of the fact that after opening to trade, in Home, competition is relatively tougher in the market for $L$ – attribute goods than in the market for $H$ – attribute goods (see Lemma (1)). Although export possibilities are better in the market for H goods than in the market for $L$ – attribute goods, overall, producers of H goods make strictly higher profits because the home market is the more important industry segment (as $\pi_H^* < \pi_H$). Consequently, all newly entering firms choose to enter the industry with the $H$ – attribute good until the new equilibrium is reached.

The most important consequence of Lemma (2) is that the long run relative welfare of $H$ – and $L$ – valuation consumers in both countries is not affected by trade: although the group which is relatively smaller gains from having access to a large set of foreign firms that produce a fitting good, this is exactly offset by the exit of domestic firms from this sector. In total, the welfare gains from trade are equal to

$$\left(\frac{P(v_H)}{P_I(v_H)}\right)^\alpha = \left(\frac{P(v_L)}{P_I(v_L)}\right)^\alpha = \left(\frac{L}{L^*} + \tau - \beta L^*\right)^\alpha.$$  

This is the same relative welfare gain that would prevail in the absence of taste heterogeneity.

We are now in position to describe the aggregate trade patterns in the long run equilibrium.

**Proposition 5 (Long Run Trade Patterns)** Assume that $\pi_H > \pi_H^*$ and that parameters are such that $n_H^T, n_H^A, n_H^A > 0, 1]. The volume of Home’s manufacturing exports is equal to $L^* = L$ for any $\pi_H, \pi_H^*$ that satisfy the stated assumptions. If $L^* > L^*$, Home has net exports of manufactured goods. If $L > L^*$, Home is a net exporter of manufactured goods. For any combination of $L$ and $L^*$ that is consistent with the stated assumptions, Home’s manufacturing exports are more $H$ – attribute intensive in the open economy equilibrium than just after trade liberalization. 

**Proof.** see Appendix. □

The first statement of Proposition 5 is intriguing: although the within-industry home market effect intensifies, the home bias vanishes completely if specialization is incomplete. While the intensifying within-industry home market effect is a straightforward consequence of the increased specialization (see the previous proposition), the intuition behind the disappearance of the consumption home bias is less clear at first sight. It is expedient to again analyze why the home bias arises in the first place at the moment of trade liberalization to understand why it later vanishes.
Consider home’s exports. The dollar volume of trade is equal to the number of firms times the sales per firm. At the moment after opening to trade, there are \( n_H^A \) \( H \)-attribute good exporters that sell relatively less than \( \tau^{-\beta} \) times their domestic sales on the export markets because competition is relatively stronger in the foreign \( H \) sector than in the domestic \( H \) sector. This is partly offset because the \( 1 - n_A^L \) \( L \)-attribute good exporters sell more than \( \tau^{-\beta} \) times their domestic sales on the export markets. However, overall, the effect overall effect on trade volume is negative since the \( H \)-sector is the more important one for the home economy.

In contrast, steady state trade flows are not affected by the underlying taste differences across nations. Since the import competition is biased towards one sector, the domestic industry concentrates into the other sector. With equally sized countries, this adjustment continues until

\[
\pi_H \left/ P^t (v_H)^{-\beta} = \pi_H^* \left/ P^{t^*} (v_H)^{-\beta}, \text{ hence implying that exports & domestic revenue per firm are the same for } H- \text{ and } L - \text{attribute firms. Thus, in the steady state heterogeneity and the composition of exports does not matter for average trade flows.}
\]

Thus, Linder’s (1961) hypothesis does not hold true in general equilibrium. It is noteworthy that his argument misses an important insight about how – in general equilibrium - trade affects a nation’s industrial structure. His hypothesis hinges on the (intuitive) notion that low domestic taste for an attribute is associated with a low volume of imports of goods embodying this attribute. This insight indeed holds if industry structure did not respond to tastes. However, in general equilibrium, the country in question loses firms that produce the type of good for which domestic demand is low, and thus the country becomes a net importer of the good. In general equilibrium, a low taste for an attribute is thus associated with a large amount of imports embodying the attribute.

6 Conclusion

In this paper, I analyze to what extent product attributes, such as the color or the quality of a good and consumers taste for these attributes influence the volume and direction of trade, as well as the welfare effect of trade liberalization. To this end, I first augment the preference structure of Dixit and Stiglitz (1977) with a model of the demand for heterogeneous products. In addition to their love of variety – which is derived from a discrete choice setup – consumers are heterogeneous in their taste for product attributes as in Mussa and Rosen (1978). Products, in turn, are heterogeneous in their attributes.

Consumers with a high taste for an attribute tend to buy from firms with a high-attribute good. The key implication of this assortative match-
ing is that firms with similar goods tend to sell to consumers with similar tastes, i.e., that the industry is endogenously segmented by product attributes. Consequently, with trade, the composition of imports matters for the composition of the domestic industry.

I next nest these preferences in a model of the international economy featuring two countries that differ in their average tastes and iceberg transportation costs. The model includes the Krugman (1980) economy as a special case without product heterogeneity, thus allowing to clearly compare the model’s insights to standard models of the international economy.

The model has three novel predictions. The first is the within-industry extension of Krugman’s (1980) "home market effect". In Krugman’s model, a country with a larger home market has more entry of firms producing for the domestic market. Thus, with open markets, this nation is also the net exporter of manufactured goods. In the model of this paper, even if two countries are characterized by an equal domestic market size for cars in general, a home bias can arise in the type of the cars these countries produce. For example, if the French consumers put relatively more emphasis on fuel efficiency, while the German consumers tend to emphasize top-speed, in general equilibrium, each country’s industry is adapted to the needs of the local population. Immediately after trade liberalization, France thus becomes a net exporter of fuel-efficient cars, while Germany becomes a net exporter of fast cars. Neither of the two economies, however, needs to become a net exporter of cars.

The model’s second prediction explains why consumption is home-biased in the sense that trade volume is lower than what would be expected on the basis of transportation costs and the elasticity of demand. At the moment of opening markets to trade, each country’s industry is optimized for the tastes of domestic consumers. The typical domestic exporter will, therefore, on average sell less on the export market than would be expected for a given level of trade costs since the typical foreign consumer is different than what the industry is optimized for: while the few German producers of fuel-efficient cars experience high demand in France, this is more than offset by the many producers of fast cars that experience low demand in France. Overall, the volume of trade is small since the German industry, which is optimized for the fast-car loving German consumer, is inappropriate for the average French consumer, who is characterized by a love for fuel efficiency.

The third prediction is that after trade liberalization, the within-industry home market effect intensifies, while the home bias of consumption disappears completely. Consider first the reasons for the trade-induced specialization and second why specialization erases the home bias. When markets are opened to trade, French firms experience relatively more import competition in the segment for fast cars than in the segment for fuel-efficient cars. Thus, domestic sales and profits are relatively higher for the makers of fuel-efficient cars. The latter effect is only partly offset by export possi-
ibilities being relatively better for the French makers of fast cars. Overall, the country thus specializes into the fuel-efficient market segment.

An intriguing result of the model is that the home bias of consumption prevailing immediately after trade liberalization vanishes in the long run although both economies specialize even more. The reason for this is that as countries specialize into the segment where they have a relatively larger domestic demand, they make room for foreign exporters in the other segment of the market. Since the latter happens to be exactly the market segment where the trade partner's industry is concentrated in, trade volume increases.

The model thus documents that Linder's (1961) hypothesis misses an important insight about how trade affects a nation's industrial structure in general equilibrium. His hypothesis hinges on the (intuitive) notion that low domestic taste for an attribute is associated with a low volume of imports of goods embodying this attribute. This insight indeed holds for a given industrial structure. However, in general equilibrium, the country in question looses many of its firms that produce such goods and the country becomes a net importer of goods embodying that attribute. Thus, directly opposed to the intuition underlying Linder's hypothesis, imports tend to over proportionally include product attributes for which the domestic consumers has a low preference for.

References


7 Appendix: Proofs

Proposition 1 (reminded) Denote the demand function of a firm with attribute $a_j$ and charging price $p_j$ by $D(a_j, p_j)$. Demand is determined by

$$D(a_j, p_j) = (1 - \alpha) \theta L \Gamma(1 - \beta) p_j^{-(1+\beta)} \int_{v \in V} \frac{\exp[\beta v a_j]}{P(v)^\beta} dv, \quad (16)$$

where $\Gamma(\cdot)$ is the beta function and $P(v)$ denotes the ideal price index for all consumers with $v_i = \tilde{v}$ and is equal to

$$P(v) = \left( \sum_{n \in J} \left( \frac{p_n}{\exp[v a_n]} \right)^{-\beta} \right)^{-1/\beta}. \quad (17)$$

Proposition 6 Proof. A consumer with valuation $\tilde{v}$ buys only from the firm offering the cheapest per unit good, adjusted for the idiosyncratic shock and the taste-attribute match, i.e., each consumer chooses $\tilde{j} = \arg \max_{i \in J} \frac{e^{a_j \tilde{v} + x_{i,j}}}{e^{a_j \tilde{v} + x_{i,j}}}$. Since the distribution of $x_{i,j}$ is continuous, the probability of ties is 0.

From the firm side, (expected) demand from consumer $\tilde{v}$ with an unknown realization of $x_{i,j}$ is then equal to the probability that the firm’s draw $x_{i,j}$, adjusted for the firm’s price and the match of $a_j$ and $\tilde{v}$ is the maximum of all adjusted draws. Since each consumer spends $(1 - \alpha) \theta_i$ on the manufacturing composite, spends it all on one variety only, sales are then equal to

$$D_j(a_j, p_j, v_i = \tilde{v}) = \frac{(1 - \alpha) \theta_i}{p_j} \int_{x_{i,j} \in X} j_x(x_{i,j}) \Pr \left( \frac{e^{a_j \tilde{v} + x_{i,j}}}{p_j} = \max_{n \in J} \left( \frac{e^{a_n \tilde{v} + x_{i,j}}}{p_n} \right) \right) dx_{i,j}$$

$$= \frac{(1 - \alpha) \theta_i}{p_j} \int_{x_{i,j} \in X} j_x(x_{i,j}) \Pr \left( \max_{n \in J, n \neq j} \left( \frac{e^{a_n \tilde{v} + x_{i,j}}}{p_n} < \frac{e^{a_j \tilde{v} + x_{i,j}}}{p_j} \right) \right) dx_{i,j}$$

$$= \frac{(1 - \alpha) \theta_i}{p_j} \int_{x_{i,j} \in X} j_x(x_{i,j}) \prod_{n \neq j} \Pr \left( \frac{e^{a_n \tilde{v} + x_{i,j}}}{p_n} < \frac{e^{a_j \tilde{v} + x_{i,j}}}{p_j} \right) dx_{i,j}$$

$$= \Pr(x_{i,n} < \ln(p_n) - \ln(p_j) + (a_j - a_n) \tilde{v} + x_{i,j}) dx_{i,j}$$

If all $x$ are distributed Gumbel with scale parameter 0 and shape parameter $1/\beta$, the following holds

$$g_x(x) = \frac{1}{\beta} \exp[-x/\beta] \exp[-\exp[-x/\beta]]$$

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and thus
\[
\Pr(x_{i,n} < \ln(p_n) - \ln(p_j) + (a_j - a_n) \bar{v} + x_{i,j})
\]
\[= \exp[- \exp[-\beta(\ln(p_n) - \ln(p_j) + (a_j - a_n) \bar{v} + x_{i,j})]]
\]
\[= \exp\left[-p_j^{-\beta} \exp[-\beta(a_j \bar{v} + x_{i,j})] \exp[\bar{v} a_n]\right]
\]
so that
\[
\prod_{n \neq j} \Pr(x_{i,n} < \ln(p_n) - \ln(p_j) + (a_j - a_n) \bar{v} + x_{i,j})
\]
\[= \exp\left[-p_j^{1/\beta} \exp[-\beta(a_j \bar{v} + x_{i,j})] \sum_{J \neq j} \left(p_n^{-\beta} \exp[\bar{v} a_n]\right)\right]
\]
and Demand from group \(\bar{v}\) can be conveniently be expressed as

\[
D_j(a_j, p_j, \bar{v}) = \frac{(1 - \alpha) \theta_i}{p_j} \int_{x_{i,j} \in X} \prod_{n \neq j} \Pr(x_{i,n} < \ln(p_n) - \ln(p_j) + (a_j - a_n) \bar{v} + x_{i,j}) \, dx_{i,j}
\]
\[= \frac{(1 - \alpha) \theta_i}{p_j} \frac{1}{\beta} \int_{x_{i,j} \in X} \exp[-\beta x_{i,j}] \exp[-\exp[-\beta x_{i,j}] \left(1 + p_j^\beta \exp[-\beta \bar{v} a_j] \sum_{J \neq j} \left(p_n^{-\beta} \exp[\bar{v} a_n]\right)\right)] \, dx_{i,j}
\]

Note that: 
\[
1 + p_j^\beta \exp[-\beta \bar{v} a_j] \sum_{J \neq j} \left(p_n^{-\beta} \exp[\bar{v} a_n]\right) = \sum_{J \neq j} \left(p_n^{-\beta} \exp[\bar{v} a_n]\right).
\]
Now, one can substitute: 
\[
z_{i,j} = p_j^\beta \exp[-\beta \bar{v} a_j] \sum_{j \in J} \left(p_n^{-\beta} \exp[\bar{v} a_n]\right) x_{i,j}
\]
leading to

\[
D_j(a_j, p_j, \bar{v}) = \frac{(1 - \alpha) \theta_i}{p_j} \frac{1}{\beta} \left(p_j^\beta \exp[-\beta \bar{v} a_j] \sum_{j \in J} \left(p_n^{-\beta} \exp[\bar{v} a_n]\right)\right)^{-1}
\]
\[
\int_{z_{i,j} \in X} \exp[-z_{i,j} \beta] \exp[-\exp[-\beta z_{i,j}]] \, dz_{i,j}
\]
Since the latter part can be expressed as the CDF of a Gumbel shock, we get demand per from a mass 1 of consumers with valuation \(\bar{v}\)

\[
D_j(a_j, p_j, \bar{v}) = \Gamma(1 - \beta) \frac{w}{p_j} \frac{p_j^{-\beta} \exp[\beta \bar{v} a_j]}{\sum_{n \in J} \left(p_n^{-\beta} \exp[\beta \bar{v} a_n]\right)}, \quad (18)
\]
To get a firm’s total demand $D_j (a_j, p_j)$, one has to integrate over all possible valuations $v$.

$$D_j (a_j, p_j) = L \int_{v \in V} j_v (v) \int_{x_{i,j} \times X} \frac{(1 - \alpha) \theta_i}{p_j} j_{x_{i,j}} (x_{i,j}) \Pr \left( \frac{e^{a_j v + x_{i,j}}}{p_j} = \max_{n \in J} \frac{e^{a_n v + x_{i,n}}}{p_n} \right) \ dx_{i,j} dv$$

$$= L \Gamma (1 - \beta) \frac{(1 - \alpha) \theta_i}{p_j} \int_{v \in V} j_v (v) \frac{p_j^{-\beta} \exp [\beta v a_j]}{\sum_{n \in J} (p_n^{-\beta} \exp [\beta v a_n])} \ dv$$

\[\square\]

**Corollary 1 (reminded).** Denote the expected welfare of consumer $i$ with $v_i = v$ and income $\theta_i$ by $E (U_i | v, \theta_i)$. If $p_O = 1$,

$$E (U_i) = (1 - \alpha) \alpha^\alpha \Gamma \left( 1 - \frac{\beta}{\alpha} \right) \left( \frac{P (v)}{\theta_i} \right)^{-\alpha}$$

where the ideal price index $P (v)$ is as defined in (6) and $\Gamma (...) \) is the gamma function.

**Corollary 2 Proof.** The consumer only buys from the draw and match-adjusted cheapest firm. Define $j^* (i) \equiv \arg \max_{j \in J} \left( \frac{\exp [v_i a_j + x_{i,j}]}{p_j} \right)$. Conditional on this $j^* (i)$, consumer $i$ maximizes

$$U_i = \max_{O_i q_{i,j^* (i)}} \left( 1 - \alpha \right) \alpha^\alpha \Gamma \left( 1 - \frac{\beta}{\alpha} \right) \left( \frac{P (v)}{\theta_i} \right)^{-\alpha} - \lambda_i \left[ O_i p_O + q_{i,j^* (i)} p_{j^* (i)} - \theta_i \right]$$

Implying that the value of the Langragian multiplier, or the marginal utility with respect to increasing income $\theta_i$, equals $\lambda_i = (1 - \alpha) \alpha^\alpha p_O^{-\left(1 - \alpha\right)} \left( \frac{p_{j^* (i)}}{\exp [v_i a_j + x_{i,j}]} \right)^{-\alpha} \theta_i$. With $p_O$ normalized to 1, the utility for a given maximum realization of $x_{i,j^* (i)} + a_{j^* (i)} v_i$ is thus

$$U_i = (1 - \alpha) \alpha^\alpha \theta_i \left( \frac{e^{x_{i,j^* (i)} + a_{j^* (i)} v_i}}{P_{j^* (i)}} \right)^{\alpha}.$$
denote the cdf of \( \tilde{U}_i = U_i / (1 - \alpha)^{1-\alpha} \alpha^\theta_i \), which is distributed

\[
F \left( \tilde{U}_i \right) = \Pr \left[ \max_{j \in J} \left( e^{x_{i,j} + a_j v_i} \right)^\alpha < \tilde{U}_i \right]
\]

\[
= \prod_{j \in J} \Pr \left[ \left( \frac{e^{x_{i,j} + a_j v_i}}{p_j} \right)^\alpha < \tilde{U}_i \right]
\]

\[
= \prod_{j \in J} \Pr \left[ x_{i,j} < \ln \tilde{U}_i^{\frac{1}{\alpha}} + \ln \left( \frac{p_j}{e^{a_j v_i}} \right) \right]
\]

\[
= \prod_{j \in J} G_{x_{i,j}} \left( \ln \tilde{U}_i^{\frac{1}{\alpha}} + \ln \left( \frac{p_j}{e^{a_j v_i}} \right) \right)
\]

\[
= \prod_{j \in J} \exp \left[ - \exp \left[ - \ln \tilde{U}_i^{\frac{1}{\alpha}} + \ln \left( \frac{p_j}{e^{a_j v_i}} \right) \right] \beta \right]
\]

\[
= \exp \left[ \tilde{U}_i^{-\frac{1}{\alpha}} \sum_{j \in J} p_j^{-\beta} e^{\beta a_j v_i} \right]
\]

\[
= \exp \left[ \left( \frac{\tilde{U}_i}{\left( \sum_{j \in J} p_j^{-\beta} e^{\beta a_j v_i} \right)^\frac{1}{\alpha}} \right)^{-\frac{1}{\alpha}} \right]
\]

\( \tilde{U}_i \) is distributed Frechet with scale parameter \( \left( \sum_{j \in J} p_j^{-\beta} e^{\beta a_j v_i} \right)^{\frac{1}{\alpha}} \) and shape parameter \( \frac{\beta}{\alpha} \).

\[
E \left( U_i \right) = (1 - \alpha)^{1-\alpha} \alpha^\theta_i E \left( \tilde{U}_i \right)
\]

\[
= (1 - \alpha)^{1-\alpha} \alpha^\theta_i \left( \sum_{j \in J} p_j^{-\beta} e^{\beta a_j v_i} \right)^{\frac{1}{\alpha}} \Gamma \left( 1 - \frac{\beta}{\alpha} \right)
\]

\[
= (1 - \alpha)^{1-\alpha} \alpha^\theta_i \Gamma \left( 1 - \frac{\beta}{\alpha} \right) P(v_i)^{-\alpha}
\]

\( \blacksquare \)

**Proposition** (Short Run Trade Volume) 3 (reminded) Assume that parameters are such that \( n_H^A, n_H^A \epsilon ]0, 1[ \). At the moment after trade liberalization, the following holds. If \( \pi_H^* = \pi_H \), the volume of trade is the same as in the absence of consumer heterogeneity and home is a net exporter of the
if $L > L^*$. If $\pi_H \neq \pi^*_H$, the volume of trade is lower than in the absence of consumer heterogeneity and decreasing in $|\pi_H - \pi^*_H|$. 

**Proof.** Denote the total value of exports at the moment after opening markets to trade by $X^S$ and $X^S*$ and the attribute specific trade flows by an additional $H$ or $L$ subscript. For each type of good, the value of trade is proportional to the number of firms of each type and the sales per such firm, i.e., Home’s export volume equals

$$X^S_H = N^A \pi^A_H \left( \frac{\pi_H L^* \tau^{-\beta} e^{\beta u_H a_H}}{P^* S (v_H)^{-\beta}} + (1 - \pi^*_H) L^* \frac{\tau^{-\beta} e^{\beta u_L a_H}}{P^* S (v_L)^{-\beta}} \right)$$

and

$$X^S_L = N^A \pi^A_L \left( \frac{\pi^*_H L^* \tau^{-\beta} e^{\beta u_L a_L}}{P^* S (v_H)^{-\beta}} + (1 - \pi^*_H) L^* \frac{\tau^{-\beta} e^{\beta u_L a_L}}{P^* S (v_L)^{-\beta}} \right).$$

The two ideal price indices for foreign $H$- and $L$-valuation consumers are given by Lemma (1) thus yielding

$$X^S = \frac{\tau^{-\beta} L^* \pi^*_H \pi^*_H}{L \pi_H + \tau^{-\beta} L^* \pi^*_H} + \frac{\tau^{-\beta} L^* (1 - \pi_H)}{L (1 - \pi_H) + \tau^{-\beta} L^* (1 - \pi^*_H)} \quad (19)$$

and

$$X^{S*} = \frac{\tau^{-\beta} L^* \pi^*_H \pi_H}{\tau^{-\beta} L \pi_H + L^* \pi_H} + \frac{\tau^{-\beta} L^* (1 - \pi_H)}{\tau^{-\beta} L (1 - \pi_H) + L^* (1 - \pi^*_H)} \quad (20)$$

Next, note that $\pi_H \frac{\tau^{-\beta} L^* \pi^*_H}{\tau^{-\beta} L \pi_H + L^* \pi_H} + (1 - \pi_H) \frac{\tau^{-\beta} L^* (1 - \pi^*_H)}{\tau^{-\beta} L (1 - \pi_H) + L^* (1 - \pi^*_H)} \mid \pi^*_H = \pi_H = \frac{1}{\tau^{-\beta} L + L^*}$, and that

$$\frac{\partial X^S}{\partial \pi_H} \begin{cases} < 0 & \text{if } \pi_H > \pi^*_H \\ 0 & \text{if } \pi_H = \pi^*_H \\ > 0 & \text{if } \pi_H < \pi^*_H \end{cases}$$

so that $\tau^{-\beta} L^* \left( \frac{\pi_H \pi^*_H}{\tau^{-\beta} L \pi_H + L^* \pi^*_H} + \frac{(1 - \pi_H)(1 - \pi^*_H)}{\tau^{-\beta} L (1 - \pi_H) + L^* (1 - \pi^*_H)} \right) < 0$ if $\pi^*_H \neq \pi_H$ and it holds that

$$X^S \begin{cases} = \frac{\tau^{-\beta} L^*}{\tau^{-\beta} L \pi_H + L^* \pi^*_H} & \text{if } \pi_H = \pi^*_H \\ < \frac{\tau^{-\beta} L^*}{\tau^{-\beta} L \pi_H + L^* \pi^*_H} & \text{if } \pi_H \neq \pi^*_H \end{cases},$$

which verifies the first two claims of proposition 3. Next, setting $\pi_H = \pi^*_H$ in (20) and (19) yields $X - X^* = \frac{\tau^{-\beta} L^*}{\tau^{-\beta} L \pi_H + L^* \pi_H} - \frac{\tau^{-\beta} L^*}{\tau^{-\beta} L \pi_H + L^* \pi_H}$, which has the described signs depending on $L, L^*$. For the second part of the claim, note that at $L = L^*$, $X - X^* = \frac{(\pi_H \pi^*_H)(1 - \pi_H)(1 - \pi^*_H) - (1 - \pi_H) \pi_H \pi^*_H}{(\pi_H \pi^*_H)(1 - \pi_H)(1 - \pi^*_H)(1 - \pi_H) \pi_H \pi^*_H}$, which is negative if $\pi_H + \pi^*_H < 1$ and $\pi_H \geq \pi^*_H$, and positive if $\pi_H + \pi^*_H < 1$ and $\pi_H \geq \pi^*_H$. The latter sign is reversed if $\pi_H < \pi^*_H$. A similar calculation for
foreign yields \((20)\). Thus, if \(L = L^*\) and \(\pi_H \neq \pi_H^*\) net trade flows satisfy

\[
X^S - X^S* \bigg|_{L = L^*; \pi_H \geq \pi_H^*} = \begin{cases} 
> 0 & \text{if } \pi_H + \pi_H^* < 1 \\
= 0 & \text{if } \pi_H + \pi_H^* = 1 \\
< 0 & \text{if } \pi_H + \pi_H^* < 1 
\end{cases}.
\]

It is noteworthy that due to the presence of the \(O\) sector, wages are equal across the two countries and thus a net trade flow in labor units is equal to a net trade flow in Dollars. ■

**Proposition** (Short Run Within-industry Home Market Effect) 4 Assume that parameters are such that \(n^H, n^H\epsilon [0, 1]\). At the moment after trade liberalization, if \(L = L^*\), Home is a net exporter of \(H - attribute\) goods iff \(\pi_H > \pi_H^*\). If \(L \neq L^*\) Home’s manufacturing exports contain a larger fraction of \(H - attribute\) goods than do Foreign’s exports.

**Proof.** Home’s net exports of \(H - attribute\) goods are equal to the number of \(H - attribute\) Home firms times exports per such firm minus the same multiplicative in Foreign

\[
X^S_H - X^S* = N^A_h n^H (\pi_H L^* \frac{\beta v_{Ha} \pi_H}{\pi_H} + (1 - \pi_H^*) L^* \frac{\beta v_{La} \pi_H}{\pi_H}) - N^A_h n^H (\pi_H L \frac{\beta v_{Ha} \pi_H}{\pi_H} + (1 - \pi_H^*) L \frac{\beta v_{La} \pi_H}{\pi_H})
\]

For the case of \(L = L^*\), taking into account the toughness of competition in Foreign as well as at Home yields

\[
X^S_H - X^S* = (\pi_H - (1 - \pi_H)) \frac{\beta v_{H} \pi_H}{\pi_H} \frac{(1 - \pi_H^*) \pi_H (1 - \pi_H^*)}{(1 - \pi_H^*) \pi_H + (1 - \pi_H^*) (1 - \pi_H^*) + \pi_H} + \Omega \pi_H \frac{(1 - \pi_H^*) (1 - \pi_H^*)}{\pi_H^* + 1 - \pi_H^*}
\]

where \(\Omega = \frac{e^{\beta v_{Ha} \pi_H} e^{\beta v_{La} \pi_H} e^{\beta v_{Ha} \pi_H} e^{\beta v_{La} \pi_H}}{e^{\beta v_{La} \pi_H} e^{\beta v_{La} \pi_H} e^{\beta v_{Ha} \pi_H} e^{\beta v_{La} \pi_H} + e^{\beta v_{Ha} \pi_H} e^{\beta v_{La} \pi_H} e^{\beta v_{Ha} \pi_H} e^{\beta v_{La} \pi_H} + e^{\beta v_{La} \pi_H} e^{\beta v_{La} \pi_H} e^{\beta v_{Ha} \pi_H} e^{\beta v_{La} \pi_H}} > 0\). Since both \(\pi_H - (1 - \pi_H)\) and \(\frac{\pi_H}{\pi_H^* + 1} - \frac{(1 - \pi_H^*)}{\pi_H^* + 1}\) are larger than 0, \(X_H - X^*_H > 0\) for \(\pi_H > \pi_H^*\).

Next, for the case of \(L \neq L^*\), to show that \(\frac{X^S_H}{X_H + X^*_L} > \frac{X^S*}{X^*_H + X^*_L}\), it suffices to
show that \( \frac{X^*_H}{X^*_{L}} > \frac{X^*_H}{X^*_{L}} \).

\[
\frac{X^*_H}{X^*_{L}} = \frac{n^A_H}{(1 - n^A_H)} \left( \pi_H \left( \frac{e^{\beta v_H a_H}}{P^S(v_H)^{\beta}} \right) + (1 - \pi_H) \frac{e^{\beta v_L a_L}}{P^S(v_L)^{\beta}} \right)
\]

\[
= \frac{\pi_H}{e^{\beta v_H a_H} - e^{\beta v_L a_L}} \pi_H - (1 - \pi_H) \frac{e^{\beta v_L a_L}}{e^{\beta v_H a_H} - e^{\beta v_L a_L}}
\]

\[
\frac{X^*_H}{X^*_{L}} = \frac{n^A_H}{(1 - n^A_H)} \left( \pi_H \left( \frac{e^{\beta v_H a_H}}{P^S(v_H)^{\beta}} \right) + (1 - \pi_H) \frac{e^{\beta v_L a_L}}{P^S(v_L)^{\beta}} \right)
\]

\[
= \frac{\pi_H}{e^{\beta v_H a_H} - e^{\beta v_L a_L}} \pi_H - (1 - \pi_H) \frac{e^{\beta v_L a_L}}{e^{\beta v_H a_H} - e^{\beta v_L a_L}}
\]

Since both \( e^{\beta v_H a_H} - e^{\beta v_L a_L} \) and \( e^{\beta v_L a_L} \) are positive, it is true that \( \frac{X^*_H}{X^*_{L}} > \frac{X^*_H}{X^*_{L}} \) for \( \pi_H > \pi^*_H \).

**Proposition 5** (reminded) Assume that \( \pi_H > \pi^*_H \) and that parameters are such that \( n^A_H, n_{H^*}^A \in [0, 1] \). The volume of Home’s manufacturing exports is equal to \( \frac{X^*_H}{X^*_{L}} \) for any \( \pi_H, \pi^*_H \) that satisfy the stated assumptions. If \( L^* = L \) Home has 0 net exports of manufactured goods. If \( L > L^* \), Home is a net exporter of manufactured goods. For any combination of \( L \) and \( L^* \) that is consistent with the stated assumptions, Home’s manufacturing exports are more \( H-attribute \) intensive than Foreign’s manufacturing exports. Home’s manufacturing exports are more \( H-attribute \) intensive in the open economy equilibrium than just after trade liberalization.

**Proof.** Home’s Exports and Imports of goods that embody the \( H-attribute \) are proportional to the number of domestic respectively foreign \( H \) firms. Home’s total exports of \( H-attribute \) goods equal

\[
X^*_H = N^T, n^*_H \left( L^* \pi^*_H \frac{\tau - \beta e^{\beta v_H a_H}}{P^T(v_H)^{\beta}} + L^* (1 - \pi^*_H) \frac{\tau - \beta e^{\beta v_L a_L}}{P^T(v_L)^{\beta}} \right)
\]

\[
= N^T, n^*_H \left( L^* \frac{\tau - \beta}{\tau - \beta N^T + N^T} \right)
\]
Similarly,

\[ X_T^L = (1 - n_H^T) N^T \frac{L^* \tau^{-\beta}}{\tau^{-\beta} NT + N^T} \]

\[ X_T^H = n_H^T N^T \frac{L^* \tau^{-\beta}}{\tau^{-\beta} NT + N^T} \]

\[ X_T^L = (1 - n_H^T) N^T \frac{L^* \tau^{-\beta}}{\tau^{-\beta} NT + N^T} \]

Because each \( H - \)attribute and \( L - \)attribute producer of a country exports the same amount, the composition of the industry does not affect the overall volume of exports.

\[ X^T = X_T^H + X_T^L = N^T \frac{L^* \tau^{-\beta}}{\tau^{-\beta} NT + N^T} = \left( L - \tau^{-\beta} L^* \right) \frac{\tau^{-\beta}}{1 - \tau^{-2\beta}} \]

\[ X_T^* = X_T^H + X_T^L = N^T \frac{L^* \tau^{-\beta}}{\tau^{-\beta} NT + N^T} = \left( L^* - \tau^{-\beta} L \right) \frac{\tau^{-\beta}}{1 - \tau^{-2\beta}} \]

and Home’s net exports of manufactured goods equal

\[ X^T - X_T^* = (L - L^*) \frac{\tau^{-\beta}}{1 + \tau^{-\beta}} \]

Home’s net exports of \( H - \)attribute equal

\[ X_T^H - X_T^H^* \]

\[ = (\pi_H - \pi_H^*) \frac{e^{3L\alpha_L}}{e^{\beta H\alpha_L} - e^{\beta H^*\alpha_L}} - ((1 - \pi_H) - (1 - \pi_H^*)) \frac{e^{3L\alpha_L}}{e^{\beta H\alpha_L} - e^{\beta H^*\alpha_L}} \]

and the labor intensity of

\[ \frac{X_T^H}{X_T^H + X_T^L} = n_H^T > n_H^T^* = \frac{X_T^H^*}{X_T^H^* + X_T^L^*} \]

Last, it is also true that

\[ \frac{X_T^H}{X_T^H + X_T^L} = n_H^T > \frac{X_T^S}{X_T^H + X_T^L} \]

Since both \( n_H^T > n_H^T \) and after trade liberalization, an \( H - \)attribute firm exports less than an \( L - \)attribute firms so that

\[ \frac{\pi_H e^{3L\alpha_H} + (1 - \pi_H) \pi_H^* e^{3L^*\alpha_H}}{\pi_H e^{3L\alpha_H} + (1 - \pi_H) \pi_H^* e^{3L^*\alpha_H}} < \frac{\pi_H^* e^{3L^*\alpha_H} + (1 - \pi_H^*) \pi_H e^{3L\alpha_H}}{\pi_H^* e^{3L^*\alpha_H} + (1 - \pi_H^*) \pi_H e^{3L\alpha_H}} \]

1. ■
7.1 The case of Complete Specialization.

What happens if parameters are such that one country specializes completely in one sector? There are two cases: complete specialization in both countries and complete specialization in only one country.

7.1.1 One Country Specializes

Assume for example that Home specializes completely in the $H$-sector, while foreign doesn’t specialize. Then, it is true that

$$\Pi(a_H) + \tau^{-\beta} \Pi^*(a_H) = F\delta\beta$$
$$\tau^{-\beta} \Pi(a_H) + \Pi^*(a_H) = F\delta\beta$$
$$\tau^{-\beta} \Pi(a_L) + \Pi^*(a_L) = F\delta\beta$$

so, one can subtract

$$\tau^{-\beta} \Pi(a_H) + \Pi^*(a_H) - F\delta\beta - \tau^{-\beta} \left( \Pi(a_H) + \tau^{-\beta} \Pi^*(a_H) - F\delta\beta \right) = 0$$

or $\Pi^*(a_H) = \frac{F\delta\beta}{1 + \tau^{-\beta}}$. So we have:

$$\Pi^*(a_H) = \frac{F\delta\beta}{1 + \tau^{-\beta}}$$

$$\Pi(a_H) + \tau^{-\beta} \Pi^*(a_H) = \Pi(a_H) + \tau^{-\beta} \frac{F\delta\beta}{1 + \tau^{-\beta}} = F\delta\beta$$

$$\Pi(a_H) = \frac{F\delta\beta}{1 + \tau^{-\beta}}$$

and

$$\tau^{-\beta} \Pi(a_L) + \Pi^*(a_L) = F\delta\beta$$

Let’s solve the free entry conditions for H-sector goods first. The price indices are:

$$P(v_H)^{-\beta} = \left( N + \tau^{-\beta} N^* n_H^* \right) e^{3v_H a_H} + \tau^{-\beta} N^* (1 - n_H^*) e^{3v_H a_L}$$
$$P(v_L)^{-\beta} = \left( N + \tau^{-\beta} N^* n_H^* \right) e^{3v_L a_H} + \tau^{-\beta} N^* (1 - n_H^*) e^{3v_L a_L}$$
$$P^*(v_H)^{-\beta} = \left( \tau^{-\beta} N + N^* n_H^* \right) e^{3v_H a_H} + N^* (1 - n_H^*) e^{3v_H a_L}$$
$$P^*(v_L)^{-\beta} = \left( \tau^{-\beta} N + N^* n_H^* \right) e^{3v_L a_H} + N^* (1 - n_H^*) e^{3v_L a_L}$$
\[ \Pi^* (a_H) = L^* \pi_H \frac{e^{\beta v a_H}}{P^* (v_H)^{-\beta}} + L^* (1 - \pi_H) \frac{e^{\beta v a_H}}{P^* (v_L)^{-\beta}} = \frac{F \delta_\beta}{1 + \tau - \beta} \]

\[ \Pi (a_H) = L \pi_H \frac{e^{\beta v a_H}}{P (v_H)^{-\beta}} + L (1 - \pi_H) \frac{e^{\beta v a_H}}{P (v_L)^{-\beta}} = \frac{F \delta_\beta}{1 + \tau - \beta} \]

or

\[ \tau^{-\beta} \Pi (a_L) + \Pi^* (a_L) \]

\[ = \tau^{-\beta} L \left( \pi_H \frac{e^{\beta v a_H}}{P (v_H)^{-\beta}} + (1 - \pi_H) \frac{e^{\beta v a_H}}{P (v_L)^{-\beta}} \right) + \]

\[ L^* \left( \pi_H^* \frac{e^{\beta v a_H}}{P^* (v_H)^{-\beta}} + (1 - \pi_H^*) \frac{e^{\beta v a_H}}{P^* (v_L)^{-\beta}} \right) \]

\[ = F \delta_\beta \]

7.1.2 Both Countries Specialize \( n_H = 1 \ n_H^* = 0 \)

\[ P (v_H)^{-\beta} = Ne^{\beta v a_H} + \tau^{-\beta} N^* e^{\beta v a_L} \]

\[ P (v_L)^{-\beta} = Ne^{\beta v a_H} + \tau^{-\beta} N^* e^{\beta v a_L} \]

\[ P^* (v_H)^{-\beta} = \tau^{-\beta} Ne^{\beta v a_H} + N^* e^{\beta v a_L} \]

\[ P^* (v_L)^{-\beta} = \tau^{-\beta} Ne^{\beta v a_H} + N^* e^{\beta v a_L} \]

\[ P (v_H)^{-\beta} = NH + \tau^{-\beta} N^* \]

\[ P (v_L)^{-\beta} = N + \tau^{-\beta} N^* H \]

\[ P^* (v_H)^{-\beta} = \tau^{-\beta} NH + N^* \]

\[ P^* (v_L)^{-\beta} = \tau^{-\beta} N + N^* H \]
Free entry in Home and Foreign satisfies

\[
F_{\delta \beta} = L \left( \frac{\pi e^{\beta v_H a_H}}{P(v_H)^{-\beta}} + (1 - \pi) \frac{e^{\beta v_L a_H}}{P(v_L)^{-\beta}} \right) \\
+ \tau^{-\beta} L^* \left( \frac{\pi^* e^{\beta v_H a_H}}{P^*(v_H)^{-\beta}} + (1 - \pi^*) \frac{e^{\beta v_L a_H}}{P^*(v_L)^{-\beta}} \right)
\]

\[
F_{\delta \beta} = L^* \left( \frac{\pi^* e^{\beta v_H a_L}}{P^*(v_H)^{-\beta}} + (1 - \pi^*) \frac{e^{\beta v_L a_L}}{P^*(v_L)^{-\beta}} \right) \\
+ \tau^{-\beta} L \left( \frac{\pi e^{\beta v_H a_L}}{P(v_H)^{-\beta}} + (1 - \pi) \frac{e^{\beta v_L a_L}}{P(v_L)^{-\beta}} \right)
\]

Let's first solve this for the absolutely symmetric case \( e^{\beta v_H a_L} = e^{\beta v_L a_H} = 1, e^{\beta v_H a_H} = e^{\beta v_L a_L} = H > 1, L = L^*, \pi_H = 1 - \pi_H^* \). Then,

\[
N = \frac{L}{F_{\delta \beta}} = N^*
\]

as in non-complete specialization.