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# The Future of Social Security

Martín Gonzalez-Eiras and Dirk Niepelt

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# The Future of Social Security\*

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## Abstract

We analyze the effect of the projected demographic transition on the political support for social security, and equilibrium outcomes. Embedding a probabilistic-voting setup of electoral competition in the Diamond (1965) OLG model, we find that intergenerational transfers arise in the absence of altruism, commitment, or trigger strategies. Closed-form solutions predict population ageing to lead to higher social security tax rates, a rising share of pensions in GDP, but eventually lower social security benefits per retiree. The response of equilibrium tax rates to demographic shocks reduces old-age consumption risk. Calibrated to match features of the U.S. economy, the model suggests that, in response to the projected demographic transition, social security tax rates will gradually increase to 16 percent; other policies that distort labor supply will become less important; and in contrast with frequently voiced fears, labor supply therefore will rise.

KEYWORDS: Social security; probabilistic voting; Markov perfect equilibrium; saving; labor supply.

JEL CLASSIFICATION CODE: E62, H55.

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“Your estimated benefits are based on current law. Congress has made changes to the law in the past and can do so at any time.”  
(Social Security Statement distributed by the U.S. Social Security Administration, 2007)

## 1 Introduction

Many countries with pay-as-you-go financed social security systems are confronted with a secular decline in population growth rates that puts increasing financial stress on these systems. Projections of social security shortfalls in those countries typically imply that solvency will eventually require increases in contribution rates or cuts in benefits (or a combination of the two). The questions are, which of these options will be implemented, and what the macroeconomic consequences will be. In this paper, we develop a robust analytical framework to address these questions. Moreover, we apply this framework to generate predictions for the medium-term outlook of the U.S. social security system.

Benefit levels and contribution rates, among many other parameters of social security systems, are politically determined and may at any time be altered in the legislative process. In order to predict adjustments of social security taxes and benefits, it is therefore essential to model the determinants of the political support for social security, and the effect of the demographic transition on these determinants. To this purpose, we introduce political choice in Diamond’s (1965) overlapping generations model. We solve for the politico-economic equilibrium and analyze the response of both policies and the allocation to an exogenously given demographic transition.

Households in the model are non-altruistic. As consumers, they take prices and policy instruments as given. As voters, they anticipate the effect of policy on equilibrium outcomes including future political choices. We assume that voters are not bound by past political decisions. The politico-economic equilibrium therefore features subgame-perfect tax and transfer choices supporting a competitive equilibrium.

Agents hold rational expectations. Voters, in particular, are fully aware of the equilibrium relationship between future state variables and policy choices, and this relationship shapes their preferences over contemporaneous policy choices. We posit that only fundamental state variables affect future policy outcomes, excluding artificial state variables of the type sustaining trigger strategy equilibria. Our underlying assumption is that, while the *existence* of social security programs may also owe to reputational forces, changes in the *size* of these programs depend more directly on the economic and political environment than through underlying trigger strategies. Focusing on the Markov perfect equilibrium, we aim at identifying the fundamental and robust forces that determine this size, without relying on arbitrary assumptions about the parameters of a trigger strategy.<sup>1</sup> In fact, the Markov perfect equilibrium we focus on is the *unique* equilibrium arising in the limit of the finite-horizon economy.

We model electoral competition under the assumption of probabilistic voting. The policy platforms of vote-seeking candidates therefore cater to the interests of all voters in society,

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<sup>1</sup>The Markov assumption also reinforces our assumption that political choices suffer from a lack of commitment, including commitment to particular trigger strategies. For a discussion of Markov perfect equilibrium, see Krusell, Quadrini and Ríos-Rull (1997). Bhaskar (1998) shows that weak informational constraints in overlapping generation games with a strictly dominant action for the old imply that the unique pure strategy equilibrium is in Markov strategies.

reflecting both support for social security benefits by the elderly and opposition against them by young tax-payers. However, young voters oppose social security less emphatically than old voters support it, and the politico-economic equilibrium therefore features a structural “bias” in favor of intergenerational transfers. This bias arises because social security taxes do not only generate a cost for workers, but also indirect benefits: By depressing savings, taxes allow to monopolize the supply of capital and thus, to manipulate the terms of trade with future, unborn generations.<sup>2</sup> The resulting bias is a robust feature. It persists even if an additional policy instrument is available that distorts labor supply and thus, depresses savings without transferring resources to the elderly.

Voters internalize only those general-equilibrium effects of transfers that materialize during their lifetimes; negative consequences borne by subsequent cohorts (due to lower capital accumulation) remain unaccounted for. The pro-transfer bias in politico-economic equilibrium therefore is a reflection of the fact that the cost of social security is partly shifted to the future. In contrast, a Ramsey government with “dynastic” welfare weights (that is, with welfare weights reflecting households’ time preference and cohort sizes) internalizes all general-equilibrium effects. The social security tax rate implemented by such a government therefore typically falls short of the tax rate in politico-economic equilibrium.

Demographic change alters factor prices and changes the relative weights that the political process attaches to the interests of old and young voters. Under standard functional form assumptions, we are able to characterize the resulting transition dynamics of the economy in closed form. (This stands in sharp contrast to most of the literature which characterizes politico-economic equilibria numerically. When we relax the functional form assumptions and thus, have to resort to numerical solutions, we find our central results to be robust.) The model predicts a slowdown of population growth to be associated with (i) higher social security tax rates, (ii) a rising share of pensions in GDP, (iii) but eventually lower social security benefits per retiree. The endogenous response of tax rates to demographic shocks also affects the allocation of consumption risk in the economy. In fact, (iv) old-age consumption risk is lower in politico-economic equilibrium than in a situation where tax rates are constant. These effects are sizeable. When calibrated to match stylized features of the U.S. economy, the closed-form solutions of the model suggest that (v) social security tax rates will gradually increase to around 16 percent. Moreover, while the social security system will absorb a growing share of GDP, (vi) the importance of other policy instruments with a distortive effect on labor supply will decline. As a result, (vii) labor supply will continue to rise until leveling off in two decades or so.

These findings have important implications for the debate about social security reform. Participants in that discussion have identified several feasible policies that would restore solvency of the U.S. social security system. According to the 2006 Annual Report of the Social Security Board of Trustees<sup>3</sup>, for example, “the projected infinite horizon shortfall [of social security] could be eliminated with an immediate increase in the combined payroll tax rate from 12.4 percent to about 16.1 percent” (p. 55). According to the well publicized reform proposal by Diamond and Orszag (2005), to name another example, tax rates could gradually be increased to around

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<sup>2</sup>In addition to altering the terms of trade, the change of savings induced by higher taxes also translates into future policy changes if policy depends on this state variable. Kotlikoff and Rosenthal (1990) discuss the incentive of young workers to monopolize the supply of capital. They assume commitment and do not model the political process. Cooley and Soares (1999) and Boldrin and Rustichini (2000), among others, stress the role of general equilibrium effects.

<sup>3</sup>The 2006 Annual Report of the Board of Trustees of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds, see <http://www.ssa.gov/OACT/TR/TR06/tr06.pdf>.

15.4 percent and benefits cut by up to 9 percent.<sup>4</sup> The model developed in the present paper predicts that equilibrium tax rates will gradually increase to a level of around 16 percent. In light of the two above-mentioned scenarios, this implies that benefits will be cut, but most likely by less than 9 percent.

Participants in the reform debate have also discussed the effects of social security tax rates on labor supply. Many observers have voiced fears that further tax hikes might imply large deadweight losses, rendering the option of raising social security taxes essentially infeasible.<sup>5</sup> The model developed in the present paper partially refutes this view. It shows that the link between social security tax rates and labor supply distortions must not be seen in isolation but in the context of a wider set of policy instruments. More specifically, it predicts that—*because* of deadweight losses—higher social security tax rates will go hand in hand with fewer other sources of labor market distortions; this will have a positive net effect on labor supply.

Our work extends a growing literature on dynamic politico-economic equilibrium, with voters sequentially choosing their preferred policies under rational expectations about the effect on future equilibrium outcomes (see, for example, Krusell et al., 1997; Hassler, Rodríguez Mora, Storesletten and Zilibotti, 2003). Moreover, it relates to an extensive literature on the sources of political support for intergenerational transfers. Typical explanations in this literature rely on altruism or commitment (see, for example, Cukierman and Meltzer, 1989; Hansson and Stuart, 1989; Conesa and Krueger, 1999; Tabellini, 2000; Persson and Tabellini, 2002). Alternatively, they view the political process as representing the interests of a tax-paying median voter who fears that the provision of future benefits hinges on the provision of current ones. According to this view, the link between current and future benefits arises because successive median voters coordinate on a sufficiently effective trigger strategy (as in Bohn (1999), Cooley and Soares (1999), Boldrin and Rustichini (2000), or Rangel (2003)), or on the self-fulfilling expectation that higher savings, due to lower current social security contributions, trigger a benefit cut (as in Forni (2005)).

Our approach differs from these models. It does not rely on altruism, commitment, expected punishments, or an infinite horizon, nor does it restrict policy choices to be binary or population growth to be sufficiently high (to render the economy dynamically inefficient, as do some previous models). Moreover, our approach generates closed-form solutions; fully characterizes the transition dynamics; and introduces multiple policy instruments, allowing to investigate (in contrast to previous literature) whether the political support for social security is robust. These advantages are largely the consequence of the probabilistic-voting assumption which provides a means to capture gradual differences in the support for social security even in a stark two-period-lived overlapping-generations environment.<sup>6</sup>

The median-voter setup does not offer all of these advantages. In median-voter models featuring a *few* overlapping generations of homogeneous agents, demographic change of plausible magnitude does not alter the identity of the median voter.<sup>7</sup> As a consequence, it induces policy adjustments only to the extent that general equilibrium effects of ageing (on prices and the social security system's internal rate of return) alter the median voter's preferred policy. But

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<sup>4</sup>Diamond and Orszag's (2005) proposal also deals with other dimensions of the social security system that are absent in our model.

<sup>5</sup>See, for example, Feldstein (2005).

<sup>6</sup>Grossman and Helpman (1998), among few other authors, have embedded probabilistic voting in a model of intergenerational transfers. Grossman and Helpman's (1998) model does not feature economic choices.

<sup>7</sup>Tabellini (2000) considers an economy with heterogeneous agents within cohorts as well as weak, bidirectional intergenerational altruism. In his model, a decrease in population growth reduces the wealth of the median voter and raises the equilibrium tax rate. Tabellini's (2000) model does not feature production.

these general equilibrium implications typically work in the direction of reducing equilibrium tax rates (Cooley and Soares, 1999; Forni, 2005), which is counterfactual in light of the historical experience.<sup>8</sup> In median-voter setups incorporating a *large* number of generations, in contrast, demographic ageing of plausible magnitude does change the identity of the median voter, introducing a mechanism to increase equilibrium tax rates (Galasso, 1999). However, these setups become analytically intractable and render the analysis of multi-dimensional policy spaces problematic.

The remainder of the paper is structured as follows: Sections 2 and 3 present the model and characterize politico-economic equilibrium. For ease of exposition, we first consider a simplified setup with inelastic labor supply and a single policy instrument (Section 2) before solving the full-fledged model in Section 3. Section 4 analyzes the Ramsey benchmark, and Section 5 contains the quantitative results. Section 6 concludes.

## 2 The Model

We consider an overlapping generations economy inhabited by cohorts of representative agents. Households live for two periods, as workers when young and retirees when old. Workers in period  $t$  inelastically supply labor at wage  $w_t$  and pay a labor income tax levied at rate  $\tau_t$ . (Later, we will relax the assumption of inelastic labor supply.) Disposable income is allocated to consumption,  $c_{1,t}$ , and savings,  $s_t$ ; the latter yields a gross rate of return,  $R_{t+1}$ . When retired, old households consume  $c_{2,t+1}$ , equal to the gross return on savings,  $s_t R_{t+1}$ , plus pension benefits,  $b_{t+1}$ . The population grows stochastically at the rate  $\nu_t - 1$ , such that the ratio of young to old households is given by  $\nu_t > 0$ .

Output is produced using an aggregate production function with constant returns to scale. Output per worker in period  $t$  positively depends on the capital-labor ratio which, in turn, is given by the ratio of per-capita savings of the cohort born in  $t - 1$ ,  $s_{t-1}$ , and the gross rate of growth of cohort  $t$ ,  $\nu_t$ . Factor markets are competitive and factor prices thus correspond to marginal products. The wage and the gross interest rate are given by  $w_t = w(s_{t-1}/\nu_t)$  and  $R_t = R(s_{t-1}/\nu_t)$ , strictly increasing and decreasing in the capital-labor ratio, respectively. Conditional on prices and policies (which may depend on the realization of  $\nu_t$ ), the indirect utility function of a worker in cohort  $t$  is given by

$$U_t = \max_{s_t} u(c_{1,t}) + \beta E_t[u(c_{2,t+1})], \quad (1)$$

subject to the budget constraints described above. The felicity function  $u(\cdot)$  is continuously differentiable, strictly increasing and concave, and satisfies  $\lim_{c \rightarrow 0} u'(c) = \infty$ . The discount factor  $\beta \in (0, 1)$ , and  $E_t[\cdot]$  denotes the expectation operator conditional on information available at time  $t$ .

The government sector consists of a social security administration running a pay-as-you-go system:<sup>9</sup> Old-age pensions are financed out of payroll taxes paid by workers such that the budget constraint of the social security administration reads

$$b_t = \tau_t \nu_t w_t. \quad (2)$$

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<sup>8</sup>Median-voter models featuring a few overlapping generations also imply, counterfactually, drastic policy reversals once the identity of the median voter changes.

<sup>9</sup>Introducing a fully funded component of social security is inconsequential, as long as the government does not force households to save more than they would voluntarily save, and investment opportunities are the same for households and the social security administration.

(The balanced budget assumption is convenient, but not important. We discuss the reasons in the Conclusions.) For the time being, we assume that the sole policy instrument of the social security administration is the payroll tax rate,  $\tau_t$ . This tax rate is determined in the political process (described in more detail below), subject to a non-negativity constraint,  $\tau_t \geq 0$ .

The timing of events is as follows: At the beginning of period  $t$ , after the realization of  $\nu_t$  has been observed, a political candidate is democratically elected to choose the contemporaneous tax rate. When deciding which candidate to support, voters anticipate how each candidate's policy platform would affect subsequent economic and political decisions. The wage rate and the return on the predetermined savings of retirees, together with the tax rate implemented by the winning candidate, determine the consumption of retirees and the disposable income of workers. Workers then turn to their role as consumers and choose how much to save.

When supporting a candidate's policy platform (as voters) and choosing savings (as consumers), workers form expectations about future benefits,  $b_{t+1}$ . In a Markovian equilibrium, these benefits depend on a set of fundamental state variables,  $\mathcal{S}_{t+1}$ :  $b_{t+1} = \nu_{t+1}w(s_t/\nu_{t+1})\tau(\mathcal{S}_{t+1})$ . Clearly,  $s_t$  and  $\nu_{t+1}$  are elements of  $\mathcal{S}_{t+1}$ , since both variables affect future wages and returns and therefore, incomes of next period's voters, and since  $\nu_{t+1}$  affects next period's benefits. Having said this, we conjecture  $s_t$  and  $\nu_{t+1}$  to be sufficient for  $\mathcal{S}_{t+1}$ , i.e.,  $\tau(\mathcal{S}_{t+1}) = \tau(s_t, \nu_{t+1})$ . We will return to this point later when discussing the political institutions in place.<sup>10</sup>

To characterize the politico-economic equilibrium, we proceed by backward induction. We start by analyzing the economic choices subject to given prices and policies, and then consider the preferences over policies (and thus, prices) and their aggregation in the political process.

## 2.1 Choice of Individual Savings

The optimal savings decision of a worker in cohort  $t$  is characterized by the Euler equation

$$u'(c_{1,t}) = \beta E_t[R_{t+1}u'(c_{2,t+1})].$$

Since households are atomistic they take aggregate savings and thus, next period's return on capital as well as social security benefits as given. Households only take into account that individual savings increase financial wealth.

Conditional on  $\tau(s_t, \nu_{t+1})$ , the Euler equation maps disposable income,  $w_t(1 - \tau_t)$ , and aggregate savings into the optimal savings of an individual household. We denote this mapping by the function

$$S^i(w_t(1 - \tau_t); s_t, \tau(s_t, \nu_{t+1})).$$

An equilibrium *aggregate* savings function,  $S(w_t(1 - \tau_t); \tau(\cdot))$ , is defined as a fixed point of the functional equation  $S(y; \tau(\cdot)) = S^i(y; S(y; \tau(\cdot)), \tau(S(y; \tau(\cdot)), \nu_{t+1})) \forall y \geq 0$ .

## 2.2 Choice of Tax Rate

To characterize society's choice of program size, we first consider the welfare implications of the choice of  $\tau_t$  for workers and retirees. These welfare implications induce group-specific preferences over policies. In a second step, we consider the aggregation of these preferences in the political process.

Retirees prefer as high a tax rate as possible. This follows directly from the fact that  $b_t$  increases in  $\tau_t$ , while  $s_{t-1}R_t$  is independent of  $\tau_t$  and the tax bill for funding the benefits is

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<sup>10</sup>We assume  $\tau(\cdot)$  to be single-valued, as is the case in the limit of the finite-horizon economy.

solely shouldered by workers. For a retiree in period  $t$ , the welfare effect of a marginal increase in the tax rate is given by

$$u'(c_{2,t})w_t\nu_t. \quad (3)$$

For workers, a change in the tax rate gives rise to more complex welfare implications. Differentiating  $U_t$  with respect to  $\tau_t$  yields

$$-u'(c_{1,t})w_t + \beta E_t \underbrace{\left[ u'(c_{2,t+1}) \left( s_t \frac{dR(s_t/\nu_{t+1})}{ds_t} + \nu_{t+1} \frac{d(w(s_t/\nu_{t+1})\tau(s_t, \nu_{t+1}))}{ds_t} \right) \right]}_{\mathcal{B}_t} \frac{dS(\cdot)}{d\tau_t}. \quad (4)$$

(An envelope argument implies that the indirect welfare effects through changes in the worker's savings cancel.) The first, negative term reflects the cost of higher tax payments. The second, “general-equilibrium” term  $\mathcal{B}_t$  reflects the welfare implications due to induced changes in aggregate savings: By shifting disposable income from workers (with a positive marginal propensity to save) to retirees (with a propensity equal to zero), an increase in the tax rate reduces aggregate savings. This increases next period's expected return on savings, with a positive welfare effect for workers, and alters expected social security benefits, with welfare effects whose sign is ambiguous in general. The total general equilibrium effect  $\mathcal{B}_t$  thus is positive, as long as the anticipated political choice of benefits does not strongly increase with aggregate savings.<sup>11</sup>

The extent to which the group-specific welfare implications of the choice of tax rate govern the equilibrium policy choice depends on the political institutions in place to aggregate voters' preferences. Previous literature has generally adopted the median-voter assumption according to which, in our context, equilibrium outcomes exclusively reflect the interests of workers (as long as  $\nu_t > 1$ ). This assumption would imply that the welfare effect for workers as given in (4) must exceed zero over some range to sustain positive taxes in equilibrium. But this condition can only be satisfied if either the interest elasticity with respect to savings is very high, or the effect of higher taxes on the subsequent political choice of benefits is strong and positive. Several authors have assumed the latter, often introducing artificial state variables that allow to sustain trigger strategies and thus, arbitrarily large elasticities of future benefits with respect to current choices of tax rate.<sup>12</sup>

Median-voter models with homogeneous agents in each cohort require a large number of overlapping generations in order to being able to capture not only the general equilibrium implications of demographic change, but also the consequences of the associated shift in political power. As discussed in the introduction, models with such a large number of overlapping generations typically have shortfalls along other dimensions. We therefore opt for a different modeling strategy, replacing the median-voter setup by the probabilistic-voting assumption. Probabilistic-voting models acknowledge the fact that voters support a candidate not only for her policy platform, but also for other characteristics like “ideology” that are orthogonal to the fundamental policy dimensions of interest.<sup>13</sup> The valuation of these other characteristics is assumed to differ across voters (even if they agree about the preferred policy platform) and to be

<sup>11</sup>Due to constant returns to scale ( $s_t dR_{t+1}/ds_t + \nu_{t+1} dw_{t+1}/ds_t = 0$ ),  $\mathcal{B}_t$  can more compactly be expressed as

$$\mathcal{B}_t = \beta E_t \left[ u'(c_{2,t+1}) \left( \nu_{t+1} (\tau_{t+1} - 1) \frac{dw(s_t/\nu_{t+1})}{ds_t} + \nu_{t+1} w_{t+1} \frac{d\tau(s_t, \nu_{t+1})}{ds_t} \right) \right] \frac{dS(\cdot)}{d\tau_t}.$$

<sup>12</sup>See, for example, Cooley and Soares (1999), Boldrin and Rustichini (2000), or Rangel (2003). Forni (2005), in contrast, restricts the set of state variables to the fundamental state variable. He finds that, over some range, multiple policy functions can exist, whose steep negative slope is sufficient to generate the desired effect.

<sup>13</sup>These characteristics are permanent and cannot be credibly altered in the course of electoral competition.

subject to random aggregate shocks, realized after candidates have chosen their platforms. This renders the probability of winning a voter’s support a continuous function of the candidate’s policy platform, in contrast to the median-voter setup, and allows to analyze multi-dimensional policy spaces (as is done in Section 3).

In the probabilistic-voting Nash equilibrium, two candidates maximizing their respective vote shares both propose the same policy platform. This platform maximizes a convex combination of the welfare of *all* voters, with the weights reflecting the group size and the sensitivity of voting behavior to policy changes.<sup>14</sup> Groups that care a lot about policy relative to the candidate’s other characteristics have more political influence since they are more likely to alter their support in response to small changes in the proposed platform. In equilibrium, these groups of “swing voters” thus tilt policy in their own favor. If all voters are equally responsive to changes in the policy platform, electoral competition implements the utilitarian optimum with respect to voters.

In the context of our model, the probabilistic-voting assumption implies that the welfare of retirees receives some weight in the objective function maximized by political candidates, even if the median voter is a worker. This implication is very realistic. Indeed, it is frequently argued in the context of social security that retirees exert *stronger* political influence per capita than workers because intergenerational transfers are more of a salient issue for old than for young voters (see, for example, Dixit and Londregan (1996, p. 1144) and Grossman and Helpman (1998, p. 1309)). While this is reassuring, our results do *not* require retirees to exert disproportionate influence.

Formalizing the foregoing discussion, the policy platform proposed by the political candidates solves the program  $\max_{\tau_t \geq 0} W(s_{t-1}, \nu_t, \tau_t; \tau(\cdot))$  where

$$W(s_{t-1}, \nu_t, \tau_t; \tau(\cdot)) \equiv \omega u(c_{2,t}) + \nu_t (u(c_{1,t}) + \beta E_t[u(c_{2,t+1})])$$

$$\text{subject to } \begin{cases} s_{t-1}, \nu_t \text{ given,} \\ s_t = S(w_t(1 - \tau_t); \tau(\cdot)), \\ \tau_{t+1} = \tau(s_t, \nu_{t+1}), \\ \text{household budget constraints.} \end{cases}$$

Here, the per-capita political weight of retirees relative to workers,  $\omega$ , reflects the sensitivity of the voting behavior of both groups; the household budget constraint incorporates the benefit, wage, and return functions; and next period’s policy choice as a function of the state is taken as given, reflecting our assumption of Markov equilibrium. An interior solution to this program is characterized by the condition that the weighted sum of (3) and (4) (where the weights are given by  $\omega$  and  $\nu_t$ , respectively),

$$w_t (\omega u'(c_{2,t}) \nu_t - \nu_t u'(c_{1,t})) + \nu_t \mathcal{B}_t, \tag{5}$$

be equal to zero. Expression (5) features two components: The direct benefit (as perceived by the political candidate) of redistributing resources from young to old households; and the indirect benefit due to the induced general equilibrium repercussions affecting young voters. Note that our earlier assumption according to which  $s_t$  and  $\nu_{t+1}$  are the only elements in  $\mathcal{S}_{t+1}$ , indeed is consistent.

In a rational expectations equilibrium, the anticipated policy function coincides with the optimal one. A rational expectations equilibrium is thus given by a fixed point  $\tau(\cdot)$  of the functional equation  $\tau(s_{t-1}, \nu_t) = \arg \max_{\tau_t \geq 0} W(s_{t-1}, \nu_t, \tau_t; \tau(\cdot)) \forall s_{t-1}, \nu_t \geq 0$ .

<sup>14</sup>See Lindbeck and Weibull (1987) and Persson and Tabellini (2000) for discussions of probabilistic voting.

## 2.3 Equilibrium

To derive closed-form solutions, we impose the following functional form assumptions:<sup>15</sup>

**Assumption 1.** Preferences are logarithmic:  $u(c) \equiv \ln(c)$ . The production function is of the Cobb-Douglas type:  $w(s/\nu) \equiv A(1 - \alpha)(s/\nu)^\alpha$ ,  $R(s/\nu) \equiv A\alpha(s/\nu)^{\alpha-1}$ ,  $A > 0, 0 < \alpha < 1$ .

Here,  $\alpha$  denotes the capital share,  $s/\nu$  the capital-labor ratio, and  $A$  the level of productivity. Under Assumption 1, a worker's savings  $s_t^i$  is characterized by the condition

$$\frac{1}{w_t(1 - \tau_t) - s_t^i} = \beta \mathbb{E}_t \left[ \frac{\alpha}{\alpha s_t^i + (1 - \alpha)\tau_{t+1}s_t} \right].$$

(We use the fact that  $\nu_{t+1}w_{t+1} = s_t R_{t+1}(1 - \alpha)/\alpha$ .) Imposing equilibrium,  $s_t^i = s_t$ , and letting  $e_t \equiv \mathbb{E}_t \left[ \frac{\alpha}{\alpha + (1 - \alpha)\tau_{t+1}} \right]$  leads to

$$s_t = A(1 - \alpha) \left( \frac{s_{t-1}}{\nu_t} \right)^\alpha (1 - \tau_t) \frac{\beta e_t}{1 + \beta e_t} \equiv \left( \frac{s_{t-1}}{\nu_t} \right)^\alpha \cdot z(\tau_t, e_t), \quad (6)$$

implying

$$\begin{aligned} c_{1,t} &= A(1 - \alpha) \left( \frac{s_{t-1}}{\nu_t} \right)^\alpha (1 - \tau_t) \left( \frac{1}{1 + \beta e_t} \right) \equiv \left( \frac{s_{t-1}}{\nu_t} \right)^\alpha \cdot \gamma(\tau_t, e_t), \\ c_{2,t} &= A s_{t-1}^\alpha \nu_t^{1-\alpha} (\alpha + (1 - \alpha)\tau_t) \equiv s_{t-1}^\alpha \nu_t^{1-\alpha} \cdot \delta(\tau_t). \end{aligned}$$

If current and future tax rates were chosen under commitment, these equations would represent the equilibrium aggregate savings and consumption functions. In politico-economic equilibrium, in contrast, these equations represent the equilibrium aggregate savings and consumption functions *only* under the condition that future tax rates are independent of inherited savings,  $\tau(s_t, \nu_{t+1}) = \tau(\nu_{t+1})$ . We conjecture, and later verify, that this is indeed the case.

Omitting terms independent of the policy choice  $\tau_t$ , the political objective function  $W(\cdot)$  then reduces to

$$W(s_{t-1}, \nu_t, \tau_t; \tau(\nu_{t+1})) \simeq \omega \ln[\delta(\tau_t)] + \nu_t \mathbb{E}_t [\ln[\gamma(\tau_t, e_t)] + \beta \ln[z(\tau_t, e_t)^\alpha]]$$

and the marginal effect of a change of tax rate on the political objective function is given by<sup>16</sup>

$$\frac{dW(\cdot)}{d\tau_t} = \frac{\omega(1 - \alpha)}{\alpha + (1 - \alpha)\tau_t} - \nu_t \frac{1 + \alpha\beta}{1 - \tau_t}.$$

<sup>15</sup>For a more general discussion, see Gonzalez-Eiras and Niepelt (2005).

<sup>16</sup>Parallel to (5), this marginal effect can be decomposed in two components, one reflecting the direct distributive effect, the other the general-equilibrium implications for workers:

$$\frac{dW(\cdot)}{d\tau_t} = \frac{\omega(1 - \alpha)}{\alpha + (1 - \alpha)\tau_t} - \nu_t \frac{1 + \beta e_t}{1 - \tau_t} + \nu_t \mathcal{B}_t,$$

where, under the maintained assumptions,

$$\mathcal{B}_t = \frac{\alpha(1 - \alpha)\beta}{1 - \tau_t} \mathbb{E}_t \left[ \frac{1 - \tau_{t+1}}{\alpha + (1 - \alpha)\tau_{t+1}} \right].$$

Note that, as a consequence of the additive separability of  $\ln[\gamma(\tau, e)]$  and  $\ln[z(\tau, e)]$ , the derivative  $dW(\cdot)/d\tau_t$  is independent of anticipated future tax rates,  $e_t$ , although  $e_t$  affects workers' savings and consumption choices.

The tax rate maximizing  $W(\cdot)$  therefore satisfies

$$\tau^{\mathcal{W}}(\nu_t) \equiv \frac{\omega(1-\alpha) - \nu_t(1+\alpha\beta)\alpha}{\omega(1-\alpha) + \nu_t(1+\alpha\beta)(1-\alpha)}.$$

If this tax rate violates the non-negativity constraint, then the constrained maximum of  $W(\cdot)$  is attained for  $\tau_t = 0$  since the objective function is strictly concave. The equilibrium tax rate therefore equals  $\tau_t^{\mathcal{W}} \equiv \max(\tau^{\mathcal{W}}(\nu_t), 0)$ , and the equilibrium policy function indeed is independent of  $s_t$ , verifying the conjecture.

This equilibrium policy function,  $\tau(s_{t-1}, \nu_t) = \tau_t^{\mathcal{W}}$ , is the *unique* equilibrium policy function if we restrict attention to (the limit of) finite-horizon economies.<sup>17</sup> To see this, consider the final period,  $T$  say. The consumption of old and young households in this period is given by

$$\begin{aligned} c_{1,T} &= w_T(1-\tau_T) = A(1-\alpha)(s_{T-1}/\nu_T)^\alpha(1-\tau_T), \\ c_{2,T} &= As_{T-1}^\alpha \nu_T^{1-\alpha}(\alpha + (1-\alpha)\tau_T), \end{aligned}$$

respectively. If  $c_{2,T}/c_{1,T}$  exceeds  $\omega$  in the absence of transfers, then the equilibrium tax rate is in a corner,  $\tau_T = 0$ . Otherwise, the tax rate is set to achieve

$$\frac{c_{2,T}}{c_{1,T}} = \omega \Rightarrow \tau_T = \frac{\omega(1-\alpha) - \nu_T\alpha}{\omega(1-\alpha) + \nu_T(1-\alpha)}.$$

It follows that the policy function in the final period satisfies  $\tau_T(s_{T-1}, \nu_T) = \max\left(\frac{\omega(1-\alpha) - \nu_T\alpha}{\omega(1-\alpha) + \nu_T(1-\alpha)}, 0\right)$  which is independent of savings in period  $T-1$ . But if the policy function in period  $T$  is independent of  $s_{T-1}$ , then the policy function in period  $T-1$  is independent of  $s_{T-2}$  as well, as shown before. In particular, this latter policy function is given by  $\tau_{T-1}^{\mathcal{W}}$ . The same logic applies in all preceding periods.

It is straightforward to show that  $\tau_t^{\mathcal{W}}$  increases in  $\omega$  and decreases in  $\alpha$ ,  $\beta$  and  $\nu_t$ . Intuitively, the political benefit of taxes increases with the relative weight the political process attaches to retirees ( $\omega$ ), while the political cost of taxes and depressed capital accumulation increases with the weight attached to the future ( $\beta$ ) and to those who care about the future ( $\nu_t$ ). At the same time, an increase in the capital share ( $\alpha$ ) lowers the marginal benefit of transfers for retirees and increases the marginal cost of taxation for workers, thereby reducing the equilibrium tax rate.

In a politico-economic equilibrium with positive tax rates, pensions as a share of GDP equal

$$\frac{w_t \tau_t^{\mathcal{W}} \nu_t}{w_t \nu_t + R_t s_{t-1}} = (1-\alpha)\tau_t^{\mathcal{W}}.$$

The model therefore predicts that the pension share is decreasing in the number of workers relative to the number of retirees. This result finds support in the data. Analyzing the rise of the welfare state in a sample of 30 countries during the 1880–1930 period, Lindert (1994) finds a significant positive relationship between the pension share and the share of the elderly. A sample of OECD countries during the 1960s and 1970s (Lindert, 1996) and a panel of 60 countries during the 1960–1998 period (Persson and Tabellini, 2003) produce similar findings.

<sup>17</sup>In the last period, the policy function is different, but also unique.

Finally, Boldrin, De Nardi and Jones (2005) report cross-section and time-series evidence of a negative relationship between fertility and social security transfers (104 countries in 1997, and post-war data for the U.S. and European countries, respectively). Boldrin et al. (2005) interpret this evidence as support for their view that increased social security transfers caused a fall in fertility. Our model suggests a mechanism with reversed causality.

The model also replicates the apparently non-monotone empirical relationship between the share of elderly in the population and public pension payments *per retiree*: For the 1880–1930 period, Lindert (1994) estimates an elasticity of the pension share in GDP with respect to the share of elderly that is larger than unity. The OECD data suggest a hump-shaped relationship between the share of the elderly and public pension payments per retiree (Lindert, 1996), and Mulligan and Sala-i-Martin (2004) conclude that there is no clear relationship.<sup>18</sup> The model can account for these observations, because it features an inverse-U shaped relationship between the share of elderly and public pension payments per retiree ( $w_t \tau_t^\mathcal{W} \nu_t$ ), and because population growth rates have on average declined over time. Our model therefore predicts population ageing to lead to a *rise* in the pension share of GDP but eventually, a *decline* of social security benefits per retiree.

Another prediction of the model relates to the extent of “risk sharing” induced by the political choice of tax rates.<sup>19</sup> Since aggregate labor and capital income shares are constant in the model, the randomness of population growth generates old-age consumption risk for a newborn worker. The endogenous response of taxes to demographic developments reduces this consumption risk. Abstracting from terms known as of time  $t$ , old-age consumption is given by

$$\ln[c_{2,t+1}] \simeq (1 - \alpha) \ln[\nu_{t+1}] - \begin{cases} 0 & \text{if } \tau_{t+1} = 0, \\ \ln[\omega + \nu_{t+1}(1 + \alpha\beta)] & \text{if } \tau_{t+1} > 0. \end{cases}$$

(The extra term in the second line reflects the dependence of  $\delta(\tau^\mathcal{W}(\nu_{t+1}))$  on population growth. If  $\tau_{t+1}$  does not vary with the population growth rate, then the same relationship applies as when  $\tau_{t+1} = 0$ .) Differentiating with respect to  $\nu_{t+1}$  yields  $\frac{d \ln[c_{2,t+1}]}{d \nu_{t+1}} = \frac{1 - \alpha}{\nu_{t+1}}$  if the tax rate is not contingent on population growth (e.g., equal to zero), and

$$\frac{d \ln[c_{2,t+1}]}{d \nu_{t+1}} = \frac{1 - \alpha}{\nu_{t+1}} - \frac{1 + \alpha\beta}{\omega + \nu_{t+1}(1 + \alpha\beta)} = \frac{(1 - \alpha)\tau_{t+1}}{\nu_{t+1}}$$

in politico-economic equilibrium with strictly positive tax rates. Since  $\tau_{t+1} < 1$ , the endogenous choice of tax rates reduces the exposure of old-age consumption to demographic risk.

For a different perspective on this political allocation of consumption risk, consider the ratio of old- to young-age consumption in a given period,

$$\frac{c_{2,t}}{c_{1,t}} = \nu_t \frac{\delta(\tau_t)}{\gamma(\tau_t, e_t)}. \quad (7)$$

If tax rates are independent of population growth, this ratio has a unitary elasticity with respect to the contemporaneous population growth rate. In politico-economic equilibrium with strictly positive tax rates, in contrast, the ratio reduces to  $c_{2,t}/c_{1,t} = \omega(1 + \beta e_t)/(1 + \alpha\beta)$  (which further simplifies to  $\omega + \alpha\beta\nu_{t+1}$  in the absence of demographic risk). That is, the consumption ratio is

<sup>18</sup>Based on data for the United States and 12 European countries over the period 1965–92, Razin, Sadka and Swagel (2002) argue that the dependency ratio is negatively related to per capita transfers.

<sup>19</sup>Note that, due to demographic shocks, risk is “shared” among state-contingent sets of households.

independent of  $\nu_t$ , but has a unitary elasticity with respect to  $(1 + \beta e_t)$ ; with i.i.d. demographic shocks, the consumption ratio is constant.

We summarize these findings in the following Proposition:

**Proposition 1.** Consider the politico-economic equilibrium under Assumption 1.

(i) There exists an equilibrium with policy functions  $\tau(\cdot)$  independent of  $s$ . These policy functions are the unique equilibrium policy functions in finite-horizon economies.

(ii) The tax functions in that equilibrium are given by  $\tau(s_{t-1}, \nu_t) = \tau_t^{\mathcal{W}}$ . These policy functions increase in  $\omega$  and decrease in  $\alpha$ ,  $\beta$  and  $\nu_t$ . They imply a pension share of GDP that increases in the fraction of retirees in the population; social security benefits per retiree that eventually decrease in this fraction; and reduced old-age consumption risk, compared to a situation with exogenous tax rates.

### 3 Elastic Labor Supply and Multiple Policy Instruments

The results in the previous section followed under the assumption that labor is supplied inelastically, and that policymakers have access to a single policy instrument (transfers from workers to retirees). Both these assumptions are not fully satisfactory. First, labor supply is likely to respond to demographic developments both directly and, through induced policy effects, indirectly. Modeling labor supply therefore is important in its own right, but also because deadweight losses might dampen the upward pressure on social security taxes. Second, the opposition of young voters against social security taxes might be stifled if additional policy instruments are available to reap the general equilibrium effects  $\mathcal{B}_t$ . To assess the robustness of our findings, it is therefore important to model the *set* of policy instruments chosen by the political process, and how this set of instruments develops during the demographic transition.

To address these concerns, we extend the model of the previous section. We introduce an endogenous labor-leisure choice as well as an additional tax on labor income, levied at rate  $\theta_t$ , whose revenue is reimbursed to young households. In the previous setup with inelastic labor supply, such a tax-cum-reimbursement would have had no effects. With elastic labor supply, in contrast, the new instrument makes it possible to monopolize labor supply and thus, depress savings of the young *without* having to transfer resources to the old. We interpret this tax as distortive labor market regulation.

We assume for tractability that a worker's felicity function is separable in consumption and leisure,  $x_t$ . The indirect utility function defined in (1) is thus replaced by

$$U_t = \max_{s_t, x_t} u(c_{1,t}) + v(x_t) + \beta E_t[u(c_{2,t+1})] \text{ s.t. household budget set,}$$

where  $v(\cdot)$  is continuously differentiable, strictly increasing and concave, and satisfies  $\lim_{x \rightarrow 0} v'(x) = \infty$ . A young household's time endowment is normalized to one. While the per-worker tax revenue  $w_t(1 - x_t)\tau_t$  continues to fund social security, the additional tax revenue  $w_t(1 - x_t)\theta_t$  funds a lump-sum transfer to workers. The budget constraint of a worker thus reads

$$w_t(1 - x_t)(1 - \tau_t - \theta_t) + T_t = c_{1,t} + s_t,$$

where, in equilibrium,  $T_t = w_t(1 - x_t)\theta_t$ . Second-period consumption is still given by  $c_{2,t+1} = s_t R_{t+1} + b_{t+1}$ , where  $b_{t+1} = \nu_{t+1} w_{t+1} \tau_{t+1} (1 - x_{t+1})$ .

Savings and labor supply of a worker are characterized by the first-order conditions

$$\begin{aligned} u'(c_{1,t}) &= \beta E_t[u'(c_{2,t+1})R_{t+1}], \\ u'(c_{1,t})w_t(1 - \tau_t - \theta_t) &= v'(x_t), \end{aligned}$$

subject to the budget set described earlier. Conditional on the anticipated values for taxes and aggregate leisure,  $\tau(s_t, \nu_{t+1})$ ,  $\theta(s_t, \nu_{t+1})$ , and  $\tilde{X}(s_t, \nu_{t+1})$ , respectively, the household's first-order conditions and budget constraint map  $(s_{t-1}, \nu_t)$ , aggregate savings and leisure, as well as the contemporaneous tax rates into the leisure and savings choice of a worker,  $X^i(\cdot)$  and  $S^i(\cdot)$ , respectively. Equilibrium *aggregate* savings and leisure functions,  $S(\cdot)$  and  $X(\cdot)$ , respectively, are defined as fixed points of the functional equations

$$\begin{aligned} S(s_{t-1}, \nu_t, \tau_t, \theta_t; \tau(\cdot), \theta(\cdot)) &= S^i(s_{t-1}, \nu_t, \tau_t, \theta_t; X(\cdot), S(\cdot), \tau(S(\cdot), \nu_{t+1}), \theta(S(\cdot), \nu_{t+1}), \tilde{X}(S(\cdot), \nu_{t+1})), \\ X(s_{t-1}, \nu_t, \tau_t, \theta_t; \tau(\cdot), \theta(\cdot)) &= X^i(s_{t-1}, \nu_t, \tau_t, \theta_t; X(\cdot), S(\cdot), \tau(S(\cdot), \nu_{t+1}), \theta(S(\cdot), \nu_{t+1}), \tilde{X}(S(\cdot), \nu_{t+1})), \\ &\forall s_{t-1}, \nu_t \geq 0, 0 \leq \tau_t, \theta_t, \tau_t + \theta_t \leq 1. \end{aligned}$$

The modified program of the political candidates reads

$$\begin{aligned} \max_{\tau_t, \theta_t \geq 0} W^\theta(s_{t-1}, \nu_t, \tau_t, \theta_t; \tau(\cdot), \theta(\cdot)), \\ W^\theta(s_{t-1}, \nu_t, \tau_t, \theta_t; \tau(\cdot), \theta(\cdot)) &\equiv \omega u(c_{2,t}) + \nu_t(u(c_{1,t}) + v(x_t) + \beta E_t[u(c_{2,t+1})]) \\ &\text{subject to } \begin{cases} s_{t-1}, \nu_t \text{ given,} \\ s_t = S(s_{t-1}, \nu_t, \tau_t, \theta_t; \tau(\cdot), \theta(\cdot)), \\ x_t = X(s_{t-1}, \nu_t, \tau_t, \theta_t; \tau(\cdot), \theta(\cdot)), \\ x_{t+1} = \tilde{X}(s_t, \nu_{t+1}), \\ \tau_{t+1} = \tau(s_t, \nu_{t+1}), \\ \theta_{t+1} = \theta(s_t, \nu_{t+1}), \\ \text{household budget constraints.} \end{cases} \end{aligned}$$

In a rational expectations equilibrium, the anticipated policy functions coincide with the optimal ones. Moreover,  $X(\cdot)$  is consistent with  $\tilde{X}(\cdot)$  if evaluated at the equilibrium policy functions.

To characterize the politico-economic equilibrium in closed form, we impose the same type of functional form assumptions as before:

**Assumption 2.** Preferences over consumption are logarithmic:  $u(c) \equiv \ln(c)$ . The production function is of the Cobb-Douglas type:  $w(s_{t-1}, \nu_t, x_t) \equiv A(1-\alpha)(s_{t-1}/(\nu_t(1-x_t)))^\alpha$ ,  $R(s_{t-1}, \nu_t, x_t) \equiv A\alpha(s_{t-1}/(\nu_t(1-x_t)))^{\alpha-1}$ .

Under Assumption 2, equilibrium savings and consumption choices are given by

$$\begin{aligned} s_t &= \left(\frac{s_{t-1}}{\nu_t}\right)^\alpha (1-x_t)^{1-\alpha} \cdot z(\tau_t, e_t), \\ c_{1,t} &= \left(\frac{s_{t-1}}{\nu_t}\right)^\alpha (1-x_t)^{1-\alpha} \cdot \gamma(\tau_t, e_t), \\ c_{2,t} &= s_{t-1}^\alpha \nu_t^{1-\alpha} (1-x_t)^{1-\alpha} \cdot \delta(\tau_t), \end{aligned}$$

where the functions  $z(\cdot)$ ,  $\gamma(\cdot)$ , and  $\delta(\cdot)$  have been defined earlier. The worker's static optimality condition yields

$$v'(x_t) = \frac{(1 - \tau_t - \theta_t)(1 + \beta e_t)}{(1 - x_t)(1 - \tau_t)}$$

and thus, an expression for leisure as a function of  $\tau_t$ ,  $\theta_t$ , and  $e_t$ , but not (directly) of  $s_{t-1}$ :

$$x_t = x(\tau_t, \theta_t, e_t). \quad (8)$$

In parallel with the approach pursued previously, we conjecture that the equilibrium policy functions  $\tau(\cdot)$  and  $\theta(\cdot)$  are independent of savings, such that  $\tau_{t+1} = \tau(\nu_{t+1})$ ,  $\theta_{t+1} = \theta(\nu_{t+1})$ , and  $de_t/ds_t = 0$ . In this case, future labor supply is unaffected by savings as well, and the objective function  $W^\theta(\cdot)$  can be expressed as

$$\begin{aligned} & W^\theta(s_{t-1}, \nu_t, \tau_t, \theta_t; \tau(\nu_{t+1}), \theta(\nu_{t+1})) \\ &= W(s_{t-1}, \nu_t, \tau_t; \tau(\nu_{t+1})) + g(x_t, \nu_t) + \text{terms unaffected by } \tau_t, \theta_t, \text{ subject to (8),} \end{aligned}$$

where the function  $W(\cdot)$  has been defined earlier and  $g(x, \nu) \equiv \ln[1-x](1-\alpha)(\omega + \nu + \alpha\beta\nu) + \nu v(x)$ . An equilibrium policy platform satisfies

$$\begin{aligned} \frac{\partial W(\cdot)}{\partial \tau_t} + \frac{\partial g(x_t, \nu_t)}{\partial x_t} \frac{\partial x_t}{\partial \tau_t} &\leq 0, \\ \frac{\partial g(x_t, \nu_t)}{\partial x_t} \frac{\partial x_t}{\partial \theta_t} &\leq 0, \end{aligned}$$

with equalities if the solution is interior.

If the optimal tax rate  $\theta_t > 0$ , then the equilibrium features the *same*  $\tau_t$  as in the main model,  $\tau_t^{\mathcal{W}}$ , since  $\partial x_t/\partial \theta_t > 0$  implies  $\partial g(x_t, \nu_t)/\partial x_t = 0$ . In this case, the optimal  $\theta_t$  is pinned down by the condition  $\partial g(x_t, \nu_t)/\partial x_t = 0$  with  $x_t$  evaluated at  $\tau_t^{\mathcal{W}}$  and  $e_t$ , the latter being a function of the potential realizations of  $\tau_{t+1}^{\mathcal{W}}$ . An increase in  $\nu_t$  or an expected increase in  $\nu_{t+1}$  then raise  $\theta_t$ . Moreover, if both tax rates are strictly positive, then  $\tau_t + \theta_t = \frac{\alpha\beta E_t[\nu_{t+1}]}{\alpha\beta E_t[\nu_{t+1}] + \omega}$ , implying that anticipated population growth increases the total tax burden.<sup>20</sup> Alternatively, if the optimal tax rate  $\theta_t = 0$ , then  $\frac{\partial x(\tau_t, 0, e_t)}{\partial \tau_t} = 0$ . The first-order condition with respect to  $\tau_t$  then implies once more the *same* choice of social security tax rate as in the basic model. Summing up, the equilibrium policy function for the social security tax rate is the same as in the basic model, and the equilibrium policy function for the purely distorting tax is independent of savings, as conjectured.

The model has interesting implications for labor supply. From the worker's static optimality condition,  $v'(x_t)(1-x_t)$  is equal to  $1+\beta e_t$  if  $\theta_t = 0$ , and equal to a function of  $\nu_t$  otherwise. Since  $v'(x_t)(1-x_t)$  decreases in  $x_t$ , equilibrium labor supply increases with future population growth but is unaffected by contemporaneous demographics if  $\theta_t = 0$ . Intuitively, with an elasticity

<sup>20</sup>The condition  $\partial g(\cdot)/\partial x_t = 0$  can be rewritten as  $\nu_t v'(x(\cdot))(1-x(\cdot)) = (1-\alpha)(\omega + \nu_t + \alpha\beta\nu_t)$ . The latter condition uniquely pins down  $x_t$  and thus, conditional on  $(\tau_t, e_t)$ , also  $\theta_t$ . Letting  $Q_t = (1-\alpha)(\omega + \nu_t + \alpha\beta\nu_t)\nu_t^{-1}$  and comparing with the worker's static optimality condition, we conclude that a strictly positive tax rate  $\theta_t$  satisfies

$$\begin{aligned} Q_t &= \frac{1 - \tau_t^{\mathcal{W}} - \theta_t}{1 - \tau_t^{\mathcal{W}}} \left( 1 + \beta E_t \left[ \frac{\alpha}{\alpha + (1-\alpha)\tau_{t+1}^{\mathcal{W}}} \right] \right), \\ \Rightarrow \theta_t &= (1 - \tau_t^{\mathcal{W}}) \left( 1 - \frac{Q_t}{1 + \beta E_t \left[ \frac{\alpha}{\alpha + (1-\alpha)\tau_{t+1}^{\mathcal{W}}} \right]} \right). \end{aligned}$$

Note that  $d\theta_t/d\tau_t^{\mathcal{W}} < 0$ ,  $d\theta_t/dQ_t < 0$ , and  $d\theta_t/de_t > 0$ . Moreover, both  $\tau_t^{\mathcal{W}}$  and  $Q_t$  weakly decrease in  $\nu_t$ . The comparative statics results then follow.

of substitution equal to one (logarithmic preferences), contemporaneous social security taxes do not affect labor supply if they are the only taxes imposed; but anticipated social security benefits (which depend on  $\nu_{t+1}$ ) reduce labor supply through a wealth effect. In contrast, if  $\theta_t > 0$ , labor supply is determined by the condition  $\partial g(x_t, \nu_t)/\partial x_t = 0$  and thus, decreases in  $\nu_t$  but is unaffected by  $\nu_{t+1}$ . Intuitively, if  $\theta_t > 0$ , labor supply is under the direct control of policy makers in the current period and therefore reflects the political weight of young voters ( $\nu_t$ ); it does not reflect future population growth because (future) tax rates do not affect the cost and benefits of increased labor supply.

Combined, these findings imply that with strictly positive tax rates  $\tau$  and  $\theta$ , falling population growth rates are accompanied by *rising* social security taxes and *rising* labor supply. This result is surprising at first sight; it partially refutes common wisdom according to which social security tax hikes would imply large deadweight losses, rendering the option of raising social security taxes economically and politically infeasible.<sup>21</sup> More generally, the result shows that the link between social security tax rates and labor supply distortions must not be seen in isolation, but in the context of a wider set of policy instruments that also respond to demographic change.

Beyond the implications for the social security tax rate, other predictions of the basic model are robust as well. For example, the expression for the pension share of GDP is unchanged, as is the consumption ratio (7). The result that the political choice of tax rates reduces old-age consumption risk continues to hold (see Appendix A.2).

The equilibrium policy functions characterized above are the *unique* equilibrium policy functions if we restrict attention to (the limit of) finite-horizon economies.<sup>22</sup> This can again be seen by a backward induction argument. In the final period,  $T$ , the consumption of old and young households is given by

$$\begin{aligned} c_{1,T} &= w_T(1 - x_T)(1 - \tau_T) = A(1 - \alpha)(s_{T-1}/\nu_T)^\alpha(1 - x_T)^{1-\alpha}(1 - \tau_T), \\ c_{2,T} &= As_{T-1}^\alpha\nu^{1-\alpha}(1 - x_T)^{1-\alpha}(\alpha + (1 - \alpha)\tau_T), \end{aligned}$$

respectively. Logarithmic preferences over consumption as well as the worker's static optimality condition  $v'(x_T)(1 - x_T) = (1 - \tau_T - \theta_T)/(1 - \tau_T)$  then imply that the first-order conditions with respect to  $\tau_T$  and  $\theta_T$  are independent of  $s_{T-1}$ . Consequently, the policy and labor-supply functions in the final period only depend on the population growth rate, and the same holds true in all preceding periods, by the arguments given before.

We summarize these findings as follows:

**Proposition 2.** Consider the politico-economic equilibrium under Assumption 2.

(i) There exists an equilibrium with policy functions  $\tau(\cdot)$  and  $\theta(\cdot)$  independent of  $s$ . These policy functions are the unique equilibrium policy functions in finite-horizon economies.

(ii) The social security tax functions in that equilibrium are given by  $\tau(s_{t-1}, \nu_t) = \tau_t^{\mathcal{W}}$ , the same functions as in the main model.

(iii) Labor supply in that equilibrium increases in future population growth, but is unaffected by contemporaneous population growth if  $\theta_t = 0$ . Conversely, labor supply decreases in contemporaneous population growth, but is unaffected by future population growth if  $\theta_t > 0$ ; in that case, higher social security taxes go hand in hand with higher labor supply.

<sup>21</sup>See, for example, Feldstein (2005).

<sup>22</sup>In the last period, the policy functions are different, but also unique.

## 4 Ramsey Allocation

It is instructive to compare the politico-economic equilibrium with the allocation implemented by a benevolent government, subject to the same set of technological and competitive-equilibrium constraints. To simplify the derivations, we pursue a similar strategy as for the analysis of the politico-economic equilibrium, considering first the case with inelastic labor supply.

Conditional on a sequence of cohort-specific discount factors  $\{\rho_i\}$ , the program of the benevolent government with commitment—the Ramsey program—is then given by  $\max_{\{s_{t-1}, \nu_t, \{\tau_i\}_{i=t}^{\infty}\}} G(s_{t-1}, \nu_t, \{\tau_i\}_{i=t}^{\infty})$ , where

$$G(s_{t-1}, \nu_t, \{\tau_i\}_{i=t}^{\infty}) \equiv \rho_{t-1} \beta u(c_{2,t}) + \mathbb{E}_t \left[ \sum_{i=t}^{\infty} \rho_i (u(c_{1,i}) + \beta u(c_{2,i+1})) \right]$$

subject to  $\begin{cases} s_{t-1}, \nu_t \text{ given,} \\ s_j = S^j(w_j(1 - \tau_j); s_j, \tau_{j+1}), j \geq t, \\ \text{household budget constraints.} \end{cases}$

(We assume throughout that the sequence  $\{\rho_i\}$  is declining sufficiently quickly for this program to be well-defined.) In contrast to the program solved by the political candidates, the Ramsey program involves the choice of a *sequence* of state-contingent tax rates, due to the planner's ability to commit. This sequence need not be optimal ex post and thus, need not satisfy fixed-point conditions as in the politico-economic equilibrium. Moreover, the Ramsey government values the welfare of all households, not only of those currently alive and voting. In general, the Ramsey policy therefore internalizes many more general-equilibrium effects than the policy implemented in politico-economic equilibrium.<sup>23</sup>

In Appendix A.1, we discuss the Ramsey policy in the general case. Here, we directly turn to the characteristics of the Ramsey policy under Assumption 1. The Ramsey planner's objective function can then be expressed as<sup>24</sup>

$$\begin{aligned} G(\cdot) &\simeq \rho_{t-1} \beta \ln[\delta(\tau_t)] + \mathbb{E}_t \left[ \sum_{i=t}^{\infty} \rho_i (\ln[\gamma(\tau_i, e_i)] + \alpha \ln[s_{i-1}] + \beta \ln[\delta(\tau_{i+1})] + \alpha \beta \ln[s_i]) \right] \\ &\simeq \rho_{t-1} \beta \ln[\delta(\tau_t)] + \mathbb{E}_t \left[ \sum_{i=t}^{\infty} \rho_i (\ln[\gamma(\tau_i, e_i)] + \beta \ln[\delta(\tau_{i+1})] + \ln[z(\tau_i, e_i)] \Gamma_i) \right], \end{aligned} \quad (9)$$

where we define  $\Gamma_i \equiv \rho_i^{-1} [\alpha \beta \rho_i + (1 + \alpha \beta)(\alpha \rho_{i+1} + \alpha^2 \rho_{i+2} + \dots)]$ . ( $\Gamma_i$  is state contingent if the planner's marginal rate of substitution across generations varies with the state of nature realized in period  $i$ .) The effect of a marginal increase of  $\tau_t$  on the objective function then is given by

$$\mathbb{E}_t \left[ \rho_{t-1} \beta \frac{1 - \alpha}{\alpha + (1 - \alpha) \tau_t} - \frac{\rho_t}{1 - \tau_t} (1 + \Gamma_t) \right],$$

<sup>23</sup>Absent binding non-negativity constraints on tax rates and tax distortions, the Ramsey policy supports the social-planner allocation, see the discussion in Appendix A.1. In the setup with labor supply distortions analyzed below, this is no longer the case. To avoid confusion, we always refer to the benchmark as the “Ramsey equilibrium” rather than the “social-planner allocation.”

<sup>24</sup>Note that

$$\begin{aligned} \ln[s_i] &\simeq \alpha \ln[s_{i-1}] + \ln[z(\tau_i, e_i)] \\ &\simeq \alpha^{i-t+1} s_{t-1} + \alpha^{i-t} \ln[z(\tau_t, e_t)] + \alpha^{i-t-1} \ln[z(\tau_{t+1}, e_{t+1})] + \dots + \ln[z(\tau_i, e_i)], \quad i \geq t. \end{aligned}$$

and the tax rate maximizing  $G(\cdot)$  is given by

$$\tau^{\mathcal{G}}(\rho_t, \rho_{t-1}, E_t \Gamma_t) \equiv \frac{\beta \rho_{t-1}(1 - \alpha) - \rho_t(1 + E_t \Gamma_t)\alpha}{\beta \rho_{t-1}(1 - \alpha) + \rho_t(1 + E_t \Gamma_t)(1 - \alpha)}.$$

If this tax rate violates the non-negativity constraint, then the constrained maximum of  $G(\cdot)$  is attained for a tax rate of zero,<sup>25</sup> implying that the optimal tax rate is given by  $\tau_t^{\mathcal{G}} \equiv \max(\tau^{\mathcal{G}}(\rho_t, \rho_{t-1}, E_t \Gamma_t), 0)$ . The interior tax rate  $\tau^{\mathcal{G}}(\cdot)$  differs twofold from the corresponding tax rate in politico-economic equilibrium,  $\tau^{\mathcal{W}}(\cdot)$ . First, the weights  $\omega$  and  $\nu_t$  are replaced by  $\beta \rho_{t-1}$  and  $\rho_t$ , respectively. Second, the expression  $(1 + \alpha\beta)$  is replaced by the term  $(1 + E_t \Gamma_t)$ , reflecting the fact that the Ramsey government internalizes general equilibrium effects over a much longer horizon. The first of the two changes implies that the Ramsey tax rate is independent of the population growth rate unless the planner's welfare weight  $\rho_t$  is a function of  $\nu_t$ . (In politico-economic equilibrium, an interior tax rate always depends on  $\nu_t$ .)

Turning to the choice of tax rates in periods after the initial one,  $i > t$ , the effect of a marginal increase in  $\tau_i$  is given by

$$E_i \left[ \rho_{i-1} \beta \frac{1 - \alpha}{\alpha + (1 - \alpha)\tau_i} - \frac{\rho_i}{1 - \tau_i} (1 + \Gamma_i) + \rho_{i-1} \frac{\Gamma_{i-1} - \beta e_{i-1}}{e_{i-1}(1 + \beta e_{i-1})} \frac{\partial e_{i-1}}{\partial \tau_i} \right].$$

The new term on the right-hand side of this expression reflects the fact that with commitment, taxes affect workers' savings decision in the preceding period. However, this commitment effect vanishes if  $\tau_i = \tau^{\mathcal{G}}(\rho_i, \rho_{i-1}, E_i \Gamma_i)$ .<sup>26</sup> All interior Ramsey tax rates are therefore given by  $\tau^{\mathcal{G}}(\rho_i, \rho_{i-1}, E_i \Gamma_i)$ .<sup>27</sup> Whenever a non-negativity constraint is binding, the optimal tax rate in that period is in a corner.

In the special case where  $\{\rho_i\}_{i=t-1}^{\infty} = \{\omega, \beta \nu_t, 0, 0, \dots\}$ , the expression for  $E_t \Gamma_t$  collapses to  $\alpha\beta$  such that  $\tau_t^{\mathcal{G}} = \tau_t^{\mathcal{W}}$ . In another special case with geometric discounting,  $\rho_i/\rho_{t-1} = \rho^{i-t+1}$ , the expression for  $E_t \Gamma_t$  reduces to  $\alpha(\beta + \rho)/(1 - \alpha\rho)$  and the Ramsey tax rate falls short of the politico-economic equilibrium tax rate whenever  $\omega$  or  $\rho$  is sufficiently large or  $\beta \nu_t$  is sufficiently small:

$$\text{Geometric discounting : } \tau_t^{\mathcal{G}} \leq \tau_t^{\mathcal{W}} \Leftrightarrow \omega \rho \geq \beta \nu_t (1 - \alpha \rho).$$

Finally, with ‘‘dynastic’’ discounting, that is, if the planner's welfare weights reflect the discount factor of households as well as the cohort size,  $\rho_i/\rho_{i-1} = \beta \nu_i$ , and if the population growth rate

<sup>25</sup>The effect of a marginal increase of  $\tau_i$  on the objective function is strictly negative for any  $\tau_i > \tau^{\mathcal{G}}(\cdot)$ .

<sup>26</sup>This follows from

$$E_i \left[ \rho_{i-1} \frac{\Gamma_{i-1} - \beta e_{i-1}}{e_{i-1}(1 + \beta e_{i-1})} \frac{\partial e_{i-1}}{\partial \tau_i} \right] \propto E_i [\Gamma_{i-1} - \beta e_{i-1}] = \alpha\beta + \frac{\alpha \rho_i}{\rho_{i-1}} (1 + E_i \Gamma_i) - \beta \frac{\alpha}{\alpha + (1 - \alpha)\tau_i}.$$

The last expression equals zero for  $\tau_i = \tau^{\mathcal{G}}(\rho_i, \rho_{i-1}, E_i \Gamma_i)$ .

<sup>27</sup>Moreover,  $\beta e_i = E_i \Gamma_i$  in this case and an interior policy therefore also solves

$$\rho_{i-1} \beta \frac{1 - \alpha}{\alpha + (1 - \alpha)\tau_i} - \frac{\rho_i}{1 - \tau_i} (1 + \beta e_i) = 0,$$

which corresponds to the condition  $n_i = 0$  in the general case discussed in the Appendix. To see that  $\beta e_i = E_i \Gamma_i$ , suppose that  $E_i [\Gamma_{i-1} - \beta e_{i-1}] = 0$ . In an interior optimum, we then have

$$\begin{aligned} \beta e_{i-1} &\equiv \beta E_{i-1} \left[ \frac{\alpha}{\alpha + (1 - \alpha)\tau_i^{\mathcal{G}}} \right] = \beta E_{i-1} \left[ \frac{\alpha}{\alpha + (1 - \alpha)\tau^{\mathcal{G}}(\rho_i, \rho_{i-1}, E_i \Gamma_i)} \right] = \\ &= E_{i-1} \left[ \alpha\beta + \alpha \frac{\rho_i}{\rho_{i-1}} (1 + E_i \Gamma_i) \right] = E_{i-1} \Gamma_{i-1}. \end{aligned}$$

is i.i.d. with mean  $\bar{\nu} < (\alpha\beta)^{-1}$ , then  $E_t\Gamma_t = \alpha\beta\frac{1+\bar{\nu}}{1-\alpha\beta\bar{\nu}}$  and the Ramsey tax rate is smaller than the equilibrium tax rate whenever  $\omega$  is large or  $\beta\bar{\nu}$  is small:

$$\text{“Dynastic” discounting, i.i.d. } \nu : \tau_t^G \leq \tau_t^W \Leftrightarrow \omega \geq 1 - \alpha\beta\bar{\nu}.$$

With dynastic discounting, under interior tax rates, the Ramsey policy achieves  $c_{2,t} = c_{1,t}$  in all periods and states of nature.

With endogenous labor supply, under Assumption 2, the government’s objective function can be expressed as

$$G^\theta(s_{t-1}, \nu_t, \{\tau_i, \theta_i\}_{i=t}^\infty) = G(s_{t-1}, \nu_t, \{\tau_i\}_{i=t}^\infty) + E_t \sum_{i=t}^\infty h_i(x_i) \text{ subject to (8),}$$

where the function  $G(\cdot)$  is defined in (9) and  $h_i(x) \equiv \rho_{i-1}\beta(1-\alpha)\ln[1-x] + \rho_i(v(x) + (1-\alpha)(1+\Gamma_i)\ln[1-x])$ . The first-order optimality conditions characterizing the modified Ramsey policy include

$$\begin{aligned} \frac{\partial G(\cdot)}{\partial \tau_i} + E_i[h'_i(x_i)]\frac{\partial x_i}{\partial \tau_i} + E_i[h'_{i-1}(x_{i-1})]\frac{\partial x_{i-1}}{\partial \tau_i} + \zeta_i &= 0, \quad i > t, \\ E_i[h'_i(x_i)]\frac{\partial x_i}{\partial \theta_i} + \chi_i &= 0, \quad i > t, \end{aligned}$$

where  $\zeta_i$  and  $\chi_i$  denote the non-negative multipliers associated with the non-negativity constraints on tax rates. These conditions imply that with geometrically declining social welfare weights, the steady-state Ramsey tax rate  $\tau$  is *lower* than in the case with inelastic labor supply,  $\tau \leq \tau_t^G$ .<sup>28</sup> Intuitively, absent non-negativity constraints on tax rates, the government would like to impose the *same* social security tax rate as in the case with inelastic labor supply and eliminate the resulting distortions by setting  $\theta < 0$ . The non-negativity constraint renders such a strategy unfeasible. Balancing its allocative and distributive goals, and in contrast with the political process, the government therefore reduces the social security tax rate in response to the presence of tax distortions.

We summarize these findings in the following Proposition:

**Proposition 3.** Consider the Ramsey policy. With inelastic labor supply, under Assumption 1, tax rates are given by  $\tau_i^G$  for all  $i \geq t$ . With elastic labor supply, under Assumption 2 and with geometrically declining social welfare weights, the steady-state social security tax rate is lower than in the case with inelastic labor supply.

## 5 Quantitative Implications

In the previous sections, we have characterized the politico-economic equilibrium and the Ramsey allocation. We now apply these results to forecast the likely development of social security

<sup>28</sup>With geometrically declining social welfare weights,  $\tau$  and  $\theta$  cannot both be strictly positive. (For  $\rho = \beta\frac{1-\alpha}{\alpha(1+\beta)}$ , the unconstrained  $\tau$  and  $\theta$  both equal zero. For any other value of  $\rho$ , either  $\tau$  or  $\theta$  is in a corner.) At least one of the two tax rates must therefore be zero. If  $\tau$  is zero, the result immediately follows. Otherwise,  $\theta = 0$  and  $\tau > 0$ . The second first-order condition then implies  $h'_i(x_i) \leq 0$  (since  $\frac{\partial x_i(\tau_i, 0, \tau_{i+1})}{\partial \theta_i} > 0$ ), and the first condition implies  $\frac{\partial G(\cdot)}{\partial \tau_i} + h'_{i-1}(x_{i-1})\frac{\partial x_{i-1}}{\partial \tau_i} = 0$  (since  $\frac{\partial x_i(\tau_i, 0, \tau_{i+1})}{\partial \tau_i} = 0$ ). Since  $\frac{\partial x_i(\tau_i, 0, \tau_{i+1})}{\partial \tau_{i+1}} > 0$ , we conclude that  $\frac{\partial G(\cdot)}{\partial \tau_i} \geq 0$  and thus, due to monotonicity of  $\frac{\partial G(\cdot)}{\partial \tau_i}$ , that  $\tau \leq \max(\tau^G, 0)$ .

taxes and contributions, and to quantitatively assess the discrepancy between politico-economic equilibrium and a benchmark Ramsey allocation. We consider the case with elastic labor supply (Assumption 2), take one period in the model to correspond to thirty years in the data, and consider a risk-free environment.<sup>29</sup>

We assume the following parameter values: Based on findings in Piketty and Saez (2003), we set  $\alpha$  to 0.2815.<sup>30</sup> We set  $\nu_t$  equal to the estimated or projected thirty-year gross U.S. population growth rate, based on series reported by the U.S. Census Bureau (middle series). We compute these thirty-year growth rates for the base years 1970, 1980,  $\dots$ , 2050. This allows us to construct three sequences of model predictions, each with a period length of thirty years. The first such sequence begins in the year 1970, the second one in the year 1980, and the third one in the year 1990. When reporting the model predictions, we merge these three sequences. Figure 1 displays the postulated population growth rates. These growth rates imply that the peak of the demographic transition has been surpassed by the second half of the twenty-first century.

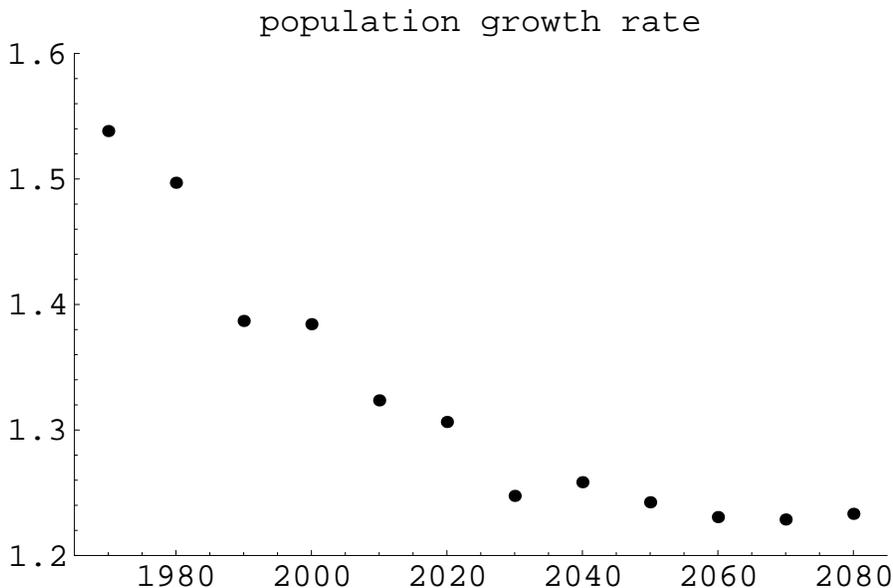


Figure 1: Population growth rate.

To calibrate  $\beta$  and  $\omega$ , we use two relationships between the model parameters that theory predicts to hold in politico-economic equilibrium. The first of the two dependencies between  $\beta$  and  $\omega$  arises from the postulated equality of the actual social security tax rate in the year 2000, 12.4 percent, and the predicted tax rate conditional on  $\alpha$  and  $\nu_{2000}$ . The second dependency between  $\beta$  and  $\omega$ ,  $\beta = \frac{\omega}{R - \nu\alpha}$ , follows from evaluating the household’s Euler equation in steady state at the equilibrium tax rate.<sup>31</sup> To exploit this second relationship, we approximate “the” annual U.S. interest rate by a weighted average of the returns on different asset classes, based on

<sup>29</sup>Since  $\tau^W$  does not depend on future population growth rates, it is unaffected by uncertainty about future population growth rates as well.

<sup>30</sup>Piketty and Saez (2003) report estimates of  $\alpha$  for post-war U.S. data. We use the average of their estimates over the period 1970–2003.

<sup>31</sup>As the economy is not in steady state in the year 2000, the condition only holds as an approximation. Robustness checks (see below) show that the validity of the simulations is not affected by this approximation.

estimates by Campbell and Viceira (2005) and the Federal Reserve Board.<sup>32</sup> We then account for the fact that real-world interest rates include a growth component that is absent in our model, to arrive at an adjusted annual gross interest rate of 1.0302.<sup>33</sup> The thirty-year gross interest rate corresponding to the variable  $R$  in the model thus is given by 2.4430.

Solving the two relationships jointly yields  $\beta = 0.4469$  and  $\omega = 0.9176$ . These values are plausible. On the one hand, they correspond to an annual discount factor of 0.9735. On the other hand, they appear consistent with the notion that old and young voters have approximately the same per-capita influence. By imposing that the time spent working and in retirement are of equal length, the model overstates the political influence of retirees; a value for  $\omega$  that is smaller than unity counterbalances this effect. (We conduct robustness checks by imposing larger or smaller values for the annual U.S. interest rate and thus, smaller or larger values for  $\beta$  and  $\omega$ . The simulation results are qualitatively unaffected; the predicted sequences for the social security tax rate are identical.)

Finally, when computing labor supply, we assume that  $v(x) \equiv m \ln(x)$  with  $m = 2.3867$ . This value implies that the long-run labor supply in politico-economic equilibrium (evaluated at the population growth rate of the year 2000) satisfies  $1 - x = 0.35$ .

Figure 2 displays the model predictions for  $\tau^W$  and  $\tau^W + \theta^W$  in the period from 1970 to 2050. The secular decrease in the population growth rate leads to a pronounced and persistent increase in the social security tax rate and thus, the pension share of GDP. This increase in intergenerational transfers is accompanied by a decrease in the purely distortive tax,  $\theta^W$ , which drops to zero by 2030.

Recall that we calibrated the model to match the social security tax rate in the year 2000. Figure 3 makes clear that, conditional on this normalization, the model does a good job in terms of replicating the actual social security tax rates between 1970 and 2000. Figure 3 also displays the out-of-sample predictions. Under the assumption of no structural change except for the demographic transition, the model predicts social security tax rates to rise for the foreseeable future until leveling off at around 16 percent by the middle of the century (see Table 1).

To assess the implications of the predicted  $\tau$ -sequence for social security benefits, we compare the model predictions with the conclusions drawn in the 2006 Annual Report of the Social Security Board of Trustees.<sup>34</sup> According to that Report, “the projected infinite horizon shortfall [of social security] could be eliminated with an immediate increase in the combined payroll tax rate from 12.4 percent to about 16.1 percent. This shortfall could also be eliminated if all current and future benefits were immediately reduced by 22 percent” (p. 55). The model, in

<sup>32</sup>Campbell and Viceira (2005) report annualized gross returns for 90-day treasury-bills (1.0152), 5-year treasury-bonds (1.0289), and stocks (1.0783) for the period 1952–2002. We approximate the average return on savings by a weighted average of these returns (1.0483) where the weights are proportional to the relative size of “deposits”, “credit market instruments”, and “equity shares at market value, directly held plus indirectly held” in the balance sheets of households and non-profit organizations (Board of Governors of the Federal Reserve System, *Flow of Funds Accounts of the United States: Annual Flows and Outstandings*, several years [we use averages for the period 1955–2002]). This yields an annual gross interest rate of  $\tilde{R} = 1.0483$ .

<sup>33</sup>Relaxing the assumption of constant productivity, we have  $A_{t+1} = A_t \gamma_A$  with  $\gamma_A > 1$ . Similarly, we have  $s_{t+1} = s_t \gamma_s$  with  $\gamma_s > 1$ . Since  $R_t \propto A_t s_{t-1}^{\alpha-1}$ , a balanced growth path with constant interest rate requires  $\gamma_s = \gamma_A^{1/(1-\alpha)}$ . Moreover, from the law of motion for savings (which is unaffected by changes in  $\gamma_A$ ) it follows that  $\gamma_s \propto A_t s_{t-1}^{\alpha-1}$ . We conclude that, on a balanced growth path,  $R$  increases by a factor of  $\gamma_A^{1/(1-\alpha)}$  if the gross growth rate of  $A$  increases from unity to  $\gamma_A$ . According to the Bureau of Labor Statistics, multifactor productivity of private businesses grew by a factor of 1.8681 between 1952 and 2002 (<http://www.bls.gov/mfp/home.htm>, series MPU740023 (K)). This implies  $\gamma_A = 1.0126$ . The result then follows.

<sup>34</sup>The 2006 Annual Report of the Board of Trustees of the Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds, see <http://www.ssa.gov/OACT/TR/TR06/tr06.pdf>.

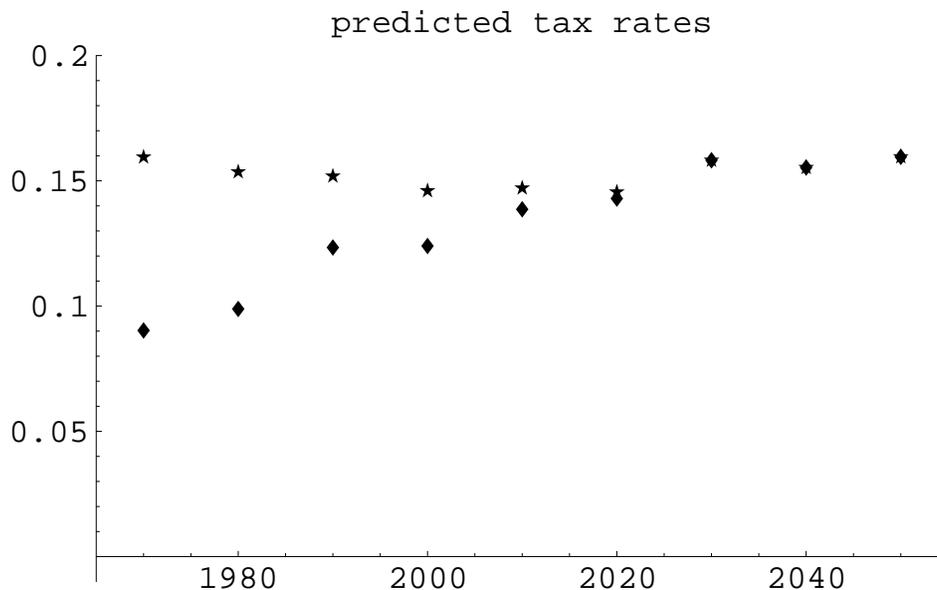


Figure 2: Predicted tax rates:  $\tau_t^W$  [●],  $\tau_t^W + \theta_t^W$  [★].

Table 1: Predicted social security tax rate

	2010	2020	2030	2040	2050	2060	2070	2080
$\tau^W$	0.1386	0.1429	0.1582	0.1553	0.1596	0.1628	0.1633	0.1620

turn, predicts  $\tau$  to reach 16 percent only after 2050, suggesting that social security benefits will fall below their promised levels, although by less than 22 percent.<sup>35</sup>

Similar conclusions follow from comparing the model predictions with the reform proposed by Diamond and Orszag (see, for example, Diamond and Orszag, 2005). These authors suggest, among a series of other measures, a gradual increase in payroll tax rates (12.5 percent in 2015, 13.2 percent in 2035, 14.2 percent in 2055, and 15.4 percent in 2078, and rising thereafter), combined with gradual benefit cuts by up to 9 percent for workers with average earnings, depending on age. The tax increases predicted by the model exceed Diamond and Orszag’s (2005) proposals, suggesting that the political process will implement only minor cuts in social security benefits.

We noted earlier that the predicted increase in the social security tax rate  $\tau^W$  is accompanied by a decrease in the purely distortive tax,  $\theta^W$ . The model predicts that as a result of these two opposing developments, the sum of the two tax rates follows a U-shaped path during the period 1970 to 2050, with a minimum being reached in the year 2020. Figure 4 displays the implications for labor supply of these trends. From 1970 until 2020, with both  $\tau_t$  and  $\theta_t$  strictly positive, the fall in the population growth rate pushes labor supply *upwards*—in contrast with

<sup>35</sup>The model abstracts from the social security trust fund whose net asset position is predicted to worsen. This effect dampens the pressure exerted on the social security system.

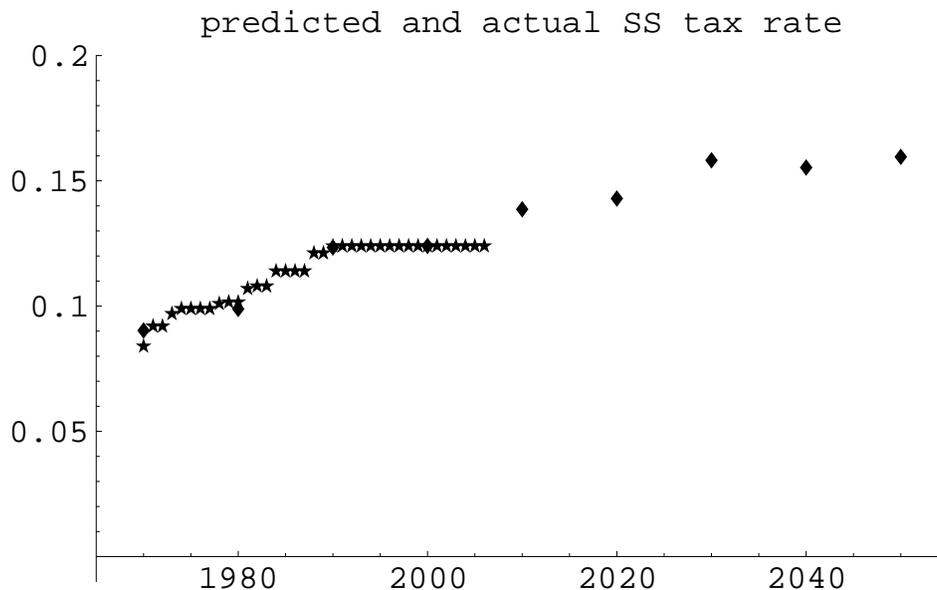


Figure 3: Social security tax rate: Predicted [●] and actual [OASDI, ★].

the common view that higher social security taxes go hand in hand with a reduction in hours (e.g., Feldstein, 2005). This implication of the model is broadly consistent with the data.<sup>36</sup> After 2020, when  $\theta_t$  drops to zero, labor supply reflects the movement of anticipated population growth rates, for reasons discussed earlier. In the medium term, labor supply therefore stabilizes on a comparatively high level.

We conclude this section with a comparison of the equilibrium outcome on the one hand and the allocation implemented by a Ramsey government with dynastic social welfare weights on the other. We consider a hypothetical steady-state with the population growth rate equal to its year-2000 value. Table 2 reports the results.

Table 2: Steady-state comparison

	Politico-economic equilibrium	Ramsey equilibrium
$\tau$	0.1240	0.0792
$\theta$	0.0355	0.0000
$c_2/c_1$	1.0918	0.9712
$1 - x$	0.3500	0.3650

Both the political process and the Ramsey government implement strictly positive social security taxes, but only the political process implements a strictly positive  $\theta$ . Relative to the

<sup>36</sup>Labor supply in the U.S. increased more strongly than predicted by the model, not least because of significant changes in the composition of labor supply across demographic groups. See, for example, McGrattan and Rogerson (2004).

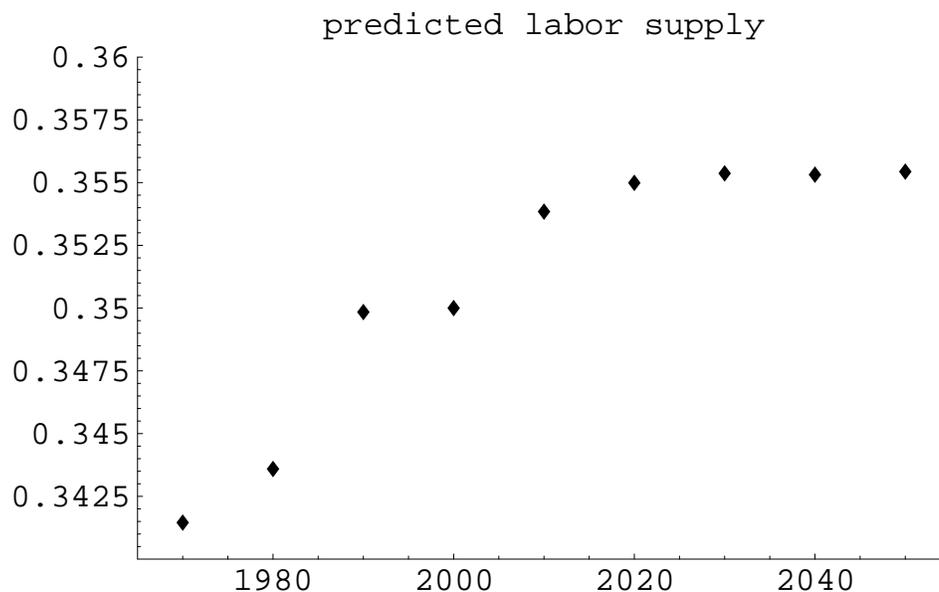


Figure 4: Predicted labor supply  $(1 - x)$ .

policy choices by the Ramsey government with dynastic welfare weights, social security tax rates in politico-economic equilibrium are 57 percent “too high,” and total tax rates are 101 percent “too high.” The politico-economic equilibrium features lower labor supply than the Ramsey allocation and a higher steady-state ratio of old- and young-age consumption. (Accordingly, the interest rate in politico-economic equilibrium is higher, and the capital-labor ratio lower.)

We conduct a series of robustness checks with respect to the fundamental model parameters. These checks confirm the central findings, according to which equilibrium social security taxes are positive and “too high” relative to the Ramsey tax rates. We also solve numerically for the equilibrium under the assumption of CIES preferences with an intertemporal elasticity of substitution,  $\varepsilon$ . With non-logarithmic preferences,  $\varepsilon < 1$  ( $\varepsilon > 1$ ), the tax function is no longer independent of  $s$ ; instead, it is negatively (positively) sloped.<sup>37</sup>

## 6 Conclusion

We have argued that the political support for intergenerational transfers reflects the interests of all voters rather than just a working median voter. The micro-political foundation for that view—probabilistic voting—is natural and has plausible implications. Introducing the probabilistic-voting assumption in the standard Diamond (1965) model preserves that model’s tractability and delivers intuitive and novel results in a strikingly transparent fashion.

Predictions of the model accord well with frequently expressed notions in the social security debate. For example, in response to population ageing, social security benefits are predicted to be cut, sooner or later. Such cuts do not herald the dismantling of pay-as-you-go social security systems, however. To the contrary, social security taxes and the GDP-share of the social security system will continue to grow. The model also gives meaning to the notion that social security constitutes a burden for future generations—even if these generations are not

<sup>37</sup>Details of these robustness checks are discussed in Gonzalez-Eiras and Niepelt (2005).

committed to honor existing social security promises; for in the model, political competition resolves the conflict between old and young voters by shifting some of the cost of the social security system to future generations. As a consequence, intergenerational transfers are too large, relative to a system balancing the interests of all generations.

We have assumed that the government's budget is balanced in each period. Relaxing this assumption introduces two further political choices in each period, one regarding the issuance of new debt, the other regarding the default rate on maturing debt. The debt issuance choice is constrained by the fact that agents investing in government debt foresee the possibility of a (partial) default by future political decision makers. Debt can therefore only be issued to the extent that a political incentive compatibility constraint is satisfied. But this implies that allowing for the issuance of defaultable government debt does not affect the politico-economic equilibrium. Since voters only care about the real allocation, the economic equivalence between social security and debt-plus-tax policies extends to the political sphere if both debt- and tax policies lack commitment.<sup>38</sup>

Since the model is very tractable, it lends itself to a variety of interesting extensions. We have analyzed one, central extension with endogenous labor supply, tax distortions, and multiple policy instruments. This extension proved the predicted support for social security to be a robust result, a novel finding in the literature. Another extension, due to Song (2005), features intragenerational heterogeneity and analyzes the interaction between social security transfers and wealth inequality. Further possible extensions might address questions relating to the interaction between social security and other policies or household choices. We leave these topics for future research.

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<sup>38</sup>However, the equilibrium choice of policy instruments is no longer uniquely pinned down in that case, since different combinations of government debt, social security transfers, and taxes support the same net transfers and the same allocation. On economic equivalence see, for example, Rangel (1997) or Niepelt (2005). On politico-economic equivalence, see Gonzalez-Eiras and Niepelt (2007).

# A Appendix

## A.1 Ramsey Policy

The effect on the Ramsey planner's objective function of a marginal increase in  $\tau_i$  is given by

$$\text{E}_t [w_i(\rho_{i-1}\beta u'(c_{2,i})\nu_i - \rho_i u'(c_{1,i})) + \mathcal{E}_i], \quad i \geq t, \quad (10)$$

where  $\mathcal{E}_i$  summarizes the general equilibrium effects of a change in  $\tau_i$ , as described below. When evaluated at  $i = t$ , this expression differs twofold from (5). First, the direct welfare effects of redistribution from young to old agents are weighted differently (the relative weight  $\omega/\nu_t$  is replaced by the relative weight  $\rho_{t-1}\beta/\rho_t$ ). Second, the general-equilibrium effects of the policy change are evaluated differently under the Ramsey policy:  $\mathcal{E}_t$  replaces  $\nu_t\mathcal{B}_t$ .<sup>39</sup>

To derive  $\mathcal{E}_i$ , we first introduce some notation. Let  $n_i \equiv \rho_{i-1}\beta u'(c_{2,i})\nu_i - \rho_i u'(c_{1,i})$  denote the net social benefit of transferring one unit of resources in period  $i$  from young to old households, and let  $I_i$  denote the effect on the government's objective function of a marginal increase in savings in period  $i$ . The contribution to  $I_i$  from general-equilibrium effects in period  $i + 1$  is given by<sup>40</sup>

$$\text{E}_t [\rho_i\beta u'(c_{2,i+1})(s_i R'_{i+1} + w'_{i+1}\nu_{i+1}\tau_{i+1}) + \rho_{i+1}u'(c_{1,i+1})w'_{i+1}(1 - \tau_{i+1})],$$

which simplifies to  $\text{E}_t[w'_{i+1}(\tau_{i+1}-1)n_{i+1}]$ . The total effect on the government's objective function of a marginal increase in savings in period  $i$ ,  $I_i$ , therefore equals

$$I_i = \text{E}_t \left[ w'_{i+1}(\tau_{i+1} - 1)n_{i+1} + \frac{dS_{i+1}}{dS_i} w'_{i+2}(\tau_{i+2} - 1)n_{i+2} + \dots \right].$$

Since under commitment, a change of tax rate also affects savings in the preceding period if  $i > t$ , the total general-equilibrium effect  $\mathcal{E}_i$  amounts to

$$\mathcal{E}_i = \frac{dS_i}{d\tau_i} I_i + \frac{\partial S_{i-1}}{\partial \tau_i} w'_i \left[ (\tau_i - 1)n_i + \frac{dS_i}{dw_i} I_i \right].$$

Notice that  $\mathcal{E}_i$  contains a term that corresponds to the expression  $\nu_t\mathcal{B}_t$  in the political program ( $dS_t/d\tau_t \cdot I_t$  includes the term  $\rho_{t-1}\beta \text{E}_t[u'(c_{2,t+1})\nu_{t+1}(\tau_{t+1} - 1)w'_{t+1}]$ ), as well as many additional terms capturing welfare effects on yet unborn generations.

These derivations imply that the marginal effect in (10) equals zero if  $n_i = 0$  for all  $i \geq t$  in all states of nature. In this case, a small change of tax rate has neither direct nor indirect effects on the government's objective as the government is indifferent at the margin between redistributing from or to workers in any period and state of nature. In fact, if it is feasible to set taxes to achieve  $n_i = 0$  for all  $i \geq t$  and all states of nature, then such a policy solves the Ramsey program. This follows from the fact that the condition  $n_i = 0$  also solves the corresponding social planner problem. (The *distribution* of consumption implemented by the social planner is governed by the same marginal rates of substitution and transformation as under the interior Ramsey policy. Moreover, savings choices of the social planner conform with households' savings choices and thus, the investment choices induced by the Ramsey policy.) An interior Ramsey policy therefore implements the social-planner allocation. By implication, it is

<sup>39</sup>No direct welfare effects from changes in savings arise since savings choices are privately optimal.

<sup>40</sup>Here,  $S_t$  denotes  $S(w_t(1 - \tau_t); \tau_{t+1})$ .  $w$  and  $R$  denote wage and return as functions of aggregate savings. A prime denotes the first derivative with respect to savings.

necessarily time-consistent. (This can also be seen from the fact that with  $n_i = 0$  for all  $i \geq t$  and all states of nature, the commitment effect

$$\frac{\partial S_{i-1}}{\partial \tau_i} w'_i \left[ (\tau_i - 1)n_i + \frac{dS_i}{dw_i} I_i \right]$$

vanishes.) As we establish in Gonzalez-Eiras and Niepelt (2005), the Ramsey policy is time consistent even if it is not interior.

## A.2 Old-Age Consumption Risk with Endogenous Labor Supply

If labor supply is endogenous, it responds to taxes and thus, population growth. This introduces a second channel for population growth to affect old-age consumption,  $c_{2,t+1}$ .

There are two cases to consider,  $\theta_{t+1} > 0$ , and  $\theta_{t+1} = 0$ . In the latter case, labor supply is unaffected by  $\tau_{t+1}$  and thus, contemporaneous population growth. The expression for  $d \ln[c_{2,t+1}]/d\nu_{t+1}$  therefore is unchanged from Section 2.

In the former case, labor supply is affected by contemporaneous population growth. This introduces a new term in the derivative  $d \ln[c_{2,t+1}]/d\nu_{t+1}$ . In particular, we have

$$\frac{d \ln[c_{2,t+1}]}{d\nu_{t+1}} = \frac{1-\alpha}{\nu_{t+1}} - \frac{1-\alpha}{1-x_{t+1}} \frac{dx_{t+1}}{d\nu_{t+1}} - \begin{cases} 0 & \text{if } \tau_{t+1} = 0, \\ \frac{1+\alpha\beta}{\omega+\nu_{t+1}(1+\alpha\beta)} & \text{if } \tau_{t+1} > 0. \end{cases}$$

Since  $\theta_{t+1} > 0$ , the first-order condition  $\frac{dg(x_{t+1}, \nu_{t+1})}{dx_{t+1}} = 0$  holds. Totally differentiating this condition yields

$$\frac{dx_{t+1}}{(1-x_{t+1})d\nu_{t+1}} = -\frac{\omega}{\nu_{t+1}(\omega+\nu_{t+1}(1+\alpha\beta))\left(\frac{v''(x_{t+1})}{v'(x_{t+1})}(1-x_{t+1})-1\right)}.$$

Substituting, we have

$$\frac{d \ln[c_{2,t+1}]}{d\nu_{t+1}} = \frac{1-\alpha}{\nu_{t+1}} \left[ 1 - \frac{\frac{\omega}{\omega+\nu_{t+1}(1+\alpha\beta)}}{1 - \frac{v''(x_{t+1})}{v'(x_{t+1})}(1-x_{t+1})} \right] - \begin{cases} 0 & \text{if } \tau_{t+1} = 0, \\ \frac{1-\alpha}{\nu_{t+1}}(1-\tau_{t+1}) & \text{if } \tau_{t+1} > 0. \end{cases}$$

Since  $v''(x) \leq 0$ , an upper bound for this expression results if  $v''(x)$  approaches  $-\infty$  and  $\tau_{t+1} = 0$ . A lower bound, in contrast, results if  $v''(x) = 0$  and  $\tau_{t+1}$  is marginally positive:

$$-\frac{\omega}{\omega+\nu_{t+1}(1+\alpha\beta)} \frac{(1-\alpha)}{\nu_{t+1}} < \frac{d \ln[c_{2,t+1}]}{d\nu_{t+1}} \leq \frac{(1-\alpha)}{\nu_{t+1}},$$

implying  $\left| \frac{d \ln[c_{2,t+1}]}{d\nu_{t+1}} \right| \leq \frac{(1-\alpha)}{\nu_{t+1}}$ . We conclude that the absolute value of the derivative is weakly smaller than the absolute value of the derivative when tax rates are constant, that is, the exposure to risk is lower than in the situation with exogenous tax rates.

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