

Larin, Benjamin

Working Paper

Bubble-driven business cycles

Working Paper, No. 143

Provided in Cooperation with:

University of Leipzig, Faculty of Economics and Management Science

Suggested Citation: Larin, Benjamin (2016) : Bubble-driven business cycles, Working Paper, No. 143, Universität Leipzig, Wirtschaftswissenschaftliche Fakultät, Leipzig

This Version is available at:

<https://hdl.handle.net/10419/127451>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

UNIVERSITÄT LEIPZIG

Wirtschaftswissenschaftliche Fakultät
Faculty of Economics and Management Science

Working Paper, No. 143

Benjamin Larin

Bubble-Driven Business Cycles

Februar 2016

ISSN 1437-9384

Bubble-Driven Business Cycles

– very preliminary draft –

Benjamin Larin*

February 8, 2016

Abstract: The 2007-2008 financial crisis highlighted that a turmoil in the financial sector including bursting asset price bubbles can cause pronounced and persistent fluctuations in real economic activity. This justifies the consideration of evolving and bursting asset price bubbles as another source of fluctuations in business cycle models. In this paper rational asset price bubbles are incorporated into a life-cycle RBC model as first developed by [Ríos-Rull \(1996\)](#). The calibration of the model to the post-war US economy and the numerical solution show that the model is able to depict plausible bubble-driven business cycles. In particular, the model generates i) a higher and empirically more plausible volatility of consumption at the cost of ii) a lower and empirically less plausible contemporaneous correlation of consumption with output than the life-cycle RBC model without bubbles.

Keywords: Computable General Equilibrium, Bubble, Asset Price, Real Activity.

JEL Classification: D58, E32, E44.

*Leipzig University, Institute for Theoretical Economics / Macroeconomics, Grimmaische Straße 12, D-04109 Leipzig, telephone: +49-341-97-33-542, e-mail: larin@wifa.uni-leipzig.de.

1. Introduction

The 2007-2008 financial crisis highlighted that a turmoil in financial markets and bursting asset price bubbles can cause pronounced and persistent fluctuations in real economic activity. This motivated renewed interest in considering financial markets within DSGE models as an impulse and a propagation mechanism for aggregate fluctuations. Although financial frictions were incorporated in RBC models over 25 years ago by [Bernanke and Gertler \(1989\)](#) and are becoming more prevalent in recent contributions on DSGE models (see, for instance, [Brunnermeier and Sannikov \(2014\)](#), [Boz and Mendoza \(2014\)](#), or [Covas and Den Haan \(2012\)](#)) asset price bubbles are rarely considered in DSGE models. Building on the contribution by [Martin and Ventura \(2012\)](#) I therefore set up and numerically solve a life-cycle RBC model with asset price bubbles in this paper. The calibration of the model to the post-war US economy and the numerical solution show that the model is able to depict plausible bubble-driven business cycles. In particular, the model generates i) a higher and empirically more plausible volatility of consumption at the cost of ii) a lower and empirically less plausible contemporaneous correlation of consumption with output than the life-cycle RBC model without bubbles.

This paper is related to the following three strands of literature. First, since the seminal contribution by [Tirole \(1985\)](#) rational asset price bubbles have been incorporated in general equilibrium models. In the [Tirole \(1985\)](#) model bubbles always crowd-out capital. Empirically it is not plausible that the capital stock and output decline during episodes of existing asset price bubbles, and increase when bubbles burst. The correlation should be the reverse. Therefore, recent models extend the [Tirole \(1985\)](#) model – mostly by financial frictions – in order to derive equilibria where bubbles create a crowding-in of capital such that capital and output increase during periods of existing bubbles (see, among others, [Galí \(2014\)](#), [Martin and Ventura \(2012\)](#), and [Farhi and Tirole \(2012\)](#)). Most of these contributions consider *two-period* overlapping generation (OLG) models and derive explicit conditions for the existence of bubbles and the characteristics of bubbles in general equilibrium. I contribute to this literature by considering rational bubbles within a *large-scale* OLG model.

Second, models that incorporate aggregate uncertainty and analyze business cycles exist

also within the literature on large-scale OLG models as initiated by [Auerbach and Kotlikoff \(1987\)](#). The first to develop a life-cycle RBC model was [Ríos-Rull \(1996\)](#), and more recent contributions are, among others, [Heer and Maußner \(2012\)](#) and [Iacoviello and Pavan \(2013\)](#). The model presented in this paper can be seen as an extension of the [Ríos-Rull \(1996\)](#) model by financial frictions and rational asset price bubbles.

Third, within the infinitely-lived agent DSGE literature a few contributions do also consider asset price bubbles. [Bernanke and Gertler \(1999\)](#) consider *irrational* asset price bubbles within an RBC model with financial frictions. As the authors state, they "use the term "bubble" [here] loosely to denote temporary deviations of asset prices from fundamental values" (p. 19). Two further contributions within this literature are [Miao et al. \(2016\)](#) and [Luik and Wesselbaum \(2014\)](#). The numerical solution in both studies is based on local perturbation methods. By log-linearizing the dynamic systems of equations only small shocks can be considered. It is hard to justify, however, that bursting bubbles presents a small shock and that local solution techniques are an accurate approximation. I therefore apply a global solution technique for solving the model in this paper.

To the best of my knowledge, this paper is the first to set up and numerically solve a life-cycle RBC model featuring *rational* bubbles. This is important for at least two reasons. First, bubbles in two-period OLG models exist at least for some 30 years, which is implausibly long. In order to depict a short-term phenomenon like asset price bubbles, one period in the model should correspond at least to one year in calendar time. Second, and more importantly, by implementing rational bubbles in a large-scale OLG model the effect of evolving and bursting bubbles as an impulse for aggregate fluctuations can be analyzed and compared to business cycles resulting from TFP shocks.

The paper is structured as follows. In section 2 the model is presented and an equilibrium is defined. In the following section 3 the calibration is explained and the model is solved numerically. Section 4 extends the model to elastic labor supply and compares the results with the model without elastic labor supply. The last section concludes.

2. The Model

2.1. Set Up

Demographics. Time is discrete and the economy exists up to infinity. The economy consists of individuals of different age groups. Each period some individuals enter and some individuals leave the economy. The age of an individual is denoted by the subscript s . Young individuals enter the economy at a real life age of 21 years which is denoted by $s = 1$. After entering the economy individuals supply inelastically one unit of labor per period for $T \geq 1$ periods. Then individuals live for further $T^R \geq 1$ periods in retirement before dying and leaving the economy. Hence, the life-span for every individual in this economy is $T + T^R$ periods. In the classic OLG-model developed by [Allais \(1947\)](#), [Samuelson \(1958\)](#), and [Diamond \(1965\)](#) $T = T^R = 1$. In the vein of [Auerbach and Kotlikoff \(1987\)](#) I will calibrate the parameters T and T^R such that one period in the model corresponds to one year.

Total population is assumed to grow at a constant rate $n \geq 0$. Let N_t^s denote the size of the cohort consisting of individuals of age s in period t . Total population is then given by $N_t = \sum_{s=1}^{T+T^R} N_t^s$. Without loss of generality total population in period $t = 0$ is set equal to one. It follows that total population is given by

$$N_t = (1 + n)^t, \quad (1)$$

and the cohort size is given by

$$N_t^s = \begin{cases} (1 + n)^{t-s+1} N_0^1, & \text{if } 0 < s \leq T + T^R \\ 0, & \text{else} \end{cases} \quad (2)$$

where $N_0^1 = \frac{1}{\sum_{s=1}^{T+T^R} (1 + n)^{1-s}}$ is the size of the cohort consisting of individuals that enter the economy in period $t = 0$.

Utility. After entering the economy individuals supply inelastically one unit of labor in every period for T periods, earning labor income w_t . For $s > T$ retirement is mandatory, households do not work and live from wealth accumulated in earlier periods of their lives.

Preferences of an individual of age $s = 1$ at period t are described by the intertemporal von Neumann-Morgenstern expected utility function

$$\mathbb{E}_t U_t^s = \mathbb{E}_t \sum_{s=1}^{T+T^R} \beta^{s-1} u(c_{t+s-1}^s), \quad (3)$$

where $0 < \beta$ is the subjective discount factor, $u(c)$ is instantaneous utility, and c_t^s denotes consumption of an individual of age s in period t . Instantaneous utility is given by the CRRA utility function

$$u(c_{t+s-1}^s) = \frac{(c_{t+s-1}^s)^{1-\epsilon} - 1}{1-\epsilon}, \quad (4)$$

where $\epsilon > 0$ is a constant preference parameter that equals the inverse of the intertemporal elasticity of substitution as well as the Arrow-Pratt measure of relative risk-aversion.

Firms. Firms of mass one employ two production factors – capital K_t and labor L_t – to produce a homogeneous final output good according to the Cobb-Douglas technology

$$Y_t = A_t K_t^\alpha (E_t L_t)^{1-\alpha}, \quad (5)$$

where $0 < \alpha < 1$ is the capital income share. Technological progress is labor augmenting, deterministic and expressed as $E_{t+1} = (1+g)E_t$. The variable A_t depicts exogenous and stochastic total factor productivity (TFP) shocks evolving according to an AR(1) process:

$$\ln A_{t+1} = \rho \ln A_t + z_{t+1}^a, \quad (6)$$

where $z_t^a \sim N(0, \sigma^a)$ is the innovation term and $0 \leq \rho < 1$ measures the persistence of TFP-shocks.

Competitive profit maximization yields wages w_t and interest rates r_t

$$w_t = (1-\alpha) \frac{Y_t}{L_t}, \quad (7)$$

$$r_t = \alpha \frac{Y_t}{K_t} - \delta, \quad (8)$$

where δ is the capital depreciation rate.

Financial friction. Besides differences in age individuals further differ with respect to their investment efficiency. A small share of the population is comparably productive at investing in capital used for production. Think of these productive individuals as the entrepreneurs in the economy or individuals with higher financial literacy. The population share of productive individuals is constant and given by $0 \leq \eta \leq 1$. These productive individuals are able to transform one unit of the final output good one-to-one into productive capital used in production. The remainder of the population, $1 - \eta$, is unproductive at investing and has to incur costs of $1 - \sigma$ when investing one unit of the final output good, where $0 \leq \sigma \leq 1$. The gross return to investment in capital faced by productive individuals is then equal to $1 + r_t$ while it is $\sigma(1 + r_t)$ for unproductive individuals. Individuals are born as either productive or unproductive investors and stay productive or unproductive investors throughout their whole life.

If financial markets were frictionless, unproductive individuals would lend all their wealth to productive individuals such that both groups would be better off, the allocation of investment would be efficient, and the capital stock in the economy would be larger than in an economy without lending between unproductive and productive individuals. However, it is assumed – as in [Martin and Ventura \(2012\)](#) – that unproductive individuals lending resources to productive individuals face a default probability greater than $1 - \sigma$ such that it is never profitable for unproductive individuals to lend resources to productive individuals. The result is a financial friction: no borrowing or lending takes place between productive and unproductive individuals, investments are inefficiently allocated, and a wedge is drawn between the returns to capital faced by productive and unproductive individuals.

Bubbles. Individuals can save labor income in order to smooth consumption and finance consumption in retirement when no labor income is earned. Therefore individuals can either supply capital $a_t^{j,s}$ for the production of a final output good, or purchase a zero-dividend asset $b_t^{j,s}$, where $a_t^{j,s}$ ($b_t^{j,s}$) denotes the capital stock (zero-dividend assets) held by an individual of productivity type $j \in \{U, P\}$ and age s at the beginning of period t .¹ Note, $b_t^{j,s}$ denotes the value of the zero-dividend asset in terms of the final output

¹[Tirole \(1985\)](#) first introduced rational asset price bubble in a simple two-period OLG model.

good. An asset price can be decomposed into a fundamental and a bubble component, where the fundamental component is the sum of the discounted stream of expected future dividends or rents.² The zero-dividend asset's fundamental is by definition zero such that its price contains a bubble when b_t is greater than zero. Following [Tirole \(1985\)](#) I refer to the zero-dividend asset just as “bubble” in the remainder.

The aggregate bubble B_t is obtained by summing up all the bubbles held by individuals in the economy

$$B_t = \sum_{s=1}^{T+T^R} N_t^s \left[\eta b_t^{P,s} + (1 - \eta) b_t^{U,s} \right], \quad (9)$$

where $b_{t+1}^{j,s+1}$ is the quantity of bubbles purchased by an individual of productivity type $j \in \{U, P\}$ and of age s at the end of period t . The size of the bubbles held by all productive (unproductive) individuals of age s in period t is $\eta N_t^s b_t^{P,s}$ ($(1 - \eta) N_t^s b_t^{U,s}$). Summing up the bubbles of all cohorts and productivity types gives B_t as described by (9).

In a given period t bubbles can be initiated and existing bubbles can burst.³ Following [Martin and Ventura \(2012\)](#) the creation and destruction of bubbles is governed by investors' sentiment $z_t^b \in \{0, 1\}$. When investors are optimistic $z_t^b = 1$, they believe that the zero-dividend asset has a positive value, new bubbles can be initiated, and $b_t \geq 0$. When investors are pessimistic $z_t^b = 0$, they want to sell all their zero-dividend assets, they will not purchase further zero-dividend assets, and $b_t = 0$. Investors' sentiment z_t^b follows a simple Markov-process with the transition matrix \mathbf{P} , as depicted in [Table 1](#). If investors are optimistic in period t , $z_t^b = 1$, the probability that investors are also optimistic in the next period $t + 1$ is given by the constant parameter $0 \leq o \leq 1$. The probability of investors becoming pessimistic after being optimistic is hence $1 - o$. Similarly, if investors are pessimistic in period t , $z_t^b = 0$, the probability that investors are also pessimistic in the next period $t + 1$ is given by the constant parameter $0 \leq p \leq 1$. The probability of switching from pessimism to optimism is then $1 - p$.

When investors are optimistic, $z_t = 1$, working productive individuals create new bubbles \hat{b}_t^N and sell them to other individuals in the same period. For the sake of simplicity

²See, for instance, [Blanchard and Watson \(1983, 2f.\)](#), or, more recently, [Brunnermeier \(2008, 3\)](#).

³[Weil \(1987\)](#) first added a stochastic probability of bursting bubbles to the [Tirole \(1985\)](#) model.

Table 1: Transition matrix \mathbf{P}

	$z_{t+1}^b = 1$	$z_{t+1}^b = 0$
$z_t^b = 1$	o	$1 - o$
$z_t^b = 0$	$1 - p$	p

it is assumed that each working productive individual creates the same amount of new bubbles. The creation of new bubbles is exogenous and given by

$$\hat{b}_t^N = z_t^b \theta E_t, \quad (10)$$

where $\theta \geq 0$ is the size of newly created bubbles when technological growth is absent ($E_t = 1$) and investors are optimistic ($z_t^b = 1$). The parameter θ has to be multiplied by labor productivity E_t in order for a balanced-growth path to exist. The aggregate value of newly created bubbles is then

$$B_t^N := \hat{b}_t^N \sum_{s=1}^T \eta N_t^s = \frac{\eta}{1 + \phi} N_t \hat{b}_t^N, \quad (11)$$

where $\phi := \frac{\sum_{s=T+1}^{T+T^R} N_t^s}{\sum_{s=1}^T N_t^s}$ is the constant old-age dependency ratio, $\frac{1}{1+\phi}$ is the constant population share of all working individuals, and $\frac{\eta}{1+\phi} N_t$ is the size of all productive working individuals.

The ex-post realized return to bubbles is given by

$$1 + q_t = \begin{cases} 1 + q_t^o = \frac{B_{t+1}}{B_t + B_t^N} & \text{if } z_t^b = 1, \\ 0 & \text{if } z_t^b = 0, \end{cases} \quad (12)$$

or equivalently as $(1 + q_t) = z_t^b(1 + q_t^o)$, where q_t^o is the return to bubbles under the realization $z_t^b = 1$.

The timing of events is described in detail in appendix [A.1](#).

Household optimization. For $j \in \{P, U\}$ the households' budget constraints read

$$\begin{aligned}
c_t^{j,s} + a_{t+1}^{j,s+1} + b_{t+1}^{j,s+1} &= w_t^j + R_t^j a_t^{j,s} + z_t^b(1 + q_t^o)b_t^{j,s} & s = 1, \dots, T \\
c_t^{j,s} + a_{t+1}^{j,s+1} + b_{t+1}^{j,s+1} &= R_t^j a_t^{j,s} + z_t^b(1 + q_t^o)b_t^{j,s} & s = T + 1, \dots, T + T^R \\
0 &= a_t^{j,1} = b_t^{j,1} = a_t^{j,T+T^R+1} = b_t^{j,T+T^R+1}. & (13)
\end{aligned}$$

The first equation depicts the budget constraints of all working individuals and the second equation the budget constraint of all retired individuals. Productive working individuals earn not only labor income, but also income from creating new bubbles, i.e. $w_t^P = w_t + z_t^b(1 + q_t^o)\hat{b}_t^N$, while unproductive individuals earn only labor income, i.e. $w_t^U = w_t$. Due to the financial friction, productive individuals face higher returns to capital than unproductive individuals such that $R_t^P = 1 + r_t < R_t^U = \sigma(1 + r_t)$. The third line of the budget constraints states that individuals enter the economy with zero wealth and that individuals leave no debt after death.

The households optimization problem then reads

$$\begin{aligned}
&\max_{\{c_{t+s-1}^{j,s}, a_{t+s}^{j,s+1}, b_{t+s}^{j,s+1}\}_{s=1}^{T+T^R}} \mathbb{E}_t \sum_{s=1}^{T+T^R} \beta^{s-1} u(c_{t+s-1}^{j,s}), \\
&\text{s.t.} \quad (13) \quad \text{and} \quad b_{t+1}^{j,s+1} \geq 0.
\end{aligned}$$

The non-negativity constraint on bubbles is imposed because bubbles are assumed to be freely disposable. It is possible, however, that households run temporary into debt, i.e. $a_t^{j,s} < 0$.

The first order conditions consist of two stochastic Euler equations – the first is derived from the first order condition for optimal capital and the second from the first order condition for optimal bubbles – a complementary slackness condition, and the budget constraints:

$$\begin{aligned}
(c_t^{j,s})^{-\epsilon} &= \beta \mathbb{E}_t \left\{ R_{t+1}^j (c_{t+1}^{j,s+1})^{-\epsilon} \right\} & s = 1, \dots, T + T^R - 1 \\
(c_t^{j,s})^{-\epsilon} &= \beta \mathbb{E}_t \left\{ (1 + q_{t+1}) (c_{t+1}^{j,s+1})^{-\epsilon} \right\} + \omega_{t+1}^{j,s+1} & s = 1, \dots, T + T^R - 1 \\
\omega_{t+1}^{j,s+1} b_{t+1}^{j,s+1} &= 0 \quad \text{and} \quad (13), & (14)
\end{aligned}$$

where $\omega_{t+1}^{j,s+1}$ is the multiplier associated with the non-negativity constraint of bubbles.

2.2. Equilibrium

A competitive equilibrium consists of a sequence of individual consumption, capital, and bubbles $\left\{ \left\{ c_t^{P,s}, c_t^{U,s}, a_t^{P,s}, a_t^{U,s}, b_t^{P,s}, b_t^{U,s} \right\}_{s=1}^{T+T^R+1} \right\}_{t=0}^{\infty}$ satisfying the FOC's of the household optimization problem as given by (14), a sequence of prices $\{w_t, r_t, q_t^o\}_{t=0}^{\infty}$ satisfying (7) and (12), a sequence of shocks $\{z_t^a, z_t^b\}_{t=1}^{\infty}$ drawn from their respective distributions and initial values $\{a_0^{P,s}, a_0^{U,s}, b_0^{P,s}, b_0^{U,s}\}_{s=1}^{T+T^R-1}, z_0^a, z_0^b, q_0^o$ such that

- the labor market clears

$$L_t = \sum_{s=1}^T N_t^s = \frac{1}{1+\phi} N_t, \quad (15)$$

- the capital market clears

$$K_{t+1} = \sum_{s=1}^{T+T^R} N_{t+1}^{s+1} \left[\eta a_{t+1}^{P,s+1} + \sigma(1-\eta) a_{t+1}^{U,s+1} \right], \quad (16)$$

- the market for bubbles clears

$$(1+q_t)(B_t + B_t^N) = \sum_{s=1}^{T+T^R} N_{t+1}^{s+1} \left[\eta b_{t+1}^{P,s+1} + (1-\eta) b_{t+1}^{U,s+1} \right], \quad (17)$$

- the goods market clears⁴

$$Y_t = C_t + \Delta K_{t+1} + \delta K_t + \underbrace{\sum_{s=1}^{T+T^R} (1-\eta)(1-\sigma) N_t^s a_{t+1}^{U,s+1}}_{\text{output-loss due to the financial friction}}, \quad (18)$$

- capital does not become negative (or, equivalently, the bubble does not become too

⁴Although it is not necessary to state this equation – it is implied by the budget constraints – it is useful as a consistency check in the numerical solution. Note further that aggregate consumption is defined

as $C_t = \sum_{s=1}^{T+T^R} N_t^s \left(\eta c_t^{P,s} + (1-\eta) c_t^{U,s} \right)$.

large)⁵

$$\sum_{s=1}^{T+T^R} \eta N_t^s a_{t+1}^{P,s+1} \geq 0 \quad \text{and} \quad \sum_{s=1}^{T+T^R} (1 - \eta) N_t^s a_{t+1}^{U,s+1} \geq 0 \quad (19)$$

- and bubbles are freely disposable

$$B_t \geq 0. \quad (20)$$

3. Numerical Solution

The model economy grows asymptotically at a rate $g + n + gn$. Some variables have therefore to be normalized in order to yield a stationary system. The notation of normalized variables is as follows: A normalized aggregate variable X_t is defined by $x_t := \frac{X_t}{E_t N_t}$. A normalized individual-level variable $x_t^{j,s}$ is defined by $\tilde{x}_t^{j,s} := \frac{x_t^{j,s}}{E_t}$. The normalized system of equations is derived in appendix A.2.

3.1. Calibration

The model is calibrated with respect to the postwar US economy. Whenever possible I use values common in the literature.

Demographics. Periods in the model correspond to years in real-time. I therefore set the number of years individuals spend working to 40 ($T = 40$) and the number of years in retirement to 20 ($T^R = 20$). The values for T and T^R are taken from Heer and Maußner (2012), who calibrate a life-cycle RBC model to quarters and set $T = 160$ and $T^R = 80$. The annual population growth rate n is set equal to 1 percent since the US was inhabited by 159 million individuals in 1950 and by 230 million in 2013 (UN, 2013, 61).

Preferences. The subjective discount factor is set below, but close to unity as common in the literature, i.e. $\beta = 0.9975$ (see, for instance, Heer and Maussner (2009)). The intertemporal elasticity of substitution $1/\epsilon$ cannot be inferred from time series of economic variables. According to a recent empirical meta-analysis by Havránek (2015) it should be far below unity. I choose a value of 0.25 such that $\epsilon = 4$.

⁵Due to the financial friction the total capital of unproductive or productive individuals cannot be negative. If, for example, total capital of unproductive individuals would be negative that would imply that unproductive individuals borrow resources from productive individuals at the return $\sigma(1 + r_t)$ contradicting the imposed financial friction.

Table 2: Set of parameters

Parameter	Value	Parameter	Value
T	40	ρ	0.814
T^R	20	σ^a	0.0142
n	0.01	σ	0.6
β	0.9975	η	0.05
ϵ	4	o	0.8
α	0.34	p	0.9
δ	0.08	θ	0.15
g	0.02		

Production. The capital income share and the capital depreciation rate are set in accordance with [Prescott \(1986\)](#) implying $\alpha = 0.34$ and $\delta = 0.08$. The growth rate g is determined by the population growth rate n and the growth rate of aggregate output. In the period 1960-2014 real US-GDP grew on average by approximately 3 percent as calculated from the US Bureau of Economic Analysis. Aggregate output in the model economy grows at the rate $n + g + ng$. The growth rate of E_t is then given by $g = (0.03 - n)/(1 + n) \approx 0.02$. The parameters for the AR(1) process for technology are set equal to $\rho = 0.814$ and $\sigma^a = 0.0142$. These parameters correspond to a quarterly AR(1) process with $\rho = 0.95$ and $\sigma^a = 0.00763$ as given in [Prescott \(1986\)](#).⁶

Financial friction. The parameters η and σ are not common in the literature on quantitative macroeconomic models and empirical time series for calibrating these parameters do not exist. I assume that 5 percent of the population is much more efficient in investing than the rest of the population. Further, σ is chosen such that the wedge between the interest rate of productive and unproductive individuals is sufficiently large for expansionary bubbles to exist. Therefore, $\eta = 0.05$ and $\sigma = 0.6$.

Bubbles. The choice of the parameters governing the duration, creation, bursting, and size of bubbles is difficult to infer from observed data. The parameters governing investors' sentiment are chosen to be $o = 0.8$ and $p = 0.9$ giving an average duration of a bubbly episode of 4 years and an average duration of a bubble-less episode of 9 years. The creation of new bubbles – governed by θ – is set such that the bubble-to-capital ratio is

⁶See [Heer and Maussner \(2009, Ch. 10, appendix 5\)](#) for how to derive parameters for the annual AR(1) process based on parameters of a quarterly AR(1) process.

approximately equal to 1 percent in the deterministic steady state, implying a value of $\theta = 0.15$. Large values of θ can result in bubbles becoming too large and violating the equilibrium conditions. [Table 2](#) summarizes the calibration of the model.

3.2. Deterministic steady state

The deterministic steady state is a hypothetical trajectory where the stochastic exogenous state-variables z_t^a and z_t^b are equal to their expected values with probability one. The computation of the deterministic steady state is necessary for the numerical solution of the dynamic system in [subsection 3.3](#).

The limiting distribution of the Markov process of investors' sentiment, π , is given by

$$\pi = \begin{bmatrix} \frac{1-p}{(1-o)+(1-p)} \\ \frac{1-o}{(1-o)+(1-p)} \end{bmatrix}. \quad (21)$$

The expected value of z_t^b is then given by $\bar{z} := \lim_{T \rightarrow \infty} \mathbb{E}_t[z_T^b] = \frac{1-p}{(1-o)+(1-p)}$. In the deterministic steady state $z_t^b = \bar{z}$, $z_t^a = 0$, and $A_t = 1$ for all t .

The stochastic Euler equations given by [\(33\)](#) become deterministic and read

$$\begin{aligned} \left(\frac{(1+g)\tilde{c}^{j,s+1}}{\tilde{c}^{j,s}} \right)^\epsilon &= \beta R^j & s = 1, \dots, T + T^R - 1 \\ \left(\frac{(1+g)\tilde{c}^{j,s+1}}{\tilde{c}_t^{j,s}} \right)^\epsilon &= \beta(1+q) + \tilde{\omega}^{j,s+1} (\tilde{c}^{j,s+1})^\epsilon & s = 1, \dots, T + T^R - 1 \\ \tilde{\omega}^{j,s+1} \tilde{b}^{j,s+1} &= 0 & \text{and } (29), \end{aligned} \quad (22)$$

In what follows I assume $\theta > 0$, as in the calibration, such that new bubbles are created in the deterministic steady state. What is now the relation between the certain return to bubbles q and the return to capital r ?

First, when $(1+q) = R^j$ individuals are indifferent with respect to the allocation of their wealth between productive capital $a^{j,s}$ and bubbles $b^{j,s}$, otherwise individuals hold only productive capital or only bubbles. The equilibrium condition for bubbles [\(17\)](#) necessitates that q is large enough such that some individuals in the economy choose to purchase bubbles, because new bubbles are created in each period. This rules out

$1 + q < \sigma(1 + r)$, because otherwise the newly created bubbles would not be purchased and the equilibrium condition (17) would be violated.

Second, assume $(1 + q) = R^U = \sigma(1 + r)$. Unproductive individuals hold bubbles and their non-negativity restriction does not bind, i.e. $\tilde{\omega}^{U,s} = 0$. Productive individuals would not purchase bubbles because $1 + q < R^P = (1 + r)$ and their non-negativity restriction would bind, i.e. $\tilde{\omega}^{P,s} > 0$, and $\tilde{b}^{P,s} = 0$. Unproductive individuals are indifferent with respect to the amount of bubbles purchased. Any distribution of the existing bubble across all unproductive individuals would satisfy the household optimality and market equilibrium conditions. I assume that each individual holds the same amount of bubbles, i.e. $b^{U,s} = b^U$.

Third, only if $(1 + q) \geq (1 + r)$ all individuals in the economy would hold bubbles, i.e. $\tilde{\omega}^{j,s} = 0$. The discussion is summarized by

$$1 + q = \begin{cases} = \sigma(1 + r) & \text{if } b < \sum_{s=1}^{T+T^R} (1 - \eta) N_0^{s+1} \tilde{s}^{U,s} \\ \in [\sigma(1 + r), (1 + r)] & \text{if } b = \sum_{s=1}^{T+T^R} (1 - \eta) N_0^{s+1} \tilde{s}^{U,s} \\ = (1 + r) & \text{if } b > \sum_{s=1}^{T+T^R} (1 - \eta) N_0^{s+1} \tilde{s}^{U,s}, \end{cases} \quad (23)$$

where total individual wealth $\tilde{s}^{j,s}$ is defined as $\tilde{s}^{j,s} := \tilde{a}^{j,s} + \tilde{b}^{j,s}$ for $j \in \{U, P\}$.

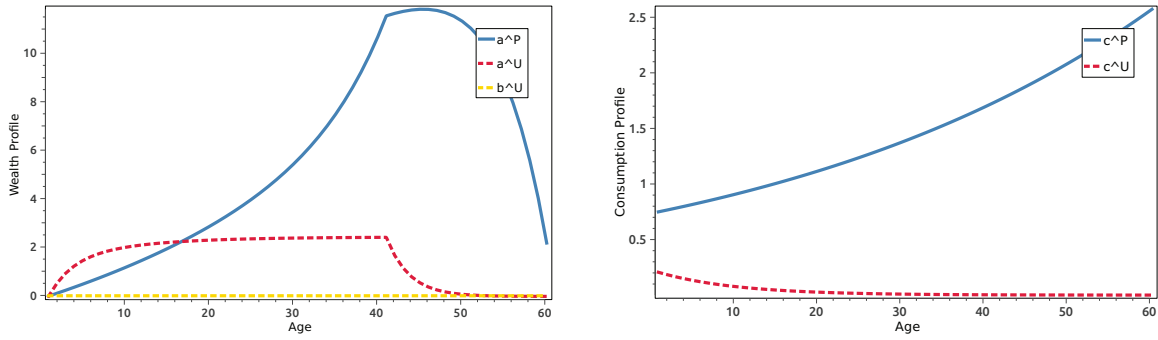
In the simulations I consider only the case where $1 + q = \sigma(1 + r)$, because the model would otherwise yield empirically implausible bubbles where the capital stock declines when bubbles emerge and increases when bubble burst. Therefore it is always checked that the wealth of the unproductive individuals is smaller or equal than the existing bubble such that (23) is satisfied. The size of individual bubbles is then given by $\tilde{b}^{P,s} = 0$ and

$$\tilde{b}^U = \frac{1}{(1 - \eta)(1 - N_0^1)} b. \quad (24)$$

The numerical procedure that has been employed to solve for the steady state is closely related to Auerbach and Kotlikoff (1987) and similar to the “direct computation method” as described in Heer and Maussner (2009). The solution algorithm is described in detail in appendix C.1.

In the steady state the aggregate (normalized) capital stock and the aggregate (nor-

Figure 1: Age profiles



malized) bubble are given by $k = 1.162$ and $b = 0.01201$. The returns to capital differ strongly between productive and unproductive individuals and are given by $R^P = 1.172$ and $R^U = 0.7032$. The bubble is positive and considerably smaller than the aggregate wealth of all unproductive individuals – which is equal to 1.511 – hence (23) is satisfied. Figure 1 depicts the individual wealth and consumption profiles. The consumption profile is increasing for productive individuals and decreasing for unproductive individuals. This is a direct result of the financial friction which draws a wedge between the interest rates faced by productive and unproductive individuals. Consumption of a productive individual grows at a constant rate of 2 percent over a life span while consumption of an unproductive individual declines at constant rate of 10 percent.⁷ The increasing consumption profile of productive individuals therefore also implies a much steeper wealth profile, because productive individuals need to accumulate more wealth for higher consumption levels at an older age than unproductive individuals.

3.3. Dynamics

The model is solved with the deterministic extended path (DEP) method. This procedure was first applied to DSGE models by Gagnon (1990). As shown by Heer and Maußner (2008) the DEP method yields the highest accuracy in the computation of the standard business cycle model at the cost of longer computational time when compared with log-linear solution methods, second-order approximations, the parameterized expectations approach, Galerkin projections, and value function iterations. In contrast to local

⁷The constant growth rate of individual consumption is obtained by rearranging the Euler equation such that $\frac{c^{j,s+1}}{c^{j,s}} - 1 = \frac{(\beta R^j)^{\frac{1}{\epsilon}}}{1+g} - 1$.

methods like the log-linear or higher-order approximations as well as linear-quadratic approximations the DEP method is a global method and therefore better suited for the present model, because the shocks due to evolving and bursting bubbles can shift the economy far away from its steady state.

The DEP method works as follows: For given endogenous and exogenous (stochastic) state variables in period t the conditional expectations of the stochastic state variables for the entire time span of the simulation is calculated and plugged into the stochastic Euler equations. The result is a deterministic dynamic system of equations. This deterministic system is solved for a time-span large enough for the economy to be very close to its deterministic steady state. From the solution the endogenous state variables at $t + 1$ and the control variables at t are stored. In $t + 1$ a new realization of the shocks is drawn and the whole procedure is repeated. Proceeding in this manner one obtains an approximation of a realized time path of the true stochastic model which – as shown by [Heer and Maubner \(2008\)](#) – can be very accurate.

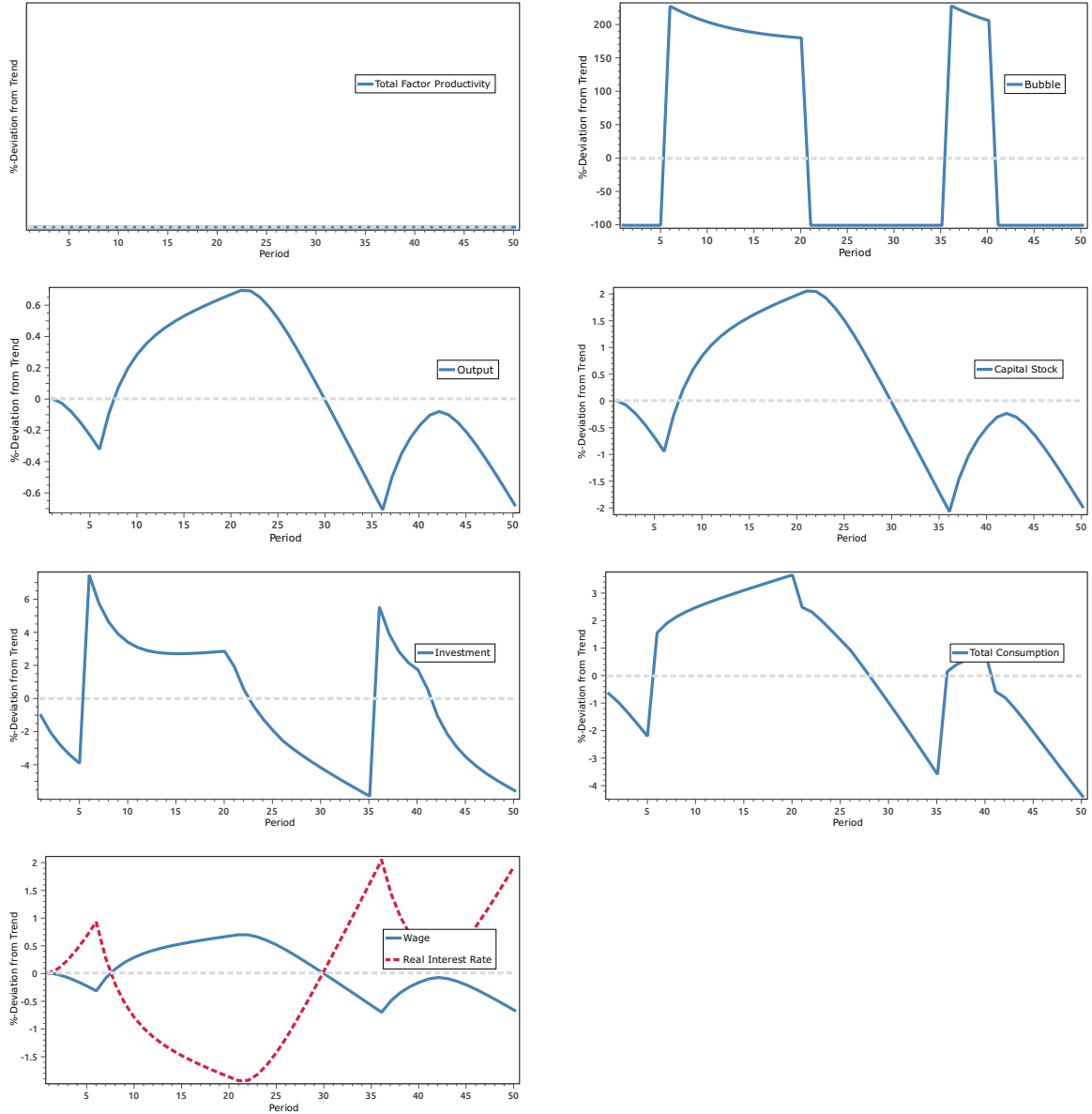
The deterministic counterpart of the true stochastic dynamic system of equations as well as the solution algorithm are described in appendix [C.2](#).

I will compare three different numerical solutions in the following: i) an economy without bubbles, i.e. θ is set equal to zero, ii) an economy with bubbles but without TFP shocks, and iii) an economy with both types of shocks.

[Figure 4](#) in the appendix depicts the impulse response functions for the model economy without bubbles. For all $t = 2, \dots, 30$ z_t^a is equal to zero, except for $t = 1$, where a positive shock of one standard deviation occurs. When the positive TFP shock materializes in the first period output, wages, interest rates, consumption, and investment increase immediately, whereas capital reacts with a lag of one period. Consumption increases by 0.45 percent, output by 1.4 percent, and investment by more than 8 percent. All variables converge back to their steady states, but consumption converges very slowly reflecting the households' desire for distributing the positive income shock evenly over their life-span.

[Figure 2](#) depicts a simulation for an economy with bubbles and without TFP shocks. During the 50 years simulated two episodes of asset price bubbles occur. When bubbles emerge capital stock, output, investment, and consumption increase and stay above their

Figure 2: Bubble-driven business cycles



respective trend. When bubbles burst all four variables decline. From the height of the first bubble-driven cycle to the subsequent trough output declines by approximately 1.5 percentage points, capital stock by four percentage points and consumption by almost eight percentage points. Note, aggregate consumption is much more volatile than in the bubble-less model economy depicted in [Figure 4](#). The reason is that bursting bubbles destroy a part of individual wealth, whereas TFP shocks imply a reduction in wages or interest rates leaving current wealth unchanged.

How does the model fare with respect to second moments? To answer this question second moments of empirical time series of aggregate output, consumption, investment, and hours for the US are reported in [Table 3](#) and second moments of the time series

Table 3: Business cycles statistics of the US-economy

Variable	Standard deviation	Relative standard deviation	First-order auto-correlation	Contemporaneous correlation with output
Output	0.014	1.000	0.221	1.000
Consumption	0.011	0.819	0.287	0.917
Investment	0.047	3.382	0.171	0.943
Hours	0.014	1.006	0.302	0.890

Table 4: Business cycles statistics of the model

Variable	Standard deviation	Relative standard deviation	First-order auto-correlation	Contemporaneous correlation with output
<i>TFP-shocks</i>				
Output	0.009	1.000	0.127	1.000
Consumption	0.002	0.241	0.172	0.985
Investment	0.057	6.263	0.020	0.933
<i>Bubble-shocks</i>				
Output	0.001	1.000	0.459	1.000
Consumption	0.007	12.100	0.052	-0.242
Investment	0.020	35.850	0.032	-0.447
<i>Both shocks</i>				
Output	0.009	1.000	0.115	1.000
Consumption	0.007	0.784	0.088	0.313
Investment	0.060	6.588	0.075	0.858

generated by the model are reported in [Table 4](#). The empirical time series are from the US Bureau of Economic Analysis and cover the period 1954–2011. All variables – the empirical time series as well as the generated time series from the model – are expressed in logarithms and HP-filtered with an annual smoothing parameter of 6.25 as proposed by [Ravn and Uhlig \(2002\)](#). The time series generated by the model are obtained from simulating the model for 500 periods and dropping the first 100 observations in order to make sure that initial conditions do not effect the calculated second moments.⁸

The empirical time series reveal standard business cycle facts,⁹ namely that i) investment is three times more volatile than output, ii) aggregate hours are as volatile as output, iii) consumption is less volatile than output, and iv) all four variables are procyclical (strong contemporaneous correlation with output). Since I am looking at annual data the first-order autocorrelation is of course lower than for quarterly data implying less persistent fluctuations at the annual frequency.

⁸On a Intel® Core™ i7 processor with four times 2.80GHz the computation of the model for 500 periods takes approximately 30 minutes.

⁹See, for instance, [King and Rebelo \(1999, ch. 2.2\)](#) for a description of stylized facts of US business cycles.

The results for the life-cycle RBC model without bubbles is depicted in the first four rows of [Table 4](#). The model generates slightly insufficient volatility in output, slightly to strong volatility in investment and far to low volatility in consumption. The relative volatility of consumption with respect to output is almost four times smaller than in the empirical time series. The persistence of all variables is to small while their contemporaneous correlation with output matches the empirical moments quite well.

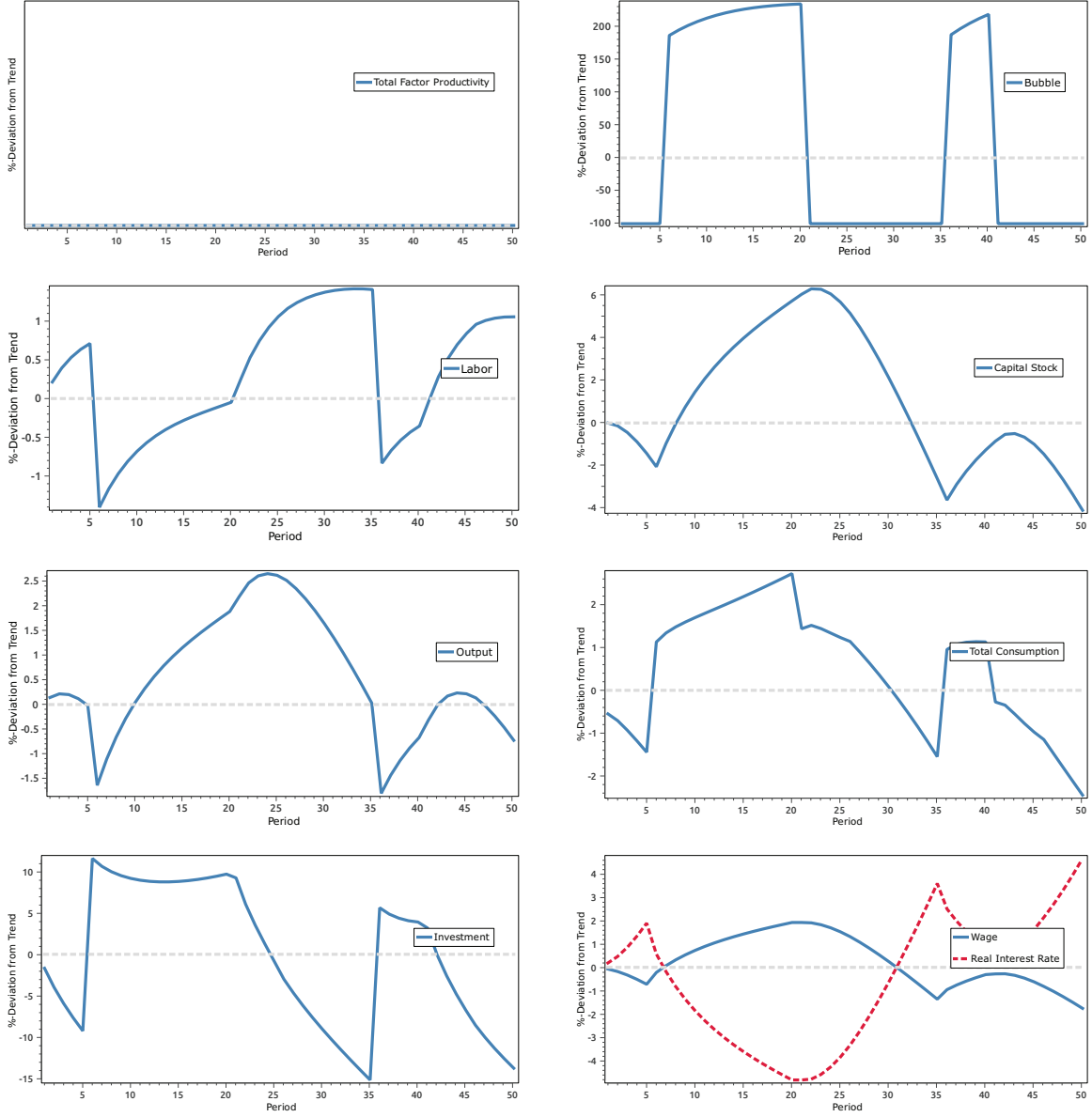
Considering the model economy with shocks from evolving and bursting bubbles and without TFP-shocks in [Table 4](#) reveals implausible business cycle statistics. By construction of the rather small bubble (1 percent of the capital stock in the steady state) bubble-driven business cycles imply rather small deviations of output, consumption and investment from their respective trends. Consumptions and investment are way to volatile in relation to output. This is a result of the transmission channel of bubbles, which imply drastic changes in individual wealth. Taken as a sole source of business cycle fluctuations evolving and bursting bubbles seem rather implausible. Adding asset price bubbles to the model economy with TFP-shocks, however, results in second moments that better match their empirical counterparts. Most importantly, the model can generate much higher and empirically more plausible volatility of consumption. Relative to output the volatility of consumption in the model (0.784) is very close to its empirical counterpart (0.819). This better performance in terms of consumption volatility comes at the cost of lower contemporaneous correlation of consumption with output, as can be seen in the last column of [Table 4](#).

4. The model with elastic labor supply

In RBC models, another important propagation mechanism besides capital accumulation is labor supply. I therefore loosen the assumption of an inelastic labor supply in this section. The changes to the model are shown in [appendix B](#).

The computation of the steady state is similar to the model with inelastic labor supply, except that i) besides k also l has to be guessed and updated initially and ii) also $l^{j,s}$ has to be computed in the solution of the household problem. The solution algorithm is provided in [appendix D](#). In the steady state $k = 0.4338$, $l = 0.399$, $b = 0.01474$,

Figure 3: Bubble-driven business cycles (endogenous labor supply)



$R^P = 1.2515$, $R^U = 0.7509$, and the bubble is considerably smaller than aggregate wealth of all unproductive individuals which is equal to 0.5814.

The impulse-response function for a TFP-shock in the model with elastic labor supply and without bubbles is shown in Figure 5 in the appendix. The graphs depict the difference of a time-path where the error term z_t^a is equal to zero for all t and a time-path where $z_t^a = 0$ for all t except for $t = 1$, where z_t^a is equal to one standard-deviation σ^a . Two qualitative differences to the model with inelastic labor supply are striking. First, the lagged increase of capital stock is much more pronounced such that output first jumps to a higher level and afterwards increases even further before converging back to its steady state level. Second, aggregate consumption drops initially, stays below its steady state

for 15 years and stays above its steady state afterwards for a very long period before converging back to its steady state. Individuals smooth consumption and therefore even forgo higher consumption and leisure in the first periods in order to earn and save more and finance higher consumption in later periods of their lives. Compared to the model with inelastic labor supply individuals can now better smooth consumption via adjusting their labor supply which results in a flatter consumption curve. Although the drop in consumption is quantitatively very small (0.1 percent) it stands in contrast to empirical business cycle facts. All variables except consumption are much more volatile in the model with elastic labor supply.

Figure 3 depicts the results of a simulation with two bubble-episodes and without TFP shocks. When bubbles emerge capital, consumption, and investment increase, and when bubbles burst capital, consumption, and investment decline, similar to the model with inelastic labor supply. However, in this model individuals react to the shocks of evolving and bursting bubbles by adjusting their labor supply. The result is that labor and output respond slower to the bubble shocks and first increase when bubbles burst before decreasing. Consumption is slightly less volatile than output, output is more volatile than labor and capital and investment are most volatile. Similar to the TFP-shock depicted in Figure 5 in the appendix all variables except consumption are more volatile than in the model with elastic labor supply.

Second moments generated from the model with elastic labor supply are depicted in Table 5. The model without bubbles and with TFP-shocks generates volatility in output which is very close to its empirical counterpart, but consumption and working hours are far too stable and investment is far too volatile. Consumption is countercyclical and investment is acyclical. Both should be procyclical. The model with bubble-shocks and without TFP-shocks generates higher volatilities in consumption, investment and hours, lower volatility in output (due to the rather small bubbles) and countercyclical investment. Adding bubble-shocks to the economy with TFP-shocks yields higher, but still too low, volatilities in consumption and working hours. The contemporaneous correlation with output is also higher and consumption is now almost acyclical.

Table 5: Business cycles statistics for the model with elastic labor supply

Variable	Standard deviation	Relative standard deviation	First-order auto-correlation	Contemporaneous correlation with output
<i>TFP-shocks</i>				
Output	0.013	1.000	0.139	1.000
Consumption	0.0004	0.033	0.371	-0.749
Investment	0.113	8.645	0.026	0.922
Hours	0.001	0.470	0.145	0.100
<i>Bubble-shocks</i>				
Output	0.003	1.000	0.127	1.000
Consumption	0.005	1.604	0.059	-0.849
Investment	0.034	10.650	0.067	-0.904
Hours	0.004	1.201	0.066	0.962
<i>Both shocks</i>				
Output	0.013	1.000	0.168	1.000
Consumption	0.005	0.380	0.064	-0.276
Investment	0.118	8.948	0.113	0.774
Hours	0.007	0.541	0.116	0.938

5. Conclusion

Given their empirical relevance for business cycles it is important to integrate asset price bubbles into DSGE models. This paper presents an important step towards this goal. The computation of a life-cycle RBC model featuring rational asset price bubbles shows that under an empirically reasonable parameter constellation asset price bubbles lead to strong fluctuations in output, consumption, labor supply, capital stock and investment. The simulated time paths show that the model is able to depict i) evolving asset price bubbles together with increasing capital, investment, consumption, labor and output and ii) bursting asset price bubbles leading to decreasing capital, investment, consumption, labor and output. However, when labor supply is endogenous bursting bubbles lead to first increasing labor supply before it starts to decline, such that output even shortly increases before decreasing in the subsequent periods.

The model did not consider a government and hence also no taxes, pensions and monetary policy reacting to evolving and bursting bubbles. Adding a social security system with pay-as-you-go pensions to the model would result in lower aggregate savings and a lower capital stock, increasing the interest rate and possibly making the existence of rational bubble impossible. However, only a part of the US-pension system is based on governmental pay-as-you-go transfers, while the major part of pension is either occupational or private. Further, since World War II investment in the US has been heavily

driven by capital imports as the US run mostly current account deficits. The model economy is a closed economy abstracting from international capital flows. Considering international capital flows would increase the capital stock and make bubbles more likely. Including further relevant macroeconomic aspects into the model to see how bubbles can exist and conducting a welfare analyses remains therefore of future interest.

References

- ALLAIS, M. (1947). *Economie et intérêt*, vol. 1 and 2. Paris: Imprimerie Nationale.
- AUERBACH, A. and KOTLIKOFF, L. (1987). *Dynamic Fiscal Policy*. Cambridge University Press.
- BERNANKE, B. and GERTLER, M. (1989). Agency costs, net worth, and business fluctuations. *The American Economic Review*, **79** (1), pp. 14–31.
- and — (1999). Monetary policy and asset price volatility. *Economic Review-Federal Reserve Bank of Kansas City*, **84** (4), 17.
- BLANCHARD, O. J. and WATSON, M. W. (1983). *Bubbles, Rational Expectations and Financial Markets*. NBER Working Paper ID 226909, Social Science Research Network, Rochester, NY.
- BOZ, E. and MENDOZA, E. G. (2014). Financial innovation, the discovery of risk, and the us credit crisis. *Journal of Monetary Economics*, **62**, 1–22.
- BRUNNERMEIER, M. K. (2008). Bubbles. In L. Blume and S. Durlauf (eds.), *The new Palgrave dictionary of economics*, vol. 2, Palgrave MacMillan.
- and SANNIKOV, Y. (2014). A macroeconomic model with a financial sector. *American Economic Review*, **104** (2), 379–421.
- COVAS, F. and DEN HAAN, W. J. (2012). The role of debt and equity finance over the business cycle*. *The Economic Journal*, **122** (565), 1262–1286.
- DIAMOND, P. A. (1965). National debt in a neoclassical growth model. *The American Economic Review*, pp. 1126–1150.
- FARHI, E. and TIROLE, J. (2012). Bubbly Liquidity. *The Review of Economic Studies*, **79** (2), 678–706.
- GAGNON, J. E. (1990). Solving the stochastic growth model by deterministic extended path. *Journal of Business & Economic Statistics*, **8** (1), 35–36.

- GALÍ, J. (2014). Monetary policy and rational asset price bubbles. *American Economic Review*, **104** (3), 721–52.
- HAVRÁNEK, T. (2015). Measuring intertemporal substitution: The importance of method choices and selective reporting. *Journal of the European Economic Association*.
- HEER, B. and MAUSSNER, A. (2008). Computation of business cycle models: A comparison of numerical methods. *Macroeconomic Dynamics*, **12** (05), 641–663.
- and MAUSSNER, A. (2009). *Dynamic general equilibrium modeling: computational methods and applications*. Springer.
- and MAUSSNER, A. (2012). The burden of unanticipated inflation: analysis of an overlapping-generations model with progressive income taxation and staggered prices. *Macroeconomic Dynamics*, **16** (02), 278–308.
- IACOVIELLO, M. and PAVAN, M. (2013). Housing and debt over the life cycle and over the business cycle. *Journal of Monetary Economics*, **60** (2), 221–238.
- KING, R. G. and REBELO, S. T. (1999). Resuscitating real business cycles. *Handbook of macroeconomics*, **1**, 927–1007.
- LUIK, M.-A. and WESSELBAUM, D. (2014). Bubbles over the us business cycle: A macroeconometric approach. *Journal of Macroeconomics*, **40**, 27–41.
- MARTIN, A. and VENTURA, J. (2012). Economic Growth with Bubbles. *American Economic Review*, **102** (6), 3033–3058.
- MIAO, J., WANG, P. and XU, Z. (2016). A bayesian dsge model of stock market bubbles and business cycles. *Forthcoming in Quantitative Economics*.
- PRESCOTT, E. (1986). *Theory ahead of business cycle measurement*. Staff Report 102, Federal Reserve Bank of Minneapolis.
- RAVN, M. O. and UHLIG, H. (2002). On adjusting the hodrick-prescott filter for the frequency of observations. *Review of economics and statistics*, **84** (2), 371–376.

- RÍOS-RULL, J.-V. (1996). Life-cycle economies and aggregate fluctuations. *The Review of Economic Studies*, **63** (3), 465–489.
- SAMUELSON, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *The journal of political economy*, **66** (6), 467–482.
- TIROLE, J. (1985). Asset bubbles and overlapping generations. *Econometrica: Journal of the Econometric Society*, pp. 1499–1528.
- UN (2013). World population prospects: The 2012 revision. *Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat*, New York.
- WEIL, P. (1987). Confidence and the Real Value of Money in an Overlapping Generations Economy. *The Quarterly Journal of Economics*, **102** (1), 1–22.

Appendix

A. The basic model

A.1. Timing of events

Within a period t the exact timing of events is as follows:

- The sequence of capital $\left\{a_t^{P,s}, a_t^{U,s}\right\}_{s=1}^{s=T+T^R}$ and bubbles $\left\{b_t^{P,s}, b_t^{U,s}\right\}_{s=1}^{s=T+T^R}$ as well as the return to bubbles in the optimistic state q_t^o are predetermined from the previous period.
- The two shocks z_t^a and z_t^b materialize.
- Given total factor productivity A_t production takes place and labor and capital are rewarded with their respective factor prices.
- If $z_t^b = 1$ bubbles from the previous period and newly created bubbles are traded. If $z_t^b = 0$ the bubble bursts.
- Given capital income and labor income as well as the income from selling bubbles individuals choose consumption $c_t^{j,s}$ and adjust their stock of capital $a_{t+1}^{j,s+1}$ and bubbles $b_t^{j,s+1}$.

A.2. Normalization

Firms. Normalized output, wages, and interest rates are given by

$$\begin{aligned} y_t &= A_t \left[\alpha k_t^\kappa + (1 - \alpha) \left(\frac{1}{1 + \phi} \right)^\kappa \right]^{\frac{1}{\kappa}} \\ \tilde{w}_t &:= \frac{w_t}{E_t} = (1 - \alpha) A_t^\kappa [(1 + \phi) y_t]^{1 - \kappa} \\ r_t &= \alpha A_t^\kappa \left(\frac{y_t}{k_t} \right)^{1 - \kappa} - \delta. \end{aligned}$$

Bubbles. The equations for bubbles in normalized variables change as follows

$$b_t = \sum_{s=1}^{T+T^R} N_0^s \left[\eta \tilde{b}_t^{P,s} + (1-\eta) \tilde{b}_t^{U,s} \right] \quad (25)$$

$$\tilde{b}_t^N := \frac{\hat{b}_t^N}{E_t} = z_r^b \theta \quad (26)$$

$$b_t^N = \frac{\eta}{1+\phi} \tilde{b}_t^N \quad (27)$$

$$1 + q_t = z_t^b (1 + q_t^o). \quad (28)$$

Note, $\frac{N_t^s}{N_t} = \frac{N_0^s}{N_0} = N_0^s$ is the time-constant population share of cohort s .

Household optimum. The budget constraints now read

$$\begin{aligned} \tilde{c}_t^{j,s} + (1+g) \left(\tilde{a}_{t+1}^{j,s+1} + \tilde{b}_{t+1}^{j,s+1} \right) &= \tilde{w}_t^j + R_t^j \tilde{a}_t^{j,s} + (1+q_t) \tilde{b}_t^{j,s} & s = 1, \dots, T \\ \tilde{c}_t^{j,s} + (1+g) \left(\tilde{a}_{t+1}^{j,s+1} + \tilde{b}_{t+1}^{j,s+1} \right) &= R_t^j \tilde{a}_t^{j,s} + (1+q_t) \tilde{b}_t^{j,s} & s = T+1, \dots, T+T^R \\ 0 &= \tilde{a}_t^{j,1} = \tilde{b}_t^{j,1} = \tilde{a}_t^{j,T+T^R+1} = \tilde{b}_t^{j,T+T^R+1}, & \end{aligned} \quad (29)$$

with $\tilde{w}_t^P := \tilde{w}_t + \tilde{b}_t^N$ and $\tilde{w}_t^U := \tilde{w}_t$.

The FOCs in normalized variables are given by

$$\begin{aligned} (1+g)^\epsilon \left(\tilde{c}_t^{j,s} \right)^{-\epsilon} &= \beta \mathbb{E}_t \left\{ R_{t+1}^j \left(\tilde{c}_{t+1}^{j,s+1} \right)^{-\epsilon} \right\} & s = 1, \dots, T+T^R-1 \\ (1+g)^\epsilon \left(\tilde{c}_t^{j,s} \right)^{-\epsilon} &= \beta \mathbb{E}_t \left\{ (1+q_{t+1}) \left(\tilde{c}_{t+1}^{j,s+1} \right)^{-\epsilon} \right\} + \tilde{\omega}_{t+1}^{j,s+1} & s = 1, \dots, T+T^R-1 \\ \tilde{\omega}_{t+1}^{j,s+1} \tilde{b}_{t+1}^{j,s+1} &= 0 \quad \text{and} \quad (29), & \end{aligned} \quad (30)$$

with $\tilde{\omega}_{t+1}^{j,s+1} := E_{t+1}^\epsilon \omega_{t+1}^{j,s+1}$.

Equilibrium. The equilibrium conditions in normalized variables are given by

$$\begin{aligned} \frac{L_t}{N_t} &= \frac{1}{1+\phi} \\ k_{t+1} &= \sum_{s=1}^{T+T^R} N_0^{s+1} \left[\eta \tilde{a}_{t+1}^{P,s+1} + \sigma(1-\eta) \tilde{a}_{t+1}^{U,s+1} \right] \\ (1+q_t)(b_t + b_t^N) &= (1+g)(1+n) \sum_{s=1}^{T+T^R} N_0^{s+1} \left[\eta \tilde{b}_{t+1}^{P,s+1} + (1-\eta) \tilde{b}_{t+1}^{U,s+1} \right] \end{aligned}$$

$$y_t + (1 - \delta)k_t = c_t + (1 + n)(1 + g)k_{t+1} + (1 + g)(1 - \eta)(1 - \sigma) \sum_{s=1}^{T+T^R} N_0^s \tilde{a}_{t+1}^{U,s+1}$$

$$\sum_{s=1}^{T+T^R} \eta N_0^s \tilde{a}_{t+1}^{P,s+1} \geq 0 \quad \text{and} \quad \sum_{s=1}^{T+T^R} (1 - \eta) N_0^s \tilde{a}_{t+1}^{U,s+1} \geq 0.$$

B. The model with elastic labor supply

Each young individual has an time endowment of one unit per period. Individual leisure is given by $l_t^{j,s}$, individual labor supply by $n_t^{j,s}$ and $1 = l_t^{j,s} + n_t^{j,s}$. The instantaneous utility function is given by

$$u(c, l) = \frac{(cl^\psi)^{1-\epsilon} - 1}{1-\epsilon}, \quad (31)$$

where $\psi > 0$ is the relative weight given to leisure and ϵ is the inverse of the intertemporal elasticity of substitution.

The budget constraints now read

$$\begin{aligned} \tilde{c}_t^{j,s} + (1+g) \left(\tilde{a}_{t+1}^{j,s+1} + \tilde{b}_{t+1}^{j,s+1} \right) &= \tilde{w}_t^j n_t^{j,s} + R_t^j \tilde{a}_t^{j,s} + (1+q_t) \tilde{b}_t^{j,s} & s = 1, \dots, T \\ \tilde{c}_t^{j,s} + (1+g) \left(\tilde{a}_{t+1}^{j,s+1} + \tilde{b}_{t+1}^{j,s+1} \right) &= R_t^j \tilde{a}_t^{j,s} + (1+q_t) \tilde{b}_t^{j,s} & s = T+1, \dots, T+T^R \\ 0 &= \tilde{a}_t^{j,1} = \tilde{b}_t^{j,1} = \tilde{a}_t^{j,T+T^R+1} = \tilde{b}_t^{j,T+T^R+1}, \end{aligned} \quad (32)$$

and the FOC now also contain an intra-temporal condition reflecting the leisure-consumption trade-off:

$$\begin{aligned} \psi \frac{\tilde{c}_t^{j,s}}{\tilde{l}_t^{j,s}} &= w_t \\ (1+g)^\epsilon (\tilde{c}_t^{j,s})^{-\epsilon} (\tilde{l}_t^{j,s})^{\psi(1-\epsilon)} &= \beta \mathbb{E}_t \left\{ R_{t+1}^j (\tilde{c}_{t+1}^{j,s+1})^{-\epsilon} (\tilde{l}_{t+1}^{j,s+1})^{\psi(1-\epsilon)} \right\} & s = 1, \dots, T+T^R-1 \\ (1+g)^\epsilon (\tilde{c}_t^{j,s})^{-\epsilon} (\tilde{l}_t^{j,s})^{\psi(1-\epsilon)} &= \beta \mathbb{E}_t \left\{ (1+q_{t+1}) (\tilde{c}_{t+1}^{j,s+1})^{-\epsilon} (\tilde{l}_{t+1}^{j,s+1})^{\psi(1-\epsilon)} \right\} + \tilde{\omega}_{t+1}^{j,s+1} & s = 1, \dots, T+T^R-1 \\ \tilde{\omega}_{t+1}^{j,s+1} \tilde{b}_{t+1}^{j,s+1} &= 0 \quad \text{and} \quad (32), \end{aligned} \quad (33)$$

Output, wages, and interest rates can be written as

$$\begin{aligned} y_t &= A_t [\alpha k_t^\kappa + (1-\alpha) l_t^\kappa]^{\frac{1}{\kappa}} \\ \tilde{w}_t &= (1-\alpha) A_t^\kappa \left(\frac{y_t}{l_t} \right)^{1-\kappa} \\ r_t &= \alpha A_t^\kappa \left(\frac{y_t}{k_t} \right)^{1-\kappa} - \delta \end{aligned}$$

where $l_t := \frac{L_t}{N_t}$ is the average labor supply. Furthermore, the labor-market clearing con-

dition changes to:

$$l_t = \sum_{s=1}^{T+T^R} N_0^s \left[\eta n_t^{P,s} (1 - \eta) n_t^{U,s} \right]. \quad (34)$$

All other equations remain unchanged.

Based on [Ríos-Rull \(1996\)](#), the new utility parameter capturing the weight of leisure in the utility function, ψ , is set equal to 2 in the computation.¹⁰

¹⁰ [Ríos-Rull \(1996\)](#) uses a slightly different instantaneous utility function. By defining $\psi := \frac{1 - \alpha_{\text{Ríos-Rull}}}{\alpha_{\text{Ríos-Rull}}}$ and $\epsilon := 1 - \alpha_{\text{Ríos-Rull}}(1 - \sigma_{\text{Ríos-Rull}})$, where $\alpha_{\text{Ríos-Rull}}$ and $\sigma_{\text{Ríos-Rull}}$ are parameters of the utility function in [Ríos-Rull \(1996\)](#), the utility function in [Ríos-Rull \(1996\)](#) can be transformed into the utility function of this paper. In [Ríos-Rull \(1996\)](#) α is set equal to 0.33 resulting in $\psi = 2.03$.

C. Computation methods

C.1. Solution algorithm for the steady state

1. Define all parameter values, set A equal to one, and compute \bar{z} as well as N_0^1 .
2. Provide an initial guess for k .
3. Given k obtain the values for (obey the order) r, \tilde{w}, q, q^1 and

$$\begin{aligned} b &= \frac{\theta(1+q)\eta}{(1+\phi)[(1+n)g+n-q]} \\ b^N &= \frac{\eta\theta}{1+\phi} \\ \tilde{b}^U &= \frac{1}{(1-\eta)(1-N_0^1)}b, \quad \text{and} \quad \tilde{b}^P = 0. \end{aligned}$$

4. Solve for $\{\tilde{s}^{P,s}, \tilde{s}^{U,s}\}_{s=1}^{T+T^R-1}$ and by using the FOC conditions (22). For each $j \in \{P, U\}$:

- Guess $\tilde{c}^{j,1}$
- Compute all $\{\tilde{c}^{j,s}\}_{s=2}^{T+T^R}$ by iterating over the Euler equation
- Given $\{\tilde{c}^{j,s}\}_{s=1}^{T+T^R}$ obtain all $\{\tilde{s}^{j,s+1}\}_{s=1}^{T+T^R}$ from the budget constraints, where $\tilde{s}^{j,1}0$ is set equal to zero, but $\tilde{s}^{j,T+T^R+1}$ is not predetermined
- If $\tilde{s}^{j,T+T^R+1}$ is not close enough to zero as defined by chosen tolerance criterion repeat the last three steps with an updated $\tilde{c}^{j,1}$ according to the Secant Method (Newton's Method does also work, but it is slower):

$$\tilde{c}_{i+2}^{j,1} = \tilde{c}_{i+1}^{j,1} - \frac{\tilde{c}_{i+1}^{j,1} - \tilde{c}_i^{j,1}}{\tilde{s}_{i+1}^{j,T+T^R+1} - \tilde{s}_i^{j,T+T^R+1}} \tilde{s}_{i+1}^{j,T+T^R+1} \quad (35)$$

5. Compute $\{\tilde{a}^{j,s+1}\}_{s=1}^{T+T^R-1}$ by $\tilde{a}^{j,s} = \tilde{s}^{j,s} - \tilde{b}^j$.
6. Compute k' by making use of the capital and labor market equilibrium condition

$$k' = \frac{1}{1+n} \sum_{s=1}^{T+T^R} N_0^s [\eta \tilde{a}^{P,s+1} + \sigma(1-\eta) \tilde{a}^{U,s+1}]$$

7. Compare both k' with k . If the difference is close to zero (defined by a tolerance criterion) stop. Otherwise update the initial guess of k and go back to step 3.

In the last step of the solution algorithm the updating of the initial guess for k is linear, i.e. a linear combination of k' and the previous initial guess k (l). The steady state solution is robust to small changes in parameters and initial guesses and takes only a few seconds.

C.2. Solution of the dynamic stochastic system

The conditional expectation of the bubble shocks is equal to the condition probability that $z_{t+k}^b = 1$ (the other possibility, $z_{t+k}^b = 0$, drops out due to the zero):

$$\tilde{z}_{t+k}^b := \mathbb{E}_t[z_{t+k}^b] = (1 - z_t^b)P_{21}^{(k)} + z_t^b P_{11}^{(k)}, \quad (36)$$

where $P_{ij}^{(k)}$ is the entry in the i -th row and j -th column of P^k .

The conditional expectation of the TFP-shock is equal to zero, i.e. $\tilde{z}_{t+k}^a := 0$ for all $k > 0$. The conditional expectation of A from period t on is given by

$$\tilde{A}_{t+k} := A_t^{\rho^k}. \quad (37)$$

This deterministic dynamic system is now obtained by replacing all z_{t+k}^b by \tilde{z}_{t+k} and all A_{t+k} by \tilde{A}_{t+k} for $k > 0$. In the firm sector A_{t+k} is replaced by \tilde{A}_{t+k} for all $k > 1$ in the equations for output, wages, and interest rates (25).

The households' optimization problem is now a problem under certainty with the FOC:

$$\begin{aligned} (1+g)^\epsilon (\tilde{c}_t^{j,s})^{-\epsilon} &= \beta R_{t+1}^j (\tilde{c}_{t+1}^{j,s+1})^{-\epsilon} & s = 1, \dots, T+T^R-1 \\ (1+g)^\epsilon (\tilde{c}_t^{j,s})^{-\epsilon} &= \beta(1+q_{t+1}) (\tilde{c}_{t+1}^{j,s+1})^{-\epsilon} + \tilde{\omega}_{t+1}^{j,s+1} & s = 1, \dots, T+T^R-1 \\ \tilde{\omega}_{t+1}^{j,s+1} \tilde{b}_{t+1}^{j,s+1} &= 0 \quad \text{and} \quad (29), & \end{aligned} \quad (38)$$

where the investment-efficiency-specific returns $R_{t+1}^P = (1+r_{t+1})$ and $R_{t+1}^U = \sigma(1+r_{t+1})$

are deterministic, as well as the return to bubbles as given by

$$1 + q_{t+1} = \tilde{z}_{t+1}(1 + q_{t+1}^o). \quad (39)$$

The same arguments as in the deterministic steady state apply with respect to the return to bubbles (see (23)). I consider only bubbles that potentially crowd-in capital such that the return to bubbles is given by

$$\tilde{z}_{t+k}^b(1 + q_{t+k}^o) = \sigma(1 + r_{t+k}) \quad (40)$$

for all $k > 0$.

Solution Algorithm

1. Initialization:

- Define parameter values and initial values $\{\tilde{a}_1^{P,s}, \tilde{b}_1^{P,s}, \tilde{a}_1^{U,s}, \tilde{b}_1^{U,s}\}_{s=1}^{T+T^R-1}, q_1^o$
- Choose nn numbers of periods for the simulation
- Draw a random sequence of the shocks $\{z_t^a, z_t^b\}_{t=1}^{nn}$
- Choose a number of transitional periods N large enough for the (deterministic) system to be close to its steady state
- Calculate the deterministic steady state as explained in [subsection 3.2](#)

2. At each point in time $t = 1, \dots, nn$:

- Calculate the conditional expectations \tilde{z}_{t+k}^a and $\tilde{z}_{t+k}^b \forall k = 1, \dots, N$ as described by (37) and (36)
- Transform the whole dynamic stochastic equation system into a deterministic system by replacing (z_{t+k}^a, z_{t+k}^b) with their expected values $(\tilde{z}_{t+k}^a, \tilde{z}_{t+k}^b)$ and solve this deterministic model for the N periods by applying the direct computation method, i.e. guess $\{k_{t+k}\}_{k=1}^T$ (that gives also $\tilde{w}_t, r_t, q_t, q_t^1, b_t, \tilde{b}_t^U$, and b_t^N), solve for the individual problem, and update the initial guess $\{k_{t+k}\}_{k=1}^T$ until convergence.¹¹

¹¹The household problem is solved similarly by guessing $c_t^{j,1}$, computing all other c 's by iterating over the

- From the solution store $\{\tilde{a}_{t+1}^{P,s}, \tilde{a}_{t+1}^{U,s}, \tilde{b}_{t+1}^{P,s}, \tilde{b}_{t+1}^{U,s}, \tilde{c}_t^{P,s}, \tilde{c}_t^{U,s}\}_{s=1}^{T+T^R}$ and q_{t+1}^o
- Use $\{\tilde{a}_{t+1}^{P,s}, \tilde{a}_{t+1}^{U,s}, \tilde{b}_{t+1}^{P,s}, \tilde{b}_{t+1}^{U,s}\}_{s=1}^{T+T^R}$ and q_{t+1}^o as initial values for period $t + 1$.

C.3. Solution algorithm for the steady state with elastic labor supply

1. Define all parameter values, set A equal to one, and compute \bar{z} as well as N_0^1 .
2. Provide an initial guess for k and l .
3. Given k and l obtain the values for (obey the order) $r, \tilde{w}, \tilde{d}, q, q^1$ and

$$b = \frac{\gamma_1(1+q)\eta}{(1+n)(1+g)(1+\phi) - (1+q)(1+\phi + \gamma_2\eta)}$$

$$b^N = \frac{\eta}{1+\phi} (\gamma_1 + \gamma_2 b)$$

$$\tilde{b}^U = \frac{1}{(1-\eta)(1-N_0^1)} b, \quad \text{and} \quad b^P = 0.$$

4. Solve for $\{\tilde{s}^{P,s}, \tilde{s}^{U,s}\}_{s=1}^{T+T^R-1}$ and $\{\tilde{l}^{P,s}, \tilde{l}^{U,s}\}_{s=1}^T$ by using the Euler equations. For each $j \in \{P, U\}$:
 - Guess $\tilde{c}^{j,1}$
 - Compute all $\{\tilde{c}^{j,s}\}_{s=2}^{T+T^R}$ by iterating over the Euler equations
 - Obtain all $\{\tilde{s}^{j,s+1}\}_{s=1}^{T+T^R}$ from the budget constraints
 - If $\tilde{s}^{j,T+T^R+1}$ is not close enough to zero (as defined by some chosen tolerance) repeat the last 3 steps with an updated \tilde{c}^1 (here the Secant Method, Newton's Method does also work):

$$\tilde{c}_{i+2}^{j,1} = \tilde{c}_{i+1}^{j,1} - \frac{\tilde{c}_{i+1}^{j,1} - \tilde{c}_i^{j,1}}{\tilde{s}_{i+1}^{j,T+T^R+1} - \tilde{s}_i^{j,T+T^R+1}} \tilde{s}_{i+1}^{j,T+T^R+1} \quad (41)$$

5. Compute $\{\tilde{a}^{s+1,U}\}_{s=1}^{T+T^R-1}$ by $\tilde{a}^{s,U} = \tilde{s}^{s,U} - \tilde{b}^U$ ($\tilde{a}^{s,P} = \tilde{s}^{s,P}$).
6. Compute $\{\tilde{l}^{P,s}, \tilde{l}^{U,s}\}_{s=1}^T$ from the intra-temporal FOC

Euler equation, and computing all s 's by making use of the budget constraints. The last $\tilde{s}_t^{j,T+T^R+1}$ should then be equal to zero, otherwise the initial guess of $\tilde{c}_t^{j,1}$ is updated until convergence.

7. Compute k' and l' by making use of the capital and labor market equilibrium condition

$$k' = \frac{1}{1+n} \sum_{s=1}^{T+T^R} N_0^s [\eta \tilde{a}^{P,s+1} + \sigma(1-\eta) \tilde{a}^{U,s+1}]$$

$$l' = \sum_{s=1}^T N_0^s [\eta(1-l^{P,s}) + (1-\eta)(1-l^{U,s})].$$

8. Compare both k' and l' with k and l , respectively. If the largest of both differences is close to zero stop. Otherwise update the initial guess of k and l and go back to step 3.

D. Solution algorithm for the dynamic stochastic system with elastic labor supply

1. Initialize

- Define parameter values and initial values $\{\tilde{a}_1^{s,P}, \tilde{b}_1^{s,P}, \tilde{a}_1^{s,U}, \tilde{b}_1^{s,U}\}_{s=1}^{T+T^R-1}, q_1^1$ (I choose the deterministic steady state)
- Choose nn numbers of periods for the simulation
- Draw a random sequence of the shocks $\{z_t^a, z_t^b\}_{t=1}^{nn}$
- Choose a number of transitional periods N large enough for the (deterministic) system to be close to its steady state
- Calculate the deterministic steady state for z_t^a and z_t^b equal to the stationary expected value of the Markov process, \bar{z}^a and \bar{z}^b

2. At each point in time $t = 1, \dots, nn$:

- Calculate the conditional expectations \tilde{z}_{t+k}^a and $\tilde{z}_{t+k}^b \forall k = 1, \dots, N$ as given by (37) and (36)
- Transform the whole dynamic stochastic equation system into a deterministic system by replacing (z_{t+k}^a, z_{t+k}^b) with their expected values $(\tilde{z}_{t+k}^a, \tilde{z}_{t+k}^b)$ and solve this deterministic model for the N periods by applying the direct computation method, i.e. guess $\{k_{t+k}, l_{t+k}\}_{k=1}^T$ (that gives also $\tilde{w}_t, r_t, q_t, q_t^1, b_t, \tilde{b}_t^U, b_t^N$,

\tilde{d}_t), and solve for the individual problem, update the initial guess $\{k_{t+k}, l_{t+k}\}_{k=1}^T$ until convergence.¹²

- From the solution store $\{\tilde{a}_{t+1}^{s,P}, \tilde{a}_{t+1}^{s,U}, \tilde{b}_{t+1}^{s,P}, \tilde{b}_{t+1}^{s,U}, \tilde{c}_t^{s,P}, \tilde{c}_t^{s,U}\}_{s=1}^{T+T^R}$ and q_{t+1}^1
- Use $\{\tilde{a}_{t+1}^{s,P}, \tilde{a}_{t+1}^{s,U}, \tilde{b}_{t+1}^{s,P}, \tilde{b}_{t+1}^{s,U}\}_{s=1}^{T+T^R}$ and q_{t+1}^1 as initial values for period $t + 1$.

¹²The household problem is solved similarly by guessing $c_t^{j,1}$, computing all other c 's by iterating over the Euler equation where l is substituted, computing all s 's by making use of the budget constraints. The last $s_t^{j,T+T^R+1}$ should then be equal to zero, otherwise the initial guess of $c_t^{j,1}$ is updated until convergence.

E. Figures

Figure 4: Impulse response functions for a TFP shock in a bubble-less economy

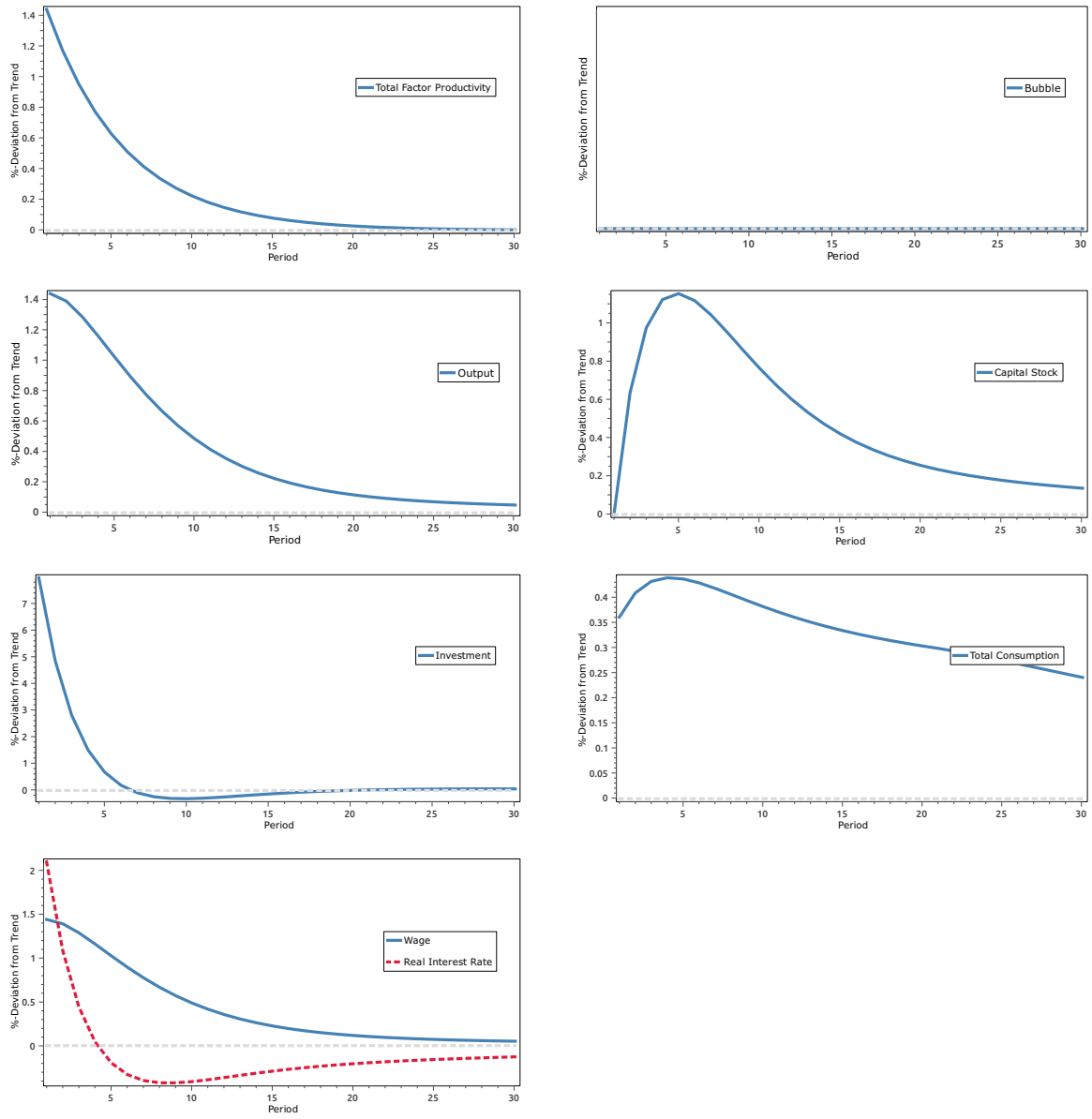
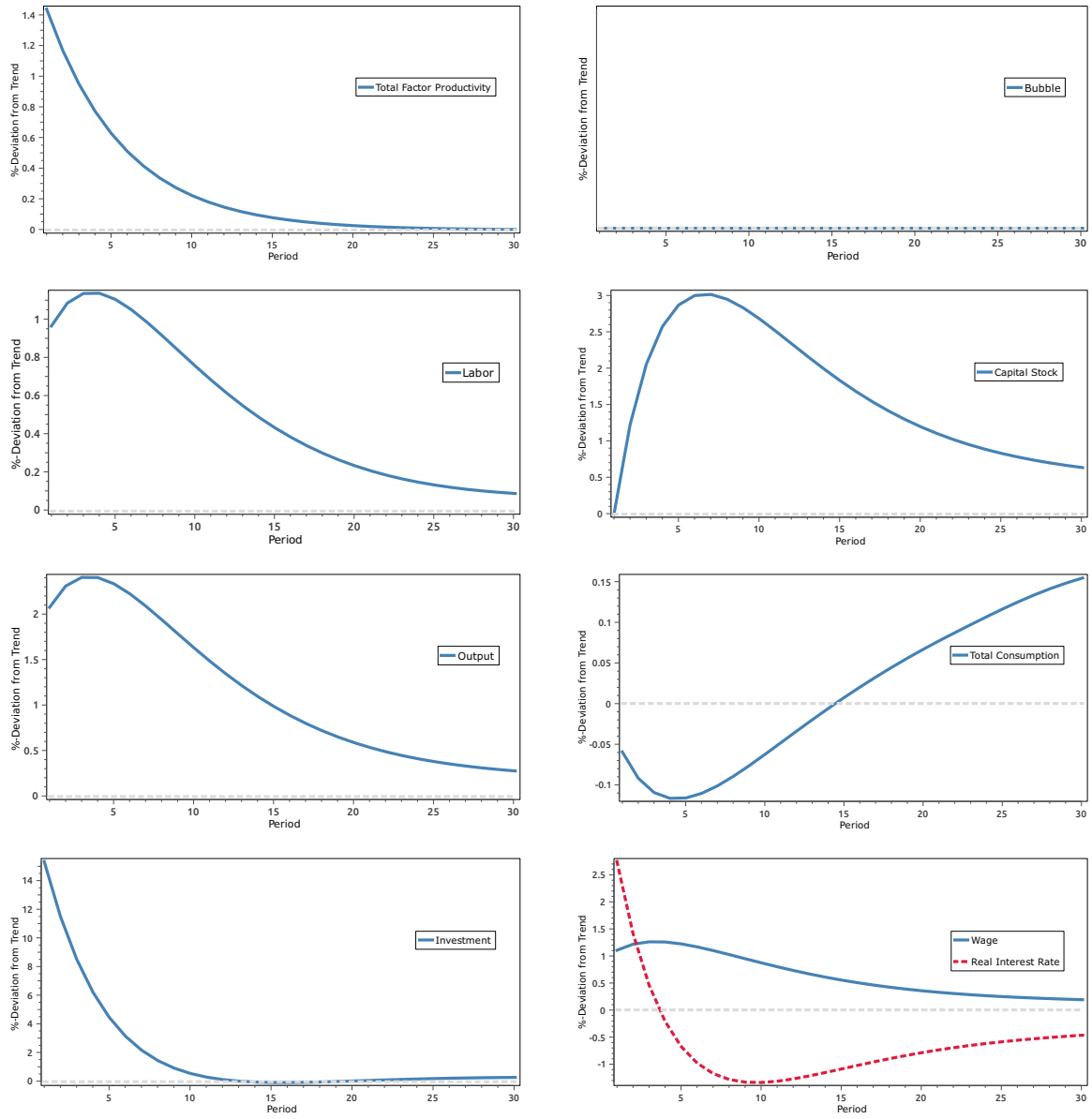


Figure 5: Impulse response functions for a TFP shock in a bubble-less economy with elastic labor supply



Universität Leipzig

Wirtschaftswissenschaftliche Fakultät

Nr. 1	Wolfgang Bernhardt	Stock Options wegen oder gegen Shareholder Value? Vergütungsmodelle für Vorstände und Führungskräfte 04/1998
Nr. 2	Thomas Lenk / Volkmar Teichmann	Bei der Reform der Finanzverfassung die neuen Bundesländer nicht vergessen! 10/1998
Nr. 3	Wolfgang Bernhardt	Gedanken über Führen – Dienen – Verantworten 11/1998
Nr. 4	Kristin Wellner	Möglichkeiten und Grenzen kooperativer Standortgestaltung zur Revitalisierung von Innenstädten 12/1998
Nr. 5	Gerhardt Wolff	Brauchen wir eine weitere Internationalisierung der Betriebswirtschaftslehre? 01/1999
Nr. 6	Thomas Lenk / Friedrich Schneider	Zurück zu mehr Föderalismus: Ein Vorschlag zur Neugestaltung des Finanzausgleichs in der Bundesrepublik Deutschland unter besonderer Berücksichtigung der neuen Bundesländer 12/1998
Nr. 7	Thomas Lenk	Kooperativer Förderalismus – Wettbewerbsorientierter Förderalismus 03/1999
Nr. 8	Thomas Lenk / Andreas Mathes	EU – Osterweiterung – Finanzierbar? 03/1999
Nr. 9	Thomas Lenk / Volkmar Teichmann	Die fiskalischen Wirkungen verschiedener Forderungen zur Neugestaltung des Länderfinanzausgleichs in der Bundesrepublik Deutschland: Eine empirische Analyse unter Einbeziehung der Normenkontrollanträge der Länder Baden-Württemberg, Bayern und Hessen sowie der Stellungnahmen verschiedener Bundesländer 09/1999
Nr. 10	Kai-Uwe Graw	Gedanken zur Entwicklung der Strukturen im Bereich der Wasserversorgung unter besonderer Berücksichtigung kleiner und mittlerer Unternehmen 10/1999
Nr. 11	Adolf Wagner	Materialien zur Konjunkturforschung 12/1999
Nr. 12	Anja Birke	Die Übertragung westdeutscher Institutionen auf die ostdeutsche Wirklichkeit – ein erfolg-versprechendes Zusammenspiel oder Aufdeckung systematischer Mängel? Ein empirischer Bericht für den kommunalen Finanzausgleich am Beispiel Sachsen 02/2000
Nr. 13	Rolf H. Hasse	Internationaler Kapitalverkehr in den letzten 40 Jahren – Wohlstandsmotor oder Krisenursache? 03/2000
Nr. 14	Wolfgang Bernhardt	Unternehmensführung (Corporate Governance) und Hauptversammlung 04/2000
Nr. 15	Adolf Wagner	Materialien zur Wachstumsforschung 03/2000
Nr. 16	Thomas Lenk / Anja Birke	Determinanten des kommunalen Gebührenaufkommens unter besonderer Berücksichtigung der neuen Bundesländer 04/2000
Nr. 17	Thomas Lenk	Finanzwirtschaftliche Auswirkungen des Bundesverfassungsgerichtsurteils zum Länderfinanzausgleich vom 11.11.1999 04/2000
Nr. 18	Dirk Büttel	Continuous linear utility for preferences on convex sets in normal real vector spaces 05/2000
Nr. 19	Stefan Dierkes / Stephanie Hanrath	Steuerung dezentraler Investitionsentscheidungen bei nutzungsabhängigem und nutzungsunabhängigem Verschleiß des Anlagenvermögens 06/2000
Nr. 20	Thomas Lenk / Andreas Mathes / Olaf Hirschfeld	Zur Trennung von Bundes- und Landeskompetenzen in der Finanzverfassung Deutschlands 07/2000
Nr. 21	Stefan Dierkes	Marktwerte, Kapitalkosten und Betafaktoren bei wertabhängiger Finanzierung 10/2000
Nr. 22	Thomas Lenk	Intergovernmental Fiscal Relationships in Germany: Requirement for New Regulations? 03/2001
Nr. 23	Wolfgang Bernhardt	Stock Options – Aktuelle Fragen Besteuerung, Bewertung, Offenlegung 03/2001

Nr. 24	Thomas Lenk	Die „kleine Reform“ des Länderfinanzausgleichs als Nukleus für die „große Finanzverfassungs-reform“? 10/2001
Nr. 25	Wolfgang Bernhardt	Biotechnologie im Spannungsfeld von Menschenwürde, Forschung, Markt und Moral Wirtschaftsethik zwischen Beredsamkeit und Schweigen 11/2001
Nr. 26	Thomas Lenk	Finanzwirtschaftliche Bedeutung der Neuregelung des bundestaatlichen Finanzausgleichs – Eine allokoative und distributive Wirkungsanalyse für das Jahr 2005 11/2001
Nr. 27	Sören Bär	Grundzüge eines Tourismusmarketing, untersucht für den Südraum Leipzig 05/2002
Nr. 28	Wolfgang Bernhardt	Der Deutsche Corporate Governance Kodex: Zuwahl (comply) oder Abwahl (explain)? 06/2002
Nr. 29	Adolf Wagner	Konjunkturtheorie, Globalisierung und Evolutionsökonomik 08/2002
Nr. 30	Adolf Wagner	Zur Profilbildung der Universitäten 08/2002
Nr. 31	Sabine Klinger / Jens Ulrich / HansJoachim Rudolph	Konjunktur als Determinante des Erdgasverbrauchs in der ostdeutschen Industrie? 10/2002
Nr. 32	Thomas Lenk / Anja Birke	The Measurement of Expenditure Needs in the Fiscal Equalization at the Local Level Empirical Evidence from German Municipalities 10/2002
Nr. 33	Wolfgang Bernhardt	Die Lust am Fliegen Eine Parabel auf viel Corporate Governance und wenig Unternehmensführung 11/2002
Nr. 34	Udo Hielscher	Wie reich waren die reichsten Amerikaner wirklich? (US-Vermögensbewertungsindex 1800 – 2000) 12/2002
Nr. 35	Uwe Haubold / Michael Nowak	Risikoanalyse für Langfrist-Investments Eine simulationsbasierte Studie 12/2002
Nr. 36	Thomas Lenk	Die Neuregelung des bundesstaatlichen Finanzausgleichs auf Basis der Steuerschätzung Mai 2002 und einer aktualisierten Bevölkerungsstatistik 12/2002
Nr. 37	Uwe Haubold / Michael Nowak	Auswirkungen der Renditeverteilungsannahme auf Anlageentscheidungen Eine simulationsbasierte Studie 02/2003
Nr. 38	Wolfgang Bernhard	Corporate Governance Kodex für den Mittel-Stand? 06/2003
Nr. 39	Hermut Kormann	Familienunternehmen: Grundfragen mit finanzwirtschaftlichen Bezug 10/2003
Nr. 40	Matthias Folk	Launhardtsche Trichter 11/2003
Nr. 41	Wolfgang Bernhardt	Corporate Governance statt Unternehmensführung 11/2003
Nr. 42	Thomas Lenk / Karolina Kaiser	Das Prämienmodell im Länderfinanzausgleich – Anreiz- und Verteilungsmittelnwirkungen 11/2003
Nr. 43	Sabine Klinger	Die Volkswirtschaftliche Gesamtrechnung des Haushaltsektors in einer Matrix 03/2004
Nr. 44	Thomas Lenk / Heide Köpping	Strategien zur Armutsbekämpfung und –vermeidung in Ostdeutschland: 05/2004
Nr. 45	Wolfgang Bernhardt	Sommernachtsfantasien Corporate Governance im Land der Träume. 07/2004
Nr. 46	Thomas Lenk / Karolina Kaiser	The Premium Model in the German Fiscal Equalization System 12/2004
Nr. 47	Thomas Lenk / Christine Falken	Komparative Analyse ausgewählter Indikatoren des Kommunalwirtschaftlichen Gesamt-ergebnisses 05/2005
Nr. 48	Michael Nowak / Stephan Barth	Immobilienanlagen im Portfolio institutioneller Investoren am Beispiel von Versicherungsunternehmen Auswirkungen auf die Risikosituation 08/2005

Nr. 49	Wolfgang Bernhardt	Familiengesellschaften – Quo Vadis? Vorsicht vor zu viel „Professionalisierung“ und Ver-Fremdung 11/2005
Nr. 50	Christian Milow	Der Griff des Staates nach dem Währungsgold 12/2005
Nr. 51	Anja Eichhorst / Karolina Kaiser	The Institutional Design of Bailouts and Its Role in Hardening Budget Constraints in Federations 03/2006
Nr. 52	Ullrich Heilemann / Nancy Beck	Die Mühlen der Ebene – Regionale Wirtschaftsförderung in Leipzig 1991 bis 2004 08/2006
Nr. 53	Gunther Schnabl	Die Grenzen der monetären Integration in Europa 08/2006
Nr. 54	Hermut Kormann	Gibt es so etwas wie typisch mittelständige Strategien? 11/2006
Nr. 55	Wolfgang Bernhardt	(Miss-)Stimmung, Bestimmung und Mitbestimmung Zwischen Juristentag und Biedenkopf-Kommission 11/2006
Nr. 56	Ullrich Heilemann / Annika Blaschzik	Indicators and the German Business Cycle A Multivariate Perspective on Indicators of Ifo, OECD, and ZEW 01/2007
Nr. 57	Ullrich Heilemann	“The Soul of a new Machine” zu den Anfängen des RWI-Konjunkturmodells 12/2006
Nr. 58	Ullrich Heilemann / Roland Schuhr / Annika Blaschzik	Zur Evolution des deutschen Konjunkturzyklus 1958 bis 2004 Ergebnisse einer dynamischen Diskriminanzanalyse 01/2007
Nr. 59	Christine Falken / Mario Schmidt	Kameralistik versus Doppik Zur Informationsfunktion des alten und neuen Rechnungswesens der Kommunen Teil I: Einführende und Erläuternde Betrachtungen zum Systemwechsel im kommunalen Rechnungswesen 01/2007
Nr. 60	Christine Falken / Mario Schmidt	Kameralistik versus Doppik Zur Informationsfunktion des alten und neuen Rechnungswesens der Kommunen Teil II Bewertung der Informationsfunktion im Vergleich 01/2007
Nr. 61	Udo Hielscher	Monti della città di firenze Innovative Finanzierungen im Zeitalter Der Medici. Wurzeln der modernen Finanzmärkte 03/2007
Nr. 62	Ullrich Heilemann / Stefan Wappler	Sachsen wächst anders Konjunkturelle, sektorale und regionale Bestimmungsgründe der Entwicklung der Bruttowertschöpfung 1992 bis 2006 07/2007
Nr. 63	Adolf Wagner	Regionalökonomik: Konvergierende oder divergierende Regionalentwicklungen 08/2007
Nr. 64	Ullrich Heilemann / Jens Ulrich	Good bye, Professir Phillips? Zum Wandel der Tariflohndeterminanten in der Bundesrepublik 1952 – 2004 08/2007
Nr. 65	Gunther Schnabl / Franziska Schobert	Monetary Policy Operations of Debtor Central Banks in MENA Countries 10/2007
Nr. 66	Andreas Schäfer / Simone Valente	Habit Formation, Dynastic Altruism, and Population Dynamics 11/2007
Nr. 67	Wolfgang Bernhardt	5 Jahre Deutscher Corporate Governance Kodex Eine Erfolgsgeschichte? 01/2008
Nr. 68	Ullrich Heilemann / Jens Ulrich	Viel Lärm um wenig? Zur Empirie von Lohnformeln in der Bundesrepublik 01/2008
Nr. 69	Christian Groth / Karl-Josef Koch / Thomas M. Steger	When economic growth is less than exponential 02/2008
Nr. 70	Andreas Bohne / Linda Kochmann	Ökonomische Umweltbewertung und endogene Entwicklung peripherer Regionen Synthese einer Methodik und einer Theorie 02/2008
Nr. 71	Andreas Bohne / Linda Kochmann / Jan Slavík / Lenka Slavíková	Deutsch-tschechische Bibliographie Studien der kontingenten Bewertung in Mittel- und Osteuropa 06/2008

Nr. 72	Paul Lehmann / Christoph Schröter-Schlaack	Regulating Land Development with Tradable Permits: What Can We Learn from Air Pollution Control? 08/2008
Nr. 73	Ronald McKinnon / Gunther Schnabl	China's Exchange Rate Impasse and the Weak U.S. Dollar 10/2008
Nr. 74	Wolfgang Bernhardt	Managervergütungen in der Finanz- und Wirtschaftskrise Rückkehr zu (guter) Ordnung, (klugem) Maß und (vernünftigem) Ziel? 12/2008
Nr. 75	Moritz Schularick / Thomas M. Steger	Financial Integration, Investment, and Economic Growth: Evidence From Two Eras of Financial Globalization 12/2008
Nr. 76	Gunther Schnabl / Stephan Freitag	An Asymmetry Matrix in Global Current Accounts 01/2009
Nr. 77	Christina Ziegler	Testing Predictive Ability of Business Cycle Indicators for the Euro Area 01/2009
Nr. 78	Thomas Lenk / Oliver Rottmann / Florian F. Woitek	Public Corporate Governance in Public Enterprises Transparency in the Face of Divergent Positions of Interest 02/2009
Nr. 79	Thomas Steger / Lucas Bretschger	Globalization, the Volatility of Intermediate Goods Prices, and Economic Growth 02/2009
Nr. 80	Marcela Munoz Escobar / Robert Holländer	Institutional Sustainability of Payment for Watershed Ecosystem Services. Enabling conditions of institutional arrangement in watersheds 04/2009
Nr. 81	Robert Holländer / WU Chunyou / DUAN Ning	Sustainable Development of Industrial Parks 07/2009
Nr. 82	Georg Quaas	Realgrößen und Preisindizes im alten und im neuen VGR-System 10/2009
Nr. 83	Ulrich Heilemann / Hagen Findeis	Empirical Determination of Aggregate Demand and Supply Curves: The Example of the RWI Business Cycle Model 12/2009
Nr. 84	Gunther Schnabl / Andreas Hoffmann	The Theory of Optimum Currency Areas and Growth in Emerging Markets 03/2010
Nr. 85	Georg Quaas	Does the macroeconomic policy of the global economy's leader cause the worldwide asymmetry in current accounts? 03/2010
Nr. 86	Volker Grossmann / Thomas M. Steger / Timo Trimborn	Quantifying Optimal Growth Policy 06/2010
Nr. 87	Wolfgang Bernhardt	Corporate Governance Kodex für Familienunternehmen? Eine Widerrede 06/2010
Nr. 88	Philipp Mandel / Bernd Süßmuth	A Re-Examination of the Role of Gender in Determining Digital Piracy Behavior 07/2010
Nr. 89	Philipp Mandel / Bernd Süßmuth	Size Matters. The Relevance and Hicksian Surplus of Agreeable College Class Size 07/2010
Nr. 90	Thomas Kohstall / Bernd Süßmuth	Cyclic Dynamics of Prevention Spending and Occupational Injuries in Germany: 1886-2009 07/2010
Nr. 91	Martina Padmanabhan	Gender and Institutional Analysis. A Feminist Approach to Economic and Social Norms 08/2010
Nr. 92	Gunther Schnabl / Ansgar Belke	Finanzkrise, globale Liquidität und makroökonomischer Exit 09/2010
Nr. 93	Ulrich Heilemann / Roland Schuhr / Heinz Josef Münch	A "perfect storm"? The present crisis and German crisis patterns 12/2010
Nr. 94	Gunther Schnabl / Holger Zemanek	Die Deutsche Wiedervereinigung und die europäische Schuldenkrise im Lichte der Theorie optimaler Währungsräume 06/2011
Nr. 95	Andreas Hoffmann / Gunther Schnabl	Symmetrische Regeln und asymmetrisches Handeln in der Geld- und Finanzpolitik 07/2011
Nr. 96	Andreas Schäfer / Maik T. Schneider	Endogenous Enforcement of Intellectual Property, North-South Trade, and Growth 08/2011
Nr. 97	Volker Grossmann / Thomas M. Steger / Timo Trimborn	Dynamically Optimal R&D Subsidization 08/2011

Nr. 98	Erik Gawel	Political drivers of and barriers to Public-Private Partnerships: The role of political involvement 09/2011
Nr. 99	André Casajus	Collusion, symmetry, and the Banzhaf value 09/2011
Nr. 100	Frank Hüttner / Marco Sunder	Decomposing R^2 with the Owen value 10/2011
Nr. 101	Volker Grossmann / Thomas M. Steger / Timo Trimborn	The Macroeconomics of TANSTAAFL 11/2011
Nr. 102	Andreas Hoffmann	Determinants of Carry Trades in Central and Eastern Europe 11/2011
Nr. 103	Andreas Hoffmann	Did the Fed and ECB react asymmetrically with respect to asset market developments? 01/2012
Nr. 104	Christina Ziegler	Monetary Policy under Alternative Exchange Rate Regimes in Central and Eastern Europe 02/2012
Nr. 105	José Abad / Axel Löffler / Gunther Schnabl / Holger Zemanek	Fiscal Divergence, Current Account and TARGET2 Imbalances in the EMU 03/2012
Nr. 106	Georg Quaas / Robert Köster	Ein Modell für die Wirtschaftszweige der deutschen Volkswirtschaft: Das "MOGBOT" (Model of Germany's Branches of Trade)
Nr. 107	Andreas Schäfer / Thomas Steger	Journey into the Unknown? Economic Consequences of Factor Market Integration under Increasing Returns to Scale 04/2012
Nr. 108	Andreas Hoffmann / Björn Urbansky	Order, Displacements and Recurring Financial Crises 06/2012
Nr. 109	Finn Marten Körner / Holger Zemanek	On the Brink? Intra-euro area imbalances and the sustainability of foreign debt 07/2012
Nr. 110	André Casajus / Frank Hüttner	Nullifying vs. dummifying players or nullified vs. dummified players: The difference between the equal division value and the equal surplus division value 07/2012
Nr. 111	André Casajus	Solidarity and fair taxation in TU games 07/2012
Nr. 112	Georg Quaas	Ein Nelson-Winter-Modell der deutschen Volkswirtschaft 08/2012
Nr. 113	André Casajus / Frank Hüttner	Null players, solidarity, and the egalitarian Shapley values 08/2012
Nr. 114	André Casajus	The Shapley value without efficiency and additivity 11/2012
Nr. 115	Erik Gawel	Neuordnung der W-Besoldung: Ausgestaltung und verfassungsrechtliche Probleme der Konsumtionsregeln zur Anrechnung von Leistungsbezügen 02/2013
Nr. 116	Volker Grossmann / Andreas Schäfer / Thomas M. Steger	Migration, Capital Formation, and House Prices 02/2013
Nr. 117	Volker Grossmann / Thomas M. Steger	Optimal Growth Policy: the Role of Skill Heterogeneity 03/2013
Nr. 118	Guido Heineck / Bernd Süßmuth	A Different Look at Lenin's Legacy: Social Capital and Risk Taking in the Two Germanies 03/2013
Nr. 119	Andreas Hoffmann	The Euro as a Proxy for the Classical Gold Standard? Government Debt Financing and Political Commitment in Historical Perspective 05/2013
Nr. 120	Andreas Hoffmann / Axel Loeffler	Low Interest Rate Policy and the Use of Reserve Requirements in Emerging Markets 05/2013
Nr. 121	Gunther Schnabl	The Global Move into the Zero Interest Rate and High Debt Trap 07/2013
Nr. 122	Axel Loeffler / Gunther Schnabl / Franziska Schobert	Limits of Monetary Policy Autonomy and Exchange Rate Flexibility by East Asian Central Banks 08/2013
Nr. 123	Burkhard Heer / Bernd Süßmuth	Tax Bracket Creep and its Effects on Income Distribution 08/2013
Nr. 124	Hans Fricke / Bernd Süßmuth	Growth and Volatility of Tax Revenues in Latin America 08/2013
Nr. 125	Ulrich Volz	RMB Internationalisation and Currency Co-operation in East Asia 09/2013

Nr. 126	André Casajus / Helfried Labrenz	A property rights based consolidation approach 02/2014
Nr. 127	Pablo Duarte	The Relationship between GDP and the Size of the Informal Economy: Empirical Evidence for Spain 02/2014
Nr. 128	Erik Gawel	Neuordnung der Professorenbesoldung in Sachsen 03/2014
Nr. 129	Friedrun Quaas	Orthodoxer Mainstream und Heterodoxe Alternativen Eine Analyse der ökonomischen Wissenschaftslandschaft 04/2014
Nr. 130	Gene Callahan / Andreas Hoffmann	The Idea of a Social Cycle 05/2014
Nr. 131	Karl Trela	Klimaanpassung als wirtschaftspolitisches Handlungsfeld 06/2014
Nr. 132	Erik Gawel / Miquel Aguado	Neuregelungen der W-Besoldung auf dem verfassungsrechtlichen Prüfstand 08/2014
Nr. 133	Ulf Papenfuß / Matthias Redlich / Lars Steinhauer	Forschend und engagiert lernen im Public Management: Befunde und Gestaltungsanregungen eines Service Learning Lehrforschungsprojektes 10/2014
Nr. 134	Karl Trela	Political climate adaptation decisions in Germany - shortcomings and applications for decision support systems 11/2014
Nr. 135	Ulf Papenfuß / Lars Steinhauer / Benjamin Friedländer	Beteiligungsberichterstattung der öffentlichen Hand im 13-Länder-Vergleich: Erfordernisse für mehr Transparenz über die Governance und Performance öffentlicher Unternehmen 02/2015
Nr. 136	Gunther Schnabl	Japans Lehren für das Schweizer Wechselkursdilemma 02/2015
Nr. 137	Ulf Papenfuß / Christian Schmidt	Determinants of Manager Pay in German State-Owned Enterprises and International Public Policy Implications: 3-Year Study for Sectors, Performance and Gender 02/2015
Nr. 138	Philipp Mandel / Bernd Süßmuth	Public education, accountability, and yardstick competition in a federal system 05/2015
Nr. 139	Gunther Schnabl	Wege zu einer stabilitäts- und wachstumsorientierten Geldpolitik aus österreichischer Perspektive 06/2015
Nr. 140	Ulf Papenfuß / Matthias Redlich / Lars Steinhauer / Benjamin Friedländer	Forschend und engagiert lernen im Public Management: Befunde und Gestaltungsanregungen eines Service Learning Lehrforschungsprojektes – 2. aktualisierte Auflage 08/2015
Nr. 141	Friedrun Quaas / Georg Quaas	Hayeks Überinvestitionstheorie 10/2015
Nr. 142	Bastian Gawellek / Marco Sunder	The German Excellence Initiative and Efficiency Change among Universities, 2001-2011 01/2016
Nr. 143	Benjamin Larin	Bubble-Driven Business Cycles 02/2016