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On the foundation of a general theory of stocks

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Abstract:
This essay develops the “concept of stocks” – a conceptual notion designed to enable a clearer understanding of the interaction between the dynamics of ecosystems and the economy. The notion of stocks is formulated in a general manner based on set theory. The central attribute of a stock is its temporal durability. Seen thus, stocks are suitable for depicting the influences a system’s history has on its present – and hence for analysing temporal developments. Since permanency is a temporal attribute, the concept of stocks is not specifically limited to individual scientific disciplines and is suitable for interdisciplinary analysis. The notion is applied to economic and ecological examples and generalised for stochastic sets. The hierarchical structure of actual ecological-economic systems can be analysed by distinguishing the stock perspective from a system view. The theory of stocks is a building block for the conceptual foundations of ecological economics.

Keywords: Dynamics, permanency, system, time scales, population, persistence

JEL classification: A12, B40, Q20

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1 Introduction

1.1 Background
The human lifetime is unsteady and finite. For the duration of their lives, humans strive to safeguard, as long as is possible, their existence. They do so by using natural structures, and by creating additional ones which feature a certain durability and thus help to maintain this existence. Put very generally, such structures can be termed ‘stocks’. Regarded thus, stocks reflect mankind’s concern over the preservation – and hence, the continuation of its existence. In the economy, for example, natural stocks such as the available water, air and soil, deposits of ore and coal, and the available wood, grain and livestock are used to produce useful consumer goods (like food and clothing) and capital goods (like tools, machines and buildings). Over time, the latter goods accumulate to form intentionally created stocks. However, production also results in unintentional by-products such as sewage, used air and solid waste which, unless naturally broken down, affect natural stocks and endanger the foundations of human life.

1.2 The aim of a general theory of stocks
Instead of investigating a certain environmental problem, in this essay we intend to tackle a general question: how can we develop a general terminology which enables us to better understand the interaction between the dynamics of ecosystems and the economy? We plan to do so by introducing a general concept of stocks. The objective is to start by developing the common language, and a common conceptual scheme, of economics and ecology so that we may describe and analyse the dynamics of coupled systems, made up of economic and natural components.

This common language is essential if problems are to be tackled which lie at the intersection of different disciplines. This applies particularly to environmental problems, which are caused by human activities, affect ecosystems and in turn, have repercussions for mankind. Hence, environmental problems belong to the spheres of ecology (the study of the interaction between life forms and their environment) and economics (the study of human economic decisions). Accordingly, environmental problems and their underlying dynamics cannot be analysed from one angle only. They must be regarded from all sides. Our approach toward a theory of stocks is therefore designed to form a module for the conceptual foundations of ecological economics.

In many cases, environmental problems are of a long-term nature. In other words, they possess a lengthy time structure when compared to the time scales at which humans act. If environmental problems and their formation are to be understood, we need to take into account, not only the momentum of the individual systems in isolation, but also the dynamics of interaction between ecological and economic systems, especially the dynamics induced by human activities.
The concept of stocks developed in this paper represents an important module for considering such dynamics. Dynamics are manifested by temporal developments, i.e. generally speaking, by the development of variables over time.\(^1\) Change, however, is the opposite of being permanent, and is therefore usually measured relative to permanency. As a result, permanency may be regarded as a ‘basis’ against which changes can be analysed. And it is precisely the attribute of permanency which is expressed by the concept of stocks, as developed here.

The attribute of permanency can generally refer to both quantifiable entities, and to those which cannot be described quantitatively – both may possess the attribute of permanency. For formal reasons, however, we shall restrict ourselves to quantifiable stocks in the majority of this paper (especially in the formal definitions).

1.3 The essay’s structure

Section 2 demonstrates that a scientific investigation of temporal phenomena is impossible without subjective factors on the part of the observer, such as the time horizon chosen. This is particularly important for our notion of stocks. Section 3 provides formal definitions of the notion of stocks and discusses its attributes. Section 4 gives a number of concrete examples of stocks taken from economic and ecological systems. Section 5 goes into more detail regarding the dynamics of stocks in the sphere of ecological systems, and stochastically extends the notion of stocks introduced. Section 6 distinguishes between different classes of stocks depending on their degree of human influence. The paper closes with a summary in Section 7.

2 Subjective factors – the level of the observer

Any model-based description of reality – and hence also our attempts to develop a theory of stocks – is influenced by subjective factors on the part of observers, especially their cognitive interest (cf. Baumgärtner and Schiller, 2001: 374–375). By its very nature, a model-based abstraction of reality has to simplify actual circumstances. The choice of simplifications is generally up to the observer and is geared towards the problem’s structural necessities, arising from the observer’s cognitive interest.\(^2\) The same applies to the notion of stocks developed below; the help of which, we hope, will enable us to describe the temporal development of both economic and ecological systems.

At this point, we should take a closer look at two particular aspects of the observer’s role, since they are very important in what follows: the selection or definition of the object of observation itself, and the selection of the observer’s time horizon and the time scale of observation at which the dynamics of the object of observation are to be investigated.

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\(^1\) In addition to movement, dynamics in its original sense also includes the causes of this movement. However, in this essay the two aspects are for didactic reasons dealt with separately. In particular, the notion of stock dynamics used in this essay initially refers solely to the change of variables (cf Definition 3.6).

\(^2\) In addition, there is generally also a whole host of other limitations, such as the limits of measuring accuracy and different types of ignorance.
2.1 Definition of the object of observation

We intend to view the object of observation as a structure comprising elements which usually interact and which we can regard as a system. Principally each element can, in turn, be regarded as a conglomerate of interacting sub-elements (assuming there is a cognitive interest in doing so), i.e. each element can itself be viewed as a system of elements at a lower hierarchical level. On the other hand, a structure regarded as a system at a given hierarchical level can be seen at a higher level, as an element of a ‘super-system’.³

This hierarchical structure results in a large number of possible perspectives, even if the object of observation has already been defined. For example, we can always distinguish between macroscopic and microscopic perspectives. A macroscopic perspective corresponds to the level of aggregation of the entire system, whereas a microscopic view addresses individual elements of the system.

If a system is regarded as a system of interacting elements, it follows automatically that a system is always structured. All living, mental and even non-living phenomena can in this sense be regarded as systems, i.e. they feature system attributes in the sense of structuredness. On the other hand, the notion of stocks developed in this essay is, by its very nature, abstracted from the internal structuredness of the object regarded as a stock. Whenever we refer to anything as a ‘stock’, we shall disregard the internal structuredness with regard to certain attributes of the elements of the stock.⁴ Whether an object of observation is regarded as a system or a stock, does not indicate any objective attributes of the phenomena but instead, depends on the observer’s viewpoint.

For example, a highly complex system such as a human can also be regarded as a stock of cells, while a hierarchical structure of employees in a company can also be seen as a stock of personnel. However, viewing an object as a stock, i.e. abstracting from its internal structuredness, implies that it is embedded in an external structure. Hence generally, it only makes sense to regard stocks as part of a meta-structure or a system, in the way that, for example people, are part of the ecological system ‘biotope’, or a company is viewed as part of the social system ‘national economy’. Then we can consider systems of interacting stocks.⁵

2.2 The time horizon and the time scale of observation

As well as defining the object of observation, a dynamic description needs to be preceded by some idea of what period of time should be covered by the modelling of the temporal development of the system under investigation. Let us call this period the time horizon \( T \).

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³ Alongside the definition of the object of observation, simplifications of the interrelations observed between elements of the system, are generally required. Baumgärtnert and Schiller (2001: 368) view this as the selection of a system representation which provides the basis for the model-based description of reality.

⁴ This is expressed for example in the attribute of appertainment which the elements of a stock have to have (cf. Section 3). With regard to this attribute a stock is homogeneous, i.e. it features no internal structure.

⁵ Hence a certain hierarchy exists between the terms ‘stock’ and ‘system’. Stocks usually exist within the framework of systems. The reverse is not true: within a stock the structuredness (i.e. the system attributes) are neglected. Observing the system structure within stocks would by contrast require a different, more detailed level of observation.
In addition, a temporal description of the object of study cannot be infinitely precise on the time axis. Instead there must be some idea about the time ‘resolution’ the investigation should take. Let us dub this resolution the time scale of observation $\tau$. The length of the time units involved need not be precisely defined; instead it suffices to set the time scale of observation by specifying a temporal order of magnitude for which the dynamics of the system observed are to be described. For example, the development of a meteorological system can be described on a time scale of hours, which would correspond to an investigation of the weather. However, if its development is regarded on a time scale of months, years or even decades, the approach focuses on climatic development. These are two different problems which generally require different approaches.

The choice of the time horizon $T$ and the time scale of observation $\tau$ are essentially only limited by the trivial description $\tau < T$, and in all other respects are independent of each other. For reasons of practicability, however, the two variables are often linked. So that, whereas in the example from above, it would theoretically be possible to have model systems with a high spatial and temporal resolution (with a very small $\tau$) for weather forecasting over a very long time horizon of decades, this would be far too time-consuming, not to mention unusable for predicting actual weather phenomena (owing to the uncertainty implied). Conversely, the dynamic modelling of the ‘climate’ over a short time horizon $T$ of a few months with a time scale of observation $\tau$ of the same order of magnitude would be equally pointless – for this would be akin to a static observation. This enables the above condition for a suitable choice of $\tau$ and $T$ to be expressed more exactly as $\tau \ll T$, where for example for reasons of practicability a lower limit also usually exists for $\tau$ relative to $T$.

Important though, is that neither the time horizon, nor the time scale of observation is objectively certain; instead they result from the observer’s cognitive interest. They are subjective factors at the level of the observer.

2.3 The context of investigation

Let us refer to the summary of the subjective factors specified by the observer – especially the object of observation, time horizon and time scale of observation – as the context of an investigation. The context (and hence the subjective factors mentioned) is determined by the cognitive interest of the researcher and generally refers to a specific problem.

The fact that the context summarises subjective factors does not, by any means, indicate that it is random. During scientific research, the context is determined intersubjectively to meet the requirements of the scientific community. Generally speaking, the quality of science partly depends on the context being suitable chosen. This means, for example, that it should reflect any experience gained of the problem concerned, or that evaluations by the scientific community – regarding, say, suitable time horizons – should be taken into account. One example of the criteria for choosing a suitable context is the conditions for the relationship between the time horizon and the time scale of observation discussed in the previous section.
Below it will become apparent that a context is essential for the exact description of our notion of stocks. Conditions are formulated in the definition, the applicability of which, can only be determined against the background of a certain context.

### 3 Our conception of stocks

The aim now, is to investigate how the phenomenon permanency can be conceptually expressed. We want to assume that the context of an investigation has been given; particularly that the time horizon and time scale have been decided. How the context influences the attribute of permanency is already apparent from everyday linguistic usage. The weather is unchanging (i.e. ‘permanent’) if it remains the same for a number of days. By contrast, a river that changed its course after each bout of winter flooding would usually be described as impermanent, even though the weather actually changes more frequently than the river’s course. This is because most rivers change their course much more slowly, encouraging observers to choose much longer time scales of observation.

#### 3.1 Definitions

We now plan to develop a concept of ‘stocks’ which is based on set theory, thereby making it generally applicable.\(^6\) However, before stocks can be defined, a number of additional terms need to be explained.

**Definition 3.1 (set, attribute of appertainment):** A set is a group of elements that feature a certain common attribute. We call the attribute which distinguishes the set, the *attribute of appertainment*.

The attribute of appertainment of a set is hence the attribute which, by definition, must be met by each element.\(^7\) It is thus an attribute which characterises the individual elements of the set observed and is therefore based on a microscopic perspective. In addition, the attribute of appertainment characterises the set as a whole, i.e. it can also be seen as an attribute from the macroscopic perspective.

We regard the attribute of appertainment of a stock as an *objectifiable* attribute of the elements – in that, intersubjective criteria can be found upon which the decision, as to whether the attribute of appertainment is met, can be based. For instance, the whereabouts of an object can serve as an attribute of appertainment because they can be objectively determined. Should the object’s colour be an attribute of appertainment, if need be a convention must be created by which the wavelength range of visible light is assigned to the exact colour concerned. Wherever possible, purely subjectively perceived attributes of potential elements (such as the

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\(^6\) In Section 5.1 we shall somewhat limit this generality when investigating the dynamics of stocks and call for a spatial definition of stocks (cf Section 3.2).

\(^7\) Regarding other attributes the elements may be different, i.e. heterogeneous.
‘beauty’ of works of art) would first need to be objectified in order to serve as an attribute of appertainment.8

Furthermore, a suitable attribute of appertainment is always chosen depending on the context of the investigation carried out by the observer. Consider for example a number of machines in a factory building. If we intend to investigate how much electricity is consumed by production activities in the factory building, a suitable attribute of appertainment of the set of machines in the building would consist of the following elements: (i) the machines are located inside the building and (ii) they need electricity to work. This, therefore, does not include any machines which are man powered machines. On the other hand, if the assets of the factory’s proprietor are to be investigated in terms of the machines inside the building, the attribute of appertainment regarding the set of machines would consist in the machines that belong to the factory proprietor himself and are located inside the building; leased machines would be left out of the set. This example shows that the context of investigation is crucial when choosing a suitable attribute of appertainment.

From this example it can be seen that the elements of a set can also be characterised by more than one attribute of appertainment (cf i and ii). Below, however, we shall refer to just one attribute of appertainment, which generally covers a number of sub-attributes.

This brings us to the definition of the concept of stocks. For us, the central characteristic of a stock is its attribute of permanency. A set can only be referred to as a stock if it exists for a certain time. The kernel of our concept of stocks is represented by the temporal aspect. This basic idea brings us to the following definition.

**Definition 3.2 (permanency, stock):** In the context of an investigation, a set is permanent at time \( t_0 \) if it is not empty for a period of time beginning at \( t_0 \), which is different from 0 on the time scale \( \tau \) given by the context. A permanent set is known as a stock.

The central attribute of permanency for stocks can only be given relative to a given time scale of observation. It represents a yardstick against which permanency is measured. With respect to the exact period for which the set observed may not be empty, Definition 3.2 needs further particularisation. At this point, the first thing is to demand a positive period for the non-empty set (measured on the time scale of observation \( \tau \)) for permanency as of the time of observation. The definition is discussed and particularised in this respect in Sections 3.2.4–3.2.6.

The time scale of observation underlying Definition 3.2 is given by the context of investigation, i.e. by the cognitive interest of the observer. It is a factor given subjectively by the observer and not an objective characteristic of the stock observed. When describing stocks, however, a second period of time plays an important role: the duration or the remaining duration of the stock to time \( t_0 \).

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8 Here, however, a limitation of this conception of stocks emerges. For example, the ‘stock’ of beautiful pictures in the world can hardly be quantified.
Definition 3.3 (duration of a stock, remaining duration): The duration of a stock denotes its entire period of existence. The remaining duration of a stock $T_{r0}$ at the time of observation $t_0$ is the duration of its existence remaining at time $t_0$.

In contrast to the time scale of observation, the duration of a stock is an objective characteristic of a stock (i.e. independent of the observer) based at the level of the object.\(^9\) The duration of a stock corresponds to the remaining duration at the time at which the stock begins to exist. The role of this duration within the conception of stocks is dealt with in more detail in Section 3.2 below.

Thus far, we have characterised stocks in two areas: (i) The attribute of appertainment must, by definition, be met, from the microscopic perspective by the individual elements of a stock. Hence, the attribute of appertainment distinguishes the stock at the macropscopic perspective from its environment and characterises it. (ii) The duration of the stock characterises it in its temporal dimension.

However, when investigating concrete problems using the notion of stocks, we are also interested (iii) in other attributes of the stock from a macroscopic perspective. These could be attributes expressed by means of a quantitative measure or those, for which no suitable quantitative measure exists. To differentiate we define a quantitative attribute.

Definition 3.4 (quantitative attribute): A quantitative attribute of a set is an attribute of the set which at any time within time horizon $T$ can be expressed on a cardinal scale. A quantitative attribute $A$ can be measured within the time interval $[t_0, T]$ by the function $f_A$: $[t_0, T] \rightarrow \mathbb{R}$.

In the example above, the number of electrical machines in the factory building is a quantitative attribute of the stock, as is the total power consumption of all the machines. Let us refer to such quantitative attributes as stock variables.

Definition 3.5 (stock variable): A stock variable is a quantitative attribute of the stock considered.

If, for example, a sand pile is regarded as a stock; its temperature, mass and weight on the earth’s surface all represent different stock variables. In a population of fish, the number of individuals and their entire biomass are different stock variables. Generally, it will be possible to view a variety of stock variables for any given stock.

When investigating a concrete problem, we therefore need to whittle down a large number of possible stock variables to the few which are suitable for the problem at hand. Alongside the time scale of observation and the definition of the of object investigated, stock variables

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\(^9\) Stating the duration of a stock assumes that the stock has been defined, i.e. that an attribute of appertainment has been defined. The definition of the stock is determined by the context of the investigation, i.e. by subjective factors. This makes the ‘objectivity’ of the attribute ‘duration’ non-absolute, i.e. the duration is not independent of the context. However, it is independent of the – purely subjective – time scale of observation. If the time of observation $t_0$ has also been fixed, the remaining duration of a stock is objective in the sense mentioned.
which are suitable for describing and analysing the stock concerned emerge from the given context.

If, for example, the above-mentioned machines in a factory are observed, both the total power consumption of machines during operation and their entire monetary value represent examples of different stock variables. If the context of investigation comprises a study of production profitability, the financial value of the machinery is an important stock variable. To contrast: if the context of investigation focuses on assessing the climatic relevance of production in the factory, the entire power consumption is a relevant stock variable – but the financial value is not to an equal extent.

Before discussing a few more attributes of our stock definition in Section 3.2 below, let us first introduce the notion of stock dynamics. ‘Stock dynamics’ is used here to refer to the variables of a stock. As already explained, the selection of relevant stock variables depends on the context of investigation – and consequently the same is also true for the stock dynamics.

**Definition 3.6 (stock dynamics):** On the time scale of observation, given by the context of investigation, the stock dynamics of a stock encompass the temporal developments of those stock variables which are relevant within the given time horizon.\(^{10}\)

In this definition, it should be noted that we refer to stock dynamics even if the stock does not change at all during the period considered, i.e. the stock variables are constant over time.\(^{11}\) Furthermore, our notion of dynamics is relative to the time scale of observation and the time horizon (cf Section 2.2). Both ‘cut a window’ out of the temporal developments in the stock variables – generally occurring on different time scales – for consideration in the investigation. Temporal development which is more rapid than the time scale of observation is neglected, as is temporal development which is so slow that it is not gauged within the time horizon.\(^{12}\)

Below we shall discuss some attributes of our concept of stocks.

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\(^{10}\) Similar to the abstraction from the internal structuredness of a stock (cf Section 2.1), our notion of the dynamics regarding stocks, does not include the actual causes of the stock variables’ temporal development at the micro-level. The notion is thus phenomenologically orientated and includes for example phenomenological regularities. The reasons for such regularities can be observed (assuming corresponding cognitive interest) at a lower level of application during a complementary investigation (cf the discussion in Section 2.1).

\(^{11}\) The use of ‘dynamics’ here simply refers to whatever changes or remains constant over time. This notion of dynamics is alien to everyday linguistic usage, which generally associates dynamics with changes to at least one stock variable. However, when investigating the temporal development of an overall system containing the stock observed (cf Section 2.1), both changes to and the constancy of the stock are usually important.

\(^{12}\) Expressed mathematically or technically, this corresponds to a Fourier transform of the actual temporal development of the stock variables observed into the frequency range and the subsequent suppression of frequency components which are too high or too low.
3.2 Attributes of our notion of stocks

3.2.1 Stocks’ spatial dimension

Definition 3.2 formulated the notion of the stock solely using the dimension of time without explicitly mentioning the spatial dimension. Generally, however, a stock also has a spatial attribute. This is self-evident for ecological and for most economic stocks. A population always has a certain area within which it can expand (at most, the whole planet). Similarly, geographical information is part of the description of nearly all economic goods. Consider for example a stock of capital goods – coal-mining machinery, say. Exactly where they can be used is of interest for virtually all questions (e.g. “all the mining machinery in the Lusatian coalfield”).

Formally speaking, any spatial dimension of stocks, in our understanding of the notion, is regarded as a sub-attribute of the attribute of appertainment and is not distinguished from other attributes. However, when we investigate the fundamental mechanisms of stock dynamics in Section 5, we shall deal with the spatial dimension separately – in order to be able to discuss flows across the spatial boundaries of stocks.

3.2.2 Quantitative and qualitative attributes of stocks

Our notion of a stock refers to the temporal attribute of permanency. In particular – given an attribute of appertainment – it does not depend on which attributes of the stock are quantitative (i.e. can be described using stock variables) and which ones are qualitative. Stocks frequently have attributes which cannot be adequately expressed using stock variables. For example, if the legal system is regarded as a stock of laws, the stock variables include the number of laws, the number of paragraphs, the number of printed pages, etc. However, even though the legal system’s fairness is not a stock variable, it still plays a crucial role with regard to social structure, and hence, the stability of society.

In order to keep things manageable, we shall concentrate below on stock attributes which can be quantitatively described, i.e. using stock variables. It should be noted that this represents a certain limitation of the range of use. After all, particularly socio-economic systems may contain numerous entities which feature the attribute of permanency and are therefore important for dynamic observations of such systems, but which are described incompletely with (quantitative) stock variables.

3.2.3 The reasons for permanency: durability and reproduction

Above, we defined the attribute of permanency as a central attribute of stocks. Permanency is an attribute of the externally observable dynamics of a stock, i.e. an attribute seen from the macroscopic perspective. However, it does not say anything about how this permanency came

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13 This stems from the general view of the notion based on set theory. It makes sense since a spatial dimension need not necessarily exist depending on the cognitive interest. For instance, regarding social systems, there are stocks which only exist in time, but to the extent that they are stocks do not have a spatial dimension. One example is monetary stocks, such as bank deposits.
into being. In particular, merely identifying a set’s character of being permanent is by no means enough to identify the determinants of its dynamics, and hence, the reasons for its permanency. In order to identify these causes, it may be necessary to analyse at the level of the component elements. This may, for instance, include interactions with other elements (such as inflows, in the form of investments, and outflows, as the usage of yield, from a capital) or internal dynamic processes (such as the radioactive decay of uranium and other substances).

Below, we shall therefore address two processes from the microscopic perspective – i.e. at the level of individual elements – which are responsible for the attribute of permanency at the macro-level. These are processes which characterise the internal dynamics of the stock: \(^{14}\) (i) the durability of individual elements of the set and (ii) the reproduction of elements of the set. Both processes reflect two fundamental mechanisms – permanency through survival and permanency through reproduction, i.e. through the generation of new elements with the same attributes.

Durability (in the sense intended here) means that the individual elements exist, without interruption, on the time scale of observation and do not change with respect to the attribute of appertainment. The second process which may lead to permanency, and is essential particularly in stocks featuring living elements, is the reproduction of elements of a set. We talk of reproduction when one or more elements produce a new element of the same type. The reproduction of elements may result in the permanency of a stock without the individual elements having to be durable on the time scale of observation. Examples of both cases are given in Section 4.

3.2.4 Three forms of time in connection with our notion of stocks

A total of three expressions of time are important for our notion of stocks. A stock refers (i) to a certain time of observation \(t_0\) for which it is stated whether a set is permanent or not. Also important, in order to define the attribute of permanency, is (ii) the time scale of observation \(\tau\) on which permanency in the sense of continuity must exist. The third expression of time in connection with our concept of stocks was formulated as an objective characteristic of a stock, namely (iii) the remaining duration of a stock at the time of observation \(t_0\). By comparing these three expressions of time, our concept of stocks can now be alternatively formulated and more precisely defined.

**Proposition 3.1 (equivalent definition of permanency):** At time \(t_0\) a set is a stock if the remaining duration of the observed set \(T_{B}^{t_0}\) at time \(t_0\) is much larger than the time scale of observation \(\tau\) given by the context, i.e. \(T_{B}^{t_0} \geq A \tau\), where \(A >> 1\).

Since the time scale of observation \(\tau\) forms the temporal ‘resolution’ with which the stock dynamics are gauged by the observer, the condition in Proposition 3.1 means that the

\(^{14}\) Cf also Section 5.1.
remaining duration of the stock extends over a number of unit lengths. \(^{15}\) This is a reflection of the permanency constituting the kernel of the definition of stocks. Hence such a comparison of the periods can serve as a ‘test’ of whether or not a set represents a stock. Proposition 3.1 can thus assist the empirical determination of permanency of the sets observed.

3.2.5 Empirical Test I: Stocks have a history

Above we introduced the notion of stocks as a theoretical concept in which the conditions under which reliable information is available for identifying the permanency of a concrete set are immaterial. \(^{16}\) This changes if we want to determine empirically whether or not a concrete object of investigation is a stock.

According to Definition 3.2 and Proposition 3.1, if the time of observation \(t_0\) is interpreted as the present, the assessment of permanency of an observed set requires information about its future temporal development. Such information is of course always beset by a degree of uncertainty or even ignorance. In order to be able to provide meaningful information \textit{ex ante} about the permanency of concrete objects of investigation, we need additional information about the stock dynamics.

In many concrete problems, one possible source of this additional information is past experience. Although Section 3.2.6 shows that knowledge of the history of a stock is not \textit{by itself} enough to answer the question of current permanency, indications are normally provided by a stock’s history. \(^{17}\)

When something is referred to today as a stock, this implies that, as a \textit{currently} existing stock, it should necessarily continue to survive in the future and hence, exert influence on it. If the time of observation \(t_0\) is shifted into the past, it follows that we will encounter stocks nowadays which originated in the past and which influence the present. By virtue of its permanency, a stock connects the past via the present to the future on the temporal scale of the time scale of observation given by the context. Hence stocks act as bearers of influence from the past into the present – and thus as factors of history. \(^{18}\)

3.2.6 Empirical Test II: Information about stocks’ dynamics

The empirically determined permanency of a set in the past, cannot by itself, always provide information about a stock’s continued survival in the future, i.e. about the current or future

\(^{15}\) The exact size of the constant \(A\) need not be determined in this general case; it will usually depend on the problem observed.

\(^{16}\) In addition, information about permanency from an \textit{ex post} perspective (i.e. with reference to the past) does not in general represent a problem since in this case complete information can be assumed.

\(^{17}\) Although a stock with no history whatsoever (i.e. a ‘stock’ which begins existing \textit{exactly} at the time of observation \(t_0\)) can, in accordance with Definition 3.2, still be regarded as a stock, it really is a borderline case and does not correspond to the essence which the concept of stocks is designed to express.

\(^{18}\) Schiller (2002: Chapters 3.1 and 4.1) analyses how the present is influenced by the past with the help of a concept introduced by him, namely “causal-factual influences”. He identifies stocks as bearers of these influences and hence as a central element of economic-ecological dynamics.
character of being permanent. As a rule, the dynamics of the set observed and its underlying mechanisms need to be analysed in order to obtain such information. Basically, this can take place at both a causal and a phenomenological level. Owing to the uncertainties existing *ex ante*, stochastic concepts are often helpful – as will become clear in Section 5.

Consider for example living populations. The number of individuals in an ecosystem must not drop below a certain threshold if the constant reproducibility, and hence, the durable continuation of the population is to be guaranteed. As a result, the mere knowledge of the *existence* of the population over a certain period in the past, does not say anything about its current permanency. Other information is needed to provide indications about future population dynamics – and hence indications concerning current and future permanency. Sections 5.2 and 5.3 contain a formal example of how information about the dynamics of stocks can be derived from knowledge of structural circumstances in theoretical ecology.

### 4 Examples of stocks

In accordance with Definition 3.2, many things may be regarded as Stocks. These things can belong to different spheres; here, we are especially interested in stocks from economic and ecological spheres. In the context of typical economic questions, examples of stocks include existing sets of coal, oil, ore, wood, cotton, grain, coffee, pearls, diamonds, gold coins, machines, computers, tools, buildings, roads, ports, ships, refrigerators, cameras and the railway network. All these objects are generally reserves or capital goods, since their usage directly or indirectly assists mankind. Economic stocks also exist which are not reserves or capital goods; such as sets of domestic waste, sludge from ports, car wrecks, obsolete machines, ruins of buildings, carbon dioxide, heavy metal in rivers, etc. Whether a stock also represents capital depends on how it is used. For example, uranium only became a capital good once humans found a use for it – namely in the form of fuel rods following the introduction of nuclear technology.

The attribute of permanency of most of the above-mentioned stocks is based on the durability of the individual elements (cf Section 3.2.3). In other words, their permanency does not involve reproduction. Instead, the permanency of for example pearls, diamonds and gold coins is based on the fact that the individual elements – for example gold coins – have a long duration.

Examples of ecological stocks include lakes, rivers, forests, and deserts, habitats and biotopes, and animal and plant populations. There is a large overlap between economic and ecological stocks. After all, sludge from ports, ruins of buildings, carbon dioxide stocks and heavy metal in rivers can be regarded as both economic and ecological stocks.

In order to exemplify ecological stocks further, we shall now take a closer look at two below: a fox population and a population of butterflies. The context of observation is an ecological field study of the long-term dynamics of the respective populations. It emerges that the time scale of observation \( \tau \) for the fox population is a matter of years with a time horizon \( T \) of at
least a number of decades. For the butterfly population the time scale of observation \( \tau \) is a matter of months with a time horizon \( T \) of about a decade.\(^{19}\)

A fox population is a reproductive community of foxes. The attribute of appertainment, i.e. the attributes which the elements of the set have to fulfil in order to belong to the stock, is in this case, therefore, the possibility to reproduce with other members of the community – in other words membership of the fox species. At the time of observation \( t_0 \) a population is permanent if it will continue to exist for a certain period of time after \( t_0 \). This information must be provided against the background of the time scale of observation given by the context – in this case as mentioned above one or more decades.

A population of butterflies may at first glance seem to die out every winter and hence not feature any permanency. This way of looking at the situation is based on the classical definition of a population and only takes into account the reproductive phase of the life of an individual – the phase of imagoes. Although all imagoes do indeed die in winter, the pupae – from which the butterflies emerge in spring – survive. Hence, if in contrast to the classical population definition, the attribute of appertainment encompasses the individuals in all phases of life, the permanency of the population is given. Although the individuals’ durability does not continue beyond the end of the year, permanency is still assured by reproduction.

5 The dynamics of stocks

Having defined and discussed the notion of stocks, we shall turn our attention to stock dynamics. Stock dynamics include the temporal development of the relevant stock variables belonging to the stock.\(^{20}\) When investigating real phenomena, stocks can always be regarded as interacting elements of a higher system (cf Section 2.1). In order to understand this interaction – and hence the development of the system as a whole – we have to know the dynamics of the stocks involved. Both are essential before controlled intervention can be carried out in the system.

5.1 General mechanisms of stock dynamics

Owing to the diversity of stocks and their interaction, the concrete determinants of stock dynamics cannot be given for all stocks. Instead, in Section 5.1 we intend to identify phenomenologically a number of general mechanisms which may determine the dynamics of a stock.\(^{21}\) For this purpose, the generality of our concept of stocks based on set theory, as taken from Section 3, needs to be restricted slightly with respect to the attribute of appertainment, by implementing a spatial definition of the stock as part of the attribute of appertainment.

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\(^{19}\) In order to predict the dynamics of a population, it needs to be observed over many reproduction cycles. This results in the time scale of observation given in the text.

\(^{20}\) We should remind ourselves that the time scale of observation and what the relevant stock variables are have already been established within the context of the investigation.

\(^{21}\) In Section 5.2 we shall study these mechanisms in more detail in an ecological system.
In this case, the attribute of appertainment of the stock observed (Definition 3.1) comprises a spatial definition and other attributes. However, this is by no means a serious limitation of generality when regarding concrete investigations of economic and ecological stocks. In fact, both economic and ecological stocks do in many cases feature a spatial dimension. For example, economic goods are usually tied to geographical coordinates (since transport is not free of charge and cannot take place infinitely quickly), while ecological stocks are often linked to a certain, spatially defined ecosystem.

In case a stock is spatially defined, there are four general mechanisms determining its dynamics: (i) the import of elements, (ii) the export of elements, (iii) the formation of elements, and (iv) the decay of elements (cf Fig. 1). Mechanisms (i) and (ii) can be directly traced back to the stock’s interaction with the outside world, since they are constituted by flows of elements across the stock’s spatial boundaries. Mechanisms (iii) and (iv) describe the change to the stock variables as caused by internal processes; and are referred to as ‘internal dynamics’ below. Each of the two pairs harbours a process of increase and one of decrease. The mechanisms of internal dynamics are also related via the interaction of the stock with elements outside the stock’s boundaries. For example, the reproduction of elements of an ecological stock depends on the food available, which is not part of the stock. The same goes for the decay of stock elements (iv), which also depends on external circumstances.

Fig. 1: Sketch of the four general mechanisms of stock dynamics when defining a spatial attribute of appertainment: the import (Element 1), export (Element 2), formation (Element 3) and decay (Element 4) of elements of a stock. The circles represent the spatial boundary of a stock at two different times. The squares symbolise (for clarity’s sake distinguishable) elements of a stock. Element 3 doubles between \( t_1 \) and \( t_2 \) to form the two Elements 3a and 3b, whereas Element 4 ceases to exist.

Mechanisms (iii) and (iv) depict processes which are central for the dynamics of a stock. In order to make predictions about these mechanisms with respect to a specific stock, its individual elements need to be observed. This micro-observation requires a change of perspective. If the processes leading to the formation and decay of the individual elements are

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22 Without the spatial definition of the object of observation, there could be no flows across the spatial boundaries and hence no ‘import’ or ‘export’. Therefore, under the general notion of stocks based on set theory in Definition 3.2, the mechanisms ‘formation’ and ‘import’ as well as ‘decay’ and ‘export’ are equivalent – the newly arrived elements for example simply meet the attribute of appertainment.
analysed, the set of elements will be viewed not as a homogeneous stock, but rather as a system of interacting elements.\textsuperscript{23}

In Section 3.2 we identified two reasons for permanency at the level of individual elements: durability and the reproduction of elements. These causes correspond to mechanisms (iii) and (iv) of stock dynamics: reproduction tallies with mechanism (iii), the formation of elements; while the durability, is the exact opposite of the decay of elements (mechanism (iv)). Mechanisms (iii) and (iv) hence make up the kernel of the attribute of permanency. This does not rule out the possibility of the dynamics of a specific stock being dominated by mechanisms (i) and (ii), i.e. import and export, one example being reserves.

5.2 An example: populations and their dynamics

So far we have dealt with the conceptualisation of central terms such as ‘stock’, ‘permanency’ and ‘stock dynamics’. The aim was to develop a common terminology to make it easier for ecologists and economists to analyse and communicate with each other about the dynamic aspects and interrelations of ecological and economic systems. Below we will apply our general concept of stocks to a well-studied example from the sphere of ecology, namely populations and their dynamics, in order to find out just how good it is. We will show that the concept is able to describe important aspects of population dynamics. We shall also see that, depending on the particular question at hand, it may be necessary to sometimes regard a set as a (homogeneous) stock, and in other cases as a system of interacting individual elements.

In a second step (starting with Section 5.3) we shall extend the notion of stocks to create a concept which can also depict stochastic processes. This is necessary to enable a description of phenomena of population dynamics which are of general relevance, and in this essay particularly, concerns the issue of when permanency is exhibited.

5.2.1 The description of a population as a stock

In this section, we consider populations using the definition customary in ecology.\textsuperscript{24} A population is a stock in our sense if it exists without interruption from the start of the investigation $t_0$ until the time $t_0 + A\tau$ (where $\tau$ is the time scale of observation, and $A \gg 1$) and does not die out in between (cf Definition 3.2 and Proposition 3.1). A suitable stock variable is the population size $N_t$, i.e. the number of individuals capable of reproduction at time $t$. In accordance with Definition 3.6, the stock dynamics can then be expressed by the time series \{\(N_{t_0}, N_{t_0+\tau}, N_{t_0+2\tau}, \ldots, N_{t_0+T}\)\} of the population size $N_t$, which starts at time $t_0$ of the investigation period, covers the specified time horizon $T$ of the investigation, and follows the temporal development of the population on the given time scale $\tau$.

\textsuperscript{23} If all the elements of a stock behave identically and non-complexly, further analysis at the micro-level may, under certain circumstances, no longer be necessary and only the stock level will need to be considered. One example is a stock whose elements undergo neither reproduction nor processes of ageing and decay, and so is only affected by import and export.
Such a description of the population dynamics, using solely the population size \( N_t \), entails the idealising assumption that the population forms a homogeneous unit. Differences between individuals (in terms of, for instance, age, state of nutrition or location) are neglected in this approach; all attributes are depicted as effective attributes of a ‘mean’ individual. This population-ecology approach corresponds by its nature to our concept of stocks, which also neglects the internal structure of the stock observed (cf Section 2.1).

The dynamics of closed populations without the exchange of individuals can be described by a growth equation of the form

\[
\frac{dN_t}{dt} = f_t(N_t),
\]

(1)

where the function \( f_t(N_t) \) denotes the population’s total growth rate. Assuming a homogeneous population, this description corresponds to the stock perspective if \( f \) represents a cumulation of the individual reproductive behaviour.

### 5.2.2 A population as a system: internal determinants of population dynamics

If, however, the cognitive interest is not directed towards a phenomenological \textit{ex post} description of the population dynamics at this level of aggregation, but instead for instance, towards understanding the reasons for the dynamics necessary for \textit{ex ante} forecasts, another perspective will have to be taken. The population is then regarded as a system of interacting elements, with the assumption of homogeneity on the part of the individuals being abandoned under certain circumstances.

The starting point for our considerations is in the central approach of population ecology, according to which changes to the population size can be attributed to the four fundamental demographic processes of the birth, death, immigration and emigration of individuals (Wissel, 1989 as well as Begon and Mortimer, 1986). These four processes are forms of the four general mechanisms of the dynamics of stocks identified in Section 5.1.25

The transition to a system perspective is only complete when \( f \) from Equation (1) is expressed as a function of \textit{individual} reproduction rates \( r_i \), which may differ among the individuals \( i \) of the population and can be subdivided into ‘individual number of offspring’ \( b_i \) and ‘individual probability of dying’ \( d_i \). This constitutes an analysis of the fundamental processes of birth and death at the level of individuals.

These individual quantities may be dependent on parameters such as the current population size \( N_t \), since a growing population size \( N_t \) results in harder competition among the individuals for existing resources. The resulting weakening in the position of individuals is generally

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24 A population is seen as a “set of individuals of species existing in genetic exchange” (e.g. Wissel, 1989: 8), which implicitly presupposes that only individuals actually capable of reproduction are considered (cf also Section 4).

25 Immigration and emigration are only features of ‘open’ populations interacting with the outside world (e.g. other populations). By contrast, solely reproduction processes (birth and death) take place in ‘closed’ populations, where there is no exchange of individuals through immigration or emigration.
reflected in a reduced number of offspring or a higher probability of dying. In this way, self-regulation results in the population size $N_t$ remaining at values corresponding to the habitat’s capacity and the resources it contains. Hence, in order to arrive at an understanding of population dynamics from the perspective of a stock, in this case the population needs to be regarded as a system of interacting individuals.

5.2.3 The influence of the outside world on the population: a system of stocks

The driving forces of population development at the level of individuals, $b_t$ and $d_t$, (individuals’ number of offspring and probability of dying), are generally not constant over time. For example, the probability of dying may show a temporal course which follows the dynamics of a population of predators at the same place.\(^{26}\) This makes the dynamics of the population analysed – and hence the dynamics of the stocks – dependent on the dynamics of the population’s abiotic and biotic environment. This environment may include the food supply or the weather, as well as other populations of the same or different species. Generally, these factors of influence are also stocks.\(^{27}\)

If the observer’s cognitive interest is directed towards understanding population dynamics, analysing the population as an individual stock in isolation is usually pointless. Instead a system-based view of the interaction between the population and its ‘outside world’ (i.e. the interaction between the stock being analysed and other stocks) is needed. In this case, the stock is regarded as part of a higher system.

5.3 Long-term effects on population dynamics: extending the notion of stocks

In the previous sections the current population size and the state of the outside world were identified as driving forces behind population dynamics. As a result, the growth behaviour of a population continuously adapts over time to the changing conditions within the population and in the outside world. This has repercussions for the predictability of the population’s long-term dynamics. As the outside world of a population is itself dynamic and is generally a complex system, over relatively long time horizons unpredictable changes to the outside world may occur at any time, affecting individual growth rates. This means that for longer periods of time, often no definitive predictions can be made regarding the development of individual populations.

\(^{26}\) Regarding dependency on weather effects (e.g. temperature and humidity), often encountered in insects, the number of offspring and probability of dying also fluctuate unpredictably, causing them to effectively act like random variables.

\(^{27}\) We can distinguish between direct and indirect effects of the outside world depending on how they affect the individual number of offspring and probability of dying. Direct effects are those which have an immediate effect on the variables, such as the ‘harvesting’ of a population, which results in an immediate increase in the probability of dying. By contrast, indirect effects include those which change a population’s environment and hence influence the effective reproduction rates. For example, reducing the size of a population’s habitat is an indirect intervention because it primarily constitutes a reduction of the resources and hence intensifies competition, which alters the reproduction rates as a secondary effect.
5.3.1 Persistence

Below we shall show that despite the typical fluctuations of a population’s size, stochastic concepts can be used to make certain predictions about a population’s character of being permanent. Such predictions are no longer possible solely on the basis of empirical data, but need a model which can be used to simulate ex ante the development of a population. Modelling stochastic population dynamics is a field of population ecology which has been well investigated (Nisbet and Gurney, 1982, Goel and Richter-Dyn, 1974, Goodman, 1987, Wissel and Stöcker, 1991, Frank et al., 1994 as well as Wissel et al., 1994).

Stochastic population models exhibit typical dynamic behaviour. A simulated population with a certain initial size \( N_{t_0} = N_0^n \) will either die out very quickly or else attain an ‘established state’ characterised by typical fluctuations in the population size \( N_t \) and a constant probability of extinction per unit of time. This is reflected in the structural law of stochastic population dynamics (Verboom, 1991, Stelter et al., 1997 as well as Grimm and Wissel, 2002): the probability \( S_n(t) \) of a population with initially \( n \) individuals surviving until time \( t \) can be approximated by the following exponential law:

\[
S_n(t) = c_n \cdot e^{-t/T_P},
\]

where \( c_n \) is the probability of reaching an established state from an initial population size of \( n \) individuals, and \( T_P \) is the mean lifetime of the population in the established state. The variable \( c_n \) subsumes all the effects stemming from the initial condition \( N_0 = n \), whereas \( T_P \) contains all the effects from the phase of the established state.

This structural law highlights a basic problem. At any time \( t \) there is a certain risk that the population may have already become extinct: \( S_n(t) < c_n \leq 1 \) holds for all \( t > 0 \). This is why population ecology introduced the term ‘persistence’ as a measure of a population’s survival.

**Definition 5.1 (persistence):** Let \( t' > t_0 \) be any point in time until which the population’s survivability is to be observed, and let \( \varepsilon \) be the maximum risk of extinction the observer is willing to accept. A population is then persistent if its probability of survival \( S_n(t') \) at this time \( t' \) is greater than or equal to \( 1 - \varepsilon \), i.e. if \( S_n(t') \geq 1 - \varepsilon \).

There are clear parallels between the ecological concept of persistence and the concept of permanency we have introduced. Both notions refer to a period of time specified by the observer and require the stocks to have an uninterrupted existence. The difference is that persistence refers to stochastic populations, and in this sense, only predictions of probability –

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28 ‘Stochastic population dynamics’ means population dynamics which are subject to random fluctuations, such as those induced by unpredictable environmental influences.

29 Below we intend to assume \( t_0 = 0 \) without restricting generality.

30 The analysis of the survivability of populations is the subject of population viability analysis, an independent branch of population ecology (for a survey see Burgman and Possingham, 2000, Possingham, 2001 and Frank et al, 2002: Chapter 13).

31 For an overview of the concept, see Grimm and Wissel (1997) as well as Frank et al (2002: Chapter 13).
but no absolute predictions – can be made. The notion of persistence can hence be regarded as a stochastic variant of the notion of permanency with respect to populations. Whether a population can be rated as persistent also depends on the accepted risk of extinction \( \varepsilon \). The lower \( \varepsilon \), the higher the threshold value the probability of survival must exceed (i.e. the more restrictive the definition of persistence).

If the stochastic extension of our general concept of stocks, i.e. the concept of persistence, is to be used in an investigation, we need some idea of the accepted risk of extinction \( \varepsilon \). This variable is not an objectifiable attribute of the population analysed but is another subjective factor and therefore belongs to the context of investigation (cf Section 2.3). The level of the accepted risk of extinction \( \varepsilon \) arises from the observer’s subjective assessment; it is influenced by the knowledge existing in society, its willingness to take risks, and many other factors. The accepted risk of extinction can thus change over the course of time – for example due to the alteration of preferences expressed by a growing awareness of ecological concerns. Hence the determination of the accepted risk of extinction largely concerns the spheres of politics and economics.

### 5.3.2 Persistence and permanency

During the development of our general concept of stocks, in Proposition 3.1 we gave a necessary condition for the permanency of a set which was formulated depending on the remaining duration of the stock \( T_B^0 \). Now we intend to investigate the extent to which a population (subject to statistical fluctuations) can also be regarded as a stock. This is not \textit{a priori} clear, for the theoretical possibility that a stochastically behaving population may be extinct at any time also holds in particular for the time \( t = A \tau \) (where \( A >> 1 \), at which the population according to Proposition 3.1 must still exist in order to be viewed as permanent in the sense of Section 3 – and hence to constitute a stock. To solve this problem, the obvious approach would be to merge the concept of persistence developed for stochastic populations with our concept of stocks, and hence, extend our concept of stocks for populations which behave stochastically.

If persistent stochastic populations are simultaneously regarded as permanent in the sense of Section 3, the formal condition for the permanency of a stochastic population results from Definition 5.1:

\[
S_n(A \tau) \geq 1 - \varepsilon. \tag{3}
\]

We shall start by only considering populations which reach the established state. For these populations, \( c_n = 1 \) can be assumed.\textsuperscript{32} Equation (2) then establishes a functional relationship between the population’s probability of survival \( S_n(\tau) \) at time \( \tau \) and its mean lifetime \( T_p \). If Expression (2) is inserted into the definition of persistence (3), it turns out that a population can only be permanent if \( e^{-A \tau / T_p} \geq 1 - \varepsilon \). Calculating the logarithm of both sides results in the

\textsuperscript{32} The case of \( c_n < 1 \) is dealt with later on in the text.
following necessary condition for the existence of permanency on the part of a stochastic population which has already reached the established state:

\[ T_p \geq \left\{ \frac{1}{\ln(1-\varepsilon)} \right\} \cdot A \tau. \] (4)

Condition (4) implies that a population is only permanent if its mean lifetime \( T_p \) exceeds the ‘test period’ for permanency from Proposition 3.1 \( A \tau \) by a factor of \( \left\{ \frac{1}{\ln(1-\varepsilon)} \right\} > 0 \).

Let us now compare the condition for permanency initially given in Proposition 3.1 for non-stochastic sets, with the condition derived in Equation (4) for the permanency of a stochastic population:

\[ T_{B0} \geq A \tau \] (Proposition 3.1)

\[ T_p \geq \left\{ \frac{1}{\ln(1-\varepsilon)} \right\} \cdot A \tau. \] (4)

Both equations have the same structure: the duration of a non-stochastic stock, \( T_{B0} \), or the mean lifetime of a stochastic stock, \( T_p \), must (as objectively given quantities) exceed a certain period of comparison determined by subjective factors. It therefore appears logical to identify the two quantities \( T_{B0} \) and \( T_p \) with each other and to define the mean lifetime as the remaining duration of a stochastic population.

However, stochastic populations and non-stochastic stocks differ in that the remaining duration of stochastic populations which are to be regarded as stocks must, according to Equation (4), exceed the ‘test period’ \( A \tau \) corrected by a certain factor. The exact correction factor depends on the subjective quantity residual risk and is shown in Fig. 2 as depending on \( \varepsilon \). It expresses a subjective assessment of the risk caused by the stochastic nature of the population.

The lower the accepted risk of extinction \( \varepsilon \), the higher the period of comparison (for a given time scale of observation \( \tau \)) which the mean lifetime of the population must exceed in order for the population to be a stock. In the special case \( \varepsilon = 1 - e^{-1} \approx 0.63 \), the correction factor has a value of 1, i.e. the threshold value for stochastic populations exactly matches that for non-stochastic stocks.

On the other hand, if the accepted risk of extinction approaches zero (i.e. \( \varepsilon \to 0 \)), the threshold value which has to be exceeded by the mean lifetime of a stochastic population for the population to be a stock tends towards infinity. We have thus proved the following proposition:

**Proposition 5.1**: If the accepted risk of extinction given in the context of an investigation is zero, stochastically behaving populations cannot be regarded as stocks in the sense of Section 3.
In the discussion of this section, we have shown that stochastically behaving populations can be regarded as stocks if the accepted risk of extinction is really greater than zero. The attribute of persistence stemming from ecological theory then corresponds to our general notion of permanency.

### 5.3.3 Reaching the established state

When deriving Equation (4) in the previous section, it was assumed that the population definitely reaches the established state, i.e. $c_n = 1$. If this assumption is abandoned, i.e. if we consider the case $c_n < 1$, an additional necessary condition for the existence of permanency arises from Equation (2) and Definition 5.1, namely $c_n > 1 - \varepsilon$. In other words, the probability of reaching the established state must exceed the threshold value $1 - \varepsilon$. Like Equation (4), this condition ties objective attributes of the population to subjective assessments on the part of the observer. An appraisal of the condition indicates that the initial population must be above a critical threshold value $n_c$ in order to be able to attain permanency.\(^3^3\)

### 5.3.4 Mechanisms for reaching persistence: the example of ‘floaters’

In this section, a mechanism is to be examined which in real ecosystems can guarantee the persistence or permanency of populations. To ensure persistence, the population size $N$ has to be maintained above a critical threshold value $n_c$. This can be done by means of the buffer mechanism described below.

Consider a fictitious population of territorial wild animals such as badgers. Inside populations, only the ‘territory owners’ can reproduce, i.e. in this case the effective population size $n$ is given by the number of occupied territories. All the other animals living in this area are unable

\(^3^3\) $c_n = 1 - (b/d)^n$ holds for constant birth and death rates which are the same for all individuals $b_i = b$ and $d_i = d$ (Wissel, 1989), resulting in the condition $n > n_c = -\ln(\varepsilon) / \ln(b/d)$. Permanency can then only be achieved if the
to reproduce and are constantly in search of unoccupied territories, or some way of driving out territory owners in order to take their place. Although these ‘floaters’ make no immediate contribution to reproduction, they are still relevant for population dynamics since they compensate for deaths among territory owners by taking over vacant positions – and hence, make sure the territories concerned are continuously occupied. This means the effective population size $N$ is maintained at a sufficiently high level above $n_c$. As long as there are enough ‘floaters’ to take over vacant territories, the population can be buffered against adverse events. Negative interventions can thus be compensated for, ensuring the population’s long-term survival.

The example of ‘floaters’ indicates that if the dynamics of such a population are to be understood, instead of considering a homogeneous stock of badgers, two independent stocks – territory owners and ‘floaters’ – need to be regarded as living in close connection with each other.

5.4 The significance of the ecological example for our concept of stocks

In the previous sections we showed using the example of ecological systems how our general notion of stocks can be employed for specific questions. In Section 5.2 we examined populations and their dynamics from various perspectives and discovered that the different perspectives each shed light on different aspects of population dynamics. We showed how on the one hand a population can be regarded as a stock, but that on the other, the population can also be regarded as a system of interacting elements, for the purpose of understanding its dynamics. It became apparent that when regarded as a stock, a population is in itself again part of a higher system, which has important repercussions for its temporal development. In Section 5.2 we used this to illustrate how our concept of stocks can contribute to structuring hierarchical links in real systems.

In Section 5.3 we extended the notion of stocks originally developed for non-stochastic sets for the analysis of stochastic populations. This was done by using the concept of persistence stemming from ecological theory as a basis. Although we initially only showed how the notion can be extended for the specific statistics of populations (Equations (2) and (3)), this technique can always be used if the stochastics of the set observed can be described by a mean lifetime.

Extending the notion of stocks to stochastic entities provides a crucial condition for the notion’s universal applicability since the ex ante analysis of real problems is usually beset by ignorance, meaning that in such cases statistical concepts need to be used. One important finding in this respect is that when the stochastic notion of stocks is used, the accepted residual risk $\varepsilon$ plays a crucial role as another subjective factor.

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time population size $n$ exceeds a minimum $n_c$, which depends on the demographic attributes of the individuals ($b$ and $d$) and the accepted residual risk $\varepsilon$. 

6 Natural, intentional and unintentional stocks

So far in this essay we have developed a concept of stocks which can be used to analyse the dynamic attributes of economic and ecological systems. We have applied this concept to examples taken from theoretical ecology and in doing so, shown that the concept can also be used to examine stochastic systems. In our opinion, the great thing about the concept of stocks is that, in addition to the formal analysis of the dynamics of economic and ecological systems, it also allows analysis to specifically include the goal orientation of human activity, which plays a decisive role in economic systems (but is completely absent in purely ecological systems). A detailed analysis of this topic would, however, go beyond the scope of this paper. In this section, we shall therefore restrict ourselves to a brief outlook.

Human economic activities are inevitably associated with influencing multiple kinds of stocks (cf e.g. Schiller 2002, Chapter 7.3.1). Therefore, it seems reasonable to classify stocks in terms of the extent to which they are influenced by human activity. We can start by differentiating between stocks which arose as a result of human activities (i.e. which were created by humans) and those which developed naturally. Stocks created by humans can then be further classified in terms of whether they were created intentionally or unintentionally. This criterion is important for further analysis since it directs attention towards the actors’ intentions (cf also Faber, Petersen and Schiller, 2002).

We distinguish between the following types of stocks:

1. **Natural stocks**, which arose in nature without human influence. Examples include ecosystems and deposits of natural raw materials.

2. **Intentional stocks** created deliberately by humans. Examples of intentional stocks include reserves and capital goods such as machines, roads and buildings.

3. **Unintentional stocks** created involuntarily through the influence of human activities, i.e. those responsible for them did not intend their creation. Examples include CFCs in the stratosphere and nitrates in groundwater.

The dynamics and attributes of all the stocks of types 2 and 3 as well as many stocks of type 1 are directly or indirectly influenced by humans. This means that classification in terms of human influence is not clear-cut; instead human influence on the development of stocks can vary in intensity on a continuous scale from non-existent to determinative.

All three types of stocks play an important role for human economic activity:

- Many natural stocks perform a whole range of services for mankind. For example, plants are responsible (through photosynthesis) for maintaining the level of oxygen in the atmosphere necessary to support life. Natural stocks are also used as resources, such as in the mining of coal and metal ore.

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34 In this definition, ‘natural’ is regarded as the opposite of ‘being influenced by humans’. The term ‘nature’ is hence not used here in the comprehensive sense of ‘everything subject to the laws of nature’.
Intentional stocks include above all reserves and capital goods. Reserves are created in order to provide for the future. Capital goods are used to produce goods. In contrast to reserves, capital goods are not used up in the production process. This is not to say that capital goods do not wear down over time. However, this is a process which takes place on a much longer time scale compared to their usage for the production of goods.

Unintentional stocks arise as a result of human, thereof mainly economic, activities such as the production of goods – even though they are not an intended consequence. Although the creation of unintentional stocks is sometimes accepted as an inevitable associated process, frequently those responsible for them are not aware of their formation. Examples of unintentional stocks include the presence of cadmium in sediment in German rivers resulting from galvanisation, and the carbon dioxide in the atmosphere caused by the use of fossil fuels.

Fig. 3 illustrates our classification of stocks by showing the way in which their dynamics and other attributes are determined by human activities.

These three different types of stocks are mainly studied by different scientific disciplines. Natural stocks – populations, ecosystems, geological information, groundwater reserves, sediment etc – are studied by the natural sciences, chiefly geology and ecology. Intentionally created artificial stocks – capital goods, reserves, know-how – are usually formed as a result of economic motivation and are therefore the subject of economics. Unintentional stocks are usually not at the focus of attention of any particular discipline, which is why they often cannot be unambiguously assigned to the traditional disciplines. Instead, they are often analysed from the viewpoints of various disciplines and are hence the subject of interdisciplinary research. In particular, unintentional products and their effects are often perceived as environmental problems, making them a subject of environmental sciences such as planning sciences.

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geography, hydrology, ecological economics and environmental economics – the latter of which both deal with stocks of pollutants and other unwanted by-products.36

7 Just how good is the notion of stocks?
In our view, the notion of stocks developed in this essay provides a good basis for the integrated ecological-economic study of the dynamics of coupled ecological and economic systems. This is mainly due to the following characteristics of the concept of stocks.

Dynamics: Our conception of stocks enables an analytical characterisation of a large number of entities – namely through the attribute of permanency. This is central to an understanding of the dynamics of both economic and ecological systems. Together with the interaction of individual stocks, it is their durability that determines the development of the embedding systems over time.

Universality: As permanency is a temporal attribute and hence does not specifically refer to individual scientific disciplines, the concept of permanency is universally applicable. Accordingly, our notion of stocks has been kept so general that it can be used to characterise both economic and ecological systems (as was shown by examples). As well as its usage in ecology and economics, we believe the notion of stocks could theoretically also be applied to other questions and disciplines. Alongside other natural sciences, these include in particular the social sciences, such as sociology and political science. Hence our notion of stocks contributes to a common language in the study of ecological and economic systems, which is needed for the analysis of interdisciplinary problems.

Compatibility: Despite its universality, our concept of stocks is compatible with a number of concepts already introduced in economics and ecology. It directly corresponds to the concept of the capital stock in economics, which plays a key role in the dynamic analysis of economic systems. And the notion is a generalisation of the concept of persistence introduced in theoretical ecology, making it suitable for direct links with ecology as well.

Identification of subjective factors: Analysing ecological and economic systems with the help of the concept of stocks developed initially requires reflection of the context of the investigation with the observer’s subjective factors being summarised. This identification of the subjective factors – for example the definition of the object of observation and the selection of the time horizon and time scale of observation – results in the reflection of methods, a process which is always essential for interdisciplinary and transdisciplinary work. This identification also enables the object of investigation to be seen from a number of different angles.

Temporal scales: Our concept of stocks enables different temporal scales to be explicitly taken into account. By its very nature, the notion of stocks refers to a time of observation and

36 Thermodynamically speaking, each production operation is associated with coupled production, i.e. the formation of unwanted by-products (Baumgärtner, 2000). This prompts the question of the responsibility of human activity (cf Baumgärtner et al, 2001).
a time scale of observation. When using the concept these, frequently only implicit elements, therefore need to be explicitly examined and determined, which makes analysis of a large number of different time scales possible.

**Differentiation between system view and stock perspective:** Using our notion of stocks enables a distinction to be drawn between the system view and the stock perspective when investigating an object of observation. This change in perspective allows the central attributes of real ecological-economic systems to be identified. As shown by the discussion in Sections 5.2 and 5.3: the hierarchical structure of such complex systems can only be understood if on a given level of observation simplifications are made at the microscopic perspective and simultaneously the interaction of the stocks within the framework of a system (system view) is taken into account.

Theoretically, the concept of stocks developed here, enables the various complicated links between ecological and economic systems to be depicted. Our concept of stocks is thus an important basic concept for integrative, dynamic interdisciplinary environmental research.

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**8 Literature**


