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Philippine Institute for Development Studies
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Consistent GDP Aggregation and Purchasing Power Parity

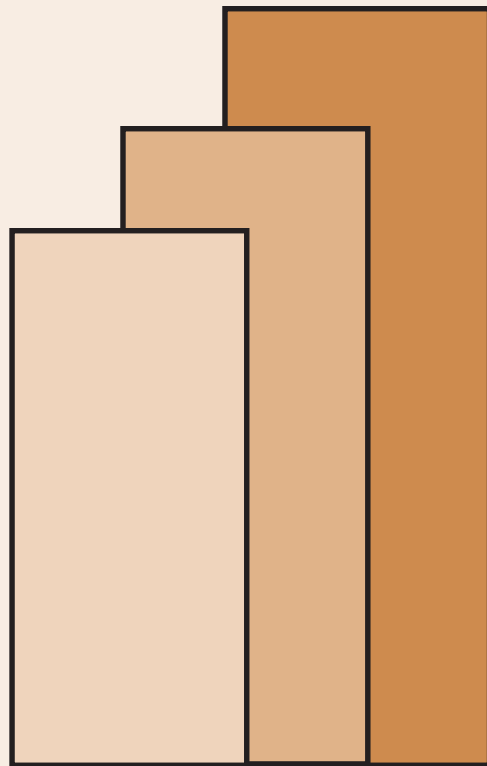
Jesus C. Dumagan

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Consistent GDP Aggregation and Purchasing Power Parity

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Abstract

Consistent aggregation ensures that real GDP *level* and *growth* do not change as the existing GDP components are merely rearranged. Otherwise, level or growth changes are spurious. This paper proposes a framework for consistent aggregation where components are converted to “purchasing power parity” (PPP) values that “add up exactly” to the same real GDP regardless of the grouping of components. This PPP framework applies to GDP either in constant prices or in chained prices. PPP is applied to US GDP in chained prices based on the Fisher index to (i) reduce US non-additivity residuals to zero; (ii) correct misleading contributions to GDP growth computed by the US Bureau of Economic Analysis; and (iii) show that GDP quantity indexes in PPP are consistent in aggregation although the Fisher formula is inconsistent. Moreover, PPP implications on GDP measurement for some areas of economic research (e.g., income inequality and poverty incidence) are discussed.

Key Words: Exact growth decomposition; consistent level aggregation

JEL classification: C43

1. Introduction

The theoretical requirement of consistent GDP aggregation ensures that in practice the *level* and *growth* of real GDP are unaffected by arbitrary regroupings of the existing components. Therefore, inconsistency in aggregation implies that observed changes in real GDP level or growth are partly spurious, being mere artifacts of the aggregation procedure. This underscores the importance of consistent aggregation in practice.

In current practice, real GDP and its subaggregates are measured either in constant prices or in chained prices. The existing criterion for consistent aggregation (or *additivity*) is simple addition (i.e., no weights) of real subaggregates that results in equality with real GDP [Balk,

2010, equation (18a)]. By this criterion, real subaggregates mimic nominal subaggregates in that the simple sum of the latter equals nominal GDP and the level of nominal GDP remains the same while the number of subaggregates is changed. Following the above criterion, GDP in constant prices is consistent in aggregation because the *fixed-base* formula for Paasche deflators that generate GDP and subaggregates in constant prices has the mathematical property of consistency in aggregation. In contrast, GDP in chained prices is inconsistent because formulas of chained deflators lack the above consistency property [Balk, 2010, equation (20)].

However, this paper takes issue with simple addition of subaggregates in constant prices or in chained prices as the criterion for consistency in aggregation because this criterion ignores differences in relative prices between subaggregates that need to be taken into account. Consider that in current practice, given $j = 1, 2, \dots, M$ subaggregates of GDP, there are $M + 1$ distinct deflators for each subaggregate and for GDP. Therefore, there are M ratios of subaggregate deflators to the GDP deflator. These ratios reflect relative price differences between subaggregates that this paper argues should be used as their weights in aggregation. However, these ratios are ignored in existing aggregation procedures.

As a weight, the ratio of a subaggregate deflator to the GDP deflator multiplies a real subaggregate and, as a result, converts it to “purchasing power parity” (PPP) value in the same unit of measure as real GDP because the GDP deflator is the common denominator. It turns out that the PPP value of a subaggregate equals its nominal value (i.e., in current prices) divided by the GDP deflator. It follows that the weighted sum of real subaggregates equals the simple sum of their PPP values that, in turn, equals GDP in current prices divided by the GDP deflator, which is real GDP. Hence, real GDP in PPP values is consistent in aggregation in the same way that GDP in current prices is consistent.

Simple addition of real subaggregates in constant prices or in chained prices is a special case of their weighted addition when relative prices are constant so that all weights equal one. Therefore, since relative prices do change in reality, simple addition of PPP values is a more realistic and more theoretically appealing criterion for consistency in aggregation since this criterion puts no restriction on the weights.

As background to the PPP framework, Section 2 of this paper revisits the chained Paasche deflators originally devised by Balk and Reich (2008). Dumagan (2011a) derived the same chained Paasche deflators and showed that they yield weights for consistent aggregation as

illustrated in a follow-up implementation (2011b). In this case, real GDP equals the sum of the weighted real subaggregates where the weights are ratios of the corresponding subaggregate deflators to the GDP deflator.

Upon closer examination, however, the deflators in the above weights are unsatisfactory because they are chained from the reference period 0 only up to $t - 1$, where t is the current period, due to the constraint of Paasche deflation with $t - 1$ as a *fixed* base. In effect, the above chained Paasche deflation framework allows relative prices to change from 0 to $t - 1$ but fixes relative prices from $t - 1$ to t . This feature makes the framework inconsistent and, therefore, the above weights need correction to allow relative prices to change from 0 to t .

Section 3 presents the corrected weights by chaining the deflators from 0 to t . These weights are exactly the same as those discussed earlier that yield PPP values as basis for this paper's PPP framework for consistent aggregation of real GDP.

Section 4 applies the PPP framework to US GDP in chained dollars based on the Fisher index. The application converts non-additive subaggregates to additive PPP values, thus, reducing to zero non-additivity residuals. Moreover, the application reveals misleading (i.e., wrong signs) growth contributions by the US Bureau of Economic Analysis. Finally, the implicit Fisher quantity indexes are computed to show that they are consistent in aggregation.

Section 5 explores the implications of PPP on the GDP index framework in current practice and on some areas of economic research, for example, income inequality and poverty incidence.

Section 6 concludes this paper.

2. Paasche Deflation Framework

Let there be $j = 1, 2, \dots, M$ subaggregates of GDP and let each j have $k = 1, 2, \dots, N$ components over a period $t = 1, 2, \dots, T$. Each component has a price p_t^{kj} and quantity q_t^{kj} . These data yield GDP in current prices,

$$(1) \quad Y_{t-1}^j = \sum_k p_{t-1}^{kj} q_{t-1}^{kj} \quad ; \quad Y_{t-1} = \sum_j Y_{t-1}^j \quad ; \quad Y_t^j = \sum_k p_t^{kj} q_t^{kj} \quad ; \quad Y_t = \sum_j Y_t^j .$$

Let the Paasche price indexes going from $t - 1$ to t be P_t for GDP and P_t^j for a subaggregate. Using the prices and quantities in (1), these indexes are,

$$(2) \quad P_t = \frac{\sum_j \sum_k p_t^{kj} q_t^{kj}}{\sum_j \sum_k p_{t-1}^{kj} q_t^{kj}} \quad ; \quad P_t^j = \frac{\sum_k p_t^{kj} q_t^{kj}}{\sum_k p_{t-1}^{kj} q_t^{kj}} \quad ; \quad P_t = \sum_j s_t^j P_t^j \quad ; \quad s_t^j = \frac{\sum_k p_{t-1}^{kj} q_t^{kj}}{\sum_j \sum_k p_{t-1}^{kj} q_t^{kj}} .$$

Let X_t and X_t^j represent real GDP and a real subaggregate in t measured in $t - 1$ prices. These are computed by,

$$(3) \quad X_t = \frac{Y_t}{P_t} = \frac{\sum_j \sum_k p_t^{kj} q_t^{kj}}{\left(\frac{\sum_j \sum_k p_t^{kj} q_t^{kj}}{\sum_j \sum_k p_{t-1}^{kj} q_t^{kj}} \right)} = \sum_j \sum_k p_{t-1}^{kj} q_t^{kj} ;$$

$$(4) \quad X_t^j = \frac{Y_t^j}{P_t^j} = \frac{\sum_k p_t^{kj} q_t^{kj}}{\left(\frac{\sum_k p_t^{kj} q_t^{kj}}{\sum_k p_{t-1}^{kj} q_t^{kj}} \right)} = \sum_k p_{t-1}^{kj} q_t^{kj} ;$$

$$(5) \quad X_t = \frac{Y_t}{P_t} = \sum_j X_t^j = \sum_j \frac{Y_t^j}{P_t^j} = \sum_j \sum_k p_{t-1}^{kj} q_t^{kj} , \text{ for } t - 1 \text{ as a fixed base.}$$

It appears from (5) that real GDP equals the *simple* sum of subaggregates and this sum is invariant to changes in k and j given Y_t . However, simple addition holds when the base period is fixed but does not hold in a chaining framework, as shown below.

Let the chained Paasche price indexes starting from the reference period 0 to the current period t be P_t^c for GDP and P_t^{cj} for a subaggregate. These are obtained by multiplying the indexes in (2) beginning with P_t and P_t^{cj} , $t = 1$ so that $t - 1 = 0$, which is the starting reference period. By convention, the index value is set equal to 1 in the reference period. Therefore,

$$(6) \quad P_t^c = (P_0 \times P_1 \times P_2 \times \dots \times P_{t-1}) \times P_t = P_{t-1}^c P_t \quad ; \quad P_0 = 1 ;$$

$$(7) \quad P_t^{cj} = (P_0^j \times P_1^j \times P_2^j \times \dots \times P_{t-1}^j) \times P_t^j = P_{t-1}^{cj} P_t^j \quad ; \quad P_0^j = 1 .$$

From the above results, divide both sides of (5) by P_{t-1}^c and then multiply and divide the right-hand side by P_{t-1}^{cj} to obtain,

$$(8) \quad \frac{Y_t}{P_{t-1}^c P_t} = \sum_j \left(\frac{P_{t-1}^{cj}}{P_{t-1}^c} \right) \left(\frac{Y_t^j}{P_{t-1}^{cj} P_t^j} \right) .$$

Finally, (8) may be rewritten as,

$$(9) \quad X_t^c = \sum_j \left(\frac{P_{t-1}^{cj}}{P_{t-1}^c} \right) X_t^{cj} \quad ; \quad X_t^c = \frac{Y_t}{P_t^c} \quad ; \quad X_t^{cj} = \frac{Y_t^j}{P_t^{cj}} \quad ; \quad X_t^c \neq \sum_j X_t^{cj} \quad ; \quad \frac{P_{t-1}^{cj}}{P_{t-1}^c} \neq 1 .$$

Except for differences in notation, (9) is identical to the result in equation (21) of Dumagan (2011a) where P_{t-1}^{cj}/P_{t-1}^c is the weight of subaggregates in chained prices ($X_t^{cj} = Y_t^j/P_t^{cj}$) that make their weighted sum equal GDP in chained prices ($X_t^c = Y_t/P_t^c$). The computations of X_t^{cj}

and X_t^c in (9) are employed in current practice but without weights so that non-additivity results from simple addition (Aspden, 2000; Ehemann, Katz, and Moulton, 2002; Schreyer, 2004).

However, the chained Paasche deflation framework in (8) or (9) is inconsistent because it recognizes changes in relative prices from the reference period 0 to $t - 1$ by chaining but ignores them from $t - 1$ to t by employing (5) that has $t - 1$ as a fixed reference period.¹ It may be noted that (5) violates the equation for GDP in current prices unless relative prices are constant because it yields, using (3),

$$(10) \quad Y_t = \sum_j \frac{P_t}{P_t^j} Y_t^j \neq \sum_j Y_t^j \quad \text{if } \frac{P_t}{P_t^j} \neq 1, \text{ for some } j.$$

If all prices change at the same rate ρ (i.e., relative prices are constant), $P_t = P_t^j = 1 + \rho$, all j . Unless this holds, (5) is invalid for the violation in (10). Therefore, (5) should be removed as a constraint to make (9) consistent by allowing changes in relative prices from 0 up to t .

3. PPP Framework for Consistent Aggregation

After removing (5) as a constraint and allowing relative prices to change from 0 to t , (8) and (9) become,

$$(11) \quad \frac{Y_t}{P_{t-1}^c P_t} = \sum_j \left(\frac{P_{t-1}^{cj} P_t^j}{P_{t-1}^c P_t} \right) \frac{Y_t^j}{P_{t-1}^{cj} P_t^j} \quad ; \quad \frac{Y_t}{P_t^c} = \sum_j \left(\frac{P_t^{cj}}{P_t^c} \right) \frac{Y_t^j}{P_t^{cj}} = \sum_j \frac{Y_t^j}{P_t^c}.$$

The above result corrects the violation by (5) in (10) because it implies $Y_t = \sum_j Y_t^j$.

The weights (P_t^{cj}/P_t^c) in (11) reflect price differences using the GDP deflator as a common base. Therefore, all $(P_t^{cj}/P_t^c)X_t^{cj}$ are values in the same unit of measure as real GDP so that the rate of exchange between them is “one to one.” That is, they are measured in “purchasing power parity” (PPP) values in terms of real GDP. In principle, the procedure in (11)

¹ Equation (31) of Balk and Reich (2008) may be rewritten to yield (9) above. Dumagan (2011a) derived the same deflators in (9) starting from the fact that real GDP in period t equals GDP in the reference period 0 multiplied by the chained Laspeyres quantity index in t . However, the latter equals its chained value from 0 to $t - 1$ multiplied by its value from $t - 1$ to t . From this, the deflators can be derived by first *expanding* the Laspeyres quantity index from $t - 1$ to t based on its consistency in aggregation property and then invoking value index decomposition at the subaggregate level (i.e., dividing GDP in current prices in t by the Paasche price index from $t - 1$ to t equals GDP in current prices in $t - 1$ multiplied by the Laspeyres quantity index from $t - 1$ to t).

is similar to real GDP aggregation across countries using PPP-adjusted nominal exchange rates, except that this exchange rate equals one in the same country. Therefore,

$$(12) \quad \left(\frac{P_t^{cj}}{P_t^c} \right) X_t^{cj} = \frac{P_t^{cj} X_t^{cj}}{P_t^c} = \frac{Y_t^j}{P_t^c} = \frac{\text{subaggregate in current prices}}{\text{GDP deflator}} = \text{real PPP value}.$$

While the PPP equation in (11) is the result of chaining from 0 to t , this equation holds generally. The deflators (P_t, P_t^j) could just be implicit deflators computed from data on GDP in current prices (Y_t, Y_t^j) and real GDP (X_t, X_t^j) in constant or in chained prices. Hence, the PPP framework may now proceed from the following definitions and relations,²

$$(13) \quad Y_t = \sum_j Y_t^j \quad ; \quad P_t = \frac{Y_t}{X_t} \quad ; \quad P_t^j = \frac{Y_t^j}{X_t^j} \quad ; \quad s_t^j = \frac{Y_t^j}{Y_t} \quad ; \quad w_t^j = \frac{X_t^j}{X_t} \quad ; \quad r_t^j = \frac{P_t^j}{P_t} = \frac{s_t^j}{w_t^j} ;$$

$$(14) \quad X_t = \sum_j \frac{P_t^j}{P_t} X_t^j = \sum_j r_t^j X_t^j \quad ; \quad X_{t-1} = \sum_j r_{t-1}^j X_{t-1}^j.$$

Note from (12) and (14) that weighted subaggregates $(P_t^j/P_t)X_t^j = Y_t^j/P_t$ are PPP values that could be in constant or in chained prices depending on the GDP deflator P_t . This shows that the simple sum of subaggregates in constant or in chained prices X_t^j equals real GDP X_t only in the special case when relative prices are constant so that the weights are all equal to one, i.e., $P_t^j/P_t = 1$, all j . In the general case when relative prices change, weighted addition of these subaggregates is the rule but this rule is equivalent to simple addition of their PPP values. Now that real GDP equals the simple sum of subaggregates in PPP values, real GDP is consistent in aggregation because this simple sum is invariant to the number of subaggregates.

From (14), real GDP growth g_t and subaggregate growth g_t^j are related to the GDP quantity index Q_t and subaggregate quantity index Q_t^j by,

$$(15) \quad Q_t = \frac{X_t}{X_{t-1}} \quad ; \quad g_t = \frac{X_t}{X_{t-1}} - 1 = Q_t - 1 \quad ; \quad Q_t^j = \frac{X_t^j}{X_{t-1}^j} \quad ; \quad g_t^j = \frac{X_t^j}{X_{t-1}^j} - 1 = Q_t^j - 1.$$

By duality, the decomposition of g_t into contributions of g_t^j yields the aggregation of Q_t^j to Q_t .

The growth decomposition may be done by combining (13) to (15) to obtain,

² It may be noted that the PPP real GDP in (14) with the “common” GDP deflator P_t is the same as the economy-wide real output employed by Tang and Wang (2004), Diewert (2010), and Dumagan (2011c) to define aggregate labor productivity (ALP). In these three studies, ALP equals (14) divided by economy-wide labor employment.

$$(16) \quad g_t = \sum_j w_{t-1}^j [r_t^j g_t^j + (r_t^j - r_{t-1}^j)].$$

Adding and subtracting $\sum_j s_{t-1}^j g_t^j$ to (16) yields a more detailed growth decomposition,³

$$(17) \quad g_t = \sum_j [s_{t-1}^j g_t^j + w_{t-1}^j (r_t^j - r_{t-1}^j) g_t^j + w_{t-1}^j (r_t^j - r_{t-1}^j)].$$

Finally, (15) and (17) yield the aggregation of Q_t^j to equal Q_t by,

$$(18) \quad Q_t = \sum_j w_{t-1}^j r_t^j Q_t^j \quad ; \quad \sum_j w_{t-1}^j r_t^j \neq 1 \quad \text{if} \quad r_t^j \neq 1.$$

It may be noted that r_t^j and r_{t-1}^j permit a breakdown of growth contributions into three components in (17). However, if there is no need for a breakdown, the whole contribution [inside square brackets] can be computed simply as follows. Consider from (13) and (14) that,

$$(19) \quad r_t^j X_t^j = \frac{Y_t^j}{P_t} = s_t^j \frac{Y_t}{P_t} \quad ; \quad r_{t-1}^j X_{t-1}^j = s_{t-1}^j \frac{Y_{t-1}}{P_{t-1}}.$$

The left-hand sides of (19) were used to obtain the breakdown above but the whole itself can be obtained by using the right-hand sides. In the latter case, earlier calculations simplify to,

$$(20) \quad Q_t = \frac{X_t}{X_{t-1}} = \frac{Y_t/P_t}{Y_{t-1}/P_{t-1}} \quad ; \quad Q_t^j = \frac{s_t^j (Y_t/P_t)}{s_{t-1}^j (Y_{t-1}/P_{t-1})} = (s_t^j / s_{t-1}^j) Q_t^j ;$$

$$(21) \quad Q_t = \frac{X_t}{X_{t-1}} = \sum_j s_{t-1}^j Q_t^j \quad ; \quad g_t = \frac{X_t - X_{t-1}}{X_{t-1}} = \sum_j s_{t-1}^j (Q_t^j - 1).$$

Therefore, the whole growth contribution of a subaggregate from (21) is,

$$(22) \quad s_{t-1}^j (Q_t^j - 1) = \frac{Y_t^j}{Y_t} \frac{X_t}{X_{t-1}} - \frac{Y_{t-1}^j}{Y_{t-1}}.$$

4. Application of the PPP Framework to US GDP in Chained Prices

The PPP framework consists of three interrelated equations: (14) for consistent real GDP *level* aggregation; (17) for exact decomposition of real GDP *growth*; and (18) for consistent GDP quantity *index* aggregation. These three equations may now be applied to US GDP in chained prices based on the Fisher index.

³ In (17), the first and second terms equal zero if subaggregate growth $g_t^j = 0$. While this may not be obvious, it can be verified that the third term is also zero if $g_t^j = 0$, all j so that GDP growth $g_t = 0$ as should be the case.

3-A. Consistent Real GDP Level Aggregation

US GDP data in Table 1 will suffice for illustration. Following the notations above, the data set $(Y_{t-1}, Y_{t-1}^j, Y_t, Y_t^j)$ refers to US GDP in current prices and $(X_{t-1}, X_{t-1}^j, X_t, X_t^j)$ refers to US GDP in chained 2005 dollars in $t - 1 = 2009$ and in $t = 2010$.

Table 1. US GDP by Major Industry Groups

	GDP in current dollars (billions)		GDP in chained 2005 dollars (billions)	
	2009	2010	2009	2010
Gross domestic product	14,119.0	14,660.4	12,880.6	13,248.2
Agriculture, forestry, fishing, and hunting	133.1	154.1	136.2	137.3
Mining	240.8	281.4	263.3	271.2
Utilities	268.1	275.7	206.6	209.3
Construction	537.5	505.6	447.4	430.9
Durable goods	867.2	961.2	857.4	942.7
Nondurable goods	717.6	756.3	613.1	618.0
Wholesale trade	780.8	807.7	810.5	844.3
Retail trade	819.6	862.8	789.7	830.8
Transportation and warehousing	389.5	406.5	341.6	347.6
Information	639.3	670.3	658.8	691.3
Finance and insurance	1,171.6	1,235.2	1,094.0	1,129.4
Real estate and rental and leasing	1,868.7	1,858.5	1,700.8	1,712.7
Professional, scientific, and technical services	1,068.5	1,103.9	950.5	972.2
Management of companies and enterprises	246.5	256.3	217.3	220.3
Administrative and waste management services	386.3	411.8	348.3	367.2
Educational services	154.9	162.6	122.1	122.4
Health care and social assistance	1,057.9	1,111.7	932.5	958.6
Arts, entertainment, and recreation	127.3	131.2	114.7	120.9
Accommodation and food services	385.8	399.9	324.0	334.5
Other services, except government	335.4	343.8	283.5	287.8
Federal government	611.5	637.7	533.4	548.6
State and local government	1,311.0	1,326.1	1,119.4	1,112.2

Source: US Bureau of Economic Analysis (BEA).

The consistent aggregation of real GDP by PPP values in (14) simplifies to,

$$(23) \quad X_t = \frac{Y_t}{P_t} = \sum_j \frac{Y_t^j}{P_t} \quad ; \quad X_{t-1} = \frac{Y_{t-1}}{P_{t-1}} = \sum_j \frac{Y_{t-1}^j}{P_{t-1}}.$$

For the US, the common chained Fisher GDP deflators can be computed from Table 1 by,

$$(24) \quad P_t = \frac{Y_t}{X_t} = 1.1066 \quad , \quad t = 2010 \quad ; \quad P_{t-1} = \frac{Y_{t-1}}{X_{t-1}} = 1.0961 \quad , \quad t - 1 = 2009 .$$

The results of applying (23) and (24) to US GDP in Table 1 are shown in Table 2.

In Table 2, the values of X_t^j are in the columns headed by BEA. The PPP values computed by $r_t^j X_t^j = Y_t^j / P_t$ are in columns headed by PPP. Notice that in 2009 and 2010 the difference between GDP in chained 2005 dollars X_t and the sum of X_t^j is non-zero, i.e., the residual due to non-additivity of X_t^j . In contrast, PPP values are additive, i.e., with zero residuals.

Table 2. US GDP in Chained Dollars by Major Industry Groups

	GDP in chained 2005 dollars (billions)				Percent Change from BEA to PPP	
	BEA	PPP	BEA	PPP	2009	2010
	2009	2009	2010	2010		
Gross domestic product	12,880.6	12,880.6	13,248.2	13,248.2		
<i>Agriculture, forestry, fishing, and hunting</i>	136.2	121.4	137.3	139.3	-10.8	1.4
Mining	263.3	219.7	271.2	254.3	-16.6	-6.2
Utilities	206.6	244.6	209.3	249.1	18.4	19.0
Construction	447.4	490.4	430.9	456.9	9.6	6.0
Durable goods	857.4	791.1	942.7	868.6	-7.7	-7.9
Nondurable goods	613.1	654.7	618.0	683.4	6.8	10.6
<i>Wholesale trade</i>	810.5	712.3	844.3	729.9	-12.1	-13.6
Retail trade	789.7	747.7	830.8	779.7	-5.3	-6.2
Transportation and warehousing	341.6	355.3	347.6	367.3	4.0	5.7
Information	658.8	583.2	691.3	605.7	-11.5	-12.4
Finance and insurance	1,094.0	1,068.8	1,129.4	1,116.2	-2.3	-1.2
Real estate and rental and leasing	1,700.8	1,704.8	1,712.7	1,679.5	0.2	-1.9
Professional, scientific, and technical services	950.5	974.8	972.2	997.6	2.6	2.6
Management of companies and enterprises	217.3	224.9	220.3	231.6	3.5	5.1
Administrative and waste management services	348.3	352.4	367.2	372.1	1.2	1.3
<i>Educational services</i>	122.1	141.3	122.4	146.9	15.7	20.0
Health care and social assistance	932.5	965.1	958.6	1,004.6	3.5	4.8
<i>Arts, entertainment, and recreation</i>	114.7	116.1	120.9	118.6	1.3	-1.9
Accommodation and food services	324.0	352.0	334.5	361.4	8.6	8.0
Other services, except government	283.5	306.0	287.8	310.7	7.9	8.0
Federal government	533.4	557.9	548.6	576.3	4.6	5.0
State and local government	1,119.4	1,196.0	1,112.2	1,198.4	6.8	7.7
Non-additivity Residual	16	0	38	0		

Source: US GDP in chained dollars under the column PPP are the author's calculations based on (23) and (24) applied to data in Table 1.

The last two columns show the effects of the weights by the percent change from BEA to PPP, i.e., from X_t^j to $r_t^j X_t^j = Y_t^j / P_t$. The changes in the subaggregates in bold italics show that the directions of change could reverse from 2009 to 2010 and the percent changes could be large in absolute size. These indicate the significance of weights in current practice.

3-B. Exact Decomposition of Real GDP Growth

Table 3 presents the PPP growth decomposition of US GDP growth from Table 1. CPC represents “contribution to percent change,” which is the term for industry contribution to US

GDP growth by BEA. BEA CPC is compared to the results from the PPP decomposition in (17), which yields PPP CPC = IGC +GPI + RPC where for each industry:

$$(25) \text{ IGC, industry growth contribution} = s_{t-1}^j g_t^j;$$

$$(26) \text{ GPI, growth and price interaction contribution} = w_{t-1}^j (r_t^j - r_{t-1}^j) g_t^j;$$

$$(27) \text{ RPC, relative price change contribution} = w_{t-1}^j (r_t^j - r_{t-1}^j).$$

Notice that for all industries, BEA CPC = 2.54 percent while PPP CPC = 2.85 percent, the actual US GDP growth in 2010.

Table 3. Contributions to Percent Change (CPC) in Real GDP by Major Industry Groups

	BEA CPC (percent) 2010	IGC (percent) 2010 (1)	GPI (percent) 2010 (2)	RPC (percent) 2010 (3)	PPP CPC (percent) 2010 (1)+(2)+(3)
Gross domestic product	2.54	2.555	0.007	0.291	2.854
Agriculture, forestry, fishing, and hunting	0.01	0.008	0.001	0.130	0.138
Mining	0.05	0.051	0.006	0.211	0.269
Utilities	0.02	0.025	0.000	0.010	0.035
Construction	-0.14	-0.140	0.005	-0.124	-0.260
Durable goods	0.61	0.611	-0.001	-0.009	0.601
Nondurable goods	0.04	0.041	0.001	0.181	0.224
Wholesale trade	0.23	0.231	-0.004	-0.090	0.136
Retail trade	0.30	0.302	-0.003	-0.051	0.248
Transportation and warehousing	0.05	0.048	0.001	0.044	0.093
Information	0.22	0.223	-0.002	-0.046	0.175
Finance and insurance	0.27	0.269	0.003	0.096	0.368
Real estate and rental and leasing	0.09	0.093	-0.002	-0.287	-0.197
Professional, scientific, and technical services	0.17	0.173	0.000	0.004	0.177
Management of companies and enterprises	0.02	0.024	0.000	0.028	0.052
Administrative and waste management services	0.15	0.148	0.000	0.004	0.153
Utilities	0.00	0.003	0.000	0.041	0.044
Health care and social assistance	0.21	0.210	0.003	0.094	0.307
Arts, entertainment, and recreation	0.05	0.049	-0.002	-0.028	0.019
Accommodation and food services	0.09	0.089	0.000	-0.015	0.073
Other services, except government	0.04	0.036	0.000	0.000	0.036
Federal government	0.12	0.123	0.001	0.019	0.143
State and local government	-0.06	-0.060	-0.001	0.078	0.018

Source: BEA CPC is regularly released by BEA together with US GDP data in Table 1. Note that BEA publishes CPC only up to two decimal places as shown above. The other results in Table 3 are the author's calculations by applying the data in Table 1 to the PPP growth decomposition in (17) for *industry growth contribution* (IGC), *growth-price interaction* (GPI), and *relative price change* (RPC) growth contributions when real GDP is in chained prices.

It appears that BEA CPC equals IGC when rounded to two decimal places. Therefore, BEA CPC excludes the effects of changes in relative prices measured by GPI and RPC. These exclusions could have adverse effects. For instance, PPP CPC shows that by including GPI and RPC the growth contributions of *mining* (0.27 percent) and *nondurable goods* (0.22 percent) are over five times those by BEA CPC (0.05 percent and 0.04 percent, respectively). More significantly, exclusions could lead to sign reversals. For example, BEA CPC shows that *real*

estate and rental and leasing had a positive contribution (0.09 percent) but PPP CPC shows that they had a negative contribution (-0.20 percent) if the negative GPI and RPC components are included. For another example, BEA CPC shows that *state and local government* had a negative contribution (-0.06 percent) but PPP CPC shows that they had a positive contribution (0.02 percent) including the negative GPI and positive RPC components.

However, the above inaccuracies in BEA CPC have an analytic explanation. Seskin and Parker (1998) shows the formula for BEA CPC from an exact decomposition of the growth rate $Q_t^F - 1$. The formula for BEA CPC can be derived from the additive decomposition of Q_t^F which exists as shown by Dumagan (2002) and Balk (2004).

The inaccuracies are due to the fact that Q_t^F in BEA decomposition is the *explicit* Fisher quantity index while the decomposition of GDP growth is a decomposition of the growth of the *implicit* Fisher quantity index, $[(X_t/X_{t-1}) - 1] = Q_t^F - 1$, where Q_t^F is given by (28) from (18),

$$(28) \quad \frac{X_t}{X_{t-1}} = Q_t^F = \sum_j w_{t-1}^j r_t^j Q_t^{jF} \quad ; \quad Q_t^{jF} = \frac{X_t^j}{X_{t-1}^j} \quad ; \quad \sum_j w_{t-1}^j r_t^j \neq 1 \quad \text{if} \quad r_t^j \neq 1.$$

The difference is that the Fisher index formula does not capture the same effects of changes in relative prices captured by the implicit Fisher index in (28). Therefore, BEA may consider the PPP growth decomposition in (17) in place of its existing decomposition formula.

3-C. Consistent GDP Quantity Index Aggregation

From the data in Table 1, the Fisher quantity indexes Q_t^F and Q_t^{jF} underlying US GDP are computed by (28) and the results are presented in Table 4 under the columns PPP. To understand the results, consider that prices and quantities existing in $t - 1 = 2009$ and $t = 2010$ are used all at the same time in the “first” stage to compute the GDP Fisher quantity index $Q_t^F = X_t/X_{t-1} = 1.0285$. In the “second” stage, $M = 22$ distinct subaggregates (i.e., industries) are created. Moreover, some industries are aggregated in the “third” stage to form a total of $M = 15$ distinct industry groups. In each case, an industry index Q_t^{jF} is computed from Table 1 by $Q_t^{jF} = X_t^j/X_{t-1}^j$ and shown in the first column for $M = 22$ and in the third column for $M = 15$.

Finally, $Q_t^F = \sum_j w_{t-1}^j r_t^j Q_t^{jF}$ is computed with weights $w_{t-1}^j r_t^j$ where from (7), $w_{t-1}^j = X_{t-1}^j/X_{t-1}$, $r_t^j = P_t^j/P_j = s_t^j/w_t^j$, and $s_t^j = Y_t^j/Y_t$. The results show consistency in aggregation

since the value of the Fisher GDP index in the first stage remains the same, $Q_t^F = 1.0285$ for $M = 22$ and for $M = 15$ while in both cases, $\sum_j w_{t-1}^j r_t^j = 1.0029$.

Table 4. Consistent Aggregation of US GDP Implicit Fisher Quantity Indexes

	Twenty-two industries		Fifteen industries	
	PPP Implicit Fisher quantity indexes 2010	PPP Weighted Implicit Fisher quantity indexes 2010	PPP Implicit Fisher quantity indexes 2010	PPP Weighted Implicit Fisher quantity indexes 2010
Gross domestic product	1.0285		1.0285	
Agriculture, forestry, fishing, and hunting	1.0081	0.0108	1.0081	0.0108
Mining	1.0300	0.0197	1.0300	0.0197
Utilities	1.0131	0.0193	1.0131	0.0193
Construction	0.9631	0.0355	0.9631	0.0355
Manufacturing			1.0576	0.1205
Durable goods	1.0995	0.0674		
Nondurable goods	1.0080	0.0531		
Wholesale trade	1.0417	0.0567	1.0417	0.0567
Retail trade	1.0520	0.0605	1.0520	0.0605
Transportation and warehousing	1.0176	0.0285	1.0176	0.0285
Information	1.0493	0.0470	1.0493	0.0470
Finance, insurance, real estate, rental, and leasing			1.0169	0.2170
Finance and insurance	1.0324	0.0867		
Real estate and rental and leasing	1.0070	0.1304		
Professional and business services			1.0286	0.1243
Professional, scientific, and technical services	1.0228	0.0774		
Management of companies and enterprises	1.0138	0.0180		
Administrative and waste management services	1.0543	0.0289		
Edu. services, health care, and social assistance			1.0248	0.0894
Educational services	1.0025	0.0114		
Health care and social assistance	1.0280	0.0780		
Arts, entertainment, rec., accom., and food services			1.0374	0.0373
Arts, entertainment, and recreation	1.0541	0.0092		
Accommodation and food services	1.0324	0.0281		
Other services, except government	1.0152	0.0241	1.0152	0.0241
Government			1.0046	0.1378
Federal government	1.0285	0.0447		
State and local government	0.9936	0.0930		
Sum of weighted Fisher subaggregate quantity indexes		1.0285		1.0285

Source: Author's calculations of (28) employing US GDP data in Table 1. The GDP for the new subaggregates in bold italics in Table 4 are not included in Table 1 but are readily available from the BEA website.

It is important to note at this juncture that the results in Table 4 follow the stages of computation by Vartia (1974, 1976). Also, the consistency results in Table 4 follow from the fact that (28) holds for implicit indexes regardless of whether or not the underlying index formula is consistent in aggregation.

Therefore, the finding above that *implicit* (i.e., computed value) US GDP Fisher indexes are consistent in aggregation does not disprove Diewert's (1978) finding of Fisher inconsistency since this result applies to the *explicit* (i.e., formula) Fisher index. Specifically, the formula of the Fisher index as the geometric mean of the Laspeyres and Paasche indexes makes it

impossible to rewrite the Fisher index exactly as the weighted sum of subaggregate Fisher indexes. In contrast, the Laspeyres and Paasche indexes can be expressed in the above manner as in (2) for the Paasche index.

5. PPP Implications on GDP Indexes and on Some Areas of Economic Research

In the PPP framework for consistent aggregation, the unit of analysis is $r_t^j X_t^j = Y_t^j / P_t =$ real PPP value. This shows that all subaggregate deflators P_t^j cancel out in the aggregation of real GDP. Thus, the choice of index formula matters only for the GDP price index or for the GDP quantity index in so far as aggregation is concerned.

Unless there is a need to study a subaggregate in *isolation*—in which case the subaggregate of interest may be looked at as if it is GDP—there is no need for a subaggregate index. Moreover, this index is necessary only if the subaggregate has at least two lower level components. In this case, the PPP framework implies that the subaggregate index applies as well at the lower level as a common deflator. However, once the subaggregates are brought together, $r_t^j X_t^j$ may be employed if there is a need to decompose their growth contributions further into three components using (17) or there is a need to disaggregate the GDP quantity index using (18). If there is no such need, the whole growth contribution of each subaggregate and the GDP quantity index may be computed using Y_t^j / P_t in which case subaggregate deflators are unnecessary.

The value $r_t^j X_t^j = Y_t^j / P_t$ is essentially the real subaggregate (i.e., real industry GDP) in the PPP framework. As such, it may be rationalized for use in determining growth contributions as follows. The GDP deflator P_t corrects Y_t^j for the effects of general inflation or changes in the overall price level while retaining in Y_t^j some price effects internal to the subaggregate. Therefore, $r_t^j X_t^j = Y_t^j / P_t$ and $r_{t-1}^j X_{t-1}^j = Y_{t-1}^j / P_{t-1}$ incorporate changes in relative prices that should be taken into account in growth analysis. After all, changes in relative prices from r_{t-1}^j to r_t^j induce resource reallocation that affects real GDP growth. Thus, the PPP growth decomposition in (17) is able to decompose growth contributions further to isolate price and growth interaction effects.

GDP in constant prices ignores relative price differences but given that these differences exist weights also exist for converting subaggregates in constant prices to PPP values according

to (14). This implies that the current practice of simple addition (i.e., no weights) of subaggregates in constant prices is objectionable because it means addition of values that are not PPP values, i.e., not in the same units of measure as real GDP because the GDP deflator is not the common denominator. In other words, the quest for simple addition is achieved at the expense of economic sensibility because addition of non-PPP values is equivalent to the proverbial nonsense of “adding apples to oranges.” Moreover, it now appears that the quest for simple addition of subaggregates in chained prices is unachievable unless relative prices are constant.

In current practice, over forty countries now implement GDP in chained prices with Canada and the US employing Fisher price and Fisher quantity indexes while all other countries (e.g., Australia, Japan, Netherlands, and United Kingdom) employ Paasche price and Laspeyres quantity indexes.⁴ Therefore, the lessons from the PPP application to US GDP in this paper are directly transferrable to Canada. Moreover, they are transferrable to all other countries simply by replacing the Fisher quantity index and the Fisher price index wherever they appeared by the Laspeyres quantity index and Paasche price index. In both cases, the PPP framework makes GDP in chained prices in current practice consistent in aggregation.

In the PPP framework, weighted addition is the rule both under constant prices and under chained prices since in both cases the weights arise from relative price differences that exist and need to be recognized. However, in (11) and (12), weighted addition of subaggregates either in constant or in chained prices is equivalent to simple addition of PPP values that equals real GDP. For this reason, PPP is the key to consistency in aggregation of real GDP. Moreover, conversion of heterogeneous subaggregates to common PPP values permits meaningful comparative analysis between distinct components of real GDP.

In light of the preceding results, this paper’s PPP framework has significant implications on ongoing research in income inequality and poverty incidence in all countries regardless of how

⁴ Magtulis (2010) listed forty-three countries that have implemented GDP in chained prices from IMF (2009). For specific country practices, see Aspden (2000) for Australia, Brueton (1999) for the UK, Chevalier (2003) for Canada, Landefeld and Parker (1997) for the US, Maruyama (2005) for Japan, and Schreyer (2004) for EU and OECD countries. Brueton (1999) noted that the European System of National Accounts 1995 recommended Paasche price and Laspeyres quantity indexes as easier and more practical than the theoretically superior Fisher price and Fisher quantity indexes recommended by the System of National Accounts 1993, produced jointly by the EU (2007), IMF (2009), WB, OECD, and UN.

real GDP is measured. For example, if real GDP is not measured in PPP, inward (or outward) shifts in Lorenz curves cannot be unambiguously interpreted to mean decrease (or increase) in income inequality. Also, it is not warranted to make welfare comparisons between households in different income strata unless real income is measured in PPP. By implication, the differentiation between the “chronic poor” and the “transient poor” is not clear cut because movements above or below a “poverty threshold” is ill-defined if the reference real income level is not measured in PPP.⁵

6. Conclusion

The PPP framework in this paper consists of three interrelated equations. One is for consistent real GDP level aggregation. The other is for exact decomposition of real GDP growth. Another is for consistent aggregation of the GDP quantity index. The linchpin of these equations are the weights of subaggregates in constant or in chained prices that convert them into PPP values in the same unit of measure as real GDP.

By the property that PPP values are common measures of real GDP and its components, these values permit meaningful economic analysis between GDP subaggregates at a point in time and also across time given the same base or reference period. Therefore, considering that real subaggregates in current practice—either in constant prices or in chained prices—are not PPP values, their continuing use in economic research—for example, in income inequality and poverty incidence—is questionable without conversion to PPP.

In summary, this paper argues that relative price differences—now ignored in practice—should be taken into account as weights of subaggregates either in constant prices or in chained prices. It turns out that the weighted sum of these subaggregates equals the simple sum of their PPP values. Moreover, this simple sum equals real GDP and remains the same regardless of the number of subaggregates. Therefore, this paper’s PPP framework permits consistent aggregation of real GDP and provides a more logical platform for economic analysis of growth, inequality, and poverty using GDP in PPP values.

⁵ These comments are directed at no specific study because they apply to any study that does not use real values in PPP. However, for studies not using PPP and focused on income inequality and poverty incidence in the Philippines, the reader may go the websites of the Philippine Institute for Development Studies and the National Statistical Coordination Board for specific examples.

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