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Working Paper

Implementing Weights for Additivity of Chained Volume Measures in the National Accounts

PIDS Discussion Paper Series, No. 2011-09

Provided in Cooperation with:

Philippine Institute for Development Studies (PIDS), Philippines

Suggested Citation: Dumagan, Jesus C. (2011) : Implementing Weights for Additivity of Chained Volume Measures in the National Accounts, PIDS Discussion Paper Series, No. 2011-09, Philippine Institute for Development Studies (PIDS), Makati City

This Version is available at:

<https://hdl.handle.net/10419/126850>

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Philippine Institute for Development Studies
Surian sa mga Pag-aaral Pangkaunlaran ng Pilipinas

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Jesus C. Dumagan

DISCUSSION PAPER SERIES NO. 2011-09

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May 2011

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Implementing weights for additivity of chained volume measures in the national accounts

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10 May 2011

In current practice, subaggregate chained volume measures (CVMs) are neither weighted nor additive. This paper derives and implements “weights” for weighted subaggregate CVMs to be additive (i.e., their sum equals aggregate CVM) because without weights, non-additivity permits the nonsensical result that a subaggregate CVM could exceed aggregate CVM. The weights are ratios of subaggregate to aggregate chained price deflators that exceed, equal, or fall below 1 depending on relative prices. CVMs in current practice are additive in the special case of constant relative prices when all weights equal 1. If relative prices change, weights do not equal 1 and their use avoids non-additivity and the above nonsensical result. Empirically, they have widespread implications because CVM is now implemented in over forty countries. Application to actual GDP data shows significant distortions of GDP composition due to non-additivity of subaggregate CVMs from ignoring relative price changes. Among this paper’s formulas for additive weighted subaggregate CVMs, the one based on Paasche price and Laspeyres quantity indexes is recommended for practical implementation.

Key Words: Additivity; Chained indexes; Chained volume measures; GDP in chained prices

JEL classification: C43

1. Introduction

Balk and Reich (2008) posited a “novel” Paasche price deflation procedure for additive subaggregate CVMs. This paper takes off from Balk and Reich’s seminal additivity result to derive and implement “weights” that make subaggregate CVMs additive in their original framework and also in other frameworks employing different price and quantity indexes. This paper is organized according to specific objectives described below.

Section 2 puts this paper’s objectives in context by outlining the CVM framework for the national accounts (i.e., GDP) based on alternative pairs of price and quantity indexes.¹ Over forty countries now implement CVM with Canada and the US employing Fisher price and Fisher quantity indexes while *all other* countries (e.g., Australia, Japan, Netherlands, and United Kingdom) employ Paasche price and Laspeyres quantity indexes.²

¹ CVM measures real GDP and its components in “chained prices” in place of “constant prices.” GDP in chained prices was earlier shown by Al, P. G., *et al.* (1986).

² Magtulis (2010) listed forty-three countries that have implemented CVM according to IMF World Economic Outlook Database (October 2009). Brueton (1999) noted that the European System of National Accounts 1995 recommended Paasche price and Laspeyres quantity indexes as easier and more practical than the theoretically superior Fisher price and Fisher quantity indexes recommended by the System of National Accounts 1993, produced jointly by the EU, IMF, WB, OECD, and UN.

Section 3 examines CVM based on Paasche price and Laspeyres quantity indexes. The Balk-Reich Paasche price deflators for additive subaggregate CVMs are derived analytically by invoking the property of the Laspeyres quantity index of being consistent in aggregation. The derivation implies that CVMs are not inherently non-additive. This contradicts the prevailing theory of inherent non-additivity of CVMs underlying current practice (Aspden, 2000; Whelan, 2002; Ehemann, Katz, and Moulton, 2002; Chevalier, 2003; Schreyer, 2004; Maruyama, 2005; European Union, 2007; Balk and Reich, 2008; Balk, 2010).³

From the above additivity result, this paper derives weights that make subaggregate CVMs—as they are computed in current practice—additive. For example, the sum of weighted subaggregate CVMs of GDP equals CVM of GDP. These weights are ratios of subaggregate to aggregate chained price deflators that exceed, equal, or fall below 1 depending on relative prices. Weights equal 1 only when relative prices are constant.⁴ In the general case when relative prices change, weights do not equal 1 and, therefore, are required for additivity.

While non-additivity is tacitly accepted in current practice, this paper shows that non-additivity is logically objectionable because it permits the nonsensical result that a subaggregate CVM could exceed the aggregate CVM, a result avoidable only by using this paper's weights. These weights have widespread implications for current practice because they apply to CVMs based on Paasche price and Laspeyres quantity indexes that, as earlier noted, are implemented in over forty countries except in Canada and the US.

Section 4 shows that CVMs based on Fisher price and Fisher quantity indexes are additive at the *lowest* level of aggregation, i.e., at the level of the quantity relative used to construct the index. Above this level, this paper derives weights (similar to those in Section 3) and shows that weighted Fisher subaggregate CVMs can only be *approximately* additive because the Fisher index is not consistent in aggregation (Diewert, 1978). Nevertheless, these CVMs are analytically superior to CVMs in current practice in Canada and the US.

Section 5 illustrates the above analytic results with actual GDP data to draw out their implications for current practice of CVM in all countries. The empirical results show significant distortions of GDP composition due to non-additivity of subaggregate CVMs.

Section 6 concludes this paper with a summary of major findings and a recommendation for practical implementation.

³ Balk and Reich (2008) themselves affirmed non-additivity at the outset and, thus, gave the impression that their additivity result is an exception. But by deriving their result from the CVM framework, this paper finds that it is well-grounded in theory and, therefore, should be considered much more than an exception.

⁴ If relative prices are constant, prices grow at the same rate from the start (e.g., a reference period or a fixed base). Hence, chained price indexes equal fixed-base price indexes and this equality holds between chained indexes for subaggregates and the aggregate. In this case, all weights above equal 1.

2. CVM framework for GDP⁵

For illustration, “aggregate CVM” refers to CVM of GDP and “subaggregate CVMs” refer to subaggregate CVMs of GDP. To illustrate CVM over periods $(0, 1, 2, \dots, T)$, it is instructive to begin with two *adjoining* periods s and t , i.e., $t = s + 1$. Let price-quantity data in each period be (p_{is}, q_{is}) and (p_{it}, q_{it}) for $i = 1, 2, \dots, N$ GDP components. Also, let GDP in current prices be Y_s and Y_t ,

$$Y_s = \sum_{i=1}^N p_{is} q_{is} \quad ; \quad Y_t = \sum_{i=1}^N p_{it} q_{it} . \quad (1)$$

The ratio Y_t / Y_s is the value index that may be decomposed into a price index multiplied by a quantity index.

Let the superscripts L , P , and F stand for Laspeyres, Paasche, and Fisher index formulas. These formulas define *chain-type* price (P) and quantity (Q) indexes,

$$P_{st}^L \equiv \frac{\sum_i^N q_{is} p_{it}}{\sum_i^N q_{is} p_{is}} \quad ; \quad P_{st}^P \equiv \frac{\sum_i^N q_{it} p_{it}}{\sum_i^N q_{it} p_{is}} \quad ; \quad P_{st}^F \equiv (P_{st}^L P_{st}^P)^{\frac{1}{2}} ; \quad (2)$$

$$Q_{st}^L \equiv \frac{\sum_i^N p_{is} q_{it}}{\sum_i^N p_{is} q_{is}} \quad ; \quad Q_{st}^P \equiv \frac{\sum_i^N p_{it} q_{it}}{\sum_i^N p_{it} q_{is}} \quad ; \quad Q_{st}^F \equiv (Q_{st}^L Q_{st}^P)^{\frac{1}{2}} . \quad (3)$$

From (1) to (3), value index decomposition is achieved by,

$$\frac{Y_t}{Y_s} = P_{st}^P Q_{st}^L = P_{st}^L Q_{st}^P = P_{st}^F Q_{st}^F . \quad (4)$$

CVM utilizes *chained* indexes formed by multiplying succeeding values of a chain-type index. For illustration, let D_t denote the chained price index generated by a chain-type price index P_{st} with succeeding values $P_{01}, P_{12}, \dots, P_{(t-1)t}$. Hence,

$$D_t = D_s P_{st} \quad ; \quad D_0 = 1 \quad ; \quad 0 = \text{reference period} . \quad (5)$$

D_t takes the value $D_0 = 1$ (or 100) in the reference period, which may be chosen arbitrarily.

From (5), $D_1 = D_0 P_{01} = 1 \times P_{01}$; and $D_2 = D_1 P_{12} = 1 \times P_{01} \times P_{12}$. In general,

$$D_t = 1 \times P_{01} \times P_{12} \times \dots \times P_{(t-2)(t-1)} \times P_{st} = D_s P_{st} \quad ; \quad t - 1 = s \quad ; \quad D_0 = 1 . \quad (6)$$

Following (5) and (6), let J_t be the chained quantity index generated by a chain-type quantity index Q_{st} . Hence, J_t expands to,

$$J_t = 1 \times Q_{01} \times Q_{12} \times \dots \times Q_{(t-2)(t-1)} \times Q_{st} = J_s Q_{st} \quad ; \quad t - 1 = s \quad ; \quad J_0 = 1 . \quad (7)$$

Therefore, substituting (2) into (6) and (3) into (7) yields chained price and quantity indexes,

$$D_t^L = D_s^L P_{st}^L \quad ; \quad D_t^P = D_s^P P_{st}^P \quad ; \quad D_t^F = D_s^F P_{st}^F ; \quad (8)$$

$$J_t^L = J_s^L Q_{st}^L \quad ; \quad J_t^P = J_s^P Q_{st}^P \quad ; \quad J_t^F = J_s^F Q_{st}^F . \quad (9)$$

⁵ Sections 2 and 3 in this paper are adopted from Dumagan (2011) which condensed the same sections from Dumagan (2010).

Finally, recall Y_t in (1) and let $Y_0 = \sum_{i=1}^N p_{i0} q_{i0}$. In this case, it can be verified that value index decomposition using chained indexes in (6) to (9) yields,

$$\frac{Y_t}{Y_0} = D_t^P J_t^L = D_t^L J_t^P = D_t^F J_t^F. \quad (10)$$

From (10), CVM of GDP is obtained either by deflating (dividing) Y_t by a chained price index or inflating (multiplying) Y_0 by the corresponding chained quantity index where Y_0 is a *scalar*. As noted earlier, Canada and the US employ Fisher price and Fisher quantity indexes while all other countries employ Paasche price and Laspeyres quantity indexes. That is,

$$\text{CVM of GDP (Canada, US)} \equiv \frac{Y_t}{D_t^F} = Y_0 J_{st}^F \quad ; \quad \text{CVM of GDP (others)} \equiv \frac{Y_t}{D_t^P} = Y_0 J_{st}^L. \quad (11)$$

The second formula in (11) is examined in detail below.

3. CVMs based on Paasche price and Laspeyres quantity indexes

Using (4) and (8), the second formula for CVM of GDP in (11) becomes,

$$\frac{Y_t}{Y_s} = \frac{D_t^P}{D_s^P} Q_{st}^L \quad ; \quad \frac{Y_t}{D_t^P} = \frac{Y_t}{D_s^P P_{st}^P} = \frac{Y_s}{D_s^P} Q_{st}^L. \quad (12)$$

Being consistent in aggregation, the Laspeyres quantity index Q_{st}^L can be expressed as a weighted sum of subaggregate indexes. For illustration, it is sufficient to start with two mutually exclusive subaggregates A and B given by,

$$Y_s = Y_s^A + Y_s^B \quad ; \quad Y_s = \sum_{i=1}^N p_{is} q_{is} \quad ; \quad N = N^A + N^B \quad ; \quad (13)$$

$$Y_s^A = \sum_{j=1}^{N^A} p_{js}^A q_{js}^A \quad ; \quad Y_s^B = \sum_{k=1}^{N^B} p_{ks}^B q_{ks}^B \quad ; \quad i = (j, k) \quad ; \quad j \neq k. \quad (14)$$

Subaggregate shares w_s^{LA} and w_s^{LB} and Laspeyres quantity indexes Q_{st}^{LA} and Q_{st}^{LB} are,

$$w_s^{LA} = \frac{Y_s^A}{Y_s} \quad ; \quad w_s^{LB} = \frac{Y_s^B}{Y_s} \quad ; \quad Q_{st}^{LA} \equiv \frac{\sum_{j=1}^{N^A} p_{js}^A q_{jt}^A}{\sum_{j=1}^{N^A} p_{js}^A q_{js}^A} \quad ; \quad Q_{st}^{LB} \equiv \frac{\sum_{k=1}^{N^B} p_{ks}^B q_{kt}^B}{\sum_{k=1}^{N^B} p_{ks}^B q_{ks}^B}. \quad (15)$$

From above, it can be verified that,

$$Q_{st}^L \equiv \frac{\sum_{i=1}^N p_{is} q_{it}}{\sum_{i=1}^N p_{is} q_{is}} = \sum_{i=1}^N w_{is}^L \left(\frac{q_{it}}{q_{is}} \right) = w_s^{LA} Q_{st}^{LA} + w_s^{LB} Q_{st}^{LB} \quad ; \quad (16)$$

$$w_{is}^L = \frac{p_{is} q_{is}}{\sum_{i=1}^N p_{is} q_{is}} \quad ; \quad \sum_{i=1}^N w_{is}^L = w_s^{LA} + w_s^{LB} = 1 \quad ; \quad (17)$$

$$\frac{Y_t}{D_t^P} = \frac{Y_s}{D_s^P} (w_s^{LA} Q_{st}^{LA} + w_s^{LB} Q_{st}^{LB}) \quad ; \quad (18)$$

$$\frac{Y_t^A}{Y_s^A} = P_{st}^{PA} Q_{st}^{LA} \quad ; \quad \frac{Y_t^B}{Y_s^B} = P_{st}^{PB} Q_{st}^{LB}. \quad (19)$$

In (19), P_{st}^{PA} and P_{st}^{PB} are subaggregate chain-type Paasche price indexes similar to the aggregate index P_{st}^P in (2).

Therefore, combining (12) to (19) yields additive subaggregate CVMs,

$$\frac{Y_t}{D_t^P} = \frac{Y_t}{D_s^P P_{st}^P} = \frac{Y_t^A}{D_s^P P_{st}^{PA}} + \frac{Y_t^B}{D_s^P P_{st}^{PB}} . \quad (20)$$

Except for different starting premises and notations, it is important to recognize that (20) is the same as the Balk-Reich (2008) additivity result.⁶ Specifically, the corresponding aggregate and subaggregate chained Paasche price deflators (or denominators) are the same as the Balk-Reich deflators.

It is important to note in (20) that additivity involves “double deflation.” The intuitive explanation is that price changes affecting subaggregates—in the framework of “chained prices”—are of two types. The first consists of changes in “specific” prices of components while the second consists of “general” price changes affecting the “real” value of nominal magnitudes in a way similar to the purchasing power of money (Balk and Reich, 2008).

The first type of specific price changes requires deflation of aggregate GDP in current prices, Y_t , by the corresponding aggregate chain-type Paasche price index, P_{st}^P , and deflation of the GDP subaggregates in current prices, Y_t^A and Y_t^B , by their corresponding subaggregate chain-type Paasche price indexes, P_{st}^{PA} and P_{st}^{PB} . This yields,

$$\frac{Y_t}{P_{st}} = \frac{Y_t^A}{P_{st}^A} + \frac{Y_t^B}{P_{st}^B} ; \quad \sum_i^N p_{is} q_{it} = \sum_j^{N^A} p_{js}^A q_{jt}^A + \sum_k^{N^B} p_{ks}^B q_{kt}^B . \quad (21)$$

In principle, the first deflation step adjusts for specific price changes from period s to t . This yields additive “volume” measures or quantities in period t denominated in period s prices given by the second equation in (21). However, by definition, CVMs are denominated in prices of the reference period 0. These CVMs are obtained by the second deflation step that adjusts for general price changes occurring from period 0 to s . This involves further deflation of the volume measures in (21) by a *common* aggregate chained Paasche price index D_s^P to convert them into the *same unit of measure* in period 0 prices ($D_0^P = 1$). This second deflation step yields the additive CVMs in (20).

To compare this paper’s additivity result in (20) to current practice, note that (8) yields the subaggregate chained Paasche price deflators,

$$D_t^{PA} = D_s^{PA} P_{st}^{PA} ; \quad D_t^{PB} = D_s^{PB} P_{st}^{PB} . \quad (22)$$

Combining (20) and (22), this paper’s additivity result becomes,

⁶ Equation (20) is rewritten into the “weighted” equation (23). Later in this paper, (23) is generalized to K subaggregates in equation (27), which is equivalent to equation (31), p. 175, in Balk and Reich (2008).

$$\frac{Y_t}{D_t^P} = \frac{Y_t}{D_S^P P_{st}^P} = \frac{D_S^{PA}}{D_S^P} \left(\frac{Y_t^A}{D_S^{PA} P_{st}^{PA}} \right) + \frac{D_S^{PB}}{D_S^P} \left(\frac{Y_t^B}{D_S^{PB} P_{st}^{PB}} \right) = \frac{D_S^{PA}}{D_S^P} \left(\frac{Y_t^A}{D_t^{PA}} \right) + \frac{D_S^{PB}}{D_S^P} \left(\frac{Y_t^B}{D_t^{PB}} \right). \quad (23)$$

In current practice, CVM of GDP and CVMs of GDP subaggregates are computed by,

$$\text{CVM of GDP} \equiv \frac{Y_t}{D_t^P} \quad ; \quad \text{CVM of A} \equiv \frac{Y_t^A}{D_t^{PA}} \quad ; \quad \text{CVM of B} \equiv \frac{Y_t^B}{D_t^{PB}}. \quad (24)$$

It follows from (23) that the subaggregate CVMs in (24) are not additive if relative prices change. That is,

$$\frac{Y_t}{D_t^P} \neq \frac{Y_t^A}{D_t^{PA}} + \frac{Y_t^B}{D_t^{PB}} \quad ; \quad \frac{D_S^{PA}}{D_S^P} \neq \frac{D_S^{PB}}{D_S^P} \neq 1. \quad (25)$$

However, (23) shows that (25) becomes additive by using as weights the ratios of subaggregate to aggregate chained price indexes D_S^{PA}/D_S^P and D_S^{PB}/D_S^P to adjust for relative price differences. If relative prices are constant, price indexes are equal or these weights equal 1 (footnote 4) so that unweighted subaggregate CVMs are additive. But if relative prices change, these weights exceed or fall below 1 and, therefore, are required for additivity.

The weights above are necessary because without them, non-additivity permits a subaggregate CVM to exceed aggregate CVM which is nonsensical. For example, consider that Y_t^A cannot exceed Y_t , i.e., $Y_t > Y_t^A$ given $Y_t^B > 0$, but prices of Y_t^A may grow slower on average than overall prices of Y_t so that $D_t^P > D_t^{PA}$. Hence, it is possible that,⁷

$$\frac{Y_t}{D_t^P} < \frac{Y_t^A}{D_t^{PA}} \quad ; \quad \frac{Y_t}{D_t^P} < \frac{Y_t^A}{D_t^{PA}} + \frac{Y_t^B}{D_t^{PB}}. \quad (26)$$

While the nonsensical results in (26) may not actually happen, their mere theoretical possibility renders “unweighted” subaggregate CVMs in (25) logically unacceptable.

Finally, consistency in aggregation of Q_{st}^L permits expanding (23) to $k = 1, 2, \dots, K$ subaggregates while maintaining additivity. Therefore,

$$\frac{Y_t}{D_t^P} = \sum_{k=1}^K Y_t^{k*} = \sum_{k=1}^K \frac{D_S^{Pk}}{D_S^P} \left(\frac{Y_t^k}{D_t^{Pk}} \right) \quad ; \quad Y_t^{k*} = \frac{D_S^{Pk}}{D_S^P} \left(\frac{Y_t^k}{D_t^{Pk}} \right). \quad (27)$$

In (27), Y_t^{k*} is this paper’s additive weighted subaggregate CVM with the weight D_S^{Pk}/D_S^P multiplying the subaggregate CVM in current practice Y_t^k/D_t^{Pk} .

4. CVMs based on Fisher price and Fisher quantity indexes

The additive decomposition property of the Fisher index (Dumagan, 2002; Balk, 2004) implies that CVMs in the Fisher index framework are additive at the *lowest* level of

⁷ A numerical example showing the possibility of (26) is available from the author upon request. However, this example satisfies (23), which holds regardless of (26).

aggregation, using the quantity relatives at the commodity level. However, Fisher CVMs can only be approximately additive *above* the commodity level—i.e., if the *starting* unit of analysis is a subaggregate—because the Fisher index is only approximately consistent in aggregation (Diewert, 1978).

Using the price indexes in (2) as weights, Dumagan (2002) showed that the additive decomposition of the Fisher quantity index Q_{st}^F in (3) can be expressed as,

$$Q_{st}^F \equiv (Q_{st}^L Q_{st}^P)^{\frac{1}{2}} = \sum_{i=1}^N w_{it}^F \left(\frac{q_{it}}{q_{is}} \right) ; \quad (28)$$

$$w_{it}^F = \left(\frac{P_{st}^F}{P_{st}^L + P_{st}^F} \right) w_{is}^L + \left(\frac{P_{st}^L}{P_{st}^L + P_{st}^F} \right) w_{is}^P ; \quad (29)$$

$$w_{is}^L = \frac{p_{is} q_{is}}{\sum_{i=1}^N p_{is} q_{is}} ; \quad w_{is}^P = \frac{p_{it} q_{is}}{\sum_{i=1}^N p_{it} q_{is}} ; \quad (30)$$

$$\sum_{i=1}^N w_{it}^F = \sum_{i=1}^N w_{is}^L = \sum_{i=1}^N w_{is}^P = 1. \quad (31)$$

In (31), w_{is}^L , w_{is}^P , and w_{it}^F are the Laspeyres, Paasche, and Fisher weights.

It follows from (4), (8), and (28) that,

$$\frac{Y_t}{D_t^F} = \frac{Y_s}{D_s^F} Q_{st}^F = \sum_{i=1}^N \frac{w_{it}^F Y_s}{D_s^F} \left(\frac{q_{it}}{q_{is}} \right) = \sum_{i=1}^N y_{it}^F ; \quad y_{it}^F = \frac{w_{it}^F Y_s}{D_s^F} \left(\frac{q_{it}}{q_{is}} \right). \quad (32)$$

In (32), y_{it}^F is the *additive* contribution of component i to CVM of GDP. The term $w_{it}^F Y_s$ apportions Y_s according to each component's weight w_{it}^F . Since $w_{it}^F Y_s$ is denominated in period s prices, it is inflated by the component's quantity index $\left(\frac{q_{it}}{q_{is}} \right)$ to obtain a “volume” measure in period t still in period s prices. Moreover, the latter is deflated by the aggregate chained Fisher price index D_s^F to convert it to reference period 0 prices, making y_{it}^F a CVM.

Since the category of each y_{it}^F in (32) is known, subaggregates can be computed by grouping y_{it}^F based on definitions in GDP accounts (e.g., *Consumption*, *Investment*, *Net Exports*, and *Government Expenditures* in the expenditure side of GDP or *Agriculture*, *Industry*, and *Services* in the product side). Since y_{it}^F is an additive CVM, the subaggregates when added together will equal the CVM of GDP given by Y_t/D_t^F in (32).

It may be noted that (28) yields,

$$Q_{st}^F - 1 = \sum_{i=1}^N w_{it}^F \left(\frac{q_{it}}{q_{is}} - 1 \right) ; \quad g_{it}^F = w_{it}^F \left(\frac{q_{it}}{q_{is}} - 1 \right). \quad (33)$$

In (33), g_{it}^F is a component's growth contribution to the growth of the Fisher quantity index. This is exactly the same as the component's growth contribution to the growth of CVM of GDP. This follows from (32) which yields,

$$\frac{Y_t/D_t^F}{Y_s/D_s^F} - 1 = Q_{st}^F - 1. \quad (34)$$

It can be verified that g_{it}^F is equivalent to the formula for component contributions to growth of US GDP in chained dollars (Seskin and Parker, 1998) since this GDP is a CVM based on the Fisher index. Hence, a component contributing g_{it}^F in (33) to *growth* of US GDP in chained dollars is also contributing y_{it}^F in (32) to the *level* of the same GDP. Presently, however, the US implements g_{it}^F but not y_{it}^F .⁸

Given the same subaggregates as before, the Fisher subaggregate shares w_t^{FA} and w_t^{FB} and subaggregate indexes Q_{st}^{FA} and Q_{st}^{FB} corresponding to (29) and (30) are,

$$\sum_{i=i}^N w_{it}^F = w_t^{FA} + w_t^{FB} = 1 \quad ; \quad N = N^A + N^B \quad ; \quad (35)$$

$$w_t^{FA} = \sum_{j=1}^{N^A} w_{jt}^F \quad ; \quad w_t^{FB} = \sum_{k=1}^{N^B} w_{kt}^F \quad ; \quad i = (j, k) \quad ; \quad j \neq k \quad ; \quad (36)$$

$$Q_{st}^{FA} \equiv (Q_{st}^{LA} Q_{st}^{PA})^{\frac{1}{2}} \quad ; \quad Q_{st}^{FB} \equiv (Q_{st}^{LB} Q_{st}^{PB})^{\frac{1}{2}}. \quad (37)$$

Since the Fisher index is only approximately consistent in aggregation (Diewert, 1978),

$$Q_{st}^F \approx w_t^{FA} Q_{st}^{FA} + w_t^{FB} Q_{st}^{FB}. \quad (38)$$

Therefore, combining (32) and (38),

$$\frac{Y_t}{D_t^F} = \frac{Y_t}{D_s^F P_{st}^F} = \frac{Y_s}{D_s^F} Q_{st}^F \approx \frac{Y_s}{D_s^F} (w_t^{FA} Q_{st}^{FA} + w_t^{FB} Q_{st}^{FB}). \quad (39)$$

By value index decomposition,

$$\frac{Y_t^A}{Y_s^A} = Q_{st}^{FA} P_{st}^{FA} \quad ; \quad \frac{Y_t^B}{Y_s^B} = Q_{st}^{FB} P_{st}^{FB}. \quad (40)$$

Combining (39) and (40) yields approximately additive Fisher subaggregate CVMs,

$$\frac{Y_t}{D_t^F} \approx \frac{Y_s w_t^{FA}}{Y_s^A} \frac{Y_t^A}{D_s^F P_{st}^{FA}} + \frac{Y_s w_t^{FB}}{Y_s^B} \frac{Y_t^B}{D_s^F P_{st}^{FB}}. \quad (41)$$

To compare (41) with current practice, note that (8) yields,

$$D_t^{FA} = D_s^{FA} P_{st}^{FA} \quad ; \quad D_t^{FB} = D_s^{FB} P_{st}^{FB}. \quad (42)$$

From (41) and (42), this paper's approximately additive Fisher subaggregate CVMs become,

$$\frac{Y_t}{D_t^F} \approx \frac{Y_s w_t^{FA}}{Y_s^A} \frac{D_s^{FA}}{D_s^F} \left(\frac{Y_t^A}{D_t^{FA}} \right) + \frac{Y_s w_t^{FB}}{Y_s^B} \frac{D_s^{FB}}{D_s^F} \left(\frac{Y_t^B}{D_t^{FB}} \right). \quad (43)$$

In contrast to (43), CVMs in the Fisher framework are computed in current practice by,

$$\text{CVM of GDP} \equiv \frac{Y_t}{D_t^F} \quad ; \quad \text{CVM of A} \equiv \frac{Y_t^A}{D_t^{FA}} \quad ; \quad \text{CVM of B} \equiv \frac{Y_t^B}{D_t^{FB}}. \quad (44)$$

⁸ Dumagan (2008) implemented both g_{it}^F and y_{it}^F using Philippine GDP data.

The US uses the formulas in (44) (Ehemann, Katz, and Moulton, 2002; Whelan, 2002). Like those in (25), they yield non-additive subaggregate CVMs if relative prices change. That is,

$$\frac{Y_t}{D_t^F} \neq \frac{Y_t^A}{D_t^{FA}} + \frac{Y_t^B}{D_t^{FB}} \quad ; \quad \frac{D_s^{FA}}{D_s^F} \neq \frac{D_s^{FB}}{D_s^F} \neq 1. \quad (45)$$

In the special case when relative prices are constant, the weights in (43) equal 1 so that the equality or additivity applies in (43) or in (45). However, if relative prices change, the weights do not equal 1 and, therefore, are necessary to avoid a similar nonsensical possibility in (26) that non-additivity will permit a subaggregate CVM to exceed aggregate CVM.

Finally, (44) may be expanded to $k = 1, 2, \dots, K$ subaggregates to yield,

$$\frac{Y_t}{D_t^F} \approx \sum_{k=1}^K Y_t^{k**} = \sum_{k=1}^K \frac{Y_s w_t^{Fk} D_s^{Fk}}{Y_s^k D_s^F} \left(\frac{Y_t^k}{D_t^{Fk}} \right) \quad ; \quad Y_t^{k**} = \frac{Y_s w_t^{Fk} D_s^{Fk}}{Y_s^k D_s^F} \left(\frac{Y_t^k}{D_t^{Fk}} \right). \quad (46)$$

In (46), Y_t^{k**} is this paper's weighted subaggregate Fisher CVM with the weight $Y_s w_t^{Fk} / Y_s^k D_s^{Pk} / D_s^P$ multiplying the subaggregate CVM in current practice Y_t^k / D_t^{Fk} .

5. Comparison of CVMs using actual GDP data

This paper's procedures and those in current practice are compared by applying them to Philippine GDP using data in current prices and in constant 1985 prices.⁹

For *illustration* purposes, the GDP data in current prices (Table 1) are treated as $(p_{is}q_{is}, p_{it}q_{it})$ and those in constant 1985 prices (Table 2) are treated as $(p_{ib}q_{is}, p_{ib}q_{it})$. Hence, cross-products $(p_{is}q_{it}, p_{it}q_{is})$ and price and quantity ratios—to compute the chain-type indexes in (2) and (3)—can be obtained as follows,

$$\frac{p_{is}q_{is}}{p_{ib}q_{is}} = \frac{p_{is}}{p_{ib}} \quad ; \quad \frac{p_{is}}{p_{ib}} p_{ib}q_{it} = p_{is}q_{it} \quad ; \quad \frac{p_{it}q_{it}}{p_{ib}q_{it}} = \frac{p_{it}}{p_{ib}} \quad ; \quad \frac{p_{it}}{p_{ib}} p_{ib}q_{is} = p_{it}q_{is}; \quad (47)$$

$$\frac{p_{it}q_{it} / p_{is}q_{is}}{p_{ib}q_{it} / p_{ib}q_{is}} = \frac{p_{it}}{p_{is}} \quad ; \quad \frac{p_{ib}q_{it}}{p_{ib}q_{is}} = \frac{q_{it}}{q_{is}}. \quad (48)$$

Notice that the constant prices p_{ib} cancel out in the computations of price and quantity cross-products and ratios between the adjoining periods s and t . Therefore, data similar to those in Table 1 and Table 2 will suffice for *illustrative* CVM computations.

The subaggregate indexes represent major sectors by combining production sources: *Agriculture* (agriculture, fishery, and forestry), *Industry* (mining, quarrying, manufacturing, construction, electricity, gas, and water), and *Services* (transport, communication, storage, trade, finance, ownership of dwellings, real estate, private and government services). The

⁹ A major motivation for this application is to encourage countries to adopt CVM or convert their GDP from constant to chained prices and also to adopt the CVM procedures in this paper.

reference period (year) is 1985 so that subaggregate CVMs and CVM of GDP are measured in “chained 1985 prices” to permit comparison with old GDP in “constant 1985 prices.”¹⁰

Table 1. Philippine GDP in current prices (billion pesos)

	2002	2003	2004	2005	2006	2007	2008	2009
Agriculture and fishery	595.61	629.70	730.70	774.12	851.11	939.12	1,098.42	1,140.29
Forestry	1.81	2.27	3.47	4.25	4.34	4.13	4.33	4.32
Mining and quarrying	33.52	43.57	52.89	63.64	75.56	108.29	110.99	119.00
Manufacturing	915.19	1,004.00	1,122.88	1,264.65	1,381.17	1,459.13	1,656.52	1,555.61
Construction	185.66	194.13	212.77	210.19	235.19	299.85	346.30	378.67
Electricity, gas and water	124.12	137.17	155.82	196.67	216.06	230.91	235.62	241.85
Transport, communication and storage	276.89	313.18	367.35	413.85	446.22	478.39	508.83	519.68
Trade	556.30	602.77	681.74	776.95	877.54	981.45	1,088.20	1,144.94
Finance	170.49	185.98	215.66	263.45	311.95	361.97	404.86	448.62
Ownership of dwellings and real estate	252.86	270.07	292.21	320.41	350.68	373.90	412.65	423.21
Private services	484.91	556.49	653.33	741.98	830.15	936.91	1,036.92	1,112.75
Government services	362.30	377.07	382.74	413.88	452.64	473.29	519.58	580.21
GDP in current prices	3,959.65	4,316.40	4,871.55	5,444.04	6,032.62	6,647.34	7,423.21	7,669.14

Source: National Statistical Coordination Board.

Table 2. Philippine GDP in constant 1985 prices (billion pesos)

	2002	2003	2004	2005	2006	2007	2008	2009
Agriculture and fishery	206.46	214.41	225.09	229.57	238.51	249.99	258.06	258.24
Forestry	0.70	0.87	1.33	1.38	1.33	1.32	1.34	1.33
Mining and quarrying	15.29	17.86	18.33	20.03	18.81	23.71	24.16	29.26
Manufacturing	252.55	263.26	278.62	293.33	306.84	315.71	329.32	312.44
Construction	46.67	47.11	48.72	45.85	49.21	60.90	65.67	69.51
Electricity, gas and water	34.17	35.26	36.75	37.66	40.07	42.75	45.87	44.58
Transport, communication and storage	80.81	87.75	97.61	104.77	111.41	120.70	125.81	128.12
Trade	170.79	180.46	192.69	203.55	215.96	233.86	236.71	243.54
Finance	48.92	51.80	56.92	64.60	72.01	81.31	83.36	89.31
Ownership of dwellings and real estate	48.95	50.96	53.65	56.51	59.73	63.22	66.85	66.20
Private services	78.03	84.37	93.35	100.37	107.92	116.37	122.02	126.69
Government services	49.64	50.99	51.24	53.82	55.24	56.65	59.78	62.77
GDP in constant 1985 prices	1,032.97	1,085.07	1,154.30	1,211.45	1,277.04	1,366.49	1,418.95	1,431.98

Source: National Statistical Coordination Board.

This paper’s CVMs and those in current practice are presented in Table 3 and Table 4. In the last panel of each table, the “Residual” is CVM of GDP less the sum of sectoral CVMs. Thus, additivity yields zero residuals while non-additivity yields non-zero residuals.

Table 3 shows that subaggregate (sectoral) CVMs from this paper’s procedure (23) or (27) are additive while those from current-practice procedure (24) are non-additive.

¹⁰ The above GDP data are available during 1982-2009 so that it is possible to compute CVMs in chained 1985 prices. However, due to space limitations, only the results during 2002-09 are reported but all results for earlier years are available from the author upon request.

Table 3. CVMs based on Paasche price and Laspeyres quantity indexes

Values in billion pesos	2002	2003	2004	2005	2006	2007	2008	2009
GDP in constant 1985 prices	1,032.97	1,085.07	1,154.30	1,211.45	1,277.04	1,366.49	1,418.95	1,431.98
GDP in CVM (chained 1985 prices)	1,028.50	1,080.03	1,148.74	1,207.38	1,274.16	1,362.48	1,417.85	1,433.91
Agriculture CVM								
This paper (additive)	155.13	161.25	166.28	176.59	179.27	189.33	199.56	210.77
Current practice (non-additive)	208.71	216.88	228.06	232.63	241.58	253.15	261.30	261.48
Industry CVM								
This paper (additive)	322.38	339.91	363.07	377.27	403.08	430.42	451.64	439.60
Current practice (non-additive)	347.48	361.33	380.24	393.92	412.60	440.69	462.81	453.38
Services CVM								
This paper (additive)	550.99	578.87	619.39	653.53	691.80	742.73	766.64	783.54
Current practice (non-additive)	466.33	494.01	530.40	566.90	603.42	649.07	673.27	695.53
Residual								
This paper (additive)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Current practice (non-additive)	5.975	7.814	10.037	13.937	16.544	19.561	20.456	23.525

Source: Author's calculations based on data in Tables 1 and 2 and this paper's additive procedure (23) or (27), in contrast to current-practice non-additive procedure (24) in Section 3 of this paper.

Table 4 shows that subaggregate CVMs from this paper's procedure in (32)—starting at the lowest level and aggregated to the sector level—are additive. But this paper's CVMs starting at the subaggregate level, from (43) or (46), are only approximately additive while the CVMs in current practice from (44) are non-additive.

Table 4. CVMs based on Fisher price and Fisher quantity indexes

Values in billion pesos	2002	2003	2004	2005	2006	2007	2008	2009
GDP in constant 1985 prices	1,032.97	1,085.07	1,154.30	1,211.45	1,277.04	1,366.49	1,418.95	1,431.98
GDP in CVM (chained 1985 prices)	1,028.49	1,080.12	1,148.88	1,207.57	1,274.15	1,362.63	1,417.76	1,433.95
Agriculture CVM								
This paper (additive)	155.153	159.701	169.722	174.634	179.977	191.353	205.084	212.394
This paper (approximately additive)	155.153	159.701	169.723	174.634	179.977	191.353	205.084	212.394
Current practice (non-additive)	208.773	216.940	228.109	232.675	241.634	253.202	261.362	261.534
Industry CVM								
This paper (additive)	324.626	342.492	363.664	381.103	403.030	430.285	450.165	434.373
This paper (approximately additive)	324.628	342.492	363.664	381.102	403.030	430.285	450.165	434.371
Current practice (non-additive)	347.562	361.456	380.351	394.057	412.580	440.853	462.921	453.329
Services CVM								
This paper (additive)	548.709	577.925	615.494	651.831	691.144	740.990	762.516	787.180
This paper (approximately additive)	548.709	577.925	615.495	651.831	691.144	740.990	762.515	787.179
Current practice (non-additive)	465.988	493.711	530.227	566.748	603.246	648.906	672.977	695.262
Residual								
This paper (additive)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
This paper (approximately additive)	-0.002	0.000	-0.001	0.000	0.000	0.000	0.001	0.002
Current practice (non-additive)	6.166	8.011	10.194	14.087	16.691	19.666	20.505	23.822

Source: Author's calculations based on data in Tables 1 and 2 and this paper's additive procedure (32) and approximately additive procedure (43) or (46), in contrast to current-practice non-additive procedure (44) in Section 4 of this paper.

Notice in Table 4 that the residuals in current practice are largely non-zero while this paper's non-zero residuals may round off to zero in about two decimal places, indicating that this paper's CVMs are close to being additive. This finding has special implications for the US because the CVMs of US GDP subaggregates are now calculated in the same way as the CVMs in current practice in Table 4. Specifically, the US has two better alternatives which are either this paper's additive or approximately additive CVMs.

Tables 3 and 4 uniformly show that non-additive procedures in current practice *overstate* the CVMs of Agriculture and Industry but *understate* the CVMs of Services by relatively large amounts compared to this paper's additive formulas. In 2002, for example, current practice CVMs are 209 billion for Agriculture and 466 billion for Services while this paper's CVMs are 155 billion for Agriculture and around 550 billion for Services. The same pattern is observable during 2002-09.

The explanation for the above overstatement or understatement by CVMs in current practice is precisely because they do not have relative price "weights" proposed by this paper. Recall the weighted subaggregate CVMs– Y_t^{k*} in (27) from the Paasche price-Laspeyres quantity index framework and Y_t^{k**} in (46) from the Fisher price-Fisher quantity index framework—which are reproduced below,

$$Y_t^{k*} = \frac{D_s^{Pk}}{D_s^P} \left(\frac{Y_t^k}{D_t^{Pk}} \right) \quad ; \quad Y_t^{k**} = \frac{Y_s W_t^{Fk} D_s^{Fk}}{Y_s^k D_s^F} \left(\frac{Y_t^k}{D_t^{Fk}} \right). \quad (49)$$

The *absence* of relative price weights D_s^{Pk}/D_s^P or D_s^{Fk}/D_s^F largely explains the *overstatement* of Agriculture CVMs and *understatement* of Services CVMs by current practice procedures Y_t^k/D_t^{Pk} in Table 3 or Y_t^k/D_t^{Fk} in Table 4 because 2002-09 data show relative prices *falling* ($D_s^{Pk}/D_s^P < 1$ and $D_s^{Fk}/D_s^F < 1$, ranging from 0.73 to 0.82) against Agriculture but *rising* ($D_s^{Pk}/D_s^P > 1$ and $D_s^{Fk}/D_s^F > 1$, ranging from 1.13 to 1.17) in favour of Services. The above absence also explains the inevitable non-additivity of CVMs in current practice.

Clearly, the quantitative effects of non-additivity amount to relatively large distortions of the sectoral composition of CVM of GDP regardless of the underlying index formula. Over time, non-additivity distorts the picture of sectoral transformation of the economy.

For the above reasons, this paper unequivocally recommends Y_t^{k*} and Y_t^{k**} over their counterparts in current practice. However, (49) shows that Y_t^{k*} is simpler to compute than Y_t^{k**} although, in theory, Y_t^{k**} is superior to Y_t^{k*} since the former is based on "superlative" Fisher price and quantity indexes while the latter is based on Paasche price and Laspeyres quantity indexes that are not superlative (Diewert, 1978). But empirically, Table 3 and Table 4 show that Y_t^{k*} and Y_t^{k**} yield very close CVMs at the subaggregate and aggregate levels.

Hence, the theoretical superiority of Y_t^{k**} may not be a compelling factor over the computational simplicity of Y_t^{k*} . Therefore, on balance, Y_t^{k*} —this paper’s additive weighted subaggregate CVM based on the Paasche price and Laspeyres quantity indexes—is the more practical alternative.¹¹

6. Conclusion

In current practice, CVMs are neither weighted nor additive. This paper derived and implemented “weights” for weighted subaggregate CVMs to be additive (i.e., their sum equals aggregate CVM) because without weights, non-additivity permits the nonsensical result that a subaggregate CVM could exceed aggregate CVM. The weights are ratios of subaggregate to aggregate chained price deflators that exceed, equal, or fall below 1 depending on relative prices. It turns out that CVMs in current practice are additive in the special case of constant relative prices when all weights equal 1. In the general case of changing relative prices, these weights do not equal 1 and their use avoids non-additivity and the above nonsensical result. Empirically, they have widespread implications because CVM is now implemented in over forty countries. Application to actual GDP data shows significant distortions of GDP composition due to non-additivity of sectoral CVMs from ignoring relative price changes. The analytical and empirical results support adoption of this paper’s additive procedures in place of those in current practice. More specifically, however, this paper’s additive weighted subaggregate CVM based on the Paasche price and Laspeyres quantity indexes is recommended for practical implementation.

Acknowledgements

The author thanks the Bangko Sentral ng Pilipinas, Statistical Research and Training Center, National Statistics Office, and Bureau of Agricultural Statistics, all in the Philippines, for inviting him as lecturer at the *Workshops on the Operationalization of Chain-Type GDP and Price Indexes* (June 2 and July 8-9, 2010, Bangko Sentral ng Pilipinas) that provided him the opportunity to pursue the CVM additivity issue in this paper. He also thanks seminar participants at the Philippine Institute for Development Studies for their encouraging comments on a longer discussion paper version from which this paper was extracted. However, he is solely responsible for the contents of this paper.

¹¹ Recall from footnote 2 that the EU System of National Accounts 1995 also recommended Paasche price and Laspeyres quantity indexes to underpin CVM. However, the EU formula for a subaggregate CVM is not the same as $Y_t^{k*} = D_s^{Pk}/D_s^P (Y_t^k/D_t^{Pk})$ in (49) but simply Y_t^k/D_t^{Pk} .

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