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Philippine Institute for Development Studies
Surian sa mga Pag-aaral Pangkaunlaran ng Pilipinas

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Deriving weights for additivity of chained volume measures in the national accounts

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Abstract

In current practice in all countries, subaggregate chained volume measures (CVMs) are not weighted and, thus, not additive. However, weights are necessary because without them, non-additivity permits the nonsensical result that a subaggregate CVM could exceed the aggregate CVM. This paper derives weights to make the sum of weighted subaggregates equal the aggregate (i.e., additivity) and avoid this nonsensical result. The weights are ratios of subaggregate to aggregate chained price deflators that exceed, equal, or fall below 1 depending on relative prices. CVMs in current practice are additive only in the special case of constant relative prices when all weights equal 1. Without weights, they are not additive when relative prices change and, in this case, empirical results show that non-additivity could significantly distort the sectoral composition of GDP.

Key Words: Additivity weight; Chained index; Consistency in aggregation

JEL classification: C43

1. Introduction

Over forty countries now implement CVMs, with Canada and US employing Fisher price and Fisher quantity indexes and all other countries—among which are Australia, France, Germany, Japan, Netherlands, and UK—employing Paasche price and Laspeyres quantity indexes.¹ However, non-additivity of CVMs prevails in *all* countries. That is, in the national accounts, the sum of CVMs of GDP subaggregates does not equal the CVM of GDP.

Section 2 of this paper examines the more common CVM framework employing Paasche price and Laspeyres quantity indexes. It is shown that the property of the Laspeyres quantity index of being consistent in aggregation is the key to additivity by yielding the

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¹ Magtulis (2010) found forty-three countries in the IMF World Economic Outlook Database (October 2009) that have implemented CVM. Brueton (1999) noted that the European System of National Accounts 1995 recommended Paasche price and Laspeyres quantity indexes as easier and more practical for CVM than the theoretically superior Fisher price and Fisher quantity indexes recommended by the System of National Accounts 1993, produced jointly by the EU, IMF, WB, OECD, and UN.

weights that make subaggregate CVMs in current practice additive.² Due to space limitations, the additivity issue in the CVM framework based on Fisher price and quantity indexes is not addressed here.³ Section 3 concludes this paper.

2. CVM based on Paasche price and Laspeyres quantity indexes

To illustrate CVM over periods $(0, 1, 2, \dots, T)$, it is instructive to begin with two *adjoining* periods s and t , i.e., $t = s + 1$. Let price-quantity data in each period be (p_{is}, q_{is}) and (p_{it}, q_{it}) for $i = 1, 2, \dots, N$ GDP components. Also, let GDP in current prices be Y_s and Y_t ,

$$Y_s = \sum_{i=1}^N p_{is} q_{is} \quad ; \quad Y_t = \sum_{i=1}^N p_{it} q_{it} . \quad (1)$$

Let P_{st}^P be the Paasche price index and Q_{st}^L be the Laspeyres quantity index. The *chain-type* formulas of these indexes are,

$$P_{st}^P \equiv \frac{\sum_i^N q_{it} p_{it}}{\sum_i^N q_{it} p_{is}} \quad ; \quad Q_{st}^L \equiv \frac{\sum_i^N p_{is} q_{it}}{\sum_i^N p_{is} q_{is}} . \quad (2)$$

From (1) and (2), it can be verified that,

$$\frac{Y_t}{Y_s} = P_{st}^P Q_{st}^L . \quad (3)$$

2.1 CVM of GDP

CVM of GDP requires chaining (3) starting from the *reference* period 0 to the *current* period t . For this purpose, *chained* Paasche price and Laspeyres quantity indexes are needed.

Let D_t^P be the chained Paasche price index spanning from 0 to t . It is defined by multiplying the succeeding values of the chain-type Paasche price index P_{st}^P given by $P_{01}^P, P_{12}^P, \dots, P_{(t-1)t}^P$. That is,

$$D_t^P = 1 \times P_{01}^P \times P_{12}^P \times \dots \times P_{(t-2)(t-1)}^P \times P_{(t-1)t}^P = D_s^P P_{st}^P \quad ; \quad t - 1 = s \quad ; \quad D_0^P = 1. \quad (4)$$

By convention, the index value equals 1 (or 100) in period 0.

² These weights are consistent with the additivity result obtained by Balk and Reich (2008) shown later by equation (15).

³ The reader interested in CVM additivity in the Fisher index framework may see Dumagan (2010 and 2011). These papers showed that the additive decomposition property of the Fisher quantity index yields additive CVMs at the *lowest* level, i.e., level of quantity relatives used to construct the index. They also showed that—because the Fisher index is only *approximately* consistent in aggregation—Fisher subaggregate CVMs can only be *approximately* additive although these are superior to CVMs in current practice (e.g., in the US).

Let J_t^L be the chained Laspeyres quantity index generated by the chain-type Laspeyres quantity index Q_{st}^L . Hence, following (4),

$$J_t^L = 1 \times Q_{01}^L \times Q_{12}^L \times \cdots \times Q_{(t-2)(t-1)}^L \times Q_{st}^L = J_s^L Q_{st}^L ; \quad t - 1 = s ; \quad J_0^L = 1. \quad (5)$$

Finally, recall Y_t in (1) and let $Y_0 = \sum_{i=1}^N p_{i0} q_{i0}$. In this case, (3), (4), and (5) yield,

$$\frac{Y_t}{Y_0} = \frac{Y_1}{Y_0} \times \frac{Y_2}{Y_1} \times \cdots \times \frac{Y_{(t-1)}}{Y_{(t-2)}} \times \frac{Y_t}{Y_{(t-1)}} = D_t^P J_t^L ; \quad \text{CVM of GDP} \equiv \frac{Y_t}{D_t^P} = Y_0 J_{st}^L. \quad (6)$$

By definition, (6) shows that CVM of GDP is obtained either by deflating (dividing) Y_t , the nominal GDP in the current period, by the chained Paasche price index D_t^P or inflating (multiplying) Y_0 , the nominal GDP in the reference period, by the corresponding chained Laspeyres quantity index J_{st}^L where Y_0 is a *scalar*.

2.2. Additive weighted subaggregate CVMs

Combining (3) to (6), CVM of GDP becomes,

$$\frac{Y_t}{Y_s} = \frac{D_t^P}{D_s^P} Q_{st}^L ; \quad \text{CVM of GDP} \equiv \frac{Y_t}{D_t^P} = \frac{Y_t}{D_s^P P_{st}^P} = \frac{Y_s}{D_s^P} Q_{st}^L. \quad (7)$$

Being consistent in aggregation (Diewert, 1978), the Laspeyres quantity index Q_{st}^L can be expressed as a weighted sum of subaggregate indexes. For illustration, it is sufficient to start with two mutually exclusive subaggregates A and B given by,

$$Y_s = Y_s^A + Y_s^B ; \quad Y_s = \sum_{i=1}^N p_{is} q_{is} ; \quad N = N^A + N^B ; \quad (8)$$

$$Y_s^A = \sum_{j=1}^{N^A} p_{js}^A q_{js}^A ; \quad Y_s^B = \sum_{k=1}^{N^B} p_{ks}^B q_{ks}^B ; \quad i = (j, k) ; \quad j \neq k. \quad (9)$$

Subaggregate shares w_s^{LA} and w_s^{LB} and Laspeyres quantity indexes Q_{st}^{LA} and Q_{st}^{LB} are,

$$w_s^{LA} = \frac{Y_s^A}{Y_s} ; \quad w_s^{LB} = \frac{Y_s^B}{Y_s} ; \quad Q_{st}^{LA} \equiv \frac{\sum_{j=1}^{N^A} p_{js}^A q_{jt}^A}{\sum_{j=1}^{N^A} p_{js}^A q_{js}^A} ; \quad Q_{st}^{LB} \equiv \frac{\sum_{k=1}^{N^B} p_{ks}^B q_{kt}^B}{\sum_{k=1}^{N^B} p_{ks}^B q_{ks}^B}. \quad (10)$$

From above, it can be verified that,

$$Q_{st}^L \equiv \frac{\sum_{i=1}^N p_{is} q_{it}}{\sum_{i=1}^N p_{is} q_{is}} = \sum_{i=1}^N w_{is}^L \left(\frac{q_{it}}{q_{is}} \right) = w_s^{LA} Q_{st}^{LA} + w_s^{LB} Q_{st}^{LB} ; \quad (11)$$

$$w_{is}^L = \frac{p_{is} q_{is}}{\sum_{i=1}^N p_{is} q_{is}} ; \quad \sum_{i=1}^N w_{is}^L = w_s^{LA} + w_s^{LB} = 1; \quad (12)$$

$$\frac{Y_t}{D_t^P} = \frac{Y_s}{D_s^P} (w_s^{LA} Q_{st}^{LA} + w_s^{LB} Q_{st}^{LB}); \quad (13)$$

$$\frac{Y_t^A}{Y_s^A} = P_{st}^{PA} Q_{st}^{LA} ; \quad \frac{Y_t^B}{Y_s^B} = P_{st}^{PB} Q_{st}^{LB}. \quad (14)$$

In (14), P_{st}^{PA} and P_{st}^{PB} are subaggregate chain-type Paasche price indexes similar to the aggregate index P_{st}^P in (2).

Therefore, combining (7) to (14) yields additive subaggregate CVMs,

$$\frac{Y_t}{D_t^P} = \frac{Y_t}{D_s^P P_{st}^P} = \frac{Y_t^A}{D_s^P P_{st}^{PA}} + \frac{Y_t^B}{D_s^P P_{st}^{PB}} . \quad (15)$$

Except for different starting premises and notations, it is important to recognize that (15) is the same as the additivity result by Balk and Reich (2008). Specifically, the corresponding aggregate and subaggregate chained Paasche price deflators (or denominators) are the same as their deflators.⁴

To compare (15) with current practice, note that the subaggregate chained Paasche price deflators corresponding to the aggregate deflator in (4) are,

$$D_t^{PA} = D_s^{PA} P_{st}^{PA} \quad ; \quad D_t^{PB} = D_s^{PB} P_{st}^{PB} . \quad (16)$$

Combining (15) and (16) yields this paper's additive weighted subaggregate CVMs,

$$\frac{Y_t}{D_t^P} = \frac{Y_t}{D_s^P P_{st}^P} = \frac{D_s^{PA}}{D_s^P} \left(\frac{Y_t^A}{D_s^{PA} P_{st}^{PA}} \right) + \frac{D_s^{PB}}{D_s^P} \left(\frac{Y_t^B}{D_s^{PB} P_{st}^{PB}} \right) = \frac{D_s^{PA}}{D_s^P} \left(\frac{Y_t^A}{D_t^{PA}} \right) + \frac{D_s^{PB}}{D_s^P} \left(\frac{Y_t^B}{D_t^{PB}} \right) . \quad (17)$$

In current practice, CVM of GDP and CVMs of GDP subaggregates are computed by (Schreyer, 2004),

$$\text{CVM of GDP} \equiv \frac{Y_t}{D_t^P} \quad ; \quad \text{CVM of A} \equiv \frac{Y_t^A}{D_t^{PA}} \quad ; \quad \text{CVM of B} \equiv \frac{Y_t^B}{D_t^{PB}} . \quad (18)$$

Based on (17), the subaggregate CVMs in (18) are not additive if relative prices change, i.e.,

$$\frac{Y_t}{D_t^P} \neq \frac{Y_t^A}{D_t^{PA}} + \frac{Y_t^B}{D_t^{PB}} \quad ; \quad \frac{D_s^{PA}}{D_s^P} \neq \frac{D_s^{PB}}{D_s^P} \neq 1 . \quad (19)$$

However, (17) shows that (19) becomes additive by using as *weights* the ratios of subaggregate to aggregate chained price indexes D_s^{PA}/D_s^P and D_s^{PB}/D_s^P to adjust for relative price differences. In the special case of constant relative prices, price indexes are equal or these weights equal 1 so that the CVMs in (19) are additive.⁵ However, in the general case of changing relative prices, these weights exceed or fall below 1 and, therefore, are required for additivity.

The weights above are necessary because without them, non-additivity permits a subaggregate CVM to exceed aggregate CVM which is nonsensical. For example, consider

⁴ Equation (15) is rewritten into the "weighted" equation (17) and generalized to K subaggregates in equation (21), which is equivalent to equation (31), p. 175, in Balk and Reich (2008).

⁵ Constant relative prices means that prices grow at the same rate from the same starting point, (e.g., a fixed base). Hence, chained price indexes equal fixed-base price indexes and this equality holds between the chained indexes for subaggregates and the aggregate. In this case, all weights above equal 1.

that Y_t^A cannot exceed Y_t , i.e., $Y_t > Y_t^A$ given $Y_t^B > 0$, but prices of Y_t^A may grow slower on average than overall prices of Y_t so that $D_t^P > D_t^{PA}$. Hence, it is possible to show by numerical example that,

$$\frac{Y_t}{D_t^P} < \frac{Y_t^A}{D_t^{PA}} \quad ; \quad \frac{Y_t}{D_t^P} < \frac{Y_t^A}{D_t^{PA}} + \frac{Y_t^B}{D_t^{PB}}. \quad (20)$$

While the nonsensical results in (20) may not actually happen, their mere theoretical possibility renders “unweighted” subaggregate CVMs in (19) logically unacceptable.

Finally, consistency in aggregation of Q_{st}^L permits expanding (17) to $k = 1, 2, \dots, K$ subaggregates while maintaining additivity. Therefore,

$$\frac{Y_t}{D_t^P} = \sum_{k=1}^K Y_t^{k*} = \sum_{k=1}^K \frac{D_s^{Pk}}{D_s^P} \left(\frac{Y_t^k}{D_t^{Pk}} \right) \quad ; \quad Y_t^{k*} = \frac{D_s^{Pk}}{D_s^P} \left(\frac{Y_t^k}{D_t^{Pk}} \right). \quad (21)$$

In (21), Y_t^{k*} is this paper’s additive weighted subaggregate CVM with the weight D_s^{Pk}/D_s^P multiplying the subaggregate CVM in current practice Y_t^k/D_t^{Pk} .

Empirically, D_s^{Pk}/D_s^P could be relatively significant. For example, Dumagan (2011) calculated from Philippine GDP data that in 2009, $D_s^{Pk}/D_s^P = 0.81$ for $k = \text{Agriculture}$ and $D_s^{Pk}/D_s^P = 1.13$ for $k = \text{Services}$.⁶ Substituting these values into (21), the results show that compared to this paper’s additive CVMs, the non-additive CVMs in current practice yield,

$$\frac{Y_t^k/D_t^{Pk}}{Y_t^{k*}} - 1 = \frac{1}{0.81} - 1 = 23.5 \% \text{ overestimate of Agriculture CVM}; \quad (22)$$

$$\frac{Y_t^k/D_t^{Pk}}{Y_t^{k*}} - 1 = \frac{1}{1.13} - 1 = -11.5 \% \text{ underestimate of Services CVM}. \quad (23)$$

These results indicate that non-additivity significantly distorts the sectoral composition of the CVM of GDP and, therefore, is detrimental to understanding the transformation of the economy.

3. Conclusion

CVMs in current practice are not additive because they do not have weights. However, weights are necessary because without them, non-additivity permits the nonsensical result that a subaggregate CVM could exceed the aggregate CVM. In the national income accounts, for example, weights make the sum of weighted CVMs of GDP subaggregates equal the CVM of GDP (i.e., additivity) and avoid this nonsensical result. The weights derived in this

⁶ These numbers can be calculated from Dumagan (2011) by substituting the appropriate 2009 Agriculture and Services CVMs from Table 3, p.11, into the first equation in (49), p. 12.

paper are ratios of subaggregate to aggregate chained price deflators that exceed, equal, or fall below 1 depending on relative prices. CVMs in current practice are additive only in the special case of constant relative prices when all weights equal 1. Without weights, they are not additive in the general case of changing relative prices. In the latter case, non-additivity distorts the sectoral composition of the CVM of GDP and, therefore, is detrimental to understanding the transformation of the economy.

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