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Sharing a River with Downstream Externalities*

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Abstract: We consider the problem of efficient emission abatement in a multi polluter setting, where agents are located along a river in which net emissions accumulate and induce negative externalities to downstream riparians. Assuming a cooperative transferable utility game, we seek welfare distributions that satisfy all agents' participation constraints and, in addition, a fairness constraint implying that no coalition of agents should be better off than it were if all non-members of the coalition would not pollute the river at all. We show that the downstream incremental distribution, as introduced by Ambec and Sprumont (2002), is the only welfare distribution satisfying both constraints. In addition, we show that this result holds true for numerous extensions of our model.

Keywords: Downstream externalities, downstream incremental distribution, optimal emission abatement, river pollution

JEL-Classification: C71, D62, Q53

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1 Introduction

Industries and cities all around the world have historically been concentrated along rivers, since rivers provide means of transportation, food production, energy generation and drinking water. Because of this intensive utilization, many rivers and streams have been and still are being heavily polluted. Excessive pollution worsens water quality, which reduces economic profits and negatively impacts wildlife and human health. One specific characteristic of rivers is that pollutants discharged into the river are carried downriver. As a consequence, it is the downstream riparians rather than the polluter himself who bears the negative consequences of the emissions discharged into the river. Moreover, if upstream polluters and downstream riparians belong to different jurisdictions, polluters may have little incentive to abate their emissions, because they cannot be held liable for the pollution damage caused in other jurisdictions.

In this paper, we consider the problem of efficient emission abatement among agents located along a river, where upstream emissions cause negative externalities to all downstream agents. This setting can be characterized as a cooperative transferable utility game with two sources of externalities. First, upstream emissions impose negative externalities on downstream agents. Second, cooperative behavior among a subset of agents (a so-called coalition) imposes positive externalities upon agents located in between different connected subsets of this coalition. Due to this second kind of externalities, the core is, in general, empty. As a consequence, we restrict our attention to the non-cooperative core, i.e. the set of partitions which consists of one coalition and only singletons otherwise. The non-cooperative core imposes cost upper bounds for any coalition, which can be interpreted as a participation constraint that has to be satisfied by any cost distribution to be acceptable to all agents. In addition, we impose cost lower bounds, which are inspired by the aspiration welfare principle, i.e. no coalition of agents should have lower costs than it can secure for itself if all non-members of the coalition would not pollute the river at all. We show that the downstream incremental distribution, as introduced by Ambec and Sprumont (2002), is the only distribution simultaneously satisfying the non-cooperative core upper cost bounds and the aspiration lower cost bounds.

The existing literature on transboundary pollution in river basins mainly focusses on the case of two jurisdictions. Notable exceptions include Ni and Wang (2007) and Gengenbach et al. (2010). Ni and Wang (2007) derive cooperative sharing rules for the costs of cleaning a river from two principles of international water law: *Absolute Territorial Sovereignty* (ATS) claims that every jurisdiction has exclusive rights to use the water on its territory, while *Unlimited Territorial Integrity* UTI expands these exclusive use rights to all water originating within and upstream of a respective jurisdiction. They adapt these principles to the case of pollution

responsibility and derive axioms characterizing the two resulting cost sharing principles. They also show that these cost-sharing principles correspond to the Shapley value solutions of the corresponding cost-sharing games. However, Ni and Wang (2007) assume exogenously given costs for cleaning the river. Thus, they are only concerned with the distribution of these costs. In contrast, pollution levels in our model are endogenously determined by the actions of the agents. Thus, we are concerned about finding cost sharing distributions that are acceptable to all agents and, at the same time, give incentives to choose efficient emission abatement levels in the first place.

In line with the literature on international environmental agreements, Gengenbach et al. (2010) model river pollution as a two-stage-cartel-formation game. In the first stage, agents decide whether to join a coalition, while pollution abatement levels are chosen in the second stage. In the absence of a supranational authority, abatement levels are in general inefficiently low, as all agents have an incentive to free ride on the abatement efforts of their upstream neighbors. Analyzing the formation of stable coalitions they find that the location of agents has no impact on coalition stability but rather impacts on environmental outcomes. In contrast to Gengenbach et al. (2010), we employ a cooperative game setting.

In fact, our paper is most closely related to Ambec and Sprumont (2002) and Ambec and Ehlers (2008), who apply an axiomatic cooperative game theoretic approach to the efficient sharing of water along a river basin. In Ambec and Sprumont (2002), agents derive strictly increasing benefits from water consumption, Ambec and Ehlers (2008) generalize the results to agents which may exhibit satiation in water consumption. Ambec and Ehlers (2008) show that the downstream incremental distribution is the only welfare distribution satisfying the non-cooperative core lower bounds and the aspiration welfare upper bounds. Several other papers propose alternative sharing rules to the downstream incremental distribution in settings similar to the one proposed by Ambec and Sprumont (2002). Interpreting the river sharing problem as a line-graph game, Van den Brink et al. (2007) derive four different efficient solutions including the downstream incremental distribution by imposing various properties with respect to deleting edges of the line-graph. However, they do not address fairness issues and consider non-satiable agents. Allowing for multiple springs and satiable agents with respect to water consumption, Van den Brink et al. (2012) propose a class of weighted hierarchical welfare distributions based on the Territorial Integration of all Basin States (TIBS) principle, which includes the downstream incremental distribution as a special case. Ansink and Weikard (2012) concentrate on reallocations of the resource itself instead of the reallocation of welfare by an appropriate transfer scheme. In case of water scarcity, the agents' overlapping claims to river water render it a contested resource similar to a bankruptcy problem. They propose a class of sequential sharing rules based on bankruptcy theory and compare them to other sharing rules, including the downstream

incremental distribution. Demange (2004) considers hierarchies without externalities and shows that the *hierarchical outcome* satisfies the core bounds for all connected coalitions for all super-additive cooperative games. However, the hierarchical outcome may violate core bounds for non-connected coalitions. If the hierarchy is a river, then the hierarchical outcome corresponds to the counterpart of the downstream incremental distribution.

Our paper can be interpreted as a generalization of the results of Ambec and Ehlers (2008) to commodities with public good properties. While water consumption is a purely private good, emission abatement exhibits public good characteristics, as it imposes negative externalities on all downstream agents. These additional externalities impose non-trivial complications for proving that the downstream incremental distribution satisfies the non-cooperative cost upper bounds and the aspiration cost lower bounds in the formulation of our river pollution model.

2 A River Sharing Model with Downstream Pollution Externalities

Consider a set of agents $N = \{1, ..., n\}$, which are located along a river. Without loss of generality, agents are numbered from upstream to downstream, i.e. $i < j$ indicates that agent j is located downriver of agent i. We follow Ambec and Sprumont (2002) and Ambec and Ehlers (2008) in defining the set of agents preceding agent i by $P_i = \{1, ..., i\}$, with the strict predecessors of agent i indicated as $P_i \backslash i = \{1, ..., i-1\}$. Analogously, the set of agents following agent i is defined by $F_i = \{i, ..., n\}$, where $F_i \backslash i = \{i+1, ..., n\}$ denotes the set of agents strictly located downriver of agent i.

Each agent i along the river produces gross emissions in exogenously given amount e_i. An agent i may choose to abate the amount x_i with $0 \leq x_i \leq e_i$, the costs of which are given by the strictly increasing, twice differentiable and strictly convex abatement cost function $c_i(x_i)$. Without loss of generality, we assume that abating nothing induces no abatement costs, i.e. $c_i(0) = 0$. Net emissions $e_i - x_i$ are passed into the river where they accumulate and are carried along its course. Assuming that net emissions of agent i are discharged into the river after agent i's but before agent $i+1$'s location, and that there is no pollution at the rivers' source, the ambient pollution level q_i at the location of agent i is given by the sum of net emissions of all strict predecessors of agent i:

$$q_i = \sum_{j \in P_i \backslash i} \gamma_{ji}(e_j - x_j) \ , \quad \forall \ i \in N \ . \tag{1}$$

with $0 < \gamma_{ji} \leq 1$. γ_{ji} represents the assimilative capacity of the river, i.e. what fraction of the net emissions released by agent j actually reach agent i. As the vector of abatement efforts

$x = (x_1, \ldots, x_n)$, together with the vector of exogenously given emissions $e = (e_1, \ldots, e_n)$, fully determine the vector of ambient pollution levels, we shall often write the ambient pollution levels as a function of the vector x:

$$q(x) = \big(q_1(x), \ldots, q_n(x)\big). \tag{2}$$

The ambient pollution level q_i causes damage costs to agent i, the amount of which is given by the increasing, twice differentiable and convex damage cost function $d_i(q_i)$. Thus, the net emissions $e_i - x_i$ released by agent i induce negative externalities for all downriver agents $j > i$, but not for agent i himself or all upstream agents $j < i$.

The total costs k_i agent i faces are the sum of abatement and damage costs:

$$k_i(x_i, q_i) = d_i(q_i) + c_i(x_i)\ . \tag{3}$$

A river sharing problem is characterized by (N, e, c, d), where $c = (c_1, \ldots, c_n)$ and $d = (d_1, \ldots, d_n)$ denote the vectors of abatement and damage cost functions. Given a river sharing problem, the distribution of total costs k_i among all agents i is determined by the emission abatement allocation x. Our assumptions about the accumulation of emissions along the river, as described in the previous paragraph, imply the following proposition.

Proposition 1 (No abatement is dominant strategy)
Given a river sharing problem (N, e, c, d) and for given emission abatement levels of all agents $j \in N\backslash i$ it is a dominant strategy for agent i not to abate at all, i.e. $x_i = 0$.

Proof: The damage costs of agent i only depend on q_i which are not influenced by x_i. As costs c_i are strictly increasing in the amount of emission abatement x_i, given q_i, total costs are minimized by setting $x_i = 0$. □

Proposition 1 states that agents who only consider their own total costs will never abate. In particular, this implies that if the river sharing problem (N, e, c, d) is considered to be a non-cooperative game among the agents $i \in N$, the unique Nash equilibrium is given by $\hat{x}_i = 0$ for all $i \in N$ (no matter whether agents are considered to decide sequentially or simultaneously). However, this outcome is, in general, inefficient. In particular, if we assume that money transfers between agents are possible and agents have unbounded resources for such transfers, the efficient emission abatement allocation $x^\star$ minimizes the sum of total costs k_i among all agents. The following proposition establishes that such an allocation exists and is also unique.

Proposition 2 (Existence and uniqueness of efficient allocation)
Given a river sharing problem (N, e, c, d) there exists a unique vector $x^\star$ which is the solution to the following constrained minimization problem:

$$\min_{\{x_i\}_{i=1}^n} \sum_{i=1}^{n} k_i(x_i, q_i(x)) \quad \textit{subject to} \tag{4a}$$

$$q_i(x) = \sum_{j \in P_i \setminus i} \gamma_{ji}(e_j - x_j) \ , \quad \forall \ i \in N \ , \tag{4b}$$

$$0 \leq x_i \leq e_i \ , \quad \forall \ i \in N \ . \tag{4c}$$

Proof: Existence and uniqueness follow directly from the strict convexity of the total costs functions $k_i(x_i, q_i)$. □

Let t_i denote the money payments. We impose $\sum_{i=1}^n t_i = 0$ and define agent i's after transfer costs z_i as:

$$z_i = k_i(x_i, q_i) + t_i \ . \tag{5}$$

Obviously, any vector $z = (z_1, \ldots, z_n)$ with $\sum_{i=1}^n z_i = \sum_{i=1}^n k_i(x_i^\star, q_i(x^\star))$ is an efficient cost distribution, as it implies a unique vector of transfer payments $t_i = z_i - k_i(x_i^\star, q_i(x^\star))$ with $\sum_{i=1}^N t_i = 0$ (no waste of money) and achieves the cost minimum $\sum_{i=1}^N k_i(x_i^\star, q_i(x^\star))$. In the following, we call any efficient cost distribution a *river sharing agreement*. The main problem will be which one to choose among this infinite set.

3 Coalitions and Cost Upper Bounds

A non-empty subset of agents $S \subset N$ is called a coalition if the agents of S choose their emission abatements such as to minimize the sum of total costs among all coalition members. Denoting by $minS$ and $maxS$ the most upstream, respectively the most downstream member of coalition S, the coalition S is *connected* or *consecutive* if all agents j with $minS < j < maxS$ are also members of the coalition S.

We define the secure costs $v(S)$ of a coalition S as the minimum value of the sum of the total costs k_i over all members of the coalition:

$$v(S) = \sum_{i \in S} k_i(x_i^v(S), q_i(x_i^v(S))) \ , \tag{6}$$

where $x^v(S) = (x_1^v(S), \ldots, x_n^v(S))$ denotes the solution to

$$\min_{\{x_i\}_{i\in S}} \sum_{i\in S} k_i(x_i(S), q_i(x(S))) \quad \text{subject to} \tag{7a}$$

$$q_i(x) = \sum_{j\in P_i\setminus i} \gamma_{ji}(e_j - x_j) \ , \quad \forall\ i \in N \ , \tag{7b}$$

$$0 \le x_i \le e_i \ , \quad \forall\ i \in S \ , \tag{7c}$$

$$x_j \text{ given} \ , \quad \forall\ j \notin S \ . \tag{7d}$$

It is obvious from the above definition that both the allocation of abatement efforts $x^v(S)$ and the secure costs $v(S)$ of the coalition S depend, in general, on the behavior of the agents not belonging to the coalition S. As an example, consider the coalition $S = \{k, \ldots, n\}$. In particular the pollution level q_k (but also the pollution levels q_i with $i > k$) depends on the amount of emission abatement undertaken by the agents i with $i < k$. According to Proposition 1, if these agents $i < k$ only minimize their own sum of abatement and damage costs, they would not abate at all, implying a pollution level of $q_k = \sum_{j\in P_k\setminus k} \gamma_{jk} e_j$. If however, the agents 1 to $k-1$ form a coalition T and minimize their joint total costs, they will, in general, choose $x_j > 0$ for at least some $j \in 1, \ldots, k-1$. This implies a pollution level of $q_k < \sum_{j\in P_k\setminus k} \gamma_{jk} e_j$ which reduces the minimal costs $v(S)$ coalition S can secure for itself. Thus, analogously to Ambec and Ehlers (2008), cooperation exerts a positive externality on the coalition S.

In the following, we restrict our attention to the *non-cooperative core*, i.e. we assume that all non-members of a coalition S behave non-cooperatively, which according to Proposition 1 implies that they do not abate at all. Then, condition (7d) is replaced by $x_j = 0$ for all $j \notin S$, and the secure costs $v(S)$ of a coalition S are well defined and unique (as the resulting optimization problem is a subproblem of the one analyzed in Proposition 2). The reason is like in Ambec and Ehlers (2008): the structure of the river sharing problem (N, e, c, d), as described in detail in Section 2, is such that only the non-cooperative core is guaranteed to be non-empty.

Like Ambec and Ehlers (2008), we impose the secure costs as the participation constraint of any coalition S. A coalition S will only agree to a river sharing agreement if it is not worse off than without the agreement. Thus, a river sharing agreement should at most assign the secure costs $v(S)$ to any coalition S as otherwise the coalition would block the agreement knowing that it can achieve at least $v(S)$ on its own. Hence, $v(S)$ defines cost upper bounds for any coalition S a river sharing agreement must satisfy in order not to be blocked.

4 Cost Lower Bounds

Ambec and Ehlers (2008) also impose welfare upper bounds that are inspired by the *unlimited territorial integrity* (UTI) doctrine. In case of water consumption, UTI claims that all agents are entitled to consume the full stream of water originating upstream from their location and, thus, have a legitimate claim to the corresponding welfare level such a consumption generates. As such claims are, in general, incompatible if water is scarce, Ambec and Sprumont (2002) and Ambec and Ehlers (2008) interpret them as welfare upper bounds agents may legitimately aspire to.

The straightforward translation of these aspiration welfare upper bounds to the case of our river pollution model is to define the minimal costs a coalition S can ensure if all non-members of the coalition would abate all their emissions, and thus, not pollute the river at all. Formally, these cost lower bounds $a(S)$ are given by:

$$a(S) = \sum_{i \in S} k_i(x_i^a(S), q_i(x_i^a(S))) \ , \tag{8}$$

where $x^a(S) = \big(x_1^a(S), \ldots, x_n^a(S)\big)$ denotes the solution to

$$\min_{\{x_i\}_{i \in S}} \sum_{i \in S} k_i(x_i(S), q_i(x(S))) \quad \text{subject to} \tag{9a}$$

$$q_i(x) = \sum_{j \in P_i \setminus i} \gamma_{ji}(e_j - x_j) \ , \quad \forall \ i \in N \ , \tag{9b}$$

$$0 \leq x_i \leq e_i \ , \quad \forall \ i \in S \ , \tag{9c}$$

$$x_j = e_j \ , \quad \forall \ j \notin S \ . \tag{9d}$$

The cost lower bounds $a(S)$ can be interpreted as a fairness condition: no coalition S should enjoy lower costs than the costs it could secure itself if all non-members of the coalition would not pollute the river at all.

5 The Downstream Incremental Distribution

Like in Ambec and Sprumont (2002) and Ambec and Ehlers (2008), there is a connection between the non-cooperative core upper bounds $v(S)$ and the cost lower bounds $a(S)$: For the coalition of all predecessors of agent i they coincide, i.e. $v(P_i) = a(P_i)$. Thus, for any coalition of predecessors P_i it is clear that the only river sharing agreement satisfying both the cost upper and cost lower bounds is the so called *downstream incremental distribution*

(*DID*) defined by

$$z_i^\star = v(P_i) - v(P_i \backslash i) \ , \quad \forall \ i \in N \ . \tag{10}$$

The *DID* assigns every agent his marginal contribution to the coalition composed of his predecessors along the river. As a consequence, the *DID* is the only candidate for a river sharing agreement that at the same time satisfies the non-cooperative core upper bounds $v(S)$ and the cost lower bounds $a(S)$ for any coalition S. The following theorem establishes that the *DID*, in fact, satisfies the non-cooperative core upper bounds $v(S)$ and the cost lower bounds $a(S)$ for any coalition S.

Theorem 1 (Only *DID* satisfies cost upper and lower bounds)
The downstream incremental distribution (DID) $z^\star$ is the only river sharing agreement satisfying the non-cooperative core upper bounds $v(S)$ and the cost lower bounds $a(S)$ for any coalition S.

Proof: The proof is split into three parts. In the first part, we show that the *DID* satisfies the non-cooperative core upper bounds for any coalition S. In part two, we proof that the *DID* also satisfies the cost lower bounds for any coalition S and, finally, in the third part, we show that any river sharing agreement that satisfies the cost upper and lower bounds for an arbitrary coalition S is identical to the *DID*.

We proof that the *DID* satisfies the non-cooperative core upper bounds for any coalition S by induction. The idea is that any coalition S can be created from the grand coalition N by consecutively deleting all non-members $m_j \in \{m_1, \ldots, m_z\}$ of S starting with the most downstream agent m_z. This procedure creates a sequence of intermediate coalitions $N = S_z, S_{z-1}, \ldots, S_1, S$. We show that the *DID* satisfies the core upper bounds for any intermediate coalition S_j, $j \in 1, \ldots, z$ and also for S.

For the first part of the proof we need the following proposition, the proof of which is given in the Appendix.

Proposition 3
For any $T \subset N$ with $minT > j$ and any $j \in N$ the following inequality holds:

$$v(P_{m_j} \cup T) - v(P_{m_j} \backslash m_j \cup T) \leq v(P_{m_j}) - v(P_{m_j} \backslash m_j) \ . \tag{11}$$

For the grand coalition $N = S_z$, the non-cooperative core upper bounds are satisfied. Now, suppose the *DID* satisfies the non-cooperative core upper bounds for some intermediate

coalition S_j, i.e.

$$\sum_{i \in S_j} z_i^\star \leq v(S_j) \ . \tag{12}$$

We generate the intermediate coalition S_{j-1} by deleting the non-member m_j from the intermediate coalition S_j. By construction the intermediate coalition S_{j-1} consists of all strict predecessors of agent m_j and all agents $i > m_j$ who belong to the coalition S. Rearranging inequality (12) and applying the definition of the *DID* implies

$$\sum_{i \in S_{j-1}} z_i^\star \leq v(S_j) - z_{m_j}^\star = v(S_j) - v(P_{m_j}) + v(P_{m_j} \backslash m_j) \ . \tag{13}$$

We have to show that the *DID* satisfies the non-cooperative core upper bounds for the intermediate coalition S_{j-1}, i.e.

$$\sum_{i \in S_{j-1}} z_i^\star \leq v(S_j) - v(P_{m_j}) + v(P_{m_j} \backslash m_j) \leq v(S_{j-1}) \ . \tag{14}$$

Rearranging this inequality yields

$$v(S_j) - v(S_{j-1}) \leq v(P_{m_j}) - v(P_{m_j} \backslash m_j) \ . \tag{15}$$

If the coalition S does not have any members $i > m_j$, then the inequality is trivially satisfied as then $S_j = P_{m_j}$ and $S_{j-1} = P_{m_j} \backslash m_j$. Otherwise, define the set T consisting of all members i of the coalition S with $i > m_j$. Then, $S_j = P_{m_j} \cup T$ and $S_{j-1} = P_{m_j} \backslash m_j \cup T$ and by virtue of Proposition 3, inequality (15) holds.

For the second part of the proof, the following proposition is needed

Proposition 4
For any $S \subset T \subset N$ and $i \notin S, T$ the following inequality holds:

$$a(S \cup i) - a(S) \leq a(T \cup i) - a(T). \tag{16}$$

The proof of Proposition 4 is given in the Appendix.
To show that the *DID* satisfies the cost lower bounds for any coalition S, we employ $v(P_i) = a(P_i)$ to rewrite the definition of the *DID*:

$$z_i^\star = v(P_i) - v(P_i \backslash i) = a(P_i) - a(P_i \backslash i) \ . \tag{17}$$

Summing up over all agents $i \in S$ and employing Proposition 4 yields

$$\sum_{i \in S} z_i^\star = \sum_{i \in S} a(P_i) - a(P_i \backslash i) \geq \sum_{i \in S} a(P_i \cap S) - a(P_i \backslash i \cap S) \ . \tag{18}$$

The right hand side of the inequality simplifies to

$$\begin{aligned} \sum_{i \in S} a(P_i \cap S) - a(P_i \backslash i \cap S) &= a(P_{minS}) + a(\{P_{minS}, P_{minS}+1\}) - a(\{P_{minS}\}) + ... \\ &+ a(\{P_{minS}, ..., P_{maxS}\}) - a(\{P_{minS}, ..., P_{maxS}-1\}) \\ &= a(\{P_{minS}, ..., P_{maxS}\}) = a(S) \ . \end{aligned} \tag{19}$$

Thus, we obtain

$$\sum_{i \in S} z_i^\star = a(P_i) - a(P_i \backslash i) \geq \sum_{i \in S} a(P_i \cap S) - a(P_i \backslash i \cap S) = a(S) \ , \tag{20}$$

which proofs that the *DID* satisfies the cost lower bounds for any coalition S:

$$\sum_{i \in S} z_i^\star \geq a(S) \ . \tag{21}$$

Finally, we proof that the *DID* is the only river sharing agreement that simultaneously satisfies the cost upper and lower bounds for any coalition S. Therefore, we have to show that whenever a river sharing agreement z satisfies both the cost upper and lower bounds, then for each agent i it holds that $z_i = z_i^\star$. Again, the proof is by induction.

Similar to Ambec and Ehlers (2008), for agent 1, any river sharing agreement z fulfilling both constraints satisfies $v(\{1\}) \geq z_1 \geq a(\{1\})$. As $v(\{1\}) = a(\{1\})$ this implies $z_1 = z_1^\star$. Now, suppose that $z_i = z_i^\star$ holds for all agents i upstream of some agent j, i.e. $i \leq j < n$. Summing up over all $i \in P_j$, we obtain

$$\sum_{i \in P_j} z_i = \sum_{i \in P_j} z_i^\star = v(P_j) \ . \tag{22}$$

As $v(P_{j+1}) = a(P_{j+1})$ and because any river sharing agreement z satisfies both the cost upper and lower bounds, $\sum_{i \in P_{j+1}} z_i = v(P_{j+1}) = a(P_{j+1})$ has to hold. Hence,

$$z_{j+1} = \sum_{i \in P_{j+1}} z_i - \sum_{i \in P_j} z_i = v(P_{j+1}) - v(P_j) = z_{j+1}^\star. \tag{23}$$

Therefore, the cost distribution z is identical to the *DID*. □

Theorem 1 is the exact counterpart to Theorem 1 of Ambec and Ehlers (2008). However, it is neither obvious nor straightforward to prove that the *DID* is the only distribution satisfying the cost upper and lower bounds in case of our river pollution model. The main challenge in Ambec and Ehlers (2008) arose from the fact that cooperation among agents impose positive externalities on any coalition S. As a consequence, the welfare level a coalition could secure for itself crucially depends on the partition of all non-members. The same is true for our river pollution model. Cooperative behavior among non-members of a coalition S induces, in general, positive abatement levels, which benefits the members of the coalition.

In contrast to Ambec and Sprumont (2002) and Ambec and Ehlers (2008), however, the decision variable in our model is emission abatement not water consumption. While water consumption only benefits the consumer and, thus, is a purely private commodity, emission abatement is not. In fact, in our model emission abatement does not benefit the abating agent but only all downstream agents, as it reduces the river's downstream pollution level. Thus, emission abatement imposes positive downstream externalities, i.e. pollution abatement is a commodity with public good properties. This is also reflected in the agents' welfare: agents' welfare in the water consumption models of Ambec and Sprumont (2002) and Ambec and Ehlers (2008) is simply given by some benefit function $b_i(x_i)$ which depends on the water consumption x_i of agent i. In our model, the costs agent i faces consist of two parts: first, the abatement costs $c_i(x_i)$, which only depend on the emission abatement of agent i and, second, the damage cost function $d_i(q_i)$ depending on the pollution level q_i, which itself is a function of the emission abatement levels of all upstream agents.

6 Discussion and Extensions

The model detailed in Section 2 relies on a number of assumptions which can be relaxed without impairing the statement of Theorem 1. First, we assumed that there is no initial pollution at the source of the river and that the net emissions of agent i do not harm agent i himself but only all downstream agents. As a consequence, agent 1 does not face any pollution and the specification of agent 1's damage function d_1 is optional. The first assumption simplified the specification of the pollution level q_i, while the latter assumption implied that in the non-cooperative Nash equilibrium no agent would abate at all. However, the proof of Theorem 1 does not draw on these assumptions and would still be valid if the pollution level agent i faces would be defined as

$$q_i = q_0 + \sum_{j \in P_i} \gamma_{ji}(e_j - x_j) \ , \tag{24}$$

where q_0 denotes an initial pollution level at the source of the river.

Second, we framed the model as a pollution abatement model. Obviously, emissions and the corresponding pollution levels are prime examples for downstream externalities, yet there are many other contexts to which our model is applicable. As an example, think of the case of flooding. Then, e_i corresponds to the water discharges from the territory of agent i into the river and x_i denotes the amount of water agent i withdraws from the stream (e.g. by the controlled flooding of designated flooding areas) and q_i is the amount of excess water at agent i's location. In this interpretation it would also be reasonable to assume that the water withdrawn x_i is not limited by the discharge e_i but could sum up to the total amount of excess water in the river basin, i.e.

$$0 \leq x_i \leq q_i \ . \tag{25}$$

These modifications would also not impact on the validity of Theorem 1.

Third, particularly in case of flood protection, agents may have different means of protection. While the withdrawal of water induces costs to agent i and benefits all his downstream agents, there are other protection techniques which are purely private goods. As an example, consider that agent i could build a levee that protects the own territory from flooding, but does not induce any positive externalities to the downstream agents. Then, the damage to agent i does not only depend on the total amount of water q_i but also on the agent's investment into private damage protection m_i, i.e. $d_i = d_i(q_i, m_i)$. Assuming that an interior solution is optimal, i.e. $m_i^\star > 0$, the optimal level of private protection $m_i^\star(q_i)$ is given by the solution of the first order condition

$$\frac{\partial d_i(q_i, m_i)}{\partial m_i} = 0 \ . \tag{26}$$

Thus, we can re-write $d_i(q_i, m_i)$ as $d_i\big(q_i, m_i^\star(q_i)\big)$. Whenever these newly specified damage functions $d_i\big(q_i, m_i^\star(q_i)\big)$ are increasing, twice differentiable and convex in q_i, we are back at the model specification introduced in Section 2.

7 Conclusion

We showed that the main result of Ambec and Ehlers (2008) that the downstream incremental distribution is the only welfare distribution that satisfied the non-cooperative core bounds and the aspiration welfare bounds simultaneously, can be generalized to the case of commodities with public good characteristics. Like their water consumption model, our river pollution problem is a cooperative game with externalities, since cooperation among non-members imposes a positive externality to the members of any coalition S. However,

our model comprises an additional source of externalities because the emissions discharged into the river induce negative externalities on all downstream agents. In addition, our results are robust to various extensions of our baseline model.

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Appendix

Proof of proposition 3:
Set $S_j = P_{m_j} \cup T$ and $S_{j-1} = P_{m_j \backslash m_j} \cup T$. Let us parameterize the damage functions for agents $j > m_j$ with a parameter $\alpha \in [0, \infty)$. Due to this parametrization, the secure costs $v(S_{j-1}, \alpha)$ of the intermediate coalitions S_{j-1} now depend on the parameter α and amount to

$$\begin{aligned} v(S_{j-1}, \alpha) = & \sum_{i \in P_{m_j} \backslash m_j} k_i(x^v(S_{j-1}, \alpha)) + \sum_{i \in F_{m_j} \backslash m_j \cap S} c_i(x^v(S_{j-1}, \alpha)) + \\ & \alpha \cdot \sum_{i \in F_{m_j} \backslash m_j \cap S} d_i(q_i(x^v(S_{j-1}, \alpha))). \end{aligned} \tag{A.1}$$

Furthermore, inequality (11) changes to

$$v(S_j, \alpha) - v(S_{j-1}, \alpha) \leq v(P_{m_j}) - v(P_{m_j} \backslash m_j) \quad \forall \alpha \in [0, \infty). \tag{A.2}$$

By showing that (A.2) holds for all $\alpha \in [0, \infty]$, then it holds, in particular, for $\alpha = 1$ and inequality (11) is satisfied.

Thus, in a next step, we show that inequality (A.2) holds for all $\alpha \in [0, \infty]$. For $\alpha = 0$, we have $v(S_j, 0) = v(P_{m_j})$ and $v(S_{j-1}, 0) = v(P_{m_j} \backslash m_j)$, therefore inequality (A.2) holds with equality. For all other α, we differentiate inequality (A.2) with respect to α, i.e.

$$\frac{\partial v(S_j, \alpha)}{\partial \alpha} - \frac{\partial v(S_{j-1}, \alpha)}{\partial \alpha} \leq 0.$$

Hence, we partially differentiate $v(S_j, \alpha)$ with respect to α and apply the envelope theorem, i.e.

$$\begin{aligned} \frac{\partial v(S_j, \alpha)}{\partial \alpha} &= \frac{\partial v(x^v(S_j, \alpha), \alpha)}{\partial \alpha} + \underbrace{\frac{\partial v(x^v(S_j, \alpha), \alpha)}{\partial x(S_j, \alpha)}}_{0} \frac{\partial x(S_j, \alpha)}{\partial \alpha} \\ &= \frac{\partial v(x^v(S_j, \alpha), \alpha)}{\partial \alpha} \\ &= \sum_{i \in F_{m_j} \backslash m_j \cap S_j} d_i(q_i(x^v(S_j, \alpha))) \end{aligned} \tag{A.3}$$

and analogously,

$$\begin{aligned}\frac{\partial v(S_{j-1},\alpha)}{\partial \alpha} &= \frac{\partial v(x^v(S_{j-1},\alpha),\alpha)}{\partial \alpha} \\ &= \sum_{i\in F_{m_j}\setminus m_j\cap S_{j-1}} d_i(q_i(x^v(S_{j-1},\alpha))).\end{aligned}$$

Hence, inequality (A.3) can be rewritten to

$$\sum_{i\in F_{m_j}\setminus m_j\cap S_j} d_i(q_i(x^v(S_j),\alpha)) \leq \sum_{i\in F_{m_j}\setminus m_j\cap S_{j-1}} d_i(q_i(x^v(S_{j-1},\alpha))). \tag{A.4}$$

Clearly, this inequality is satisfied whenever

$$\sum_{l\in P_k} x_l^v(S_{j-1},\alpha) \leq \sum_{l\in P_k} x_l^v(S_j,\alpha), \forall k \in S_{j-1}, S_j, \tag{A.5}$$

which is stated in the following lemma.

Lemma 1
For any agent $k \in S,T$ the following inequality is satisfied

$$\sum_{l\in P_k} x_l^v(S_{j-1},\alpha) \leq \sum_{l\in P_k} x_l^v(S_j,\alpha), \forall k \in S_{j-1}, S_j. \tag{A.6}$$

Proof of Lemma 1
Consider two coalitions S and $T = S \cup m$ and an agent $m \notin S$. Given this notation, inequality (A.6) changes to

$$\sum_{j\in P_k\cap S} x_j^v(S,\alpha) \leq \sum_{j\in P_k\cap T} x_j^v(T,\alpha), \forall k \in S,T.$$

Let us proof Lemma 1 by contradiction, i.e. assume that

$$\sum_{j\in P_k\cap T} x_l^v(T,\alpha) < \sum_{j\in P_k\cap S} x_l^v(S,\alpha). \tag{A.7}$$

According to the parameterized minimization problem, the following first order conditions have to be satisfied

$$\begin{aligned} c_i'(x_i) &\leq \sum_{j \in F_i \setminus i \cap T \cap P_m} d_j'(q_j(x_j)) + \alpha \sum_{j \in F_m \setminus m \cap T} d_j'(q_j(x_j)) \\ &\leq \sum_{j \in F_i \setminus i \cap T \cap P_m} d_j' \left(\sum_{k \in P_j \setminus j} \gamma_{kj} e_k - \sum_{k \in P_j \setminus j \cap T} \gamma_{kj} x_k \right) \\ &\quad + \alpha \cdot \sum_{j \in F_m \setminus m \cap T} d_j' \left(\sum_{k \in P_j \setminus j} \gamma_{kj} e_k - \sum_{k \in P_j \setminus j \cap T} \gamma_{kj} x_k \right), \forall i \in T, \end{aligned} \tag{A.8}$$

and

$$\begin{aligned} c_i'(x_i) &\leq \sum_{j \in F_i \setminus i \cap S \cap P_m \setminus m} d_j'(q_j(x_j)) + \alpha \sum_{j \in F_m \setminus m \cap S} d_j'(q_j(x_j)) \\ &\leq \sum_{j \in F_i \setminus i \cap S \cap P_m \setminus m} d_j' \left(\sum_{k \in P_j \setminus j} \gamma_{kj} e_k - \sum_{k \in P_j \setminus j \cap S} \gamma_{kj} x_k \right) \\ &\quad + \alpha \sum_{j \in F_m \setminus m \cap S} d_j' \left(\sum_{k \in P_j \setminus j} \gamma_{kj} e_k - \sum_{k \in P_j \setminus j \cap S} \gamma_{kj} x_k \right), \forall i \in S \end{aligned} \tag{A.9}$$

Due to assumption (A.7), the right hand side of (A.8) for $i \in T$ is higher than the right hand side of (A.9) for $i \in S$. This implies $c_i'(x_i(T)) \geq c_i'(x_i(S))$ for all agents $i \in S, T$ and thus, due to the characteristics of the cost function $c_i(.)$, $x_i^v(T) \geq x_i^v(S)$, $\forall i$. This, however, implies

$$\sum_{j \in P_k \cap T} x_j(T) > \sum_{j \in P_k \cap S} x_j(S).$$

Therefore, by contradiction, inequality $\sum_{j \in P_k \cap S} x_j(S) > \sum_{j \in P_k \cap S} x_j(T)$ cannot hold.

Hence, given Lemma 1, inequality (A.2) is satisfied for all $\alpha \in [0, \infty]$, thus also for $\alpha = 1$ implying that inequality (11) holds for the coalition S_{j-1}. By induction, inequality (11) holds for all intermediate coalitions S_j, $j = 1, ..., z$ with $S_1 = S$, therefore the *DID* is stable for all non-consecutive coalitions S.

Proof of proposition 4

For the proof of Proposition 4 the following lemma is required.

Lemma 2

For any two coalitions S, T, the following relationships among the abatement levels of an

agent $j \in T, S$ *hold*

$$x_j^a(T \cup i) \geq x_j^a(S \cup i) \geq x_j^a(S) \text{ and } x_j^a(T \cup i) \geq x_j^a(T).$$

Proof of Lemma 2

It suffices to show that these inequalities hold for two coalitions S, T, with $T = S \cup t$, $t \in N \backslash S$. Let us first establish that $x_j^a(G \cup i) \geq x_j^a(G)$ for all coalitions $G = T, S$. The first order conditions for an agent $j \in G$ respectively $j \in G \cup i$ read

$$c_j'(x_j) \leq \sum_{k \in F_j \backslash j \cap G \cup i} d_k'(\sum_{m \in P_k \backslash k \cap G \cup i} \gamma_{mk}(e_m - x_m)) \tag{A.10}$$

respectively

$$c_j'(x_j) \leq \sum_{k \in F_j \backslash j \cap G} d_k'(\sum_{m \in P_k \backslash k \cap G} \gamma_{mk}(e_m - x_m)). \tag{A.11}$$

The right hand side of the first order condition in (A.10) is either larger than the right hand side of (A.11), if $j \leq i$, or equal to it, if $j > i$. Thus, $x_j^a(G \cup i) \geq x_j^a(G), \forall j \in G, G \cup i$ and $G = T, S$. Due to $T \cup i = S \cup i \cup t$, it follows that $x_j^a(T \cup i) \geq x_j^a(S \cup i)$.

Recall inequality (16) in Proposition 4. We restate the two differences in the inequality in the following way

$$\begin{aligned} a(T \cup i) - a(T) &= k_i(x_i^a(T \cup i)) + \sum_{j \in T} k_j(x_j^a(T \cup i)) - k_j(x_j^a(T)) \\ &= k_i(x_i^a(T \cup i)) + \sum_{j \in T \backslash S} k_j(x_j^a(T \cup i)) - k_j(x_j^a(T)) + \\ &\quad \sum_{j \in S} k_j(x_j^a(T \cup i)) - k_j(x_j^a(T)) \end{aligned}$$

and

$$a(S \cup i) - a(S) = k_i(x_i^a(S \cup i)) + \sum_{j \in S} k_j(x_j^a(S \cup i)) - k_j(x_j^a(S)).$$

Thus, by rearranging and using the above expressions, inequality (16) can expressed as

$$\begin{aligned} &k_i(x_i^a(T\cup i)) - k_i(x_i^a(S\cup i)) + \sum_{j\in T\setminus S} k_j(x_j^a(T\cup i)) - k_j(x_j^a(T)) + \\ &\sum_{j\in S} k_j(x_j^a(T\cup i)) - k_j(x_j^a(T)) + \sum_{j\in S} k_j(x_j^a(S)) - k_j(x_j^a(S\cup i)) \geq 0. \end{aligned} \tag{A.12}$$

To proof that inequality (A.12) is satisfied, we divide the terms into three groups I, II, III as represented in the following

$$\begin{aligned} &\underbrace{\sum_{j\in S} k_j(x_j^a(T\cup i)) - k_j(x_j^a(T)) + \sum_{j\in S} k_j(x_j^a(S)) - k_j(x_j^a(S\cup i)) +}_{I} \\ &\underbrace{\sum_{j\in T\setminus S} k_j(x_j^a(T\cup i)) - k_j(x_j^a(T))}_{II} + \underbrace{k_i(x_i^a(T\cup i)) - k_i(x_i^a(S\cup i))}_{III} \geq 0. \end{aligned} \tag{A.13}$$

In the three lemmas presented below, we will show that for all subgroups I, II, III we have $I, II, III \geq 0$. As a result, we conclude that inequality (A.12) holds.

Lemma 3

Given the terms in subgroup I of (A.13), it holds that $I \geq 0$, i.e.

$$\sum_{j\in S} k_j(x_j^a(T\cup i)) - k_j(x_j^a(T)) + \sum_{j\in S} k_j(x_j^a(S)) - k_j(x_j^a(S\cup i)) \geq 0. \tag{A.14}$$

Proof of Lemma 3

Let us rewrite inequality (A.14) by splitting it into cost and damage functions, i.e.

$$\begin{aligned} &\sum_{j\in S} c_j(x_j^a(T\cup i)) - c_j(x_j^a(T)) + c_j(x_j^a(S)) - c_j(x_j^a(S\cup i)) \\ &\quad + \sum_{j\in S} d_j(q_j(x_j^a(T\cup i))) - d_j(q_j(x_j^a(T))) + d_j(q_j(x_j^a(S))) - d_j(q_j(x_j^a(S\cup i))) \geq 0. \end{aligned}$$

We proof the inequality above graphically. Due to the convexity of both the damage and cost functions of each agent $j \in S$ and due to the relationships $x_j^a(T\cup i) \geq x_j^a(S\cup i) \geq x_j^a(S)$ and $x_j^a(T\cup i) \geq x_j^a(T)$ established in Lemma 2, for each agent j it holds that

$$\begin{aligned} c_j(x_j^a(T\cup i)) - c_j(x_j^a(S\cup i)) &= m, m \geq 0 \\ c_j(x_j^a(S)) - c_j(x_j^a(T)) &= n, n \leq 0, \end{aligned}$$

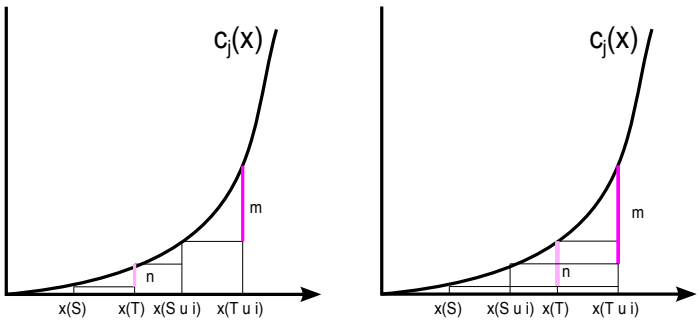

Figure 1: Cost functions

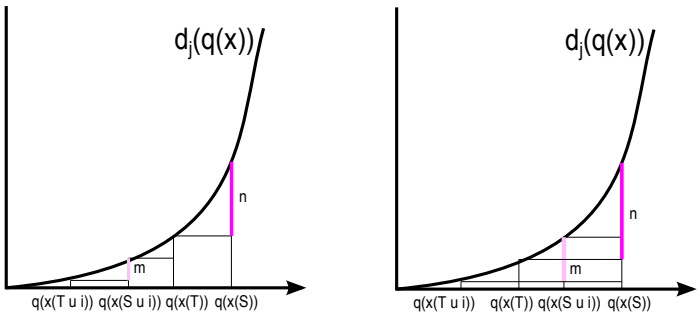

Figure 2: Damage cost functions

with $|m| \geq |n|$ as depicted[1] in Figure 1. Similarly,

$$d_j(q_j(x_j^a(T \cup i))) - d_j(q_j(x_j^a(S \cup i))) = m, m \leq 0$$
$$d_j(q_j(x_j^a(S))) - d_j(q_j(x_j^a(T))) = n, n \geq 0, \forall j$$

with $|n| \geq |m|$ as depicted in Figure 2. Thus, we conclude that inequality (A.14) holds.

Lemma 4

Given the terms in subgroup II of (A.13), it holds that $II \geq 0$, *i.e.*

$$\sum_{j \in T \setminus S} k_j(x_j^a(T \cup i)) - k_j(x_j^a(T)) \geq 0. \tag{A.15}$$

[1] As no general relationship can be established for $x_j^a(T)$ and $x^a(S \cup i)$, both cases are depicted in the Figures 1 and 2.

Proof of Lemma 4
The following first order conditions have to be satisfied

$$c_j'(x_j(T)) \leq \sum_{k \in F_j \backslash j \cap T} d_k' \left(\sum_{t \in P_k \backslash k \cap T} \gamma_{tk}(e_t - x_t(T)) \right), \forall j \in T, \tag{A.16}$$

and

$$c_j'(x_j(T \cup i)) \leq \sum_{k \in F_j \backslash j \cap T \cup i} d_k' \left(\sum_{t \in P_k \backslash k \cap T} \gamma_{tk} e_t + \gamma_{ik} e_i - \gamma_{tk} x_t(T) - \sum_{t \in P_k \backslash k \cap T \cup i} \gamma_{tk} \Delta x_t \right), \forall j \in T \cup i, \tag{A.17}$$

with $\sum_{t \in P_k \backslash k \cap T \cup i} \gamma_{tk} \Delta x_t = \sum_{t \in P_k \backslash k \cap T \cup i} \gamma_{tk} x_j^a(T \cup i) - \sum_{t \in P_k \backslash k \cap T} \gamma_{tk} x_t^a(T)$.
As $x_j^a(T \cup i) \geq x_j^a(T)$ derived in Lemma 2, the right hand side of inequality (A.17) has to be larger than the right hand side of (A.16). In order for this to be satisfied, the following needs to hold

$$\gamma_{ik} e_i \geq \sum_{t \in P_k \backslash k \cap T \cup i} \gamma_{tk} \Delta x_t = \sum_{t \in P_k \backslash k \cap T \cup i} \gamma_{tk} x_j^a(T \cup i) - \sum_{t \in P_k \backslash k \cap T} \gamma_{tk} x_t^a(T).$$

Consequently,

$$d_j \left(\sum_{k \in P_j \backslash j \cap (T \cup i)} \gamma_{kj}(e_j - x_j^a(T \cup i)) \right) \geq d_j \left(\sum_{k \in P_j \backslash j \cap T} \gamma_{kj}(e_j - x_j^a(T)) \right), \forall j \in T. \tag{A.18}$$

In addition, due to $x_j^a(T \cup i) \geq x_j^a(T)$ and $c_i(x_i)$ increasing and convex, we have $c_j(x_j^a(T \cup i)) \geq c_j(x_j^a(T))$. Combining this with (A.18) implies $k_j(x_j^a(T \cup i)) \geq k_j(x_j^a(T))$. Summing up over all $j \in T \backslash S$ yields the desired inequality (A.15).

Lemma 5
Given the terms in subgroup III of (A.13), it holds that $III \geq 0$, *i.e.*

$$k_i(x_i^a(T \cup i)) - k_i(x_i^a(S \cup i)) \geq 0.$$

Proof of Lemma 5
Recall that for each $j \in S \cup i$, the following first order condition must hold

$$c_j'(x_j(S \cup i)) \leq \sum_{k \in F_j \backslash j \cap S \cup i} d_k' \left(\sum_{t \in P_k \backslash k \cap S \cup i} \gamma_{tk}(e_t - x_t(S \cup i)) \right). \tag{A.19}$$

Similarly, for each $j \in T \cup i$ it must hold that

$$c'_j(x_j(T \cup i)) \leq \sum_{k \in F_j \setminus j \cap T \cup i} d'_k\Big(\sum_{t \in P_k \setminus k \cap S \cup i} \gamma_{tk} e_t + \sum_{t \in P_k \setminus k \cap T \setminus S} \gamma_{tk} e_t - \sum_{t \in P_k \setminus k \cap S \cup i} \gamma_{tk} x_t(S \cup i) - \sum_{t \in P_k \setminus k \cap T \cup i} \gamma_{tk} \Delta x_t\Big). \tag{A.20}$$

Given that $x_j^a(T \cup i) \geq x_j^a(S \cup i)$ derived in Lemma 2, the left hand side of inequality (A.20) is larger than the left hand side of inequality (A.19). Consequently, it has to hold that

$$\sum_{t \in P_k \setminus k \cap T \setminus S} \gamma_{tk} e_t \geq \sum_{t \in P_k \setminus k \cap T \cup i} \gamma_{tk} \Delta x_t = \sum_{t \in P_k \setminus k \cap T \cup i} \gamma_{tk} x_t^a(T \cup i) - \sum_{t \in P_k \setminus k \cap S \cup i} \gamma_{tk} x_t^a(S \cup i).$$

The agents $j \in T \cup i$ do not abate more than the additional pollution flow passing through their region compared to what they would optimally abate if they belong to the smaller coalition $S \cup i$. As a result,

$$d_i\left(\sum_{j \in P_i \setminus i \cap T \cup i} \gamma_{ji}(e_j - x_j^a(T \cup i))\right) \geq d_i\left(\sum_{j \in P_i \setminus i \cap S \cup i} \gamma_{ji}(e_j - x_j^a(S \cup i))\right). \tag{A.21}$$

In addition, as $x_j^a(T \cup i) \geq x_j^a(S \cup i)\ \forall j$ derived in Lemma 2 and $c_i(x_i)$ is increasing and convex, we have

$$c_i(x_i^a(T \cup i)) \geq c_i(x_i^a(S \cup i)). \tag{A.22}$$

Combining (A.21) with (A.22) implies

$$k_i(x_i(T \cup i)) \geq k_i(x_i(S \cup i)).$$

Proposition 4 then follows from combining Lemmas 3, 4 and 5.