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Central Bank Purchases of Government Bonds

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Central Bank Purchases of Government Bonds*

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Abstract

We develop a dynamic general equilibrium model to analyze the effects of central bank purchases of government bonds by investigating the following three questions: Under what conditions are these purchases socially desirable, what incentive problems do they mitigate, and how large are these effects? We show that by purchasing government bonds, central banks induce agents to increase their demand for money, which increases the value of money and thereby improves the allocation and welfare. We then analyze the post-crisis period and show that implementing the zero lower bound was optimal and worth 0.014 percent of total consumption.

Keywords: Monetary theory, over-the-counter markets, open market operations, money demand, pecuniary externality.

JEL Classification: D52, D62, E31, E40, E50, G11, G12, G28.

1 Introduction

Governments generally issue two intrinsically useless objects: money and bonds. While money is perfectly liquid, divisible, and widely accepted as a medium of exchange, bonds are “intentionally handicapped (hence discounted)” due to “physical or legal characteristics that render them less liquid than money” (Andolfatto, 2011, p.133). By conducting monetary policy, central banks have a major control over the composition of these issued objects. Concretely, by means of open market operations (OMOs, hereafter), central banks purchase government bonds in exchange for money in order to achieve an interest rate target.

In this paper, we take a closer look at OMOs. That is, we analyse under what conditions they are desirable and prove competent in implementing an interest rate target, what the optimal

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degree of activity in OMOs is, and whether it was optimal to implement the zero lower bound in the aftermath of the global financial crisis.

For this purpose, we construct a microfounded monetary model, where trading in financial markets is essential. In our model, agents face idiosyncratic liquidity shocks, and they hold a portfolio composed of money and government bonds. Money can be directly used to purchase goods and thus serves as a medium of exchange. In contrast, government bonds cannot be used as a medium of exchange, but are a superior store of value.¹ The idiosyncratic liquidity shocks generate an ex post inefficient allocation of the medium of exchange: Some agents will hold money, but have no current need for it, while other agents will hold insufficient amounts of money in order to satisfy their liquidity needs. A secondary financial market allows agents to trade money for bonds and so improves the allocation of the medium of exchange. The secondary financial market is an over-the-counter market, that embeds the recent advances in search theory. OMOs are modelled as a central bank purchase of government bonds funded by the issuance of money; i.e., we abstract from the issuance of interest-bearing reserves, financial intermediaries and repurchase agreements, and assume that the central bank directly controls the bond-to-money ratio. We derive our results in the monetary steady state equilibrium; i.e., there are no aggregate shocks. Furthermore, we focus on the optimal degree of OMOs in an economy, where the efficient allocation is not attainable; i.e., on an economy with inflation rates above the Friedman rule.

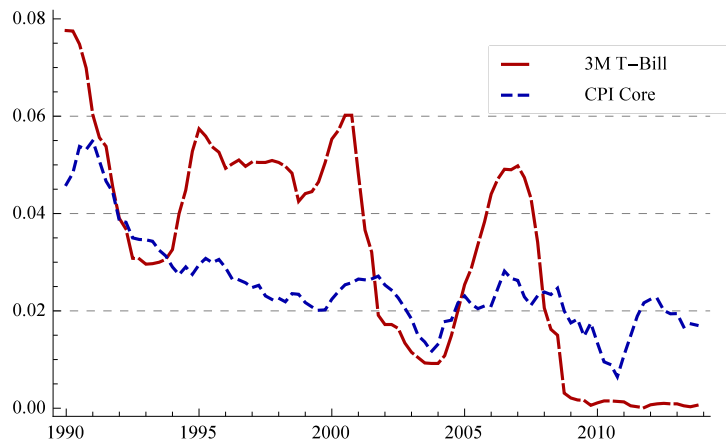


FIGURE 1: U.S. SHORT-TERM INTEREST RATES VS. INFLATION

Our main findings is that OMOs mitigate a pecuniary externality and so improve the allocation and welfare. The externality arises because in our incomplete market model the resulting equilibria are not constrained efficient. In such an environment, government interventions can be

¹It is socially beneficial that government bonds cannot be used as a medium of exchange. Otherwise, bonds would be perfect substitutes for money and thus be redundant. See Kocherlakota (2003), Andolfatto (2011), Berentsen and Waller (2011), and Berentsen et al. (2014) for a more detailed discussion.

welfare-improving.² In our model, the secondary financial market reduces the incentive to self-insure against liquidity shocks, and agents attempt to benefit from money held by other market participants. As a result, the aggregate demand for money is too low, and OMOs can mitigate this externality. The reasoning behind this result is as follows. OMOs reduce the bond-to-money ratio, which makes bonds scarce and increases their price above their fundamental value. This induces agents to demand more money, which marginally increases the value of money and so improves the insurance for all market participants. However, we show that this result is only valid for low inflation rates. The reason is that for higher inflation rates, the opportunity cost of holding money rises, and as a consequence, the incentive to increase the demand for money vanishes.

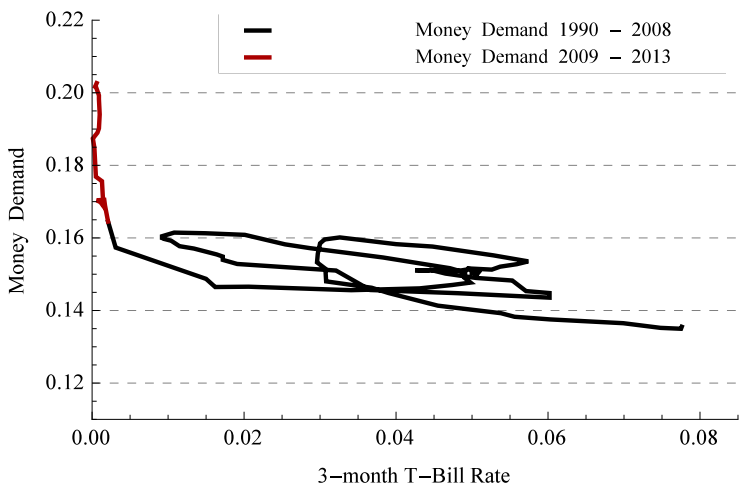


FIGURE 2: U.S. MONEY DEMAND

We calibrate the model to U.S. data in order to estimate the impact of OMOs on quantities and prices. We focus our analysis on the post-crisis period, since this was the first time that the Federal Reserve Bank (Fed, hereafter) implemented the zero lower bound and the core inflation rate remained above the short-term interest rates for a prolonged period of time. As an illustration, we show in Figure 1 the development of the core inflation rate and the annualized yield on U.S. government bonds with a maturity of three months (referred to as T-Bill in Figure 1).

Our numerical analysis suggests that the implementation of the zero lower bound in late 2008 was optimal and worth 0.014 percent of total consumption. The economic mechanism behind this finding is that the implementation of the zero lower bound results in an increase in the demand for money, which in turn raises the value of money and so improves consumption. In Figure 2, we show the demand for money with respect to the 3-month T-Bill rate for the United States.³

²See Greenwald and Stiglitz (1986) and Berentsen et al. (2015b) for a more detailed discussion.

³Money demand is measured as the ratio of M1 adjusted for retail sweeps divided by the nominal gross domestic

As implied by our model, the demand for money increased subsequent to the implementation of the zero lower bound in late 2008.

Furthermore, we show that OMOs are welfare-improving for quarterly inflation rates of up to 3.2 percent. The optimal degree of OMOs reduces the yield on short-term government bonds close to zero for any quarterly inflation rate below 3.2 percent. Since the opportunity cost of holding money increases in the inflation rate, the required amount of OMOs to implement the optimal policy decreases in the inflation rate.

2 Literature

Our paper is in the field of the “New Monetarist Economics,” a branch of literature that builds on Kiyotaki and Wright (1989) and especially Lagos and Wright (2005).⁴ In our model, types are alternating, which generates an ex post inefficient allocation of money and generates an endogenous role for a financial market, where agents can adjust their portfolios. In this sense, our paper is related to Berentsen et al. (2007) and Berentsen and Waller (2011). In contrast to these studies, we do not assume competitive pricing in the secondary financial market and model the exchange process in more detail. Concretely, we assume over-the-counter trading with search and bargaining frictions in the spirit of Duffie et al. (2005).⁵

Unlike the above-mentioned studies, our main focus is on central bank purchases of government bonds and their implications for the allocation and welfare. More closely related to what we do are the studies by Williamson (2012, 2014*a, b*). While we assume that only money can be used as a medium of exchange, Williamson assumes that also claims to bonds are acceptable in some meetings. That is, Williamson assumes that agents do not directly trade money for bonds, but rather that banks collateralize their deposits with bonds and that agents can trade these claims for goods in monitored meetings.⁶ In the exchange process, Williamson assumes take-it-or-leave-it offers, while we assume over-the-counter trading. This allows us to understand the importance of search and bargaining frictions, and it also has interesting implications regarding the optimality of OMOs, which are not present under the assumption of take-it-or-leave-it offers. Williamson models some features in more detail than we do, such as financial intermediation and the issuance of private debt, while we focus more explicitly on other issues. That is, we show under what conditions central banks succeed in controlling the short-term interest rates by

product. Throughout the paper, we follow Berentsen et al. (2015*a*) and use M1 adjusted for retail sweeps instead of M1, because it represents the stock of transaction media more accurately.

⁴A detailed overview of major contributions to this field can be found in Williamson and Wright (2010), Nosal and Rocheteau (2011), and Lagos et al. (2015).

⁵There is a rapidly growing literature which builds on the seminal contribution of Duffie et al. (2005). See, for instance, Ashcraft and Duffie (2007), Duffie et al. (2008), Lagos and Rocheteau (2009), Lagos et al. (2011), Mattesini and Nosal (2013), Trejos and Wright (2014), Rocheteau and Wright (2013), Geromichalos and Herrenbrueck (2015), Lagos and Zhang (2015), Berentsen et al. (2015*b*), and Huber and Kim (2015).

⁶A more detailed discussion about the acceptability of illiquid assets can be found in Shi (2008), Lagos and Rocheteau (2008), Lester et al. (2012), Hu and Rocheteau (2013), and Rocheteau et al. (2015). Collateralization is discussed in Ferraris and Watanabe (2008), He et al. (2015), Li and Li (2012), Gu et al. (2013), Bethune et al. (2014), Andolfatto et al. (2015), and Rocheteau et al. (2015).

purchasing government bonds and when such a policy measure is socially beneficial. To do this, we do not solely focus on equilibria where bonds are scarce and exhibit a liquidity premium, but also focus on equilibria where bonds are plentiful, which allows us to obtain interesting insights.⁷ For instance, we find that in equilibria where bonds and money are plentiful, welfare can always be improved by conducting OMOs. Furthermore, we find that OMOs are not socially beneficial if inflation is too high. Both arguments are missing in the studies by Williamson.

Gertler and Karadi (2013) present a model which is similar to Williamson (2012, 2014*a, b*) in order to analyze the effects of central bank purchases of long-maturity government bonds or private loans. They find that central banks, as opposed to private intermediaries, obtain funds elastically; i.e., they can fund the purchase of long-term securities by issuing short-term debt. This provides central banks with a channel for large-scale asset purchases to be effective in reducing borrowing costs if financial assets are scarce. In contrast to Gertler and Karadi (2013), we focus on central bank purchases of short-maturity bonds in order to control the short-term interest rates. Furthermore, we focus in more detail on the societal implications of such a policy measure.

Similar to what we do, Herrenbrueck (2014) integrates an over-the-counter market in the spirit of Duffie et al. (2005) into a New Monetarist framework. Like Williamson (2012, 2014*a, b*), Herrenbrueck assumes take-it-or-leave-it offers in the secondary financial market, while we assume that agents bargain over the terms of trade in bilateral meetings. Herrenbrueck finds that central bank purchases of government bonds are temporarily welfare-improving, because they transfer wealth to agents who value it more; i.e., to agents who want to sell financial assets for money in order to consume. Opposed to Herrenbrueck, we find that the main welfare-improving aspect of such a policy measure is that it corrects a pecuniary externality, and not that it transfers wealth to agents who value it more. In other words, it is not the resulting liquidity premium on bonds that improves welfare, but the incentive to increase the demand for money.

Most closely related to our study is the work by Rocheteau et al. (2015). Similar to Williamson (2012, 2014*a, b*), the authors assume that bonds are accepted as a medium of exchange in some meetings, while they are not in others. That is, Rocheteau et al. (2015) focus on the acceptability and pledgability of bonds in an equilibrium where bonds are scarce, while we simplify according to this rationale and model the exchange process in more detail in order to analyze the welfare implications of OMOs. In contrast to us, Rocheteau et al. (2015) do not model a secondary financial market explicitly. Consequently, they cannot evaluate the role of search and bargaining frictions in secondary bond markets. Since search frictions are empirically well documented in the market for U.S. government bonds, we believe that it is important to incorporate these frictions in order to analyse the social implications of OMOs.⁸ Doing so allows us to show that search frictions are essential for OMOs to be welfare-improving. Furthermore, we do not solely focus on equilibria where bonds are scarce, which allows to state that OMOs

⁷The existence of liquidity premia is discussed in Geromichalos et al. (2007), Lagos and Rocheteau (2008), Lagos (2010*a, b*, 2011), Jacquet and Tan (2012), Lester et al. (2012), Nosal and Rocheteau (2013), Lagos and Zhang (2015), Berentsen et al. (2014, 2015b), and Huber and Kim (2015).

⁸See Duffie et al. (2005) and Krishnamurthy (2002) for a more detailed discussion about the search frictions in the market for U.S. government bonds.

are only welfare-improving for low inflation rates.

Our paper is also related to the literature that focuses on correcting pecuniary externalities, such as Berentsen et al. (2014 and 2015b). Berentsen et al. (2014) find that adding search frictions to a competitive secondary financial market can be welfare-improving for high inflation rates. The reason is that adding search frictions increases the demand for money, which is welfare-improving, but it also increases consumption variability, since only some agents have access to the secondary financial market. In a similar framework to ours, Berentsen et al. (2015b) find that the demand for money is too low in an equilibrium where trading is unconstrained and that imposing a financial transaction tax can correct this externality. However, imposing a financial transaction tax requires the central bank to operate the secondary financial market in order to perfectly enforce tax payment. In contrast, OMOs are a conventional policy tool of central banks and it is crucial to understand the economic mechanisms behind such a policy instrument.

3 Environment

A $[0, 1]$ -continuum of agents live forever in discrete time. In each period, there are three markets that open sequentially.⁹ The first market is a secondary bond market, where agents trade money for nominal bonds. The second market is a goods market, where agents produce or consume market-2 goods. The third market is a centralized market, where all agents consume and produce market-3 goods, and financial contracts are redeemed. This market is called the primary bond market. All goods are perfectly divisible and non-storable.

At the beginning of each period, agents receive two i.i.d shocks: a preference shock and an entry shock. The preference shock determines whether an agent is a producer or a consumer in the goods market. With probability n , an agent can produce but not consume, and with probability $1 - n$, he can consume but not produce. Throughout the paper, we assume $n \in (0.5, 1)$. The entry shock determines whether an agent has access to the secondary bond market. With probability δ he has access, and with the counter probability $1 - \delta$ he has not.

In the goods market, trading is frictionless; i.e., agents trade against the market and take prices as given. Consumers get utility $u(q)$ from q consumption, where $u'(q), -u''(q) > 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Producers incur a utility cost $c(q) = q$ from producing q units of market-2 goods.

In the primary bond market, trading is also perfectly frictionless and competitive. The market-3 good is produced and consumed by all agents using a linear production technology; i.e., h units of time produce h units of goods.¹⁰ Agents get utility $U(x)$ from x consumption, where $U'(x), -U''(x) > 0$, $U'(0) = \infty$, and $U'(\infty) = 0$. Agents discount between, but not within, periods. The term $\beta = (1 + r)^{-1}$ denotes the discount factor between two consecutive periods, where $r > 0$ represents the real interest rate. A central bank operates in the primary bond market

⁹Our framework is similar to Berentsen et al. (2015b), which builds on Lagos and Wright (2005). However, the contribution of Berentsen et al. (2015b) is different. In particular, the authors investigate the social benefits of a financial transaction tax on bond transactions in an equilibrium where trading is unconstrained.

¹⁰The assumption of quasi-linear preferences in the primary bond market results in a degenerate end-of-period distribution of money holdings, which makes the model tractable (see Lagos and Wright, 2005).

and issues two perfectly divisible and storable assets: money and one-period bonds. Both assets are intrinsically useless. Bonds are issued at a discount, and pay off one unit of money in the next-period primary bond market. Bonds are intangible objects; i.e., no physical object exists. In the goods market, agents cannot commit, and there is a lack of record-keeping. These two frictions imply that producers ask for immediate compensation from consumers. As bonds are intangible objects, only money can serve as a medium of exchange in the goods market.¹¹ The per-capital stock of money is denoted by M_t , and the per-capita stock of newly issued bonds is denoted by B_t at the end of period t . The issuance price of bonds in the primary bond market is denoted by ρ_t . Thus, the change in the stock of money in period t is given by

$$M_t - M_{t-1} = \tau_t M_{t-1} + B_{t-1} - \rho_t B_t, \quad (1)$$

which is determined by three components: the lump-sum money injections, $\tau_t M_{t-1}$, the money created to redeem previously issued bonds, B_{t-1} , and the money withdrawal from selling B_t units of bonds at price ρ_t . We assume that there is a strictly positive initial stock of money M_0 and bonds B_0 , where $B_0/M_0 = \mathcal{B}$ denotes the bond-to-money ratio. For $\tau_t < 0$, the central bank must be able to extract money via lump-sum taxes from the economy.

At the beginning of each period, and after the realization of the two i.i.d shocks, agents can trade money for bonds in the secondary bond market. Consumers and producers meet at random in bilateral meetings and bargain over the terms of trade. The probability that a consumer meets a producer is denoted by $\delta \in [0, 1]$, and the probability that a producer meets a consumer by $\delta_p = \delta(1-n)n^{-1}$, since $n \geq 0.5$. Agents who have a match in this market are called *active* and those who are not are called *passive*.

3.1 Efficient Allocation

As a benchmark exercise, we present the allocation chosen by a social planner who dictates consumption and production. The planner treats all agents symmetrically and his optimization problem is

$$W = \max_{h,x,q} [(1-n)u(q) - nq_p] + U(x) - h, \quad (2)$$

subject to the feasibility constraint $h \geq x$ and the market clearing condition $nq_p \geq (1-n)q$. The efficient allocation satisfies $U'(x^*) = 1$, $u'(q^*) = 1$, and $h^* = x^*$.

¹¹The necessary assumptions that make money essential are discussed in more detail in Kocherlakota (1998), Wallace (2001), Lagos and Wright (2005) and Shi (2006). A more detailed discussion about why bonds cannot be used as a medium of exchange can be found in Kocherlakota (2003), Andolfatto (2011), Berentsen and Waller (2011), or Berentsen et al. (2014 and 2015b). This assumption is crucial, since it allows us to break the irrelevance theorem proposed in Wallace (1981).

4 Agent's Decisions

For notational simplicity, we omit time subscript t going forward. Next-period variables are indexed by $+1$, and previous-period variables by -1 . In what follows, we study the agents' decisions in a representative period t and work backwards from the last market (primary bond market) to the first market (secondary bond market).

4.1 Primary Bond Market

In the primary bond market, agents can acquire any amount of money and newly issued bonds at price ρ . Agents want to hold money, because they will use it in the next-period goods market, if they turn out to be consumers. In contrast, bonds cannot be used as a medium of exchange; i.e., they are illiquid, but they can be traded for money in the next-period secondary bond market, which opens before the goods market. Furthermore, agents can produce and consume the market-3 goods using a linear production technology; they receive money for maturing bonds; can trade money for market-3 goods; and they receive the lump-sum money transfer T from the central bank. An agent entering the primary bond market with m units of money and b units of bonds has the value function $V_3(m, b)$. He solves the following decision problem:

$$V_3(m, b) = \max_{x, h, m_{+1}, b_{+1}} [U(x) - h + \beta V_1(m_{+1}, b_{+1})], \quad (3)$$

subject to

$$x + \phi m_{+1} + \phi \rho b_{+1} = h + \phi m + \phi b + \phi T, \quad (4)$$

where ϕ is the price of money in terms of market-3 goods, and h denotes hours worked. The first-order conditions with respect to m_{+1} , b_{+1} and x are $U'(x) = 1$, and

$$\frac{\beta \partial V_1}{\partial m_{+1}} = \rho^{-1} \frac{\beta \partial V_1}{\partial b_{+1}} = \phi. \quad (5)$$

According to (5), the marginal cost of taking one additional unit of bonds into the next period, $\rho\phi$, is lower than that for money, ϕ , for any $\rho < 1$. The reason is that bonds are only beneficial to agents who will be active consumers in the next period. Therefore, bonds exhibit a lower marginal benefit than money for $\rho < 1$, which is denoted by $\beta \partial V_1 / \partial b_{+1}$. Due to the quasi-linear preferences, the choice of m_{+1} and b_{+1} is independent of m and b . As a result, each agent exits the primary bond market with the same amount of money and bonds. The envelope conditions are

$$\frac{\partial V_3}{\partial m} = \frac{\partial V_3}{\partial b} = \phi. \quad (6)$$

The above equation states that at the beginning of the primary bond market, the marginal value of money and bonds is equal to ϕ . This is because bonds are redeemed at their face value in this market.

4.2 Goods Market

We assume competitive pricing in the goods market; i.e., all agents take prices as given and trade against the market. Consider the consumer's decision problem, where p denotes the price of one unit of the market-2 good q :

$$V_2^c(m, b) = \max_q \left[\begin{array}{l} u(q) + V_3(m - pq, b) \\ s.t. m \geq pq. \end{array} \right]. \quad (7)$$

The constraint states that a consumer cannot spend more money than the amount he brings into this market. If the constraint is non-binding, we have $\frac{\partial q}{\partial m} = 0$ and $u'(q) = 1$. If the constraint is binding, we have $\frac{\partial q}{\partial m} = \phi$ and $u'(q) > 1$. In this case, the buyer's envelope conditions are

$$\frac{\partial V_2^c}{\partial m} = \phi u'(q) \quad \text{and} \quad \frac{\partial V_2^c}{\partial b} = \phi, \quad (8)$$

where we have used the envelope conditions in the primary bond market.

The producer's value function in the goods market is

$$V_2^p(m, b) = \max_{q_p} [-c(q_p) + V_3(m + pq_p, b)], \quad (9)$$

where q_p satisfies the market clearing condition

$$(1 - n)[\delta \hat{q} + (1 - \delta)q] = nq_p, \quad (10)$$

and where the symbol “ $\hat{\cdot}$ ” denotes the quantities traded by active agents. According to (10), the produced quantity of producers, nq_p , equals the consumed quantity of active consumers, $(1 - n)\delta \hat{q}$, plus the consumed quantity of passive consumers, $(1 - n)(1 - \delta)q$. The market clearing condition states that the consumed quantities of active and passive consumers are different, since active consumers could adjust their portfolio in the secondary bond market and thus can consume more in the goods market.

It is easy to see that $p\phi = c'(q_p) = 1$ holds in any monetary equilibrium. The reason is the following. For $p\phi < c'(q_p) = 1$, there is no trade, because it is suboptimal for producers to trade in the good market. For $p\phi > c'(q_p) = 1$, each single producer has an incentive to sell more goods by (9). Hence, selling any finite amount of goods is suboptimal; i.e., the optimal strategy by producers is not supported by the market clearing condition (10). Therefore, the equilibrium price must be $p\phi = c'(q_p) = 1$.

Taking the total derivative of (9) with respect to m and b and using (6) yields the envelope conditions of the producer:

$$\frac{\partial V_2^p}{\partial m} = \frac{\partial V_2^p}{\partial b} = \phi. \quad (11)$$

Because producers cannot use money or bonds in this market, their marginal benefit equals the price of money in the primary bond market.

4.3 Secondary Bond Market

In the secondary bond market, the terms of trade are determined by the proportional bargaining solution proposed by Kalai (1977), which is increasingly popular in monetary economics due to its monotonicity properties.¹² Consumers and producers are matched pairwise and bargain over the terms of trade. Let (m_j, b_j) , (\hat{m}_j, \hat{b}_j) denote the portfolios of an active agent before and after trading in the secondary bond market, respectively. By the market clearing condition, we have

$$\hat{m}_c - m_c = -(\hat{m}_p - m_p) \quad \text{and} \quad \hat{b}_p - b_p = -(\hat{b}_c - b_c).$$

Let $d_m \equiv \hat{m}_c - m_c$ and $d_b \equiv \hat{b}_p - b_p$ be the trading amounts of money and bonds in the secondary bond market. Hence, we have the budget constraints for producers and consumers,

$$\phi m_p \geq \phi d_m \quad \text{and} \quad \phi b_c \geq \phi d_b, \quad (12)$$

which state that producers cannot offer more money than they have, and consumers cannot offer more bonds than they have.

The Kalai constraint states that the trade surplus is split among producers and consumers according to their bargaining power. It is given by

$$(1 - \eta) [u(\hat{q}) - u(q) - \phi d_m] = \eta \phi (d_b - d_m), \quad (13)$$

where η denotes the bargaining power of a consumer, and $1 - \eta$ is the bargaining power of a producer. The trade surplus in the secondary bond market is $u(\hat{q}) - u(q) - \phi d_m$, where $u(\hat{q}) - u(q) - \phi d_b$ is the consumer's surplus and $d_b - d_m$ is the producer's surplus. An active agent's decision problem is

$$K(m_c, m_p, b_c, b_p) \equiv \max_{d_m, d_b} [u(\hat{q}) - u(q) - \phi d_m] \quad \text{s.t.} \quad (12) \text{ and } (13). \quad (14)$$

Note that if $K(m_c, m_p, b_c, b_p)$ is differentiable with respect to $x = m_c, m_p, b_c, b_p$, then

$$\frac{\partial K}{\partial x} = u'(\hat{q}) \frac{\partial \hat{q}}{\partial x} - u'(q) \frac{\partial q}{\partial x} - \phi \frac{\partial d_m}{\partial x}. \quad (15)$$

If the budget constraints for producers and consumers (12) are non-binding; i.e., if $\phi m_p > \phi d_m$ and $\phi b_c > \phi d_b$, then the first-order condition of the maximization problem in (14) with respect to d_m is

$$u'(\hat{q}) \frac{\partial \hat{q}}{\partial d_m} - \phi = \phi [u'(\hat{q}) - 1] = 0, \quad (16)$$

which means that active consumers can consume the efficient quantity, such that $u'(\hat{q}) = 1$. Finally, we can derive the value function of a consumer and a producer before entering the

¹²Kalai bargaining is discussed in more detail in Aruoba et al. (2007), or Rocheteau and Wright (2005). For its application to financial markets, see Geromichalos and Herrenbrueck (2015), Berentsen et al. (2015), and Huber and Kim (2015). A detailed discussion of its monotonicity properties is provided by Chun and Thomson (1988).

secondary bond market as

$$\begin{aligned} V_1^c(m_c, b_c) &= \delta\eta K(m_c, m, b_c, b) + V_2^c(m_c, b_c), \\ V_1^p(m_p, b_p) &= \delta_p(1 - \eta)K(m, m_p, b, b_p) + V_2^p(m_p, b_p), \end{aligned} \tag{17}$$

when the trading partner has a portfolio (m, b) .

5 Monetary Equilibrium

We focus on symmetric, stationary monetary equilibria, where all agents follow identical strategies and where real variables are constant over time. The gross growth rate of bonds is denoted by $\zeta \equiv B/B_{-1}$, and the gross growth rate of the money supply is denoted by $\gamma \equiv M/M_{-1}$. In a stationary monetary equilibrium, the real stock of money and bonds must be constant; i.e., $\phi M = \phi_{+1}M_{+1}$ and $\phi B = \phi_{+1}B_{+1}$, which implies $\gamma = \zeta = \phi/\phi_{+1}$.

In what follows, we present three stationary monetary equilibria. In the first equilibrium, labeled type-I, the producer's cash constraint and the consumer's bond constraint (12) are non-binding in the secondary bond market. In the second equilibrium, labeled type-II, the producer's cash constraint is binding and the consumer's bond constraint is non-binding. In the third equilibrium, labeled type-III, the producer's cash constraint is non-binding and the consumer's bond constraint is binding. All proofs are relegated to the Appendix.

5.1 Type-I Equilibrium

A type-I equilibrium is characterized by

$$u'(\hat{q}) = 1, \tag{18}$$

$$\phi m_p > \phi d_m, \tag{19}$$

$$\phi b_c > \phi d_b. \tag{20}$$

Equations (19) and (20) simply mean that the constraints of money and bond holdings are non-binding in the secondary bond market.

Proposition 1 *A type-I equilibrium is a list $\{\hat{q}, q, q_p, \rho\}$ satisfying (10) and*

$$1 = u'(\hat{q}), \tag{21}$$

$$\frac{\gamma}{\beta} = (1 - n)\delta\eta [u'(\hat{q}) - u'(q)] + (1 - n)u'(q) + n, \tag{22}$$

$$\rho = \frac{\beta}{\gamma}. \tag{23}$$

Equation (21) is obtained from the first-order condition in the secondary bond market (16) and states that active agents consume the efficient quantity; i.e., such that $u'(\hat{q}) = 1$. Equation (22) is derived from the marginal value of money in the secondary bond market. The left-hand side of

(22) represents the marginal cost of acquiring one additional unit of money in the primary bond market, and the right-hand side of the equation denotes the marginal benefit. With probability $1 - n$ the agent will be a consumer in the goods market, in which case he has the marginal utility $u'(q)$. With probability $(1 - n)\delta$ he will be an active consumer and additionally obtain a fraction η of the surplus $[u'(\hat{q}) - u'(q)]$. Finally, with probability n the agent will be a producer in the goods market, in which case he receives a marginal utility of 1. Equation (23) is derived from the marginal value of bonds in the secondary bond market and states that bonds are priced at their fundamental value, β/γ , in the primary bond market.

5.2 Type-II Equilibrium

A type-II equilibrium is characterized by

$$u'(\hat{q}) > 1, \quad (24)$$

$$\phi m_p = \phi d_m, \quad (25)$$

$$\phi b_c > \phi d_b. \quad (26)$$

Equation (24) means that an active consumer does not consume the optimal amount of goods in the goods market, because the constraint on the producer's money holdings is binding in the secondary bond market (25). As in the type-I equilibrium, the meaning of (26) is that the constraint on the consumer's bond holdings is non-binding in the secondary bond market.

Proposition 2 *A type-II equilibrium is a list $\{\hat{q}, q, q_p, \rho\}$ satisfying (10) and*

$$\hat{q} = 2q, \quad (27)$$

$$\frac{\gamma}{\beta} = (1 - n)\delta\eta [u'(\hat{q}) - u'(q)] + (1 - n)u'(q) + n + n\delta^p(1 - \eta) [u'(\hat{q}) - 1], \quad (28)$$

$$\rho = \frac{\beta}{\gamma}. \quad (29)$$

Equation (27) is a direct consequence of the active consumers' binding cash-constraint in the goods market and the fact that active producers transfer all their money to their trading partner in the secondary bond market. Equation (28) is similar to (22), except for the last term on the right-hand side of the equation. Because the cash-constraint of an active producer is binding in the secondary bond market, he can earn a strictly positive surplus on his money holdings $[u'(\hat{q}) - 1]$, according to his bargaining power $1 - \eta$. Like in the type-I equilibrium, bonds are priced at their fundamental value, (29), because the bond constraint of an active consumer is non-binding.

5.3 Type-III Equilibrium

A type-III equilibrium is characterized by

$$u'(\hat{q}) > 1, \quad (30)$$

$$\phi m_p > \phi d_m, \quad (31)$$

$$\phi b_c = \phi d_b. \quad (32)$$

Also in the type-III equilibrium, an active consumer does not consume the efficient quantity. Equation (31) means that the producer's cash-constraint is non-binding in the secondary bond market, while (32) means that the consumer's bond constraint is binding in the secondary bond market.

Proposition 3 *A type-III equilibrium is a list $\{\hat{q}, q, q_p, \rho\}$ satisfying (10) and*

$$\mathcal{B} = \frac{(1 - \eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q)}{q}, \quad (33)$$

$$\frac{\gamma}{\beta} = (1 - n) \left[\delta u'(\hat{q}) \frac{\eta + (1 - \eta)u'(q)}{\eta + (1 - \eta)u'(\hat{q})} + (1 - \delta)u'(q) \right] + n, \quad (34)$$

$$\rho = \frac{\beta}{\gamma} \left[1 + (1 - n)\delta\eta \frac{u'(\hat{q}) - 1}{\eta + (1 - \eta)u'(\hat{q})} \right]. \quad (35)$$

Equation (33) is derived from the Kalai condition (13). Equation (34) is derived from the marginal value of money in the secondary bond market. With probability $(1 - n)\delta$, an agent will be an active consumer, in which case he obtains a share of the surplus equal to $u'(\hat{q}) [\eta + (1 - \eta)u'(q)] [\eta + (1 - \eta)u'(\hat{q})]^{-1}$. With probability $(1 - n)(1 - \delta)$, he will be a passive consumer and obtain the marginal utility $u'(q)$. With probability n , he will be a producer in the goods market, in which case he receives a marginal utility of 1, because his cash-constraint is non-binding in the secondary bond market. Equation (35) is derived from the marginal value of bonds in the secondary bond market and states that bonds exhibit a liquidity premium equal to $(1 - n)\delta\eta [u'(\hat{q}) - 1] [\eta + (1 - \eta)u'(\hat{q})]^{-1}$. The reason for this result is the binding bond constraint of consumers in the secondary bond market.¹³

6 Regions of Existence

Hereafter, we derive the regions of existence of each type of equilibrium with respect to the inflation rate γ and the bond-to-money ratio $\mathcal{B} = \frac{B}{M} = \frac{b}{m}$. We focus on an economy where the efficient allocation is not attainable; i.e, $\gamma > \beta$ and $\delta < 1$.¹⁴ Note that $\phi b = \frac{b}{m}\phi m = \mathcal{B}\phi m = \mathcal{B}q$

¹³See Geromichalos and Herrenbrueck (2015) for a more detailed analysis under what conditions a liquidity premium exists in the primary bond market.

¹⁴It is easy to show that under competitive pricing, the type-I equilibrium coincides with the type-I equilibrium under Kalai bargaining for $\eta = 1$ (see Berentsen et al. 2014 and 2015b, and Huber and Kim 2015). With

holds in any type of equilibrium. For now, we follow the literature that builds on Lagos and Wright (2005) and assume that the fiscal policy is purely passive. That is, we assume that any change in \mathcal{B} has no effect on the inflation rate γ . We will show later what implications it has when we relax this assumption.

The following three Propositions guarantee existence of equilibria, even though they might not be unique, which is clarified in the subsequent four lemmas.

Proposition 4 *There exists a constant γ_{12} and a function $\mathcal{B}_{13}(\gamma)$ such that the type-I equilibrium is supported if, and only if, $\gamma < \gamma_{12}$ and $\mathcal{B} > \mathcal{B}_{13}(\gamma)$.*

Proposition 5 *There exists a function $\mathcal{B}_{23}(\gamma)$ such that the type-II equilibrium is supported if, and only if, $\gamma > \gamma_{12}$ and $\mathcal{B} > \mathcal{B}_{23}(\gamma)$.*

Proposition 6 *There exists a function $\mathcal{B}_{32}(\gamma)$ such that the type-III equilibrium is supported if, and only if, (i) $\gamma \leq \gamma_{12}$ and $\mathcal{B} < \mathcal{B}_{13}(\gamma)$; or (ii) $\gamma > \gamma_{12}$ and $\mathcal{B} < \mathcal{B}_{32}(\gamma)$.*

The following lemma characterizes the properties of $\mathcal{B}_{13}(\gamma)$, $\mathcal{B}_{23}(\gamma)$, and $\mathcal{B}_{32}(\gamma)$.

Lemma 7 *$\mathcal{B}_{13}(\gamma)$, $\mathcal{B}_{23}(\gamma)$, and $\mathcal{B}_{32}(\gamma)$ satisfy the following properties;*

- (i) $\mathcal{B}_{13}(\gamma)$ is increasing in $\gamma < \gamma_{12}$,
- (ii) $\mathcal{B}_{23}(\gamma)$ is increasing in $\gamma > \gamma_{12}$,
- (iii) $\mathcal{B}_{32}(\gamma)$ is increasing in $\gamma > \gamma_{12}$,
- (iv) $\lim_{\gamma \rightarrow \gamma_{12}} \mathcal{B}_{13}(\gamma) = \lim_{\gamma \rightarrow \gamma_{12}} \mathcal{B}_{23}(\gamma) = \lim_{\gamma \rightarrow \gamma_{12}} \mathcal{B}_{32}(\gamma)$.

The functions and critical values described in Propositions 4-6 are visualized in Figure 3 for ease of understanding.

The chart on the left-hand side of Figure 3 shows that there exists a region with $\mathcal{B}_{32}(\gamma) > \mathcal{B}_{23}(\gamma)$ for $\gamma > \gamma_{12}$. In such a case, the region between $\mathcal{B}_{23}(\gamma)$ and $\mathcal{B}_{32}(\gamma)$ supports the type-II and the type-III equilibrium.¹⁵ The chart on the right-hand side of Figure 3 shows that there

$\delta = 1$ and $\eta = 1$, the type-I equilibrium only exists at the Friedman rule ($\gamma_{12} = \beta$); i.e., the efficient allocation, $u'(q^*) = 1$, is only attainable at $\gamma = \beta$. In contrast, under Kalai bargaining, it holds that $q = \hat{q} = q^*$ for $\delta = 1$ and $\eta < 1$ for $\beta < \gamma < \text{Min}(\gamma_{12}, \gamma_{13})$, with $\gamma_{12} = \beta \{(1-n)\eta + (1-n)(1-\eta)u'(0.5q^*) + n\}$, and with $\gamma_{13} = \beta \{(1-n)\eta + (1-n)(1-\eta)u'(q) + n\}$, where $\mathcal{B}q = (1-\eta)[u(q^*) - u(q)] + \eta(q^* - q)$ by Proposition 4. Hence, for $\delta = 1$ and $\eta < 1$, the efficient allocation is attainable for inflation rates above the Friedman rule, which is shown by Geromichalos and Herrenbrueck (2015).

¹⁵When $\mathcal{B}_{23}(\gamma) \leq \mathcal{B}_{32}(\gamma)$ for $\gamma > \gamma_{12}$, the region between $\mathcal{B}_{23}(\gamma)$ and $\mathcal{B}_{32}(\gamma)$ also supports an equilibrium in which the bond constraint of active consumers and the money constraint of active producers are binding simultaneously. This equilibrium is different from the type-II and the type-III equilibrium. It is a list $\{\hat{q}, q, q_p, \rho\}$ satisfying (10) and $\hat{q} = 2q$, $\mathcal{B}q = (1-\eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q)$, $\frac{\gamma}{\beta} \leq (1-n)\delta\eta[u'(\hat{q}) - u'(q)] + (1-n)u'(\hat{q}) + n + n\delta^p(1-\eta)[u'(\hat{q}) - 1]$, $\frac{\gamma}{\beta} \geq (1-n)\left[\delta u'(\hat{q})\frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} + (1-\delta)u'(q)\right] + n$, $\rho \geq \frac{\beta}{\gamma}$, $\rho \leq \frac{\beta}{\gamma}\left[1 + (1-n)\delta\eta\frac{u'(\hat{q}) - 1}{\eta + (1-\eta)u'(\hat{q})}\right]$. The proof is available on request.

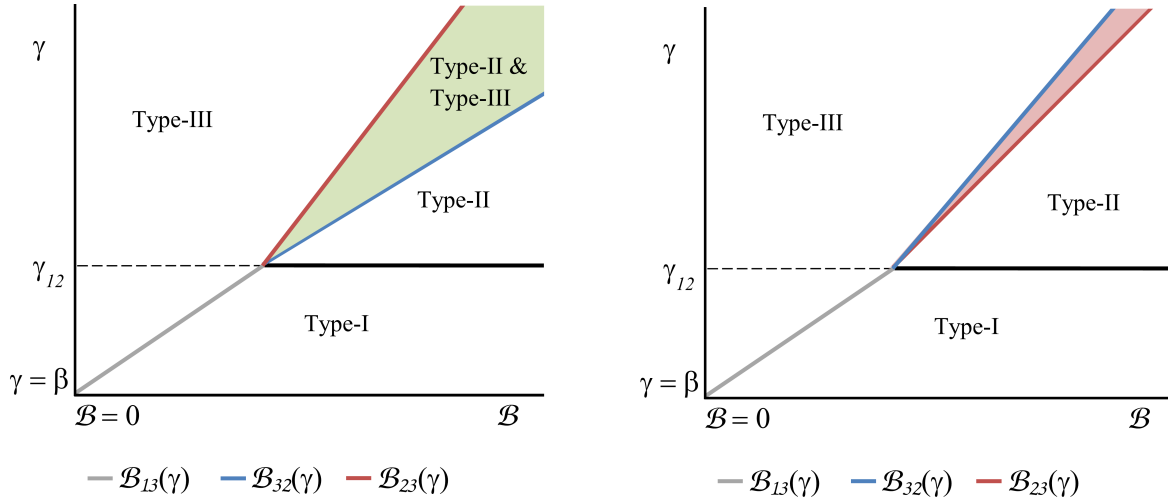


FIGURE 3: REGIONS OF EXISTENCE

exists a region with $\mathcal{B}_{32}(\gamma) < \mathcal{B}_{23}(\gamma)$ for large $\gamma > \gamma_{12}$. In such a case, the region between $\mathcal{B}_{32}(\gamma)$ and $\mathcal{B}_{23}(\gamma)$ does not support trading in the secondary financial market, so that active and passive agents consume the same quantity.¹⁶

The following two lemmas show the existence of an overlapping region supporting the type-II and the type-III equilibrium, which is highlighted in green in the chart on the left-hand side of Figure 3.

Lemma 8 *If $u'(2q) - \eta u'(q) + \eta > 0$ for all $q < \hat{q}$, then $\mathcal{B}_{32}(\gamma) > \mathcal{B}_{23}(\gamma)$ for all $\gamma > \gamma_{12}$.*

Lemma 9 *If there exists a constant $a < \frac{q^*}{2}$ with $u'(q^*) = 1$, such that $u'(2q) - \eta u'(q) + \eta > 0$ for all $a < q < \frac{q^*}{2}$, then $\mathcal{B}_{32}(\gamma) > \mathcal{B}_{23}(\gamma)$ for small $\gamma > \gamma_{12}$.*

The following lemma shows the existence of a region which does not support trading in the secondary financial market for large $\gamma > \gamma_{12}$. This region is highlighted in red in the chart on the right-hand side of Figure 3.

Lemma 10 *If there exists a constant a , such that $u'(2q) - \eta u'(q) + \eta < 0$ for all $q < a$, then $\mathcal{B}_{32}(\gamma) < \mathcal{B}_{23}(\gamma)$ for large $\gamma > \gamma_{12}$.*

7 Optimal Degree of OMOs

In this section, we present one of the main results of our paper; i.e., how to find the optimal bond-to-money ratio, which coincides with the optimal degree of OMOs. For this purpose, we

¹⁶More precisely, this equilibrium is a list $\{\hat{q}, q, q_p, \rho\}$ satisfying (10) and $\hat{q} = q$, $\frac{\gamma}{\beta} = (1-n)u'(q) + n$, $\rho = \frac{\beta}{\gamma}$.

first need to derive the welfare function:

$$(1 - \beta)\mathcal{W} \equiv U(x^*) - x^* + (1 - n) [\delta u(\hat{q}) + (1 - \delta)u(q)] - nq_p, \quad (36)$$

where $U(x^*) - x^*$ denotes the agent's utility in the primary bond market, $(1 - n) [\delta u(\hat{q}) + (1 - \delta)u(q)]$ denotes the agent's expected utility in the goods market if he turns out to be a consumer, and $-nq_p$ denotes the expected utility of an agent if he becomes a producer in the goods market. Differentiating (36) with respect to \mathcal{B} yields

$$(1 - \beta) \frac{\partial \mathcal{W}}{\partial \mathcal{B}} = (1 - n) \left[\delta u'(\hat{q}) \frac{\partial \hat{q}}{\partial \mathcal{B}} + (1 - \delta) u'(q) \frac{\partial q}{\partial \mathcal{B}} \right] - n \frac{\partial q_p}{\partial \mathcal{B}}. \quad (37)$$

In the type-I and type-II equilibrium, we always have $\frac{\partial \hat{q}}{\partial \mathcal{B}} = \frac{\partial q}{\partial \mathcal{B}} = \frac{\partial q_p}{\partial \mathcal{B}} = 0$, and hence $(1 - \beta) \frac{\partial \mathcal{W}}{\partial \mathcal{B}} = 0$. In contrast, in the type-III equilibrium, welfare critically depends on the bond-to-money ratio. Proposition 11 formulates a condition under which it is optimal to increase or decrease the bond-to-money ratio.

Proposition 11 *Let (γ, \mathcal{B}) support the type-III equilibrium. (i) If $\Theta(\gamma, \mathcal{B}) > 0$, then welfare is increasing in \mathcal{B} ; (ii) If $\Theta(\gamma, \mathcal{B}) < 0$, then welfare is decreasing in \mathcal{B} , where*

$$\Theta(\gamma, \mathcal{B}) \equiv \delta [u'(\hat{q}(\gamma, \mathcal{B})) - 1] \frac{\partial \hat{q}}{\partial \mathcal{B}} + (1 - \delta) [u'(q(\gamma, \mathcal{B})) - 1] \frac{\partial q}{\partial \mathcal{B}},$$

$$\begin{aligned} \frac{\partial \hat{q}}{\partial \mathcal{B}} &= \frac{q(\gamma, \mathcal{B})}{\eta + (1 - \eta)u'(\hat{q}(\gamma, \mathcal{B}))} + \frac{\mathcal{B} + \eta + (1 - \eta)u'(q(\gamma, \mathcal{B}))}{\eta + (1 - \eta)u'(\hat{q}(\gamma, \mathcal{B}))} \frac{\partial q}{\partial \mathcal{B}} > 0, \\ \frac{\partial q}{\partial \mathcal{B}} &= \frac{A(\hat{q}(\gamma, \mathcal{B}), q(\gamma, \mathcal{B}))}{B(\hat{q}(\gamma, \mathcal{B}), q(\gamma, \mathcal{B}), \mathcal{B})} < 0, \end{aligned}$$

with

$$\begin{aligned} A(\hat{q}, q) &\equiv -(1 - n)\delta u''(\hat{q})q [\eta + (1 - \eta)u'(q)] \eta, \\ B(\hat{q}, q, \mathcal{B}) &\equiv (1 - n)\delta u''(\hat{q}) [\mathcal{B} + \eta + (1 - \eta)u'(q)] [\eta + (1 - \eta)u'(q)] \eta \\ &\quad + (1 - n)u''(q) [\eta + (1 - \eta)u'(\hat{q})]^2 [\eta(1 - \delta) + (1 - \eta)u'(\hat{q})]. \end{aligned}$$

In Proposition 11, we show that the contribution of $\frac{\partial \hat{q}}{\partial \mathcal{B}} > 0$, while the contribution of $\frac{\partial q}{\partial \mathcal{B}} < 0$. Hence, the contribution of $\frac{\partial q_p}{\partial \mathcal{B}}$ depends on which of the two effects dominates. The reason for $\frac{\partial q}{\partial \mathcal{B}} < 0$ is as follows. In the secondary bond market, agents have the possibility to trade money for bonds after the realization of their idiosyncratic preference shock. The possibility to do so decreases the demand for money and hence its value. Decreasing the bond-to-money ratio helps to mitigate this externality in the type-III equilibrium and induces agents to increase their demand for money.

The following theorem states that if an economy is in the type-I equilibrium, then it is always optimal to decrease the bond-to-money ratio such that the bond constraint of active consumers

becomes binding in the secondary bond market and the type-III equilibrium exists.

Theorem 12 *Let (γ, \mathcal{B}) support the type-I equilibrium. Then, welfare will be improved by decreasing \mathcal{B} .*

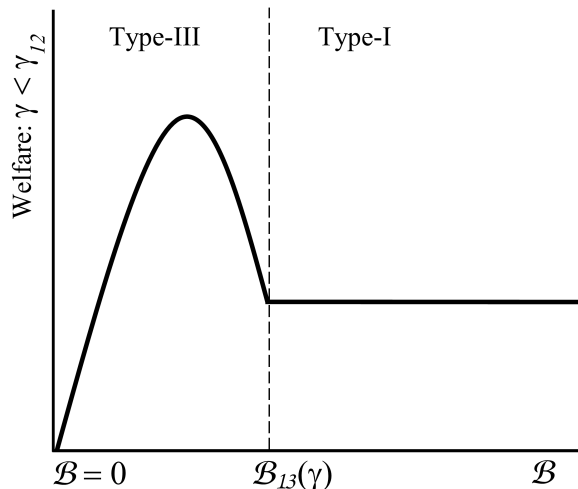


FIGURE 4: WELFARE AS A FUNCTION OF \mathcal{B} FOR $\gamma < \gamma_{12}$

The reasoning behind the result stated in Theorem 12 is as follows. In the type-I equilibrium, since $\gamma < \gamma_{12}$ and $\mathcal{B} > \mathcal{B}_{13}(\gamma)$, decreasing the bond-to-money ratio will not affect welfare. However, at $\mathcal{B} = \mathcal{B}_{13}(\gamma)$, welfare will be further improved by decreasing \mathcal{B} . For ease of understanding, Figure 4 stylistically shows the evolution of welfare as a function of \mathcal{B} for $\gamma < \gamma_{12}$.¹⁷

For inflation rates above γ_{12} , either the type-II or the type-III equilibrium exists. Since welfare is continuous in each type of equilibrium and since welfare is maximized (in terms of \mathcal{B}) in the interior of the region of the type-III equilibrium for $\gamma = \gamma_{12}$, then it must also be maximized in the interior of the region of the type-III equilibrium, when $\gamma > \gamma_{12}$ and γ is sufficiently close to γ_{12} by the above continuities. Hence, only when γ is considerably higher than γ_{12} , it is not optimal to reduce \mathcal{B} in the type-II equilibrium, such that $\mathcal{B}^* = \mathcal{B}_{23}(\gamma)$. This result is shown graphically in Figure 5 for $u'(2q) - \eta u'(q) + \eta > 0$ and consequently $\mathcal{B}_{32}(\gamma) > \mathcal{B}_{23}(\gamma)$ for all $\gamma > \gamma_{12}$.

Figure 5 shows that if $\gamma > \gamma_{12}$ and γ is considerably higher than γ_{12} , then welfare is maximized in the type-II equilibrium, and the optimal policy is to keep the bond-to-money ratio at least at the border of the type-II equilibrium. Note, that \mathcal{B}^* is not continuous between the type-II and the type-III equilibrium, hence there is a jump in \mathcal{B}^* for high inflation rates. In other words, if inflation is too high, a central bank policy of purchasing government bonds does not improve welfare, but is more likely to achieve the opposite.

¹⁷The finding presented in Theorem 12 is similar to the result in Berentsen et al. (2015b), who show that the demand for money is too low in the type-I equilibrium and that a financial transaction tax can correct this pecuniary externality.

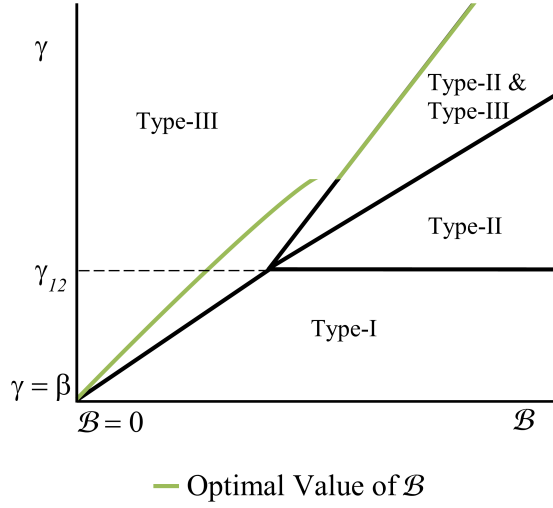


FIGURE 5: REGIONS OF EXISTENCE AND OPTIMAL \mathcal{B}

8 Discussion

In this section, we discuss the evolution of the secondary bond market price of bonds, the assumption of purely passive fiscal policy, the availability of bilateral credit, and the role of search and bargaining frictions in the secondary bond market.

The Price of Bonds in the Secondary Bond Market In our model, bonds are issued in a competitive market and are subsequently traded in a secondary bond market. Hereafter, we have a closer look at the evolution of the secondary bond market price of bonds, over which agents bargain in bilateral meetings. In particular, the secondary bond market price of bonds, labeled as φ hereafter, is given by the trading amount of money that active producers offer, ϕd_m , divided by the trading amount of bonds that active consumers want to sell, ϕd_b . Using the Kalai constraint (13) and $\phi d_m = \hat{q} - q$, we find that in each equilibrium it holds that

$$\varphi = \left[(1 - \eta) \frac{u(\hat{q}) - u(q)}{\hat{q} - q} + \eta \right]^{-1}. \quad (38)$$

From the previous section, we already know that in the type-I and in the type-II equilibrium we have $\frac{\partial \hat{q}}{\partial \mathcal{B}} = \frac{\partial q}{\partial \mathcal{B}} = 0$, and therefore also $\frac{\partial \varphi}{\partial \mathcal{B}} = \frac{\partial \rho}{\partial \mathcal{B}} = 0$. Furthermore, from Proposition 11, we have $\frac{\partial \hat{q}}{\partial \mathcal{B}} > 0$ and $\frac{\partial q}{\partial \mathcal{B}} < 0$ in the type-III equilibrium. Thus, it is easy to show that $\frac{\partial \rho}{\partial \mathcal{B}} < 0$. In the type-III equilibrium, the sign of $\frac{\partial \varphi}{\partial \mathcal{B}}$ depends on whether $\frac{\partial \hat{q}}{\partial \mathcal{B}} > 0$ or $\frac{\partial q}{\partial \mathcal{B}} < 0$ dominates. We find that for large values of η and δ , we have $\frac{\partial \varphi}{\partial \mathcal{B}} < 0$, while for low values of η and δ , we have $\frac{\partial \varphi}{\partial \mathcal{B}} > 0$. These findings are formalized in Proposition 13.

Proposition 13 *Let (γ, \mathcal{B}) support the type-III equilibrium. Then, we always have $\frac{\partial \rho}{\partial \mathcal{B}} < 0$. If the values of δ are η large, we have $\frac{\partial \varphi}{\partial \mathcal{B}} < 0$, and if the values of δ are η small, we have $\frac{\partial \varphi}{\partial \mathcal{B}} > 0$.*

In the market for U.S. Treasuries and T-Bills search and bargaining frictions are low and mainly caused by time delays due to the search for suitable counterparties.¹⁸ Consequently, the price of short-dated bonds agreed on in the secondary bond market is likely to move in the same direction as the price of bonds purchased by the Fed through OMOs; i.e., $\frac{\partial \varphi}{\partial \mathcal{B}} < 0$. However, our results indicate that for secondary bond markets that feature severe search and bargaining frictions, the opposite may occur.

Passive Fiscal Policy Up to now, we assumed that any change in \mathcal{B} has no effect on the inflation rate γ . We showed that under this assumption it can be optimal to reduce \mathcal{B} in order to improve welfare. However, one can argue that it is easier for the central bank to determine the lump-sum transfer τ and that it does not explicitly control γ . From the central bank’s budget constraint (1), we have

$$\gamma - 1 - \tau = \mathcal{B}(1 - \rho\gamma). \quad (39)$$

In the type-III equilibrium, we have $\frac{\partial \rho}{\partial \mathcal{B}} < 0$ and hence any reduction in \mathcal{B} results in an increase in ρ . Thus, reducing \mathcal{B} results in a reduction in the value of the right-hand side of the above equation and the equality can only be satisfied if γ declines. Thus, assuming that τ is constant results in a reduction of γ , which further improves welfare, since $\frac{\partial \hat{q}}{\partial \gamma} < 0$ and $\frac{\partial q}{\partial \gamma} < 0$.¹⁹

Figure 1 shows that the Fed reduced the three-month T-Bill rate from slightly more than 5 percent at the beginning of 2007 to essentially zero until the end of 2008. Equation (39) shows that under the assumption of a constant lump-sum transfer τ , such a severe intervention would result in a substantial reduction in the inflation rate. Based on this finding, and the fact that core inflation declined only slightly since 2007, we stick to the assumption of purely passive fiscal policy for the remainder of this paper.

Bilateral Credit In the 1980s and 1990s, innovations in payment and information technologies resulted in the use of new media of exchange that are not subject to the inflation tax. For instance, the sweep technology which was introduced in the early 1990s, “essentially consists of software used by banks that automatically moves funds from checking accounts to MMDAs” (Lucas and Nicolini, 2015, p. 5).²⁰ Hence, the sweep technology allows clients to earn interest on checking accounts, which was not possible before. Beside the sweep technology, an increasing number of households began to use credit cards to purchase their goods. Financial innovations such as these

¹⁸See Duffie et al. (2005) for a more detailed discussion about search frictions in financial markets, and Krishnamurthy (2002) for the empirical price effects of search frictions in the market for U.S. Treasuries.

¹⁹The proof that we have $\frac{\partial \hat{q}}{\partial \gamma} < 0$ and $\frac{\partial q}{\partial \gamma} < 0$ in the type-III equilibrium is straight-forward and available on request.

²⁰Investors in money market deposit accounts (MMDAs) earn interest, but are subject to a limited number of transfers per month.

finally resulted in a permanent downward shift in the demand for money.²¹

In our model, we assume that agents cannot commit and that no record-keeping technology exists, which rules out bilateral trade credit. In order to incorporate bilateral credit, we would need to assume that some meetings in the goods market are non-anonymous. Doing so would affect our results quantitatively, but not qualitatively, as long as some agents have no access to bilateral credit.²² According to the 2010 Federal Reserve Survey of Consumer Finances, 32 percent of all U.S. families did not have a credit card in 2010. Thus, we conclude that incorporating limited access to bilateral credit might reduce our estimates of the social benefits of OMOs, while our qualitative results continue to hold.

The Role of Search and Bargaining Frictions Hereafter, we evaluate the role of search and bargaining frictions for OMOs to be welfare-improving. In order to do this, we have also derived our model for competitive pricing in the secondary bond market. It is straight forward to show that prices and quantities characterized by Proposition 1 and 3 coincide with competitive pricing for $\eta = 1$.²³ In this case, our solution for $\Theta(\gamma, \mathcal{B})$ from Proposition 11 reduces to

$$\Theta(\gamma, \mathcal{B}) \equiv \delta [u'(\hat{q}(\gamma, \mathcal{B})) - 1] \frac{\partial \hat{q}}{\partial \mathcal{B}} + (1 - \delta) [u'(q(\gamma, \mathcal{B})) - 1] \frac{\partial q}{\partial \mathcal{B}},$$

$$\frac{\partial \hat{q}}{\partial \mathcal{B}} = q(\gamma, \mathcal{B}) + (\mathcal{B} + 1) \frac{\partial q}{\partial \mathcal{B}} > 0,$$

$$\frac{\partial q}{\partial \mathcal{B}} = -\frac{(1 - n)\delta u''(\hat{q})q}{(1 - n)[\delta u''(\hat{q})(\mathcal{B} + 1) + u''(q)(1 - \delta)]} < 0.$$

The above equation shows that our results stated in Proposition 11 and Theorem 12 continue to hold for competitive pricing with partial access to the secondary bond market ($\delta < 1$). Note, however, that for a fully competitive and frictionless secondary bond market; i.e., $\eta = \delta = 1$, our results ease to hold. To see this, note that for $\eta = \delta = 1$ the solution of \hat{q} is completely determined by equation (34), since every agent is active. Consequently, welfare is independent of \mathcal{B} . Hence, search frictions are a key ingredient for our results to persist.

9 Numerical Analysis

We illustrate the findings presented in the previous sections with a numerical analysis of the U.S. data. In particular, we have a closer look at the post-crisis period and analyze how the monetary authority affected quantities and prices when it reduced the yield on 3-month T-Bills to zero in

²¹See Anderson and Rasche (2001), VanHoose and Humphrey (2001), Teles and Zhou (2005), Cynamon et al. (2006), Rojas Breu (2013), Berentsen et al. (2015a), and Lucas and Nicolini (2015), for an extended discussion about the financial innovations that have taken place over the past few decades and how they have affected the demand for money.

²²See Berentsen et al. (2011) for a detailed analysis of the quantitative effects of incorporating bilateral trade credit in a New Monetarist framework.

²³For a formal proof of this argument, we refer the interested reader to Huber and Kim (2015).

late 2008. This was the first time in U.S. history where the short-term interest rates were reduced to zero and where the core inflation rate remained above them for an extended period of time (see Figure 1). Therefore, we think that it is crucial to understand the implications of such a policy measure and to know by how far it deviates from the optimal intervention.

We set a model period to one quarter and choose the following functional forms for preferences and technology: $u(q) = q^{1-\alpha}/(1-\alpha)$, $c(q) = q$, and $U(x) = A \log(x)$. From Lemma 8, it follows that for $u(q) = q^{1-\alpha}/(1-\alpha)$, we need $\alpha \in (0, 1)$ and $\eta \leq 2^{-\alpha}$ to have an overlapping region supporting the type-II and the type-III equilibrium for all $\gamma > \gamma_{12}$.²⁴

The following parameters need to be identified: (i) preference parameters: (β, A, α) ; (ii) the technology parameter: (n) ; (iii) search and bargaining frictions: (δ, η) ; (iv) and policy parameters (\mathcal{B}, γ) .

The parameters are identified by using U.S. data from the first quarter of 1990 to the fourth quarter of 2008.²⁵ Concretely, we use the pre-2009 data to identify the parameters and show what implications the reduction in the effective federal funds rate had on the post-2009 period. The parameters β , γ , and n can be set equal to their target values. That is, we set $\beta = 0.997$ to replicate the real discount factor in the data, measured as the difference between the quarterly 3-month T-Bill rate and the quarterly core inflation rate, $\gamma = 1.007$. We follow related studies and set $n = 0.5$ in order to maximize the number of matches. We calibrate A and α simultaneously by matching the average level of money demand, denoted as $\mathcal{MD} = 0.15$, and its elasticity with respect to the 3-month T-Bill rate, denoted as $\xi = -0.05$.²⁶

The model's money demand is

$$\mathcal{MD} = \frac{\phi M_{-1}}{Y} = \frac{\phi m}{A + (1-n)[\delta \phi \hat{m} + (1-\delta)\phi m]},$$

where $\phi m = \phi M_{-1} = q$ and $\phi \hat{m} = \hat{q}$.²⁷ The model's money demand depends on the yield on ρ , $i_\rho = 1/\rho - 1$, and on α and A via $u(q)$ and $U(x)$. We estimate the model's elasticity of money demand with respect to the yield on ρ in the same way as its empirical counterpart; i.e., by ordinary least squares and a log-log specification.

We assume that in the pre-crisis data, T-Bills were priced fundamentally; i.e., $\rho = \beta/\gamma$, and that \mathcal{B} was sufficiently high such that either the type-I or the type-II equilibrium existed. The remaining unknowns are δ and η . Since U.S. Treasuries and T-Bills are traded in over-the-

²⁴Note, that for $u(q) = q^{1-\alpha}/(1-\alpha)$, $\alpha \in (0, 1)$, and $\eta \leq 2^{-\alpha}$, we have $u'(2q) - \eta u'(q) + \eta = 2^{-\alpha} q^{-\alpha} - \eta q^{-\alpha} + \eta > 0$. Hence, $\mathcal{B}_{32}(\gamma) > \mathcal{B}_{23}(\gamma)$ for all $\gamma > \gamma_{12}$.

²⁵All data sources are provided in the Appendix. Berentsen et al. (2015a) show that due to financial innovations in the 1980s and 1990s a break in the money demand function occurred in the United States. Since the money demand function exhibits a higher stability in the post-1990 data, we choose this data set for our numerical analysis.

²⁶Remember, that money demand is measured as the ratio of M1 adjusted for retail sweeps divided by the nominal gross domestic product. The elasticity of money demand with respect to the rate on 3-month T-Bills is estimated by ordinary least squares and a log-log specification.

²⁷The model's money demand is defined as the ratio of the real stock of money, $\phi M_{-1} = \phi m = q$, divided by the real output of the economy, Y . The latter term is obtained by the sum of the real output of the primary bond market, which equals A for $U(x) = A \log(x)$, and the real output of the goods market, which equals $(1-n)[\delta \phi \hat{m} + (1-\delta)\phi m]$.

counter markets, we know that $0 < \eta < 1$ and $0 < \delta < 1$. However, it is difficult to differentiate search from bargaining frictions in the data. Therefore, we set $\delta = \eta$ and calibrate these two parameters simultaneously by matching the secondary bond market price φ . We follow Berentsen et al. (2014) and interpret φ as the price of bonds with a remaining maturity of one week; i.e., $\varphi = \rho^{7/90}$. Table 1 presents the calibration targets and the parameter values, where the model replicates both money demand targets and the target value of φ exactly.

Parameter	Target Description	Target Value	Parameter Value
β	Real interest rate	0.003 (quarterly)	0.997
γ	Core inflation	0.007 (quarterly)	1.007
n	Maximum number of matches	0.5	0.5
A	Money demand MD (simultaneous)	0.15 (annual)	0.318
α	Elasticity of money demand ξ (simultaneous)	-0.05 (annual)	0.417
δ	SBM price of bonds φ (simultaneous, $\delta = \eta$)	0.9992	0.995
η	SBM price of bonds φ (simultaneous, $\delta = \eta$)	0.9992	0.995

This table displays the calibration targets and parameter values. The parameters β , γ and n are set equal to their target values, while A , α , and $\delta = \eta$, are obtained simultaneously by matching the average money demand, its elasticity with respect to the 3-month T-Bill rate, and the secondary bond market (SBM) price $\varphi = \rho^{7/90}$.

TABLE 1: CALIBRATION TARGETS AND PARAMETER VALUES

The calibrated parameter values allow us to obtain the critical values γ_{12} , $\mathcal{B}_{23}(\gamma)$, $\mathcal{B}_{32}(\gamma)$, and to study the effect of a reduction in \mathcal{B} on consumption and asset prices.²⁸ In order to do this, we first calculate the optimal value of the T-Bill-to-money ratio, \mathcal{B}^* , which solves $\Theta(\gamma, \mathcal{B}^*) = 0$ from Proposition 11. Having obtained \mathcal{B}^* allows us to calculate the percentage of total consumption that agents would be willing to give up in order to be in a steady state with $\mathcal{B} = \mathcal{B}^*$ instead of the initial value of $\mathcal{B} > \mathcal{B}_{23}(\gamma) > \mathcal{B}_{32}(\gamma)$, which is denoted as $1 - \Delta$. Furthermore, we calculate the impact of this policy measure on the yield of bonds in the primary bond market, denoted as $\Delta_{i_\rho} = 1/\rho(\mathcal{B}^*) - 1/\rho(\mathcal{B})$. Table 2 gives a brief overview of our calibration results.

Table 2 shows that $\gamma_{12} = 0.999 < 1.007$; i.e., that the U.S. economy is in the type-II equilibrium at the calibrated value of γ for $\mathcal{B} > \mathcal{B}_{23}(\gamma) > \mathcal{B}_{32}(\gamma) > 1.001$. Furthermore, we find that the model-implied share of goods market consumption accounts for 60 percent of total consumption, which is in line with related studies.²⁹

As can be seen from Table 2, the social benefits associated with the optimal degree of OMOs (i.e., a reduction of $\mathcal{B} > \mathcal{B}_{23}(\gamma) > \mathcal{B}_{32}(\gamma) > 1.001$ to $\mathcal{B}^* = 0.05$) are low and in the region of 0.014 percent of total consumption. However, reducing \mathcal{B} to \mathcal{B}^* has a major impact on the yield of bonds in the primary bond market; i.e., it reduces the average annualized yield on 3-month T-Bills from its pre-crisis level of 3.97 percent by $\Delta_{i_\rho} = -0.0389$ to 0.08 percent. Hence, we

²⁸Note that for the baseline calibration, we have $\eta > 2^{-\alpha}$ and thus there exists a region which does not support trading in the secondary bond market for $\gamma > \gamma_{12}$ by Lemma 10.

²⁹See, for instance, Berentsen et al. (2011 and 2014).

γ_{12}	$\mathcal{B}_{23}(\gamma)$	$\mathcal{B}_{32}(\gamma)$	\mathcal{B}^*	$1 - \Delta$	Δ_{i_ρ} (annual)	s_{GM}
0.999	1.001	1.001	0.050	0.00014	-0.0389	0.60

This table displays the calibration results. Columns 1-3 show the critical values γ_{12} , $B_{23}(\gamma)$, and $B_{32}(\gamma)$ at the calibrated value of $\gamma = 1.007$. Column 4 shows the optimal bond-to-money ratio, B^* , which is calculated such that $\Theta(\gamma, B^*) = 0$. Furthermore, column 5 shows the percentage of total consumption that agents would be willing to give up in order to be in a steady state with $B = B^*$, instead of $B > B_{23}(\gamma) > B_{32}(\gamma)$, denoted as $1 - \Delta$; column 6 shows the annualized effect on the yield of ρ of such a policy measure, denoted as Δ_{i_ρ} (annual); and column 7 shows the size of the goods market, s_{GM} .

TABLE 2: CALIBRATION RESULTS

find that the optimal reduction in the T-Bill rate approaches its empirical counterpart, where we obtain an average annualized rate of 0.10 percent in the period for 2009 to 2013.

In order to better understand the relationship between OMOs and the inflation rate, we show in Figure 6 the development of \mathcal{B}^* and $i_{\rho(\mathcal{B}^*)}$ as a function of the quarterly inflation rate γ , where $i_{\rho(\mathcal{B}^*)}$ denotes the annualized yield on ρ for $\mathcal{B} = \mathcal{B}^*$.

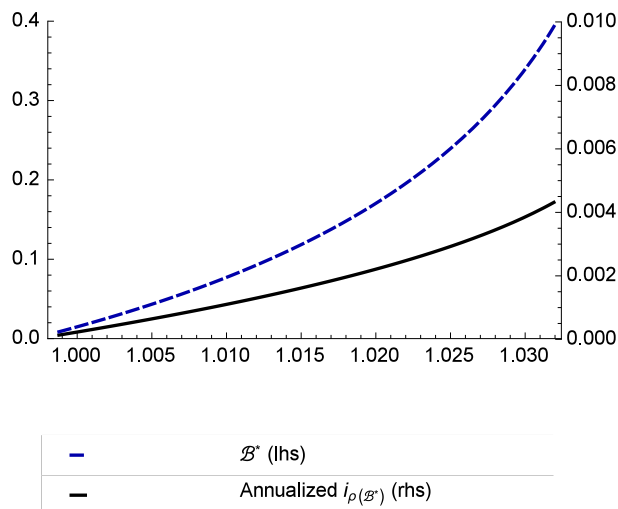


FIGURE 6: \mathcal{B}^* AND $i_{\rho(\mathcal{B}^*)}$ AS FUNCTION OF γ

We find that OMOs are welfare-improving for quarterly inflation rates of up to $\gamma = 1.032$, although the type-I equilibrium only exists for $\gamma < \gamma_{12} = 0.999$. Furthermore, the above figure shows that \mathcal{B}^* is increasing in the inflation rate, while the resulting effect on $i_{\rho(\mathcal{B}^*)}$ remains

essentially unchanged. That is, for the calibrated parameter set, the optimal value of \mathcal{B}^* reduces the yield on 3-month T-Bills close to zero for any $\beta < \gamma < 1.032$. Since the opportunity cost of holding money is increasing in the inflation rate, the required amount of OMOs, $\mathcal{B} - \mathcal{B}^*$, to achieve the optimal policy is declining. As explained in Section 7, \mathcal{B}^* is not continuous between the type-II and the type-III equilibrium; i.e., for an inflation rate which is marginally higher than $\gamma = 1.032$, the optimal bond-to-money ratio jumps from $\mathcal{B}^* = 0.393$ to $\mathcal{B}^* = \mathcal{B}_{23}(\gamma) = 1.001$.

We now investigate the development of the annualized yield on φ for $\mathcal{B} = \mathcal{B}^*$, denoted as $i_{\varphi(\mathcal{B}^*)}$. Remember, that we calibrated the model such that the annualized yield on φ equals the annualized yield on ρ for $\mathcal{B} > \mathcal{B}_{23}(\gamma) > \mathcal{B}_{32}(\gamma) > 1.001$. However, by reducing \mathcal{B} to \mathcal{B}^* , the annualized yield on φ declines by less than the yield on ρ , which is shown in Figure 7.

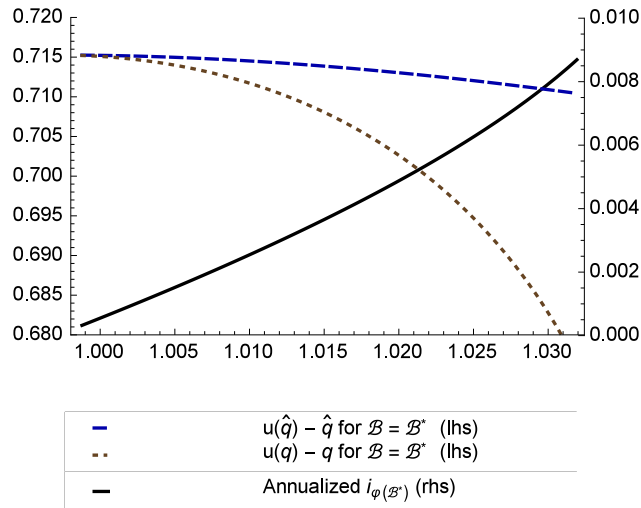


FIGURE 7: $u(\hat{q}) - \hat{q}$, $u(q) - q$, AND $i_{\varphi(\mathcal{B}^*)}$ AS FUNCTION OF γ

The reason for this result is that the trade surplus in the secondary bond market is increasing in the inflation rate. This can easily be seen from the expression for φ in (38). By rearranging terms, we can rewrite the yield on φ as

$$i_{\varphi} = \frac{1}{\varphi} - 1 = (1 - \eta) \left\{ \frac{u(\hat{q}) - \hat{q} - [u(q) - q]}{\hat{q} - q} \right\}.$$

Since the outside option, $u(q) - q$, declines by more than the benefit of trading, $u(\hat{q}) - \hat{q}$, the effect on the total trade surplus is positive for increasing inflation rates. Hence, because active producers receive a fraction $1 - \eta$ of the total trade surplus, we obtain $i_{\varphi(\mathcal{B}^*)} > i_{\rho(\mathcal{B}^*)}$.

Due to the higher opportunity cost of holding money, the incentive to increase the demand

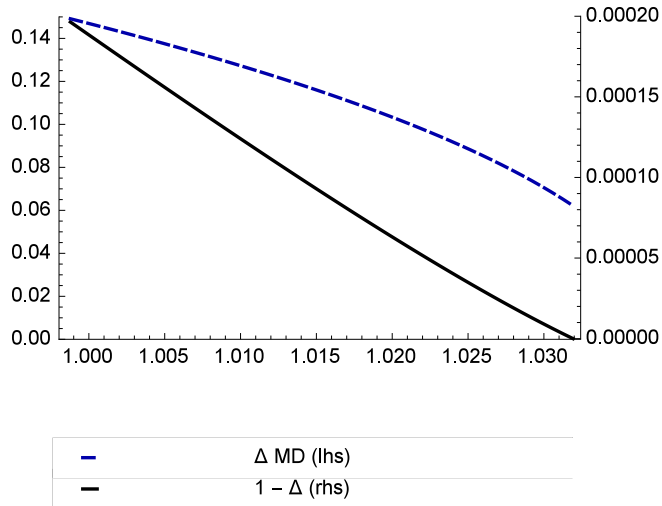


FIGURE 8: $\Delta \mathcal{M}\mathcal{D}$ AND $1 - \Delta$ AS FUNCTION OF γ

for money vanishes for higher inflation rates and so do the social benefits of OMOs in terms of total consumption, $1 - \Delta$. For ease of reference, we show in Figure 8 the change in the demand for money, $\Delta \mathcal{M}\mathcal{D} = \mathcal{M}\mathcal{D}(\mathcal{B}^*) - \mathcal{M}\mathcal{D}(\mathcal{B})$, as well as the development of $1 - \Delta$ as a function of γ .

10 Conclusion

We develop a general equilibrium model, where agents can trade money for government bonds in a secondary financial market which features search and bargaining frictions. The possibility to do so reduces the incentive to self-insure against liquidity shocks, and as a consequence agents rely on the liquidity provision by other market participants. In such an environment, OMOs can be welfare-improving for low inflation rates. The reason is that such a policy measure makes bonds scarce and less attractive, which induces agents to increase their demand for money. In turn, this marginally increases the value of money and so improves the allocation and welfare.

We calibrate the model to U.S. data in order to analyze the post-crisis period, where the Fed reduced the yield on 3-month T-Bills to zero. We find that the optimal policy implied by our model succeeds in replicating the empirical evidence to a large extent. That is, our model implies that the optimal degree of activity in OMOs succeeds in reducing the yield on 3-month T-Bills close to zero and is worth 0.014 percent of total consumption. Furthermore, we find that central bank purchases of government bonds are welfare-improving for quarterly inflation rates of up to 3.2 percent.

11 Appendix - For Online Publication

11.1 Proofs

Proof of Proposition 1. *Derivation of (21).* Since the short selling constraints (12) are not binding in a type-I equilibrium, it is obvious from (16) that (21) holds.

Derivation of (22). The marginal value of money of an agent at the beginning of a period can be written as follows:

$$\frac{\partial V_1}{\partial m} = (1 - n) \frac{\partial V_1^c}{\partial m} + n \frac{\partial V_1^p}{\partial m}.$$

Using (17), we can rewrite the above equation as

$$\frac{\partial V_1}{\partial m} = (1 - n) \left[\delta \eta \frac{\partial K}{\partial m_c} + \frac{\partial V_2^c}{\partial m_c} \right] + n \left[\delta_p (1 - \eta) \frac{\partial K}{\partial m_p} + \frac{\partial V_2^p}{\partial m_p} \right],$$

where

$$\begin{aligned} \frac{\partial K}{\partial m_c} &:= \frac{\partial K(m_c, m_p, b_c, b_p)}{\partial m_c} \Big|_{(m_c, m_p, b_c, b_p) = (m, m, b, b)}, \\ \frac{\partial K}{\partial m_p} &:= \frac{\partial K(m_c, m_p, b_c, b_p)}{\partial m_p} \Big|_{(m_c, m_p, b_c, b_p) = (m, m, b, b)}. \end{aligned}$$

We can use the envelope condition of a consumer in the goods market, (8), to replace $\frac{\partial V_2^c}{\partial m_c}$, and (11) to replace $\frac{\partial V_2^p}{\partial m_p}$. Furthermore, in a symmetric equilibrium, we have $\frac{\partial K}{\partial m_c} = \phi - \phi u'(q)$ and $\frac{\partial K}{\partial m_p} = 0$, because $\frac{\partial \hat{q}}{\partial m_c} = 0$, $\frac{\partial q}{\partial m_c} = \frac{1}{p} = \phi$, $\frac{\partial d_m}{\partial m_c} = -1$, and $\frac{\partial \hat{q}}{\partial m_p} = \frac{\partial q}{\partial m_p} = \frac{\partial d_m}{\partial m_p} = 0$ with (15). Using these expressions to replace $\frac{\partial K}{\partial m_c}$ and $\frac{\partial K}{\partial m_p}$ and (5) updated one period, we get (22).

Derivation of (23). Following the same procedure as in the derivation of (22), we can rewrite the marginal value of bonds at the beginning of a period as

$$\frac{\partial V_1}{\partial b} = (1 - n) \left[\delta \eta \frac{\partial K}{\partial b_c} + \frac{\partial V_2^c}{\partial b_c} \right] + n \left[\delta_p (1 - \eta) \frac{\partial K}{\partial b_p} + \frac{\partial V_2^p}{\partial b_p} \right].$$

Using (8) to replace $\frac{\partial V_2^c}{\partial b_c}$, (11) to replace $\frac{\partial V_2^p}{\partial b_p}$, $\frac{\partial K}{\partial b_p} = \frac{\partial K}{\partial b_c} = 0$, and (5) updated one period, we get (23). ■

Proof of Proposition 2. *Derivation of (27).* Since the cash-constraint of active and passive consumers in the goods market is binding, we have $m_c + d_m = p\hat{q}$ for an active consumer and $m_c = pq$ for a passive consumer. Furthermore, because active producers are cash-constrained in the secondary bond market, we have $m_p = d_m = M$. Using $m_p = m_c = M$ and rearranging terms, we obtain (27).

Derivation of (28). The marginal value of money of an agent at the beginning of a period

can be written as

$$\frac{\partial V_1}{\partial m} = (1 - n) \left[\delta \eta \frac{\partial K}{\partial m_c} + \frac{\partial V_2^c}{\partial m_c} \right] + n \left[\delta_p (1 - \eta) \frac{\partial K}{\partial m_p} + \frac{\partial V_2^p}{\partial m_p} \right],$$

where we used (17). We can use the envelope condition of a consumer in the goods market, (8), to replace $\frac{\partial V_2^c}{\partial m_c}$ and (11) to replace $\frac{\partial V_2^p}{\partial m_p}$. In a symmetric equilibrium, we have $\frac{\partial K}{\partial m_c} = \phi u'(\hat{q}) - \phi u'(q)$ and $\frac{\partial K}{\partial m_p} = \phi u'(\hat{q}) - \phi$, because $\frac{\partial \hat{q}}{\partial m_c} = \frac{\partial q}{\partial m_c} = \frac{1}{p} = \phi$, $\frac{\partial d_m}{\partial m_c} = 0$, and $\frac{\partial \hat{q}}{\partial m_p} = \phi$, $\frac{\partial q}{\partial m_p} = 0$ and $\frac{\partial d_m}{\partial m_p} = 1$ with (15). Using these expressions to replace $\frac{\partial K}{\partial m_c}$ and $\frac{\partial K}{\partial m_p}$ and (5) updated one period, we get (22).³⁰

Derivation of (29). The derivation of (29) is equal to the derivation of (23) and is not repeated here. ■

Proof of Proposition 3. *Derivation of (33).* Since the bond constraint of active consumers is binding in the secondary bond market, we have $d_b = b$. Since the cash-constraint of active and passive consumers is binding in the goods market, we have $m_c + d_m = p\hat{q}$ and $m_c = pq$. Using these expressions and the first-order condition of producers in the goods market, $p\phi = 1$, in (13), we obtain (33).

Derivation of (34). The marginal value of money of an agent at the beginning of a period can be written as

$$\frac{\partial V_1}{\partial m} = (1 - n) \left[\delta \eta \frac{\partial K}{\partial m_c} + \frac{\partial V_2^c}{\partial m_c} \right] + n \left[\delta_p (1 - \eta) \frac{\partial K}{\partial m_p} + \frac{\partial V_2^p}{\partial m_p} \right],$$

where we used (17). We can use the envelope condition of a consumer in the goods market, (8), to replace $\frac{\partial V_2^c}{\partial m_c}$ and (11) to replace $\frac{\partial V_2^p}{\partial m_p}$. Furthermore, we can use $\frac{\partial K}{\partial m_p} = 0$, because $\frac{\partial \hat{q}}{\partial m_p} = \frac{\partial q}{\partial m_p} = \frac{\partial d_m}{\partial m_p} = 0$ with (15),³¹ to obtain

$$\frac{\partial V_1}{\partial m} = (1 - n) \left[\delta \eta \frac{\partial K}{\partial m_c} + \phi u'(q) \right] + n\phi. \quad (40)$$

Next, we need to derive $\frac{\partial K}{\partial m_c}$. By (15), we have

$$\frac{\partial K}{\partial m_c} = u'(\hat{q}) \frac{\partial \hat{q}}{\partial m_c} - u'(q) \frac{\partial q}{\partial m_c} - \phi \frac{\partial d_m}{\partial m_c}. \quad (41)$$

³⁰Since $\hat{q} = \phi(m_c + d_m) = \phi(m_c + m_p)$ and $q = \phi m_p$, $\frac{\partial \hat{q}}{\partial m_c} = \frac{\partial \hat{q}}{\partial m_p} = \frac{\partial q}{\partial m_c} = \phi$. Since $d_m = m_p$ is binding, equation (24) holds. Thus, if a buyer has a little more (or less) m_c , agents will still trade $d_m = m_p$. Hence $\frac{\partial d_m}{\partial m_c} = 0$. If a producer has a little more (or less) m_p , then he will still trade the whole m_p . Hence $\frac{\partial d_m}{\partial m_p} = 1$. $\frac{\partial q}{\partial m_p} = 0$ is obvious.

³¹It is obvious that $\frac{\partial \hat{q}}{\partial m_p} = \frac{\partial q}{\partial m_p} = \frac{\partial d_m}{\partial m_p} = 0$, since \hat{q} , q , and d_m do not depend on the producer's money holdings, because the producer's cash constraint is non-binding in the type-III equilibrium.

Next, we have to find $\frac{\partial \hat{q}}{\partial m_c}$ and $\frac{\partial d_m}{\partial m_c}$. By (13), we have

$$(1 - \eta) \left[u'(\hat{q}) \frac{\partial \hat{q}}{\partial m_c} - u'(q) \frac{\partial q}{\partial m_c} \right] = -\eta \phi \frac{\partial d_m}{\partial m_c} \quad (42)$$

and since $\hat{q} = \phi(m_c + d_m)$, $q = \phi m_c$, we also have

$$\frac{\partial \hat{q}}{\partial m_c} = \phi + \phi \frac{\partial d_m}{\partial m_c} \quad \text{and} \quad \frac{\partial q}{\partial m_c} = \phi. \quad (43)$$

By solving the system of equations (42) and (43), we can derive

$$\frac{\partial \hat{q}}{\partial m_c} = \phi \frac{\eta + (1 - \eta)u'(q)}{\eta + (1 - \eta)u'(\hat{q})}, \quad (44)$$

$$\frac{\partial d_m}{\partial m_c} = -\frac{1 - \eta}{\phi \eta} \left[u'(\hat{q}) \frac{\partial \hat{q}}{\partial m_c} - u'(q) \frac{\partial q}{\partial m_c} \right]. \quad (45)$$

Using (44) and (45) in (41), we have

$$\begin{aligned} \frac{\partial K}{\partial m_c} &= u'(\hat{q}) \frac{\partial \hat{q}}{\partial m_c} - u'(q) \frac{\partial q}{\partial m_c} + \frac{1 - \eta}{\eta} \left[u'(\hat{q}) \frac{\partial \hat{q}}{\partial m_c} - u'(q) \frac{\partial q}{\partial m_c} \right] \\ &= \frac{1}{\eta} \left[u'(\hat{q}) \frac{\partial \hat{q}}{\partial m_c} - u'(q) \frac{\partial q}{\partial m_c} \right] = \frac{1}{\eta} \left[\phi u'(\hat{q}) \frac{\eta + (1 - \eta)u'(q)}{\eta + (1 - \eta)u'(\hat{q})} - \phi u'(q) \right], \end{aligned}$$

where we have used $\frac{\partial q}{\partial m_c} = \phi$. Using the above equation in (40) and (5) updated one period, we obtain (34).

Derivation of (35). The marginal value of bonds at the beginning of a period can be written as

$$\frac{\partial V_1}{\partial b} = (1 - n) \left[\delta \eta \frac{\partial K}{\partial b_c} + \frac{\partial V_2^c}{\partial b_c} \right] + n \left[\delta_p (1 - \eta) \frac{\partial K}{\partial b_p} + \frac{\partial V_2^p}{\partial b_p} \right]. \quad (46)$$

By (33), we can find

$$\frac{\partial \hat{q}}{\partial b} = \frac{\phi}{(1 - \eta)u'(\hat{q}) + \eta} \quad \text{and} \quad \frac{\partial d_m}{\partial b} = \frac{1}{(1 - \eta)u'(\hat{q}) + \eta},$$

since $\mathcal{B} = \frac{\phi b}{\phi m} = \frac{\phi b}{q}$. Using the above equations to replace $\frac{\partial \hat{q}}{\partial b_c}$ and $\frac{\partial d_m}{\partial b_c}$, $\frac{\partial q}{\partial b_c} = 0$, and $\frac{\partial \hat{q}}{\partial b_p} = \frac{\partial q}{\partial b_p} = \frac{\partial d_m}{\partial b_p} = \frac{\partial q}{\partial b_c} = 0$, we obtain

$$\begin{aligned} \frac{\partial K}{\partial b_c} &= \phi \frac{u'(\hat{q}) - 1}{(1 - \eta)u'(\hat{q}) + \eta}, \\ \frac{\partial K}{\partial b_p} &= 0. \end{aligned}$$

Using the two above equations in (46), (8) to replace $\frac{\partial V_2^c}{\partial b_c}$, (11) to replace $\frac{\partial V_2^p}{\partial b_p}$, and (5) updated one period, we obtain (35). ■

Proof of Proposition 4. In the type-I equilibrium, agents must hold enough money and bonds to support the optimal amount of consumption for active consumers, $u'(\hat{q}) = 1$. First, consider the following system of equations

$$\begin{aligned} u'(\hat{q}) &= 1, \\ \hat{q} &= 2q, \\ \frac{\gamma}{\beta} &= (1-n)\delta\eta + (1-n)(1-\delta\eta)u'(q) + n, \end{aligned}$$

with three variables \hat{q} , q , and γ . Let \hat{q}_{12} , q_{12} , and γ_{12} be the solution of the system of equations. The existence of the solution is immediate. By construction, γ_{12} is the threshold of the inflation rate at which the cash constraint of active producers is just binding, $\phi m_p = \phi d_m$. From the proof of proposition 2, we know that $\hat{q} = 2q$. Hence $\gamma \leq \gamma_{12}$ must hold in the type-I equilibrium.

Furthermore, the bond-to-money ratio must be high enough in order to support $u'(\hat{q}) = 1$. Hence, for a given $\gamma \leq \gamma_{12}$, consider the following system of equations

$$\begin{aligned} u'(\hat{q}) &= 1, \\ \mathcal{B}q &= (1-\eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q), \\ \frac{\gamma}{\beta} &= (1-n)\delta\eta + (1-n)(1-\delta\eta)u'(q) + n, \end{aligned}$$

with three variables \hat{q} , q , and \mathcal{B} . Let $\hat{q}_{13}(\gamma)$, $q_{13}(\gamma)$, and $\mathcal{B}_{13}(\gamma)$ be the solution of the system of equations. The existence of the solution is immediate, since u' is continuous and $u'(\infty) = 0$. By construction, $\mathcal{B}_{13}(\gamma)$ is the threshold of the bond-to-money ratio at which the bond constraint of active consumers is just binding. Hence, $\mathcal{B} \geq \mathcal{B}_{13}(\gamma)$ must hold in the type-I equilibrium. ■

Proof of Proposition 5. First, consider the following system of equations,

$$\begin{aligned} u'(\hat{q}) &= 1, \\ \hat{q} &= 2q, \\ \frac{\gamma}{\beta} &= (1-n)\delta u'(\hat{q}) + (1-n)(1-\delta\eta)u'(q) - (1-n)\delta(1-\eta) + n, \end{aligned}$$

with three variables \hat{q} , q , and γ . Let \hat{q}_{21} , q_{21} , and γ_{21} be the solution of the system of equations. With $u'(\hat{q}) = 1$, we have

$$\frac{\gamma}{\beta} = (1-n)(1-\delta\eta)u'(q) + (1-n)\delta\eta + n.$$

Hence the solution exists, and $\gamma_{21} = \gamma_{12}$. By the construction of $\gamma_{21} = \gamma_{12}$, it follows that γ_{12} is the threshold of the inflation rate at which active consumers can consume the optimal amount of goods, $u'(\hat{q}) = 1$. Hence $\gamma \geq \gamma_{12}$ must hold in the type-II equilibrium.

In the type-II equilibrium, the bond-to-money ratio must be high enough such that the bond constraint is not binding in the secondary bond market. Hence, given that $\gamma \geq \gamma_{12}$, we consider the following system of equations

$$\begin{aligned}\hat{q} &= 2q, \\ \mathcal{B}q &= (1 - \eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q), \\ \frac{\gamma}{\beta} &= (1 - n)\delta u'(\hat{q}) + (1 - n)(1 - \delta\eta)u'(q) - (1 - n)\delta(1 - \eta) + n,\end{aligned}$$

with three variables \hat{q} , q , and \mathcal{B} . Let $\hat{q}_{23}(\gamma)$, $q_{23}(\gamma)$, and $\mathcal{B}_{23}(\gamma)$ be the solution of the system of equations. The existence of the solution is immediate, since u' is continuous and $u'(0) = \infty$. By construction, $\mathcal{B}_{23}(\gamma)$ is the threshold of the bond-to-money ratio at which the bond constraint is just binding conditional on the given inflation rate γ . Hence $\mathcal{B} > \mathcal{B}_{13}(\gamma)$ must hold in the type-II equilibrium. ■

Proof of Proposition 6. *Case $\gamma \leq \gamma_{12}$:* Given $\gamma \leq \gamma_{12}$, consider the following system of equations,

$$\begin{aligned}u'(\hat{q}) &= 1, \\ \mathcal{B}q &= (1 - \eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q), \\ \frac{\gamma}{\beta} &= (1 - n)\delta u'(\hat{q}) \frac{\eta + (1 - \eta)u'(q)}{\eta + (1 - \eta)u'(\hat{q})} + (1 - n)(1 - \delta)u'(q) + n,\end{aligned}$$

with three variables \hat{q} , q , and \mathcal{B} . Let $\hat{q}_{31}(\gamma)$, $q_{31}(\gamma)$, and $\mathcal{B}_{31}(\gamma)$ be the solution of the system of equations. With $u'(\hat{q}) = 1$, we have

$$\frac{\gamma}{\beta} = (1 - n)(1 - \delta\eta)u'(q) + (1 - n)\delta\eta + n.$$

Hence the solution exists, and $\mathcal{B}_{31}(\gamma) = \mathcal{B}_{13}(\gamma)$. By the construction of $\mathcal{B}_{31}(\gamma) = \mathcal{B}_{13}(\gamma)$, it follows that $\mathcal{B}_{13}(\gamma)$ is the threshold of the bond-to-money ratio at which active consumers can consume the optimal amount of goods $u'(\hat{q}) = 1$. Hence, $\mathcal{B} \leq \mathcal{B}_{13}(\gamma)$ must hold in the type-III equilibrium conditional on $\gamma \leq \gamma_{12}$.

Case $\gamma > \gamma_{12}$: Given $\gamma > \gamma_{12}$, consider the system of equations,

$$\begin{aligned}\hat{q} &= 2q, \\ \mathcal{B}q &= (1 - \eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q), \\ \frac{\gamma}{\beta} &= (1 - n)\delta u'(\hat{q}) \frac{\eta + (1 - \eta)u'(q)}{\eta + (1 - \eta)u'(\hat{q})} + (1 - n)(1 - \delta)u'(q) + n\end{aligned}$$

with three variables \hat{q} , q , and \mathcal{B} . Let $\hat{q}_{32}(\gamma)$, $q_{32}(\gamma)$, and $\mathcal{B}_{32}(\gamma)$ be the solution of the system of equations. The existence of the solution is immediate, since u' is continuous, $\lim_{q \rightarrow 0} \frac{(1-n)\delta u'(2q)}{\eta + (1-\eta)u'(2q)} = \frac{(1-n)\delta}{(1-\eta)} \in \mathbb{R}$, and $u'(0) = \infty$. By construction, $\mathcal{B}_{32}(\gamma)$ is the threshold of the bond-to-money ratio

at which the cash constraint in the secondary bond market is also binding. Hence, $\mathcal{B} < \mathcal{B}_{32}(\gamma)$ must hold in the type-III equilibrium conditional on $\gamma > \gamma_{12}$. ■

Proof of Lemma 7. (i): Note that $\hat{q}_{13}(\gamma)$, $q_{13}(\gamma)$, and $\mathcal{B}_{13}(\gamma)$ are the solution of the system of equations

$$\begin{aligned} u'(\hat{q}) &= 1, \\ \mathcal{B}q &= (1 - \eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q), \\ \frac{\gamma}{\beta} &= (1 - n)\delta\eta + (1 - n)(1 - \delta\eta)u'(q) + n. \end{aligned}$$

From the last equation, it is easy to see that q_{13} is decreasing in γ . From the first and the second equation, we have

$$\frac{\partial \mathcal{B}}{\partial q} = \frac{(1 - \eta)(-u'(q) - \eta)}{q} - \frac{(1 - \eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q)}{q^2} < 0.$$

Therefore, as γ increases, q_{13} decreases and $\mathcal{B}_{13}(\gamma)$ increases.

(ii): Note that $\hat{q}_{23}(\gamma)$, $q_{23}(\gamma)$, and $\mathcal{B}_{23}(\gamma)$ are the solution of the system of equations

$$\begin{aligned} \hat{q} &= 2q, \\ \mathcal{B}q &= (1 - \eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q), \\ \frac{\gamma}{\beta} &= (1 - n)\delta u'(\hat{q}) + (1 - n)(1 - \delta\eta)u'(q) - (1 - n)\delta(1 - \eta) + n. \end{aligned}$$

From the first and the last equation, it is easy to see that q_{23} is decreasing in γ . From the first and the second equation, we have

$$\frac{q^2}{1 - \eta} \frac{\partial \mathcal{B}}{\partial q} = [2qu'(2q) - qu'(q)] - [u(2q) - u(q)] = [2qu'(2q) - u(2q)] - [qu'(q) - u(q)].$$

Note that $xu'(x) - u(x)$ is decreasing in x , since its derivative is

$$u'(x) + u''(x) - u'(x) < 0.$$

Hence $\frac{\partial \mathcal{B}}{\partial q} < 0$. Therefore, as γ increases, q_{23} decreases and $\mathcal{B}_{23}(\gamma)$ increases.

(iii): Note that $\hat{q}_{32}(\gamma)$, $q_{32}(\gamma)$, and $\mathcal{B}_{32}(\gamma)$ are the solution of the system of equations

$$\begin{aligned} \hat{q} &= 2q, \\ \mathcal{B}q &= (1 - \eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q), \\ \frac{\gamma}{\beta} &= (1 - n)\delta u'(\hat{q}) \frac{\eta + (1 - \eta)u'(q)}{\eta + (1 - \eta)u'(\hat{q})} + (1 - n)(1 - \delta)u'(q) + n. \end{aligned}$$

From the first and the last equation, we have

$$\frac{\gamma}{\beta} = (1-n)\delta u'(2q) \frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(2q)} + (1-n)(1-\delta)u'(q) + n.$$

If we differentiate the right-hand side of the above equation, then we obtain

$$\begin{aligned} & 2(1-n)\delta u''(2q) \frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(2q)} + (1-n)(1-\delta)u''(q) \\ & + (1-n)\delta u'(2q) \frac{(1-\eta)u''(q)}{\eta + (1-\eta)u'(2q)} - (1-n)\delta u'(2q) \frac{2[\eta + (1-\eta)u'(q)](1-\eta)u''(2q)}{[\eta + (1-\eta)u'(2q)]^2}. \end{aligned}$$

Multiplying the above formula by $[\eta + (1-\eta)u'(2q)]^2$, we obtain

$$\begin{aligned} & 2(1-n)\delta u'''(q)[\eta + (1-\eta)u'(2q)] \\ & + (1-n)(1-\delta)u'''(2q)]^2 \\ & + (1-n)\delta u'(2q)(1-\eta)u'''(2q)] \\ & - 2(1-n)\delta u'(2q)[\eta + (1-\eta)u'(q)](1-\eta)u''(2q) \\ & = 2(1-n)\delta u'''(q)]\eta \\ & + (1-n)(1-\delta)u'''(2q)]^2 \\ & + (1-n)\delta u'(2q)(1-\eta)u'''(2q)] < 0. \end{aligned}$$

The last inequality follows immediately from $u'' < 0$. Hence q_{32} is decreasing in γ . From the first and the second equation, we have

$$\frac{\partial \mathcal{B}}{\partial q} < 0,$$

as is shown in (ii). Therefore, as γ increases, q_{32} decreases and $\mathcal{B}_{32}(\gamma)$ increases.

(iv): The proof is immediate by noticing that the following three formulas are identical when $u'(\hat{q}) = 1$:

$$\begin{aligned} & (1-n)\delta\eta + (1-n)(1-\delta\eta)u'(q) + n \\ & = (1-n)\delta u'(\hat{q}) + (1-n)(1-\delta\eta)u'(q) - (1-n)\delta(1-\eta) + n \\ & = (1-n)\delta u'(\hat{q}) \frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} + (1-n)(1-\delta)u'(q) + n. \end{aligned}$$

■

Proof of Lemma 8. Given $\gamma > \gamma_{12}$, we have

$$\begin{aligned}\frac{\gamma}{\beta} &= (1-n)\delta u'(2q_{23}(\gamma)) + (1-n)(1-\delta\eta)u'(q_{23}(\gamma)) - (1-n)\delta(1-\eta) + n \\ &= (1-n)\delta u'(2q_{32}(\gamma)) \frac{\eta + (1-\eta)u'(q_{32}(\gamma))}{\eta + (1-\eta)u'(2q_{32}(\gamma))} + (1-n)(1-\delta)u'(q_{32}(\gamma)) + n.\end{aligned}$$

Let

$$\begin{aligned}f_2(q) &\equiv (1-n)\delta u'(2q) + (1-n)(1-\delta\eta)u'(q) - (1-n)\delta(1-\eta) + n, \\ f_3(q) &\equiv (1-n)\delta u'(2q) \frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(2q)} + (1-n)(1-\delta)u'(q) + n,\end{aligned}$$

then

$$f_2(q_{23}(\gamma)) = f_3(q_{32}(\gamma)) = \frac{\gamma}{\beta}.$$

Note that

$$f_2(q) - f_3(q) = \frac{(1-n)\delta(1-\eta)}{\eta + (1-\eta)u'(2q)} [u'(2q) - 1] [u'(2q) - \eta u'(q) + \eta].$$

Since by assumption $u'(2q) - \eta u'(q) + \eta > 0$, we have $f_2(q) > f_3(q)$. Moreover, because

$$\frac{\partial f_2}{\partial q} = 2(1-n)\delta u''(2q) + (1-n)(1-\delta\eta)u''(q) < 0,$$

we must have

$$q_{23}(\gamma) > q_{32}(\gamma).$$

Otherwise, we would have a contradiction, $\frac{\gamma}{\beta} = f_2(q_{23}(\gamma)) > f_3(q_{32}(\gamma)) = \frac{\gamma}{\beta}$.

By (13), the bond-to-money ratio is

$$\mathcal{B} = (1-\eta) \frac{u(2q) - u(q)}{q} + \eta.$$

Hence, if we differentiate \mathcal{B} with respect to q , we obtain

$$\frac{\partial \mathcal{B}}{\partial q} = \frac{2u'(2q) - u'(q)}{q} - \frac{u(2q) - u(q)}{q^2} < 0,$$

as we showed in the proof of Lemma 7. Therefore $\mathcal{B}_{32}(\gamma) > \mathcal{B}_{23}(\gamma)$, because $q_{23}(\gamma) > q_{32}(\gamma)$. ■

Proof of Lemma 9. As $\gamma > \gamma_{12}$ approaches γ_{12} , $q_{32}(\gamma) < \frac{q^*}{2}$ approaches $\frac{q^*}{2}$. Hence, for a given a , we have $q_{32}(\gamma) > a$ for a sufficiently small γ . From the previous proof, we have

$$f_2(q) - f_3(q) = \frac{(1-n)\delta(1-\eta)}{\eta + (1-\eta)u'(2q)} [u'(2q) - 1] [u'(2q) - \eta u'(q) + \eta].$$

For a small γ , so for $q_{32}(\gamma) > a$, we have $u'(2q_{32}(\gamma)) - \eta u'(q_{32}(\gamma)) + \eta > 0$. Therefore, it holds that $f_2(q_{32}(\gamma)) > f_3(q_{32}(\gamma))$. Because also $\frac{\partial f_2}{\partial q} < 0$, and

$$\frac{\gamma}{\beta} = f_2(q_{23}(\gamma)) = f_3(q_{32}(\gamma)) = \frac{\gamma}{\beta},$$

we must have

$$q_{23}(\gamma) > q_{32}(\gamma).$$

Otherwise, we would have a contradiction, $\frac{\gamma}{\beta} = f_2(q_{23}(\gamma)) > f_3(q_{32}(\gamma)) = \frac{\gamma}{\beta}$. At last, we have

$$\frac{\partial \mathcal{B}}{\partial q} = \frac{2u'(2q) - u'(q)}{q} - \frac{u(2q) - u(q)}{q^2} < 0.$$

Since $q_{23}(\gamma) > q_{32}(\gamma) > a$, we must have $\mathcal{B}_{32}(\gamma) > \mathcal{B}_{23}(\gamma)$. ■

Proof of Lemma 10. As γ approaches ∞ , $q_{32}(\gamma)$ approaches zero. Hence for a given a , $q_{32}(\gamma) < a$ for a sufficiently large γ . From the previous proof, we have

$$f_2(q) - f_3(q) = \frac{(1-n)\delta(1-\eta)}{\eta + (1-\eta)u'(2q)} [u'(2q) - 1] [u'(2q) - \eta u'(q) + \eta].$$

For large γ ; i.e., for $q_{32}(\gamma) < a$, we have $u'(2q_{32}(\gamma)) - \eta u'(q_{32}(\gamma)) + \eta < 0$. Therefore, it holds that $f_2(q_{32}(\gamma)) < f_3(q_{32}(\gamma))$. Because also $\frac{\partial f_2}{\partial q} < 0$, and

$$\frac{\gamma}{\beta} = f_2(q_{23}(\gamma)) = f_3(q_{32}(\gamma)) = \frac{\gamma}{\beta},$$

we must have

$$q_{23}(\gamma) < q_{32}(\gamma).$$

Otherwise, we would have a contradiction, $\frac{\gamma}{\beta} = f_2(q_{23}(\gamma)) < f_3(q_{32}(\gamma)) = \frac{\gamma}{\beta}$. At last, we have

$$\frac{\partial \mathcal{B}}{\partial q} = \frac{2u'(2q) - u'(q)}{q} - \frac{u(2q) - u(q)}{q^2} < 0.$$

Since $q_{23}(\gamma) < q_{32}(\gamma) < a$, we must have $\mathcal{B}_{32}(\gamma) < \mathcal{B}_{23}(\gamma)$. ■

Proof of Proposition 11. Consider any (γ, \mathcal{B}) that supports the type-III equilibrium. Note that $\hat{q} = \hat{q}(\gamma, \mathcal{B})$, $q = q(\gamma, \mathcal{B})$, and $q_p = q_p(\gamma, \mathcal{B})$ are completely determined by (γ, \mathcal{B}) in the type-III equilibrium, because \hat{q} , q and q_p are solutions of the system of equations,

$$nq_p = (1-n)[\delta\hat{q} + (1-\delta)q] \tag{47}$$

$$\mathcal{B}q = (1-\eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q), \tag{48}$$

$$\frac{\gamma}{\beta} = (1-n)\delta u'(\hat{q}) \frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} + (1-n)(1-\delta)u'(q) + n. \tag{49}$$

By differentiating equation (47) with respect to \mathcal{B} , we obtain

$$n \frac{\partial q_p}{\partial \mathcal{B}} = (1-n) \left[\delta \frac{\partial \hat{q}}{\partial \mathcal{B}} + (1-\delta) \frac{\partial q}{\partial \mathcal{B}} \right].$$

Hence, we can simplify (37) as

$$(1-\beta) \frac{\partial \mathcal{W}}{\partial \mathcal{B}} = (1-n) \left\{ \delta [u'(\hat{q}) - 1] \frac{\partial \hat{q}}{\partial \mathcal{B}} + (1-\delta) [u'(q) - 1] \frac{\partial q}{\partial \mathcal{B}} \right\}.$$

By differentiating equation (48) with respect to \mathcal{B} , we have

$$q + \mathcal{B} \frac{\partial q}{\partial \mathcal{B}} = (1-\eta) \left[u'(\hat{q}) \frac{\partial \hat{q}}{\partial \mathcal{B}} - u'(q) \frac{\partial q}{\partial \mathcal{B}} \right] + \eta \left(\frac{\partial \hat{q}}{\partial \mathcal{B}} - \frac{\partial q}{\partial \mathcal{B}} \right);$$

i.e., by rearranging the above equation we obtain

$$\frac{\partial \hat{q}}{\partial \mathcal{B}} = \frac{q}{\eta + (1-\eta)u'(\hat{q})} + \frac{\mathcal{B} + \eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} \frac{\partial q}{\partial \mathcal{B}}. \quad (50)$$

By differentiating equation (49) with respect to \mathcal{B} , we have

$$\begin{aligned} 0 &= (1-n)\delta u''(\hat{q}) \frac{\partial \hat{q}}{\partial \mathcal{B}} \frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} + (1-n)\delta u'(\hat{q}) \frac{(1-\eta)u''(q)}{\eta + (1-\eta)u'(\hat{q})} \frac{\partial q}{\partial \mathcal{B}} \\ &\quad - (1-n)\delta u'(\hat{q}) \frac{\eta + (1-\eta)u'(q)}{[\eta + (1-\eta)u'(\hat{q})]^2} (1-\eta)u''(\hat{q}) \frac{\partial \hat{q}}{\partial \mathcal{B}} + (1-n)(1-\delta)u''(q) \frac{\partial q}{\partial \mathcal{B}}. \end{aligned} \quad (51)$$

With the equations (50) and (51), we have

$$\begin{aligned} 0 &= (1-n)\delta u''(\hat{q}) \left[\frac{q}{\eta + (1-\eta)u'(\hat{q})} + \frac{\mathcal{B} + \eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} \frac{\partial q}{\partial \mathcal{B}} \right] \frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} \\ &\quad + (1-n)\delta u'(\hat{q}) \frac{(1-\eta)u''(q)}{\eta + (1-\eta)u'(\hat{q})} \frac{\partial q}{\partial \mathcal{B}} \\ &\quad - (1-n)\delta u'(\hat{q}) \frac{\eta + (1-\eta)u'(q)}{[\eta + (1-\eta)u'(\hat{q})]^2} (1-\eta)u''(\hat{q}) \left[\frac{q}{\eta + (1-\eta)u'(\hat{q})} + \frac{\mathcal{B} + \eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} \frac{\partial q}{\partial \mathcal{B}} \right] \\ &\quad + (1-n)(1-\delta)u''(q) \frac{\partial q}{\partial \mathcal{B}}. \end{aligned}$$

By factoring $\frac{\partial q}{\partial \mathcal{B}}$, we have

$$\begin{aligned}
& - (1-n)\delta u''(\hat{q}) \frac{q}{\eta + (1-\eta)u'(\hat{q})} \frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} \\
& + (1-n)\delta u'(\hat{q}) \frac{\eta + (1-\eta)u'(q)}{[\eta + (1-\eta)u'(\hat{q})]^2} (1-\eta)u''(\hat{q}) \frac{q}{\eta + (1-\eta)u'(\hat{q})} \\
& = \left\{ \begin{array}{l} (1-n)\delta u''(\hat{q}) \frac{\mathcal{B} + \eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} \frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} \\ \quad + (1-n)\delta u'(\hat{q}) \frac{(1-\eta)u''(q)}{\eta + (1-\eta)u'(\hat{q})} \\ - (1-n)\delta u'(\hat{q}) \frac{\eta + (1-\eta)u'(q)}{[\eta + (1-\eta)u'(\hat{q})]^2} (1-\eta)u''(\hat{q}) \frac{\mathcal{B} + \eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} \\ \quad + (1-n)(1-\delta)u''(q) \end{array} \right\} \frac{\partial q}{\partial \mathcal{B}}.
\end{aligned}$$

If we multiply the above equation by $[\eta + (1-\eta)u'(\hat{q})]^3$, we obtain

$$\begin{aligned}
& - (1-n)\delta u''(\hat{q})q [\eta + (1-\eta)u'(q)] [\eta + (1-\eta)u'(\hat{q})] \\
& + (1-n)\delta u'(\hat{q}) [\eta + (1-\eta)u'(q)] (1-\eta)u''(\hat{q})q \\
& = \left\{ \begin{array}{l} (1-n)\delta u''(\hat{q}) [\mathcal{B} + \eta + (1-\eta)u'(q)] [\eta + (1-\eta)u'(q)] [\eta + (1-\eta)u'(\hat{q})] \\ \quad + (1-n)\delta u'(\hat{q})(1-\eta)u''(q) [\eta + (1-\eta)u'(\hat{q})]^2 \\ - (1-n)\delta u'(\hat{q}) [\eta + (1-\eta)u'(q)] (1-\eta)u''(\hat{q}) [\mathcal{B} + \eta + (1-\eta)u'(q)] \\ \quad + (1-n)(1-\delta)u''(q) [\eta + (1-\eta)u'(\hat{q})]^3 \end{array} \right\} \frac{\partial q}{\partial \mathcal{B}}.
\end{aligned}$$

After simplifying the above equation, we have

$$\begin{aligned}
& - (1-n)\delta u''(\hat{q})q [\eta + (1-\eta)u'(q)] \eta \\
& = \left\{ \begin{array}{l} (1-n)\delta u''(\hat{q}) [\mathcal{B} + \eta + (1-\eta)u'(q)] [\eta + (1-\eta)u'(q)] \eta \\ \quad + (1-n)u''(q) [\eta + (1-\eta)u'(\hat{q})]^2 [\eta(1-\delta) + (1-\eta)u'(\hat{q})] \end{array} \right\} \frac{\partial q}{\partial \mathcal{B}}.
\end{aligned}$$

Hence, we can denote

$$\frac{\partial q}{\partial \mathcal{B}} = \frac{A(\hat{q}, q)}{B(\hat{q}, q, \mathcal{B})},$$

where

$$\begin{aligned}
A(\hat{q}, q) & \equiv -(1-n)\delta u''(\hat{q})q [\eta + (1-\eta)u'(q)] \eta, \\
B(\hat{q}, q, \mathcal{B}) & \equiv (1-n)\delta u''(\hat{q}) [\mathcal{B} + \eta + (1-\eta)u'(q)] [\eta + (1-\eta)u'(q)] \eta \\
& \quad + (1-n)u''(q) [\eta + (1-\eta)u'(\hat{q})]^2 [\eta(1-\delta) + (1-\eta)u'(\hat{q})].
\end{aligned}$$

Note that since $u'' < 0$, we have

$$A(\hat{q}, q) > 0, \quad \text{and} \quad B(\hat{q}, q, \mathcal{B}) < 0.$$

Hence, $\frac{\partial q}{\partial \mathcal{B}} < 0$. Moreover,

$$\begin{aligned} [\eta + (1 - \eta)u'(\hat{q})] B(\hat{q}, q, \mathcal{B}) \frac{\partial \hat{q}}{\partial \mathcal{B}} &= qB(\hat{q}, q, \mathcal{B}) + [\mathcal{B} + \eta + (1 - \eta)u'(q)] A(\hat{q}, q) \\ &= q(1 - n)u''(q) [\eta + (1 - \eta)u'(\hat{q})]^2 [\eta(1 - \delta) + (1 - \eta)u'(\hat{q})] < 0. \end{aligned}$$

Therefore, $\frac{\partial \hat{q}}{\partial \mathcal{B}} > 0$. At last, let

$$\begin{aligned} \Theta(\gamma, \mathcal{B}) &\equiv \delta [u'(\hat{q}(\gamma, \mathcal{B})) - 1] \frac{\partial \hat{q}}{\partial \mathcal{B}} + (1 - \delta) [u'(q(\gamma, \mathcal{B})) - 1] \frac{\partial q}{\partial \mathcal{B}} \\ &= \delta [u'(\hat{q}(\gamma, \mathcal{B})) - 1] \frac{q(\gamma, \mathcal{B})}{\eta + (1 - \eta)u'(\hat{q}(\gamma, \mathcal{B}))} \\ &\quad + \left\{ \delta [u'(\hat{q}(\gamma, \mathcal{B})) - 1] \frac{\mathcal{B} + \eta + (1 - \eta)u'(q(\gamma, \mathcal{B}))}{\eta + (1 - \eta)u'(\hat{q}(\gamma, \mathcal{B}))} + (1 - \delta) [u'(q(\gamma, \mathcal{B})) - 1] \right\} \\ &\quad \times \frac{A(\hat{q}(\gamma, \mathcal{B}), q(\gamma, \mathcal{B}))}{B(\hat{q}(\gamma, \mathcal{B}), q(\gamma, \mathcal{B}), \mathcal{B})}, \end{aligned}$$

then $\frac{\partial \mathcal{W}}{\partial \mathcal{B}} = \frac{1-n}{1-\beta} \Theta(\gamma, \mathcal{B})$. Thus, if $\Theta(\gamma, \mathcal{B}) > 0$, then welfare will be improved by increasing \mathcal{B} , and if $\Theta(\gamma, \mathcal{B}) < 0$, then welfare will be improved by decreasing \mathcal{B} . ■

Proof of Theorem 12. By Proposition 4, $\gamma < \gamma_{12}$, and $\mathcal{B} > \mathcal{B}_{13}(\gamma)$. Decreasing the bond-to-money ratio \mathcal{B} up to $\mathcal{B}_{13}(\gamma)$ will not change welfare, since it supports the type-I equilibrium. But at $\mathcal{B} = \mathcal{B}_{13}(\gamma)$,

$$\Theta(\gamma, \mathcal{B}_{13}(\gamma)) = (1 - \delta) [u'(q(\gamma, \mathcal{B}_{13}(\gamma))) - 1] \frac{A(\hat{q}(\gamma, \mathcal{B}_{13}(\gamma)), q(\gamma, \mathcal{B}_{13}(\gamma)))}{B(\hat{q}(\gamma, \mathcal{B}_{13}(\gamma)), q(\gamma, \mathcal{B}_{13}(\gamma)), \mathcal{B}_{13}(\gamma))} < 0.$$

The inequality comes from $A(\hat{q}, q) > 0$, $B(\hat{q}, q, \mathcal{B}) < 0$, and $u'(q) - 1 > 0$. Welfare will be improved by decreasing \mathcal{B} further at $\mathcal{B}_{13}(\gamma)$ by Proposition 11. ■

Proof of Proposition 13. In the type-III equilibrium, equation (35) holds. Thus, we always have

$$\frac{\partial \rho}{\partial \mathcal{B}} = \frac{\beta(1 - n)\delta\eta}{\gamma} \frac{u''(\hat{q}) \frac{\partial \hat{q}}{\partial \mathcal{B}}}{[\eta + (1 - \eta)u'(\hat{q})]^2} < 0,$$

since $\frac{\partial \hat{q}}{\partial \mathcal{B}} > 0$.

In the type-III equilibrium, the secondary bond market price satisfies

$$\varphi^{-1} = (1 - \eta) \frac{u(\hat{q}) - u(q)}{\hat{q} - q} + \eta,$$

since $\mathcal{B}q = (1 - \eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q)$ by equation (33). If we differentiate $\frac{u(\hat{q}) - u(q)}{\hat{q} - q}$ with

respect to \mathcal{B} , then we have

$$\frac{1}{(\hat{q} - q)} \left\{ \left[u'(\hat{q}) \frac{\partial \hat{q}}{\partial \mathcal{B}} - u'(q) \frac{\partial q}{\partial \mathcal{B}} \right] - \frac{u(\hat{q}) - u(q)}{\hat{q} - q} \left(\frac{\partial \hat{q}}{\partial \mathcal{B}} - \frac{\partial q}{\partial \mathcal{B}} \right) \right\}.$$

By the concavity of u , we have

$$0 < u'(\hat{q}) < \frac{u(\hat{q}) - u(q)}{\hat{q} - q} < u'(q).$$

Hence, it holds that

$$[u'(\hat{q}) - u'(q)] \frac{\partial \hat{q}}{\partial \mathcal{B}} < \left[u'(\hat{q}) \frac{\partial \hat{q}}{\partial \mathcal{B}} - u'(q) \frac{\partial q}{\partial \mathcal{B}} \right] - \frac{u(\hat{q}) - u(q)}{\hat{q} - q} \left(\frac{\partial \hat{q}}{\partial \mathcal{B}} - \frac{\partial q}{\partial \mathcal{B}} \right) < [u'(\hat{q}) - u'(q)] \frac{\partial q}{\partial \mathcal{B}}.$$

When $\delta = \eta = 0$, by Proposition 11 we have $\frac{\partial q}{\partial \mathcal{B}} = 0$, $\frac{\partial \hat{q}}{\partial \mathcal{B}} > 0$. Hence, we have

$$\left[u'(\hat{q}) \frac{\partial \hat{q}}{\partial \mathcal{B}} - u'(q) \frac{\partial q}{\partial \mathcal{B}} \right] - \frac{u(\hat{q}) - u(q)}{\hat{q} - q} \left(\frac{\partial \hat{q}}{\partial \mathcal{B}} - \frac{\partial q}{\partial \mathcal{B}} \right) < 0 \text{ when } \delta = \eta = 0.$$

Therefore, if δ and η are small, by continuity, we have

$$\frac{\partial \varphi^{-1}}{\partial \mathcal{B}} < 0; \text{ i.e., } \frac{\partial \varphi}{\partial \mathcal{B}} = -\varphi^2 \frac{\partial \varphi^{-1}}{\partial \mathcal{B}} > 0.$$

On the other hand, if $\delta = \eta = 1$, by Proposition 11 we have $\frac{\partial q}{\partial \mathcal{B}} < 0$, $\frac{\partial \hat{q}}{\partial \mathcal{B}} = 0$. Hence, we have

$$\left[u'(\hat{q}) \frac{\partial \hat{q}}{\partial \mathcal{B}} - u'(q) \frac{\partial q}{\partial \mathcal{B}} \right] - \frac{u(\hat{q}) - u(q)}{\hat{q} - q} \left(\frac{\partial \hat{q}}{\partial \mathcal{B}} - \frac{\partial q}{\partial \mathcal{B}} \right) > 0 \text{ when } \delta = \eta = 1.$$

Therefore if δ and η are large, by continuity, we have

$$\frac{\partial \varphi^{-1}}{\partial \mathcal{B}} \geq 0; \text{ i.e., } \frac{\partial \varphi}{\partial \mathcal{B}} = -\varphi^2 \frac{\partial \varphi^{-1}}{\partial \mathcal{B}} \leq 0.$$

Note that if $\eta = 1$, then $\frac{\partial \varphi}{\partial \mathcal{B}} = 0$ independent on δ . ■

11.2 Data Source

All the data that we used for the calibration is downloadable from the Federal Reserve Bank of St. Louis FRED[®] database. Table A.1 gives a brief overview of the respective identifiers.

Description	Identifier	Period
3-Month Treasury Bill: Secondary Market Rate	DTB3	90:Q1-13:Q4
Consumer Price Index: All Items Less Food & Energy	CPILFESL	90:Q1-13:Q4
M1 Adjusted for Retail Sweeps	M1ADJ	90:Q1-13:Q4
Gross Domestic Product	GDP	90:Q1-13:Q4

TABLE A.1: DATA SOURCE

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