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Designing Institutions for Diversity

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Abstract

This paper analyzes the design of innovation contests when the quality of an innovation depends on the research approach of the supplier, but the best approach is unknown. Diversity of approaches is beneficial because of the resulting option value. An auction induces the social optimum, while a fixed-prize tournament induces insufficient diversity. The optimal contest for the buyer is an augmented fixed-prize tournament, where suppliers can choose from a set of at most two prizes. This allows the buyer to implement any level of diversity at the lowest cost.

Keywords: Contests, tournaments, auctions, diversity, procurement.

JEL: L14, L22, L23.

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The use of contests to procure innovations has a long history, and it is becoming ever more popular. Recently, private buyers have awarded the Netflix Prize, the Ansari X Prize, and the InnoCentive prizes. Public agencies have organized, for instance, the DARPA Grand Challenges, the Lunar Lander Challenge and the EU Vaccine Prize.¹ The literature on contest design deals with the problem of providing incentives for costly innovation effort.² However, effort is by no means the only important requirement for a successful innovation. Ex ante, there are often many potentially optimal approaches to solving an innovation problem. Contest design should therefore encourage innovators to take different approaches. In this paper, we thus ask how contest design influences the diversity of research approaches.

Many practical examples illustrate the importance of the issue. First, the often cited Longitude Prize of 1714 for a method to determine a ship's longitude at sea featured two competing approaches.³ The lunar method was an attempt to use the position of the moon to calculate the position of the ship. The alternative, ultimately successful, approach relied on a clock which accurately kept Greenwich time at sea, thus allowing estimation of longitude by comparison with the local time (measured by the position of the sun). Second, when the Yom Kippur War in 1973 revealed the vulnerability of the US aircraft to Soviet-made radar-guided missiles, General Dynamics sought to resolve the issue through electronic countermeasures, while McDonnell Douglas, Northrop, and eventually Lockheed, attempted to build planes with small radar cross-section.⁴ Third, the EU Vaccine Prize was announced in 2012 with the goal of improving the so-called cold-chain vaccine technology. Interestingly, the competition rules explicitly stated that diverse innovation approaches were conceivable: "It is important to note that approaches to be taken by the participants in the competition are not prescribed and may include alternate formulations, novel packaging and/or transportation techniques, or significant improvements over existing technologies, amongst others."⁵ Finally, the announcement of the 2015 Horizon Prize for better use of antibiotics contains a similar statement.⁶

Thus, in many innovation contests both the buyer (the contest designer) and the suppliers (contestants) are aware that there are multiple conceivable approaches to innovation. Furthermore, none of the participants knows the best approach beforehand. However, after the suppliers have followed a particular approach, it is often possible to assess the quality of innovations, for instance, by looking at prototypes or detailed descriptions of research projects. In the following, we will ask whether buyers can and should do more than to appeal to suppliers to pursue diverse approaches: Can they design institutions in such a way that suppliers have incentives to provide diversity? And will they benefit from doing so?

Architectural contests share some important properties with innovation contests. A buyer who thinks about procuring a new building usually does not know what exactly the ideal building would look like, but once she examines the submitted plans, she can choose the one she prefers. Guidelines for architectural competitions explicitly recognize the need for diversity. For example, the Royal Institute of British Architects states: "Competitions enable a

¹See "Innovation: And the winner is...", *The Economist*. Aug 5, 2010.

²Section 7 discusses this literature.

³See, e.g., Che and Gale (2003) for a discussion of the Longitude Prize.

⁴See Paul Crickmore (2003), *Nighthawk F-117: Stealth Fighter*. Airline Publishing Ltd.

⁵European Commission (2012), "Prize Competition Rules." August 28, 2012 (accessed on April 3, 2015). http://ec.europa.eu/research/health/pdf/prize-competition-rules_en.pdf

⁶European Commission (2015), "Better use of antibiotics." March 24, 2015 (accessed on April 3, 2015). <http://ec.europa.eu/research/horizonprize/index.cfm?prize=better-use-antibiotics>

wide variety of approaches to be explored simultaneously with a number of designers.”⁷ Two important features are common in innovation contests and architecture contests. First, the buyer often specifies a fixed prize ex ante. Second, the number of contestants may be large. There is a tension between these observations and the results of an interesting theoretical literature that investigates the optimal design of procurement contests.⁸ This literature focuses on incentives for efforts rather than for diversity. Most papers conclude that (i) buyers should use auctions rather than fixed-prize tournaments and (ii) it is optimal for a buyer to invite only two participants. Our paper argues that neither of these conclusions can be taken for granted when the diversity of approaches is an issue.

To our knowledge, our paper is the first analysis of the design of innovation contests with multiple conceivable research approaches. We develop a model with a buyer and several suppliers. Crucially, the value of an innovation does not depend on effort, but on the difference between the chosen research approach and an ideal, but initially unknown approach. The suppliers and the buyer agree about the distribution of the ideal approach. It seems plausible that the buyer would like to induce the suppliers to choose different approaches to gain from the resulting option value. This paper studies which contests can induce such diversity. More broadly, we analyze the consequences of different institutions on buyer payoffs and total welfare.

In line with the literature on innovation contests, we assume that neither research inputs nor research outputs are verifiable,⁹ because they are both difficult to evaluate, and the relation between them is stochastic. Clearly, this assumption is more palatable in some cases than in others. For instance, in the 2005 Grand Challenge, DARPA asked participants to build an unmanned vehicle that could complete a 212 kilometer course in rugged territory as fast as possible. In principle, DARPA could have rewarded participants according to the exact time needed to complete the course, a verifiable signal of quality.¹⁰ In other contexts, for instance, when basic research (or architectural style) is considered, more judgment is necessary, and it is much harder to come up with verifiable measures of quality and, in particular, of the monetary value of quality differences. In these cases, non-verifiability seems highly plausible.

The lack of verifiability of research activity precludes any kind of contract that could be implemented by a court, which leads to a hold-up problem. One solution to this problem is to use what Che and Gale (2003) define as a contest mechanism (or simply contest). Such a contest prescribes a possible set of prices and commits the buyer to paying the price chosen by the supplier from which the innovation is procured. Examples are fixed-prize tournaments as well as auctions with or without reserve price. As argued by Che and Gale (2003), a court can easily enforce the rules of a contest, since it only needs to verify if the correct price was paid out to one of the participants. In line with the literature, we focus on contests as means of procuring innovation.

In our benchmark model, there are two homogeneous suppliers who choose a research

⁷See Royal Institute of British Architects (2013), "Design competitions guidance for clients." (accessed on Apr 3, 2015) <http://competitions.architecture.com/requestform.aspx>.

⁸See Section 7 for details on the related literature.

⁹For an extensive discussion see Che and Gale (2003) and Taylor (1995).

¹⁰Instead, DARPA only used the speed criterion to rank participants (see Section 1.6 of the DARPA Grand Challenge Rules (2004, accessed on June 24, 2015). http://archive.darpa.mil/grandchallenge05/Rules_8oct04.pdf).

approach, modeled as a point on the unit interval. The distribution of the ideal approach has a symmetric and single-peaked density. Quality depends linearly on the distance between the approach taken and the ideal approach. After the buyer has communicated the rules of the game, the suppliers choose their approaches, and qualities become common knowledge. We abstract from cost considerations, so as to focus on suppliers' incentives to diversify.

In the benchmark model, we obtain the following results.¹¹ First, maximizing the expected total payoffs of buyers and suppliers (henceforth, the social optimum) involves diversity of approaches. Second, an auction mechanism implements the social optimum. Third, fixed-prize tournaments do not induce any diversity, but nevertheless yield higher expected buyer payoffs than auctions. Fourth, the optimal contest for the buyer is an "augmented fixed-prize tournament", in which the suppliers can choose from at most two prizes; these contests allow the buyer to finetune the degree of diversification. Finally, for a uniform state distribution, the fixed-prize tournament maximizes the expected buyer payoffs.

We then provide several extensions, including the following. First, we show that, when the buyer can charge sufficiently large participation fees, she can implement the social optimum and appropriate the resulting rents completely. Second, we allow for multiple suppliers. The social optimum continues to involve diversity, and it can be implemented with an auction. Fixed-prize tournaments now induce some diversity, but less than socially optimal. However, as the number of suppliers increases, the difference between fixed-prize tournaments and auctions becomes smaller. The buyer continues to prefer tournaments to auctions. She may benefit from inviting a large number of suppliers, which is a straightforward implication of the option value provided by additional suppliers. Third, we consider exogenously heterogeneous suppliers, reflecting differences in technology or style (as in the architectural example). For substantial heterogeneity, active diversification is no longer optimal. Otherwise the main results carry through: Auctions implement the social optimum; tournaments do not; buyers nevertheless prefer fixed-prize tournaments unless there are substantial fixed costs or participation fees. Moreover, with heterogeneous suppliers, it is possible to analyze the impact of the revelation of quality information on the incentives of contestants to diversify. Revelation is crucial: Without it, the unique equilibrium of an auction is the one without any diversification.

The contest metaphor has useful interpretations beyond the procurement context. As we discuss briefly in the conclusion, our model also applies when suppliers choose products in the face of uncertain demand by a potentially large number of homogeneous buyers. Contest design then corresponds to the choice of alternative regulatory institutions. Our approach shows that unregulated markets provide incentives for suppliers to choose the socially optimal products, but at the cost of endowing them with ex-post market power. As a result, regulation may yield higher expected consumer surplus, even though it does not induce the optimal expected product quality.

In Section 1, we introduce the benchmark model with two homogeneous suppliers. In Section 2, we introduce innovation contests. In Section 3, we show that an auction induces the social optimum. Section 4 deals with the optimal design of contests from the buyer's perspective, while Section 5 extends the analysis of optimal contests beyond the benchmark model by considering fixed costs and unconditional transfers. Section 6 presents extensions

¹¹In addition, we allow for more general distribution functions and transportation costs, and we consider negotiations, contests with multiple prizes and contests with multiple approaches developed by the same supplier as alternative procurement institutions.

of the model. Section 7 discusses our paper in the context of related literature. Section 8 concludes, pointing in particular to the above-mentioned re-interpretation of our model for a world with many buyers. Short and instructive proofs are in the Appendix. The Online Appendix (available at <https://sites.google.com/site/iletina/did>) contains technical notation and omitted proofs, as well as more details about the extensions.

1 The Benchmark Model

We now describe the assumptions of our benchmark model. Its simple setting brings out the intuition most clearly. We will see in Section 6 that the main insights are much more general.

A risk-neutral buyer B needs an innovation that two risk-neutral suppliers ($i \in \{1, 2\}$) can provide. Each supplier chooses an approach $v_i \in [0, 1]$ at cost $C(v_i)$. The quality q_i of the resulting innovation depends on a state $\sigma \in [0, 1]$, which is distributed with density $f(\sigma)$, and corresponds to an (ex-post) ideal approach. We thus assume that $q_i = \Psi - \delta(|v_i - \sigma|)$, where $\Psi > 0$ and δ is an increasing function. Unless specified otherwise, we will maintain assumptions (A1)-(A3) below.

Assumption (A1) *The density function $f(\sigma)$ is (i) symmetric: $f(1/2 - \varepsilon) = f(1/2 + \varepsilon)$ $\forall \varepsilon \in [0, 1/2]$, (ii) single-peaked: $f(\sigma) \leq f(\sigma') \forall \sigma < \sigma' < 1/2$ and (iii) has full support: $f(\sigma) > 0 \forall \sigma \in [0, 1]$.*

Assumption (A2) *All approaches have the same fixed costs $C(v_i) \equiv C \geq 0$.*

Assumption (A3) *$\delta(v_i, \sigma) = b|v_i - \sigma|$ with $b \in (0, \Psi]$.*

Using (A3), we denote the quality resulting from approach v_i in state σ as $q(v_i, \sigma) = \Psi - b|v_i - \sigma|$ for some $\Psi > 0$. Thus quality is bounded below by $\Psi - b$ and bounded above by Ψ . Though most of the results are more general, we will specify the density function as follows for some specific results:

Assumption (A1)' *The density function $f(\sigma)$ is uniform.*

When (A1)' holds, all approaches are equally likely to be ideal and, by (A2), they are equally costly. Nevertheless, the market participants agree ex ante that the central approach maximizes expected quality and thus expected total profits.

As we show in the Proposition 18 in the Online Appendix, a vector maximizing social welfare (v_1^*, v_2^*) always exists and always features diversity, that is $v_1^* \neq v_2^*$. Intuitively, without diversity, the expected minimal distance to the ideal approach can be reduced by arbitrarily moving one of the two approaches away from the other. Depending on the distribution of σ , diversity can be substantial. For instance, when (A1)' holds, the social optimum is $(v_1^*, v_2^*) = (1/4, 3/4)$.¹²

¹²See Proposition 18 in the online appendix.

- Period 1:* Suppliers simultaneously select approaches $v_i \in [0, 1]$.
- Period 2:* The state is realized. All players observe $v_i \in [0, 1]$; thus q_1 and q_2 become common knowledge.
- Period 3:* Suppliers simultaneously choose prices $p_i \in \mathcal{P}$.
- Period 4:* The buyer observes prices; then she chooses a supplier $i \in \{1, 2\}$. She pays p_i to the chosen supplier and 0 to the other supplier.

2 Innovation Contests

The buyer can choose an innovation contest determining the procedure for choosing and remunerating suppliers. These contests are closely related to the procurement contests analyzed by Che and Gale (2003), where suppliers choose efforts rather than approaches.¹³ In line with these authors, we assume that neither v_i nor q_i is contractible.¹⁴ The environment (b, Ψ, C) of a contest consists of the utility and cost parameters. For now, we set $C = 0$, so as to separate the suppliers' decisions on which approach to choose from the decision whether to produce. In Section 5, we allow for positive fixed costs so as to analyze the entry decision.

The buyer chooses a set \mathcal{P} of allowable prices (bids), where \mathcal{P} is an arbitrary finite union of closed subintervals of \mathbb{R}^+ .¹⁵ We denote the minimum of \mathcal{P} as \underline{P} and the maximum, if it exists, as \overline{P} . An *innovation contest* is the extensive-form game between the buyer and the suppliers defined by the buyer's choice of \mathcal{P} and the following rules:

The assumption that all players observe v_i and σ is convenient, as it allows us to apply the subgame perfect equilibrium (SPE). However, the assumption is more restrictive than necessary. As long as all players can observe qualities, all results still hold with the SPE replaced by a Perfect Bayesian Equilibrium with suitably specified beliefs.¹⁶ Moreover, in the Subsection 6.2 we modify the observability assumption by imposing that the suppliers have to submit bids when they choose approaches; thus they cannot observe qualities when they choose prices.¹⁷

The following are examples of innovation contests:

1. $\mathcal{P} = \mathbb{R}^+$: a *procurement auction without reserve price*.
2. $\mathcal{P} = [0, \overline{P}]$: a *procurement auction with reserve price \overline{P}* .
3. $\mathcal{P} = \{A\}$, where $A \geq 0$: a *fixed-prize tournament (FPT)* with A set by the buyer.
4. $\mathcal{P} = \{A, 0\}$, where $A \geq 0$: an *augmented fixed-prize tournament*.

¹³In Section 6, we will briefly discuss an alternative class of institutions.

¹⁴For example, Che and Gale (2003) and Taylor (1995) assume that neither inputs nor outputs of innovative activity are verifiable. As an example of the verifiability problem, Che and Gale (2003) point to the protracted battle between John Harrison, the inventor of the marine chronometer, and the Board of Longitude, over whether his invention met the requirements of the 1714 Longitude Prize. See also references in Taylor (1995).

¹⁵Formally, $\mathcal{I}(\mathbb{R}^+) := \{\mathcal{P} \subseteq \mathbb{R}^+ : \mathcal{P} = \cup_{k=1}^{\bar{k}} [a_k, b_k] \text{ or } \mathcal{P} = \cup_{k=1}^{\bar{k}} [a_k, b_k] \cup [a_{\bar{k}+1}, \infty) \text{ for } a_k \leq b_k \in \mathbb{R}^+, \bar{k} \in \mathbb{N}\}$.

¹⁶Of course, if the suppliers could develop a design anew, they could achieve a better quality by incorporating what they have learned. We exclude this possibility and focus on the highest quality that can be produced at the end of the contest. One justification is that, due to unmodelled dynamic considerations, the good has to be produced in the current period.

¹⁷We cannot pursue this issue with homogeneous suppliers, as we would run into existence problems.

The first three examples are well-known. The last example will turn out to be a useful alternative for the buyer. We apply the following tie-breaking rules.

(T1) If suppliers offer the same surplus, the buyer prefers the higher quality one. If both have the same quality, the tie is randomly broken.

(T2) Given equal monetary payoffs, the suppliers prefer to win the contest.

(T1) and (T2) will be shown to guarantee that the outcomes are robust to infinitesimal changes in the reward structure.¹⁸

3 Implementing the Social Optimum

Proposition 18 in the Online Appendix demonstrates that the social optimum requires at least some diversity under very general conditions. Moreover, this diversity can be substantial, as the case of the uniform state distribution shows. We now show that a simple innovation contest implements the social optimum. We require the following notation:

Notation 1 $\bar{p}(\sigma) \equiv \max \{p \in \mathcal{P} \mid p \leq |q(v_1, \sigma) - q(v_2, \sigma)| + \underline{P}\}$.

The following result formalizes the familiar "asymmetric Bertrand" logic that low-quality firms choose minimal prices, whereas high-quality firms translate the quality differential into a price differential.

Lemma 2 *The subgame of an innovation contest corresponding to (q_i, q_j) has an equilibrium such that $p_i(q_i, q_j) = \bar{p}(\sigma)$ if $q_i \geq q_j$ and $p_i(q_i, q_j) = \underline{P}$ if $q_i < q_j$. In any SPE of any contest, $p_i(q_i, q_j) = \bar{p}(\sigma)$ if $q_i \geq q_j$.*

Lemma 2 sharpens the Bertrand logic: The price differential will only fully reflect the quality differential when this is feasible for the high-quality supplier. In many cases, the equilibrium described in Lemma 2 is unique.¹⁹ Lemma 2 is essential in the following result.

Proposition 3 *(i) The auction mechanism $(\mathcal{P} = \mathbb{R}^+)$ implements the social optimum. (ii) If f is uniform ((A1)' holds), then any equilibrium of an auction is socially optimal.*

The logic of the direct proof that we provide in the Online Appendix is very similar to the reasoning behind the optimality of Vickrey-Clarke-Groves mechanisms.²⁰ Claim (i) also follows from the more general results in Lemma 21. Here, we therefore merely explain why the auction mechanism induces diversity, as required for the social optimum. Intuitively, if the approaches are identical, the suppliers have the same quality in all states. Hence, by Lemma 2, subgame equilibrium prices are zero, and suppliers will not earn positive profits. A deviation to any other approach is thus profitable.

¹⁸For example, in an auction (T1) ensures that the higher quality supplier wins by offering the same surplus to the seller as his competitor. Even without (T1), the higher quality supplier could ensure that he wins by offering a slightly higher surplus. Similarly, (T2) will allow us to avoid the consideration of limits in the discussion of the optimal tournament.

¹⁹If P is convex and $\sup P > \bar{p}(\sigma)$ for all σ , then $p_i(q_i, q_j) = \underline{P}$ for $q_i < q_j$ in every equilibrium. To see this, note that, according to the lemma, $p_i = \bar{p}(\sigma) = \underline{P} + q(v_i, \sigma) - q(v_j, \sigma)$ in any equilibrium. If $p_j > \underline{P}$, then player i can choose a slightly higher prize, and he still wins. Hence, this is a profitable deviation.

²⁰See, e.g., Mas-Colell et al. (1995).

4 The Optimal Contest for the Buyer

The auction mechanism induces the efficient amount of diversity, but leaves rents to the successful supplier. It will turn out that, because it avoids such rents, a suitable FPT is preferable for the buyer, even though it does not induce any diversity in a two-player setting. The remaining results of the section show that the optimal innovation contest for the buyer is a (possibly augmented) FPT. For the special case of the uniform state distribution, there are no contests leading to higher buyer payoffs than the FPT with the minimal prize $A = 0$. An FPT never induces diversity:

Proposition 4 *For any $A \geq 0$, the unique equilibrium of an FPT is $(v_1, v_2) = (1/2, 1/2)$.*

The intuition is straightforward. As the size of the prize is independent of quality differences, the suppliers care only about maximizing the expected winning probability. By (A1), this requires moving to the center.²¹ We need further notation:

Notation 5 $\Delta q(v_i, v_j) \equiv |q(v_i, v_i) - q(v_j, v_i)|$ is the maximum quality difference given (v_i, v_j) .

By Lemma 2, in any subgame the successful supplier chooses the highest available price below the sum of the quality differential and the minimum bid in any subgame. We now show that, for the equilibrium choice of approaches, the bid corresponds *exactly* to the sum of the maximum quality differential and the minimum bid. This is also the price paid in all other states resulting in the maximum quality difference.

Lemma 6 *Let $v_1 \leq v_2$. (i) If a contest implements (v_1, v_2) , then $\Delta q(v_1, v_2) + \underline{P} \in \mathcal{P}$. (ii) If $\sigma \in [0, v_1] \cup [v_2, 1]$, the successful supplier bids $p_i(q_i, q_j) = \Delta q(v_i, v_j) + \underline{P}$.*

Intuitively, (i) if $\Delta q(v_1, v_2) + \underline{P} \notin \mathcal{P}$, suppliers could increase their chances of winning by small moves towards the approach of the other party, without reducing the price in those cases where they win. (ii) shows that in all states outside the interval (v_1, v_2) the buyer pays a constant price, reflecting the (maximal) quality difference between the two suppliers.

Lemma 6(i) has an immediate implication.

Corollary 7 *In an auction with a reserve price \bar{P} , the diversity in any (pure strategy) equilibrium (v_i, v_j) is bounded by the reserve price: $\bar{P} \geq \Delta q(v_i, v_j)$.*

If the maximal quality difference between the two suppliers were above the maximum feasible bid, the supplier could not charge the buyer for this quality difference. He could thus choose an approach slightly closer to the competitor to increase the chances of winning without reducing the price.

Corollary 7 embeds the polar cases treated so far, the auction without reserve price and the FPT. In an auction without reserve price, suppliers are free to choose the bid and thus

²¹For $A = 0$, uniqueness requires the tie-breaking rule (T2). Otherwise, any design choice is an equilibrium. An alternative justification of the equilibrium $(1/2, 1/2)$ is that it is the limit of the equilibria for $A \rightarrow 0$.

capture the benefits of diversification. This results in optimal diversity. By Corollary 7, reserve prices limit this possibility: They determine an upper bound on equilibrium diversity. As the reserve price approaches zero, so does the equilibrium diversity, as for an FPT with prize zero. Thus, the choice of reserve price involves a tradeoff between efficiency-increasing diversity and market power for the suppliers. As the next result shows, the buyer never resolves this tradeoff in favor of efficiency, as she prefers fixed prize tournaments to any auction with or without reserve price.

Proposition 8 *Among all contests where \mathcal{P} is convex, the buyer's payoff is maximal in an FPT with $A = 0$.*

In auctions with and without reserve price, \mathcal{P} is convex. Thus, according to Proposition 8, the buyer prefers the socially inefficient tournament without diversity to the unrestricted auction inducing socially optimal diversity and to auctions with reserve price inducing intermediate levels of diversity. The proof of Proposition 8 relies heavily on the fact that higher quality suppliers bid the sum of the quality differential and the minimum \underline{P} whenever available (Lemma 2). Thus the buyer's expected payoff is the difference between the expectation of the minimum quality and the minimum bid. The best she can do is to choose an FPT with $A = 0$, because this maximizes the minimum quality and minimizes the minimum bid.

Proposition 8 highlights an important point: Contrary to the case of effort-inducing contests (Che and Gale, 2003), the buyer can never benefit from using auctions in diversity-inducing contests. Intuitively, while diversity is desirable because of the option value resulting from different approaches, it also leads to greater quality differences ex post. The resulting increase in market power eliminates all benefits for the buyer, which has no counterpart in effort-inducing contests.

We now ask whether the buyer can do even better by using non-convex price sets. The following implementation result shows how the buyer can use augmented tournaments to fine-tune diversity.

Proposition 9 *Suppose $A = \Delta q(v_1, v_2)$ for some (v_1, v_2) such that $0 < v_1 \leq 1/2 \leq v_2 < 1$. In the augmented FPT with $\mathcal{P} = \{A, 0\}$, the strategy profiles $(v_1, v_2, p_1(\cdot), p_2(\cdot))$ such that $p_i(q_i, q_j) = A$ if $q_i - q_j \geq A$ and 0 otherwise, form an equilibrium.*

Thus, the buyer can implement any desired diversity in an augmented FPT with A as the corresponding maximal quality difference. For instance, to induce the social optimum, the buyer has to set $A = \Delta q(v_i^*, v_j^*)$. The resulting payments to the successful supplier are as low as possible: In states where the quality difference is maximal, the buyer has to pay this quality difference, which is the minimum payment consistent with Lemma 6. In all other states, the buyer pays 0, which is obviously minimal. The flexibility of the augmented FPT in inducing diversity and the low buyer payments suggest that the optimal contest is in this class. The following result confirms this intuition.

Proposition 10 *The buyer can achieve the highest possible payoff among all innovation contest equilibria with an augmented FPT.*

Proposition 10 is the central result of this section. Like the class of convex contests analyzed above, the class of augmented FPTs contains the FPT with $A = 0$. As the next result shows, this FPT is the optimal contest if the state distribution is uniform.

Corollary 11 *If (A1)' holds, the FPT with $A = 0$ is an optimal contest.*

To sum up, the buyer always prefers the optimal FPT to the auction. The class of augmented FPTs contains her optimal innovation contest. In the special case of the uniform state distribution, the FPT with $A = 0$ is optimal.

5 Fixed Costs and Participation Fees

Our analysis without fixed costs and participation fees has identified a conflict between efficiency and rent extraction. We now show to which extent the buyer can resolve this conflict when there are fixed costs and/or she can charge participation fees. For simplicity, we will assume (A1)'. Denote with $W(v_1, v_2) = E_\sigma[\max\{q(v_1, \sigma), q(v_2, \sigma)\}]$ the gross welfare generated by (v_1, v_2) . To avoid uninteresting cases, we assume that total net profits are non-negative even when the suppliers select $(v_1, v_2) = (1/2, 1/2)$.

Assumption (A4) $W(1/2, 1/2) \geq 2C$.

Suppose the buyer can charge participation fees of at most $T \geq 0$. We do not allow for discriminatory participation fees. The fee can be negative, corresponding to a subsidy. The buyer thus chooses a participation fee $t \in (-\infty, T]$ which is the same for each supplier.²² An innovation contest with participation fees consists of a pair (\mathcal{P}, t) with an environment (b, Ψ, C, T) that is common knowledge. The rules are as before, with two modifications. First, when the buyer decides on \mathcal{P} , she also fixes t . Second, after the buyer has chosen the rules of the innovation contest, the suppliers decide whether to enter, with an outside option of zero. Upon entering, C and t are sunk,²³ and the suppliers choose their approaches based only on \mathcal{P} , that is, exactly as described in Sections 2-4. The buyer chooses \mathcal{P} and t to maximize her expected payoff, which is equal to the total expected surplus, net of supplier payoffs and (possibly negative) transfers. Thereby, she indirectly selects the equilibrium levels of v_1 and v_2 and the functions $p_1(\cdot, \cdot)$ and $p_2(\cdot, \cdot)$, assigning prices to qualities. Compared with Section 2, she also has to take the participation constraints into account.

Proposition 12 *Suppose (A1)' and (A4) hold and let $v_1^* \leq v_2^*$.*

(i) *If $T \geq F(v_1^*) \Delta q(v_i^*, v_j^*) - C$, the optimal contest is given by $t = F(v_1^*) \Delta q(v_i^*, v_j^*) - C$ and $\mathcal{P} = \{\bar{p}, 0\}$ where $\bar{p} = \Delta q(v_i^*, v_j^*)$. In equilibrium, $v_i = v_i^*$; and $p_i = \bar{p}$ if and only if $q_i - q_j \geq \bar{p}$ ($i = 1, 2; j \neq i$).*

(ii) *If $T < F(v_1^*) \Delta q(v_i^*, v_j^*) - C$, the optimal contest is given by $t = T$ and $\mathcal{P} = \{\bar{p}, 0\}$ where $\bar{p} = \Delta q(\tilde{v}_i, 1 - \tilde{v}_i)$ for $\tilde{v}_i \in (v_i^*, 1/2]$ defined by $T = F(\tilde{v}_i) \Delta q(\tilde{v}_i, 1 - \tilde{v}_i) - C$. In equilibrium, $v_i = \tilde{v}_i$, $v_j = 1 - \tilde{v}_i$, and $p_i = \bar{p}$ if and only if $q_i - q_j \geq \bar{p}$ ($i = 1, 2; j \neq i$).*

If $T \geq F(v_1^*) \Delta q(v_i^*, v_j^*) - C$, the maximal participation fee is at least as high as the expected net profit that each supplier earns in the equilibrium of an augmented tournament

²²The assumption on feasible transfers is slightly more restrictive than necessary. First, T does not need to be finite. Second, the maximal possible subsidy does not need to be infinite. The result holds whenever the maximal subsidy is at least C .

²³The participation decision obliges the supplier to produce one design at cost C and to pay participation fees to the buyer. If participation fees are negative (i.e. if the buyer offers a subsidy), the participation decision obliges the buyer to pay the subsidy.

that induces efficient diversity. The buyer uses this augmented tournament, appropriating the total net surplus by charging the maximal participation fee such that each supplier expects to break even. If $T < F(v_1^*) \Delta q(v_i^*, v_j^*) - C$, the buyer charges the maximal participation fee and only induces enough diversity for suppliers to break even on expectation.²⁴ Again, she opts for rent extraction rather than efficiency.

Summing up, by Corollary 11, without fixed costs and participation fees, the buyer cannot use an appropriate choice of \mathcal{P} to increase surplus and to capture some of the increase herself. Participation fees help to capture the surplus increase. Proposition 12 shows that the buyer optimally induces the amount of diversity which increases the surplus exactly by the amount that can be captured through the participation fees.

6 Extensions

We now show that our main findings still hold for multiple suppliers, heterogeneous suppliers, and for general state distribution and distance measures. We also show that the buyer can potentially do better than in the optimal FPT by not committing to an ex-ante price policy, but instead relying on ex-post negotiations. On the contrary, FPTs with multiple prizes and contests where each buyer can submit multiple designs do not improve the outcome.

At this stage, we would also like to mention one natural extension that appears less tractable. One might want to combine our analysis which focuses on diversity of approaches with the previous literature that focuses on costly effort. Preliminary considerations suggest that such a convex combination is not straightforward, and it is not obvious that it would lead to interesting insights. While we can identify multiple equilibrium candidates as (fairly intransparent) functions of parameters, verifying that these candidates satisfy second-order conditions is an arduous task in general. The complications are at least partly due to the fact that equilibrium existence problems arise with homogeneous suppliers, so that the analysis has to be carried out in the heterogeneous supplier framework described below, which already is quite complex in itself. We thus refrained from treating this case.

6.1 Number of Suppliers

In innovation contests there are usually more than two suppliers. For example, there were 49 registered competitors in the EU Vaccine Prize, 12 of which submitted final designs for evaluation.²⁵ Our main results hold even when there are many suppliers. For simplicity, we assume that (A1)' holds.²⁶ The social optimum and the equilibria in the FPT are as follows.

Lemma 13 *Suppose there are $n > 3$ suppliers and (A1)', (A2) and (A3) hold.*

(i) *The social optimum is $(v_1^*, \dots, v_n^*) = (1/2n, 3/2n, 5/2n, \dots, (2n-1)/2n)$; in particular, there is no duplication.*

(ii) *For any $k \in \{1, \dots, n\}$, an outcome with k active approaches (r_1, \dots, r_k) can be supported in an equilibrium if*

²⁴This case contains the results of Proposition 11 as a special case (where $T = C = 0$).

²⁵European Commission (2014), "German company has won the EU's € 2 million vaccine prize." March 10, 2014 (accessed on April 3, 2015). http://ec.europa.eu/research/health/vaccine-prize_en.html

²⁶We do not consider the case $n = 3$. In this case, an FPT does not have a pure strategy equilibrium, so that comparison with the other cases would be difficult.

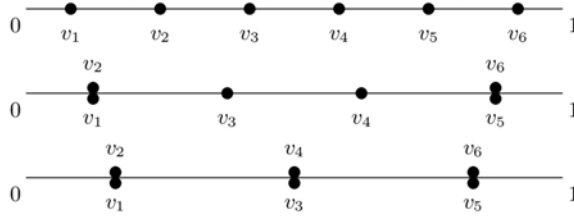


Figure 1: Equilibria when $n = 6$.

The first constellation represents the socially optimal outcome of an auction. The other two constellations represent two possible outcomes of an FPT.

(a) $k \in \{\underline{k}, \dots, \bar{k}\}$, where $\bar{k} = n - 2$ and $\underline{k} = n/2$ if n is even and $\underline{k} = (n + 1)/2$ if n is odd and

(b) $(r_1, \dots, r_k) = ((1/2k, 3/2k, 5/2k, \dots, (2k - 1)/2k))$.

Two suppliers choose the extreme approaches r_1 and r_k ; each of the intermediate approaches r_2, \dots, r_{k-1} is chosen by one or two suppliers.

Figure 1 illustrates the two previous lemmas for $n = 6$. In line with Lemma 13(i), the social optimum (depicted in the first constellation) involves evenly spread approaches. Proposition 24 (i) in the Online Appendix shows that the auction mechanism implements the social optimum. The two other constellations describing the equilibria of the FPT highlight implications of Lemma 13(ii), which are stated more generally in Proposition 24 (ii) and (iii) in the Online Appendix. First, the two most extreme approaches are not as far apart as the most extreme approaches of the social optimum; in this sense, there is less than optimal diversity. Second, as $\bar{k} < n$, there is duplication; moreover, this duplication always affects the two most extreme approaches chosen by the suppliers. Finally, depending on the specific equilibrium, there may be additional duplication for intermediate approaches.

It is important to note that the difference between the FPT and the social optimum decreases as the number of suppliers increases.²⁷

Lemma 13 has another simple but important implication when we allow for fixed costs and transfers: The optimal innovation contest may involve an arbitrarily large number of suppliers. This differs from several results for the case of contests that merely influence the suppliers' efforts, where the optimal number of participants is typically two.

Corollary 14 *Suppose the fixed costs are $C > 0$ and that (A1)' holds. Define $n_-(C) = \max \{n \in \mathbb{N} | n \leq \sqrt{b} / 2\sqrt{C}\}$ and $n_+(C) = n_-(C) + 1$. An innovation contest that maximizes total net profits is an auction with $n_-(C)$ or $n_+(C)$ suppliers.*

The result is a straightforward implication of the previous results. Lemma 13(i) characterizes the socially optimal allocation, and the auction mechanism implements this allocation.

²⁷As $r_k - r_1 = (k - 1)/k$, and the minimum for k on $\{\underline{k}, \dots, \bar{k}\}$ is \underline{k} , $r_k - r_1$ attains its minimum for even n at $k = n/2$, where it becomes $(n - 2)/n$ rather than the socially optimal maximal diversity of $(n - 1)/n$. Hence, the difference between the actual diversity and the social optimum is bounded above by $1/n$. The argument for uneven n is analogous.

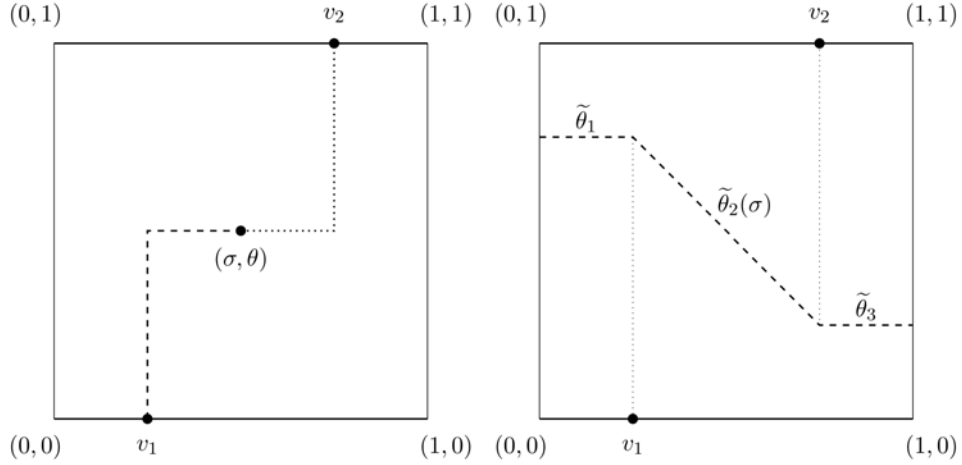


Figure 2: Quality from approaches v_1 and v_2 .

(Left panel) The transportation costs correspond to a weighted sum of the length of the vertical and the horizontal dashed (dotted) line for supplier 1(2). (Right panel) For a given realization (σ, θ) of the state, supplier 1 is the producer of higher (lower) quality if θ is below (above) some critical value $\tilde{\theta}(\sigma)$.

The condition in the Corollary describes the number of suppliers that optimally balances the gains from higher expected quality against the losses from higher fixed costs. While the corollary is stated for the socially optimal contest, it is clear that the buyer can also often benefit from inviting more than two suppliers.

6.2 Heterogeneous Suppliers

The assumption of homogeneous suppliers simplifies the analysis. In many contexts, it is nevertheless natural to allow for heterogeneity: Suppliers may differ in their approaches to innovation, because they have different kinds of expertise or research capabilities. Architects may have different and essentially fixed styles. We now show that the main results hold with heterogeneous suppliers.

Section D.2 in the Online Appendix introduces the modified set-up and the results in detail. We suppose that the state space is now a unit square, with elements (σ, θ) . The set of feasible approaches of supplier 1 is the lower edge of the square; the set of supplier 2 is the upper edge (see Figure 2). We specify the quality functions as

$$q_1(v_1, \varphi) = \Psi - a\theta - b|v_1 - \sigma| \quad (1)$$

$$q_2(v_2, \varphi) = \Psi - a(1 - \theta) - b|v_2 - \sigma| \quad (2)$$

The optimal diversification depends on the ratio $\beta \equiv b/a$.

Proposition 15 (i) *Suppose without loss of generality that $v_1 \leq v_2$. Then the social optimum with heterogeneous suppliers is:*

$$(v_1^*, v_2^*) = \begin{cases} \left(\frac{1}{2}, \frac{1}{2}\right) & \text{if } \beta < 1 \\ \left(\frac{1}{2\beta}, 1 - \frac{1}{2\beta}\right) & \text{if } 1 \leq \beta < 2 \\ \left(\frac{1}{4}, \frac{3}{4}\right) & \text{if } 2 \leq \beta \end{cases} .$$

(ii) *An auction weakly implements the social optimum.*

The intuition for (i) is simple. Whenever $\beta < 1$, exogenous differences between suppliers matter more than differences in approaches. The social optimum (which minimizes the expected horizontal transportation costs) is $(v_1, v_2) = (1/2, 1/2)$. As β increases, so does the expected value of diversity. The expected quality is maximal for $(v_1, v_2) = (1/4, 3/4)$, which is the social optimum for homogeneous suppliers. Interestingly, the auction implements the social optimum even when this does not require diversification.

As for homogeneous suppliers, we can show that fixed-prize tournaments do not induce any diversification (Proposition 32), but that buyers prefer them to auctions (Proposition 33(i)). Moreover, buyers prefer symmetric auction equilibria to asymmetric auction equilibria (Proposition 33(ii)).

The modified framework allows us to use the alternative informational assumption that suppliers cannot observe qualities when they submit bids, which is intractable for homogeneous suppliers (see Proposition 16). Thus, we can also ask: Should buyers reveal quality information to suppliers if they are the only ones who observe quality? Information revelation is indeed essential for the positive result of Proposition 15(ii). To see this, we modify the innovation contest by assuming that suppliers simultaneously choose research approaches and bids. In such an auction without information revelation, suppliers thus commit to their bids before they learn quality. We assume that β is sufficiently low, that is, suppliers are sufficiently heterogeneous.

Assumption (A5): $\beta < 2\sqrt{2} + 3$

When this assumption is violated, in particular for homogeneous suppliers, existence problems arise in auctions without information revelation. Note, however, that the upper bound on β does not rule out any of the cases discussed in Proposition 15.²⁸

Proposition 16 *For heterogeneous suppliers, an auction without information revelation has a unique equilibrium. In this equilibrium, $(v_i, p_i) = (1/2, a)$ for $i = 1, 2$.*

The degree of exogenous differentiation a fully determines prices. Moreover, the equilibrium involves minimum differentiation. Whenever $\beta > 1$, such an equilibrium is not socially optimal according to Proposition 15(ii), reflecting inefficiently low expected quality.

²⁸ In particular, Assumption (A5) is consistent with the case $\beta \geq 2$ in which the social optimum is $(v_1, v_2) = (1/4, 3/4)$, the maximum diversity that can be optimal for any set of parameters.

6.3 Generalized distributions and transportation costs

First, we show that the main results from Sections 3 and 4 are neither sensitive to the assumptions on the distribution of the optimal approach σ nor to the details of the quality function $q(v_i, \sigma)$. For simplicity, we confine the analysis to the game without transfers or fixed costs.²⁹ Proposition 34 in the Online Appendix applies to arbitrary state distributions with positive density and arbitrary quality functions that are decreasing in the distance between a given approach and the ideal one. It shows that (i) the social optimum still requires diversity of approaches, (ii) the auction mechanism implements the social optimum; and (iii) in any FPT, there is no diversity in the unique equilibrium, and both suppliers choose the approach corresponding to the median of the state distribution. The result that buyers prefer the optimal FPT to an auction does not hold under these very general conditions. However, we show in Proposition 35 in the Online Appendix that it is still valid if we relax only the assumption on state distributions (A1) or the assumption on quality functions (A3).

6.4 Multiple Prizes

In the 2005 DARPA Grand Challenge, only the winner of the contest was eligible for the prize (\$2 million), while the other contestants received nothing. This corresponds to an FPT as introduced above. However, in the subsequent DARPA contest, known as the 2007 Urban Challenge, rules specified that not only would the winner receive a prize (which was again \$2 million), but the next two participants would also receive prizes (\$1 million and \$0.5 million).³⁰ In this section, we show that a buyer is worse off in an FPT with two prizes than with a single prize.³¹

Clearly, when there are only two suppliers, a second prize has no effect, as the suppliers would consider it as an unconditional transfer, and the effective prize would be the difference between the first and the second prize. Hence, as in Section 6.1, we suppose $n > 3$. We assume that (A1)', (A2) and (A3) hold, $C = 0$, and that the two prizes are $A_1 > A_2 > 0$. In Online Appendix D.4, we prove the following result:

Lemma 17 *For any (pure strategy) equilibrium in an FPT with two prizes, there exists an equilibrium in an FPT with a single prize which makes the buyer strictly better off.*

The proof shows that any equilibrium of an FPT with two prizes involves more duplication than the chosen equilibrium of an FPT with a single prize, which leads to a lower buyer payoff. This result suggests that multiple prizes do not improve diversity.³²

6.5 Multiple Designs by the Same Supplier

We have assumed so far that each supplier can only develop a single approach. However, in the 2005 DARPA Grand Challenge, vehicles designed by the Red Team from Carnegie

²⁹This game is equivalent to the subgame following entry of both suppliers in the game with arbitrary levels of transfers and costs.

³⁰See Section 1.4 of the DARPA Urban Challenge Rules (2007) (accessed on June 24, 2015). http://archive.darpa.mil/grandchallenge/docs/Urban_Challenge_Rules_102707.pdf

³¹The results can be extended to more than two prizes.

³²Of course, there may be reasons outside of the model which would make multiple prizes a desirable choice for a contest designer. For example, if suppliers are risk averse, providing multiple prizes may be a way of increasing their expected utility.

Mellon University took the second and third place. By developing multiple designs, a supplier internalizes some of the resulting option value. It is thus natural to ask if our results still hold when suppliers develop multiple approaches. The modified model is analytically intractable, but a numerical analysis suggest that our main results are robust. We study the cases with $n \in \{2, \dots, 5\}$ suppliers, each of which can develop $m = 2$ approaches, and the case with $n = 2$ suppliers, each of which can develop $m = 3$ approaches. We assume that (A1)', (A2) and (A3) hold and that $C = 0$. We also fix values of Ψ and b .³³

Result 1 *If there are n suppliers and each develops m approaches, then: (i) in an auction there exists an equilibrium which is equivalent to the socially optimal equilibrium of an auction with $n \cdot m$ suppliers, each of which develops one approach. (ii) In an FPT, there exists an equilibrium which is identical to the maximally duplicative equilibrium of an FPT with $n \cdot m$ suppliers, each of which develops one approach.*

Proposition 24 in Online Appendix D.1 proves that the buyer prefers an FPT to an auction inducing the socially optimal outcome when there are multiple suppliers and each develops one approach. Moreover, Result 1 suggests that the case where n suppliers each develop m approaches corresponds to the case where $n \cdot m$ suppliers each develop one approach (see Section 6.1). This suggests that the buyer is better off holding an FPT than an auction also in the case when suppliers can develop multiple approaches.

6.6 Negotiations

Finally, we suppose that the buyer decides not to commit to a design contest, but instead leaves the determination of payments to negotiations, which split the surplus of the relationship, i.e., the quality differential Δq .³⁴ A share $\tau \in (0, 1)$ accrues to the buyer, the remainder to the successful supplier. This introduces a potential hold-up problem when there are fixed costs. If the suppliers are not convinced that they will have sufficient bargaining power to break even, they will not invest. If the suppliers are not deterred from investing, negotiations implement the social optimum, because they share an important feature with auctions: Suppliers are rewarded for higher quality relative to the other supplier. The distributional properties depend on τ . Independent of its value, the buyer prefers negotiations to auctions, because the latter give the entire surplus Δq to the supplier. Holding an auction thus represents a transfer of market power from the buyer to the suppliers. For sufficiently high bargaining power of the buyer, negotiations are preferable to the optimal FPT for the buyer, provided suppliers expect to break even.

³³For details and the code used to obtain numerical results, see Supplementary Material for Section VI.E, available at <https://sites.google.com/site/iletina/did>.

³⁴Somewhat relatedly, Ding and Wolfstetter (2011) consider a case where a supplier can choose to bypass the contest and negotiate with the buyer directly in an environment where innovation quality is obtained by expending costly effort.

7 Relation to the Literature

This paper contributes to the literature on optimal contest design, especially the design of innovation contests. In models of fixed-prize tournaments, Taylor (1995) shows that free entry is undesirable, and Fullerton and McAfee (1999) show that the optimal number of participants is two. Fullerton et al. (2002) find that buyers are better off using auctions than fixed-prize tournaments. In a very general framework, Che and Gale (2003) show that an auction with two suppliers is the optimal contest. Contrary to the previous literature, our paper focuses on inducing the suppliers to choose the right approaches rather than to exert as much effort as possible. We find that it can be optimal for the buyer to invite a large number of suppliers. Moreover, for any given number of suppliers, we show that the buyer often prefers fixed-prize tournaments to auctions, even though the latter implement the socially optimal diversity of research approaches.

While we are not aware of any paper that considers optimal contest design when diversity plays a role, some authors obtain related results. For instance, Schöttner (2008) considers two contestants who influence quality stochastically by exerting effort. While she does not deal directly with optimal contest design, she finds that, for large random shocks, the buyer prefers to hold a fixed-prize tournament rather than an auction to avoid the market power of a lucky seller in an auction. In our setting, the correlation of outcomes is endogenous, and diversity of approaches implies that, whenever one supplier is subject to a negative shock, the other one will likely be subject to a positive shock. Thereby, diversification generates an option value which is desirable from a social point of view. However, as we saw above, in an auction, the option value is often captured by the suppliers and not by the buyer. Thus, the buyer is better off not inducing diversity through an auction at all.

In Ganuza and Hauk (2006), suppliers choose both an approach to innovation and a costly effort.³⁵ However, these authors focus on fixed-prize tournaments, while we study the optimal contest design.³⁶ Erat and Krishnan (2012) study a fixed-prize tournament where suppliers can choose from a discrete set of approaches.³⁷ Each approach is successful with some probability that is independent of success or failure of any other approach. The qualities of successful approaches can vary. The authors find that suppliers cluster on approaches delivering the highest quality. This result is similar to our result that fixed-prize tournaments lead to a duplication of approaches in equilibrium. In addition to considering alternative contests, our model also considers correlated rather than independent qualities; it is thus meaningful to speak of similar approaches.³⁸

More broadly, our paper is related to the literature on innovation contests with exponential-bandit experimentation (see Halac, Kartik and Liu (2014) and references therein). In these

³⁵In Ganuza and Pechlivanos (2000), Ganuza (2007) and Kaplan (2012), the buyer has to choose the design or alternatively can reveal information about the preferred design.

³⁶More broadly related is Bajari and Tadelis (2001) who do not deal with innovations, but with construction projects. The issue of the right approach to the problem arises in such settings as well. The supplier obtains new information during the period when the contract is being executed, which allows him to adapt the original approach at some cost. Since the relationship is between a buyer and only one supplier, the question of diversity of approaches does not arise. This is also true for the related work by Arve and Martimort (2015) who study risk-sharing considerations in the design of contracts with ex-post adaptation.

³⁷See also Terwiesch and Xu (2008) for the effect of number of suppliers when exogenous random shocks are large. For empirical evidence see Boudreau, Lacetera and Lakhani (2011).

³⁸See also Konrad (2014) for a variant of Erat and Krishnan's model where first best is restored if the tie-breaking is decided via costly competition (for example lobbying) as opposed to randomly.

models, it is uncertain whether the innovation is feasible. Suppliers participating in the contest expend costly effort to learn the state, and they also learn from the experimentation of their opponents. The goal of the contest is to induce experimentation. However, each supplier experiments in the same way. In our model, experimentation arises at the industry level for suitable contests, as the heterogeneity of approaches allows the buyer to pick the best available choice.

Finally, our paper is related to the literature on policy experimentation. For instance, Callander and Harstad (2015) show that decentralized policy experimentation yields too much diversity. In their model, the success probabilities of different experiments are independent, no matter how similar the policies are. This assumption removes the option value of having different experiments, which is central in our model. If there existed an ideal policy (in terms of quality) as in our model, then the option value would have to be traded off against the benefits of convergence emphasized by Callander and Harstad (2015). It would be interesting to see whether and how centralization would help to resolve this trade off.

8 Conclusions and Discussion

The ideal approach to solving an innovation procurement problem is usually unknown when suppliers are asked to choose research approaches. The paper investigates the implications of this uncertainty for contest design. Under very general conditions, it is socially optimal to induce suppliers to take diverse research approaches, and the social optimum can be obtained with an auction mechanism, provided the bidding takes place after qualities are commonly known. To reduce supplier rents, the buyer nevertheless prefers a fixed-prize tournament that induces less diversification. In a two-player setting, diversification decreases with the reserve price in an auction. Moreover, the optimal contest is an augmented fixed prize tournament, which induces diversity at the lowest possible cost to the buyer.

Our paper offers a rationale for the frequent use of fixed-prize tournaments in complex procurement tasks, as in innovation and architecture. Even though other mechanisms may yield higher option value and thus higher expected quality, with suitable fixed-prize tournaments the buyer can extract all rents from the suppliers. This trade-off between efficiency and rent appropriation vanishes when the buyer can charge sufficiently high participation fees. Then the buyer can induce the social optimum, and use transfer payments or participation fees to make sure that the suppliers just break even. Moreover, when the exogenous differentiation of supplier characteristics is sufficiently pronounced, it may not even be necessary to provide additional incentives for diversification from a social point of view.

Our model can be used to analyze how institutions affect the incentives of individuals to experiment in domains where the optimal approach to solving a given problem is not known. A particularly promising application would be to think of our model as capturing product choice in markets with a unit mass of homogeneous buyers, each of which has unit demand. We can then interpret the uncertainty about the ideal state in two ways. First, it may capture uncertainty about the buyers' taste. Second, it may capture an "engineering uncertainty" where the suppliers know what the buyers would like, but are uncertain about how to achieve this. Either way, the rules of the contest translate directly into a description of the regulatory constraints in a market environment. For instance, the auction corresponds to an unregulated market environment where suppliers choose products under uncertainty

about the preferred product and charge prices once qualities are realized. The fixed prize tournament can be interpreted as a regulated market where prices are fixed ex ante: The prize A is then the profit that the firm earns in the market from selling at the regulated price to a unit mass of consumers. Similarly, auctions with reservation prices have a natural interpretation as markets where there is a price cap.

Our results suggest that an unregulated market maximizes expected total surplus, whereas the regulated market maximizes the expected consumer surplus. The unregulated market gives incentives for firms to diversify, but leaves them with market power. The trade-off resembles the trade-off between ex-ante incentives and ex-post monopoly power in the innovation literature. In our case, however, the higher expected quality from the unregulated market does not result from higher innovation incentives at the individual firm level, but rather from the higher diversification incentives at the market level. Price caps strike a balance between the goals of maximizing consumer surplus and total surplus. Augmented FPTs have no obvious counterpart in reality: They would correspond to a regulated environment where firms can select between offering two different prices depending on the realized quality levels. Our analysis suggests that, in some markets, such augmented FPTs may even be better for consumers than full price regulation. These simple considerations clearly have limitations resulting from the rather special market environment. However, the arguments suggest that the contest approach may potentially be valuable to analyze product innovations (or product selection) in market environments. A full analysis of this topic is left for future research.

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9 Appendix: Main Proofs

9.1 Proof of Proposition 4

Using (T1), in an FPT, the expected profit of supplier i , given v_j , is

$$\Pi_i(v_i|v_j) = \begin{cases} A \Pr(|v_i - \sigma| \leq |v_j - \sigma|) & \text{if } v_i \neq v_j \\ A/2 & \text{if } v_i = v_j \end{cases}.$$

Since $\Pr(|v_i - \sigma| \leq |1/2 - \sigma|) < 1/2$ for all $v_i \neq 1/2$, $(v_1, v_2) = (1/2, 1/2)$ is an equilibrium. For uniqueness, suppose an equilibrium with $v_i < v_j$ exists. Then $\Pi_i(v_i|v_j) = A \int_0^{(v_i+v_j)/2} dF(\sigma)$. Thus supplier i can increase his expected profit by marginally increasing v_i , since $d\Pi_i(v_i|v_j)/dv_i = Af((v_i+v_j)/2)/2 > 0$. The argument for $v_i > v_j$ is analogous. There can be no equilibrium with $v_i = v_j \neq 1/2$, because $\Pi_i(1/2|v_j) > A/2 = \Pi_i(v_i|v_j)$.

9.2 Proof of Lemma 6

(i) The result is trivial for $v_1 = v_2$. For $v_1 < v_2$, we show that supplier 1 can profitably deviate to some $v'_1 > v_1$ if $\Delta q(v_1, v_2) + \underline{P} \notin \mathcal{P}$. Before the deviation, by Lemma 2, if $\sigma \in [0, v_1]$, supplier 1 wins and $\bar{p}(\sigma) < \Delta q(v_1, v_2) + \underline{P}$. By continuity, $\exists v'_1 \in (v_1, v_2]$ such that $\bar{p}(\sigma) < \Delta q(v'_1, v_2) + \underline{P} < \Delta q(v_1, v_2) + \underline{P}$. By deviating to v'_1 , supplier 1 wins whenever $\sigma < (v'_1 + v_2)/2$ rather than when $\sigma < (v_1 + v_2)/2$. The set of states in which supplier 1 wins after the deviation thus is a strict superset of the set of states in which the supplier wins before the deviation. For $\sigma \in [0, v_1]$, the price is unaffected. For $\sigma \in (v_1, (v'_1 + v_2)/2]$, the price is at least as high as before the deviation. Thus, v'_1 is a profitable deviation by (T2). (ii) follows directly from Lemmas 2 and 6 (i).

9.3 Proof of Proposition 8

Denote the minimum allowable price with \underline{P} . If $v_1 \neq v_2$ in equilibrium, by Proposition 4, the contest is not an FPT. Suppose that $v_1 < v_2$. By Lemmas 2 and 6, the buyer pays $q_i - q_j + \underline{P}$ to the supplier with $q_i \geq q_j$ in equilibrium. Thus, for any σ , the buyer payoff is $\min\{q_1, q_2\} - \underline{P}$. Hence, the expected payoff of a buyer who induces $v_1 < v_2$ with \underline{P} is

$$\begin{aligned} \Pi_b(v_1, v_2; P) &= \int_0^1 \min\{q_i(v_i, \sigma), q_j(v_j, \sigma)\} dF(\sigma) - \underline{P} \\ &= \int_0^{\frac{v_1+v_2}{2}} q_2(v_2, \sigma) dF(\sigma) + \int_{\frac{v_1+v_2}{2}}^1 q_1(v_1, \sigma) dF(\sigma) - \underline{P} \end{aligned}$$

Thus

$$\frac{d\Pi_b}{dv_1} = \int_{\frac{v_1+v_2}{2}}^1 \frac{\partial q_1}{\partial v_1} dF(\sigma) > 0; \quad \frac{d\Pi_b}{dv_2} = \int_0^{\frac{v_1+v_2}{2}} \frac{\partial q_2}{\partial v_2} dF(\sigma) < 0.$$

Thus, the buyer payoff is maximal for $v_1 = v_2$ and $\underline{P} = 0$. Given $v_1 = v_2$, the buyer payoff is maximal for $v_1 = v_2 = 1/2$, the unique equilibrium of an FPT with A arbitrarily close to 0. Given (T2), it is an equilibrium for $A = 0$.

9.4 Proof of Proposition 9

Sequential rationality of $p_i(\cdot)$ follows from Lemma 2. We now show that $(v_1, p_1(\cdot))$ is a best response of player 1 to $(v_2, p_2(\cdot))$; the argument for player 2 is analogous. For $A = 0$, only $(v_1, v_2) = (1/2, 1/2)$ satisfies the above conditions. Thus, the statement for $A = 0$ follows from Proposition 4. If $v_1 < v_2$, $\Delta q(v_1, v_2) > 0$, and the probability that player 1 wins with a positive prize is $F(v_1)$. Deviating to $v'_1 < v_1$ is not profitable, because the winning probability falls to $F(\hat{v}_1)$, with $\hat{v}_1 < v_1$ implicitly defined by $q(v'_1, \hat{v}_1) - q(v_2, \hat{v}_1) = \Delta q(v_1, v_2)$, and the prize does not rise. It is not profitable to deviate to $v''_1 \in (v_1, \tilde{v})$, where $\tilde{v} = \min(2v_2 - v_1, 1) \geq 1/2$: For such deviations, $\Delta q(v''_1, v_2) < \Delta q(\tilde{v}, v_2) \leq \Delta q(v_1, v_2) \forall \sigma$, so that the probability of winning a positive prize is 0. Finally, if $\tilde{v} < 1$, deviating to $v'''_1 \in [\tilde{v}, 1]$ is not profitable, because $\tilde{v} \geq 1/2 + v_2 - v_1$ implies $1 - \tilde{v} \leq 1/2 - (v_2 - v_1) \leq v_2 - (v_2 - v_1) = v_1$ and therefore, by symmetry of the state distribution, $1 - F(v'''_1) \leq 1 - F(\tilde{v}) \leq F(v_1)$. By analogous arguments, there are no profitable deviations for supplier 2.

9.5 Proof of Proposition 10

We provide a sketch of the proof here. The complete proof is in the Online Appendix. The result follows from two lemmas. First we show that approaches maximizing buyer's payoffs satisfy $0 < v_1^B \leq \frac{1}{2} \leq v_2^B < 1$. Hence by Proposition 9, they can be implemented with an augmented FPT. To prove this claim, we show that $v_1^B \leq \frac{1}{2} \leq v_2^B$ must hold in any equilibrium, as otherwise the supplier with the lower winning probability can profitably deviate to the uncontested half of $[0, 1]$. $0 < v_1^B \leq v_2^B < 1$ must hold since approaches on the boundary are socially inefficient, and the buyer can either match or increase the total payoff without increasing the transfers to the suppliers. The second lemma shows that a suitable augmented FTP implements (v_1^B, v_2^B) at the lowest expected cost for the buyer, hence maximizing the buyer payoff. The intuition for this follows from Lemma 6, as the payments are minimal in each state.

APPENDIX

FOR ONLINE PUBLICATION

Designing Institutions for Diversity
Igor Letina and Armin Schmutzler

In this appendix we provide proofs which were omitted from the main text, as well as some additional results and discussion related to the extensions. The supplementary material is organized according to the section of the main text to which it relates.

A Supplementary Material for Section 3

A.1 Notation

We introduce the following notation:

1. $v^\beta(p_1, p_2 | q_1, q_2) \in \{v_1, v_2\}$ is the approach that maximizes $q_i - p_i$.³⁹
2. $\pi_i(p_i, p_j | q_i, q_j)$ is the profit that supplier i earns with prices p_1 and p_2 , conditional on qualities q_1 and q_2 , assuming that the buyer selects $v^\beta(p_1, p_2, q_1, q_2)$.
3. $p_i(q_i, q_j) \in \mathcal{P}^{[\Psi-b, \Psi]^2}$ is the price strategy function.⁴⁰
4. $\Pi_i(v_i, v_j, p_i, p_j)$ is the expectation over $\pi_i(p_i, p_j | q_i, q_j)$ when suppliers choose $v_1, v_2, p_1()$ and $p_2()$, where the expectation is taken over all pairs of quality realizations for given (v_1, v_2) .

A subgame-perfect equilibrium of the innovation contest given by \mathcal{P} consists of supplier strategies $s_i = (v_i, p_i) \in [0, 1] \times \mathcal{P}^{[\Psi-b, \Psi]^2}$ and buyer strategies $\nu \in \{v_1, v_2\}^{(\mathcal{P}^2 \times [\Psi-b, \Psi])^2}$ such that:

(DC1) $\nu = v^\beta$ (sequential rationality of the buyer).

(DC2) $\pi_i(p_i(q_i, q_j), p_j(q_j, q_i) | q_i, q_j) \geq \pi_i(p'_i, p_j(q_j, q_i) | q_i, q_j)$ for all $p'_i \in \mathcal{P}, (q_i, q_j) \in [\Psi - b, \Psi]^2$ (sequential rationality of supplier i)

(DC3) $\Pi_i(v_i, v_j, p_i(q_i, q_j), p_j(q_j, q_i)) \geq \Pi_i(v'_i, v_j, \tilde{p}_i(q_i, q_j), p_j(q_j, q_i))$ for all $v'_i \in [0, 1]$ and all $\tilde{p}_i(q_i, q_j) \in \mathcal{P}^{[\Psi-b, \Psi] \times [\Psi-b, \Psi]}$ (optimal approach of supplier i).

³⁹When $q_1 - p_1 = q_2 - p_2$, we appeal to tie-breaking rule (T1) below.

⁴⁰For sets X and Y , Y^X is the set of all mappings from X to Y .

A.2 The Social Optimum

We start with the simple result that the social optimum involves diversity (Proposition 18(i)). We then provide an equally straightforward characterization of the social optimum for a wide class of state distributions (Proposition 18(ii)). Finally, we turn towards the implementation result for auction (Proposition 3), preceded by the auxiliary result on pricing in subgames (Lemma 2).

As the costs of each approach are the same, the social optimum (v_1^*, v_2^*) must maximize the expected maximal quality $E_\sigma [\max \{q(v_1, \sigma), q(v_2, \sigma)\}]$ or, equivalently, minimize the expected distance to the ideal approach, $E_\sigma [\min \{\delta |v_1 - \sigma|, \delta |v_2 - \sigma|\}]$. With only one potential supplier i , the optimal approach would correspond to $v_i = 1/2$, as this maximizes the expected quality. With two suppliers, the optimization needs to take into account the option value generated by having different choices once qualities have been observed. The next result shows that it is always optimal to have at least some diversification, and that the required diversification can be substantial.

Proposition 18 (i) *A social optimum (v_1^*, v_2^*) exists and satisfies $v_1^* \neq v_2^*$.*
(ii) *If $f(1/2) < 2f(0)$, then the unique social optimum with $v_1 \leq v_2$ is characterized by $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$ and thus $v_2^* = 1 - v_1^*$. In particular, if (A1)' holds, then the social optima are $(v_1^*, v_2^*) = (1/4, 3/4)$ and $(3/4, 1/4)$.*

To understand (i), suppose that both suppliers choose the same approach. Starting from such a situation, suppose one of the suppliers, say, supplier 1, chooses an arbitrary alternative approach, whereas the other supplier continues to choose the same one. After this modification, the minimal distance decreases for a non-degenerate set of ideal states. There can be no σ where the expected minimal distance increases, as the initial choice of supplier 2 is still available. (ii) gives a sufficient condition for the approaches to be symmetric around 1/2. The condition states that the ideal state distribution is sufficiently dispersed.⁴¹

A.2.1 Proof of Proposition 18

(i) Suppose, without loss of generality, that $v_1 \leq v_2$. The social welfare function is given by $W(v_1, v_2) = \int_0^1 \max\{q(v_1, \sigma), q(v_2, \sigma)\} dF(\sigma)$. Hence

⁴¹The condition guarantees that the expected quality is a strictly concave function of the approaches. It is thus more restrictive than necessary. A simple *necessary* condition for the optimum to satisfy $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$ is $f(1/2) < 2f(v_1^*)$; otherwise the objective function is not even locally concave. Moreover, this condition turns out to be necessary for the existence of a social optimum with $v_2^* = 1 - v_1^*$. It is simple to provide examples where $f(1/2) < 2f(v_1^*)$ is violated. For instance, consider the kinked distribution defined by the density

$$f(\sigma) = \begin{cases} 0.6 & \text{if } \sigma \in [0, 0.45) \cup (0.5, 1] \\ 4.6 & \text{if } \sigma \in [0.45, 0.55] \end{cases} .$$

$$W(v_1, v_2) = \Psi - b \left(\begin{array}{c} \int_0^{v_1} (v_1 - \sigma) dF(\sigma) + \int_{v_1}^{(v_1+v_2)/2} (\sigma - v_1) dF(\sigma) + \\ \int_{(v_1+v_2)/2}^{v_2} (v_2 - \sigma) dF(\sigma) + \int_{v_2}^1 (\sigma - v_2) dF(\sigma) \end{array} \right).$$

This is a continuous function with a compact domain, hence it attains the maximum. To prove $v_1^* \neq v_2^*$, note that

$$\frac{\partial W(v_1, v_2)}{\partial v_1} = b(-2F(v_1) + F((v_1 + v_2)/2)) \quad (3)$$

$$\frac{\partial W(v_1, v_2)}{\partial v_2} = b(1 - 2F(v_2) + F((v_1 + v_2)/2)). \quad (4)$$

(3) and (4) imply that there are no boundary optima. To see this, first note that $\frac{\partial W(0, v_2)}{\partial v_1} > 0 \forall v_2 > 0$ and $\frac{\partial W(v_1, 1)}{\partial v_2} < 0 \forall v_1 < 1$. Moreover $(v_1, v_2) = (0, 0)$ and $(1, 1)$ are both dominated by $(1/2, 1/2)$. Thus, the optimum must satisfy

$$-2F(v_1) + F((v_1 + v_2)/2) = 0 \quad (5)$$

$$1 - 2F(v_2) + F((v_1 + v_2)/2) = 0. \quad (6)$$

Together these conditions imply $F(v_2^*) = 1/2 + F(v_1^*)$ and thus $v_1^* \neq v_2^*$.

(ii) For $v_1 \in [0, 1/2]$, let $g(v_1) = F^{-1}(F(v_1) + \frac{1}{2})$. F^{-1} is well-defined because of (A1)(iii). Inserting $v_2 = g(v_1)$ in (5) and (6), the first-order conditions hold for $(v_1, v_2) = (v_1, g(v_1))$ if

$$v_1 = F^{-1}\left(\frac{F((v_1 + g(v_1))/2)}{2}\right). \quad (7)$$

(7) has at least one solution $v_1^* \in (0, 1/2)$. This holds because both sides of (7) are strictly increasing, and the r.h.s. is positive for $v_1 = 0$ and strictly less than $1/2$ for $v_1 = 1/2$. Now consider $(v_1^*, v_2^*) = (v_1^*, g(v_1^*))$ such that $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$. Thus $F(v_2^*) = F(v_1^*) + 1/2$. Moreover, symmetry implies $v_1^* + v_2^* = 1$ and thus the r.h.s. of (7) is $F^{-1}(\frac{1}{4})$, so that the first-order condition holds for (v_1^*, v_2^*) .

Finally, consider the Hessian matrix

$$\begin{aligned} H &= \begin{bmatrix} \frac{\partial^2 W}{\partial v_1^2} & \frac{\partial^2 W}{\partial v_1 \partial v_2} \\ \frac{\partial^2 W}{\partial v_1 \partial v_2} & \frac{\partial^2 W}{\partial v_2^2} \end{bmatrix} \\ &= \begin{bmatrix} -2f(v_1) + \frac{1}{2}f((v_1 + v_2)/2) & \frac{1}{2}f((v_1 + v_2)/2) \\ \frac{1}{2}f((v_1 + v_2)/2) & -2f(v_2) + \frac{1}{2}f((v_1 + v_2)/2) \end{bmatrix}. \end{aligned}$$

First, H is negative definite at (v_1^*, v_2^*) if and only if $f(1/2) < 2f(v_1^*)$. To see this, note that $f(v_1^*) = f(v_2^*)$ and $f((v_1^* + v_2^*)/2) = f(1/2)$. Hence,

$$-2f(v_1^*) + \frac{1}{2}f((v_1^* + v_2^*)/2) = -2f(v_1^*) + \frac{1}{2}f(1/2) < 0 \Leftrightarrow f(1/2) < 4f(v_1^*).$$

In addition,

$$|H| = 4f(v_1^*)f(v_2^*) - (f(v_1^*) + f(v_2^*))f((v_1^* + v_2^*)/2) = 4f(v_1^*)^2 - 2f(v_1^*)f(1/2).$$

This condition holds if and only if $f(1/2) < 2f(v_1^*)$.

Second, H is negative definite $\forall (v_1, v_2)$ if $f(1/2) < 2f(0)$. To see this, note that $f(v)$ is minimized at $v = 0$ and maximized at $v = 1/2$. Hence, $f(1/2) < 2f(0) < 4f(0)$ implies

$$-2f(v_i) + \frac{1}{2}f\left(\frac{v_1 + v_2}{2}\right) \leq -2f(0) + \frac{1}{2}f\left(\frac{1}{2}\right) < 0 \quad \forall i \in \{1, 2\}.$$

and

$$|H| = f(v_1) \left(2f(v_2) - f\left(\frac{v_1 + v_2}{2}\right) \right) + f(v_2) \left(2f(v_1) - f\left(\frac{v_1 + v_2}{2}\right) \right) > 0.$$

Therefore, $f(1/2) < 2f(0)$ is a sufficient condition for (v_1^*, v_2^*) to be the unique global optimum. The statement for the uniform distribution follows.

A.3 Implementation of the Social Optimum

A.3.1 Proof of Lemma 2

Consider the equilibrium for the subgame defined by (v_1, v_2, σ) and the resulting quality vector (q_1, q_2) . If $q_1 = q_2$, the standard Bertrand logic implies that $(\bar{p}(\sigma), \bar{p}(\sigma)) = (\underline{P}, \underline{P})$ is the unique equilibrium. Now suppose $q_i > q_j$. Clearly, the suggested strategy profile is a subgame equilibrium. To see that i must bid $\bar{p}(\sigma)$ in equilibrium, first suppose $p_i > \bar{p}(\sigma)$. If $p_i > p_j + q(v_i, \sigma) - q(v_j, \sigma)$, player j wins. By setting $p_i = \bar{p}(\sigma) \leq p_j + q(v_i, \sigma) - q(v_j, \sigma)$, player i can ensure that he wins, which is a profitable deviation by (T2). If $p_i > \bar{p}(\sigma)$ and $p_i \leq p_j + q(v_i, \sigma) - q(v_j, \sigma)$, player i wins. By setting $p_j = \underline{P}$, player j can profitably deviate. If $p_i < \bar{p}(\sigma)$, player i can deviate upwards to $\bar{p}(\sigma)$. He then still wins by (T1), and profits are higher.

A.3.2 Proof of Proposition 3

We use the following notation:

Notation 19 $\Pi(v_i|v_j) = \Pi_i(v_i, v_j, p_i, p_j)$, when $p_i()$ and $p_j()$ are the subgame equilibria as in Lemma 2.

We now proceed with the proof.

(i) By Lemma 2, the unique equilibrium of the pricing subgame induced by q_1 and q_2 is $p_i = \max\{q_i - q_j, 0\}$ for $i, j \in \{1, 2\}; j \neq i$. Suppose that an auction does not implement the social optimum (v_1^*, v_2^*) . Then, for some i , there exists $\bar{v}_i \neq v_i^*$ such that $\Pi_i(\bar{v}_i|v_j^*) > \Pi_i(v_i^*|v_j^*)$. Let $\Theta_i(v_i, v_j) = \{\sigma \in [0, 1] | q(v_i, \sigma) \geq q(v_j, \sigma)\}$ and $\Theta_{-i}(v_i, v_j) = [0, 1] \setminus \Theta_i(v_i, v_j)$. Thus $\Pi_i(\bar{v}_i|v_j^*) > \Pi_i(v_i^*|v_j^*)$ if and only if

$$\int_{\Theta_i(\bar{v}_i, v_j^*)} (q(\bar{v}_i, \sigma) - q(v_j^*, \sigma)) dF(\sigma) > \int_{\Theta_i(v_i^*, v_j^*)} (q(v_i^*, \sigma) - q(v_j^*, \sigma)) dF(\sigma),$$

or equivalently

$$\begin{aligned} & \int_{\Theta_i(\bar{v}_i, v_j^*)} (q(\bar{v}_i, \sigma) - q(v_j^*, \sigma)) dF(\sigma) + \int_0^1 q(v_j^*, \sigma) dF(\sigma) > \\ & \int_{\Theta_i(v_i^*, v_j^*)} (q(v_i^*, \sigma) - q(v_j^*, \sigma)) dF(\sigma) + \int_0^1 q(v_j^*, \sigma) dF(\sigma) \end{aligned}$$

Splitting $[0, 1]$ into $\Theta_i(\bar{v}_i, v_j^*)$ and $\Theta_{-i}(\bar{v}_i, v_j^*)$ in the first line and into $\Theta_i(v_i^*, v_j^*)$ and $\Theta_{-i}(v_i^*, v_j^*)$ in the second line and simplifying, this is equivalent with

$$\begin{aligned} & \int_{\Theta_i(\bar{v}_i, v_j^*)} q(\bar{v}_i, \sigma) dF(\sigma) + \int_{\Theta_{-i}(\bar{v}_i, v_j^*)} q(v_j^*, \sigma) dF(\sigma) > \\ & \int_{\Theta_i(v_i^*, v_j^*)} q(v_i^*, \sigma) dF(\sigma) + \int_{\Theta_{-i}(v_i^*, v_j^*)} q(v_j^*, \sigma) dF(\sigma). \end{aligned}$$

and thus

$$\int_0^1 \max\{q(\bar{v}_i, \sigma), q(v_j^*, \sigma)\} dF(\sigma) > \int_0^1 \max\{q(v_i^*, \sigma), q(v_j^*, \sigma)\} dF(\sigma),$$

contradicting optimality of (v_1^*, v_2^*) .

(ii) By Lemma 2, the unique equilibrium of the pricing subgame is $p_i = \max\{q_i - q_j, 0\}$ for $i, j \in \{1, 2\}$. This immediately implies that there is no equilibrium of the auction with $v_1 = v_2$. Suppose that $v_1 < v_2$ in an auction equilibrium. Given the equilibrium pricing, we can write the expected profits as follows

$$\begin{aligned} \Pi_1(v_1|v_2) &= \int_0^{\frac{v_1+v_2}{2}} (q(v_1, \sigma) - q(v_2, \sigma)) dF(\sigma) \\ &= \int_0^{v_1} b(v_2 - v_1) d\sigma + \int_{v_1}^{\frac{v_1+v_2}{2}} b(v_1 + v_2 - 2\sigma) d\sigma \end{aligned}$$

$$\begin{aligned} \Pi_2(v_2|v_1) &= \int_{\frac{v_1+v_2}{2}}^1 (q(v_2, \sigma) - q(v_1, \sigma)) dF(\sigma) \\ &= \int_{\frac{v_1+v_2}{2}}^{v_2} b(2\sigma - v_1 - v_2) d\sigma + \int_{v_2}^1 b(v_2 - v_1) d\sigma \end{aligned}$$

The FOCs for an interior equilibrium are thus

$$\begin{aligned} \frac{\partial \Pi_1(v_1|v_2)}{\partial v_1} &= b \left(- \int_0^{v_1} d\sigma + \int_{v_1}^{\frac{v_1+v_2}{2}} d\sigma \right) \\ &= b \left(-2F(v_1) + F\left(\frac{v_1+v_2}{2}\right) \right) = 0 \end{aligned}$$

$$\begin{aligned}\frac{\partial \Pi_2(v_2|v_1)}{\partial v_1} &= b \left(- \int_{\frac{v_1+v_2}{2}}^{v_2} d\sigma + \int_{v_2}^1 d\sigma \right) \\ &= b \left(-2F(v_2) + F\left(\frac{v_1+v_2}{2}\right) + 1 \right) = 0\end{aligned}$$

Using (A1)', the two conditions are equivalent to

$$-2v_1 + \frac{v_1+v_2}{2} = 0 = -2v_2 + \frac{v_1+v_2}{2} + 1.$$

The unique solution to the above system is $(v_1, v_2) = (1/4, 3/4)$, which is the social optimum (see Proposition 18). This solution also satisfies the second-order conditions. Boundary solutions clearly do not exist.

A.4 General Implementation of the Social Optimum

We prove, in a general setting, that a mechanism which i) selects the highest quality ex-post and ii) transfers a positive fraction of the difference between best and second-best quality to the winner always implements the socially optimal choice of approaches.

Let $N = \{1, \dots, n\}$ be the set of suppliers. Let $V \subseteq \mathbb{R}$ be a compact and convex set of possible approaches. Let $\Theta \subseteq \mathbb{R}$ be a compact and convex set of possible supplier characteristics. The actual supplier characteristics $\theta_1, \dots, \theta_n \in \Theta$ are commonly known. Let the ideal approach $\sigma \in V$ and the ideal supplier characteristic $\theta \in \Theta$ be distributed according to continuous probability density functions $f_\sigma(\sigma)$ and $f_\theta(\theta)$, respectively, with corresponding cumulative distribution functions F_σ and F_θ . For any combination of v_i and θ_i , and any (θ, σ) , the quality of supplier i is given by the function $q(v_i, (\theta, \sigma)) := \tilde{q}(\delta_\theta(|\theta_i - \theta|), \delta_\sigma(|v_i - \sigma|))$ where δ_θ and δ_σ are differentiable functions satisfying $\delta'_\theta > 0$, $\delta'_\sigma > 0$, and q is a differentiable function satisfying $\partial q / \partial \delta_\theta < 0$ and $\partial q / \partial \delta_\sigma < 0$. Here, $\delta_\theta(\cdot)$ is the reduction in quality due to suboptimal supplier characteristic and $\delta_\sigma(\cdot)$ is the reduction in quality due to a suboptimal approach. Let q , δ_θ and δ_σ be continuous functions.

Let the vector of approaches be $\mathbf{v} = [v_1, \dots, v_n]$. Then the expected social welfare is given by $W(\mathbf{v}) = \int_V \int_\Theta \max_{i \in N} \{q(v_i, (\theta, \sigma))\} dF_\theta dF_\sigma$. Denote with $\tilde{\Theta}_i(\mathbf{v}, \sigma) \subseteq \Theta$ the set of states θ for which supplier i offers the highest quality. Ties are broken so that the demand is assigned to the supplier with a lower index.⁴² Thus, $\tilde{\Theta}_i(\mathbf{v}, \sigma) \cap \tilde{\Theta}_j(\mathbf{v}, \sigma) = \emptyset$ for any $i \neq j$ and $\cup_N \tilde{\Theta}_i(\mathbf{v}, \sigma) = \Theta$.

Definition 20 *In the context just described, a procurement institution is a game as follows: The suppliers simultaneously select approaches $v_i \in [0, 1]$. Then the buyer selects a supplier $i \in N$. When choosing his approach, the selected supplier correctly anticipates a transfer payment $p_i(q_i, \mathbf{q}_{-i})$ conditional on being selected; the transfer is zero otherwise.*

A procurement institution is thus sufficiently general to encompass innovation contests and negotiations as discussed in this paper.

⁴²This tie breaking assumption is different from the one made in baseline model, where given equal qualities and prices, the ties are broken randomly. This is done in order to simplify the definition of sets $\tilde{\Theta}$. The tie breaking is immaterial in this case since, in the institution we will consider, all suppliers make positive profits in expectation, while the transfer when qualities are equal are zero. Hence, incentives do not change regardless of the tie breaking rule.

A.4.1 An Implementation Result

Lemma 21 is a statement on the SPE of a procurement institution as defined above.

Lemma 21 *Any procurement institution that always selects the ex-post optimal quality and uses the transfer rule*

$$t_i(q_i, \mathbf{q}_{-i}) = c \left(q_i - \max_{j \neq i} \{q_j\} \right) \quad (8)$$

if $q_i > \max_{j \neq i} \{q_j\}$ for all $j \in N$, and $t_i(q_i, \mathbf{q}_{-i}) = 0$ otherwise, (weakly) implements the social optimum for any $c > 0$.

Proof. Step 1: W has a maximum $v^* \in V^n$.

V^n is compact and W is continuous. Thus, a maximum exists.

Step 2: \mathbf{v}^* is an equilibrium of the game defined by the proposed procurement institution.

Suppose not. Then, there exists $v_i \in V$ such that

$$\begin{aligned} \Pi_i(v_i | \mathbf{v}_{-i}^*) &= \int_V \int_{\tilde{\Theta}_i(v_i, \mathbf{v}_{-i}^*, \sigma)} c \left(q_i(v_i | \theta, \sigma) - \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} \right) dF_\theta dF_\sigma > \\ &\int_V \int_{\tilde{\Theta}_i(v_i^*, \mathbf{v}_{-i}^*, \sigma)} c \left(q_i(v_i^* | \theta, \sigma) - \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} \right) dF_\theta dF_\sigma = \Pi_i(v_i^* | \mathbf{v}_{-i}^*). \end{aligned} \quad (9)$$

Multiply by $1/c$ and add the maximum expected quality of suppliers other than i to both sides:

$$\begin{aligned} &\int_V \int_{\tilde{\Theta}_i(v_i, \mathbf{v}_{-i}^*, \sigma)} q_i(v_i | \theta, \sigma) - \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} dF_\theta dF_\sigma \\ &+ \int_V \int_{\Theta} \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} dF_\theta dF_\sigma > \int_V \int_{\tilde{\Theta}_i(v_i^*, \mathbf{v}_{-i}^*, \sigma)} q_i(v_i^* | \theta, \sigma) \\ &- \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} dF_\theta dF_\sigma + \int_V \int_{\Theta} \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} dF_\theta dF_\sigma. \end{aligned}$$

We can choose to split the set Θ in a convenient way:

$$\begin{aligned} &\int_V \int_{\tilde{\Theta}_i(v_i, \mathbf{v}_{-i}^*, \sigma)} q_i(v_i | \theta, \sigma) - \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} dF_\theta dF_\sigma + \\ &\int_V \int_{\tilde{\Theta}_i(v_i, \mathbf{v}_{-i}^*, \sigma)} \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} dF_\theta dF_\sigma + \\ &\int_V \int_{\tilde{\Theta}_{-i}(v_i, \mathbf{v}_{-i}^*, \sigma)} \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} dF_\theta dF_\sigma > \\ &\int_V \int_{\tilde{\Theta}_i(v_i^*, \mathbf{v}_{-i}^*, \sigma)} q_i(v_i^* | \theta, \sigma) - \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} dF_\theta dF_\sigma + \\ &\int_V \int_{\tilde{\Theta}_i(v_i^*, \mathbf{v}_{-i}^*, \sigma)} \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} dF_\theta dF_\sigma + \\ &\int_V \int_{\tilde{\Theta}_{-i}(v_i^*, \mathbf{v}_{-i}^*, \sigma)} \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} dF_\theta dF_\sigma. \end{aligned}$$

Simplifying, we obtain:

$$\begin{aligned}
& \int_V \int_{\tilde{\Theta}_i(v_i, \mathbf{v}_{-i}^*, \sigma)} q_i(v_i | \theta, \sigma) dF_\theta dF_\sigma + \\
& \int_V \int_{\tilde{\Theta}_{-i}(v_i, \mathbf{v}_{-i}^*, \sigma)} \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} dF_\theta dF_\sigma > \\
& \int_V \int_{\tilde{\Theta}_i(v_i^*, \mathbf{v}_{-i}^*, \sigma)} q_i(v_i^* | \theta, \sigma) dF_\theta dF_\sigma + \\
& \int_V \int_{\tilde{\Theta}_{-i}(v_i^*, \mathbf{v}_{-i}^*, \sigma)} \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} dF_\theta dF_\sigma.
\end{aligned}$$

By definition of $\tilde{\Theta}$ we have:

$$\begin{aligned}
& \int_V \int_{\Theta} \max \left\{ q_i(v_i | \theta, \sigma), \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} \right\} dF_\theta dF_\sigma > \\
& \int_V \int_{\Theta} \max \left\{ q_i(v_i^* | \theta, \sigma), \max_{j \neq i} \{q_j(v_j^* | \theta, \sigma)\} \right\} dF_\theta dF_\sigma
\end{aligned} \tag{10}$$

Thus $W(v_i, \mathbf{v}_{-i}^*) > W(v_i^*, \mathbf{v}_{-i}^*)$, contradicting the optimality of \mathbf{v}^* ■

B Supplementary Material for Section 4

We now provide the details of Proposition 10, which states that the optimal contest in the baseline model is an augmented FPT. Moreover, we supply the calculations for the case of the uniform distribution (Corollary 11).

B.0.2 Proof of Proposition 10

The statement of the proposition follows from two lemmas. First we show that approaches maximizing buyer's payoffs can be implemented with an augmented FPT. Next, we show that an appropriately chosen augmented FPT does so at the lowest expected cost for the buyer, hence maximizing the buyer payoff.

Lemma 22 *If (v_1^B, v_2^B, p_1, p_2) is an innovation contest equilibrium that maximizes the buyer's expected payoffs, then $0 < v_1^B \leq \frac{1}{2} \leq v_2^B < 1$.*

We prove this lemma in two steps.

Step 1: *If $(v_1^B, v_2^B, p_1^B, p_2^B)$ is a contest equilibrium maximizing buyer's expected payoff where w.l.o.g. $v_1^B \leq v_2^B$, then $v_1^B \leq 1/2 \leq v_2^B$.*

Proof: We will show that $v_1 \leq 1/2 \leq v_2$ must hold in any contest equilibrium. Suppose that $v_1 \leq v_2 < 1/2$. The case that $1/2 < v_1 \leq v_2$ follows analogously. Let p_1, p_2 be the associated pricing strategies. Then, the expected profit of supplier 1 is $\Pi_1(v_1, v_2) = \int_0^{\frac{v_1+v_2}{2}} p_1(q_1(\sigma), q_2(\sigma)) dF(\sigma)$. Consider the deviation $v_1' = 2v_2 - v_1 < 1$ with the same pricing function. Supplier 1 now wins whenever $\sigma > (v_2 + v_1')/2$. We can write the expected profit as $\Pi_1(v_1', v_2) = \int_{\frac{v_1'+v_2}{2}}^{2v_2} p_1(q_1(\sigma), q_2(\sigma)) dF(\sigma) + \int_{2v_2}^1 p_1(q_1(\sigma), q_2(\sigma)) d\sigma$. Clearly,

$(v_1 + v_2)/2 = 2v_2 - \frac{v'_1 + v_2}{2}$. Moreover, for all $\sigma' \in [0, (v_1 + v_2)/2]$, there exists a unique $\sigma'' \in [(v'_1 + v_2)/2, 2v_2]$ such that $q(v_1, \sigma') - q(v_2, \sigma') = q(v_1, \sigma'') - q(v_2, \sigma'')$ and $f(\sigma') \leq f(\sigma'')$.

Thus $\int_0^{\frac{v_1 + v_2}{2}} p_1(q_1(\sigma), q_2(\sigma)) dF(\sigma) \leq \int_{\frac{v'_1 + v_2}{2}}^{2v_2} p_1(q_1(\sigma), q_2(\sigma)) dF(\sigma)$. Hence, $\Pi_1(v_1, v_2) \leq \Pi_1(v'_1, v_2)$ and v'_1 leads to strictly higher probability of winning, hence v'_1 is a profitable deviation.⁴³ Thus, $v_1 \leq 1/2 \leq v_2$ must hold in any equilibrium; in particular, therefore $v_1^B \leq 1/2 \leq v_2^B$.

Step 2: If $(v_1^B, v_2^B, p_1^B, p_2^B)$ is a contest equilibrium maximizing the buyer's payoff, then $0 < v_i^B < 1$ for $i \in \{1, 2\}$.

Proof: By Step 1, we know that $v_1^B \leq 1/2 \leq v_2^B$. Suppose $v_1^B = 0$ and $v_2^B = 1$. By single-peakedness (A1), $v_1 = v_2 = 1/2$ results in weakly higher social welfare than (v_1^B, v_2^B) . As this outcome can be implemented with $A = 0$, the buyer would be strictly better off in an FPT with $A = 0$ than in any contest implementing $v_1^B = 0$ and $v_2^B = 1$ where the suppliers earn positive profits. Finally, observe that $v_1^B = 0$ and $v_2^B = 1$ cannot be implemented so that the suppliers earn zero profits, as the suppliers could increase their probability of winning by deviating to the interior, which by (T2) would be a profitable deviation.

Next, suppose $v_1^B = 0$ and $v_2^B < 1$ (the case that $v_1^B > 0$ and $v_2^B = 1$ follows analogously). Since $dW(v_1^B, v_2^B)/dv_1^B|_{v_1^B=0} = bF((v_1 + v_2)/2) > 0$, there exists some $\bar{\varepsilon} > 0$ such that $W(\varepsilon, v_2^B) > W(0, v_2^B)$ for every $\varepsilon \in (0, \bar{\varepsilon})$. If supplier 1 earns $\Pi_1(0, v_2^B, p_1^B, p_2^B) = 0$ in the contest implementing $v_1^B = 0$ and $v_2^B < 1$, then he could increase his probability of winning by deviating to the interior, which by (T2) would be a profitable deviation. Therefore $\Pi_1(0, v_2^B, p_1^B, p_2^B) > 0$. Hence there exists $\varepsilon \in (0, \bar{\varepsilon})$ such that $F(\varepsilon)\Delta q(\varepsilon, v_2^B) \leq \Pi_1(0, v_2^B, p_1^B, p_2^B)$. Thus, an augmented FPT with $P = \{\Delta q(\varepsilon, v_2^B), 0\}$ implements $v_1 = \varepsilon$, $v_2 = v_2^B$, thereby increasing social welfare and decreases total transfers (transfers to supplier 2 decrease by Lemma 2). Therefore, this augmented FPT increases the buyer's payoff, which is a contradiction.

Lemma 23 If $(v_1^B, v_2^B, p_1^B, p_2^B)$ is an equilibrium of a contest maximizing the buyer's payoff, then it can be implemented by a contest with $P = \{A, 0\}$.

Proof: From Proposition 9 and the lemma above, we know that the augmented FPT with $A = \Delta q(v_1^B, v_2^B)$ implements (v_1^B, v_2^B) . It remains to be shown that the buyer cannot implement (v_1^B, v_2^B) with lower prices with any other contest. To see this, note that by Lemmas 2 and 6, in any contest that implements (v_1^B, v_2^B) the price paid by the buyer is exactly $\Delta q(v_1^B, v_2^B) + \underline{P}$ if $\sigma \in [0, v_1^B] \cup [v_2^B, 1]$, and the transfer is at least 0 if $\sigma \in (v_1^B, v_2^B)$. On the other hand, in the augmented FTP with $A = \Delta q(v_1^B, v_2^B)$, the price paid by the buyer is exactly $\Delta q(v_1^B, v_2^B)$ if $\sigma \in [0, v_1^B] \cup [v_2^B, 1]$, and the price is 0 if $\sigma \in (v_1^B, v_2^B)$. Therefore, no other design contest that implements (v_1^B, v_2^B) can do so with lower expected prices than the augmented FTP with $A = \Delta q(v_1^B, v_2^B)$.

B.0.3 Proof of Corollary 11

Suppose not. Then, by Proposition 10, there exists an $A > 0$ such that $\mathcal{P} = \{A, 0\}$ induces an equilibrium $v_1 < v_2$ for which the buyer payoff is greater than in the FPT with equilibrium $v_1^T = v_2^T = 1/2$ and $A = 0$.

⁴³Given the tie-breaking rule T2, this is even true for $p = 0$.

Step 1: The total surplus for $v'_1 = v'_2 = v' \equiv (v_1 + v_2)/2$ is weakly greater than the buyer surplus in the outcome (v_1, v_2) of the augmented contest with $\mathcal{P} = \{A, 0\}$.

Proof. For any $\sigma \in [0, v_1] \cup [v_2, 1]$, the buyer surplus in the contest with $\mathcal{P} = \{A, 0\}$ is $\min\{q_1(v_1, \sigma), q_2(v_2, \sigma)\}$ while the total surplus for $v'_1 = v'_2$ is given by $q(v', \sigma)$.⁴⁴ For any $\sigma \in [0, v_1]$, $q(v_1, \sigma) > q(v', \sigma) > q_2(v_2, \sigma)$. For any $\sigma \in [v_2, 1]$, $q(v_1, \sigma) < q(v', \sigma) < q(v_2, \sigma)$. Thus for any $\sigma \in [0, v_1] \cup [v_2, 1]$ the total surplus for $v'_1 = v'_2$ is strictly greater than the buyer surplus in the outcome (v_1, v_2) of the augmented contest. It thus suffices to show that the expected buyer surplus from states $\sigma \in (v_1, v_2)$ with $\mathcal{P} = \{A, 0\}$ equals the expected total surplus from these states for (v'_1, v'_2) with $v'_1 = v'_2$. The expected buyer surplus from states $\sigma \in (v_1, v')$ is given by $\int_{v_1}^{v'} q(v_1, \sigma) d\sigma$ with $\mathcal{P} = \{A, 0\}$; and the expected total surplus is given by $\int_{v_1}^{v'} q(v', \sigma) d\sigma$ for (v'_1, v'_2) with $v'_1 = v'_2$. Hence, $\int_{v_1}^{v'} q(v_1, \sigma) d\sigma = \int_{v_1}^{v'} q(v', \sigma) d\sigma$. By a similar argument, the expected buyer surplus from states $\sigma \in (v', v_2)$ for $\mathcal{P} = \{A, 0\}$ is identical to the expected total surplus from these states when a (v'_1, v'_2) with $v'_1 = v'_2$ is implemented. ■

Step 2: The buyer surplus in the FPT with $v_1^T = v_2^T = 1/2$ is (weakly) greater than the total surplus that can be obtained by a (v'_1, v'_2) with $v'_1 = v'_2 = v' \equiv (v_1 + v_2)/2$.

Proof. In an FPT with $A = 0$, buyer surplus and total surplus are equal. If $v' = \frac{1}{2}$, the result holds with equality. If $v' \neq \frac{1}{2}$, the inequality is strict, since the total surplus, assuming $v'_1 = v'_2$, is maximized for $v'_1 = v'_2 = 1/2$. ■

C Supplementary Material for Section 5

The maximization problem for the buyer is to choose $(v_1, v_2, p_1, p_2, \mathcal{P}, t) \in [0, 1]^2 \times \mathcal{P}^{[\Psi-b, \Psi]^2} \times \mathcal{I}(\mathbb{R}^+) \times (-\infty, T]$ so as to maximize

$$W(v_1, v_2) - \Pi_1(v_1, v_2, p_1, p_2) - \Pi_2(v_1, v_2, p_1, p_2) + 2t$$

such that, for all $i \in \{1, 2\}$ and $j \neq i$, (D1)-(D3) hold and

$$\Pi_i(v_i, v_j, p_i, p_j) - t - C \geq 0 \text{ for all } i, j \in \{1, 2\} \text{ and } i \neq j. \quad (11)$$

C.0.4 Proof of Proposition 12

We will prove the two cases in turn.

(i) Suppose $T \geq F(v_1^*) \Delta q(v_1^*, v_2^*) - C$. From Proposition 10 we know that for the proposed $\mathcal{P} = \{\bar{p}, 0\}$, (v_1^*, v_2^*) emerges in equilibrium, with the pricing strategies p_1 and p_2 given by Proposition 10. The buyer's payoff in the proposed equilibrium, net of uncontingent transfers, is

$$\begin{aligned} & W(v_1^*, v_2^*) - \Pi_1(v_1^*, v_2^*, p_1, p_2) - \Pi_2(v_1^*, v_2^*, p_1, p_2) + 2t \\ &= W(v_1^*, v_2^*) - 2F(v_1^*) \Delta q(v_1^*, v_2^*) + 2(F(v_1^*) \Delta q(v_1^*, v_2^*) - C) = W(v_1^*, v_2^*) - 2C \end{aligned} \quad (12)$$

This is the highest payoff that the buyer can achieve without violating the suppliers' participation constraints.

⁴⁴It is not necessary for this statement that (v'_1, v'_2) can actually be implemented by an innovation contest.

(ii) Suppose $T < F(v_1^*) \Delta q(v_1^*, v_2^*) - C$. The proof for this case relies on the fact that implementation with minimal expected contingent payments uses augmented tournaments. It shows that (v_1^B, v_2^B) must satisfy $v_1^B + v_2^B = 1$. Among all the augmented tournaments implementing (v_1, v_2) with $v_1 \leq v_2$ and $v_1 + v_2 = 1$, the buyer has highest expected payoffs at $(1/2, 1/2)$, whereas the supplier payoffs are single-peaked at $(v_1, v_2) = (v_1^*, v_2^*)$, the welfare optimum. Using these facts, the proof shows that the buyer always chooses the minimal value of the non-contingent transfer t , and she just implements enough diversity so that the suppliers (who benefit from some diversity) break even on expectation.

Step 1: *The outcome of an optimal contest can be implemented by $\mathcal{P} = \{p, 0\}$ for some $p \geq 0$.*

First, $v_1 \leq v_2 < 1/2$ cannot hold in any contest equilibrium. This follows directly from (ii) Step 1 in the proof of Proposition 10.

Second, if the contest maximizes the buyer's payoff, then $0 < v_i^B < 1$ for $i \in \{1, 2\}$. By an argument as in Step (ii) of the proof of Proposition 10, for any contest implementing $v_i^B \in \{0, 1\}$, there always exists an augmented FPT which weakly increases the total surplus and strictly increases the buyer's surplus. Thus, the buyer can use this FPT with suitable unconditional transfers to make the suppliers equally well off as in any contest implementing $v_i^B \in \{0, 1\}$, while increasing own payoff.

Thus, the optimal outcome for the buyer satisfies $0 < v_1^B \leq 1/2 \leq v_2^B < 1$. By Proposition 10, there exists $P = \{p, 0\}$ which maximizes buyer payoff conditional on implementing v_1^B and v_2^B . As the buyer can always transfer part of her own payoff to the suppliers via the unconditional transfers, any feasible combination of payoffs of the buyer and suppliers which can be obtained in a contest that implements v_1^B and v_2^B can also be obtained in an augmented FPT with appropriately chosen unconditional transfer t .

Step 2: *In an optimal contest $v_1^B + v_2^B = 1$.*

Consider any (v_1, v_2) such that $v_1 + v_2 < 1$. We show that $(v_1, v_2) \neq (v_1^B, v_2^B)$; the case $v_1 + v_2 > 1$ follows analogously. By Step 1, the optimal outcome can be implemented by some $\mathcal{P} = \{p, 0\}$. The equilibrium values of p_i are zero whenever $\sigma \in (v_1, v_2)$. Hence, the participation constraint for supplier 1 implies that $F(v_1)p - t \geq C$; thus $v_1 + v_2 < 1$ implies $(1 - F(v_2))p - t > C$. Now suppose the buyer implements $(v_1 + \varepsilon, v_2 + \varepsilon)$, where ε is sufficiently small. We know that $(v_1 + \varepsilon, v_2 + \varepsilon)$ can also be implemented with $P = \{p, 0\}$. Thus, we can write the buyer's expected payoffs as

$$\Pi_b(\varepsilon) = W(v_1 + \varepsilon, v_2 + \varepsilon) - F(v_1 + \varepsilon)p - (1 - F(v_2 + \varepsilon))p + 2t$$

for $\varepsilon \geq 0$. Thus

$$\frac{d\Pi_b(\varepsilon)}{d\varepsilon} = dW(v_1 + \varepsilon, v_2 + \varepsilon)/d\varepsilon - p + p = dW(v_1 + \varepsilon, v_2 + \varepsilon)/d\varepsilon.$$

Maximizing social welfare is equivalent to minimizing the expected distance

$$\begin{aligned} D(v_1 + \varepsilon, v_2 + \varepsilon) &= \int_0^{v_1 + \varepsilon} (v_1 + \varepsilon - \sigma) d\sigma + \int_{v_1 + \varepsilon}^{\frac{v_1 + v_2}{2} + \varepsilon} (\sigma - v_1 - \varepsilon) d\sigma \\ &\quad + \int_{\frac{v_1 + v_2}{2} + \varepsilon}^{v_2 + \varepsilon} (v_2 + \varepsilon - \sigma) d\sigma + \int_{v_2 + \varepsilon}^1 (\sigma - v_2 - \varepsilon) d\sigma. \end{aligned}$$

The first order condition is

$$\begin{aligned}\frac{dD(v_1 + \varepsilon, v_2 + \varepsilon)}{d\varepsilon} &= \int_0^{v_1 + \varepsilon} d\sigma - \int_{v_1 + \varepsilon}^{\frac{v_1 + v_2}{2} + \varepsilon} d\sigma + \int_{\frac{v_1 + v_2}{2} + \varepsilon}^{v_2 + \varepsilon} d\sigma - \int_{v_2 + \varepsilon}^1 d\sigma \\ &= F(v_1 + \varepsilon) - (1 - F(v_2 + \varepsilon)) = v_1 + v_2 - 1 + 2\varepsilon,\end{aligned}$$

which is always negative for small enough ε . Thus $W(v_1 + \varepsilon, v_2 + \varepsilon)$ increases in ε for sufficiently small ε , while the total expected supplier profits and transfers are independent of ε . If the participation constraints are satisfied, then expected buyer profits are thus strictly increasing in ε . Finally, the participation constraints hold, because $F(v_1 + \varepsilon)p - t \geq F(v_1)p - t$ and there exists ε small enough such that $(1 - F(v_2 + \varepsilon))p - t \geq C$.

Step 3: For the innovation contest inducing an equilibrium (v_1, v_2) such that $v_1 + v_2 = 1$ with the highest expected payoffs for the buyer, supplier profits are increasing in v_1 for $v_1 \in (0, v_1^*)$ and decreasing for $v_1 \in (v_1^*, 1/2)$.

For such a contest, the successful supplier earns $p = \Delta q(v_1, v_2) = b(1 - 2v_1)$ for $\sigma \in [0, v_1^*]$ and 0 otherwise. Thus, the expected profit of supplier 1 in such a contest is $\Pi_1(v_1, 1 - v_1, p_1, p_2) = bv_1(1 - 2v_1)$. Since $v_1^* = 1/4$, the result follows.

Step 4: For $v_2 = 1 - v_1$ and $\varepsilon \geq 0$, the expected buyer payoff when $(v_1 + \varepsilon, v_2 - \varepsilon)$ is implemented with an augmented FPT, $\Pi_b(v_1 + \varepsilon, v_2 - \varepsilon, p_1, p_2)$, increases in ε .

For any $\sigma \in [0, v_1 + \varepsilon]$, the buyer's payoff equals $q_2(v_2 - \varepsilon, \sigma)$, which increases in ε . For any $\sigma \in (v_1 + \varepsilon, 1/2]$, the buyer's payoff equals $q_1(v_1 + \varepsilon, \sigma)$ which also increases in ε . By similar arguments, the buyer's payoff increases for any $\sigma \in (1/2, v_2 - \varepsilon]$ and any $\sigma \in (v_2 - \varepsilon, 1]$.

Step 5: $v_1^B \geq v_1^*$.

Consider (v_1, v_2) such that $v_1 < v_1^*$ and $v_2 = 1 - v_1$. By Step 4, moving from (v_1, v_2) to (v_1^*, v_2^*) the buyer could increase her payoff while by Step 3 the participation constraint would remain satisfied.

Step 6: For $v_2 = 1 - v_1$, $v_1 > v_1^*$ and $\varepsilon \geq 0$, $W(v_1 + \varepsilon, v_2 - \varepsilon)$ decreases in ε .

Using the same argument as in Step 2, the derivative of the expected distance is

$$\begin{aligned}\frac{dD(v_1 + \varepsilon, v_2 - \varepsilon)}{d\varepsilon} &= \int_0^{v_1 + \varepsilon} d\sigma - \int_{v_1 + \varepsilon}^{\frac{v_1 + v_2}{2}} d\sigma - \int_{\frac{v_1 + v_2}{2}}^{v_2 - \varepsilon} d\sigma + \int_{v_2 - \varepsilon}^1 d\sigma \\ &= 1 - 2(F(v_2 - \varepsilon) - F(v_1 + \varepsilon)) > 0\end{aligned}$$

Since the derivative is positive, the total expected distance increases in ε , and the total expected surplus decreases.

Step 7: It is optimal for the buyer to set $t = T$.

Suppose that a buyer implements (v_1, v_2) and $t < T$. By implementing $(v_1 - \varepsilon, 1 - v_2 + \varepsilon)$, she could increase total surplus by Step 6. By choosing a suitable $t' > t$, the payoffs of the suppliers would remain the same. Thus, the objective function of the buyer would strictly increase.

Step 8: The optimal (v_1^B, v_2^B) for the buyer satisfies $T = F(v_1^B) \Delta q(v_1^B, v_2^B) - C$ and $v_2^B = 1 - v_1^B$.

By Step 7, t is fixed at $t = T$. In the augmented tournament which implements any $(v_1, 1 - v_1)$ at minimal costs for the buyer, the supplier earns $F(v_1) \Delta q(v_1, 1 - v_1)$. Thus, for any $v_1' > v_1 \geq v_1^*$, the participation constraint is not met if $(v_1', 1 - v_1')$ is implemented. For

any $v_1' < v_1$, the expected buyer payoffs decrease by Step 4 for fixed t , and since t is already at the lower bound, the objective function decreases.

Step 9: $\mathcal{P} = \{\bar{p}, 0\}$ where $\bar{p} = \Delta q(v_1^B, 1 - v_1^B)$.

By Step 1, this contest implements $(v_1^B, 1 - v_1^B)$ given in Step 8. Using the arguments in Proposition 10, it does so at the lowest possible expected costs to the buyer.

D Supplementary Material for Section 6

In this section, we provide precise statements and proofs of the results summarized in Sections 6.1-6.4 and 6.6. For the details of the numerical analysis in Section 6.5, we refer the reader to Supplementary Material for Section VI.E, available at <https://sites.google.com/site/iletina/did>.

D.1 Proofs from Subsection 6.1

We first provide results on the implication of different mechanisms in the n-player setting. All proofs follow below.

Proposition 24 *Suppose there are $n > 3$ suppliers and (A1)', (A2) and (A3) hold.*

(i) *The auction mechanism implements the social optimum.*

(ii) *In any equilibrium $\mathbf{v}^T = (v_1^T, \dots, v_n^T)$ of an FPT, there is duplication of approaches, and the maximal diversity is smaller than in the social optimum, that is,*

$$\max_{i \in N} v_i^T - \min_{j \in N} v_j^T < \max_{i \in N} v_i^* - \min_{j \in N} v_j^*.$$

(iii) *For any number of active approaches in an FPT, there exists an equilibrium inducing this number of active approaches which the buyer prefers to the socially optimal equilibrium induced by an auction.*

D.1.1 Proof of Lemma 13(i)

By Proposition 24 (i), $v_i^* \neq v_j^*$ for all $i \neq j \in \{1, \dots, n\}$. Thus

$$W(\mathbf{v}) = \int_0^{\frac{v_1+v_2}{2}} q_1(v_1, \sigma) d\sigma + \sum_{k=2}^{n-1} \int_{\frac{v_{k-1}+v_k}{2}}^{\frac{v_k+v_{k+1}}{2}} q_k(v_k, \sigma) d\sigma + \int_{\frac{v_{n-1}+v_n}{2}}^1 q_n(v_n, \sigma) d\sigma$$

The maximum of this function exists and it obviously does not involve corner solutions. Hence, it is given by first order conditions. The first order conditions are given by

$$\frac{\partial W(\mathbf{v})}{\partial v_1} = -bv_1 + b\frac{v_2 - v_1}{2} = 0 \quad (13)$$

$$\frac{\partial W(\mathbf{v})}{\partial v_k} = -b\frac{v_k - v_{k-1}}{2} + b\frac{v_{k+1} - v_k}{2} = 0 \quad (14)$$

for $k \in \{2, \dots, n-1\}$

$$\frac{\partial W(\mathbf{v})}{\partial v_n} = -b\frac{v_n - v_{n-1}}{2} + b(1 - v_n) = 0 \quad (15)$$

(14) can be rearranged to give $v_k - v_{k-1} = v_{k+1} - v_k \equiv \Delta^v$ for $k = 2, \dots, n-1$. (13) and (15) give $v_1 = 1 - v_n = \Delta^v/2$. Inserting these equations into $v_1 + (v_2 - v_1) + \dots + (v_n - v_{n-1}) + (1 - v_n) = 1$ gives $\Delta^v = \frac{1}{n}$. Thus, $v_1 = \frac{1}{2n}$ and $v_k = \frac{1}{2n} + \frac{k-1}{n} = \frac{2k-1}{2n}$ for $k \in \{2, \dots, n\}$.

D.1.2 Proof of Lemma 13(ii)

Step 1: In any equilibrium of the FPT, $v_1 = v_2$ and $v_{n-1} = v_n$. This implies that there are at most $\bar{k} = n - 2$ active approaches.

Suppose $v_1 < v_2$. Then the expected profit of supplier 1 is $A \frac{v_1 + v_2}{2}$. For $v'_1 = v_1 + \varepsilon$, $\varepsilon > 0$, such that $v'_1 < v_2$, the expected profit is $A \frac{v'_1 + v_2}{2} > A \frac{v_1 + v_2}{2}$. A similar argument holds for $v_{n-1} < v_n$.

Step 2: No approach is developed by more than two suppliers in any equilibrium of the FPT. This implies there are at least $\underline{k} \geq n/2$ active approaches.

Suppose there are m suppliers developing the same approach. The expected profit of each of those suppliers is $\left(\tilde{P}/m\right) A$, for some probability \tilde{P} . However, a sufficiently small deviation in at least one direction for any of those suppliers yields a payoff of at least $\left(\tilde{P}/2\right) A > \left(\tilde{P}/m\right) A$ for any $m \geq 3$.

Step 3: Suppose n is even and $k = n/2$. Then any choice of r_1, \dots, r_k as in Lemma 13 is an equilibrium.

In the suggested equilibria, the active approaches are equidistant, and, by Step 2, two suppliers supply each approach. Also, $r_1 = 1/n$ and $r_{n/2} = 1 - 1/n$. For any $1 < m < n/2$, $r_m - r_{m-1} = 2/n$. Simple calculations show that any of the active approaches offers the highest quality with probability $2/n$ so that each supplier has a payoff of $\Pi_i = A/n$. Deviating to any other active approach leads to payoff of $2A/3n$; hence it is not profitable. A deviation to $[0, r_1)$ or $(r_{n/2}, 1]$ results in a winning probability strictly lower than $1/n$, so this is not a profitable deviation either. Finally, consider a deviation to $v \in (r_{m-1}, r_m)$, $m \in \{2, \dots, n/2\}$. The deviating supplier wins if and only if σ is in the set $\left[\frac{v+r_{m-1}}{2}, \frac{v+r_m}{2}\right]$, so that the winning probability is $1/n$ and this is also not a profitable deviation.

Step 4: Now let $k > n/2$. Then any choice of r_1, \dots, r_k as in Lemma 13 is an equilibrium.

Arguing as in Step 3, any of the active approaches offers the highest quality with probability $1/k$, so that if there are two suppliers using an approach, each of them wins with probability $1/2k$, and if there is only one supplier using this approach, he wins with probability $1/k$. Consider a supplier who wins with probability $1/2k$. By the same argument as in Step 3, if he deviates to $[0, r_1)$ or $(r_k, 1]$, he wins with probability strictly lower than $1/2k$. Deviating to any approach in some interval (r_l, r_{l+1}) ; $l \in \{1, \dots, k-1\}$, he wins with probability $1/2k$; hence such a deviation is not profitable either. If he deviates to any active approach, he wins with a probability of at most $1/2k$. Thus, such suppliers do not have profitable deviations. Finally consider a deviation by a supplier who is the only one to develop some r_m , where $1 < m < k$. Any deviation to $[0, r_{m-1})$ or $[r_{m+1}, 1]$ leads to strictly lower payoffs, by the same argument as above. For any approach $v \in (r_{m-1}, r_{m+1})$, he wins whenever $\sigma \in \left[\frac{v+r_{m-1}}{2}, \frac{v+r_{m+1}}{2}\right]$, so that the winning probability is $\frac{v+r_{m+1}}{2} - \frac{v+r_{m-1}}{2} = \frac{r_{m+1}-r_{m-1}}{2} = 1/k$. Hence, this is not a profitable deviation either.

D.1.3 Proof of Proposition 24

(i) This follows as a corollary to Lemma 21 below.

(ii) Let $\mathbf{v} = [v_1, \dots, v_n]$ be the vector of approaches, ordered so that $v_1 \leq \dots \leq v_n$. For the first claim, by Lemma 13 there is duplication in any FPT. We prove the second claim in several steps. For any supplier i , let $P_{\sigma < v_i}^i$ ($P_{\sigma > v_i}^i$) be the probability that supplier i wins and,

in addition, $\sigma < v_i$ ($\sigma > v_i$). Let $P^i = P_{\sigma < v_i}^i + P_{\sigma > v_i}^i$ be the total probability that supplier i wins.

Step 1: If suppliers i and j are both using the same approaches or the same as some supplier $k \neq i, j$, then $P_{\sigma < v_i}^i = P_{\sigma > v_i}^i = P_{\sigma < v_j}^j = P_{\sigma > v_j}^j$ in any equilibrium.

Suppose first that $P_{\sigma < v_i}^i \neq P_{\sigma > v_i}^i$ for some supplier i using the same approach as another one. Suppose that $P_{\sigma < v_i}^i > P_{\sigma > v_i}^i$ (the opposite case is analogous). Then, a deviation to $v_i - \varepsilon$ for some sufficiently small $\varepsilon > 0$ leads to a winning probability of $2P_{\sigma < v_i}^i > P_{\sigma < v_i}^i + P_{\sigma > v_i}^i$,⁴⁵ which is a profitable deviation. Next, suppose that $P_{\sigma > v_i}^i < P_{\sigma < v_j}^j$ (the opposite case is analogous). Then, a deviation of player i to $v_j - \varepsilon$ for sufficiently small $\varepsilon > 0$ leads to a winning probability of $2P_{\sigma < v_j}^j > P_{\sigma < v_i}^i + P_{\sigma > v_i}^i$,⁴⁶ which is a profitable deviation.

Step 2: In any equilibrium of an FPT with n suppliers, $P^1 = P^2 = P^{n-1} = P^n \geq \frac{1}{2(n-2)}$.

By Lemma 13, all extreme approaches are duplicate. The three equalities thus follow from Step 1. Suppose that the inequality does not hold. Then $P^1 + P^2 + P^{n-1} + P^n < \frac{2}{n-2}$ which in turn implies that $\sum_{j=3}^{n-2} P^j \geq \frac{n-4}{n-2}$. But then there exist at least one $k \in \{3, \dots, n-2\}$ such that $P^k \geq \frac{1}{n-2}$. A deviation by any supplier $1, 2, n-1$ or n to v_k would give him the probability of winning of at least $\frac{1}{2(n-2)}$, which would be a profitable deviation.

Step 3: Any equilibrium of an FPT with n suppliers satisfies $\max_i v_i^T - \min_i v_i^T \leq \frac{n-3}{n-2}$.

Suppose not. As $\frac{2(n-2)-1}{2(n-2)} - \frac{2-1}{2(n-2)} = \frac{n-3}{n-2}$, there exists an equilibrium of an FPT such that either $\max_i v_i^T > \frac{2(n-2)-1}{2(n-2)}$ or $\min_i v_i^T < \frac{2-1}{2(n-2)}$ or both. If $\max_i v_i^T > \frac{2(n-2)-1}{2(n-2)}$, then Step 1 implies $P^n < \frac{1}{2(n-2)}$, which is impossible by Step 2. If $\min_i v_i^T < \frac{2-1}{2(n-2)}$, then $P^1 < \frac{1}{2(n-2)}$ by Step 1, which is again impossible by Step 2.

Step 4: The socially optimal diversity is greater than $\frac{n-3}{n-2}$.

This follows directly from Lemma 13.

Step 3 and Step 4 immediately imply the result.

(iii) Any number of active approaches $k \in \{\underline{k}, \dots, \bar{k}\}$ can be implemented in an equilibrium of an FPT where active approaches are equidistant, given as in Lemma 13(ii). Since the buyer payoff in an FPT is the best available quality, her payoff in these equilibria decreases as the number of active approaches decreases. We will show that even in an equilibrium of an FPT satisfying the condition in Lemma 13(ii) with the minimum number of active approaches, the buyer has higher expected payoff than in an auction which implements the socially optimal outcome. The buyer's payoff in an equilibrium of an FPT satisfying the condition in Lemma 13(ii) when n is odd and $k = (n+1)/2$ is the same as if $n' = n+1$ and $k' = k = n'/2$. Moreover, the buyer's payoff is strictly greater in an auction with n' suppliers than in an auction with n suppliers. Hence, if the buyer prefers the FPT equilibrium to the auction equilibrium with (even) n' , she will also prefer FPT equilibrium to the auction equilibrium with (odd) n . Thus, it is sufficient to look at n even and $k = n/2$.

Lemma 13 implies $v_k^* = (2k-1)/2n$ for $k \in \{1, \dots, n\}$, and Lemma 13 implies $r_m = (2m-1)/n$ for $m \in \{1, \dots, n/2\}$. First suppose $\sigma \leq r_1$. Since $\frac{v_1^* + v_2^*}{2} = \frac{1}{n}$, the highest quality approach in the auction is v_1^* . Arguing as in Lemma 2 for the case of two suppliers, the equilibrium price corresponds to the difference between the highest and the second-highest quality. The net payoff of the buyer for $\sigma \leq r_1$ is thus $q(v_2^*, \sigma) < q(r_1, \sigma)$ which is the same

⁴⁵The winning probability is approximately $2P_{\sigma < v_i}^i$ if $v_i = \min\{v_1, \dots, v_n\}$.

⁴⁶The winning probability is approximately $2P_{\sigma < v_j}^j$ if $v_j = \min\{v_1, \dots, v_n\}$.

as the payoff in an FPT. Similar arguments holds for $\sigma \geq r_k$. Now suppose $\sigma \in (r_1, r_k)$. Let the highest available quality in the auction correspond to v_i^* . Then $\sigma \in \left(\frac{v_{i-1}^*+v_i^*}{2}, \frac{v_i^*+v_{i+1}^*}{2}\right)$. We distinguish two cases.

Case 1: l is even. In the FPT, the highest quality corresponds to $\frac{v_{l-1}^*+v_l^*}{2} = \frac{l-1}{n} = r_{l/2}$. If $\sigma \in \left(\frac{v_{l-1}^*+v_l^*}{2}, v_l^*\right)$, the buyer payoff in the auction is provided by v_{l-1}^* . Since $(\sigma - r_{l/2}) - (\sigma - v_{l-1}^*) = -1/2n$, at each state in this interval the buyer payoff corresponds to a quality which is lower in the auction than in the FPT by $b/2n$. If $\sigma \in \left(v_l^*, \frac{v_l^*+v_{l+1}^*}{2}\right)$, the buyer payoff in the auction corresponds to a quality determined by v_{l+1}^* . Note that $v_{l+1}^* - \sigma - (\sigma - r_{l/2}) = v_{l+1}^* + r_{l/2} - 2\sigma = \frac{4l-1}{2n} - 2\sigma > -1/2n$ for all $\sigma < \frac{v_l^*+v_{l+1}^*}{2}$. Hence, the loss from the auction if $\sigma \in \left(\frac{v_{l-1}^*+v_l^*}{2}, v_l^*\right)$ is larger than the gain if $\sigma \in \left(v_l^*, \frac{v_l^*+v_{l+1}^*}{2}\right)$. Thus, the expected buyer payoff if $\sigma \in \left(\frac{v_{l-1}^*+v_l^*}{2}, \frac{v_l^*+v_{l+1}^*}{2}\right)$ is larger in the FPT than in the auction whenever l is even.

Case 2: l is odd. In the FPT, the buyer payoff corresponds to the quality of the approach $\frac{v_l^*+v_{l+1}^*}{2} = \frac{l}{n} = r_{(l+1)/2}$. Using an analogous argument as in Case 1, we can show that for all $\sigma \in \left(v_l, \frac{v_l^*+v_{l+1}^*}{2}\right)$, the buyer payoff in the FPT is $b/2n$ higher than in the auction. For all $\sigma \in \left(\frac{v_{l-1}^*+v_l^*}{2}, v_l\right)$, the auction provides higher buyer payoff, but by less than $b/2n$.

Combining Cases 1 and 2, the expected buyer payoff conditional on $\sigma \in (r_1, r_k)$ is higher in the FPT than in the auction. Thus, over the whole domain of σ , the FPT also provides higher expected buyer payoff.

D.1.4 Proof of Corollary 14

According to Lemma 13, the social optimum is given by the choices $v_k^* = (2k-1)/2n$ ($k \in \{1, \dots, n\}$). The average quality in the social optimum is thus $\Psi - b/4n$. Therefore the social surplus is $\Psi - b/4n - nC$. The maximum of this expression in \mathbb{R}^+ is $n = \sqrt{b}/2\sqrt{C}$. By concavity of the objective function, the optimal choice of $n \in \mathbb{N}$ is thus given by $n_-(C)$ or $n_+(C)$. According to Proposition 24, the social optimum for any given number of players can be implemented with an auction.

D.2 Proofs from Subsection 6.2

D.2.1 The Set-Up

We modify the set-up of Section 1 as follows.

- (H1) The ideal state $\varphi = (\sigma, \theta)$ is distributed uniformly on $\Phi = [0, 1] \times [0, 1] \subset \mathbb{R}^2$.
- (H2) Supplier i can choose an approach $s_i = (v_i, i-1)$ from $S_i = [0, 1] \times \{i-1\} \subset [0, 1] \times [0, 1]$.

Thus we can think of the set of possible approaches of each supplier as embedded into the state space, with the set of supplier 1 corresponding to the lower edge and the set of supplier 2 corresponding to the upper edge (see Figure 2). Hence, the approach of a supplier depends on its identity as well as on its active choice. When we focus on the latter, we can identify a supplier's approach with the first (horizontal) coordinate v_i . For $s_i = (v_i, i-1)$, $i = 1, 2$,

$\varphi = (\sigma, \theta)$ and suitable constants Ψ , $a \geq 0$ and $b > 0$, we specify the quality function as $q_i(v_i, \varphi) = \Psi - a|i - 1 - \theta| - b|v_i - \sigma|$. Thus

$$q_1(v_1, \varphi) = \Psi - a\theta - b|v_1 - \sigma| \quad (16)$$

$$q_2(v_2, \varphi) = \Psi - a(1 - \theta) - b|v_2 - \sigma| \quad (17)$$

Hence, for each approach $(v_i, i - 1)$, there is one state ($\varphi = (v_i, i - 1)$) under which this approach yields maximal quality, and this maximal quality is the same for all approaches, given as Ψ . Moreover, a captures the degree of exogenous differentiation between suppliers: For $v_1 = v_2 = v$, it measures the maximum possible difference in willingness-to-pay between $(v, 0)$ and $(v, 1)$. $a = 0$ corresponds to the benchmark case of homogeneous suppliers. As before, the parameter b captures the degree to which the supplier can affect quality by choosing different approaches. To emphasize the dependence of each supplier's quality on the realization of the state, we often drop v_i and write $q_i(\sigma, \theta) \equiv q_i(v_i, \sigma, \theta)$.

D.2.2 Notation and Preliminary Results

We first provide some definitions and simple results concerning states and relative qualities.

Figure 2 helps to understand the following notation. Essentially, for each $\sigma \in [0, 1]$, (i) describes the value of θ for which quality levels of both suppliers are the same and, for each $\theta \in [0, 1]$, (ii) describes the value of σ for which quality levels are the same. The slightly more complicated formulations in the notation below account for the fact that, for some values of σ (θ), there may be no value of $\theta \in [0, 1]$ ($\sigma \in [0, 1]$) for which qualities are identical, in which case an adequate boundary solution is required.

Notation: Suppose (v_1, v_2) are such that $v_1 \leq v_2$.

(i) For $\sigma \in [0, 1]$, $\tilde{\theta}(\sigma)$ is the value of θ that is minimal with the property that there exists no $\theta < \tilde{\theta}(\sigma)$ such that $q_2 > q_1$.

(ii) For $\theta \in [0, 1]$, $\tilde{\sigma}(\theta)$ is the value of σ that is minimal with the property that there exists no $\sigma < \tilde{\sigma}(\theta)$ such that $q_2 > q_1$.

In all states of the world (σ, θ) with $\theta < \tilde{\theta}(\sigma)$ or $\sigma < \tilde{\sigma}(\theta)$ buyers prefer to choose supplier 1 if they pay the same price in both cases.

Whenever $\tilde{\theta}(\sigma)$ takes a value in $(0, 1)$, then it can also be defined by the simple requirement that $q_1(\sigma, \tilde{\theta}(\sigma)) = q_2(\sigma, \tilde{\theta}(\sigma))$. Though we suppress this in the notation, $\tilde{\theta}(\sigma)$ and $\tilde{\sigma}(\theta)$ also depend on v_1 and v_2 . Note the following identities:

For $\sigma \in [0, v_1]$,

$$\tilde{\theta}(\sigma) \equiv \tilde{\theta}_1 = \begin{cases} \frac{1}{2} + \frac{b(v_2 - v_1)}{2a} & \text{if } \frac{1}{2} + \frac{b(v_2 - v_1)}{2a} < 1 \\ 1 & \text{if } \frac{1}{2} + \frac{b(v_2 - v_1)}{2a} \geq 1 \end{cases} . \quad (18)$$

For $\sigma \in [v_1, v_2]$,

$$\tilde{\theta}_2(\sigma) = \begin{cases} 0 & \text{if } \frac{1}{2} + \frac{b(v_2 + v_1 - 2\sigma)}{2a} \leq 0 \\ \frac{1}{2} + \frac{b(v_2 + v_1 - 2\sigma)}{2a} & \text{if } 0 < \frac{1}{2} + \frac{b(v_2 + v_1 - 2\sigma)}{2a} < 1 \\ 1 & \text{if } \frac{1}{2} + \frac{b(v_2 + v_1 - 2\sigma)}{2a} \geq 1 \end{cases} , \quad (19)$$

For $\sigma \in [v_2, 1]$,

$$\tilde{\theta}(\sigma) \equiv \tilde{\theta}_3 = \begin{cases} 0 & \text{if } \frac{1}{2} - \frac{b(v_2 - v_1)}{2a} \leq 0 \\ \frac{1}{2} - \frac{b(v_2 - v_1)}{2a} & \text{if } 0 < \frac{1}{2} - \frac{b(v_2 - v_1)}{2a} \end{cases} . \quad (20)$$

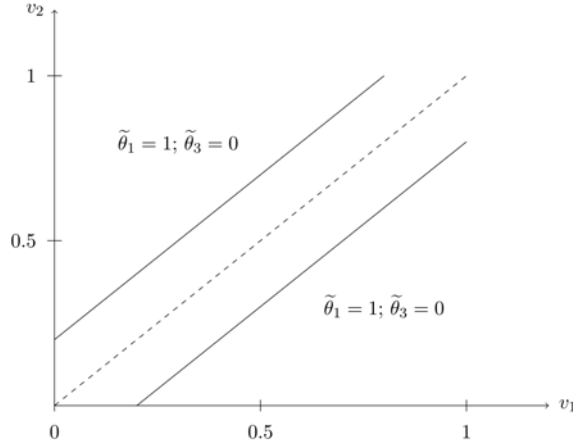


Figure 3: Interior and boundary values for conditional demand.

Whenever $\tilde{\sigma}(\theta) \in (0, 1)$, then it can also be defined by the simple requirement that $q_1(\tilde{\sigma}(\theta), \theta) = q_2(\tilde{\sigma}(\theta), \theta)$. Hence

$$\tilde{\sigma}(\theta) = \frac{v_1 + v_2}{2} + \frac{a}{2b}(1 - 2\theta) \quad (21)$$

We will frequently use the following results.

- Lemma 25** (i) If there exists $\sigma \in [0, 1]$ such that $\tilde{\theta}(\sigma) \notin (0, 1)$, then $\tilde{\theta}_1 = 1$ and $\tilde{\theta}_3 = 0$.
(ii) Suppose $\tilde{\theta}_1 = 1$ and $\tilde{\theta}_3 = 0$. Then $v_1 \leq \tilde{\sigma}(1) < \tilde{\sigma}(0) \leq v_2$.
(iii) There exists $\sigma \in [0, 1]$ such that $\tilde{\theta}(\sigma) \notin (0, 1)$ if and only if $v_2 - v_1 \geq a/b$.

Proof. (i) First note that from (18) and (20) that the conditions for $\tilde{\theta}_1$ and $\tilde{\theta}_3$ to take boundary values are identical. Thus, if one is at the boundary, so is the other. Furthermore, $\tilde{\theta}_2$ lies between $\tilde{\theta}_1$ and $\tilde{\theta}_3$. Hence, the result follows.

(ii) If $\tilde{\theta}_1 = 1$, then $v_1 \leq \tilde{\sigma}(1)$. If $\tilde{\theta}_3 = 0$, then $\tilde{\sigma}(0) \leq v_2$. $\tilde{\sigma}(1) < \tilde{\sigma}(0)$ follows directly from (21) for $\beta > 0$.

(iii) This follows directly from (i) and (18) and (20). ■

For later use of Lemma 25, we refer to the regions where $v_2 - v_1(<) \geq a/b$ as R1 and R2, respectively. Figure 3 illustrates Lemma 25(iii). For parameter values a, b such that $a/b = 0.2$, it depicts the choices of v_1 and v_2 for which $\tilde{\theta}(\sigma) \notin (0, 1)$ for some σ .⁴⁷

D.2.3 Proof of Proposition 15

Statement (ii) follows from Lemma 21. We prove statement (i) below.

Step 1: Social welfare and its derivatives

The social planner chooses (v_1, v_2) to maximize social welfare,

$$W(v_1, v_2) = \int_0^1 \int_0^1 \max\{\Psi - a\theta - b|v_1 - \sigma|, \Psi - a(1 - \theta) - b|v_2 - \sigma|\} d\theta d\sigma.$$

⁴⁷The figure does not restrict attention to $v_1 \leq v_2$.

We will show below that for each parameter value there is a unique local maximizer of the social welfare function. By Lemma 25, the social planner can choose (v_1, v_2) so that $\tilde{\theta}_1$ and $\tilde{\theta}_3$ are both interior (R1) or so that not neither $\tilde{\theta}_1$ nor $\tilde{\theta}_3$ is interior (R2). We will consider the two cases in Steps 1.1 and 1.2, respectively. Assume w.l.o.g. that $v_1 \leq v_2$.

Step 1.1: Social welfare in R1

Denoting the restriction of the social welfare function to R1 as W^1 , we obtain

$$\begin{aligned} W^1(v_1, v_2) = & \int_0^{v_1} \left(\int_0^{\hat{\theta}_1} (\Psi - a\theta - b(v_1 - \sigma)) d\theta + \int_{\hat{\theta}_1}^1 (\Psi - a(1 - \theta) - b(v_2 - \sigma)) d\theta \right) d\sigma \\ & + \int_{v_1}^{v_2} \left(\int_0^{\hat{\theta}_2(\sigma)} (\Psi - a\theta - b(\sigma - v_1)) d\theta + \int_{\hat{\theta}_2(\sigma)}^1 (\Psi - a(1 - \theta) - b(v_2 - \sigma)) d\theta \right) d\sigma \\ & + \int_{v_2}^1 \left(\int_0^{\hat{\theta}_3} (\Psi - a\theta - b(\sigma - v_1)) d\theta + \int_{\hat{\theta}_3}^1 (\Psi - a(1 - \theta) - b(\sigma - v_2)) d\theta \right) d\sigma. \end{aligned}$$

Using (18)-(20), the first order conditions are:

$$\frac{\partial W^1}{\partial v_1} = b \frac{a - 2av_1 + bv_1 - bv_2 + bv_1^2 + bv_2^2 - 2bv_1v_2}{2a} = 0 \quad (22)$$

$$\frac{\partial W^1}{\partial v_2} = b \frac{a - 2av_2 - bv_1 + bv_2 - bv_1^2 - bv_2^2 + 2bv_1v_2}{2a} = 0. \quad (23)$$

The second order conditions hold globally.

Step 1.2: Social welfare in R2

Lemma 25(ii) implies that $v_1 \leq \tilde{\sigma}(1) \leq \tilde{\sigma}(0) \leq v_2$ in this case. Denoting the restriction of the social welfare function to R2 as W^2 , we obtain

$$\begin{aligned} W^2(v_1, v_2) = & \int_0^{v_1} \left(\int_0^1 (\Psi - a\theta - b(v_1 - \sigma)) d\theta \right) d\sigma + \int_{v_1}^{\tilde{\sigma}(1)} \left(\int_0^1 (\Psi - a\theta - b(\sigma - v_1)) d\theta \right) d\sigma \\ & + \int_{\tilde{\sigma}(1)}^{\tilde{\sigma}(0)} \left(\int_0^{\hat{\theta}_2(\sigma)} (\Psi - a\theta - b(\sigma - v_1)) d\theta + \int_{\hat{\theta}_2(\sigma)}^1 (\Psi - a(1 - \theta) - b(v_2 - \sigma)) d\theta \right) d\sigma \\ & + \int_{\tilde{\sigma}(0)}^{v_2} \left(\int_0^1 (\Psi - a(1 - \theta) - b(v_2 - \sigma)) d\theta \right) d\sigma \\ & + \int_{v_2}^1 \left(\int_0^1 (\Psi - a(1 - \theta) - b(\sigma - v_2)) d\theta \right) d\sigma \end{aligned}$$

Using (18)-(20), the first order conditions are:

$$\frac{\partial W^2}{\partial v_1} = \frac{1}{2}bv_2 - \frac{3}{2}bv_1 = 0 \quad (24)$$

$$\frac{\partial W^2}{\partial v_2} = b + \frac{1}{2}bv_1 - \frac{3}{2}bv_2 = 0 \quad (25)$$

The second order conditions hold globally.

Step 1.3: Social welfare is continuous at the regime boundary between R1 and R2.

Inserting $v_2 = v_1 + a/b$ in W^1 and W^2 shows that these functions are identical at the regime boundary.

Step 2: *Whenever $b < a$, $(v_1, v_2) = (1/2, 1/2)$ is a global maximum.*

$(v_1, v_2) = (1/2, 1/2)$ solves the system of first order conditions (22) and (23). If $b < a$, $v_2 - v_1 < \frac{a}{b}$ holds $\forall (v_1, v_2) \in [0, 1] \times [0, 1]$. Thus $(1/2, 1/2)$ is a global maximum if it maximizes W^1 on the set of all (v_1, v_2) such that $v_2 \leq 1$ and $v_1 \leq v_2$. As $(1/2, 1/2)$ satisfies the FOC and W^1 is globally concave, the result follows.

Step 3: *Whenever $a \leq b < 2a$, $(v_1, v_2) = (a/2b, 1 - a/2b)$ is a global maximum.*

$(v_1, v_2) = (a/2b, 1 - a/2b)$ solves the system of first order conditions (24) and (25). $(a/2b, 1 - a/2b)$ satisfies $v_1 \leq v_2$ if and only if $a \leq b$. $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are interior by Lemma 25(iii) if and only if $v_2 - v_1 < a/b$, which is equivalent to $b < 2a$. W^1 is globally concave in R1, so that $(a/2b, 1 - a/2b)$ is a maximum on this subset. The remainder of the proof of this step shows that, given the parameter restriction $b < 2a$, the best design vector on R2 must lie on the boundary $v_2 - v_1 = a/b$, so that, by continuity, it yields lower expected welfare than the proposed optimum. To see that $v_2 - v_1 = a/b$, first note that by the conditions derived in Step 1.2, the only candidate for an equilibrium that is in the interior of the set of all (v_1, v_2) for which $v_2 - v_1 \geq a/b$ is $(v_1, v_2) = (1/4, 3/4)$. Given the parameter restriction $b < 2a$, however, this candidate violates $v_2 - v_1 \geq a/b$. Next, it is straightforward to show that $\frac{\partial W^2}{\partial v_1} > 0$ for $v_1 = 0$ and $\frac{\partial W^2}{\partial v_2} < 0$ for $v_2 = 1$ and $v_1 < 1$. Thus, the maximal value of $W = W^2$ on the regime where $v_2 - v_1 \geq a/b$ cannot be obtained on these boundaries. It must therefore lie on the boundary where $v_2 - v_1 = a/b$ (see Figure 3).

Step 4: *Whenever $b \geq 2a$, the global maximum is $(1/4, 3/4)$.*

The candidate optimum satisfies $v_2 - v_1 \geq a/b$ for $b \geq 2a$, so that W^2 applies by Lemma 25(iii). $(1/4, 3/4)$ is the unique point where the first-order conditions for W^2 hold. As the second-order conditions hold globally in R2, $(1/4, 3/4)$ is the unique maximizer of W^2 in this regime. Now consider the alternative of choosing designs such that $v_2 - v_1 < a/b$, so that Step 1.1 applies. The first order conditions are (22) and (23). For $b \geq 2a$, this system has no interior solutions. Next, we show that the constrained maximization problem

$$\max_{(v_1, v_2)} W^1 \text{ s.t. (i) } v_2 - v_1 < a/b; \text{ (ii) } v_1 \leq v_2 \text{ (iii) } v_1 \geq 0; \text{ (iv) } v_2 \leq 1 \quad (26)$$

has no solutions for which $v_1 = 0$ or $v_2 = 1$. For $v_1 = 0$, (22) implies that $\frac{\partial W^1}{\partial v_1} = b \frac{a - bv_2 + bv_2^2}{2a} > 0$ for $v_2 = v_2 - v_1 \leq a/b$. For $v_2 = 1$, (23) implies that $\frac{\partial W^1}{\partial v_2} = b \frac{-a + bv_1 - bv_1^2}{2a} < 0$ for $1 - v_1 = v_2 - v_1 \leq a/b$. Next consider the diagonal $v_1 = v_2$. Along this diagonal, W^1 is increasing for $v_1 < 1/2$ and decreasing for $v_1 > 1/2$. Therefore, the constrained maximum on this diagonal is obtained at $(v_1, v_2) = (1/2, 1/2)$. Simple but tedious calculations show that $W^1(\frac{1}{2}, \frac{1}{2}) = \frac{1}{4}\Psi - \frac{1}{4}b - \frac{1}{4}a$, whereas, in the suggested optimum, $W^2(\frac{1}{4}, \frac{3}{4}) = \frac{1}{24b} (2a^2 - 6ab + 3b^2 + 18\Psi b)$. As a result, $W^2(\frac{1}{4}, \frac{3}{4}) - W^1(\frac{1}{2}, \frac{1}{2}) = \frac{1}{24b} (2a^2 - 6ab + 3b^2 + 18\Psi b)$. This expression is positive for $b = 2a$ and increasing in b for $b = 2a$; hence, it is everywhere positive. Thus, the suggested deviation is not profitable either.

D.2.4 Proof of Proposition 16

Part 1: Preliminary Remarks We first introduce some notation and results that help us to identify the candidate equilibrium. Like the notation introduced in Section D.2.2, this

notation is useful to identify critical state values for which the buyer is indifferent between the two suppliers (with the proviso that the notation captures boundary values in cases where no such critical states exist).

We suppose w.l.o.g. that $v_1 \leq v_2$.

Notation (i) In an auction without information revelation, for $\sigma \in [0, 1]$, $\hat{\theta}(\sigma)$ is the value of θ that is minimal with the property that there exists no $\theta < \hat{\theta}(\sigma)$ such that $q_2 - p_2 > q_1 - p_1$
(ii) In auction without information revelation, for $\theta \in [0, 1]$, $\hat{\sigma}(\theta)$ is the value of σ that is minimal with the property that there exists no $\sigma < \hat{\sigma}(\theta)$ such that $q_2 - p_2 > q_1 - p_1$

We suppress the dependence of $\hat{\theta}(\sigma)$ and $\hat{\sigma}(\theta)$ on designs and prices. We obtain:

If $\sigma \in [0, v_1]$,

$$\hat{\theta}(\sigma) \equiv \hat{\theta}_1 = \begin{cases} 0 & \text{if } \frac{1}{2} + \frac{p_2 - p_1}{2a} + \frac{b(v_2 - v_1)}{2a} \leq 0 \\ \frac{1}{2} + \frac{p_2 - p_1}{2a} + \frac{b(v_2 - v_1)}{2a} & \text{if } 0 < \frac{1}{2} + \frac{p_2 - p_1}{2a} + \frac{b(v_2 - v_1)}{2a} < 1 \\ 1 & \text{if } \frac{1}{2} + \frac{p_2 - p_1}{2a} + \frac{b(v_2 - v_1)}{2a} \geq 1 \end{cases} \quad (27)$$

If $\sigma \in [v_1, v_2]$,

$$\hat{\theta}(\sigma) \equiv \hat{\theta}_2(\sigma) = \begin{cases} 0 & \text{if } \frac{1}{2} + \frac{p_2 - p_1}{2a} + \frac{b(v_2 + v_1 - 2\sigma)}{2a} \leq 0 \\ \frac{1}{2} + \frac{p_2 - p_1}{2a} + \frac{b(v_2 + v_1 - 2\sigma)}{2a} & \text{if } 0 < \frac{1}{2} + \frac{p_2 - p_1}{2a} + \frac{b(v_2 + v_1 - 2\sigma)}{2a} < 1 \\ 1 & \text{if } \frac{1}{2} + \frac{p_2 - p_1}{2a} + \frac{b(v_2 + v_1 - 2\sigma)}{2a} \geq 1 \end{cases} \quad (28)$$

If $\sigma \in [v_2, 1]$,

$$\hat{\theta}(\sigma) \equiv \hat{\theta}_3 = \begin{cases} 0 & \text{if } \frac{1}{2} + \frac{p_2 - p_1}{2a} - \frac{b(v_2 - v_1)}{2a} \leq 0 \\ \frac{1}{2} + \frac{p_2 - p_1}{2a} - \frac{b(v_2 - v_1)}{2a} & \text{if } 0 < \frac{1}{2} + \frac{p_2 - p_1}{2a} - \frac{b(v_2 - v_1)}{2a} < 1 \\ 1 & \text{if } \frac{1}{2} + \frac{p_2 - p_1}{2a} - \frac{b(v_2 - v_1)}{2a} \geq 1 \end{cases} \quad (29)$$

Moreover, note that, when $\hat{\sigma}(\theta) \in (0, 1)$, it solves $q_1(\sigma, \theta) - p_1 = q_2(\sigma, \theta) - p_2$. Thus, for the candidate equilibrium choice of player 1 $(v_1, p_1) = (1/2, a)$,

$$\hat{\sigma}(\theta) = \frac{a(1 - 2\theta) + p_2 - p_1 + b(v_2 + v_1)}{2b} \quad (30)$$

We can use (27)-(29) to obtain the probability that the design of supplier 1 is chosen by the buyer as

$$P(p_1, v_1, p_2, v_2) = \int_0^{\hat{\theta}_1} \hat{\theta}_1 d\sigma + \int_{v_1}^{\hat{\theta}_2(\sigma)} \hat{\theta}_2(\sigma) d\sigma + \int_{v_2}^1 \hat{\theta}_3 d\sigma. \quad (31)$$

If $\hat{\theta}_l \in (0, 1)$ for all $l = 1, 2, 3$, therefore, (27)-(29) imply

$$P(p_1, v_1, p_2, v_2) = \frac{a + p_2 - p_1 + b(v_1 - v_2) + b(v_2^2 - v_1^2)}{2a}. \quad (32)$$

Part 2: The candidate equilibrium It is straightforward to show that $(v_i, p_i) = (1/2, a)$ for $i = 1, 2$ is the only strategy profile for which $\hat{\theta}_1, \hat{\theta}_2$ and $\hat{\theta}_3$ take interior values, and first- and second-order conditions hold. From (27)-(29), $\hat{\theta}_1, \hat{\theta}_2$ and $\hat{\theta}_3$ take interior values ($\hat{\theta}_1 = \hat{\theta}_2 = \hat{\theta}_3 = 1/2$) for this equilibrium candidate. Using $\Pi_1 = p_1 P(p_1, v_1, p_2, v_2)$ and $\Pi_2 = p_2 (1 - P(p_1, v_1, p_2, v_2))$, (32) implies

$$\frac{d\Pi_i}{dv_i} = \frac{b - 2bv_i}{2a} p_i.$$

Thus, for any $p_i > 0$, $d\Pi_i/dv_i = 0$ holds if and only if $v_i = 1/2$. Also using (32), it follows that, for $i = 1, 2, j \neq i$,

$$\frac{d\Pi_i}{dp_i} = \frac{a - 2p_i + p_j + bv_i - bv_i^2 - bv_j + bv_j^2}{2a}.$$

Substituting $v_i = v_j = 1/2$ into this expression gives the first-order conditions

$$\frac{d\Pi_i}{dp_i} = a - 2p_i + p_j = 0. \quad (33)$$

These two conditions can only be satisfied simultaneously if $p_1 = p_2 = a$. The second-order conditions clearly hold for $p_1 = p_2 = a$ and $v_1 = v_2 = 1/2$. Finally, there can be no equilibrium where either v_1 or v_2 takes boundary values. To see this, it suffices to note that $v_1 = 0$ cannot arise in equilibrium because $d\Pi_i/dv_i = bp_i/2a > 0$.

Part 3: No profitable deviations The remainder of the proof proceeds by showing that there are no profitable deviations. By symmetry, it suffices to consider deviations of player 2 such that $v_2 \geq 1/2$.

We first note that, for $v_1 = 1/2$ and $p_1 = a$, (27)-(29) imply the following relation between the deviation choices v_2 and p_2 and the values of $\hat{\theta}_1$ and $\hat{\theta}_3$.

BC1 $\hat{\theta}_1 = 0$ if and only if $p_2 \leq -bv_2 + b/2$.

BC2 $\hat{\theta}_1 = 1$ if and only if $p_2 \geq -bv_2 + 2a + b/2$.

BC3 $\hat{\theta}_3 = 0$ if and only if $p_2 \leq bv_2 - b/2$.

BC4 $\hat{\theta}_3 = 1$ if and only if $p_2 \geq bv_2 + 2a - b/2$

The following general observations are helpful:

BC5 At $v_2 = 0.5$, $bv_2 + 2a - b/2 = -bv_2 + 2a + b/2$ and $bv_2 - b/2 = -bv_2 + b/2 = 0$

BC6 At $v_2 = 0.5 + a/b$, $-bv_2 + 2a + b/2 = bv_2 - b/2$

BC7 For $v_2 > 0.5$, $p_2 > -bv_2 + b/2$ (so that $\hat{\theta}_1 > 0$) and $bv_2 + 2a - b/2 > -bv_2 + 2a + b/2$

BC8 For $v_2 > 0.5$, $bv_2 + 2a - b/2 > \max\{-bv_2 + b/2, -bv_2 + 2a + b/2, bv_2 - b/2\}$

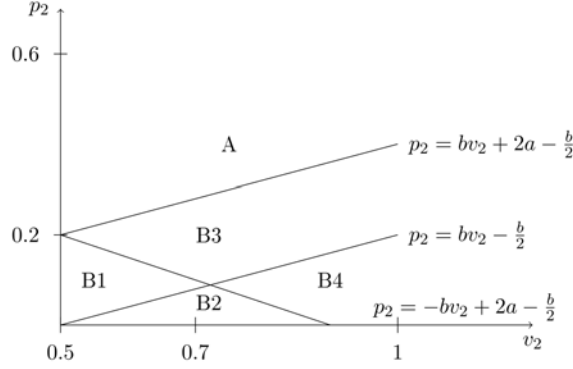


Figure 4: Deviation regions in the beauty contest.

Table 1: Type-B regions in the beauty contest

	$p_2 \leq -bv_2 + 2a + b/2$	$p_2 > -bv_2 + 2a + b/2$
$p_2 \geq bv_2 - b/2$	Region B1	Region B3
$p_2 < bv_2 - b/2$	Region B2	Region B4

As a result of (BC7), there is no feasible deviation with $v_2 \geq v_1 = 1/2$ and $\hat{\theta}_1 = 0$. Also, by (BC7) $\hat{\theta}_1 \in (0, 1)$ implies that $\hat{\theta}_3 = 0$ or $\hat{\theta}_3 \in (0, 1)$. $\hat{\theta}_1 = 1$ is compatible with $\hat{\theta}_3 = 0$, $\hat{\theta}_3 \in (0, 1)$ and $\hat{\theta}_3 = 1$. The set of conceivable deviations can thus be divided into five parameter regions which correspond to the possible combinations of $\hat{\theta}_1 = 0$, $\hat{\theta}_1 \in (0, 1)$, $\hat{\theta}_1 = 1$ and $\hat{\theta}_3 = 0$, $\hat{\theta}_3 \in (0, 1)$, $\hat{\theta}_3 = 1$. Figure 4 shows these regions for $a = 0.1$ and $b = 0.5$.

Note, however, from (BC6) that Region B4 disappears if $2a \geq b$.

We distinguish between those deviations where $\hat{\theta}_3 = 1$ and those where $\hat{\theta}_3 < 1$.

Definition 26 (i) A type-A deviation satisfies $p_2 \geq bv_2 + 2a - b/2$.
(ii) Type B-deviations satisfy $p_2 < bv_2 + 2a - b/2$

Type A deviations are obviously non-profitable.

Lemma 27 *There exist no profitable type-A deviations.*

Proof. For a Type-A deviation, $\hat{\theta}_3 = 1$ and thus, by BC 7, $\hat{\theta}_1 = 1$. Thus the probability that supplier 2 wins is equal to 0, hence the expected profits are 0. Thus this cannot be a profitable deviation. ■

We have to analyze Type B-deviations more carefully. In Table 1 (and Figure 4), we distinguish between four subregions:

We now consider Type-B deviations into each of the regions 1-4. As an important preliminary remark, note that the deviation profits are continuous at the boundary between the different regions.

Lemma 28 *There exist no profitable deviations into region B1.*

Proof. In the interior of B1, $\hat{\theta}_l \in (0, 1)$ for all $l = 1, 2, 3$ by (27)-(29). Thus, we can apply (32). There exists no critical point of the deviation profit in the interior of B1. For $p_j = a$, $v_j = 1/2$, the solution to the first-order conditions (33) for p_i is a for arbitrary values of $v_i > 0$ and the solution for v_i is $1/2$ for $p_i = a$. Moreover, the second-order conditions hold globally. Thus, deviation profits are globally concave in region B1 and they are maximized at the candidate equilibrium. Therefore, there are no profitable deviations into region B1. ■

Lemma 29 *There exist no profitable type-B deviations into region B2.*

Proof. In region B2, $\hat{\theta}_3 = 0$. Thus, the probability that supplier 2 wins is given by:

$$\begin{aligned} 1 - P(p_1, v_1, p_2, v_2) &= \int_0^{v_1} (1 - \hat{\theta}_1) d\sigma + \int_{v_1}^{\hat{\sigma}(0)} (1 - \hat{\theta}_2(\sigma)) d\sigma + (1 - \hat{\sigma}(0)) \\ &= \frac{-4b^2v_2^2 - 4bp_2 - 4p_2^2 - 4b^2v_2 + 32ab + 3b^2 - 8bp_2v_2}{32ab}. \end{aligned}$$

and the expected profit of supplier 2 from the deviation is

$$E\Pi_{s_2}(p_1, v_1, p_2, v_2) = p_2 \frac{-4b^2v_2^2 - 4bp_2 - 4p_2^2 - 4b^2v_2 + 32ab + 3b^2 - 8bp_2v_2}{32ab}.$$

Therefore

$$\begin{aligned} \frac{d}{dp_2} E\Pi_{s_2} &= \frac{-4b^2v_2^2 - 8bp_2 - 12p_2^2 - 4b^2v_2 + 32ab + 3b^2 - 16bp_2v_2}{32ab} \\ \frac{d}{dv_2} E\Pi_{s_2} &= -\frac{p_2(b + 2p_2 + 2bv_2)}{8a} < 0. \end{aligned}$$

Thus there will exist no maximum in the interior of region B2, and v_2 is strictly increasing towards the left boundary, which is the boundary to region B1. We have already seen that there are no profitable deviations into region B1 and, in particular, no deviations to its boundary with B2. By continuity of the deviation profits, therefore, there are no profitable deviations to B2. ■

Lemma 30 *There exist no profitable deviations of supplier 2 into region B3.*

Proof. We first consider the interior of region B3. We shall prove the following two statements:

- (i) *Deviation profits have a local maximum in the interior of region B3 if and only if $b \in ((32/10)a, 5a)$.*
- (ii) *A deviation to this local maximum from the proposed equilibrium is never profitable.*
- (i) The derivatives of the deviation profits of supplier 2 are

$$\frac{dE\Pi_{s_2}}{dp_2} = \tag{34}$$

$$\frac{16a^2 + 24ab - 7b^2 + 8p_2(2bv_2 - 3b - 4a) + 12p_2^2 + v_2(20b^2 - 16ab) - 12b^2v_2^2}{32ab} \tag{35}$$

$$\frac{dE\Pi_{s_2}}{dv_2} = \frac{p_2(-4a + 5b + 2p_2 - 6bv_2)}{8a} \tag{36}$$

There are four solutions to the system of first order conditions:

$$\begin{aligned} p_2 &= 0, v_2 = \frac{1}{2b}(-4a + b) \\ p_2 &= \frac{4a + b}{6}, v_2 = \frac{8b - 4a}{9b} \\ p_2 &= 2a + \frac{b}{2}, v_2 = 1 \\ p_2 &= 0, v_2 = \frac{1}{6b}(4a + 7b) \end{aligned}$$

For $p_2 = 0$, expected profits are zero, hence the deviation is not be profitable. Of the remaining two solutions, only $(\bar{v}_2, \bar{p}_2) = (\frac{8b-4a}{9b}, \frac{4a+b}{6})$ satisfies the second-order conditions and is thus a candidate for a local maximum in the interior of Region B3. It remains to be shown that it satisfies the four conditions (a) $\bar{v}_2 \leq 1$, (b) $\bar{p}_2 < b\bar{v}_2 + 2a - b/2$, (c) $\bar{p}_2 > b\bar{v}_2 - b/2$ and (d) $\bar{p}_2 > -b\bar{v}_2 + 2a + b/2$.

Conditions (a) and (b) hold without any parameter restrictions (except $a > 0$ and $b > 0$). (c) holds if and only if $b < 5a$; (d) holds if and only if $b > \frac{16}{5}a$.

(ii) The expected profits in the locally optimal deviation described in (i) are $(4a + b)^3 / 324ab$ as opposed to $a/2$ for the equilibrium choices p_2^*, v_2^* . Hence, the deviation is not profitable if and only if

$$E\Pi_{s_2}(p_2^*, v_2^*; p_1^*, v_1^*) - E\Pi_{s_2}(\bar{p}_2, \bar{v}_2; p_1^*, v_1^*) = -\frac{64a^3 - 114a^2b + 12ab^2 + b^3}{324ab} \geq 0$$

We show that the function $g(b) = 64a^3 - 114a^2b + 12ab^2 + b^3$ is negative for $b \in [\frac{32}{10}a, 5a]$, for any value of $a > 0$. First, $g''(b) = 24a + 6b > 0$. Hence the function is convex. Second, $g(\frac{32}{10}a) < 0$ and $g(5a) < 0$, thus it is negative for all values of $b \in [\frac{32}{10}a, 5a]$ and consequently there are no profitable deviations for these values of b .

We thus have proven statements (i) and (ii). It remains to be shown that there are no profitable deviations to the two boundaries of region B3 belonging to the set.

There is no profitable deviation into the subregion of B3 where $p_2 = bv_2 - b/2$

This is the boundary between B3 and region B4. Simple calculations show that this boundary is non-degenerate if and only if $2a \leq b$.⁴⁸ It consists of all points $(\frac{1}{2} + \frac{p_2}{b}, p_2)$ for which $p_2 \in [a, b/2]$. The first-order condition for the maximal profit on this line is

$$\frac{d}{dp_2} E\Pi_{s_2} \left(p_2, v_2 = \frac{1}{2} + \frac{p_2}{b}; p_1, v_1 \right) = \frac{a + b - 4p_2}{2b}.$$

For $b \in [2a, 3a]$ the expected profit on the boundary is maximized by setting $p_2 = a$, while for $b > 3a$ it is maximized by the unique solution to the first order condition: $p_2 = (a + b)/4$. Next, we compare the expected profits. For $b \in [2a, 3a]$, the maximum deviation profit is given by

$$E\Pi_{s_2} \left(p_2 = a, v_2 = \frac{1}{2} + \frac{a}{b}; p_1, v_1 \right) = \frac{ab - a}{2b}.$$

⁴⁸Recall from (BC6) that for $2a > b$, the intersection point between the lines defining the lower boundary of region B3 is no longer in the interval $[0, 1]$.

Since $(b - a) / b < 1$, the expected deviation profit is never larger than the equilibrium profit $a/2$. For $b > 3a$ the maximum deviation profit is given by

$$E\Pi_{s_2} \left(\bar{p}_2 = \frac{a+b}{4}, \bar{v}_2 = \frac{a+3b}{4b}; p_1, v_1 \right) = \frac{(a+b)^2}{16b}$$

and the difference between equilibrium and deviation profits is

$$E\Pi_{s_2}(p_2^*, v_2^*; p_1^*, v_1^*) - E\Pi_{s_2}(\bar{p}_2, \bar{v}_2; p_1^*, v_1^*) = \frac{6ab - a^2 - b^2}{16b}.$$

Thus, the deviation profits are larger than the equilibrium profits if and only if $b > a(2\sqrt{2} + 3)$, which is never true by Assumption 1.

There is no profitable deviation of player 2 for which $v_2 = 1$.

Simple calculations show that (v_2, p_2) is in Region B3 for $v_2 = 1$ if and only if $p_2 \in [b/2, 2a + b/2]$. Evaluating $\frac{d}{dp_2} E\Pi_{s_2}$ at $v_2 = 1$ yields the first-order condition

$$16a^2 + 8ab - 32ap_2 + b^2 - 8bp_2 + 12p_2^2 = 0.$$

This equation has two solutions $p_2 = 2a + b/2$ and $p_2 = 2a/3 + b/6$. The first point is a local minimum. Thus consider $p_2 = 2a/3 + b/6$. It is in the interval $[b/2, 2a + b/2]$ only if $b/2 \leq a$. Using (36), $\frac{d}{dv_2} E\Pi_{s_2} < 0$ for $(v_2, p_2) = (1, 2a/3 + b/6)$. Hence, this point cannot be an optimal deviation either. If $b/2 > a$, the maximum expected profit in the intersection of region B3 and the line $v_2 = 1$ is achieved at $(v_2, p_2) = (1, b/2)$. The expected profit is

$$E\Pi_{s_2}(\bar{p}_2, \bar{v}_2; p_1, v_1) = \frac{a}{4} < \frac{a}{2}.$$

Thus this is never a profitable deviation. ■

Lemma 31 *There exist no profitable deviation of player 2 into region B4.*

Proof. In this region, $\hat{\theta}_3 = 0$ and $\hat{\theta}_1 = 1$. The probability that supplier 2 wins is thus given by

$$1 - P(p_1, v_1, p_2, v_2) = \int_{\sigma(1)}^{\sigma(0)} (1 - \hat{\theta}_2(\sigma)) d\sigma + (1 - \sigma(0)) = \frac{2a + 3b - 2p_2 - 2bv_2}{4b}.$$

and the expected profit of supplier 2 from the deviation is

$$E\Pi_{s_2}(p_1, v_1, p_2, v_2) = p_2 \frac{2a + 3b - 2p_2 - 2bv_2}{4b}.$$

As $\frac{d}{dv_2} E\Pi_{s_2} < 0$, there can be no point in Region B4 which yields higher deviation profits than the best deviation in the remaining part of Region B. ■

Part 4: Uniqueness We have already seen in Part 2 that the candidate equilibrium is the only equilibrium for which the $\widehat{\theta}_l$ are interior for $l = 1, 2, 3$. It remains to be shown that there are no equilibria where not all $\widehat{\theta}_i$ are interior. We distinguish between four cases.

Step 1: No equilibria where $\widehat{\theta}_1 = 1$ and $\widehat{\theta}_3 = 0$

Using (30), we obtain

$$\widehat{\sigma}(0) = \frac{b(v_1 + v_2) + p_2 - p_1 + a}{2b} \quad (37)$$

$$\widehat{\sigma}(1) = \frac{b(v_1 + v_2) + p_2 - p_1 - a}{2b}. \quad (38)$$

The expected payoff of player 1 is thus $\pi_1 = p_1 (\widehat{\sigma}(0) + (\widehat{\sigma}(1) - \widehat{\sigma}(0)) / 2)$ and therefore

$$\pi_1 = p_1 \frac{b(v_1 + v_2) + p_2 - p_1}{2b}.$$

This immediately shows that $\partial\pi_1/\partial v_1 > 0$, ruling out all equilibria with $v_1 < v_2$ where $\widehat{\theta}_1 = 1$ and $\widehat{\theta}_3 = 0$ before and after the increase. By (27) and (29), a strategy profile with $v_1 = v_2$ cannot satisfy $\widehat{\theta}_1 = 1$ and $\widehat{\theta}_3 = 0$.

Now consider a strategy profile such that $\widehat{\theta}_3 = 0$, but $\widehat{\theta}_3$ becomes interior after a marginal increase of v_1 , while $\widehat{\theta}_1 = 1$. After the marginal increase, the winning probability of player 2 in a situation where $\widehat{\theta}_3 \in (0, 1)$ and $\widehat{\theta}_1 = 1$ is $P_2 = (1 - \widehat{\theta}_3) ((1 - v_2) + (v_2 - \widehat{\sigma}(1)) / 2)$. Using (30) and (38),

$$\widehat{\theta}_3 = \frac{a + p_2 - p_1 - b(v_2 - v_1)}{2a} \quad (39)$$

$$v_2 - \widehat{\sigma}(1) = \frac{b(v_2 - v_1) - p_2 + p_1 + a}{2b} \quad (40)$$

we immediately obtain

$$P_2 = \left(1 - \frac{a + p_2 - p_1 - b(v_2 - v_1)}{2a}\right) \left(1 - v_2 + \left(\frac{b(v_2 - v_1) - p_2 + p_1 + a}{4b}\right)\right)$$

Thus, $\pi_1 = p_1 (1 - P_2)$ and

$$\frac{\partial\pi_1}{\partial v_1} = \frac{p_1}{4a} (a + 2b + p_1 - p_2 - bv_1 - bv_2).$$

Using (39), $\widehat{\theta}_3 = 0$ implies $p_1 - p_2 = a - b(v_2 - v_1)$. Hence $\frac{\partial\pi_1}{\partial v_1} = \frac{p_1}{2a} (a + b - bv_2) > 0$. Thus, there is no equilibrium such that $\widehat{\theta}_1 = 1$ and $\widehat{\theta}_3 = 0$ and $\widehat{\theta}_3 = 0$ is interior after a marginal increase of v_1 .

Step 2: No equilibria where $\widehat{\theta}_1 \in (0, 1)$ and $\widehat{\theta}_3 = 0$

The winning probability of player 1 in a situation where $\widehat{\theta}_1 \in (0, 1)$ and $\widehat{\theta}_3 = 0$ is $P_1 = \widehat{\theta}_1 v_1 + (\widehat{\sigma}(0) - v_1) / 2$. From (37),

$$\widehat{\sigma}(0) - v_1 = \frac{a + p_2 - p_1 + b(v_2 - v_1)}{2b}.$$

From this and (27), we can calculate player 2's profits as

$$\pi_2 = p_2(1 - P_1) = \left[1 - (a + p_2 - p_1 + b(v_2 - v_1)) \left(\frac{v_1}{2a} + \frac{1}{4b} \right) \right]$$

and thus

$$\frac{\partial \pi_2}{\partial v_2} = -bp_2 \left(\frac{v_1}{2a} + \frac{1}{4b} \right) < 0$$

Hence, there can be neither an interior best response of player 2 nor a best response at $v_2 = 1$. Thus, there is no equilibrium of the type described.

Step 3: No equilibria where $\hat{\theta}_1 = 1$ and $\hat{\theta}_3 \in (0, 1)$

This is the same situation as in Step 2, with the roles of suppliers 1 and 2 reversed. Thus, the same arguments show that there can be no such equilibrium.

Step 4: No equilibria where $\hat{\theta}_1 = 1$ and $\hat{\theta}_3 = 1$ (or with $\hat{\theta}_1 = 0$ and $\hat{\theta}_3 = 0$)

In such an equilibrium, one supplier i earns a positive profit, whereas the other supplier (supplier j) does not. Supplier j can always deviate and imitate supplier i . As a result, both suppliers will share the profit, so that the deviation is profitable.

D.2.5 Fixed Prize Tournaments

Now consider the FPT for some arbitrary $A \geq 0$.

Proposition 32 *An FPT with heterogeneous suppliers has a unique equilibrium. In this equilibrium $v_1 = v_2 = 1/2$.*

As in the auction without revelation of quality information, minimum differentiation arises in equilibrium for any value of the prize A . Thus, the suppliers choose the same approach in the tournament as in the auction without information revelation. The *optimal FPT* for the buyer has $A = 0$ (or $A = 2C$ if fixed costs are taken into account). We do not provide the proof, since it follows the one of Proposition 16 (in the Online Appendix) closely. The analysis is considerably simpler, however: Only v_1 and v_2 need to be determined, whereas p_1 and p_2 can be replaced by A . Moreover, the structure of conceivable deviations is much simpler, as only one-dimensional deviations are feasible.

D.2.6 Buyer Payoffs in Auctions and Fixed-Prize Tournaments

As in the case of homogeneous suppliers, when there are no fixed costs and the buyer cannot use transfers, she is better off not inducing diversification.

Proposition 33 *Suppose suppliers are heterogeneous.*

(i) *The expected payoff of a buyer in any asymmetric equilibrium of an auction without a reserve price and with full information revelation is lower than in the (symmetric) equilibrium of the optimal FPT.*

(ii) *The expected payoff of a buyer in any asymmetric equilibrium of an auction without a reserve price and with full information revelation is lower than in the symmetric equilibrium of the auction.*

We omit the proof, as the intuition is exactly as for the case of homogeneous suppliers, relying on the fact that buyers always have to pay the quality differential resulting from diversity.

D.3 Proofs from Subsection 6.3

We now prove the claim from Section 6.3 that the main results from Sections 3 and 4 are neither sensitive to the assumptions on the distribution of the optimal approach σ nor to the exact nature of the quality reduction from suboptimal approaches. For simplicity, we confine the analysis to the game without transfers or fixed costs. This game is equivalent to the subgame following entry of both suppliers in the game with arbitrary levels of transfers and costs. We relax assumptions (A1) and (A3) as follows.

Assumption (A1)'' $f(\sigma) > 0 \forall \sigma > 0$.

Assumption (A3)' $q(v, \sigma) = \Psi - \delta(|v - \sigma|)$ where $\delta' > 0$.

Note that (A1)'' dispenses with the assumption that the distribution is symmetric and single-peaked, while (A3)' dispenses with the linearity assumption. Thus, we allow for arbitrary distributions of the state with positive density, and quality is an arbitrary decreasing function of the distance between a given approach and the ideal one.

Proposition 34 *Suppose that (A1)'', (A2) and (A3)' hold. Then, (i) the social optimum satisfies $v_1^* \neq v_2^*$; (ii) the auction mechanism ($\mathcal{P} = \mathbb{R}^+$) implements the social optimum; and (iii) in any FPT ($\mathcal{P} = \{A\}$ for some $A \geq 0$) the unique equilibrium is such that $v_1 = v_2$ and $F(v_i) = 1/2$ for $i = 1, 2$.*

Proof. Step 1: $v_1^* \neq v_2^*$. The proof is identical to the proof of Proposition 18.

Step 2: *The auction mechanism ($\mathcal{P} = \mathbb{R}^+$) implements the social optimum.*

This follows from Lemma 21 below.

Step 3: *The unique equilibrium in an FPT is such that $v_1 = v_2$ and $F(v_i) = 1/2$ for $i = 1, 2$.*

First, we show that the suggested (v_1, v_2) emerges as an equilibrium. Denote the prize with A . Let v_j be such that $F(v_j) = 1/2$. Since f is everywhere positive, such a v_j is unique. Now if supplier $i \in \{1, 2\}$ plays $v_i = v_j$, his expected profit is $\Pi_i(v_i|v_j) = A/2$. For any $v_i < v_j$ the expected profit is $\Pi_i(v_i|v_j) = AF((v_i + v_j)/2) < A/2$. Similarly, for any $v_i > v_j$ the expected profit is $\Pi_i(v_i|v_j) = A(1 - F((v_i + v_j)/2)) < A/2$. Thus, $v_i = v_j$ is an equilibrium. Second, $v'_i = v'_j$ is an equilibrium only if $F(v'_j) = 1/2$. Suppose not. Then, a supplier i can profitably deviate to v_i such that $F(v_i) = 1/2$, since his expected profit will be $\Pi_i(v_i|v_j) > A/2$. Third, $v_i \neq v_j$ is never an equilibrium. Suppose it was. Let $v_1 < v_2$. Then, the expected profit of supplier 1 is $\Pi_1(v_1|v_2) = AF((v_1 + v_2)/2)$, while deviating to $(v_1 + v_2)/2$ leads to a payoff of $AF((v_1 + 3v_2)/4) > AF((v_1 + v_2)/2)$. ■

The result that the buyer prefers the optimal tournament to an auction requires somewhat stronger conditions. Proposition 8 can be generalized either by relaxing the assumption on the distribution of ideal approaches or by relaxing the assumption on quality reduction from non-ideal approaches, but not both simultaneously.

Proposition 35 *Suppose that (i) (A1)'', (A2) and (A3) or (ii) (A1), (A2) and (A3)' hold. Then the buyer prefers the optimal FPT ($\mathcal{P} = 0$) to an auction.*

Proof. (i) Let $v_1 < v_2$ be the outcome of an auction. Since, by Lemma 2, the buyer pays the quality difference in equilibrium, the buyer payoff, is the lower quality in each state. That is, for all $\sigma \in [0, (v_1 + v_2)/2]$, the buyer payoff is equal to $q(v_2, \sigma)$, and it is equal to $q(v_1, \sigma)$ everywhere else. For any (v'_1, v'_2) with $v'_1 = v'_2 = (v_1 + v_2)/2$, total expected payoffs are $q((v_1 + v_2)/2, \sigma) > \min(q(v_1, \sigma), q(v_2, \sigma))$, which is higher than the expected buyer payoff for $v_1 < v_2$. For all outcomes where $v'_1 = v'_2 = v'$, the expected total profit is $\Pi_b(v') = \int_0^{v'} q(v', \sigma) dF(\sigma) + \int_{v'}^1 q(v', \sigma) dF(\sigma)$. The derivative is $d\Pi_b(v')/dv' = -\delta'F(v') + \delta'(1 - F(v'))$. Since $\delta' = b$ by (A3), the expected total payoff among all (v'_1, v'_2) with $v'_1 = v'_2$ is maximized for v' such that $F(v') = 1/2$. From Proposition 34 we know this is the outcome of an FPT, and that she can appropriate the entire surplus. Thus, her payoff is higher than the total surplus at (v'_1, v'_2) with $v'_1 = v'_2 = (v_1 + v_2)/2$ and thus higher than her expected payoff at $v_1 < v_2$.

(ii) By (A1), $f(\sigma)$ is symmetric, unimodal and everywhere positive, and q is as in (A3)'. The argument is as in Step 1, except that

$$d\Pi_b(v')/dv' = \int_0^{v'} -\delta'(v' - \sigma) dF(\sigma) + \int_{v'}^1 \delta'(\sigma - v') dF(\sigma).$$

Suppose $v' < 1/2$. For every $\sigma' \in [0, v')$, there exist $\sigma'' = 2v' - \sigma' \in (v', 2v']$, such that $\delta'(v' - \sigma') = \delta'(\sigma'' - v')$. Since $v' < 1/2$, by unimodality and symmetry of f , we obtain $f(\sigma') \leq f(\sigma'')$, which implies $\int_0^{v'} -\delta'(v' - \sigma) dF(\sigma) + \int_{v'}^{2v'} \delta(\sigma - v') dF(\sigma) \geq 0$. Since $\int_{2v'}^1 \delta(\sigma - v') dF(\sigma) > 0$, it follows that $d\Pi_b(v')/dv' > 0$. Similar argument shows that $d\Pi_b(v')/dv' < 0$ if $v' > 1/2$ and $d\Pi_b(v')/dv' = 0$ if $v' = 1/2$. Hence, among the class of approaches such that $v'_i = v'_j$ the buyer profit is maximized for $v'_i = v'_j = 1/2$. By symmetry, $F(1/2) = 1/2$, so that this outcome is implemented in an FPT. ■

D.4 Proofs from Subsection 6.4

This section provides the proof of Lemma 17 from Section 6.4. Suppose that there are n suppliers and that assumptions (A1)', (A2) and (A3) hold. Consider an FPT with two prizes $A_1 > A_2 > 0$, where the supplier with the highest quality receives A_1 and the supplier with the second-highest quality receives A_2 .⁴⁹ For notational convenience, suppose that $v_1 \leq v_2 \leq \dots \leq v_n$. We first provide an intermediate result.

Lemma 36 *If v_1, v_2, \dots, v_n is an equilibrium of an FPT with two prizes, then $v_1 = v_2 = v_3$ and $v_{n-2} = v_{n-1} = v_n$.*

Proof. We will prove that $v_1 = v_2 = v_3$. The other claim follows by an analogous argument.

Step 1: $v_1 = v_2$. Suppose not. Then $v_1 < v_2$. Thus, the expected profit of supplier 1 is

$$\Pi_1(v_1|v_{-1}) = \frac{v_1 + v_2}{2}A_1 + \frac{v_3 - v_2}{2}A_2.$$

Therefore, a deviation to any $v'_1 \in (v_1, v_2)$ increases the probability of winning the first prize, while not affecting the probability of winning the second prize. Hence, it is profitable.

⁴⁹Ties are broken randomly, with equal chance of winning for each firm with the respective quality.

Step 2: $v_1 = v_2 < v_3 = v_4$ cannot be an equilibrium. Denote with $P_{\sigma < v_i}^{i,1}$ the probability that supplier i wins the first prize when $\sigma < v_i$. Analogously define the probabilities of winning when the state is greater than the chosen approach and the probabilities of winning the second prize. By random tie breaking we have $P_{\sigma < v_1}^{1,1} = P_{\sigma < v_2}^{2,1} = P_{\sigma < v_1}^{1,2} = P_{\sigma < v_2}^{2,2}$ and $P_{\sigma > v_1}^{1,1} = P_{\sigma > v_2}^{2,1} = P_{\sigma > v_1}^{1,2} = P_{\sigma > v_2}^{2,2}$. We will show that $P_{\sigma < v_1}^{1,1} = P_{\sigma > v_1}^{1,1}$. Suppose that this was not true. First, suppose $P_{\sigma < v_1}^{1,1} > P_{\sigma > v_1}^{1,1}$. Then, there exist $\varepsilon, \varepsilon', \varepsilon'' > 0$ arbitrarily small such that a deviation $v'_1 = v_1 - \varepsilon$ leads to expected profits

$$\Pi_1(v'_1|v_{-1}) = 2 \left(P_{\sigma < v_1}^{1,1} - \varepsilon' \right) A_1 + 2 \left(P_{\sigma > v_1}^{1,1} - \varepsilon'' \right) A_2.$$

For sufficiently small ε this constitutes a profitable deviation. The case $P_{\sigma < v_1}^{1,1} < P_{\sigma > v_1}^{1,1}$ follows by an analogous argument, but the incentives to deviate are even stronger.

Now suppose that $P_{\sigma < v_1}^{1,1} = P_{\sigma > v_1}^{1,1}$ and $v_1 = v_2 < v_3 = v_4$. We will show that this cannot be an equilibrium. In the proposed equilibrium $P_{\sigma < v_1}^{1,1} = v_1/2$ and $P_{\sigma < v_1}^{1,1} + P_{\sigma > v_1}^{1,1} = P_{\sigma < v_1}^{1,2} + P_{\sigma > v_1}^{1,2} = v_1$. Hence, the expected profit is

$$\Pi_1(v_1|v_{-1}) = v_1 A_1 + v_1 A_2.$$

For any deviation $v'_1 \in (v_2, v_3)$ the probability of winning the first prize is

$$\frac{v'_1 + v_3}{2} - \frac{v'_1 + v_2}{2} = \frac{v_3 - v_2}{2} = v_1$$

where the last equality follows from $P_{\sigma < v_1}^{1,1} = P_{\sigma > v_1}^{1,1}$. Using $v_3 = v_4$, the probability of winning the second prize is

$$\frac{v_2 + v'_1}{2} > v_1$$

thus it follows that $\Pi_1(v'_1|v_{-1}) > \Pi_1(v_1|v_{-1})$.

Step 3: $v_1 = v_2 < v_3 < v_4$ cannot be an equilibrium. The expected profit of supplier 1 is

$$\Pi_1(v_1|v_{-1}) = \frac{v_1}{2} A_1 + \frac{v_3 - v_1}{4} A_1 + \frac{v_3 + v_1}{4} A_2 + \frac{v_4 - v_3}{4} A_2. \quad (41)$$

Consider a deviation to $v'_1 \in (v_1, v_3)$. The expected profit is

$$\Pi_1(v'_1|v_{-1}) = \frac{v_3 - v_1}{2} A_1 + \frac{v'_1 + v_1}{2} A_2 + \frac{v_4 - v_3}{2} A_2.$$

If $\Pi_1(v'_1|v_{-1}) > \Pi_1(v_1|v_{-1})$, then this is a profitable deviation. If $\Pi_1(v'_1|v_{-1}) \leq \Pi_1(v_1|v_{-1})$ is equivalent with

$$\frac{v_1}{2} A_1 - \frac{v_3 - v_1}{4} A_1 + \frac{v_3 - v_1}{4} A_2 - \frac{v'_1}{2} A_2 - \frac{v_4 - v_3}{4} A_2 \geq 0 \quad (42)$$

But consider in that case a deviation to $v''_1 = v_1 - \varepsilon$ for small positive ε . The expected profit is

$$\Pi_1(v''_1|v_{-1}) = \frac{v''_1 + v_1}{2} A_1 + \frac{v_3 - v_1}{2} A_2$$

and $\lim_{\varepsilon \rightarrow 0} \Pi_1(v_1''|v_{-1}) = v_1 A_1 + \frac{v_3 - v_1}{2} A_2$. Together with (41), this implies

$$\lim_{\varepsilon \rightarrow 0} \Pi_1(v_1''|v_{-1}) - \Pi_1(v_1|v_{-1}) = \frac{v_1}{2} A_1 + \frac{v_3 - v_1}{4} A_2 - \frac{v_3 - v_1}{4} A_1 - \frac{v_1}{2} A_2 - \frac{v_4 - v_3}{4} A_2.$$

Since $v_1' > v_1$, (42) implies $\lim_{\varepsilon \rightarrow 0} \Pi_1(v_1''|v_{-1}) - \Pi_1(v_1|v_{-1}) > 0$. Hence, there always exists $\varepsilon > 0$ small enough such that $\Pi_1(v_1''|v_{-1}) - \Pi_1(v_1|v_{-1}) > 0$. ■

The lemma implies that the maximal number of active approaches in an FPT with two prizes is $n - 4$. By Lemma 13(ii), an FPT with a single prize implements an equilibrium with $n - 2$ active approaches. By Lemma 13(i), it is possible to implement the socially optimal allocation with $n - 2$ approaches in an FPT with a single prize. Implementing this equilibrium in a single-prize FPT, where the prize size is the sum of the two prizes in an FPT with two prizes, strictly increases the total payoff. On the other hand, the payoff of the sellers remains the same, as the total size of the fixed prize remains the same. Hence, the expected buyer payoff strictly increases.

D.5 Supplementary Material for Subsection 6.6

In this subsection, we provide a more detailed version of the discussion in Section 6.6, with slightly more emphasis on the formalism.

We have seen in Sections 3 and 4 that the buyer can induce the social optimum with an auction, but the optimal innovation contest from her own perspective nevertheless is an FPT (possibly augmented). In the following, we therefore ask whether alternative procurement institutions might help the buyer to increase her expected payoffs by inducing diversity.

A commonly held assumption in the literature on innovation contests is that the buyer is a monopsonist with complete market power. Hence, without the expectation of receiving sufficiently high transfers, suppliers would not invest in developing innovations, as the buyer would appropriate the entire surplus ex post. In innovation contests, the buyer commits to paying one of the prices selected by the suppliers. It is exactly this commitment that resolves the hold-up problem. In this section, we consider the case when the buyer does not commit to a pricing rule ex ante. Ignoring the participation constraint for a moment, we show that ex post price negotiations implement the socially optimal equilibrium for any bargaining power that the buyer holds. Moreover, as long as the participation constraint holds, abstaining from commitment can have favorable effects on the buyer's expected payoffs.

More formally, the *environment* of a negotiation is a tuple (b, Ψ, τ) . The only difference to the environment for an innovation contest is an additional parameter $\tau \in (0, 1)$ capturing the bargaining power of the buyer. As noted before, $\tau = 1$ would be equivalent to the environment where the buyer is a monopsonist, as in the preceding section. On the other hand, $\tau = 0$ would be equivalent to a situation where the buyer commits to an auction without reserve price. The negotiation is characterized by an *allocation mapping* $A^\tau : (q_1, q_2) \mapsto (\{1, 2\}, p_1, p_2)$ assigning to each quality vector a selected supplier and potentially quality-dependent payments from the buyer to the suppliers. We think of this function as mapping the quality vectors onto the equilibrium payoffs of an unmodelled negotiation game \mathcal{N} .

We make the following assumptions on this negotiation game:

1. In equilibrium, the buyer selects the high-quality supplier.

2. The payments are such that the buyer obtains an exogenously determined fraction τ of the additional surplus Δq generated by the high-quality supplier, and this supplier receives the remaining fraction $1 - \tau$. Here τ is a black box parameter capturing the bargaining power of the buyer.

Ignoring fixed costs and transfers, negotiations thus determine payoffs as

$$U^B(q_1, q_2) = \tau \Delta q + \min\{q_1, q_2\} \quad (43)$$

for the buyer and

$$\Pi^i(q_1, q_2) = \begin{cases} (1 - \tau) \Delta q & \text{if } q_i > q_j \\ 0 & \text{if } q_i \leq q_j \end{cases} \quad (44)$$

for the supplier.

These assumptions are consistent with the understanding that quality is observable, but not verifiable: We can think of U^B and Π^i as equilibrium payoffs in an unmodelled negotiation game with full information about parameters that are relevant to the division of the surplus, but it is not necessary that a court can observe payoffs. The following result is an immediate implication of Lemma 21.

Corollary 37 *Price negotiations (weakly) implement the social optimum (v_1^*, v_2^*) .*

The distributional properties depend on τ , which is exogenous. No matter what τ is, however, the buyer always prefers negotiations to auctions, because of the outside option of going back to the alternative supplier. In this sense, holding an auction represents a transfer of market power from the buyer to the suppliers. Clearly, for sufficiently high bargaining power of the buyer, negotiations are preferable to the optimal FPT for the buyer. Of course, the anticipated bargaining power of the buyer has to remain sufficiently low that the suppliers expect to break even.