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Centralized Trading of Corporate Bonds

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Centralized Trading of Corporate Bonds*

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Abstract

In the post-crisis period, increased regulation of financial intermediaries led to a significant decline in corporate bond market liquidity. In order to stabilize these markets, policy makers recently proposed that the trading of corporate bonds should be more centralized. In this paper, we show that a centralization of corporate bond markets always leads to an inferior outcome when compared with the initial over-the-counter structure. The regulator may achieve a superior allocation only if it is feasible for him to also affect market liquidity, by either increasing or decreasing it.

Keywords: Monetary theory, over-the-counter markets, financial regulation, corporate bonds, liquidity.

JEL Classification: D47, D52, D62, E31, E44, E50, G11, G12, G28.

1 Introduction

Most corporate bonds are traded in over-the-counter markets, which feature search and bargaining frictions. In this context, the degree of search frictions is more commonly referred to as liquidity; i.e., corporate bond markets with low (high) search frictions are called liquid (illiquid). Since the global financial crisis struck the U.S. economy, liquidity in corporate bond markets declined considerably. Among other causes, this decline in liquidity has been traced back to the increased regulation of financial intermediaries that was intended to strengthen the financial system. In particular, regulations such as the Dodd-Frank Act and Basel III increased the capital and liquidity requirements of banks and limited their leverage ratio. The focus of these regulations was to ensure that banks are sufficiently well capitalized, but they also resulted in unintended consequences. Concretely, since the leverage rules apply independent of the risk profile of the

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underlying assets, banks committed less capital to balance-sheet-intensive activities such as fixed income trading. In Figure 1, we show the development of the fixed income trading assets as a percentage of the total assets for the U.S. commercial banking industry, as reported by the Federal Reserve Bank of New York.¹ Figure 1 shows that the total capital committed to the

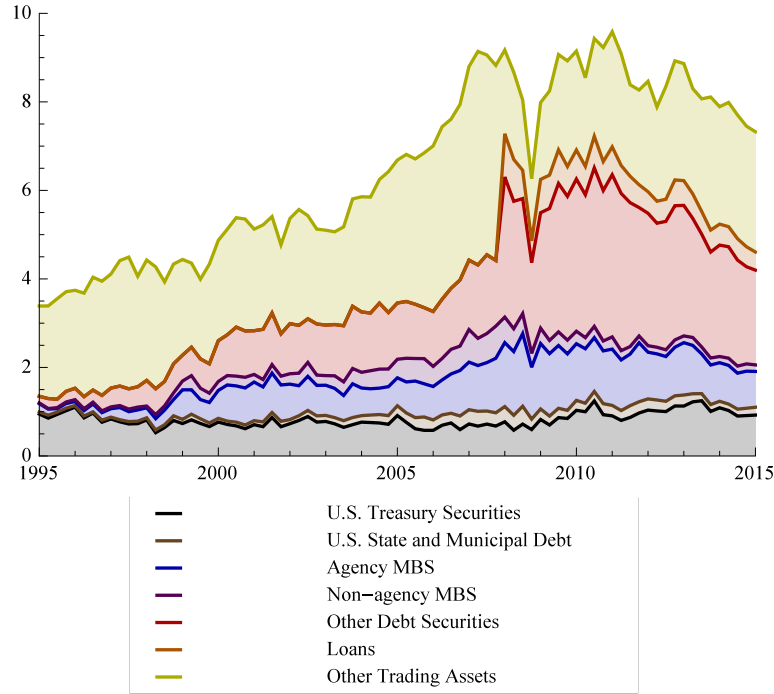


FIGURE 1: TRADING ASSETS AS PERCENTAGE OF TOTAL ASSETS

trading desks for fixed income securities declined from its pre-crisis level of 9 percent in 2007 to 7 percent in late 2014. Further evidence that the new capital requirements reduced the ability of financial intermediaries to provide liquidity by facilitating trades (i.e., by lending securities or taking them on inventory) is indicated by the decline in primary dealers' net inventories relative to the size of the U.S. corporate bond market, which is shown in Figure 2.² Primary dealers have been large liquidity providers in corporate bond markets by holding securities on inventory and thereby absorbing demand and supply imbalances.³ However, holding bonds on inventory increases the balance sheet and is thus subject to the new regulatory constraints. Since the peak

¹The data for Figure 1 is downloadable at www.newyorkfed.org/research/banking_research/quarterly_trends.html (as of 31.10.2015).

²A more detailed discussion about how financial intermediaries facilitate trades is provided by Madhavan (2000), Duffie (2012), the Committee on the Global Financial System (2014), and Fender and Lewrick (2015). Fleming (2003) provides an overview of different measures for assessing liquidity in fixed income markets.

³A primary dealer is a large financial institution which is entitled to directly purchase government securities from the U.S. Federal Reserve in order to subsequently sell these to its clients.

in 2007, primary dealers’ net inventories of corporate bonds dropped by nearly 80 percent, while the size of the corporate bond market increased by nearly 50 percent.⁴ However, a decrease in primary dealers’ net inventories might also be caused by their reduced willingness to hold bonds on inventory and thereby risk losses in volatile market environments, and does not necessarily mean that the new regulatory constraints are binding.⁵

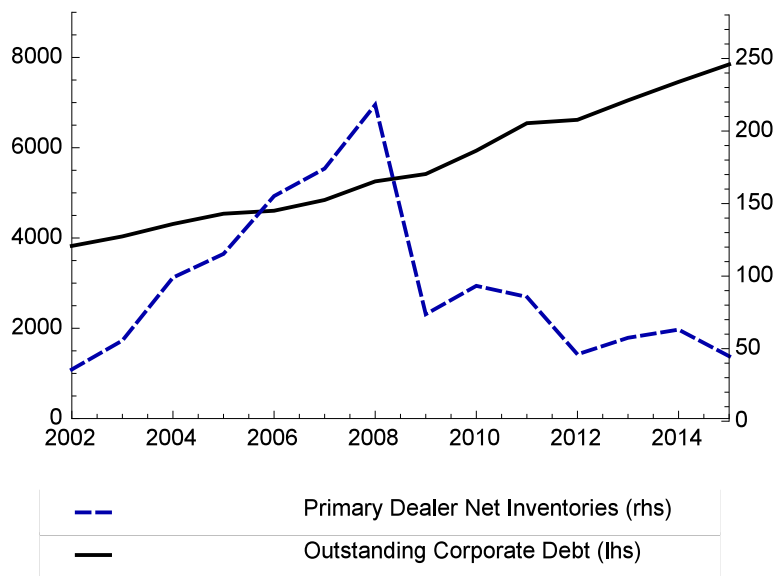


FIGURE 2: U.S. CORPORATE BOND MARKET

Although corporate bond market liquidity is difficult to assess, policy makers became aware of the looming liquidity squeeze and proposed new measures to stabilize these markets. On September 16, 2014, the U.S. Securities and Exchange Commission (SEC, hereafter) promoted a more standardized issuance of corporate bonds in order to centralize corporate bond markets:

“I will not pretend that migrating fixed income transactions from the over-the-counter markets is a small task; however, I will note that in the early part of the twentieth century, there was an active market in corporate and municipal bonds on the New York Stock Exchange.” (Gallagher, 2014)

In this paper, we take a closer look at the normative implications of centralizing secondary

⁴Both time series shown in Figure 2 are measured in USD bn. The source for the primary dealers’ net inventories is www.marketaxess.com/research/market-insights/ (as of 31.10.2015) and the source of the corporate bond market size is www.sifma.org/research/statistics.aspx (as of 31.10.2015).

⁵See Adrian et al. (2013), the Committee on the Global Financial System (2014), and Fleming and Lewrick (2015) for a more detailed discussion about this argument.

corporate bond markets in the post-crisis period. That is, we account for this specific market environment which is characterized by high search frictions and expansionary monetary policy. The latter factor has prompted the introduction of the zero lower bound in late 2008 and subsequent maturity extension programs, causing corporate bonds to be priced above their fundamental value, which in turn has spurred corporate bond issuance.

In our model, companies issue bonds in a centralized market in order to finance their investment projects. Corporate bonds are book-keeping entries and therefore they cannot serve as a medium of exchange; i.e., only money does. Nevertheless, agents want to hold corporate bonds since they are a superior store of value and since they can trade them for money in a secondary bond market according to their consumption needs. In contrast to the primary bond market, the secondary bond market is an over-the-counter market, which features search and bargaining frictions. We assume that agents meet in bilateral meetings and negotiate about the terms of trade. Hence, although financial intermediaries play an important role in secondary corporate bond markets in reality, we abstract from such intermediations in our model. In practice, financial intermediaries mainly facilitate trades by cushioning demand and supply imbalances and thereby affect the degree of search frictions. In our model, we simplify along this dimension and represent the degree of search frictions using an exogenous matching probability. We refer to agents who have a match in the secondary bond market as *active* and to those who do not as *passive*.

We show that when secondary bond markets feature an over-the-counter structure, agents who want to sell bonds only obtain a certain fraction of the bargaining power, while they receive all of it in the case of a centralization. Hence, market centralization provokes a shift in bargaining power, which generally increases the demand for bonds by such a degree that bonds become scarce and exhibit a liquidity premium. In turn, this reduces the funding costs of companies, which curbs corporate bond issuance. Due to the higher supply, agents trade more bonds in the secondary bond market, although the number of matches remains unchanged. The possibility to do so induces agents to reduce their demand for money, which depresses the value of money by such an extent that active and passive agents consume less after the centralization of the secondary bond market.

We show that a centralized bond market may result in a superior outcome compared to the initial over-the-counter structure only on the condition that the regulator also succeeds in changing search frictions. In particular, decreasing as well as increasing search frictions may improve the allocation and welfare. A reduction in the search frictions, i.e., an increase in market liquidity, results in a further increase in bond prices and hence in corporate bond issuance. In turn, this further depresses the value of money such that active and passive agents consume less. However, reducing search frictions implies that there are more active agents, and since they can consume more than passive agents, this results in an overall increase in welfare. In contrast, an increase in the search frictions, i.e., a reduction in market liquidity, results in a reduction in the demand for bonds such that bonds are priced at their fundamental value. Due to the lower probability of being able to trade in the secondary bond market, agents increase their demand for money, which raises the value of money and thereby increases consumption. Thus, although an increase in search frictions results in less active agents, overall welfare increases. Our findings

indicate that a centralization of secondary bond markets may only succeed in improving welfare if the regulator can also affect search frictions. Otherwise, centralization always results in an inferior outcome as compared to the initial over-the-counter structure.

We then analyze whether an intervention exists which is always superior to the initial over-the-counter structure. We find that a reduction in search frictions, i.e., an increase in market liquidity, combined with a reduction in the bargaining power of agents who want to sell bonds, always improves welfare. The reason is that a reduction in bargaining power incentivizes agents to increase their demand for money, while a reduction in search frictions increases the number of trades in the secondary bond market. The combination of both effects results in an optimal intervention for an economy which features low inflation rates, such as the United States in the post-crisis period.

2 Literature

Our paper is part of the “New Monetarist Economics,” building on Kiyotaki and Wright (1989) and especially Lagos and Wright (2005).⁶ Similar to Berentsen et al. (2014), agent-types are alternating in our model, which generates an endogenous role for a secondary financial market. The main finding of Berentsen et al. (2014) is that increasing search frictions in a centralized secondary financial market can be welfare-improving. However, Berentsen et al. (2014) state that it is never optimal to shut the secondary financial market down. In contrast, we focus on the optimal market structure of secondary financial markets; i.e., whether any benefits are associated with centralizing corporate bond markets. We show that centralizing corporate bond markets reduces overall welfare as it incentivizes agents to reduce their demand for money. However, once markets are centralized, as in Berentsen et al. (2014), welfare can be improved by either granting full access or by shutting the secondary financial market down. The reason for this difference arises because Berentsen et al. (2014) analyse an equilibrium where both money and bonds are scarce, which only exists for high inflation rates. Since the post-crisis period is rather characterized by low inflation rates, we focus on low-inflationary environments and involve the existence of further equilibria in our analysis.

Research that is more closer related to the present paper is the study by Geromichalos and Herrenbrueck (2015), who also integrate an over-the-counter market in the spirit of Duffie et al. (2005) into a New Monetarist framework.⁷ The authors find that for secondary financial markets which do not exhibit any search frictions, i.e., where everybody is able to trade, bargaining achieves a superior outcome to that of centralized trading. We generalize the result of Geromichalos and Herrenbrueck (2015) to corporate bond markets which feature search frictions and show that centralization always results in a reduction in welfare. These authors, however, do not investigate one of the central theses of our paper; i.e., that the regulator may only achieve

⁶For an overview of this literature, see Williamson and Wright (2010), Nosal and Rocheteau (2011), and Lagos et al. (2015).

⁷There are many other studies that integrate over-the-counter financial markets in a New Monetarist framework. Here, we refer to the studies of Lagos and Rocheteau (2009), Lagos et al. (2011), Rocheteau and Wright (2013), Trejos and Wright (2014), Lagos and Zhang (2015), Berentsen et al. (2015), and Huber and Kim (2015).

a superior outcome compared to the initial over-the-counter structure on the condition that he also succeeds in affecting search frictions.

Along the same lines, Lagos and Zhang (2015) present a model where they explicitly incorporate the role of financial intermediaries in the secondary financial market. Concretely, Lagos and Zhang (2015) assume that agents do not directly trade with each other, but that they trade with a financial intermediary which has access to a competitive interdealer market. The authors find that the higher the bargaining power of financial intermediaries, the lower the asset prices become. However, the authors do not focus on the social implications of such a shift in bargaining power. Compared to our framework, an increase in the bargaining power of financial intermediaries coincides with a decrease in the bargaining power of agents who want to sell bonds. Similar to Lagos and Zhang (2015), we find that such a shift in bargaining power generally reduces asset prices; however, we also show that it improves welfare. Furthermore, Lagos and Zhang (2015) mainly focus on asset pricing, while we focus on the optimal design of the market structure; i.e., whether centralizing the secondary financial market proves effective in improving welfare compared to an over-the-counter structure.

Our paper is also related to the literature that studies the markets for risk-sharing by means of insurance contracts (e.g., Parlour and Rajan 2001, Thompson 2010, Duffie and Zhu 2011, Leitner 2013, Acharya and Bisin 2014, and Stephens and Thompson 2014). These papers are mainly concerned about the moral hazard of the involved counterparties in the insurance contract and how over-the-counter markets affect their behaviour. Moral hazard arises, because agents do not need to disclose trade information publicly in over-the-counter markets. In contrast, in centralized markets this information is revealed, thereby preventing moral hazard (see Leitner 2013, and Acharya and Bisin 2014). Duffie and Zhu (2011) and Stephens and Thompson (2014) also analyze the benefits of centralized markets. In contrast to the previous studies, their focus is on the netting of positions and pooling of risk through centralized markets. Duffie and Zhu (2011) show that centralized trading can be beneficial due to the netting of trades, which reduces counterparty risk. In contrast, Stephens and Thompson (2014) show that the pooling of risk through centralized markets results in an adverse selection of insurance providers, as everybody wants to benefit from the low insurance premia of bad providers. Our study differs from this literature, because neither counterparty risk nor asymmetric information exists among agents. We show that a centralization of corporate bond markets is welfare-decreasing, since it reduces the incentive of agents to self-insure against idiosyncratic liquidity shocks. In our model, the regulator may only achieve a superior outcome on the condition that he also succeeds in controlling search frictions.

3 Environment

For our model specifications, we apply the basic environment analyzed in Huber and Kim (2015).⁸ Time is discrete and the economy is populated by a $[0, 1]$ -continuum of infinitely lived agents. In each period, there are three markets that open sequentially. In the first market, agents trade money for corporate bonds. In the second market, agents produce or consume market-2 goods. In the third market, financial contracts are redeemed and agents produce and consume market-3 goods. We refer to these three markets as the *secondary bond market*, the *goods market*, and the *primary bond market*.

At the beginning of each period, agents receive two i.i.d shocks: a preference shock and an entry shock. The preference shock determines the ability to produce or consume the market-2 good in the goods market: An agent can produce but not consume with probability n , while he can consume but not produce with probability $1 - n$. Throughout the paper, we assume $n \in (0.5, 1)$. The entry shock determines whether or not an agent has access to the secondary bond market, denoted respectively as probability δ , or probability $1 - \delta$.

In the goods market, trading is frictionless and competitive. Consumers enjoy utility $u(q)$ from q consumption, where $u'(q), -u''(q) > 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Producers incur a utility cost $c(q) = q$ from producing q units of the market-2 good. No record-keeping technology exists and agents cannot commit. As a consequence, producers ask for immediate compensation from consumers; i.e., a medium of exchange is required.

The primary bond market is frictionless, and agents can produce and consume the market-3 good. Agents receive utility $U(x)$ from x consumption, where $U'(x), -U''(x) > 0$, $U'(0) = \infty$, and $U'(\infty) = 0$. The market-3 good is produced by means of a linear production technology; i.e., h hours of work produce h units of the market-3 good. Agents discount between two consecutive periods with the discount factor $\beta \in (0, 1)$. In this third market, a central bank supplies a perfectly divisible, storable, and intrinsically useless object called money. The change in the stock of money is given by $M_{t+1} = \gamma M_t$, where γ denotes the gross growth rate of money and M_t (M_{t+1}) denotes the stock of money in period t ($t + 1$). The central bank injects (withdraws) money through a lump-sum transfer T_t to all agents, where $T_t = M_{t+1} - M_t = (\gamma - 1)M_t$. For notational simplicity, we omit the time subscript t going forward and denote next-period variables by $+1$, and previous-period variables by -1 .

Our model assumes a continuum $\ell > 0$ of homogeneous firms who issue a storable and perfectly divisible object called a bond in the primary bond market.⁹ In contrast to money, bonds are intangible objects; i.e., no physical object exists. As a consequence, only money can serve as a medium of exchange in the goods market.¹⁰ Any firm can enter the market, but there is a cost

⁸Our model builds on Lagos and Wright (2005) and especially on Berentsen et al. (2007, 2014, and 2015). The main difference to our previous work (Huber and Kim 2015) is that we analyze the optimal structure of corporate bond markets instead of central bank purchases of government bonds.

⁹Our firms have some similarities to the ones studied in Berentsen et al. (2011), Mortensen and Pissarides (1994), and Williamson (1987, 2012). However, we deviate in several dimensions from these studies, as will be shown further on in the paper.

¹⁰An extended discussion about the frictions that make money essential can be found, for instance, in Kocherlakota (1998), Wallace (2001), Lagos and Wright (2005) and Shi (2006). A more detailed explanation about why

C in terms of market-3 goods to enter the market. A generic firm issues a quantity of b_f private bonds at price $\rho < 1$ to finance its project. Bonds can only be purchased by market-3 goods, since firms use these goods as an input for their projects. If the project is successful during the period, then the firm honors its obligation and pays back 1 unit of the market-3 good in the next period's primary bond market. If a firm cannot repay its obligation, then the firm defaults and will be forced permanently out of the market. The period payoff of a generic firm is $\frac{f(\rho\phi b_f)}{\ell}z$, where $f' > 0$, $f'' < 0$, $f'(0) = \infty$, and $z \in [0, 1]$. We denote $G(z)$ as the distribution of $z \in [0, 1]$ with a density function $g(z)$, and an expectation value $\mathbb{E}(z)$. A projects payoff z is realized before the next period's primary bond market opens, but after the secondary bond market closes. Firms cannot issue more bonds than \bar{b}_f , where $f(\rho\phi\bar{b}_f) = \ell\phi\bar{b}_f$, since it is certain that agents' bonds cannot be redeemed for $\phi b_f > \phi\bar{b}_f$ for any realization of z .

At the beginning of each period, and after the realization of the two i.i.d shocks, agents can trade money for bonds in the secondary bond market. Consumers and producers meet at random in bilateral meetings and bargain over the quantity of money and bonds to be exchanged. A consumer meets a producer with probability $\delta \in [0, 1]$, and a producer meets a consumer with probability $\delta^p = \delta(1 - n)n^{-1}$, since $n \geq 0.5$. Agents who are able to trade in this market are called *active* and those who are not are called *passive*.

4 Agent's Decisions

Hereafter, we focus on the agents' decision problem in a representative period t . We first present the last market (primary bond market) and work backwards to the first market (secondary bond market).

4.1 Primary Bond Market

In the primary bond market, agents can produce and consume the market-3 good and receive the lump-sum money transfer T from the central bank. A firm's project is successful with probability $1 - \sigma$, in which case it honors its obligations and pays back 1 unit of the market-3 good for each maturing bond. With probability σ , a firm's project is not successful, in which case the firm defaults, and consequently a bond-holder receives zero. Old and new firms issue bonds at price $\rho < 1$ in this market and agents decide about the amount of money and bonds that they will take into the next period. An agent entering the primary bond market with m units of money and b units of bonds has the value function $V_3(m, b)$ and solves the following decision problem:

$$V_3(m, b) = \max_{x, h, m_{+1}, b_{+1}} [U(x) - h + \beta V_1(m_{+1}, b_{+1})], \quad (1)$$

subject to

$$x + \phi m_{+1} + \phi \rho b_{+1} = h + \phi m + (1 - \sigma)\phi b + \phi T, \quad (2)$$

bonds cannot serve as a medium of exchange is provided by Kocherlakota (2003), Andolfatto (2011), Berentsen and Waller (2011), and Berentsen et al. (2014 and 2015).

where ϕ denotes the price of money in terms of market-3 goods, and h denotes hours worked. Due to the linear production technology, the choice of m_{+1} and b_{+1} is independent of m and b . As a result, each agent exits the market with the same amount of money and bonds. Furthermore, all agents buy the same amount of bonds from each firm; i.e., all agents hold an identical fraction of defaulting bonds in their portfolios.¹¹

4.1.1 Goods Market

In the goods market, all agents take prices as given and trade against the market. A consumer entering the goods market has the following decision problem:

$$V_2^c(m, b) = \max_q \left[\begin{array}{l} u(q) + V_3(m - pq, b) \\ s.t. \ m \geq pq. \end{array} \right], \quad (3)$$

where $V_2^c(m, b)$ denotes his value function, and where p denotes the price of one unit of the market-2 good q . The budget constraint simply states that a consumer cannot spend more money than he has.

If the budget constraint is non-binding, then we have $\frac{\partial q}{\partial m} = 0$, and consequently consumers are able to consume the efficient quantity q^* , which is the solution of $u'(q^*) = 1$. If the constraint is binding, then we have $\frac{\partial q}{\partial m} = \phi$ and $u'(q) > 1$. Hence, consumers spend all their money and consumption is inefficiently low. In equilibrium, *active* consumers will hold more money than *passive* consumers and are therefore able to consume more in this market. Hereafter, quantities associated to active agents are denoted with a hat “^”.

A producer entering the goods market has the following decision problem:

$$V_2^p(m, b) = \max_{q_p} [-c(q_p) + V_3(m + pq_p, b)], \quad (4)$$

where q_p satisfies the market clearing condition

$$(1 - n)[\delta \hat{q} + (1 - \delta)q] = nq_p. \quad (5)$$

The market clearing condition states that the produced quantity of a consumer, q_p , equals the sum of the quantities consumed by active, $(1 - n)\delta \hat{q}$, and passive consumers, $(1 - n)(1 - \delta)q$, divided by the number of producers in the market. Due to the assumption of competitive pricing, it is easy to show that producers do not earn any surplus; i.e., that the marginal cost of producing q_p units of market-2 goods equals the market price $p\phi$.

4.2 Secondary Bond Market

In the secondary bond market, consumers and producers meet at random in bilateral meetings, and the trade surplus is split according to the Kalai (1977) solution. The Kalai trading protocol

¹¹To see this, consider the following example: $b_f = 9$ and $\ell = 10/9$ according to decisions of the firms. Then, we have $\ell\phi b_f = \phi b = 10$; i.e., each firm offers 9 bonds, but each agent holds 10 bonds. In other words, agents must be able to buy the same number of bonds from each firm.

has the property of being strongly monotonic; i.e., no agent is made worse off by an expansion of the bargaining set.¹² An agent enters the secondary bond market with a portfolio (m_j, b_j) and leaves it with a portfolio (\hat{m}_j, \hat{b}_j) , where $j = c$ for consumers and $j = p$ for producers. By the market clearing condition, we obtain

$$\hat{m}_c - m_c = -(\hat{m}_p - m_p) \quad \text{and} \quad \hat{b}_p - b_p = -(\hat{b}_c - b_c).$$

An active producer spends $d_m \equiv \hat{m}_c - m_c$ units of money and an active consumer sells $d_b \equiv \hat{b}_p - b_p$ units of bonds, which implies that the budget constraints of a producer and a consumer are given by

$$\phi m_p \geq \phi d_m \quad \text{and} \quad \phi b_c \geq \phi d_b. \quad (6)$$

The total trade surplus in this market is $u(\hat{q}) - u(q) - \phi d_m$, where $u(\hat{q}) - u(q) - \phi d_b$ is the consumer's surplus and $d_b - d_m$ is the producer's surplus. The Kalai constraint splits the total trade surplus among matched agents according to their bargaining power

$$(1 - \eta) [u(\hat{q}) - u(q) - \phi d_b] = \eta \phi (d_b - d_m), \quad (7)$$

where η is the bargaining power of a consumer, and $1 - \eta$ that of a producer. An active agent's decision problem is

$$K(m_c, m_p, b_c, b_p) \equiv \max_{d_m, d_b} [u(\hat{q}) - u(q) - \phi d_m] \quad s.t. \quad (6) \text{ and } (7). \quad (8)$$

That is, the Kalai protocol maximizes the total trade surplus and is therefore efficient. If $K(m_c, m_p, b_c, b_p)$ is differentiable with respect to $x = m_c, m_p, b_c, b_p$, then we have

$$\frac{\partial K}{\partial x} = u'(\hat{q}) \frac{\partial \hat{q}}{\partial x} - u'(q) \frac{\partial q}{\partial x} - \phi \frac{\partial d_m}{\partial x}. \quad (9)$$

If the active agent's budget constraints (6) are non-binding, then the first-order condition of (8) with respect to d_m is $u'(\hat{q}) = 1$; i.e., active consumers can consume the efficient quantity. The value functions of consumers and producers entering the secondary bond market are given by

$$\begin{aligned} V_1^c(m_c, b_c) &= \delta \eta K(m_c, m, b_c, b) + V_2^c(m_c, b_c), \\ V_1^p(m_p, b_p) &= \delta_p (1 - \eta) K(m, m_p, b, b_p) + V_2^p(m_p, b_p), \end{aligned} \quad (10)$$

where (m, b) denotes the portfolio of the trading partner.

¹²For a formalization of Kalai bargaining, see Rocheteau and Wright (2005), and Aruoba et al. (2007). Recent studies that apply Kalai bargaining to financial markets are, for example, Geromichalos and Herrenbrueck (2015), Berentsen et al. (2015), and Huber and Kim (2015). A study of the monotonicity properties of Kalai can be found in Chun and Thomson (1988).

5 Firm's Decisions

Firms only interact with agents in the primary bond market. If their project was successful during the period, they honor their obligation and repay 1 unit of the market-3 good for each maturing bond. Furthermore, old firms which were successful in the previous period and new firms which can afford to pay the entry cost C in terms of market-3 goods issue a quantity b_f of corporate bonds at price $\rho < 1$ to finance their new projects. A firm cannot issue more bonds than \bar{b}_f , where $f(\rho\phi\bar{b}_f) = \ell\phi\bar{b}_f$, because otherwise agents' bonds would not be redeemed, independent of the realization of the projects' payoff $z \in [0, 1]$. Hereafter, we denote $\mathbb{b} \equiv \phi b_f$ and $\bar{\mathbb{b}} \equiv \phi\bar{b}_f$. A firm has the value function V_f and solves the following decision problem

$$V_f = \max_{\mathbb{b} \in [0, \bar{\mathbb{b}}]} \left\{ \int_0^{\frac{\ell\mathbb{b}}{f(\rho\mathbb{b})}} \frac{f(\rho\mathbb{b})}{\ell} z dG(z) + \int_{\frac{\ell\mathbb{b}}{f(\rho\mathbb{b})}}^1 \left[\frac{f(\rho\mathbb{b})}{\ell} z - \mathbb{b} + \beta V_{f+} \right] dG(z) \right\},$$

where $\ell > 0$ denotes the set of firms in the market, $\frac{f(\rho\mathbb{b})}{\ell} z$ denotes the period payoff of a generic firm, and $G(z)$ denotes the distribution function of z . A firm with a project realization of $z \in \left[0, \frac{\ell\mathbb{b}}{f(\rho\mathbb{b})}\right)$ is not able to honor its debt and will consequently be forced to default and to be permanently excluded from the market. A firm with a project realization of $z \in \left[\frac{\ell\mathbb{b}}{f(\rho\mathbb{b})}, 1\right]$ can fulfill its obligations \mathbb{b} and has the continuation value βV_{f+} . The firm's decision problem depends on monetary policy through the issue price of bonds in the primary bond market, ρ , which affects the number of firms in the market, ℓ , and the real quantity of bonds issued by a generic firm, \mathbb{b} .

In steady state, it holds that $V_{f+} = V_f$ and thus, we can simplify a firm's value function to

$$V_f = \frac{1}{1 - \beta(1 - \sigma)} \left[\frac{f(\rho\mathbb{b})}{\ell} \mathbb{E}(z) - \phi b_f (1 - \sigma) \right],$$

where $\mathbb{E}(z)$ denotes the expectation value of a firm's project payoff z and $\sigma = G\left(\frac{\ell\mathbb{b}}{f(\rho\mathbb{b})}\right)$ denotes the default probability. By the entry condition, we also have $V_f = C$ in equilibrium. A firm will not strategically default if $\beta V_f \geq \mathbb{b}$.

Going forward, we focus on the case where the density at $z = 0$ equals $g(0) = g$ and the density at $z = 1$ equals $g(1) = 1 - g$; i.e., a firm's period operation is fully successful with probability $1 - g$ and not successful with probability g . In such a case, a firm's value function simplifies to

$$V_f = \frac{1 - g}{1 - \beta(1 - g)} \left[\frac{f(\rho\mathbb{b})}{\ell} - \mathbb{b} \right].$$

From the above equation, it is easy to see that the optimal real quantity of bonds issued by a

firm \mathbb{b} satisfies

$$\rho f'(\rho \mathbb{b}) = \ell. \quad (11)$$

Furthermore, in equilibrium we also have $\sigma = g$ and $V_f = C$. Hence, a firm will not strategically default if $\beta C \geq \mathbb{b}$, or equivalently if

$$\beta C = \frac{\beta(1-g)}{1-\beta(1-g)} \left[\frac{f(\rho \mathbb{b})}{\ell} - \mathbb{b} \right] \geq \mathbb{b}. \quad (12)$$

Equation (11) and (12) state that the number of firms in the market, ℓ , and the real quantity of bonds issued by a firm, \mathbb{b} , is completely determined by the endogenous price of bonds in the primary bond market, ρ , the production technology of firms, $f(x)$, and the exogenous terms C , β , and g . By the resource constraint, it also holds in equilibrium that $\phi b = \ell \mathbb{b}$; i.e., that the total real quantity of bonds held by agents, ϕb , equals the total real quantity of bonds issued by firms, $\ell \mathbb{b}$.

6 Equilibria

Hereafter, we characterize two stationary monetary equilibria, where all agents follow identical strategies and where real variables are constant over time. In the first equilibrium, trading is unconstrained; i.e., (6) is non-binding. We label this equilibrium as type-A. In the second equilibrium, producers are unconstrained, while the bond constraint of consumers is binding. This equilibrium is labeled as type-B. All proofs are relegated to the Appendix.

6.1 Type-A Equilibrium

In a type-A equilibrium, the bond constraint of active consumers is non-binding, $\phi b_c > \phi d_b$, which allows them to consume the efficient quantity, $u'(\hat{q}) = 1$. Furthermore, the cash constraint of active producers is non-binding, $\phi m_p > \phi d_m$, which means that they are indifferent between holding money or bonds.

Proposition 1 *Given $\{\gamma, \beta, \delta, \eta, n, C, g\}$, a type-A equilibrium is a list $\{\hat{q}, q, q_p, \rho, \mathbb{b}, \ell\}$ satisfying (5), (11), (12), and*

$$1 = u'(\hat{q}), \quad (13)$$

$$\frac{\gamma}{\beta} = (1-n)\delta\eta [u'(\hat{q}) - u'(q)] + (1-n)u'(q) + n, \quad (14)$$

$$\rho = \frac{\beta}{\gamma}(1-g). \quad (15)$$

Equation (13) states that active consumers can consume the efficient quantity, because the budget constraints (6) are non-binding in the secondary bond market. Equation (14) states that the marginal benefit of acquiring one additional unit of money equals the sum of three terms: If an agent turns out to be a consumer (*active* or *passive*), he has the marginal utility $u'(q)$; if

an agent turns out to be an *active* consumer, he additionally receives a fraction η of the trade surplus $u'(\hat{q}) - u'(q)$ such that he can consume the efficient quantity $u'(\hat{q}) = 1$; if an agent turns out to be a producer (*active* or *passive*), he obtains a marginal utility of 1. Equation (15) states that bonds compensate for the marginal cost of holding money, γ/β , and the default probability of firms, $\sigma = g$.

6.2 Type-B Equilibrium

In a type-B equilibrium, the bond constraint of active consumers is binding, $\phi b_c = \phi d_b$; i.e., they would like to sell more bonds than they have in order to satisfy their consumption needs. Hence, they cannot achieve the efficient quantity and $u'(\hat{q}) > 1$ holds. As in the type-A equilibrium, the cash constraint of active producers is non-binding, $\phi m_p > \phi d_m$.

Proposition 2 *Given $\{\gamma, \beta, \delta, \eta, n, C, g\}$, a type-B equilibrium is a list $\{\hat{q}, q, q_p, \rho, \mathbb{b}, \ell\}$ satisfying (5), (11), (12), and*

$$\ell \mathbb{b} = (1 - \eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q), \quad (16)$$

$$\frac{\gamma}{\beta} = (1 - n) \left[\delta u'(\hat{q}) \frac{\eta + (1 - \eta)u'(q)}{\eta + (1 - \eta)u'(\hat{q})} + (1 - \delta)u'(q) \right] + n, \quad (17)$$

$$\rho = \frac{\beta}{\gamma} \left[1 - g + (1 - n)\delta\eta \frac{u'(\hat{q}) - 1}{\eta + (1 - \eta)u'(\hat{q})} \right]. \quad (18)$$

Equation (16) is based on the Kalai constraint, where we used $\phi b = \ell \mathbb{b}$. Equation (17) states that the marginal benefit of acquiring one additional unit of money equals 1 for producers (*active* or *passive*), it equals the marginal utility $u'(q)$ for *passive* consumers, and it equals the share of the surplus $u'(\hat{q}) [\eta + (1 - \eta)u'(q)] [\eta + (1 - \eta)u'(\hat{q})]^{-1}$ for *active* consumers. Equation (18) states that bonds are priced above their fundamental value, $\beta(1 - g)/\gamma$, and exhibit a liquidity premium equal to $(1 - n)\delta\eta [u'(\hat{q}) - 1] [\eta + (1 - \eta)u'(\hat{q})]^{-1}$. Hence, because bonds are scarce in this equilibrium, $\phi b_c = \phi d_b$, they do not compensate for the marginal cost of holding money and the default probability of firms.

7 Regions of Existence

Hereafter, we specify two non-overlapping regions in which the type-A and the type-B equilibria exist, as a function of the inflation rate γ and the entry cost C . In order to do this, we first need to derive the reaction of firms in equilibrium. Let $\alpha(x) = -\frac{xf''(x)}{f'(x)}$ denote the coefficient of risk aversion of $f(x)$, and let $\epsilon(x) = \frac{xf'(x)}{f(x)}$ denote the elasticity of $f(x)$. In equilibrium, firms' decisions are completely determined by the endogenous bond price ρ and the exogenous terms C , g , and β through (11) and (12). In Lemma 3, we derive the effect on the real quantity of bonds issued by a generic firm, \mathbb{b} , the number of firms in the market, ℓ , and the total real quantity of bonds issued, $\ell \mathbb{b} = \phi b$, in response to a marginal increase in the exogenous terms C , g , and β .

Lemma 3 *In equilibrium,*

$$\begin{aligned} \frac{\partial \mathbb{b}}{\partial C} &> 0, & \frac{\partial \mathbb{b}}{\partial g} &> 0, & \frac{\partial \mathbb{b}}{\partial \beta} &< 0, \\ \frac{\partial \ell}{\partial C} &< 0, & \frac{\partial \ell}{\partial g} &< 0, & \frac{\partial \ell}{\partial \beta} &> 0, \end{aligned}$$

and

$$\frac{\partial \ell \mathbb{b}}{\partial C}, \frac{\partial \ell \mathbb{b}}{\partial g}, -\frac{\partial \ell \mathbb{b}}{\partial \beta} \begin{cases} < 0, & \text{if } \alpha(\rho \mathbb{b}) > 1, \\ = 0, & \text{if } \alpha(\rho \mathbb{b}) = 1, \\ > 0, & \text{if } \alpha(\rho \mathbb{b}) < 1. \end{cases}$$

Lemma 3 shows that the response of the total real quantity of bonds issued, $\ell \mathbb{b} = \phi b$, to a marginal increase in the exogenous terms C , g , and β , critically depends on the coefficient of risk aversion of $f(x)$. In Lemma 4, we derive the effect on \mathbb{b} , ℓ , and $\ell \mathbb{b} = \phi b$, in response to a marginal increase in the endogenous bond price ρ .

Lemma 4 *In equilibrium,*

$$\begin{aligned} \frac{\partial \mathbb{b}}{\partial \rho} \begin{cases} < 0, & \text{if } \alpha(\rho \mathbb{b}) > 1 - \epsilon(\rho \mathbb{b}), \\ = 0, & \text{if } \alpha(\rho \mathbb{b}) = 1 - \epsilon(\rho \mathbb{b}), \\ > 0, & \text{if } \alpha(\rho \mathbb{b}) < 1 - \epsilon(\rho \mathbb{b}), \end{cases} \\ \frac{\partial \ell}{\partial \rho} > 0, \end{aligned}$$

and

$$\frac{\partial \ell \mathbb{b}}{\partial \rho} \begin{cases} < 0, & \text{if } \alpha(\rho \mathbb{b}) > 1, \\ = 0, & \text{if } \alpha(\rho \mathbb{b}) = 1, \\ > 0, & \text{if } \alpha(\rho \mathbb{b}) < 1. \end{cases}$$

Note, that by the incentive constraint

$$\frac{\beta(1-g)}{1-\beta(1-g)} \left[\frac{f(\rho \mathbb{b})}{\ell} - \mathbb{b} \right] \geq \mathbb{b},$$

we have

$$\epsilon(\rho \mathbb{b}) \equiv \frac{f'(\rho \mathbb{b})\rho \mathbb{b}}{f(\rho \mathbb{b})} \leq \beta(1-g) < 1;$$

i.e., the elasticity of $f(x)$ is bounded above by $\beta(1-g)$. With the results from Lemma 3 and Lemma 4, we are able to derive the regions of existence of the type-A and the type-B equilibrium. In Proposition 5, we derive these regions for an economy where the entry cost is relatively low $C < \bar{C}$, and where the production technology $f(x)$ satisfies $\alpha(x) = -\frac{xf''(x)}{f'(x)} < 1$ and $\epsilon(x) =$

$$\frac{xf'(x)}{f(x)} \leq \beta(1-g) \text{ for all } x \geq 0.^{13}$$

Proposition 5 *Let $C < \bar{C}$, $\alpha(x) < 1$ and $\epsilon(x) \leq \beta(1-g)$ for all $x > 0$. If $\gamma < \gamma_{AB}$, equilibrium prices and quantities are characterized by Proposition 1; and if $\gamma > \gamma_{AB}$, equilibrium prices and quantities are characterized by Proposition 2.*

Figure 3 shows the regions of existence as a function of the search and bargaining frictions. The chart on the left-hand side shows the respective regions for $0 < \delta\eta < 1$, while the chart on the right-hand side shows them for $\delta\eta = 1$. In the proof of Proposition 5, we show that at $C = \bar{C}$, the critical inflation rate that separates the type-A from the type-B equilibrium equals

$$\gamma_{AB,C=\bar{C}} = \beta \left\{ (1-n) [\delta\eta + (1-\delta\eta)u'(q^*/2)] + n \right\}. \quad (19)$$

From the above equation, it is clear that for $\delta\eta = 1$ we obtain $\gamma_{AB,C=\bar{C}} = \beta$, and hence for any $C < \bar{C}$ only the type-B equilibrium exists. Furthermore, we find that \bar{C} is decreasing in $\delta\eta$; i.e., for $\delta\eta = 1$ we obtain a lower value of \bar{C} as compared to $0 < \delta\eta < 1$.

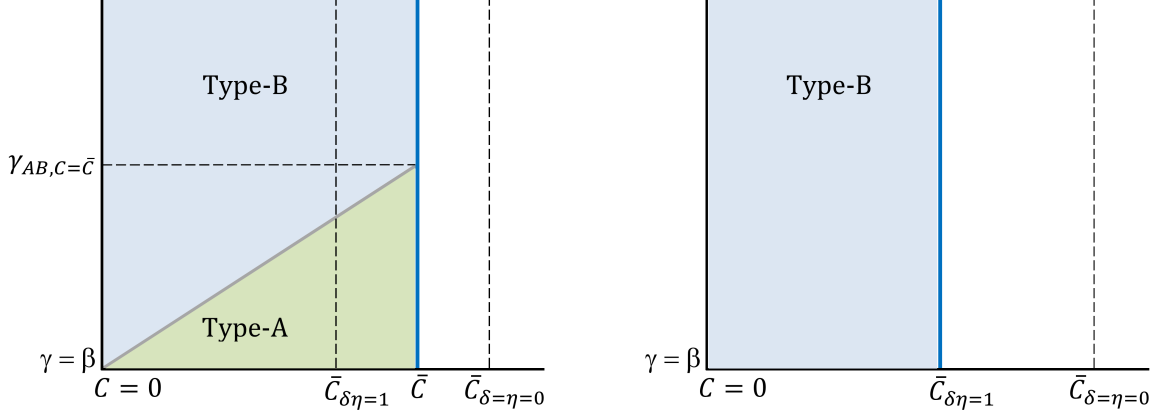


FIGURE 3: REGIONS OF EXISTENCE

In the subsequent section, we analyze the normative implications of centralizing corporate bond markets in an economy similar to the one observed in the post-crisis data of the United States. That is, we assume that the expansive monetary policy in the post-crisis period, such as central bank purchases of government bonds, resulted in a scarcity of corporate bonds such that they exhibit a liquidity premium.¹⁴ However, integrating such central bank interventions goes beyond the scope of this paper, and we refer the interested reader to Williamson (2012)

¹³For instance, a standard production function of the form $f(x) = Ax^{1-\alpha}$ with $1-\alpha \leq \beta(1-g)$, $\alpha < 1$, and $A > 0$ satisfies these properties, since $-\frac{xf''(x)}{f'(x)} = \alpha < 1$ and $\epsilon(x) = \frac{xf'(x)}{f(x)} = 1-\alpha$.

¹⁴The existence of liquidity premia is discussed in more detail in Geromichalos et al. (2007), Lagos and Rocheteau (2008), Lagos (2010a, b, 2011), Jacquet and Tan (2012), Lester et al. (2012), Nosal and Rocheteau (2013), Lagos and Zhang (2014), and Berentsen et al. (2014, 2015).

for a formal proof of this argument. For a liquidity premium to exist, we require low entry costs, $C < \bar{C}_{\delta\eta=1}$, and $\gamma > \gamma_{AB}$ with γ being sufficiently close to γ_{AB} in order to account for the low inflation rates in the post-crisis data. Furthermore, we assume that the secondary bond market initially features search and bargaining frictions; i.e., $0 < \delta\eta < 1$. The policy measures that we investigate are defined as follows. We first examine what happens if the regulator succeeds in changing the market structure of the secondary bond market from an over-the-counter structure to an exchange-traded market which is fully competitive. Secondly, we propose a novel policy measure to enhance welfare in a secondary bond market which is initially characterized by $0 < \delta\eta < 1$.

8 Centralizing the Secondary Bond Market

Hereafter, we analyze the social implications of centralizing secondary corporate bond markets and its effects on quantities and prices. In order to do so, we first need to derive the type-A and the type-B equilibrium for competitive pricing in the secondary bond market, which is shown by Proposition 6.

Proposition 6 *Equilibrium prices and quantities characterized by Proposition 1 and 2 coincide with competitive pricing in the secondary bond market if, and only if, $\eta = 1$.*

Proposition 6 states that centralizing the secondary bond market implies that consumers obtain the entire trade surplus in the type-A and the type-B equilibrium in the secondary bond market. To understand the implications of such a policy measure, we show in Figure 4 the regions of existence in terms of the search (δ) and bargaining frictions (η), for $\gamma > \gamma_{AB}$ and $C < \bar{C}_{\delta\eta=1}$. To draw this figure, we assume that the economy is initially characterized by $\delta = \eta = 0.5$ and that the entry cost $C < \bar{C}_{\delta\eta=1}$ is sufficiently high, such that the type-A equilibrium is supported for $\delta = \eta = 0$.¹⁵ The figure shows that the economy is initially in the type-B equilibrium with γ being close to γ_{AB} , which is visible by the small distance to the type-A equilibrium. Increasing η from $\eta = 0.5$ to $\eta = 1$ moves the economy to the right in Figure 4; i.e, it reduces $\gamma_{AB,C=\bar{C}}$, which is directly visible from (19), where we obtain $\frac{\partial \gamma_{AB,C=\bar{C}}}{\partial \eta} = \beta(1-n)\delta[1-u'(q^*/2)] < 0$. Furthermore, the figure shows that the type-B equilibrium is supported for large values of δ and η . To see this, note that by (14) we know that in the type-A equilibrium it holds that

$$u'(q) - 1 = \frac{\gamma - \beta}{\beta(1-n)(1-\delta\eta)},$$

so q is close to \hat{q} when γ is close to β , since $u'(\hat{q}) = 1$ by (13). For a given γ , the right-hand side of the above equation grows infinitely large, as $\delta\eta$ approaches to one. Since the left-hand side of

¹⁵This condition is satisfied for a wide range of parameter values and functional forms of the utility- and production functions.

the equation is bounded above by $u'\left(\frac{\hat{q}}{2}\right) - 1$, the type-A equilibrium no longer exists for large $\delta\eta$.¹⁶

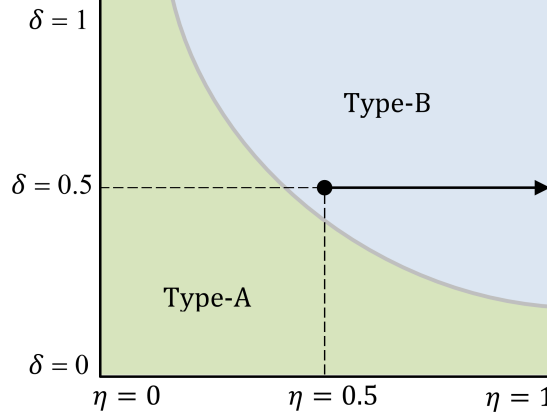


FIGURE 4: CENTRALIZING THE SECONDARY BOND MARKET

In order to analyze the social implication of increasing η in the type-B equilibrium, we first need to define the welfare function, which is given by

$$(1 - \beta)\mathcal{W} \equiv U(x^*) - x^* + (1 - n)[\delta u(\hat{q}) + (1 - \delta)u(q)] - nq_p. \quad (20)$$

The term $U(x^*) - x^*$ denotes an agent's expected utility in the primary bond market; the term $(1 - n)[\delta u(\hat{q}) + (1 - \delta)u(q)]$ denotes an agent's expected utility in the goods market if he becomes a consumer; and the term $-nq_p$ denotes an agent's expected disutility if he becomes a producer in the goods market.

Proposition 7 summarizes the effects of a policy of increasing η ; i.e., a policy that results in a fully centralized and competitive secondary bond market when η approaches one.

Proposition 7 *Let δ and η be large, and γ be small in the type-B equilibrium. Then, we have*

$$\frac{\partial \rho}{\partial \eta} > 0, \quad \frac{\partial(\ell b)}{\partial \eta} > 0, \quad \frac{\partial \hat{q}}{\partial \eta} < 0, \quad \frac{\partial q}{\partial \eta} < 0, \quad \frac{\partial \mathcal{W}}{\partial \eta} < 0.$$

Proposition 7 states that increasing the consumer's bargaining power η results in a higher liquidity premium on bonds in the primary bond market; i.e., ρ increases. As a consequence, the total real quantity of bonds ℓb issued by firms increases, which implies that there is a higher trading volume in the secondary bond market, even though the number of matches $\delta(1 - n)$ remains unchanged. The drawback of this intervention is that each agent holds less money $\frac{q}{\phi}$, such that *active* and *passive* agents will consume less in the goods market, which results in a decrease in welfare.

¹⁶Note that $\hat{q} = 2q$ holds at $\gamma_{AB,C=\bar{C}}$. See the proof of Proposition 5 for more details.

With the knowledge that centralizing the secondary bond market without affecting the market access probability δ is welfare-decreasing, we next analyse what happens if it is feasible for the regulator to change δ at $\eta = 1$. The regulator can either increase δ which moves the economy further in the type-B equilibrium or he can reduce it, which moves the economy in the type-A equilibrium. In Figure 5, we visualize both possibilities by the vertical arrows.

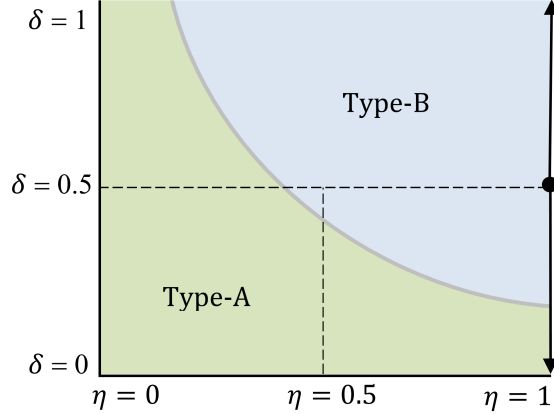


FIGURE 5: INCREASING OR DECREASING δ

We first analyze the case where the regulator increases δ . Such an intervention moves the economy upwards in Figure 5; i.e., it reduces $\gamma_{AB,C=\bar{C}}$, since $\frac{\partial \gamma_{AB,C=\bar{C}}}{\partial \delta} = \beta (1 - n) \eta [1 - u'(q^*/2)] < 0$. The effects of a policy of increasing δ are summarized in Proposition 8.

Proposition 8 *Let δ and η be large, and γ be small in the type-B equilibrium. Then, we have*

$$\frac{\partial \rho}{\partial \delta} > 0, \quad \frac{\partial(\ell b)}{\partial \delta} > 0, \quad \frac{\partial \hat{q}}{\partial \delta} < 0, \quad \frac{\partial q}{\partial \delta} < 0, \quad \frac{\partial \mathcal{W}}{\partial \delta} > 0.$$

Increasing the market access probability δ results in a higher total real quantity of bonds ℓb issued by firms in the primary bond market. This implies that there is an increase in trading volume in the secondary bond market which is caused by two effects: (i) more agents enter the secondary bond market; (ii) more bonds are traded in each match. The drawback of an increase in δ is that each agent holds less money and therefore *active* and *passive* agents will consume less in the goods market. Whether this intervention is welfare-improving depends on which of these effects dominates. For low inflation rates, increasing δ is welfare-improving, because even though each consumer consumes less, there are more trades in the secondary bond market.

Hence, centralizing the secondary bond market might indeed improve welfare as compared to an over-the-counter structure, if the regulator succeeds in increasing the market access probability. The aftermath of the global financial crisis showed that increasing liquidity in secondary corporate bond markets might be difficult to achieve or not be feasible at all. Therefore, we analyze in the next paragraph what happens if the regulator decreases δ in a centralized secondary bond market

($\eta = 1$), which is shown by the downward pointing arrow in Figure 5. Decreasing δ moves the economy to the type-A equilibrium, and Proposition 9 summarizes the effects on welfare of this intervention.

Proposition 9 *In the type-A equilibrium, we have $\frac{\partial \mathcal{W}}{\partial \delta} > 0$ for small values of η , and $\frac{\partial \mathcal{W}}{\partial \delta} < 0$ for large values of η .*

It is clear that $\frac{\partial \hat{q}}{\partial \delta} = 0$, since $u'(\hat{q}) = 1$ for any δ in the type-A equilibrium. Because ρ only depends on β , γ , and g , we also have $\frac{\partial \rho}{\partial \delta} = 0$. Therefore, it also holds that $\frac{\partial \ell b}{\partial \delta} = 0$. Proposition 9 states that for a centralized secondary bond market, welfare is increased by reducing the market access probability δ . The reason is that for high values of η , the demand for money is too low in the type-A equilibrium. Thus, reducing δ makes bonds less attractive and induces all agents to increase their money holdings, which is welfare-improving. For ease of understanding, we show in Figure 6, the development of welfare as a function of the market access probability δ for $\eta = 1$. We find that for $\eta = 1$, welfare at $\delta = 1$ coincides with welfare at $\delta = 0$; i.e., $\mathcal{W}_{\delta=1} = \mathcal{W}_{\delta=0}$, while

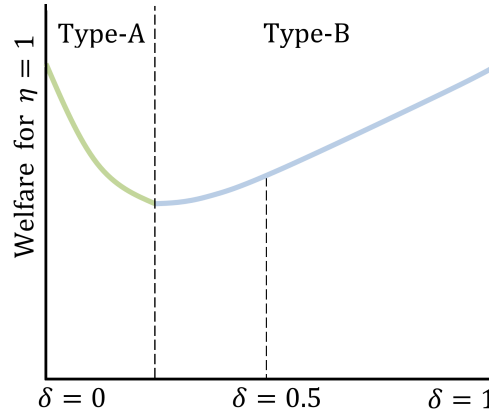


FIGURE 6: WELFARE AS FUNCTION OF δ FOR $\eta = 1$

for $\eta < 1$ we obtain $\mathcal{W}_{\delta=1} > \mathcal{W}_{\delta=0}$. This finding is formalized in the following lemma.

Lemma 10 *Given η , let $\delta = 0$ support the type-A equilibrium, and $\delta = 1$ support the type-B equilibrium. If $\eta = 1$, then $\delta = 0$ and $\delta = 1$ result in an identical value of welfare. Otherwise, $\delta = 1$ generates a higher value of welfare than $\delta = 0$.*

That is, for centralized bond markets welfare is the same with full access or with zero access to the secondary bond market, while for $\eta < 1$ welfare is maximized by granting full access to the secondary bond market. Hence, although granting full access maximizes welfare for $\eta < 1$, depending on the initial search and bargaining frictions, welfare can also be improved by reducing δ . This finding is important, since it might not be feasible for regulators to increase the market access probability.

In the next paragraph, we analyze an economy which features high search frictions (low δ), such that $\gamma < \gamma_{AB}$ for $\eta = 0.5$ and $C < \bar{C}_{\delta\eta=1}$. In contrast to the previous analysis, an increase in the bargaining power of consumers to $\eta = 1$ keeps the economy in the type-A equilibrium. Figure 7 visualizes this policy measure by the black arrow. Figure 7 shows that increasing η moves the economy to the right and results in a reduction of γ_{AB} . Proposition 11 summarizes the effects on welfare of changing η in the type-A equilibrium.

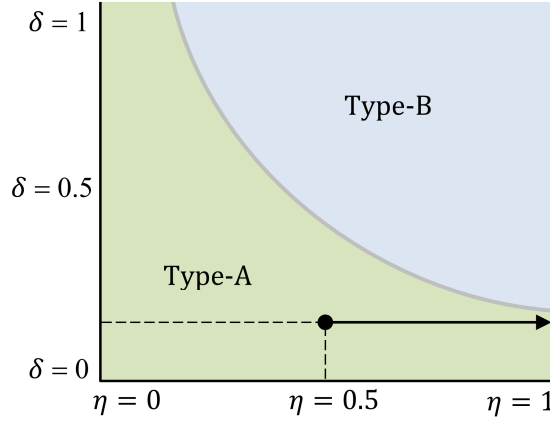


FIGURE 7: INCREASING η IN THE TYPE-A EQUILIBRIUM

Proposition 11 *In the type-A equilibrium, it always holds that $\frac{\partial \mathcal{W}}{\partial \eta} < 0$.*

Following the same reasoning as for the analysis with respect to δ , we have $\frac{\partial \hat{q}}{\partial \eta} = 0$, $\frac{\partial \rho}{\partial \eta} = 0$, and $\frac{\partial \ell b}{\partial \eta} = 0$ in the type-A equilibrium.

Our findings show that simply by centralizing a secondary bond market, i.e., by increasing the consumer's bargaining power to $\eta = 1$, welfare is always decreased in a type-A and a type-B equilibrium. Only if the regulator succeeds in changing the market access probability (by either increasing or decreasing it), he may eventually achieve a welfare-superior allocation.

8.1 Optimal Policy

Hereafter, we show how regulators might intervene in corporate bond markets to achieve an allocation which is always superior to the ones discussed above; i.e., to centralized- and over-the-counter trading. As before, we analyze an economy which is initially characterized by $\delta = \eta = 0.5$ with $\gamma > \gamma_{AB}$ and $C < \bar{C}_{\delta\eta=1}$. Figure 8 shows the optimal intervention in such an environment by the black arrow.

Figure 8 shows that the optimal intervention moves the economy to the top-left corner in the type-A equilibrium. From Proposition 7 and 8, we already know that $\frac{\partial \mathcal{W}}{\partial \eta} < 0$ and $\frac{\partial \mathcal{W}}{\partial \delta} > 0$ in the type-B equilibrium for large values of δ and η , and small inflation rates. Furthermore, from

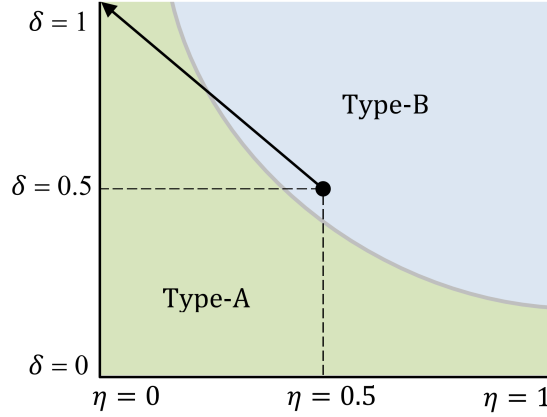


FIGURE 8: OPTIMAL POLICY

Proposition 9 and 11, we know that in the type-A equilibrium we have $\frac{\partial \mathcal{W}}{\partial \eta} < 0$ and $\frac{\partial \mathcal{W}}{\partial \delta} > 0$ for small values of η . Hence, reducing η and increasing δ both have welfare-enhancing effects in the type-A equilibrium. Reducing η improves welfare because it incentivizes agents to increase their demand for money, which in turn allows them to consume more in the goods market. Increasing δ increases the number of trades in the secondary bond market, which also results in an increase in consumption and so improves welfare. The combination of both effects ($\delta = 1$ and $\eta = 0$) results in the optimal intervention for small $\gamma > \beta$ in a secondary bond market which initially features search ($0 < \delta < 1$) and bargaining frictions ($0 < \eta < 1$).

9 Conclusion

In this paper, we present a model that shows under what conditions corporate bonds are trading above their fundamental value and why corporate bond issuance increases in such an environment. Furthermore, we incorporate an over-the-counter secondary bond market and show that it is never optimal to centralize this market, as it incentivates agents to decrease their demand for money which in turn lowers consumption and welfare. If, and only if, the regulator succeeds in also changing the market access probability, by either increasing or decreasing it, he may achieve an allocation which is superior to the one of a traditional over-the-counter market. Furthermore, we show that the optimal policy is to grant full liquidity and to increase bargaining frictions rather than to decrease them.

10 Appendix - For Online Publication

Proof of Proposition 1. *Derivation of (13).* It is obvious that (13) holds when the short-selling constraints (6) are non-binding in (8).

Derivation of (14). At the beginning of a representative period, the marginal value of money is given by

$$\frac{\partial V_1}{\partial m} = (1 - n) \frac{\partial V_1^c}{\partial m} + n \frac{\partial V_1^p}{\partial m}.$$

By (10), the above equation can be rewritten as

$$\frac{\partial V_1}{\partial m} = (1 - n) \left[\delta \eta \frac{\partial K}{\partial m_c} + \frac{\partial V_2^c}{\partial m_c} \right] + n \left[\delta_p (1 - \eta) \frac{\partial K}{\partial m_p} + \frac{\partial V_2^p}{\partial m_p} \right], \quad (21)$$

where $\frac{\partial K}{\partial m_j} := \frac{\partial K(m_c, m_p, b_c, b_p)}{\partial m_j} \big|_{(m_c, m_p, b_c, b_p) = (m, m, b, b)}$ for $j = c, p$. Next, we derive the first-order and the envelope conditions in the primary bond market and the envelope conditions in the goods market to replace $\frac{\partial V_1}{\partial m}$, $\frac{\partial V_2^c}{\partial m_c}$ and $\frac{\partial V_2^p}{\partial m_p}$, respectively. In the primary bond market, the first-order conditions with respect to m_{+1} , b_{+1} and x are $U'(x) = 1$, and

$$\frac{\beta \partial V_1}{\partial m_{+1}} = \rho^{-1} \frac{\beta \partial V_1}{\partial b_{+1}} = \phi. \quad (22)$$

It is standard in models building on Lagos and Wright (2005) that the choice of m_{+1} and b_{+1} is independent of m and b , which results in a degenerate distribution of money and bonds at the beginning of a period. Using $\frac{\beta \partial V_1}{\partial m_{+1}}$ updated one period, we can replace $\frac{\partial V_1}{\partial m}$ in (21). The envelope conditions in the primary bond market are

$$\frac{\partial V_3}{\partial m} = \frac{1}{1 - \sigma} \frac{\partial V_3}{\partial b} = \phi. \quad (23)$$

The above equation states that the marginal value of bonds is lower than the marginal value of money, because only a fraction $1 - \sigma$ will be redeemed at face value in the primary bond market, while σ firms will default and redeem zero. In the goods market, the buyer's envelope conditions are

$$\frac{\partial V_2^c}{\partial m} = \phi u'(q) \quad \text{and} \quad \frac{\partial V_2^c}{\partial b} = (1 - \sigma) \phi, \quad (24)$$

where we used (23) to replace $\frac{\partial V_3}{\partial m}$ and $\frac{\partial V_3}{\partial b}$. Following the same procedure, we obtain the producer's envelope conditions in the goods market as

$$\frac{\partial V_2^p}{\partial m} = \frac{1}{1 - \sigma} \frac{\partial V_2^p}{\partial b} = \phi. \quad (25)$$

Next, we need to derive the envelope conditions in the primary bond market to replace $\frac{\partial K}{\partial m_c}$ and

$\frac{\partial K}{\partial m_p}$ in (21). We obtain

$$\frac{\partial K}{\partial m_c} = \phi - \phi u'(q) \quad \text{and} \quad \frac{\partial K}{\partial m_p} = 0, \quad (26)$$

where we have used $\frac{\partial \hat{q}}{\partial m_c} = 0$, $\frac{\partial q}{\partial m_c} = \frac{1}{p} = \phi$, $\frac{\partial d_m}{\partial m_c} = -1$, and $\frac{\partial \hat{q}}{\partial m_p} = \frac{\partial q}{\partial m_p} = \frac{\partial d_m}{\partial m_p} = 0$ by (9). Using (22), (24), (25), and (26) in (21), we obtain (14).

Derivation of (15). We follow the same procedure as in the derivation of (21) to rewrite the marginal value of bonds at the beginning of a period as

$$\frac{\partial V_1}{\partial b} = (1 - n) \left[\delta \eta \frac{\partial K}{\partial b_c} + \frac{\partial V_2^c}{\partial b_c} \right] + n \left[\delta_p (1 - \eta) \frac{\partial K}{\partial b_p} + \frac{\partial V_2^p}{\partial b_p} \right].$$

To replace $\frac{\partial V_2^c}{\partial b_c}$ and $\frac{\partial V_2^p}{\partial b_p}$, we use (24) and (25), respectively. Because the bond constraint is non-binding in the type-A equilibrium, we have $\frac{\partial K}{\partial b_p} = \frac{\partial K}{\partial b_c} = 0$. Finally, using (22) updated one period, we obtain (15). ■

Proof of Proposition 2. *Derivation of (16).* As a consequence of the binding bond constraint of active consumers, it holds that $d_b = b$. Furthermore, since the cash constraint of consumers (*active* and *passive*) is binding in the goods market, it holds that $m_c + d_m = p\hat{q}$ and $m_c = pq$. Using these expressions in (7) and the first-order condition of producers in the goods market to replace $p\phi = 1$, we obtain (16).

Derivation of (17). At the beginning of a representative period, the marginal value of money is given by

$$\frac{\partial V_1}{\partial m} = (1 - n) \left[\delta \eta \frac{\partial K}{\partial m_c} + \frac{\partial V_2^c}{\partial m_c} \right] + n \left[\delta_p (1 - \eta) \frac{\partial K}{\partial m_p} + \frac{\partial V_2^p}{\partial m_p} \right],$$

where we used (10) to replace $V_1^c(m_c, b_c)$ and $V_1^p(m_c, b_c)$. We use the envelope conditions of consumers and producers in the goods market, (24) and (25), to replace $\frac{\partial V_2^c}{\partial m_c}$ and $\frac{\partial V_2^p}{\partial m_p}$, respectively. Furthermore, because the producer's cash constraint is non-binding, we can use $\frac{\partial \hat{q}}{\partial m_p} = \frac{\partial q}{\partial m_p} = \frac{\partial d_m}{\partial m_p} = 0$ with (9) to obtain $\frac{\partial K}{\partial m_p} = 0$. Using these expressions to rewrite the marginal value of money at the beginning of a period, we obtain

$$\frac{\partial V_1}{\partial m} = (1 - n) \left[\delta \eta \frac{\partial K}{\partial m_c} + \phi u'(q) \right] + n\phi. \quad (27)$$

The envelope condition of a consumer in the secondary bond market, $\frac{\partial K}{\partial m_c}$, is derived as follows. Using (9), we obtain

$$\frac{\partial K}{\partial m_c} = u'(\hat{q}) \frac{\partial \hat{q}}{\partial m_c} - u'(q) \frac{\partial q}{\partial m_c} - \phi \frac{\partial d_m}{\partial m_c}. \quad (28)$$

Next, we need to derive $\frac{\partial \hat{q}}{\partial m_c}$, $\frac{\partial q}{\partial m_c}$ and $\frac{\partial d_m}{\partial m_c}$. Using (7), we obtain

$$(1 - \eta) \left[u'(\hat{q}) \frac{\partial \hat{q}}{\partial m_c} - u'(q) \frac{\partial q}{\partial m_c} \right] = -\eta \phi \frac{\partial d_m}{\partial m_c} \quad (29)$$

and because $\hat{q} = \phi(m_c + d_m)$, $q = \phi m_c$, we also obtain

$$\frac{\partial \hat{q}}{\partial m_c} = \phi + \phi \frac{\partial d_m}{\partial m_c} \quad \text{and} \quad \frac{\partial q}{\partial m_c} = \phi. \quad (30)$$

Using the above expressions for $\frac{\partial \hat{q}}{\partial m_c}$ and $\frac{\partial q}{\partial m_c}$ in (29), we obtain

$$\frac{\partial \hat{q}}{\partial m_c} = \phi \frac{\eta + (1 - \eta)u'(q)}{\eta + (1 - \eta)u'(\hat{q})}, \quad (31)$$

$$\frac{\partial d_m}{\partial m_c} = -\frac{1 - \eta}{\phi \eta} \left[u'(\hat{q}) \frac{\partial \hat{q}}{\partial m_c} - u'(q) \frac{\partial q}{\partial m_c} \right]. \quad (32)$$

Using the above expressions for $\frac{\partial \hat{q}}{\partial m_c}$, $\frac{\partial d_m}{\partial m_c}$, and $\frac{\partial q}{\partial m_c} = \phi$ in (28), we obtain the following expression for $\frac{\partial K}{\partial m_c}$:

$$\begin{aligned} \frac{\partial K}{\partial m_c} &= u'(\hat{q}) \frac{\partial \hat{q}}{\partial m_c} - u'(q) \frac{\partial q}{\partial m_c} + \frac{1 - \eta}{\eta} \left[u'(\hat{q}) \frac{\partial \hat{q}}{\partial m_c} - u'(q) \frac{\partial q}{\partial m_c} \right] \\ &= \frac{1}{\eta} \left[u'(\hat{q}) \frac{\partial \hat{q}}{\partial m_c} - u'(q) \frac{\partial q}{\partial m_c} \right] = \frac{\phi}{\eta} \left[u'(\hat{q}) \frac{\eta + (1 - \eta)u'(q)}{\eta + (1 - \eta)u'(\hat{q})} - u'(q) \right], \end{aligned}$$

Using the above solution to replace $\frac{\partial K}{\partial m_c}$ in (27) and (22) updated one period to replace $\frac{\partial V_1}{\partial m}$, we obtain (17).

Derivation of (18). At the beginning of a representative period, the marginal value of bonds is given by

$$\frac{\partial V_1}{\partial b} = (1 - n) \left[\delta \eta \frac{\partial K}{\partial b_c} + \frac{\partial V_2^c}{\partial b_c} \right] + n \left[\delta_p (1 - \eta) \frac{\partial K}{\partial b_p} + \frac{\partial V_2^p}{\partial b_p} \right]. \quad (33)$$

Using (16), we can obtain

$$\frac{\partial \hat{q}}{\partial b} = \frac{\phi}{(1 - \eta)u'(\hat{q}) + \eta} \quad \text{and} \quad \frac{\partial d_m}{\partial b} = \frac{1}{(1 - \eta)u'(\hat{q}) + \eta},$$

where we used $\ell b = \phi b$ and $\phi d_m = \hat{q} - q$. We can use the above expressions to replace $\frac{\partial \hat{q}}{\partial b}$ and

$\frac{\partial d_m}{\partial b}$, respectively, $\frac{\partial q}{\partial b_c} = 0$, and $\frac{\partial \hat{q}}{\partial b_p} = \frac{\partial q}{\partial b_p} = \frac{\partial d_m}{\partial b_p} = \frac{\partial q}{\partial b_c} = 0$, in (9), to obtain

$$\begin{aligned}\frac{\partial K}{\partial b_c} &= \phi \frac{u'(\hat{q}) - 1}{(1 - \eta)u'(\hat{q}) + \eta}, \\ \frac{\partial K}{\partial b_p} &= 0.\end{aligned}$$

Using the two above expressions, we can replace $\frac{\partial K}{\partial b_c}$ and $\frac{\partial K}{\partial b_p}$ in (33); using the envelope conditions in the goods market, (24) and (25), we can replace $\frac{\partial V_2^c}{\partial b_c}$ and $\frac{\partial V_2^p}{\partial b_p}$; and using (22) updated one period we can replace $\frac{\partial V_1}{\partial b}$ to obtain (18). ■

Proof of Lemma 3. By (11) and (12), we obtain

$$\frac{1 - \beta(1 - g)}{(1 - g)}C - \frac{f(\rho \mathbb{b})}{\rho f'(\rho \mathbb{b})} + \mathbb{b} = 0.$$

Hence,

$$\begin{aligned}\frac{\partial \mathbb{b}}{\partial C} &= -\frac{1 - \beta(1 - g)}{(1 - g)} \frac{[f'(\rho \mathbb{b})]^2}{f(\rho \mathbb{b})f''(\rho \mathbb{b})} = \frac{1 - \beta(1 - g)}{(1 - g)} \frac{\epsilon(\rho \mathbb{b})}{\alpha(\rho \mathbb{b})} > 0, \\ \frac{\partial \mathbb{b}}{\partial g} &= -\frac{C}{(1 - g)^2} \frac{[f'(\rho \mathbb{b})]^2}{f(\rho \mathbb{b})f''(\rho \mathbb{b})} = \frac{C}{(1 - g)^2} \frac{\epsilon(\rho \mathbb{b})}{\alpha(\rho \mathbb{b})} > 0, \\ \frac{\partial \mathbb{b}}{\partial \beta} &= C \frac{[f'(\rho \mathbb{b})]^2}{f(\rho \mathbb{b})f''(\rho \mathbb{b})} = -C \frac{\epsilon(\rho \mathbb{b})}{\alpha(\rho \mathbb{b})} < 0.\end{aligned}$$

Using (11), we obtain

$$\frac{\partial \ell}{\partial \mathbb{b}} = \rho^2 f''(\rho \mathbb{b}) < 0.$$

Thus, we obtain

$$\frac{\partial \ell}{\partial C} < 0, \quad \frac{\partial \ell}{\partial g} < 0, \quad \frac{\partial \ell}{\partial \beta} > 0.$$

Using (11), we also obtain

$$\ell \mathbb{b} = f'(\rho \mathbb{b}) \rho \mathbb{b}.$$

Consequently, we obtain

$$\frac{\partial \ell \mathbb{b}}{\partial \mathbb{b}} = \rho f'(\rho \mathbb{b}) \left[1 + \frac{\rho \mathbb{b} f''(\rho \mathbb{b})}{f'(\rho \mathbb{b})} \right] = \rho f'(\rho \mathbb{b}) [1 - \alpha(\rho \mathbb{b})].$$

By using

$$\frac{\partial \ell \mathbb{b}}{\partial C} = \frac{\partial \ell \mathbb{b}}{\partial \mathbb{b}} \frac{\partial \mathbb{b}}{\partial C} = \rho f'(\rho \mathbb{b}) [1 - \alpha(\rho \mathbb{b})] \frac{\partial \mathbb{b}}{\partial C},$$

we obtain the desired result for $\frac{\partial \ell \mathbb{b}}{\partial C}$. It is clear that when we follow the above procedure, we can derive the results for $\frac{\partial \ell \mathbb{b}}{\partial g}$, and $\frac{\partial \ell \mathbb{b}}{\partial \beta}$. ■

Proof of Lemma 4. By (11) and (12), we obtain

$$\frac{1 - \beta(1 - g)}{(1 - g)}C - \frac{f(\rho \mathbb{b})}{\rho f'(\rho \mathbb{b})} + \mathbb{b} = 0.$$

Therefore, we obtain

$$\begin{aligned} f(\rho \mathbb{b})f''(\rho \mathbb{b})\rho^2 \frac{\partial \mathbb{b}}{\partial \rho} &= [f'(\rho \mathbb{b})]^2 \rho \mathbb{b} - f(\rho \mathbb{b})f'(\rho \mathbb{b}) - f(\rho \mathbb{b})f''(\rho \mathbb{b})\rho \mathbb{b} \\ &= f(\rho \mathbb{b})f'(\rho \mathbb{b}) \left[\frac{f'(\rho \mathbb{b})\rho \mathbb{b}}{f(\rho \mathbb{b})} - 1 - \frac{f''(\rho \mathbb{b})\rho \mathbb{b}}{f'(\rho \mathbb{b})} \right] \\ &= -f(\rho \mathbb{b})f'(\rho \mathbb{b}) [1 - \epsilon(\rho \mathbb{b}) - \alpha(\rho \mathbb{b})]. \end{aligned} \quad (34)$$

Rearranging terms yields

$$\frac{\partial \mathbb{b}}{\partial \rho} = \frac{\mathbb{b}}{\alpha(\rho \mathbb{b})\rho} [1 - \epsilon(\rho \mathbb{b}) - \alpha(\rho \mathbb{b})].$$

From $\ell \mathbb{b} = f'(\rho \mathbb{b})\rho \mathbb{b}$, we obtain

$$\frac{\partial \ell \mathbb{b}}{\partial \rho} = f''(\rho \mathbb{b})\rho \mathbb{b}^2 + f''(\rho \mathbb{b})\rho^2 \mathbb{b} \frac{\partial \mathbb{b}}{\partial \rho} + f'(\rho \mathbb{b})\mathbb{b} + f'(\rho \mathbb{b})\rho \frac{\partial \mathbb{b}}{\partial \rho}.$$

Using the above equation and (34), we obtain

$$\begin{aligned} \frac{\partial \ell \mathbb{b}}{\partial \rho} &= \frac{[f'(\rho \mathbb{b})]^2 \rho \mathbb{b}^2}{f(\rho \mathbb{b})} + f'(\rho \mathbb{b})\rho \frac{\partial \mathbb{b}}{\partial \rho} \\ &= \frac{f'(\rho \mathbb{b})}{f(\rho \mathbb{b})f''(\rho \mathbb{b})\rho} \left[f'(\rho \mathbb{b})f''(\rho \mathbb{b})\rho^2 \mathbb{b}^2 + f(\rho \mathbb{b})f''(\rho \mathbb{b})\rho^2 \frac{\partial \mathbb{b}}{\partial \rho} \right] \\ &= \frac{f'(\rho \mathbb{b})}{f(\rho \mathbb{b})f''(\rho \mathbb{b})\rho} [f'(\rho \mathbb{b})f''(\rho \mathbb{b})\rho^2 \mathbb{b}^2 + [f'(\rho \mathbb{b})]^2 \rho \mathbb{b} - f(\rho \mathbb{b})f'(\rho \mathbb{b}) - f(\rho \mathbb{b})f''(\rho \mathbb{b})\rho \mathbb{b}] \\ &= \frac{[f'(\rho \mathbb{b})]^2}{f(\rho \mathbb{b})f''(\rho \mathbb{b})\rho} [f'(\rho \mathbb{b})\rho \mathbb{b} - f(\rho \mathbb{b})] \left[1 + \frac{f''(\rho \mathbb{b})\rho \mathbb{b}}{f'(\rho \mathbb{b})} \right] \\ &= -\frac{[f'(\rho \mathbb{b})]^2}{f''(\rho \mathbb{b})\rho} [1 - \epsilon(\rho \mathbb{b})] [1 - \alpha(\rho \mathbb{b})]. \end{aligned} \quad (35)$$

Because $f(x)$ is concave, it follows that $1 - \epsilon(\rho \mathbb{b}) > 0$ and thus that

$$\frac{\partial \ell \mathbb{b}}{\partial \rho} \begin{cases} < 0, & \text{if } \alpha(\rho \mathbb{b}) > 1, \\ = 0, & \text{if } \alpha(\rho \mathbb{b}) = 1, \\ > 0, & \text{if } \alpha(\rho \mathbb{b}) < 1. \end{cases}$$

At last, note that

$$\frac{\partial \ell \mathbb{b}}{\partial \rho} = \frac{\partial \ell}{\partial \rho} \mathbb{b} + \ell \frac{\partial \mathbb{b}}{\partial \rho} = \frac{\partial \ell}{\partial \rho} \mathbb{b} + f'(\rho \mathbb{b}) \rho \frac{\partial \mathbb{b}}{\partial \rho}.$$

Using (35), we obtain

$$\frac{\partial \ell}{\partial \rho} = \frac{[f'(\rho \mathbb{b})]^2 \rho \mathbb{b}}{f(\rho \mathbb{b})} = \epsilon(\rho \mathbb{b}) f'(\rho \mathbb{b}).$$

■

Proof of Proposition 5. The critical inflation rate γ_{AB} is the value of γ , where the bond constraint of active consumers becomes just binding in the type-A equilibrium; i.e., (16) holds. For a given $C < C_1$, let q_1 , ρ_1 , ℓ_1 , and \mathbb{b}_1 satisfy

$$\begin{aligned} \frac{\gamma}{\beta} &= (1 - n) [\delta \eta + (1 - \delta \eta) u'(q_1)] + n, \\ \rho_1 &= \frac{\beta}{\gamma} (1 - g), \\ \ell_1 &= \rho_1 f'(\rho_1 \mathbb{b}_1), \\ \mathbb{b}_1 &= \frac{(1 - \eta)[u(q^*) - u(q_1)] + \eta(q^* - q_1)}{\ell_1}, \end{aligned}$$

where we used $u'(\hat{q}) = 1$. The critical inflation rate γ_{AB} is the solution to

$$C \equiv \frac{(1 - g)}{1 - \beta(1 - g)} \left[\frac{f(\rho_1 \mathbb{b}_1)}{\ell_1} - \mathbb{b}_1 \right].$$

The critical value \bar{C} is the value of C where the cash constraint of active producers becomes just binding; i.e., $m_p = d_m = M$. Furthermore, since *active* and *passive* consumers are cash-constrained in the goods market, we have $m_c + d_m = p\hat{q}$ and $m_c = pq$, respectively. Using $m_p = m_c = M$ and rearranging terms, we obtain $\hat{q} = 2q$. Furthermore, we have $u'(\hat{q}) = 1$ and

$$\begin{aligned} \gamma_2 &= \beta \{ (1 - n) [\delta \eta + (1 - \delta \eta) u'(q^*/2)] + n \}, \\ \rho_2 &= \frac{\beta}{\gamma_2} (1 - g), \\ \ell_2 &= \rho_2 f'(\rho_2 \mathbb{b}_2), \\ \mathbb{b}_2 &= \frac{(1 - \eta)[u(q^*) - u(q^*/2)] + \eta q^*/2}{\ell_2}, \end{aligned}$$

The critical entry cost \bar{C} is the solution to

$$\bar{C} \equiv \frac{(1 - g)}{1 - \beta(1 - g)} \left[\frac{f(\rho_2 \mathbb{b}_2)}{\ell_2} - \mathbb{b}_2 \right].$$

■

Proof of Proposition 6. We now assume competitive pricing in the secondary bond market,

which changes the agent's decision problem in the following way. As before, an agent enters the secondary bond market with a portfolio (m_j, b_j) and leaves it with a portfolio (\hat{m}_j, \hat{b}_j) , where $j = c$ for consumers and $j = p$ for producers. Let φ denote the price of bonds in terms of money in the secondary bond market. Then, an active agent's budget constraint satisfies

$$\phi m_j + \varphi \phi b_j = \phi \hat{m}_j + \varphi \phi \hat{b}_j. \quad (36)$$

The left-hand side of (36) is the sum of the real values of money and bonds with which the agent enters the secondary bond market, and the right-hand side is the real value of the portfolio with which the agent leaves the secondary bond market. It is obvious that $\varphi > 1$ does not support trades in the secondary bond market, since it is suboptimal for producers to trade for $\varphi > 1$. In the following analysis, we thus focus on $\varphi = 1$. Trading is further constrained by two short-selling constraints: Active agents cannot sell more bonds, and they cannot spend more money, than the amount they carry from the previous period. That is

$$\hat{m}_j \geq 0, \hat{b}_j \geq 0. \quad (37)$$

Let $V_1^j(m_j, b_j)$ denote the value function of a consumer and a producer by holding (m_j, b_j) before entering the secondary bond market. For $\varphi = 1$, producers are indifferent between trading in the secondary bond market or not; i.e.,

$$V_1^p(m_p, b_p) = V_2^p(m_p, b_p) = V_2^p(\hat{m}_p, m_p + b_p - \hat{m}_p) \quad (38)$$

for any $\hat{m}_p \geq 0, m_p + b_p - \hat{m}_p \geq 0$. Therefore, the decision problem is up to consumers; i.e.,

$$V_1^c(m_c, b_c) = (1 - \delta)V_2^c(m_c, b_c) + \delta \max_{\hat{m}_c} V_2^c(\hat{m}_c, m_c + b_c - \hat{m}_c) \quad s.t. \quad \hat{m}_c \geq 0, m_c + b_c - \hat{m}_c \geq 0.$$

Note that

$$\frac{dV_2^c(\hat{m}_c, m_c + b_c - \hat{m}_c)}{d\hat{m}_c} = \begin{cases} 0 & , \text{ if } \phi \hat{m}_c > q^* \\ \phi u'(\phi \hat{m}_c) - \phi > 0 & , \text{ if } \phi \hat{m}_c < q^*. \end{cases}$$

Thus, the optimal \hat{m}_c is

$$\hat{m}_c \begin{cases} \in \left[\frac{q^*}{\phi}, m_c + b_c \right] & , \text{ if } u'(\phi(m_c + b_c)) - 1 < 0 \\ = m_c + b_c & , \text{ if } u'(\phi(m_c + b_c)) - 1 \geq 0. \end{cases}$$

Therefore, we can formulate the value function of consumers as

$$V_1^c(m_c, b_c) = \begin{cases} (1 - \delta)V_2^c(m_c, b_c) + \delta V_2^c\left(\frac{q^*}{\phi}, m_c + b_c - \frac{q^*}{\phi}\right) & , \text{ if } u'(\phi(m_c + b_c)) < 1 \\ (1 - \delta)V_2^c(m_c, b_c) + \delta V_2^c(m_c + b_c, 0) & , \text{ if } u'(\phi(m_c + b_c)) \geq 1. \end{cases} \quad (39)$$

Type-A Equilibrium. A type-A equilibrium is characterized by $\varphi = 1$ and $\frac{1}{\varphi} > u'(\phi(m + \varphi b))$. It

is a list $\{\hat{q}, q, q_p, \rho, \varphi, \mathbb{b}, \ell\}$ satisfying (5), (11), (12), and

$$1 = \varphi, \quad (40)$$

$$1 = u'(\hat{q}), \quad (41)$$

$$\frac{\gamma}{\beta} = (1-n)\delta [u'(\hat{q}) - u'(q)] + (1-n)u'(q) + n, \quad (42)$$

$$\rho = \frac{\beta}{\gamma}(1-g). \quad (43)$$

Because the cash constraint of active producers is non-binding, it follows that $1 = \varphi$ has to hold. Equation (41) is immediate by (39). Equation (42) and (43) are derived from (38) and (39):

$$\begin{aligned} \frac{\partial V_1}{\partial m} &= (1-n) \frac{\partial V_1^c}{\partial m_c} + n \frac{\partial V_1^p}{\partial m_p} \\ &= (1-n)[(1-\delta)\phi u'(q) + \delta\phi] + n\phi \\ &= \phi [(1-n)(1-\delta)u'(q) + (1-n)\delta + n], \end{aligned}$$

and

$$\begin{aligned} \frac{\partial V_1}{\partial b} &= (1-n) \frac{\partial V_1^c}{\partial b_c} + n \frac{\partial V_1^p}{\partial b_p} \\ &= (1-n)(1-g)[(1-\delta)\phi + \delta\phi] + n(1-g)\phi \\ &= (1-g)\phi. \end{aligned}$$

Type-B Equilibrium. A type-B equilibrium is characterized by $\varphi = 1$ and $\frac{1}{\varphi} \leq u'(\phi(m + \varphi b))$. It is a list $\{\hat{q}, q, q_p, \rho, \varphi, \mathbb{b}, \ell\}$ satisfying (5), (11), (12), and

$$1 = \varphi, \quad (44)$$

$$\ell \mathbb{b} = \hat{q} - q, \quad (45)$$

$$\frac{\gamma}{\beta} = (1-n) [\delta u'(\hat{q}) + (1-\delta)u'(q)] + n, \quad (46)$$

$$\rho = \frac{\beta}{\gamma} \{1 - g + (1-n)\delta [u'(\hat{q}) - 1]\}. \quad (47)$$

Because the cash constraint of active producers is non-binding, it follows that $1 = \varphi$ has to hold. Equation (45) comes from the fact that active consumers sell all their bonds to active producers for the same amount of money, so that

$$\phi \hat{m} = \phi m + \phi b \Rightarrow \ell \mathbb{b} = \phi b = \phi \hat{m} - \phi m = \hat{q} - q.$$

Equation (46) and (47) are derived from (38) and (39):

$$\begin{aligned}\frac{\partial V_1}{\partial m} &= (1-n)\frac{\partial V_1^c}{\partial m_c} + n\frac{\partial V_1^p}{\partial m_p} \\ &= (1-n)[(1-\delta)\phi u'(q) + \delta\phi u'(\hat{q})] + n\phi \\ &= \phi [(1-n)\delta u'(\hat{q}) + (1-n)(1-\delta)u'(q) + n],\end{aligned}$$

and

$$\begin{aligned}\frac{\partial V_1}{\partial b} &= (1-n)\frac{\partial V_1^c}{\partial b_c} + n\frac{\partial V_1^p}{\partial b_p} \\ &= (1-n)(1-g)[(1-\delta)\phi + \delta\phi u'(\hat{q})] + n(1-g)\phi \\ &= \phi \{1-g + (1-n)\delta [u'(\hat{q}) - 1]\}.\end{aligned}$$

■

Proof of Proposition 7. From (17), we obtain

$$\begin{aligned}\delta u''(\hat{q})\frac{\partial \hat{q}}{\partial \eta}\frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} + \delta u'(\hat{q})\frac{1-u'(q)}{\eta + (1-\eta)u'(\hat{q})} + \delta u'(\hat{q})\frac{(1-\eta)u''(q)}{\eta + (1-\eta)u'(\hat{q})}\frac{\partial q}{\partial \eta} \\ - \delta u'(\hat{q})\frac{\eta + (1-\eta)u'(q)}{[\eta + (1-\eta)u'(\hat{q})]^2}[1-u'(\hat{q}) + (1-\eta)u''(\hat{q})\frac{\partial \hat{q}}{\partial \eta}] + (1-\delta)u''(q)\frac{\partial q}{\partial \eta} = 0.\end{aligned}$$

Hence if $\delta\eta \neq 1$, then

$$\begin{aligned}\frac{\partial q}{\partial \eta} &= -\frac{\delta u'(\hat{q})\frac{1-u'(q)}{\eta + (1-\eta)u'(\hat{q})} - \delta u'(\hat{q})\frac{\eta + (1-\eta)u'(q)}{[\eta + (1-\eta)u'(\hat{q})]^2}[1-u'(\hat{q})]}{\delta u'(\hat{q})\frac{(1-\eta)u''(q)}{\eta + (1-\eta)u'(\hat{q})} + (1-\delta)u''(q)} \\ &\quad - \frac{\delta u''(\hat{q})\frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} - \delta u'(\hat{q})\frac{\eta + (1-\eta)u'(q)}{[\eta + (1-\eta)u'(\hat{q})]^2}(1-\eta)u''(\hat{q})}{\delta u'(\hat{q})\frac{(1-\eta)u''(q)}{\eta + (1-\eta)u'(\hat{q})} + (1-\delta)u''(q)}\frac{\partial \hat{q}}{\partial \eta}.\end{aligned}\tag{48}$$

If $\delta\eta = 1$, then

$$\frac{\partial \hat{q}}{\partial \eta} = \frac{u'(\hat{q})[u'(q) - u'(\hat{q})]}{u''(\hat{q})}.\tag{49}$$

The effect on the demand of bonds is

$$\begin{aligned}\frac{\partial \{(1-\eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q)\}}{\partial \eta} &= -[u(\hat{q}) - u(q)] + \hat{q} - q \\ &\quad + [\eta + (1-\eta)u'(\hat{q})]\frac{\partial \hat{q}}{\partial \eta} - [\eta + (1-\eta)u'(q)]\frac{\partial q}{\partial \eta}.\end{aligned}$$

From (18), we obtain

$$\frac{\partial \rho}{\partial \eta} = \frac{\beta(1-n)\delta}{\gamma} \left\{ \begin{array}{l} \frac{u'(\hat{q}) - 1}{\eta + (1-\eta)u'(\hat{q})} \left[1 + \frac{\eta[u'(\hat{q}) - 1]}{\eta + (1-\eta)u'(\hat{q})} \right] \\ + \eta \frac{u''(\hat{q})}{\eta + (1-\eta)u'(\hat{q})} \left[1 - \frac{[u'(\hat{q}) - 1](1-\eta)}{\eta + (1-\eta)u'(\hat{q})} \right] \frac{\partial \hat{q}}{\partial \eta} \end{array} \right\}. \quad (50)$$

From Lemma 4, we know that the effect on the supply of bonds is

$$\frac{\partial(\ell\mathbb{b})}{\partial \eta} = \frac{\partial(\ell\mathbb{b})}{\partial \rho} \frac{\partial \rho}{\partial \eta} = \frac{f'(\rho\mathbb{b})\mathbb{b}}{\alpha(\rho\mathbb{b})} [1 - \epsilon(\rho\mathbb{b})] [1 - \alpha(\rho\mathbb{b})] \times \frac{\partial \rho}{\partial \eta},$$

where $\frac{\partial \rho}{\partial \eta}$ is given by (50). By market clearing, we obtain

$$\begin{aligned} & -[u(\hat{q}) - u(q)] + \hat{q} - q + [\eta + (1-\eta)u'(\hat{q})] \frac{\partial \hat{q}}{\partial \eta} - [\eta + (1-\eta)u'(q)] \frac{\partial q}{\partial \eta} \\ &= \frac{f'(\rho\mathbb{b})\mathbb{b}}{\alpha(\rho\mathbb{b})} [1 - \epsilon(\rho\mathbb{b})] [1 - \alpha(\rho\mathbb{b})] \\ & \times \frac{\beta(1-n)\delta}{\gamma} \left\{ \begin{array}{l} \frac{u'(\hat{q}) - 1}{\eta + (1-\eta)u'(\hat{q})} \left[1 + \frac{\eta[u'(\hat{q}) - 1]}{\eta + (1-\eta)u'(\hat{q})} \right] \\ + \eta \frac{u''(\hat{q})}{\eta + (1-\eta)u'(\hat{q})} \left[1 - \frac{[u'(\hat{q}) - 1](1-\eta)}{\eta + (1-\eta)u'(\hat{q})} \right] \frac{\partial \hat{q}}{\partial \eta} \end{array} \right\}. \end{aligned} \quad (51)$$

With (48), (49), and (51), we can explicitly derive $\frac{\partial \hat{q}}{\partial \eta}$, and $\frac{\partial q}{\partial \eta}$. For instance, if $\delta\eta = 1$, then we have

$$\begin{aligned} \frac{\partial \rho}{\partial \eta} &= \frac{\beta}{\gamma} u'(\hat{q})[u'(q) - 1] > 0 \\ \frac{\partial(\ell\mathbb{b})}{\partial \delta} &= [1 - \epsilon(\rho\mathbb{b})] [1 - \alpha(\rho\mathbb{b})] \frac{\beta}{\gamma} u'(\hat{q})[u'(q) - 1] > 0 \\ \frac{\partial q}{\partial \eta} &= -[u(\hat{q}) - u(q)] + \hat{q} - q + \frac{u'(\hat{q})[u'(q) - u'(\hat{q})]}{u''(\hat{q})} \\ & \quad - \frac{f'(\rho\mathbb{b})\mathbb{b}}{\alpha(\rho\mathbb{b})} [1 - \epsilon(\rho\mathbb{b})] [1 - \alpha(\rho\mathbb{b})] \frac{\beta}{\gamma} u'(\hat{q})[u'(q) - 1] < 0. \end{aligned}$$

By the continuity, if the values of δ and η are large enough, then

$$\frac{\partial \rho}{\partial \eta} > 0, \quad \frac{\partial(\ell\mathbb{b})}{\partial \eta} > 0, \quad \frac{\partial \hat{q}}{\partial \eta} < 0, \quad \frac{\partial q}{\partial \eta} < 0.$$

Next, we derive the effects of a change in η on welfare. From (5), we have

$$n \frac{\partial q_p}{\partial \eta} = (1-n) \left[\delta \frac{\partial \hat{q}}{\partial \eta} + (1-\delta) \frac{\partial q}{\partial \eta} \right].$$

Hence, together with (20), we obtain

$$\begin{aligned} (1-\beta) \frac{\partial \mathcal{W}}{\partial \eta} &= (1-n) \left[\delta u'(\hat{q}) \frac{\partial \hat{q}}{\partial \eta} + (1-\delta) u'(q) \frac{\partial q}{\partial \eta} \right] - n \frac{\partial q_p}{\partial \eta} \\ &= (1-n) \left[\delta [u'(\hat{q}) - 1] \frac{\partial \hat{q}}{\partial \delta} + (1-\delta) [u'(q) - 1] \frac{\partial q}{\partial \delta} \right] < 0. \end{aligned}$$

Therefore, if $\delta\eta = 1$, then $\frac{\partial \mathcal{W}}{\partial \eta} < 0$. ■

Proof of Proposition 7. From (17), we obtain

$$\begin{aligned} u'(\hat{q}) \frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} - u'(q) + \delta \frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} u''(\hat{q}) \frac{\partial \hat{q}}{\partial \delta} \\ + \delta u'(\hat{q}) \frac{(1-\eta)u''(q)}{[\eta + (1-\eta)u'(\hat{q})]} \frac{\partial q}{\partial \delta} - \delta u'(\hat{q}) \frac{[\eta + (1-\eta)u'(q)](1-\eta)u''(\hat{q})}{[\eta + (1-\eta)u'(\hat{q})]^2} \frac{\partial \hat{q}}{\partial \delta} + (1-\delta) u''(q) \frac{\partial q}{\partial \delta} = 0. \end{aligned}$$

Hence, if $\delta\eta \neq 1$, then we obtain

$$\frac{\partial q}{\partial \delta} = \frac{u'(q) - u'(\hat{q}) \frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} - \delta \frac{\eta + (1-\eta)u'(q)}{\eta + (1-\eta)u'(\hat{q})} u''(\hat{q}) \left[1 - \frac{(1-\eta)u'(\hat{q})}{\eta + (1-\eta)u'(\hat{q})} \right] \frac{\partial \hat{q}}{\partial \delta}}{\left\{ \delta u'(\hat{q}) \frac{(1-\eta)}{[\eta + (1-\eta)u'(\hat{q})]} + (1-\delta) \right\} u''(q)}. \quad (52)$$

If $\delta\eta = 1$, then we obtain

$$\frac{\partial \hat{q}}{\partial \delta} = \frac{u'(q) - u'(\hat{q})}{u''(\hat{q})} < 0. \quad (53)$$

The effect on the demand of bonds is

$$\frac{\partial \{(1-\eta)[u(\hat{q}) - u(q)] + \eta(\hat{q} - q)\}}{\partial \delta} = [\eta + (1-\eta)u'(\hat{q})] \frac{\partial \hat{q}}{\partial \delta} - [\eta + (1-\eta)u'(q)] \frac{\partial q}{\partial \delta}.$$

From (18), we have

$$\frac{\partial \rho}{\partial \delta} = \frac{\beta(1-n)\eta}{\gamma} \left\{ \frac{u'(\hat{q}) - 1}{\eta + (1-\eta)u'(\hat{q})} + \delta \frac{u''(\hat{q})}{\eta + (1-\eta)u'(\hat{q})} \left[1 - \frac{[u'(\hat{q}) - 1](1-\eta)}{\eta + (1-\eta)u'(\hat{q})} \right] \frac{\partial \hat{q}}{\partial \delta} \right\}. \quad (54)$$

From Lemma 4, we know that the effect on the supply of bonds is

$$\frac{\partial(\ell \mathbb{b})}{\partial \delta} = \frac{\partial(\ell \mathbb{b})}{\partial \rho} \frac{\partial \rho}{\partial \delta} = \frac{f'(\rho \mathbb{b}) \mathbb{b}}{\alpha(\rho \mathbb{b})} [1 - \epsilon(\rho \mathbb{b})] [1 - \alpha(\rho \mathbb{b})] \times \frac{\partial \rho}{\partial \delta},$$

where $\frac{\partial \rho}{\partial \delta}$ is given by (54). By market clearing, we have

$$\begin{aligned}
& [\eta + (1 - \eta)u'(\hat{q})] \frac{\partial \hat{q}}{\partial \delta} - [\eta + (1 - \eta)u'(q)] \frac{\partial q}{\partial \delta} \\
&= \frac{f'(\rho \mathbb{b})}{\alpha(\rho \mathbb{b})} [1 - \epsilon(\rho \mathbb{b})] [1 - \alpha(\rho \mathbb{b})] \\
&\quad \times \frac{\beta(1 - n)\eta}{\gamma} \left\{ \frac{u'(\hat{q}) - 1}{\eta + (1 - \eta)u'(\hat{q})} + \delta \frac{u''(\hat{q})}{\eta + (1 - \eta)u'(\hat{q})} \left[1 - \frac{[u'(\hat{q}) - 1](1 - \eta)}{\eta + (1 - \eta)u'(\hat{q})} \right] \frac{\partial \hat{q}}{\partial \delta} \right\}.
\end{aligned} \tag{55}$$

With (52), (53), and (55), we can explicitly derive $\frac{\partial \hat{q}}{\partial \delta}$, and $\frac{\partial q}{\partial \delta}$. For instance, if $\delta\eta = 1$, then we have

$$\begin{aligned}
\frac{\partial \rho}{\partial \delta} &= \frac{\beta}{\gamma} [u'(q) - 1] > 0 \\
\frac{\partial(\ell \mathbb{b})}{\partial \delta} &= \frac{f'(\rho \mathbb{b})}{\alpha(\rho \mathbb{b})} [1 - \epsilon(\rho \mathbb{b})] [1 - \alpha(\rho \mathbb{b})] \frac{\beta}{\gamma} [u'(q) - 1] > 0 \\
\frac{\partial q}{\partial \delta} &= \frac{u'(q) - u'(\hat{q})}{u''(\hat{q})} - \frac{f'(\rho \mathbb{b})}{\alpha(\rho \mathbb{b})} [1 - \epsilon(\rho \mathbb{b})] [1 - \alpha(\rho \mathbb{b})] \frac{\beta}{\gamma} [u'(q) - 1] < 0.
\end{aligned}$$

By the continuity, if the values of δ and η are large enough, then

$$\frac{\partial \rho}{\partial \delta} > 0, \quad \frac{\partial(\ell \mathbb{b})}{\partial \delta} > 0, \quad \frac{\partial \hat{q}}{\partial \delta} < 0, \quad \frac{\partial q}{\partial \delta} < 0.$$

Next, we derive the effects of a change in δ on welfare. From (5), we have

$$n \frac{\partial q_p}{\partial \delta} = (1 - n) \left[\delta \frac{\partial \hat{q}}{\partial \delta} + (1 - \delta) \frac{\partial q}{\partial \delta} + \hat{q} - q \right].$$

Hence, together with (20), we obtain

$$\begin{aligned}
(1 - \beta) \frac{\partial \mathcal{W}}{\partial \delta} &= (1 - n) \left[\delta u'(\hat{q}) \frac{\partial \hat{q}}{\partial \delta} + (1 - \delta) u'(q) \frac{\partial q}{\partial \delta} + u(\hat{q}) - u(q) \right] - n \frac{\partial q_p}{\partial \delta} \\
&= (1 - n) \left\{ \delta [u'(\hat{q}) - 1] \frac{\partial \hat{q}}{\partial \delta} + (1 - \delta) [u'(q) - 1] \frac{\partial q}{\partial \delta} + u(\hat{q}) - u(q) - (\hat{q} - q) \right\}.
\end{aligned}$$

Note that if $\delta\eta = 1$, then

$$\begin{aligned}
\frac{1 - \beta}{1 - n} \frac{\partial \mathcal{W}}{\partial \delta} &= \frac{[u'(q) - u'(\hat{q})][u'(\hat{q}) - 1]}{u''(\hat{q})} + u(\hat{q}) - u(q) - (\hat{q} - q) \\
&= (\hat{q} - q) \left[\frac{u(\hat{q}) - u(q)}{\hat{q} - q} - \frac{u'(\hat{q}) - u'(q)}{\hat{q} - q} \frac{u'(\hat{q}) - 1}{u''(\hat{q})} - 1 \right].
\end{aligned}$$

Furthermore, note that

$$\frac{u(q^*) - u(q)}{q^* - q} - \frac{u'(q^*) - u'(q)}{q^* - q} \frac{u''(q^*) - 1}{u''(q)} - 1 = \frac{u(q^*) - u(q)}{q^* - q} - 1 > 0,$$

so by continuity, if γ is not large (but δ and η are large), then we have

$$\frac{u(\hat{q}) - u(q)}{\hat{q} - q} - \frac{u'(\hat{q}) - u'(q)}{\hat{q} - q} \frac{u''(\hat{q}) - 1}{u''(q)} - 1 > 0;$$

i.e., welfare is improving as δ increases ($\frac{\partial \mathcal{W}}{\partial \delta} > 0$), even if all agents consume less with a higher value of δ . ■

Proof of Proposition 9. From (5) and (14), we obtain

$$\begin{aligned} n \frac{\partial q_p}{\partial \delta} &= (1-n)(1-\delta) \frac{\partial q}{\partial \delta} + (1-n)(\hat{q} - q), \\ \frac{\partial q}{\partial \delta} &= \frac{\eta[u'(q) - 1]}{(1-\delta\eta)u''(q)}. \end{aligned}$$

Using these two expressions in (20), we obtain

$$\begin{aligned} (1-\beta) \frac{\partial \mathcal{W}}{\partial \delta} &= (1-n)(1-\delta)u'(q) \frac{\partial q}{\partial \delta} - n \frac{\partial q_p}{\partial \delta} + (1-n)[u(\hat{q}) - u(q)] \\ &= \frac{(1-n)(1-\delta)\eta[u'(q) - 1]^2}{(1-\delta\eta)u''(q)} + (1-n)[u(\hat{q}) - u(q) - (\hat{q} - q)]. \end{aligned}$$

If $\eta = 0$, then

$$(1-\beta) \frac{\partial \mathcal{W}}{\partial \delta} = (1-n)[u(\hat{q}) - u(q) - (\hat{q} - q)] > 0,$$

because

$$u(\hat{q}) - u(q) - (\hat{q} - q) = \frac{1}{\hat{q} - q} \left[\frac{u(\hat{q}) - u(q)}{\hat{q} - q} - 1 \right] > \frac{1}{\hat{q} - q} [u'(\hat{q}) - 1] = 0.$$

Thus, if η is close to zero, then it is optimal to increase δ . Now consider the case with $\eta = 1$, where we have

$$\frac{1-\beta}{1-n} \frac{\partial \mathcal{W}}{\partial \delta} = \frac{[u'(q) - 1]^2}{u''(q)} + [u(\hat{q}) - u(q) - (\hat{q} - q)].$$

Note that

$$\frac{[u'(q) - 1]^2}{u''(q)} + [u(\hat{q}) - u(q) - (\hat{q} - q)] = 0$$

at $q = \hat{q}$, and

$$\begin{aligned}
& \frac{\partial}{\partial q} \left\{ \frac{[u'(q) - 1]^2}{u''(q)} + [u(\hat{q}) - u(q) - (\hat{q} - q)] \right\} \\
&= \frac{2[u'(q) - 1][u''(q)]^2 - [u'(q) - 1]^2 u'''(q)}{[u''(q)]^3} - [u'(q) - 1] \\
&= [u'(q) - 1] \left\{ 1 - \frac{[u'(q) - 1]u'''(q)}{[u''(q)]^2} \right\} > 0
\end{aligned}$$

for q close to \hat{q} . Therefore, we have

$$\frac{\partial \mathcal{W}}{\partial \delta} < 0$$

for γ close to β . ■

Proof of Lemma 10. First, we consider the case with $\delta = 0$, which supports the type-A equilibrium, $\hat{q}_A, q_A, q_{p,A}$. Using the clearing condition (5), we have $(1 - n)q_A = nq_{p,A}$, so that welfare with $\delta = 0$ is given by

$$\begin{aligned}
(1 - \beta)\mathcal{W}|_{\delta=0} &= U(x^*) - x^* + (1 - n)u(q_A) - nq_{p,A} \\
&= U(x^*) - x^* + (1 - n)[u(q_A) - q_A],
\end{aligned}$$

where q_A satisfies $\frac{\gamma}{\beta} = (1 - n)u'(q_A) + n$ by (14). Now, consider the case with $\delta = 1$, which supports the type-B equilibrium, $\hat{q}_B, q_B, q_{p,B}$. By (5), we have $(1 - n)\hat{q}_B = nq_{p,B}$, so that welfare with $\delta = 1$ is given by

$$\begin{aligned}
(1 - \beta)\mathcal{W}|_{\delta=1} &= U(x^*) - x^* + (1 - n)u(\hat{q}_B) - nq_{p,B} \\
&= U(x^*) - x^* + (1 - n)[u(\hat{q}_B) - \hat{q}_B],
\end{aligned}$$

where \hat{q}_B satisfies $\frac{\gamma}{\beta} = (1 - n)u'(\hat{q}_B) \frac{\eta + (1 - \eta)u'(q_B)}{\eta + (1 - \eta)u'(\hat{q}_B)} + n$ by (17). Note that $\frac{\eta + (1 - \eta)u'(q_B)}{\eta + (1 - \eta)u'(\hat{q}_B)} > 1$ for $\eta < 1$ and $\frac{\eta + (1 - \eta)u'(q_B)}{\eta + (1 - \eta)u'(\hat{q}_B)} = 1$ for $\eta = 1$. Therefore $\hat{q}_B > q_A$ for $\eta < 1$, and $\hat{q}_B = q_A$ for $\eta = 1$. The proof is completed by noticing that $u(x) - x$ is concave and maximized at $q^* \geq \hat{q}_B$. ■

Proof of Proposition 11. From (5) and (14), we obtain

$$\begin{aligned}
n \frac{\partial q_p}{\partial \eta} &= (1 - n)(1 - \delta) \frac{\partial q}{\partial \eta}, \\
\frac{\partial q}{\partial \eta} &= \frac{\delta[u'(q) - 1]}{(1 - \delta\eta)u''(q)}.
\end{aligned}$$

Using these two expressions in (20), we have

$$\begin{aligned}(1 - \beta) \frac{\partial \mathcal{W}}{\partial \eta} &= (1 - n)(1 - \delta)u'(q) \frac{\partial q}{\partial \eta} - n \frac{\partial q_p}{\partial \eta} \\ &= \frac{(1 - n)(1 - \delta)\delta[u'(q) - 1]^2}{(1 - \delta\eta)u''(q)} < 0.\end{aligned}$$

■

References

- [1] Acharya, V., and Bisin, A., 2014, “Counterparty Risk Externalities: Centralized versus Over-The-Counter Markets”, *Journal of Economic Theory*, 149, 153-182.
- [2] Adrian, T., Fleming, M., Goldberg, J., Lewis, M., Natalucci, F., and Wu, J., 2013, “Dealer Balance Sheet Capacity and Market Liquidity during the 2013 Selloff in Fixed-income Markets”, *Liberty Streets Economics*.
- [3] Andolfatto, D., 2011, “A Note on the Societal Benefits of Illiquid Bonds”, *Canadian Journal of Economics*, 44, 133-147.
- [4] Aruoba, B., Rocheteau, G., and Waller, C., 2007, “Bargaining and the Value of Money”, *Journal of Monetary Economics*, 54, 2636-2655.
- [5] Berentsen, A., Camera, G., and Waller, C., 2007, “Money, Credit and Banking”, *Journal of Economic Theory*, 135, 171-195.
- [6] Berentsen, A., Huber, S., and Marchesiani, A., 2014, “Degreasing the Wheels of Finance”, *International Economic Review*, 55, 735-763.
- [7] Berentsen, A., Huber, S., and Marchesiani, A., 2015, “The Societal Benefits of a Financial Transaction Tax”, *working paper (first version 2014)*, University of Zurich.
- [8] Berentsen, A., Menzio, G., and Wright, R., 2011, “Inflation and Unemployment in the Long Run”, *American Economic Review*, 101, 371-398.
- [9] Berentsen, A., and Waller, C., 2011, “Outside Versus Inside Bonds: A Modigliani-Miller Type Result for Liquidity Constrained Economies”, *Journal of Economic Theory*, 146, 1852-1887.
- [10] Chun, Y., and Thomson, W., 1988, “Monotonicity Properties of Bargaining Solutions When Applied to Economics”, *Mathematical Social Sciences*, 15, 11-27.
- [11] Committee on the Global Financial System, 2014, “Market-making and Proprietary Trading: Industry Trends, Drivers and Policy Implications”, *CGFS Papers*, 52.

- [12] Duffie, D., 2012, “Market Making under the Proposed Volcker Rule”, *working paper, Stanford University*.
- [13] Duffie, D., Gârleanu, N., and Pederson, L. H., 2005, “Over-the-Counter Markets”, *Econometrica*, 73, 1815-1847.
- [14] Duffie, D., and Zhu, H., 2011, “Does a Central Clearing Counterparty Reduce Counterparty Risk?”, *Review of Asset Pricing Studies*, 1, 74-95.
- [15] Fender, I., and Lewrick, U., 2015, “Shifting Tides - Market Liquidity and Market-Making in Fixed Income Instruments”, *BIS Quarterly Report*.
- [16] Fleming, M., 2003, “Measuring Treasury Market Liquidity”, *FRBNY Economic Policy Review*, 9, 83-108.
- [17] Gallagher, D. M., 2014, “Remarks to the Georgetown University Center for Financial Markets and Policy Conference on Financial Markets Quality”, *SEC speeches*.
- [18] Geromichalos, A., Licari, J., and Lledo, J. S., 2007, “Asset Prices and Monetary Policy”, *Review of Economic Dynamics*, 10, 761-779.
- [19] Geromichalos, A., and Herrenbrueck, L., 2015, “Monetary Policy, Asset Prices, and Liquidity in Over-the-Counter Markets”, *Journal of Money, Credit, and Banking*, *forthcoming*.
- [20] Huber, S., and Kim, J., 2015, “Central Bank Purchases of Government Bonds”, *working paper, University of Zurich*.
- [21] Jacquet, N. L., and Tan, S., 2012, “Money and Asset Prices with Uninsurable Risk”, *Journal of Monetary Economics*, 59, 784-797.
- [22] Kalai, E., 1977, “Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons”, *Econometrica*, 45, 1623-1630.
- [23] Kocherlakota, N., 1998, “Money Is Memory”, *Journal of Economic Theory*, 81, 232-251.
- [24] Kocherlakota, N., 2003, “Societal Benefits of Illiquid Bonds”, *Journal of Economic Theory*, 108, 179-193.
- [25] Kiyotaki, N., and Wright, R., 1989, “On Money As a Medium of Exchange”, *Journal of Political Economy*, 97, 927-954.
- [26] Lagos, R., 2010a, “Some Results on the Optimality and Implementation of the Friedman Rule in the Search Theory of Money”, *Journal of Economic Theory*, 145, 1508-1524.
- [27] Lagos, R., 2010b, “Asset Prices and Liquidity in an Exchange Economy”, *Journal of Monetary Economics*, 57, 913-930.

- [28] Lagos, R., 2011, “Asset Prices, Liquidity, and Monetary Policy in an Exchange Economy”, *Journal of Money, Credit and Banking*, 43, 521-552.
- [29] Lagos, R., and Rocheteau, G., 2008, “Money and Capital as Competing Media of Exchange”, *Journal of Economic Theory*, 142, 247-258.
- [30] Lagos, R., and Rocheteau, G., 2009, “Liquidity in Asset Markets with Search Frictions”, *Econometrica*, 77, 403-426.
- [31] Lagos, R., Rocheteau, G., and Weill, P., 2011, “Crises and Liquidity in Over-the-Counter Markets”, *Journal of Economic Theory*, 146, 2169-2205.
- [32] Lagos, R., Rocheteau, G., and Wright, R., 2015, “Liquidity: A New Monetarist Perspective”, *Journal of Economic Literature*, forthcoming.
- [33] Lagos, R., and Wright, R., 2005, “A Unified Framework for Monetary Theory and Policy Evaluation”, *Journal of Political Economy*, 113, 463-484.
- [34] Lagos, R., and Zhang, S., 2015, “Monetary Exchange in Over-the-Counter Markets: A Theory of Speculative Bubbles, the Fed Model, and Self-fulfilling Liquidity Crises”, *working paper (first version 2014)*.
- [35] Leitner, Y., 2013, “Inducing Agents to Report Hidden Trades: A Theory of an Intermediary”, *Review of Finance*, 16, 1013-1042.
- [36] Lester, B., Postlewaite, A., and Wright, R., 2012, “Liquidity, Information, Asset Prices and Monetary Policy”, *Review of Economic Studies*, 79, 1209-1238.
- [37] Madhavan, A., 2000, “Market Microstructure: A Survey”, *Journal of Financial Markets*, 3, 205-258.
- [38] Mortensen, D. T., and Pissarides, C. A., 1994, “Job Creation and Job Destruction in the Theory of Unemployment”, *Review of Economic Studies*, 61, 397-415.
- [39] Nosal, E., and Rocheteau, G., 2011, “Money, Payments, and Liquidity”, MIT Press.
- [40] Nosal, E., and Rocheteau, G., 2013, “Pairwise Trades, Asset Prices, and Monetary Policy”, *Journal of Economic Dynamics and Control*, 37, 1-17.
- [41] Parlour, C., and Rajan, U., 2001, “Competition in Loan Contracts”, *American Economic Review*, 91, 1311-1328.
- [42] Rocheteau, G., and Wright, R., 2005, “Money in Competitive Equilibrium, in Search Equilibrium, and in Competitive Search Equilibrium”, *Econometrica*, 73, 175-202.
- [43] Rocheteau, G., and Wright, R., 2013, “Liquidity and Asset Market Dynamics”, *Journal of Monetary Economics*, 60, 275-294.

- [44] Shi, S., 2006, “Viewpoint: A Microfoundation of Monetary Economics”, *Canadian Journal of Economics*, 39, 643-688.
- [45] Stephens, E., and Thompson, J., 2014, “CDS as Insurance: Leaky Lifeboats in Stormy Seas”, *Journal of Financial Intermediation*, 23, 279-299.
- [46] Thompson, J., 2010, “Counterparty Risk in Financial Contracts: Should the Insured Worry about the Insurer?”, *Quarterly Journal of Economics*, 125, 1195-1252.
- [47] Trejos, A., and Wright, R., 2014, “Search-Based Models of Money and Finance: An Integrated Approach”, *Journal of Economic Theory*, *forthcoming*.
- [48] Wallace, N., 2001, “Whither Monetary Economics?”, *International Economic Review*, 42, 847-869.
- [49] Williamson, S., 1987, “Financial Intermediation, Business Failures, and Real Business Cycles”, *Journal of Political Economy*, 95, 265-302.
- [50] Williamson, S., 2012, “Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach”, *American Economic Review*, 102, 2570-2605.
- [51] Williamson, S., and Wright, R., 2010 “New Monetarist Economics: Models”, in B. Friedman and M. Woodford (eds.) *Handbook of Monetary Economics*, Volume II, Amsterdam: North-Holland.