

Schneider, Florian; Schonger, Martin

Working Paper

Allais at the Horse Race: Testing models of ambiguity aversion

Working Paper, No. 207

Provided in Cooperation with:

Department of Economics, University of Zurich

Suggested Citation: Schneider, Florian; Schonger, Martin (2015) : Allais at the Horse Race: Testing models of ambiguity aversion, Working Paper, No. 207, University of Zurich, Department of Economics, Zurich,
<https://doi.org/10.5167/uzh-113552>

This Version is available at:

<https://hdl.handle.net/10419/126604>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



**University of
Zurich** ^{UZH}

University of Zurich
Department of Economics

Working Paper Series

ISSN 1664-7041 (print)
ISSN 1664-705X (online)

Working Paper No. 207

**Allais at the Horse Race:
Testing Models of Ambiguity Aversion**

Florian Schneider and Martin Schonger

October 2015

ALLAIS AT THE HORSE RACE: TESTING MODELS OF AMBIGUITY AVERSION

FLORIAN SCHNEIDER AND MARTIN SCHONGER

ABSTRACT. Most models of ambiguity aversion satisfy Anscombe-Aumann's Monotonicity axiom. This paper proposes a test of Monotonicity, the Allais Horse Race. It is an adaptation of the Allais paradox to a setting with both subjective and objective uncertainty. Viewed as a thought experiment, the Allais Horse Race allows for introspective assessment of Monotonicity. Implementing it as an incentivized experiment, we find that the modal choice of subjects violates Monotonicity in a specific, intuitive way. Overall, we find that models of ambiguity aversion that satisfy Monotonicity cannot describe the behavior of about half of all subjects.

1. INTRODUCTION

Since Ellsberg (1961) pioneered the concept of ambiguity aversion, both theorists and experimentalists have taken a keen interest in the concept. Ambiguity aversion is usually studied in the Anscombe-Aumann (1963) framework. Anscombe-Aumann proposed a Monotonicity axiom. Monotonicity prescribes, that if two acts differ only on a single state, then the preference between these two acts is given by the preference between the lotteries that are assigned to that state. The descriptive validity of this axiom is the focus of this paper.

Schneider: Department of Economics, University of Zurich, florian.schneider2@econ.uzh.ch, www.econ.uzh.ch/faculty/fschneider.html. Schonger: Center for Law and Economics, ETH Zurich, mschonger@ethz.ch, www.n.ethz.ch/~scmartin.

For helpful discussions we would like to thank Björn Bartling, Damian Kozbur, Felix Kübler, Roberto Weber, and seminar participants at EEA Mannheim, NCBEE Tampere, CUSO Bern 2015, and Zurich. We would like to thank Himanshu Jain for excellent research assistance. Financial support from the Chair of Gerard Hertig, ETH Zurich is gratefully acknowledged.

JEL classification: D81 (Criteria for Decision-Making under Risk and Uncertainty)

Keywords: ambiguity aversion, monotonicity, Anscombe-Aumann, Allais paradox, experiments.

The Monotonicity axiom is widely used in models of ambiguity aversion. Indeed, apart from basic choice theoretic axioms like Transitivity and Continuity, Monotonicity seems to be the most common axiom models of ambiguity aversion satisfy. Within the Anscombe-Aumann framework, models of ambiguity aversion that satisfy Monotonicity include Multiple priors (Gilboa and Schmeidler, 1989), Rank-Dependent/Choquet Model (Schmeidler, 1989), Smooth ambiguity preferences (Klibanoff, Marinacci, and Mukerji, 2005), Variational preferences (Maccheroni, Marinacci, and Rustichini, 2006), Confidence-function preferences (Chateauneuf and Faro, 2009), Vector Expected Utility (Siniscalchi, 2009), Uncertainty-averse preferences (Cerrei-Vioglio, Maccheroni, Marinacci, and Montrucchio, 2011), MBA-preferences (Cerrei-Vioglio, Ghirardato, Maccheroni, Marinacci, and Siniscalchi, 2011), monotone mean-dispersion preferences (Grant and Polak, 2013), and Hedging preferences (Dean and Ortoleva, forthcoming). Models that do not satisfy Monotonicity include the Anticipated utility/Recursive model (Segal, 1987), the related Recursive rank-dependent utility model (Abdellaoui and Zank, 2015), and (non-monotone) mean-dispersion preferences (Grant and Polak, 2013).

Experimental work on ambiguity aversion has focused on measuring ambiguity aversion or seeking in different domains (gains, losses), for different likelihoods, and for different sources of ambiguity¹. But we know of no method to test the Monotonicity axiom, let alone experimental work that actually does so. A study that like ours tries to distinguish between different models of ambiguity aversion is Halevy (2007), but that study focuses on the association between ambiguity aversion and failure to satisfy the axiom of Reduction of compound lotteries.²

This paper provides a thought experiment that is a test of Monotonicity. Our thought experiment is an adaptation of the classical Allais paradox. Recall that the Allais paradox is set in a world of purely objective uncertainty, and tests Expected Utility (specifically the von Neumann-Morgenstern Independence axiom).

¹For recent reviews of the experimental literature see Hey (2014); Trautmann and van de Kuilen (2015)

²Also see Abdellaoui, Klibanoff, and Placido (2015).

Previous authors³ have adapted the Allais paradox to a setting of purely subjective uncertainty, where it becomes a test of Subjective Expected Utility. We adapt the Allais paradox to a setting where there are both objective and subjective sources of uncertainty. The so-modified Allais paradox tests Monotonicity, and we call it the Allais Horse Race. The Allais Horse Race serves two roles: first it allows for introspective “testing” of Monotonicity, and second, it paves the way for the actual experimental test of the descriptive validity of Monotonicity.

We conduct an incentivized experiment implementing the Allais Horse Race. A real-world lottery is the source of objective uncertainty, while for subjective uncertainty we use future weather in a foreign city. We find that about half of all participants violate Monotonicity, and overwhelmingly do so in a specific, non-random way. The hypothesis that violations are due to random error is easily and robustly rejected by the data. The specific pattern of violations we find mirrors the pattern that has been found in experimental investigations of the Allais paradox. Towards the end of the experiment we confront participants with the original Allais paradox. It turns out that violating Independence strongly predicts violating Monotonicity.

To provide some insight into the empirical correlation between violations of Independence and Monotonicity, theorem 1 establishes that for probabilistically sophisticated decision-makers, Monotonicity and Independence are equivalent. This might explain why decision-makers in general tend to either satisfy both Independence and Monotonicity or violate both. For most decision-makers the axioms, while not equivalent, are probably similar.

The paper is organized as follows: Section 2 introduces the Allais Horse Race thought experiment, gives a formal setting, and provides the conditional equivalence of Monotonicity and Independence. Section 3 describes the design of our actual experiment, and section 4 provides the results. Section 5 concludes.

2. THE ALLAIS HORSE RACE AND MONOTONICITY

2.1. The Allais Horse Race.

³See MacCrimmon and Larsson (1979); Tversky and Kahneman (1992); Wu and Gonzalez (1999), also the discussion in Wakker (2010), pp. 134f.

$$\begin{array}{l}
1A : \$1 \text{ Million with certainty.} \\
1B : \begin{cases} \$0 & \text{Horse 1-11 wins and roulette stops on 1.} \\ \$1 \text{ Million} & \text{Horse 12-100 wins.} \\ \$5 \text{ Million} & \text{Horse 1-11 wins and roulette stops on 2-11.} \end{cases} \\
\\
2A : \begin{cases} \$0 & \text{Horse 12-100 wins.} \\ \$1 \text{ Million} & \text{Horse 1-11 wins.} \end{cases} \\
2B : \begin{cases} \$0 & \text{Horse 1-11 wins and roulette stops on 1,} \\ & \text{or Horse 12-100 wins.} \\ \$5 \text{ Million} & \text{Horse 1-11 wins and roulette stops on 2-11.} \end{cases}
\end{array}$$

FIGURE 1. Allais Horse Race

We develop the Allais Horse Race thought experiment to test Monotonicity. The purpose of the thought experiment is twofold: to help intuition of how strong Monotonicity is, and to allow for actual experimental tests. As in the classical Anscombe-Aumann story there is a horse race and a roulette wheel. 100 horses numbered from 1 to 100 are starting. You do not know the probability that a particular horse will win. At the same time as the horses are running, the roulette wheel is spun. The roulette wheel has 11 fields, which are equiprobable and numbered from 1 to 11. As shown in figure 1 the decision-maker is confronted with two choice situations, 1 and 2. In each choice situation she has the choice between two options, A and B. In choice situation 1 an intuitively plausible choice, in line with the certainty effect (Kahneman and Tversky, 1979), might be to prefer 1A, which is a million for sure, over 1B, where there is a chance of not winning anything. In choice situation 2 by contrast in both bets there is a danger of winning nothing, thus the chance of winning \$5 Million in bet 2B may make that bet more attractive than bet 2A. Such a choice pattern violates Monotonicity (see observation 1 in the next subsection). It will be helpful to have a name for this particular choice pattern, let us call these choices the intuitive paradoxical choice pattern.

Compare the Allais Horse Race to the classical Allais paradox (Allais, 1953), reproduced in figure 2. Recall that in the Allais paradox all uncertainty is ob-

$$\begin{aligned}
\text{IA} & : \text{\$1 Million with certainty.} \\
\text{IB} & : \begin{cases} \$0 & \text{Roulette stops on 1.} \\ \$1 \text{ Million} & \text{Roulette stops on 2-90.} \\ \$5 \text{ Million} & \text{Roulette stops on 91-100.} \end{cases} \\
\text{IIA} & : \begin{cases} \$0 & \text{Roulette stops on 1-89.} \\ \$1 \text{ Million} & \text{Roulette stops on 90-100.} \end{cases} \\
\text{IIB} & : \begin{cases} \$0 & \text{Roulette stops on 1-90.} \\ \$5 \text{ Million} & \text{Roulette stops on 91-100.} \end{cases}
\end{aligned}$$

FIGURE 2. Allais paradox

jective. Thus in the metaphors of the Anscombe-Aumann framework, the Allais paradox only features a roulette wheel (albeit with more fields), but no horse race. Since the Allais Horse Race and the Allais paradox only differ in the sources of uncertainty, a decision-maker who does not distinguish between objective and subjective uncertainty, i.e. a probabilistically sophisticated one, views them as the same choice situation (for a more formal statement see p. 8).

Again let us call the pattern of choosing IA over IB, and IIB over IIA the intuitive paradoxical choice pattern, as it is analogous to the intuitive paradoxical choice pattern in the Allais Horse Race (choosing 1A over 1B, and 2B over 2A). The intuitive paradoxical choice pattern is what has often been found in experimental investigations of the Allais paradox⁴, and constitutes a violation of Independence. The next subsection observes that exhibiting the intuitive paradoxical choice pattern in the Allais Horse Race behavior violates Monotonicity.

2.2. Formal setting. To formally discuss the Monotonicity axiom and its implications, let us introduce an Anscombe-Aumann framework. There are (monetary) prizes ω in an interval $[\underline{\omega}, \bar{\omega}] \subset \mathbb{R}$. We denote the space of simple probability distributions over $[\underline{\omega}, \bar{\omega}]$ by \mathcal{P} , with generic elements p, q, r, t . A degenerate lottery puts probability 1 on ω in $[\underline{\omega}, \bar{\omega}]$ and is denoted by σ_ω . There is a (finite or

⁴See for example Kahneman and Tversky (1979), Conlisk (1989), Burke, Carter, Gominiak, and Ohl (1996), Fan (2002), and Huck and Müller (2012).

infinite) set of states $\mathcal{S} = \{\dots, s, \dots\}$. An act f is a function $f : \mathcal{S} \rightarrow \mathcal{P}$ such that for some finite partition $(E_1, \dots, E_n, \dots, E_N)$ of the state space \mathcal{S} , for all n , if s', s'' in E_n then $f_{s'} = f_{s''}$. The set of all acts is denoted by \mathcal{L} . Generic acts are f, g, h . Given a partition $(E_1, \dots, E_n, \dots, E_N)$, the act $f = [\dots, f_n \text{ on } E_n, \dots]$ gives the lottery f_n on event E_n . An act that yields the same lottery p in each state is called a constant act, and, slightly abusing notation, is denoted by p when there is no risk of confusion. Given two acts f and g and a common partition, a new act can be generated by taking a convex combination of the lotteries obtained in each event: $\alpha f + (1 - \alpha) g$, where α in $(0, 1)$.⁵ Throughout we assume that objective and subjective uncertainty are resolved simultaneously. \succsim is a preference order on \mathcal{L} :

Axiom (Weak order). \succsim is complete, reflexive and transitive.

Anscombe-Aumann(1963) proposed the Monotonicity axiom which they called “monotonicity in prizes”. Monotonicity requires that if the lottery that an act assigns to an event is replaced by a preferred lottery, then the new act must be preferred:

Axiom (Monotonicity). For all constant acts p, q : if $p \succsim q$, then for any act f given a partition for f :

$$\begin{aligned} [\dots, f_{n-1} \text{ on } E_{n-1}, p \text{ on } E_n, f_{n+1} \text{ on } E_{n+1}, \dots] &\succsim \\ [\dots, f_{n-1} \text{ on } E_{n-1}, q \text{ on } E_n, f_{n+1} \text{ on } E_{n+1}, \dots]. & \end{aligned}$$

Another version of the Monotonicity axiom is often used in the literature: for any acts f, g , if $f_s \succsim g_s$ for all s then $f \succsim g$. Note that under transitivity the two versions are equivalent.

While not the focus of this paper, it is natural to ask what Monotonicity implies for the representation of preferences. Monotonicity, together with three basic choice theoretic axioms (Weak order, Mixture-continuity, and First-order stochastic dominance), ensures that the preference admits a monotone, “state-wise separable” representation. That is an act can be evaluated by a two-step procedure: A preference functional evaluates the lottery given in each state. The

⁵ $\alpha f + (1 - \alpha) g \equiv [\alpha f_1 + (1 - \alpha) g_1 \text{ on } E_1, \dots, \alpha f_N + (1 - \alpha) g_N \text{ on } E_N]$

preference functional could for example be expected utility, rank-dependent expected utility (Quiggin, 1982) or cumulative prospect theory (Tversky and Kahneman, 1992). This yields S numbers which are then aggregated by a monotone function to a single number representing the utility of the act. For a more formal discussion see the appendix.

Now we can translate the Allais Horse Race into the formal setting. Define the state space to be $S = \{1, 2, \dots, 100\}$, and partition it into the events $E_1 = \{1, \dots, 89\}$ and $E_2 = \{90, \dots, 100\}$. The four acts can be written as $1A = \sigma_{1M}$, $1B = [\sigma_{1M} \text{ on } E_1, (0; \frac{1}{11}, 5M; \frac{10}{11}) \text{ on } E_2]$, $2A = [\sigma_0 \text{ on } E_1, \sigma_{1M} \text{ on } E_2]$, and $2B = [\sigma_0 \text{ on } E_1, (0; \frac{1}{11}, 5M; \frac{10}{11}) \text{ on } E_2]$. The following observation tells us which choice patterns are consistent with Monotonicity:

Observation 1. *If \succsim satisfies Weak order and Monotonicity, then:*

$1A \succsim 1B$ and $2A \succsim 2B$, or, $1B \succsim 1A$ and $2B \succsim 2A$.

The proof, and all other proofs, are in the appendix. Note that Monotonicity rules out the intuitive paradoxical choice pattern.

2.3. Equivalence of Independence and Monotonicity under probabilistic sophistication. As argued previously, for a decision-maker who is probabilistically sophisticated, the Allais paradox and the Allais Horse Race are essentially the same problem. Behind this insight lies a more general principle, which theorem 1 in this subsection develops: for any decision-maker who is probabilistically sophisticated, the Independence and Monotonicity axioms are equivalent. Machina and Schmeidler (1992, 1995) provide an axiomatic justification for probabilistic sophistication. A probabilistically sophisticated decision-makers acts as if she has a unique subjective probability distribution over states, and she treats objective and subjective probabilities interchangeably:

Axiom (Probabilistic sophistication). *There exists a non-atomic⁶ probability measure μ such that for all acts f with partition (E_1, \dots, E_N) :*

$$f \sim \sum_{i=1}^N \mu(E_i) f_i.$$

⁶A finitely additive probability measure μ is non-atomic if for every event E with $\mu[E] > 0$ and every α in $[0, 1]$ there is an event $E' \subset E$ such that $\mu(E') = \alpha\mu(E)$.

The above axiom closely follows the Machina-Schmeidler (1995, p.114) definition of probabilistic sophistication.⁷ The probabilistic sophistication axiom neither implies nor is implied by Independence and Monotonicity. Probabilistic sophistication essentially turns the Allais Horse Race into the Allais paradox. Assuming that the decision-maker views the states as informationally symmetric, she assigns subjective probabilities of $\mu(E_1) = \frac{89}{100}$ and $\mu(E_2) = \frac{11}{100}$. Then the decision-maker is indifferent between each act in the Allais Horse Race and the corresponding act in the Allais paradox ($1A \sim IA$, $1B \sim IB$, $2A \sim IIA$, and $2B \sim IIB$). Before turning to theorem 1, let us state the axioms required:

Axiom (Independence). *For all constant acts p , q , and r , and for all α in $(0, 1)$:*

$$p \succsim q \Rightarrow \alpha p + (1 - \alpha)r \succsim \alpha q + (1 - \alpha)r.$$

Independence applies to objective uncertainty only, thus in the Anscombe-Aumann framework its domain is the set of constant acts. If one specifies as the domain the set of all acts, then one gets a stronger independence axiom, which for clarity we shall refer to as Anscombe-Aumann-Independence (AA-Independence):

Axiom (AA-Independence). *For all acts f, g, h , and for all α in $(0, 1)$:*

$$f \succsim g \Rightarrow \alpha f + (1 - \alpha)h \succsim \alpha g + (1 - \alpha)h.$$

The central insight of the theorem is, that, given probabilistic sophistication, Monotonicity and Independence are equivalent. In addition, given probabilistic sophistication, Independence implies its stronger sibling AA-Independence:

Theorem 1. *If \succsim satisfies Weak order and Probabilistic sophistication then the following statements are equivalent:*

- (i) \succsim satisfies Monotonicity.
- (ii) \succsim satisfies Independence.
- (iii) \succsim satisfies AA-Independence.

We believe that Monotonicity and Independence are similar, and that even non-probabilistically sophisticated decision-makers will tend to either satisfy or

⁷Unlike Machina-Schmeidler we require the subjective probability measure to be non-atomic. Savage (1954) was the first to provide conditions for existence of a non-atomic probability measure. For several definitions of non-atomic measures, and a brief discussion see Gilboa (2009, p.107-8).

violate both axioms. For probabilistic sophistication the similarity peaks in the form of equivalence.

Incidentally, theorem 1 suggests an alternative axiomatization of subjective expected utility (SEU). As is well-known (see Fishburn, 1970), substantive axioms that ensure SEU are AA-Independence together with Monotonicity. The theorem suggests that alternatively, if one assumes probabilistic sophistication, then adding Monotonicity (or Independence) also yields SEU.

3. EXPERIMENTAL DESIGN

To find out to what extent the Monotonicity axiom holds empirically we run an experiment. A second purpose of the experiment is to compare and relate the prevalence of potential violations of Monotonicity to those of Independence. We adapt the sources of uncertainty and prizes of the Allais Horse Race as follows: Rather than a horse race, the source of subjective uncertainty is weather in a foreign city as in Fox and Tversky (1995). Specifically, we consider the event that tomorrow’s high temperature in Mexico City is “unexpectedly high”, and its complement “not unexpectedly high”. We define “unexpectedly high” as 6° Fahrenheit or more above the current forecast. Participants are told the current forecast and are encouraged to check it on an external website, which shows forecasts and past realizations rounded to integer degrees Fahrenheit. For objective uncertainty we use a real-world lottery, the evening draw of “Texas Pick 3”. It produces the numbers between 000 and 999 with equal probability. In the Allais Horse Race we consider the last digit, in the Allais paradox the last two digits. Thus we slightly modify the objective probabilities in the Allais Horse Race (10 instead of 11 equiprobable objective events). This simplifies the objective probabilities for participants, and, in the terminology of Zizzo (2010), non-deceptively obfuscates the purpose of the experiment to participants as the Allais Horse Race and the Allais paradox become even less similar. For the non-zero prizes, rather than the thought experiment amounts of \$1 Million and \$5 Million we use \$4 and \$5.⁸ With these modifications and partitioning the state space into the event $E =$ “unexpectedly hot” and its complement E^C we get the acts

⁸Similar proportions of the two non-zero prizes in small-stakes Allais paradox experiments are used in Fan (2002) and Incekara-Hafalir and Stecher (2012).

$1A^* = \sigma_4$, $1B^* = [\sigma_4 \text{ on } E^C, (0; \frac{1}{10}, 5; \frac{9}{10}) \text{ on } E]$, $2A^* = [\sigma_0 \text{ on } E^C, \sigma_4 \text{ on } E]$, and $2B^* = [\sigma_0 \text{ on } E^C, (0; \frac{1}{10}, 5; \frac{9}{10}) \text{ on } E]$. Figure 3 shows, using the example of act $1B^*$, how the acts are displayed to participants (the temperature symbols are defined and explained to participants). Acts are called “lotteries” throughout the experiment. Note that observation 1 analogously applies to the acts in the experiment. In the case of the Allais paradox, the lotteries in the experiment ($1A^*$, $1B^*$, $2A^*$, $2B^*$) differ from the original lotteries ($1A$, $1B$, $2A$, $2B$) only in the prizes as described above.

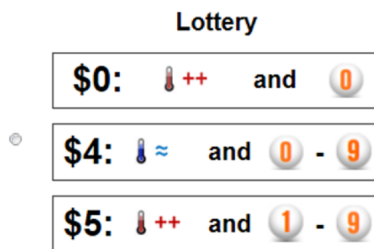


FIGURE 3. Visual representation of act $1B^*$

Participants were recruited on Amazon Mechanical Turk (AMT). We restricted participation to U.S. workers. The experiment was conducted using the Qualtrics Survey Platform, all subjects completed the study within a few hours on the same day in the second half of 2015. We incentivized participants with a substantial participation fee and prizes: The median hourly reservation wage on AMT is \$1.38⁹, the experiment was described as taking 10 to 15 minutes with a participation fee of \$0.50, to which in addition prizes from \$0 to \$5 could be won.

The sequence of the experiment is as follows¹⁰: participants are informed about the study and voluntary consent is obtained, the acts are explained including links to the external websites of weather.com and Texas Pick 3, followed by three (non-incentivized) understanding questions. As a further test of participants’ understanding the subsequent screen offers a choice between two acts, where one act first-order stochastically dominates the other¹¹. On this screen, as well as

⁹Horton and Chilton (2010)

¹⁰A web appendix gives screenshots and a detailed description of the experiment.

¹¹The notion of first-order stochastic dominance used here is the generalization of the standard notion of first-order stochastic dominance to a setting of objective/subjective uncertainty, see axiom FOSD, p.22.

the Allais Horse Race and Allais paradox screens, subjects are given a choice¹² between the acts, which are presented in random order. The next two screens each present an Allais Horse Race choice situation, where the order of the screens (choice situations) is random. This is followed by the two Allais paradox choice situations, which are again displayed in random order. Finally, using the reservation probability method¹³, participants' beliefs about the probability of the event E ("unexpectedly high temperature") are elicited in an incentivized manner. In total participants are asked to make nine choices, and the Random Lottery Incentive (RLI) system is used to make hedging impossible.¹⁴

4. EXPERIMENTAL RESULTS

Data from $N = 552$ participants was collected. Table 1 gives participants' choices in the Allais Horse Race. Consider the first row of results which concerns the full sample. Choices in columns two and three do not violate Monotonicity, choices in columns four and five do. The modal choice of participants violates Monotonicity in a particular way: 38.8 percent of participants exhibit the intuitive paradoxical choice pattern (choosing $1A^*$ over $1B^*$ but $2B^*$ over $2A^*$). Violations of Monotonicity are not random, rather there is an asymmetric pattern, as violations in the opposite way are much rarer with 9.2 percent. Following Conlisk (1989) and the subsequent literature, we test whether violations are the result of random, symmetric participant error. We can reject that hypothesis at all conventional significance levels ($Z = 11.06$, $p\text{-value} < 10^{-28}$). The remaining

¹²We use direct choice rather than willingness-to-pay to avoid the preference reversals arising from loss aversion as discovered by Trautmann, Vieider, and Wakker (2011).

¹³The reservation probability method is an adaptation of the BDM-mechanism (Becker, De-groot, and Marschak, 1964). Its advantage is that it does not require correction for the risk attitude of subjects (Schlag, Tremewan, and van der Weele, 2014). Besides offering incentives to participants, the reservation probability method has another advantage in an ambiguity aversion context: asking for a single probability estimate need not be a well-defined question for subjects who do not form a single subjective prior, but the questions of the reservation probability method are well-defined even for such subjects. The exact interpretation of the answer then naturally depends on what model of ambiguity attitudes one assumes. A conservative interpretation is to assume the multiple priors model (Gilboa and Schmeidler, 1989), then the elicited belief corresponds to the prior that assigns the lowest probability on the event tomorrow's temperature is "unexpectedly high".

¹⁴Starmer and Sugden (1991) show that the RLI is empirically valid since participants tend to bracket narrowly.

rows repeat the analysis but for different subsamples as a first robustness check. The second row of results considers only participants who got both test questions 1 and 2 correct. The third row considers only participants who in addition satisfy (generalized) first-order stochastic dominance. The remaining rows split the sample by duration. These different subsamples can be seen as different ways of restricting attention to subjects who understand the experiment and take it seriously, an issue about which one may be particularly concerned on AMT. The picture that emerges is that the distribution of violations is very similar across subsamples. Table 1 in the appendix (table 5, p.25) finds robustness across further subsamples. A more formal test of robustness is provided by the linear probability model towards the end of this section.

TABLE 1. Violations of Monotonicity (in percent)

	SATISFY		VIOLATE		Conl.-Z	N
	1A*2A*	1B*2B*	1A*2B*	1B*2A*		
Full sample	36.8	15.2	38.8	9.2	11.06	552
Robustness in subsamples:						
Testq. 1-2 correct	36.7	14.7	39.7	8.9	10.24	436
Testq. 1-2 correct & FOSD	35.3	17.2	38.1	9.4	8.33	331
Duration 1. quartile	37.0	14.5	39.9	8.7	5.85	138
Duration 2. quartile	36.2	14.5	38.4	10.9	4.99	138
Duration 3. quartile	36.2	14.5	39.1	10.1	5.31	138
Duration 4. quartile	37.7	17.4	37.7	7.2	5.97	138

A similar pattern emerges in the Allais paradox (table 2): 35.3 percent of participants exhibit the intuitive paradoxical choice pattern (choosing IA* over IB* while choosing IIB* over IIA*).¹⁵ Conlisk's test rejects the hypothesis that this violation is due to random, symmetric error ($Z= 6.92$, $p\text{-value} < 10^{-10}$). Again results are robust across subsamples, table 6 in the appendix (p.26) offers even more robustness checks by subsample yielding the same conclusion. Roughly speaking violations of Independence have similar prevalence as violations of Monotonicity,

¹⁵Other incentivized, small-stakes Allais experiments find similar results, e.g. 27.2% for that cell in Fan (2002).

in both cases about half the subjects violate the axioms, and in both cases the violations display the same asymmetric pattern.

TABLE 2. Violations of Independence (in percent)

	SATISFY		VIOLATE		Conl.-Z	N
	IA*IIA*	IB*IIB*	IA*IIB*	IB*IIA*		
Full sample	23.0	26.4	35.3	15.2	6.92	552
Robustness in subsamples:						
Testq. 1-2 correct	24.5	24.8	35.3	15.4	6.09	436
Testq. 1-2 correct & FOSD	23.9	25.7	32.9	17.5	4.04	331
Duration 1. quartile	21.7	28.3	37.7	12.3	4.50	138
Duration 2. quartile	21.7	21.7	39.1	17.4	3.54	138
Duration 3. quartile	21.7	29.0	34.1	15.2	3.26	138
Duration 4. quartile	26.8	26.8	30.4	15.9	2.55	138

Violations of Monotonicity and Independence are related: The probability for a participant to violate Monotonicity with the intuitive paradoxical choice pattern is 31.7 percent conditional on not violating Independence with the intuitive paradoxical choice pattern, but it rises to 51.8 percent conditional on violating Independence with the intuitive paradoxical choice pattern. Table 3 provides a fuller picture of the relationship: The full sample is partitioned into the four possible choice patterns in the Allais paradox, each row in the table corresponds to a pattern. The table entries give the fraction of participants exhibiting a par-

TABLE 3. Relationship between Independence and Monotonicity

		SATISFY		VIOLATE		Conl.-Z	N
		1A*2A*	1B*2B*	1A*2B*	1B*2A*		
Satisfy	IA*IIA*	52.0	3.9	37.0	7.1	5.67	127
	IB*IIB*	21.2	36.3	29.5	13.0	3.14	146
Violate	IA*IIB*	43.6	3.6	51.8	1.0	13.60	195
	IB*IIA*	25.0	22.6	27.4	25.0	0.30	84

ticular Allais Horse Race behavior among each of these subsamples. Note that

for each particular behavior in the Allais paradox the modal choice is the analogous behavior in the Allais Horse Race (except for the last and smallest group, the opposite of the intuitive paradoxical choice pattern). This is striking since the questions are presented screens apart, the order in which the acts and choice situations appear is random, and the questions are substantially different.

TABLE 4. Linear Probability Model

	(1)	(2)	(3)
Allais IB*IIA*	-0.0963 (0.0651)	-0.0969 (0.0658)	-0.0783 (0.0660)
Allais IA*IIB*	0.148*** (0.0560)	0.149*** (0.0563)	0.158*** (0.0559)
Allais IB*IIB*	-0.0756 (0.0573)	-0.0764 (0.0588)	-0.0770 (0.0588)
Testq. 1 correct		-0.0124 (0.0587)	-0.0178 (0.0569)
Testq. 2 correct		0.0476 (0.0732)	0.0367 (0.0732)
Testq. 3 correct		-0.0125 (0.0422)	-0.0112 (0.0428)
Satisfy FOSD		0.0116 (0.0469)	0.0218 (0.0475)
Duration (min.)		-0.00427 (0.00316)	-0.00501 (0.00323)
Belief \leq 10%		0.0481 (0.0503)	0.0576 (0.0514)
Belief $>$ 10%		0.0181 (0.0582)	0.0428 (0.0580)
_cons	0.370*** (0.0430)	0.353*** (0.0997)	0.324** (0.146)
N	552	552	552
R-sq	0.0435	0.0486	0.0954
Ind. Contr.	No	No	Yes

Dependent variable: Binary variable of intuitive paradoxical choice pattern (1A*2B*). Heteroskedasticity-consistent standard errors in parentheses. * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$. Omitted category: Allais IA*IIA*. Individual controls: gender, age, income, education, employment status, and ownership of stocks.

To investigate the questions of robustness of the results, and predictors of violations of Monotonicity further, table 4 provides a linear probability model¹⁶. The dependent variable is a binary variable which takes the value 1 iff a subject exhibits the intuitive paradoxical choice pattern in the Allais Horse Race. In all our specifications exhibiting the intuitive paradoxical choice pattern in the Allais paradox increases the likelihood of exhibiting the analogous pattern in the Allais Horse Race by about 15 percentage points compared to the baseline of choosing IA* and IIA*. That coefficient is significant at the 1 percent level in all specifications.

5. CONCLUSION

Data collected in this paper challenges models of ambiguity aversion that assume Anscombe-Aumann's Monotonicity axiom. About half of all our experimental subjects exhibit behavior at odds with the predictions of such models. The Monotonicity axiom is, apart from basic choice theoretic axioms, the most common assumption in models of ambiguity aversion. Monotonicity says that if the lottery that an act assigns to a particular state is replaced by a preferred lottery then the new act must be preferred to the original one. From the perspective of the representation of preferences, Monotonicity implies that the preference admits a monotone, state-wise separable representation.

This paper introduces the Allais Horse Race, which is a modification of the famous Allais paradox. In the Allais paradox all uncertainty is objective, the modification consists of making some but not all of the uncertainty subjective. Thereby the experiment ceases to be a test of Independence, and instead becomes a test of Monotonicity.

Theorem 1 relates the Monotonicity and Independence axioms, by establishing that for probabilistically sophisticated decision-makers the two axioms are equivalent. We think of Monotonicity as the cousin of Independence in the world of subjective/objective uncertainty. Even decision-makers who are not probabilistically sophisticated probably view the axioms as related. Our experimental evidence confirms this intuition: First, violations of Independence and Monotonicity are of similar prevalence, being exhibited by about one in two participants. Second,

¹⁶Employing a logit rather than a linear probability model yields essentially the same results.

violations of Independence and Monotonicity are related, violating Independence is a strong predictor of violating Monotonicity.

The Allais paradox challenged the descriptive accuracy of Independence in a setting of objective uncertainty, leading to the development of new positive theories (e.g. Gul, 1991) which replace Independence by weaker axioms such as Betweenness (see Chew, 1983 and Dekel, 1986). The Allais Horse Race tests Monotonicity, and the evidence collected on its basis in this paper shows that models that assume Monotonicity are not rich enough to describe modal subject behavior.

REFERENCES

- ABDELLAOUI, M., P. KLIBANOFF, AND L. PLACIDO (2015): “Experiments on Compound Risk in Relation to Simple Risk and to Ambiguity,” *Management Science*, 61(6), 1306–1322.
- ABDELLAOUI, M., AND H. ZANK (2015): “Recursive Non-expected Utility: Accounting for Allais and Ellsberg-type Behavior,” Discussion paper, University of Manchester.
- ALLAIS, M. (1953): “Le Comportement de l’Homme Rationnel devant le Risque: Critique des Postulats et Axiomes de l’Ecole Americaine,” *Econometrica*, 21(4), pp. 503–546.
- ANSCOMBE, F. J., AND R. J. AUMANN (1963): “A Definition of Subjective Probability,” *The Annals of Mathematical Statistics*, 34(1), pp. 199–205.
- BECKER, G. M., M. H. DEGROOT, AND J. MARSCHAK (1964): “Measuring utility by a single-response sequential method,” *Behavioral Science*, 9(3), 226–232.
- BURKE, M. S., J. R. CARTER, R. D. GOMINIAK, AND D. F. OHL (1996): “An experimental note on the allais paradox and monetary incentives,” *Empirical Economics*, 21(4), 617–632.
- CERREIA-VIOGLIO, S., P. GHIRARDATO, F. MACCHERONI, M. MARINACCI, AND M. SINISCALCHI (2011): “Rational preferences under ambiguity,” *Economic Theory*, 48(2-3), 341–375.

- CERREIA-VIOGLIO, S., F. MACCHERONI, M. MARINACCI, AND L. MONTRUCCHIO (2011): “Uncertainty averse preferences,” *Journal of Economic Theory*, 146(4), 1275 – 1330.
- CHATEAUNEUF, A., AND J. H. FARO (2009): “Ambiguity through confidence functions,” *Journal of Mathematical Economics*, 45(9 - 10), 535 – 558.
- CHEW, S. H. (1983): “A Generalization of the Quasilinear Mean with Applications to the Measurement of Income Inequality and Decision Theory Resolving the Allais Paradox,” *Econometrica*, 51(4), pp. 1065–1092.
- CONLISK, J. (1989): “Three Variants on the Allais Example,” *American Economic Review*, 79(3), pp. 392–407.
- DEAN, M., AND P. ORTOLEVA (forthcoming): “Allais, Ellsberg, and Preferences for Hedging,” *Theoretical Economics*.
- DEKEL, E. (1986): “An axiomatic characterization of preferences under uncertainty: Weakening the independence axiom,” *Journal of Economic Theory*, 40(2), 304 – 318.
- ELLSBERG, D. (1961): “Risk, Ambiguity, and the Savage Axioms,” *Quarterly Journal of Economics*, 75(4), pp. 643–669.
- FAN, C.-P. (2002): “Allais paradox in the small,” *Journal of Economic Behavior and Organization*, 49(3), 411 – 421.
- FISHBURN, P. C. (1970): *Utility Theory for Decision Making*, Publications in operations research. Wiley, New York.
- FOX, C. R., AND A. TVERSKY (1995): “Ambiguity Aversion and Comparative Ignorance,” *Quarterly Journal of Economics*, 110(3), 585–603.
- GILBOA, I. (2009): *Theory of Decision Under Uncertainty*, Econometric Society Monographs. Cambridge University Press.
- GILBOA, I., AND D. SCHMEIDLER (1989): “Maxmin expected utility with non-unique prior,” *Journal of Mathematical Economics*, 18(2), 141 – 153.
- GRANT, S., AND B. POLAK (2013): “Mean-dispersion preferences and constant absolute uncertainty aversion,” *Journal of Economic Theory*, 148(4), 1361 – 1398.
- GUL, F. (1991): “A theory of disappointment aversion,” *Econometrica*, pp. 667–686.

- HALEVY, Y. (2007): “Ellsberg Revisited: An Experimental Study,” *Econometrica*, 75(2), 503–536.
- HEY, J. D. (2014): “Chapter 14 - Choice under Uncertainty: Empirical Methods and Experimental Results,” in *Handbook of the Economics of Risk and Uncertainty*, ed. by M. Machina, and K. Viscusi, vol. 1, pp. 809 – 850. North-Holland.
- HORTON, J. J., AND L. B. CHILTON (2010): “The labor economics of paid crowdsourcing,” in *Proceedings of the 11th ACM conference on Electronic commerce*, pp. 209–218. Association for Computing Machinery.
- HUCK, S., AND W. MÜLLER (2012): “Allais for all: Revisiting the paradox in a large representative sample,” *Journal of Risk and Uncertainty*, 44(3), 261–293.
- INCEKARA-HAFALIR, E., AND J. D. STECHER (2012): “An Experimental Test of Theories of Behavior in Allais-Type Tasks,” Discussion paper, SSRN, available at <http://dx.doi.org/10.2139/ssrn.2179060>.
- KAHNEMAN, D., AND A. TVERSKY (1979): “Prospect Theory: An Analysis of Decision under Risk,” *Econometrica*, 47(2), pp. 263–292.
- KLIBANOFF, P., M. MARINACCI, AND S. MUKERJI (2005): “A Smooth Model of Decision Making under Ambiguity,” *Econometrica*, 73(6), pp. 1849–1892.
- MACCHERONI, F., M. MARINACCI, AND A. RUSTICHINI (2006): “Ambiguity Aversion, Robustness, and the Variational Representation of Preferences,” *Econometrica*, 74(6), pp. 1447–1498.
- MACCRIMMON, K., AND S. LARSSON (1979): “Utility Theory: Axioms Versus Paradoxes,” in *Expected Utility Hypotheses and the Allais Paradox*, ed. by M. Allais, and O. Hagen, vol. 21 of *Theory and Decision Library*, pp. 333–409. Springer Netherlands.
- MACHINA, M. J., AND D. SCHMEIDLER (1992): “A More Robust Definition of Subjective Probability,” *Econometrica*, 60(4), pp. 745–780.
- (1995): “Bayes without Bernoulli: Simple Conditions for Probabilistically Sophisticated Choice,” *Journal of Economic Theory*, 67(1), 106 – 128.
- QUIGGIN, J. (1982): “A theory of anticipated utility,” *Journal of Economic Behavior & Organization*, 3(4), 323–343.
- SAVAGE, L. J. (1954): *The Foundations of Statistics*. Wiley.
- SCHLAG, K., J. TREMEWAN, AND J. VAN DER WEELE (2014): “A penny for your thoughts: a survey of methods for eliciting beliefs,” *Experimental Economics*,

- pp. 1–34.
- SCHMEIDLER, D. (1989): “Subjective Probability and Expected Utility without Additivity,” *Econometrica*, 57(3), pp. 571–587.
- SEGAL, U. (1987): “The Ellsberg paradox and risk aversion: An anticipated utility approach,” *International Economic Review*, 28(1), 175–202.
- SINISCALCHI, M. (2009): “Vector Expected Utility and Attitudes Toward Variation,” *Econometrica*, 77(3), 801–855.
- STARMER, C., AND R. SUGDEN (1991): “Does the Random-Lottery Incentive System Elicit True Preferences? An Experimental Investigation,” *American Economic Review*, 81(4), pp. 971–978.
- TRAUTMANN, S., AND P. P. WAKKER (2015): “Making the Anscombe-Aumann Approach to Ambiguity Suited for Descriptive Applications,” Discussion paper, Erasmus University Rotterdam.
- TRAUTMANN, S. T., AND G. VAN DE KUILEN (2015): “Ambiguity Attitudes,” in *Handbook of Judgment and Decision Making*, ed. by G. Keren, and G. Wu. Blackwell.
- TRAUTMANN, S. T., F. M. VIEIDER, AND P. P. WAKKER (2011): “Preference Reversals for Ambiguity Aversion,” *Management Science*, 57(7), 1320–1333.
- TVERSKY, A., AND D. KAHNEMAN (1992): “Advances in prospect theory: Cumulative representation of uncertainty,” *Journal of Risk and Uncertainty*, 5(4), 297–323.
- WAKKER, P. P. (2010): *Prospect Theory: For Risk and Ambiguity*. Cambridge University Press.
- WU, G., AND R. GONZALEZ (1999): “Nonlinear Decision Weights in Choice Under Uncertainty,” *Management Science*, 45(1), 74–85.
- ZIZZO, D. (2010): “Experimenter demand effects in economic experiments,” *Experimental Economics*, 13(1), 75–98.

APPENDIX A: PROOFS

To better distinguish constant acts from lotteries, we denote the act that yields the lottery $p \in \mathcal{P}$ in all states by $[p] \in \mathcal{L}$.

Proof of observation 1: Observation 1 is equivalent to the following statement: If \succsim satisfies Weak order and Monotonicity, then (i) $1A \succ 1B$ implies $2A \succsim 2B$,

(ii) $1B \succ 1A$ implies $2B \succ 2A$, (iii) $2A \succ 2B$ implies $1A \succ 1B$, and (iv) $2B \succ 2A$ implies $1B \succ 1A$.

Proof of (i): First, observe that $1A \succ 1B$ implies $[\sigma_{1M}] \succ [(0; \frac{1}{11}, 5M; \frac{10}{11})]$, as otherwise we would have $[(0; \frac{1}{11}, 5M; \frac{10}{11})] \succ [\sigma_{1M}]$ which by Monotonicity would mean that $1B \succ 1A$, a contradiction. Second, by Monotonicity, $[\sigma_{1M}] \succ [(0; \frac{1}{11}, 5M; \frac{10}{11})]$ implies $2A \succ 2B$.

The proofs of (ii) - (iv) are analogous. ■

Proof of theorem 1: (iii) trivially implies (ii), thus it suffices to show that (ii) implies (i), and (i) implies (iii). (ii) implies (i): Consider two constant acts $[p], [q]$ with $[p] \succ [q]$. We have to show that for any act h

$$\begin{aligned} [\dots, h_{n-1} \text{ on } E_{n-1}, p \text{ on } E_n, h_{n+1} \text{ on } E_{n+1}, \dots] &\succ \\ [\dots, h_{n-1} \text{ on } E_{n-1}, q \text{ on } E_n, h_{n+1} \text{ on } E_{n+1}, \dots]. & \end{aligned}$$

By probabilistic sophistication there exists a probability measure μ such that:

$$[\dots, h_{n-1} \text{ on } E_{n-1}, p \text{ on } E_n, h_{n+1} \text{ on } E_{n+1}, \dots] \sim \left[\mu(E_n)p + \sum_{i \neq n} \mu(E_i)h_i \right].$$

The latter is a constant act that can be seen as a mixture of two constant acts, putting weight $\mu(E_n)$ on p and weight $1 - \mu(E_n)$ on $\frac{\sum_{i \neq n} \mu(E_i)h_i}{1 - \mu(E_n)}$. By an analogous argument we have

$$\begin{aligned} [\dots, h_{n-1} \text{ on } E_{n-1}, q \text{ on } E_n, h_{n+1} \text{ on } E_{n+1}, \dots] &\sim \\ \left[\mu(E_n)q + (1 - \mu(E_n)) \frac{\sum_{i \neq n} \mu(E_i)h_i}{1 - \mu(E_n)} \right]. & \end{aligned}$$

Now note that by Independence

$$\left[\mu(E_n)p + (1 - \mu(E_n)) \frac{\sum_{i \neq n} \mu(E_i)h_i}{1 - \mu(E_n)} \right] \succ \left[\mu(E_n)q + (1 - \mu(E_n)) \frac{\sum_{i \neq n} \mu(E_i)h_i}{1 - \mu(E_n)} \right].$$

(i) implies (iii): Consider three acts f, g, h with $f \succ g$ and any $\alpha \in (0, 1)$. Consider a common partition (E_1, \dots, E_N) . By probabilistic sophistication there exists a non-atomic probability measure μ such that $f \sim \left[\sum_{i=1}^N \mu(E_i)f_i \right]$, $g \sim \left[\sum_{i=1}^N \mu(E_i)g_i \right]$ and $h \sim \left[\sum_{i=1}^N \mu(E_i)h_i \right]$. As μ is non-atomic, there exists an event $E \subset \mathcal{S}$ s.t. $\mu(E) = \alpha$.

First, note that by Monotonicity:

$$\left[\sum_{i=1}^N \mu(E_i) f_i \text{ on } E, \sum_{i=1}^N \mu(E_i) h_i \text{ on } \mathcal{S} \setminus E \right] \succsim \left[\sum_{i=1}^N \mu(E_i) g_i \text{ on } E, \sum_{i=1}^N \mu(E_i) h_i \text{ on } \mathcal{S} \setminus E \right].$$

Second, by applying probabilistic sophistication twice we obtain:

$$\begin{aligned} \left[\sum_{i=1}^N \mu(E_i) f_i \text{ on } E, \sum_{i=1}^N \mu(E_i) h_i \text{ on } \mathcal{S} \setminus E \right] &\sim \\ \left[\alpha \sum_{i=1}^N \mu(E_i) f_i + (1 - \alpha) \sum_{i=1}^N \mu(E_i) h_i \right] &\sim \\ \alpha f + (1 - \alpha) h, & \end{aligned}$$

and, similarly, $\left[\sum_{i=1}^N \mu(E_i) g_i \text{ on } E, \sum_{i=1}^N \mu(E_i) h_i \text{ on } \mathcal{S} \setminus E \right] \sim \alpha g + (1 - \alpha) h.$ ■

APPENDIX B: CHARACTERIZING MODELS SATISFYING MONOTONICITY

We restrict attention to finite state spaces $\mathcal{S} = \{1, \dots, s, \dots, S\}$. The following observation connects the form of the representation with Monotonicity:

Observation 2. *If \succsim can be represented by $V(U \circ f)$, where $U : \mathcal{P} \rightarrow \mathbb{R}$ is a preference functional, and $V : U(\mathcal{P})^S \rightarrow \mathbb{R}$ is a strictly monotone¹⁷ function, then \succsim satisfies Monotonicity.*

Proof. For any two constant acts $[p], [q]$, with $[p] \succsim [q]$, since $V(U \circ f)$ represents \succsim we have $V(U(p), \dots, U(p)) \geq V(U(q), \dots, U(q))$. As V is strictly monotone: $U(p) \geq U(q)$. As V is monotone, for all acts h :

$$V(\dots, U(h_{s-1}), U(p), U(h_{s+1}), \dots) \geq V(\dots, U(h_{s-1}), U(q), U(h_{s+1}), \dots)$$

thus $[\dots, h_{s-1}, p, h_{s+1}, \dots] \succsim [\dots, h_{s-1}, q, h_{s+1}, \dots].$ ■

We call representations of the form described in observation 2 *monotone* and *state-wise separable*. A state-wise separable representation evaluates an act by first taking a preference functional U of the roulette lottery obtained in each state, and then aggregating across states using a real-valued, monotone function V . The observation raises the question whether the reverse is also true, that is whether Monotonicity guarantees that the preference will admit a monotone,

¹⁷We say that V is *strictly monotone* if for all x, y in $U(\mathcal{P})^S$: If there is no s such that $y_s > x_s$, then $V(x) \geq V(y)$; and if $x_s > y_s$ for all s , then $V(x) > V(y)$.

state-wise separable representation. Theorem 2 establishes that Monotonicity together with three basic choice theoretic axioms implies existence of a monotone, state-wise separable representation. The three basic choice theoretic axioms are Weak order, Mixture-continuity, and first-order stochastic dominance as appropriately generalized to the Anscombe-Aumann setting¹⁸.

Axiom (Mixture-continuity). *For all acts f, g, h : if $f \succ g \succ h$, then there exists α in $(0, 1)$ such that $g \sim \alpha f + (1 - \alpha) h$.*

Axiom (FOSD). *For all acts f, g : if for all states s , f_s first-order stochastically dominates g_s then $f \succeq g$, with strict preference whenever there is strict dominance in all states.*

The three basic choice theoretic axioms ensure existence of a representation:

Lemma 1. \succeq satisfies the axioms Weak order, Mixture-continuity, and FOSD if and only if, there exists a mixture-continuous¹⁹, strictly monotonic²⁰ preference functional $W : \mathcal{L} = \mathcal{P}^S \rightarrow \mathbb{R}$ representing \succeq .

Proof. First, we construct W : For all acts f , by FOSD we have: $[\sigma_{\bar{\omega}}] \succeq f \succeq [\sigma_{\underline{\omega}}]$. Consider the set

$$A(f) = \{\alpha \in [0, 1] : f \sim \alpha [\sigma_{\bar{\omega}}] + (1 - \alpha) [\sigma_{\underline{\omega}}]\}.$$

By Mixture-continuity this set is non-empty, by the strict part of FOSD it has at most one element. Thus we can define $W : \mathcal{P}^S \rightarrow [0, 1]$ by $W(f) = A(f)$.

Second, we show that W represents \succeq : For all acts f and g by FOSD:

$$W(f) \geq W(g) \Leftrightarrow W(f) [\sigma_{\bar{\omega}}] + (1 - W(f)) [\sigma_{\underline{\omega}}] \succeq W(g) [\sigma_{\bar{\omega}}] + (1 - W(g)) [\sigma_{\underline{\omega}}].$$

By definition of W we have

$$f \sim W(f) [\sigma_{\bar{\omega}}] + (1 - W(f)) [\sigma_{\underline{\omega}}], \text{ and } g \sim W(g) [\sigma_{\bar{\omega}}] + (1 - W(g)) [\sigma_{\underline{\omega}}].$$

¹⁸The axiom we give here is a slight weakening of the FOSD-Axiom in Machina and Schmeidler (1995).

¹⁹Given a convex set C (in P , \mathcal{L} , or an Euclidean space), we say that a function $\phi : C \rightarrow \mathbb{R}$ is *mixture-continuous* if for all c, c' in C with $\phi(c) < \phi(c')$: For all ϕ_0 in $(\phi(c), \phi(c'))$, there exists α in $(0, 1)$ such that $\phi_0 = \phi(\alpha c + (1 - \alpha) c')$.

²⁰We say that a preference functional W over \mathcal{L} is *strictly monotonic* if $W(f) \geq W(g)$ whenever for all s f_s first-order stochastically dominates g_s , with strict inequality if the dominance is strict on all s .

Therefore,

$$W(f)[\sigma_{\bar{\omega}}] + (1 - W(f))[\sigma_{\underline{\omega}}] \succsim W(g)[\sigma_{\bar{\omega}}] + (1 - W(g))[\sigma_{\underline{\omega}}] \Leftrightarrow f \succsim g.$$

Third, W is monotone as \succsim satisfies FOSD.

Fourth, W is mixture-continuous: Consider two acts f, g with $W(g) < W(f)$ and W_0 in $(W(g), W(f))$. By definition of W : $W_0 = W(W_0[\sigma_{\bar{\omega}}] + (1 - W_0)[\sigma_{\underline{\omega}}])$. By Mixture-continuity of \succsim , there exists α in $(0, 1)$ such that $\alpha f + (1 - \alpha)g \sim W_0[\sigma_{\bar{\omega}}] + (1 - W_0)[\sigma_{\underline{\omega}}]$, and thus $W(\alpha f + (1 - \alpha)g) = W_0$.

That the representation implies the axioms is trivial. ■

As the lemma showed the three basic choice theoretic axioms guarantee existence of a representation, adding the Monotonicity axiom makes sure that a monotone and state-wise separable representation exists:

Theorem 2. \succsim satisfies the axioms Weak order, Mixture-continuity, FOSD and Monotonicity, if and only if there exists a mixture-continuous²¹, strictly monotonic²² preference functional $U : \mathcal{P} \rightarrow \mathbb{R}$, and a mixture-continuous, strictly monotone function $V : U(\mathcal{P})^S \rightarrow \mathbb{R}$, such that $V(U \circ f)$ represents \succsim .

Proof. i) The axioms imply the representation:

First we construct U and V . Using the W from lemma 1, we define $U : \mathcal{P} \rightarrow [0, 1]$ by:

$$U(p) \equiv W([p]),$$

and $V : [0, 1]^S \rightarrow [0, 1]$ by:

$$V(x_1, \dots, x_S) \equiv W([x_1\sigma_{\bar{\omega}} + (1 - x_1)\sigma_{\underline{\omega}}, \dots, x_S\sigma_{\bar{\omega}} + (1 - x_S)\sigma_{\underline{\omega}}]).$$

To show that W represents, we prove that (given Monotonicity) $W(f)$ and $V(U \circ f)$ are identical. That is for all acts $f : W(f) = V(U(f_1), \dots, U(f_S))$.

Observe that it suffices to show that for all acts f :

$$W(f)[\sigma_{\bar{\omega}}] + (1 - W(f))[\sigma_{\underline{\omega}}] \sim V(U(f_1), \dots, U(f_S))[\sigma_{\bar{\omega}}] + (1 - V(U(f_1), \dots, U(f_S)))[\sigma_{\underline{\omega}}]$$

²¹Given a convex set C (in P , \mathcal{L} , or an Euclidean space), we say that a function $\phi : C \rightarrow \mathbb{R}$ is *mixture-continuous* if for all c, c' in C with $\phi(c) < \phi(c')$: For all ϕ_0 in $(\phi(c), \phi(c'))$, there exists α in $(0, 1)$ such that $\phi_0 = \phi(\alpha c + (1 - \alpha)c')$.

²²We say that U is *strictly monotonic* if $U(p) \geq U(q)$ whenever p first-order stochastically dominates q , with strict inequality in the case of strict dominance.

as it implies, by (the strict part of) FOSD, $W(f) = V(U(f_1), \dots, U(f_S))$.

By construction of W :

$$W(f)[\sigma_{\bar{\omega}}] + (1 - W(f))[\sigma_{\underline{\omega}}] \sim f.$$

Observe that by definition of U , for all lotteries f_s :

$[f_s] \sim U(f_s)[\sigma_{\bar{\omega}}] + (1 - U(f_s))[\sigma_{\underline{\omega}}]$, and use Monotonicity to get:

$$f \sim [U(f_1)\sigma_{\bar{\omega}} + (1 - U(f_1))\sigma_{\underline{\omega}}, \dots, U(f_S)\sigma_{\bar{\omega}} + (1 - U(f_S))\sigma_{\underline{\omega}}].$$

Finally, by definition of V :

$$\begin{aligned} &V(U(f_1), \dots, U(f_S))[\sigma_{\bar{\omega}}] + (1 - V(U(f_1), \dots, U(f_S)))[\sigma_{\underline{\omega}}] \sim \\ &[U(f_1)\sigma_{\bar{\omega}} + (1 - U(f_1))\sigma_{\underline{\omega}}, \dots, U(f_S)\sigma_{\bar{\omega}} + (1 - U(f_S))\sigma_{\underline{\omega}}]. \end{aligned}$$

As W is mixture-continuous and strictly monotonic, U is mixture-continuous and strictly monotonic, and V is mixture-continuous and strictly monotone.

(ii) The representation implies the axioms:

According to lemma 1, \succsim satisfies the axioms Weak order, Mixture-continuity and FOSD. According to observation 2, \succsim satisfies Monotonicity. ■

Note that many models of ambiguity aversion that admit a state-wise separable representation assume expected utility regarding objective uncertainty, i.e.

$$U(p) = \sum_{\omega \in [\underline{\omega}, \bar{\omega}]: p(\omega) \neq 0} p(\omega) u(\omega).$$

Adding the Independence axiom in theorem 2 yields this form (compare Trautmann and Wakker 2015, p.9).

APPENDIX C: EXPERIMENTAL RESULTS

TABLE 5. Robustness Checks Allais Horse Race

	SATISFY		VIOLATE		Conl.-Z	N
	1A*2A*	1B*2B*	1A*2B*	1B*2A*		
Full Sample	36.8	15.2	38.8	9.2	11.06	552
Testq. 1 correct	37.1	14.9	38.9	9.1	10.25	463
Testq. 2 correct	37.1	14.8	39.3	8.8	10.97	501
Testq. 3 correct	38.4	14.5	38.7	8.5	8.71	318
Testq. 1-2 correct	36.7	14.7	39.7	8.9	10.24	436
Testq. 1-3 correct	37.6	14.5	40.5	7.4	8.44	242
Satisfy FOSD	35.2	17.0	38.7	9.1	9.41	395
Testq. 1-2 correct & FOSD	35.3	17.2	38.1	9.4	8.33	331
Testq. 1-3 correct & FOSD	35.6	17.5	37.9	9.0	6.15	177
Duration 1. quartile	37.0	14.5	39.9	8.7	5.85	138
Duration 2. quartile	36.2	14.5	38.4	10.9	4.99	138
Duration 3. quartile	36.2	14.5	39.1	10.1	5.31	138
Duration 4. quartile	37.7	17.4	37.7	7.2	5.97	138
4. q.; Testq. 1-2 cor. & FOSD	34.0	20.2	37.2	8.5	4.52	94
4. q.; Testq. 1-3 cor. & FOSD	35.3	21.6	37.3	5.9	3.84	51
Belief $\leq 10\%$	35.8	16.9	39.8	7.5	8.46	254
Belief $> 10\%$	37.2	14.3	37.2	11.3	6.10	231

TABLE 6. Robustness Checks Allais Paradox

	SATISFY		VIOLATE		Conl.-Z	N
	IA*IIA*	IB*IIB*	IA*IIB*	IB*IIA*		
Full Sample	23.0	26.4	35.3	15.2	6.92	552
Testq. 1 correct	23.8	25.3	35.4	15.6	6.23	463
Testq. 2 correct	24.4	25.3	35.3	15.0	6.70	501
Testq. 3 correct	25.2	21.7	37.4	15.7	5.55	318
Testq. 1-2 correct	24.5	24.8	35.3	15.4	6.09	436
Testq. 1-3 correct	28.9	17.8	36.8	16.5	4.48	242
Satisfy FOSD	23.0	26.1	33.9	17.0	4.86	395
Testq. 1-2 correct & FOSD	23.9	25.7	32.9	17.5	4.04	331
Testq. 1-3 correct & FOSD	28.8	17.5	33.9	19.8	2.61	177
Duration 1. quartile	21.7	28.3	37.7	12.3	4.50	138
Duration 2. quartile	21.7	21.7	39.1	17.4	3.54	138
Duration 3. quartile	21.7	29.0	34.1	15.2	3.26	138
Duration 4. quartile	26.8	26.8	30.4	15.9	2.55	138
4. q.; Testq. 1-2 cor. & FOSD	25.5	29.8	26.6	18.1	1.24	94
4. q.; Testq. 1-3 cor. & FOSD	29.4	23.5	29.4	17.6	1.23	51