Tesfaselassie, Mewael F.

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The Impact of Disembodied Technological Progress on Working Hours

by Mewael F. Tesfasellassie

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Abstract:

The paper analyzes the effects of disembodied technological progress on steady state hours worked in the workhorse New-Keynesian model, which features a neoclassical labor market, and its extension that allows for equilibrium unemployment. Both versions of the model are shown to imply a positive effect of growth on hours. Thus they can rationalize the long-term trend decline in productivity growth and the average number of hours per person observed across major industrialized countries during the postwar period. In the workhorse model slower growth decreases hours worked by reducing the effective discount rate and thus increasing the price markup, which acts like a tax hike on labor supply. This effect vanishes when the inflation rate is zero, because of the constancy of the price markup. In the extended version, the price markup effect interacts with a negative capitalization effect, whereby slower growth decreases hours by reducing the effective discount rate and in turn increasing employment and the marginal rate of substitution between consumption and hours.

Keywords: E24; E31

JEL classification: Productivity growth, working hours, employment, nominal price rigidity, trend inflation.

Mewael F. Tesfaselassie
Kiel Institute for the World Economy
24106 Kiel, Germany
E-mail: mewael.tesfaselassie@ifw-kiel.de

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1 Introduction

During the postwar period industrialized countries experienced a long-term trend decline both in productivity growth and the average number of hours per person (see, e.g., OECD (1998), Ohanian, Raffo and Rogerson (2007), Gordon (2016)). An interesting question from a theoretical perspective is whether the two phenomena are related. To be specific, does a slowdown in productivity growth lead to a reduction in the number of hours worked? And what are the channels by which the former affects the latter? In order to examine these questions, the paper analyzes the effects of disembodied technological progress on aggregate hours worked (and broadly on labor market outcomes) using alternative frameworks that are commonly adopted in the business cycle literature.

The first framework is based on the workhorse New-Keynesian model, which features monopolistically competitive firms that are subject to price staggering and a neoclassical labor market so that labor adjustments are in hours—the so-called intensive margin. The advantage is that model is relatively simple and has an analytical solution. It is shown that lower trend productivity growth is associated with lower aggregate hours due to what we call the markup channel. Along a balanced growth path, slower growth implies (i) lower real rate of interest and (ii) lower future aggregate demand relative to the present one. The markup channel is a result of the interaction of (i) and (ii). Lower interest rate induces firms to raise their price markups (as it exacerbates future erosion of their price markup by ongoing inflation) while lower growth of aggregate demand induces them to lower their price markups (as it mitigates future erosion of their price markup by ongoing inflation). Under the maintained assumption that the intertemporal substitution in consumption is low (see Eriksson (1997)), the interest rate effect dominates the aggregate demand effect so that the lower is productivity growth the higher is the average price markup, which acts like a tax-hike on labor supply and thus induces households to work less hours.

However, the workhorse New-Keynesian model has a drawback because it predicts no relationship between trend productivity growth and hours when the rate of inflation is zero. The reason is that under a zero rate of inflation, the markup is constant (as in a flexible price world) and independent of trend productivity growth. In order to overcome this drawback, the second framework introduces labor market frictions into the standard

1 The workhorse model is used in an expanding literature that examines the real effects of trend inflation (see, for e.g., King and Wolman (1996) for an early discuss and Graham and Snower (2008) and Floro and Gobbi (2015) for a more recent discussion on the topic). Much of this literature abstracts either from growth considerations.

2 The tax-like effect of changes in the average price markup is well known in the New-Keynesian literature (see, e.g., Goodfriend (1997)).
New-Keynesian model, thereby allowing for adjustments in employment—the so-called extensive margin. The extended model has two-sectors—a final good sector and an intermediate good sector. Firms in the final good sector are monopolistic competitors, face price staggering and produce using the intermediate good as an input. Firms in the intermediate good sector face a perfectly competitive output market, produce using labor as an input and incur labor hiring costs.\(^3\)

The incorporation of labor market frictions into the standard New-Keynesian model gives rise to a second channel—the so-called \textit{capitalization} channel (see, e.g., Aghion and Howitt (1994))—whereby trend productivity growth affects hours. It arises due to the interaction of interest rate effect with an offsetting \textit{hiring cost} effect—along a balanced growth path, slower productivity growth implies lower future costs of hiring workers relative to the present one and thus weakens the incentive for a firm to hire workers.\(^4\) Since the capitalization channel is independent of inflation, the extended model predicts a relationship between trend productivity growth and hours even when the rate of inflation is zero. However, a drawback of the extended model is that, unlike the workhorse New-Keynesian model, there is no closed-form solution. Our results are therefore based on numerical analysis by calibrating the model to the US economy.

As in the workhorse New-Keynesian model, we find that the lower is trend productivity growth the lower is hours worked.\(^5\) The intuition is that, under the standard assumption of Nash bargaining over wages and hours (see, e.g., Shimer (2010)) hours are set such that the marginal revenue product of supplying hours equals the marginal rate of substitution between hours and consumption.

The higher is the average markup associated with slower growth, the lower is the relative price of the intermediate good and thus the lower is the marginal revenue product. Given employment, hours fall so as to decrease the marginal rate of substitution. Moreover, when inflation is low enough the markup effect is dominated by the capitalization effect so that equilibrium employment is higher the lower is trend productivity growth.\(^6\) Higher

\(^3\)The two-sector framework is standard in the business cycle literature (see, e.g., Trigari (2006), Christoffel and Kuester (2008) and Blanchard and Gali (2010)). The assumption that hiring costs are the source of labor market rigidity follows closely Blanchard and Gali (2010).

\(^4\)The capitalization effect is well known within the growth and unemployment literature (see, e.g., Pissarides (2000, ch. 3) for an overview).

\(^5\)This result is robust to alternative parameterization of the model.

\(^6\)This mimics the result in Tesfay (2014), who considers only adjustments in employment. Tesfay (2014) revisits the growth-unemployment nexus and shows that introducing nominal price rigidity helps reconcile the prediction of labor search-type models (e.g., Pissarides (2000, ch. 3)) with the experience of the 1970s. That analysis is motivated by the observation that the 1970s were characterized not only by a slowdown in productivity growth but also by higher inflation rates.
employment increases the marginal rate of substitution and implies lower hours. When the rate of inflation is zero, the marginal revenue product is constant but the indirect effect of growth on hours via employment is operative. When inflation is high enough the markup effect dominates the capitalization effect so that equilibrium employment is lower the lower is trend productivity growth. Lower employment decreases the marginal rate of substitution and implies higher hours. This effect is more than offset by the relatively strong decline in the marginal revenue product, and thus in hours, when inflation is higher.

We remark that while we focus on the relationship between productivity growth and hours worked overtime in major industrialized countries, there exists a vast literature that examines the relationship between the level of productivity (and per capita income) and hours worked across countries. For instance, in an empirical paper Bick, Fuchs-Schündeln and Lagakos (2016) document that average hours worked per adult are substantially higher in lower-income countries than in higher-income countries. To the extent that income growth is inversely related to the initial income level, as is the case under neoclassical growth models, one may be tempted to recast the level effect in terms of a growth effect. We do not emphasize this connection for reasons that has to do with the ongoing debate on cross-country growth regressions (see, e.g., Durlauf (2009)).

The paper is organized as follows. In section 2 we present our results under the workhorse New-Keynesian model (i.e., one with a neoclassical labor market), which we solve analytically. In section 3 we consider an extension of the workhorse model by incorporating labor market frictions, along the lines of the labor search literature. The analysis here is numerical, as there is no closed-form solution. For this reason, the discussion of our baseline results is followed by a sensitivity analysis. In section 4 we give concluding remarks.

2 The model with a neoclassical labor market

2.1 Households

There is a representative household whose period utility is nonseparable in consumption $C_t$ and hours worked $h_t$: $U(C_t, h_t) = \frac{c_t^{1-\sigma}V(h_t)^{1-1/\sigma}}{1-\sigma}$, where $V(h_t) = \left(1 + (\sigma - 1)\gamma h_t^{1+\phi}/(1 + \phi)\right)^{1/\sigma}$.

The specification of household utility follows Shimer (2010) and is consistent with balanced growth. The household maximizes $E_t \sum \beta^t U(C_t, h_t)$, subject to the budget constraint

$$P_t C_t + B_t = W_t h_t + R_{t-1} B_{t-1} + D_t,$$
where $\beta$ is the subjective discount factor, $P_t$ is the price level, $R_t$ is the nominal interest rate on per capita bond holdings $B_t$, $W_t$ is the nominal wage and $D_t$ is the per capita nominal profit income from the ownership of firms.

It is straightforward to derive the familiar Euler equation

$$1 = E_t\left(\frac{Q_{t,t+1}R_t}{\Pi_{t+1}}\right),$$

where $\Pi_t \equiv P_t/P_{t-1}$ is gross inflation rate and $Q_{t,t+1} \equiv \beta(C_{t+1}/C_t)^{-\sigma}V(h_{t+1})/V(h_t)$. Here $Q_{t,t+1}$ is the household’s stochastic discount factor, which is used to discount future real payoffs. It can be rewritten as

$$Q_{t,t+1} \equiv \beta \Gamma^{-\sigma} \left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} \frac{V(h_{t+1})}{V(h_t)},$$

where $c_t = C_t/A_t$. The steady state of equation (1) is $R/\Pi = \Gamma^\sigma/\beta$, which shows that higher trend productivity growth implies higher gross real rate ($R/\Pi$) and in turn a stronger discounting of future payoffs.

The first order condition for the optimal supply of hours equates the real wage $w_t = W_t/P_t$ to the marginal rate of substitution between consumption and hours. It can be rewritten in detrended form

$$w_t^d = \frac{\sigma \gamma_1 c_t h_t^\phi}{1 + (\sigma - 1)\gamma_1 h_t^{1+\phi}/(1 + \phi)},$$

where $w_t^d \equiv w_t/A_t$. As is standard we assume $C_t$ to be a Dixit-Stiglitz composite $C_t = \left(\int_{0}^{1} C_{k,t}^1 dk\right)^\mu$ where each good is indexed by $k$, $\theta$ is the elasticity of substitution between goods and $\mu \equiv \frac{\theta}{\theta - 1}$. Optimal consumption allocation across goods gives the demand equation $C_{k,t} = \left(\frac{P_{k,t}}{P_t}\right)^{-\theta} C_t$ where $P_t = \left(\int_{0}^{1} P_{k,t}^{1-\theta} dk\right)^{1/\sigma}$.

### 2.2 Firms

There is a continuum of monopolistically competitive firms with a linear technology $Y_{k,t} = A_t h_{k,t}$, where growth in labor productivity $A_t$ is assumed to be deterministic and $\Gamma = A_t/A_{t-1}$ denoted gross productivity growth. Firms face Calvo-type price staggering, whereby only a fraction $1 - \omega$ of firms can reset prices in any given period. Let $P_{k,t}$ denote firm $k$'s output price. The firm maximizes its expected lifetime profit.
$E_t \sum_{i=0}^{\infty} \omega^i Q_{t+i} (P_{k,t}/P_{t+i} - mc_{t+i}) Y_{k,t+i}$, where $mc_i$ is the real marginal cost and is equal to $w_t$. Using demand for good $k$, $Y_{k,t+i} = (P_{k,t}/P_{t+i})^{-\theta} C_{t+i}$, and the aggregate resource constraint $Y_{t+i} = C_{t+i}$ in the profit function and differentiating with respect to $P_{k,t}$ gives

$$p_t^* = \frac{E_t \sum_{i=0}^{\infty} \omega^i Q_{t+i} Y_{t+i} w_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^\theta}{E_t \sum_{i=0}^{\infty} \omega^i Q_{t+i} Y_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}}.$$  \hspace{1cm} (4)

where $p_t^* \equiv P_t^*/P_t$ is the optimal relative price. Equation (4) can be rewritten in stationary variables

$$p_t^* = \frac{E_t \sum_{i=0}^{\infty} (\beta \omega \Gamma^{1-\sigma})^i \left( \frac{c_{t+i}}{c_t} \right)^{-\sigma} V(h_{t+i}) w_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^\theta}{E_t \sum_{i=0}^{\infty} (\beta \omega \Gamma^{1-\sigma})^i \left( \frac{c_{t+i}}{c_t} \right)^{-\sigma} V(h_{t+i}) y_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}}.$$  \hspace{1cm} (5)

This is our key equation capturing the influence of trend productivity growth in the presence of price staggering. We thus discuss its relevance in more detail by looking at its steady state version

$$p^* = \frac{\mu \sum_{i=0}^{\infty} (\beta \omega \Gamma^{1-\sigma} \Pi^\theta)^i}{\sum_{i=0}^{\infty} (\beta \omega \Gamma^{1-\sigma} \Pi^{\theta-1})^i} w^d = \frac{1 - \beta \omega \Gamma^{1-\sigma} \Pi^{\theta-1}}{1 - \beta \omega \Gamma^{1-\sigma} \Pi^\theta} w^d,$$  \hspace{1cm} (6)

where for the sums to be convergent, we impose the restriction $\Pi < \Pi^{\text{max}} = (\Gamma^{\sigma-1}/(\beta \omega))^{1/\theta}$.

In the presence of trend inflation, firms choose a markup higher than that implied by a zero rate of inflation so as to mitigate the future erosion of their markup by an ongoing inflation. The underlying reason behind this markup distortion is the asymmetry in the profit function: profit declines more strongly with a markup that is below the optimum (under flexible prices) than with a markup above the optimum.\(^8\) A change in trend productivity growth has two opposing effects. On the one hand, the associated higher output growth increases future relative to present demand conditions and this leads to a higher price markup. On the other hand, the associated higher consumption growth implies higher real interest rate and this leads to a lower price markup since future demand conditions are discounted at a higher rate. The discounting effect is stronger the larger is $\sigma$. Under the more plausible assumption that $\sigma > 1$ (see, e.g., Eriksson (1997) and Shimer (2010)) the markup distortion is smaller the higher is the rate of productivity growth owing to stronger discounting effect from higher real interest rate.

\(^7\)For instance, assuming plausible parameter values—$\beta = 0.99, \sigma = 3, \omega = 0.75, \theta = 11$ and $\Gamma = 1.005$ (i.e., an annualized growth rate of 2 percent)—$\Pi^{\text{max}} = 1.028$ (i.e., an annualized inflation rate of about 11.35 percent).

\(^8\)See, e.g., Amano et. al (2009) for a detailed discussion.
The aggregate price index can be rewritten as a weighted average of optimized and non-optimized prices

\[ P_t = \left( (1 - \omega)P_{t-1}^{1-\theta} + \omega P_t^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{7} \]

Finally, by market clearing \( c_t = y_t \) while aggregate output is related to hours worked by

\[ h_t = \Delta ty_t. \tag{8} \]

where \( \Delta_t \equiv \int_0^1 (P_{k,t}/P_t)^{-\theta} df \) is a measure of price dispersion. It can be rewritten as

\[ \Delta_t = (1 - \omega)p_t^{\star -\theta} + \omega \Pi_t^{\theta} \Delta_{t-1}. \tag{9} \]

### 2.3 Steady state equilibrium and results

In steady state the price level (7) implies

\[ p^\star = p^\star(\Pi) \equiv \left( \frac{1 - \omega \Pi^{\theta-1}}{1 - \omega} \right)^{1/(1-\theta)}, \tag{10} \]

while the price dispersion (9) becomes

\[ \Delta = \Delta(\Pi) = \frac{(1 - \omega)p^{\star -\theta}}{1 - \omega \Pi^\theta}. \tag{11} \]

Both the optimal relative price and the degree of price dispersion are therefore pinned by trend inflation alone. It is easily seen that for \( \Pi \geq 1, \partial p^\star / \partial \Pi > 0 \) and \( \partial \Delta / \partial \Pi > 0 \).

Substituting equation (10) in the steady state optimal relative price (6) and rearranging we get

\[ w^d = \frac{p^\star(\Pi)(1 - \beta \omega \Gamma^{1-\sigma} \Pi^\theta)}{\mu (1 - \beta \omega \Gamma^{1-\sigma} \Pi^{\theta-1})} \equiv w(\Gamma, \Pi). \tag{12} \]

Combining equation (12) with the steady state of equation (3),

\[ w^d = \frac{\sigma \gamma_1 \Delta}{h^{-(1+\phi)} + (\sigma - 1) \gamma_1/(1 + \phi)}, \tag{13} \]

where we made use of the steady state of market clearing \( c = y \) and of equation (8) \( y = h/\Delta \) to substitute out \( c \) and \( y \), leads to an equilibrium determination of hours,

\[ \frac{w(\Gamma, \Pi)}{\Delta(\Pi)} = \frac{\sigma \gamma_1}{h^{-(1+\phi)} + (\sigma - 1) \gamma_1/(1 + \phi)}. \tag{14} \]
Equation (14) relates equilibrium hours to productivity growth $\Gamma$ and steady state inflation $\Pi$. Note that under zero trend rate of inflation (i.e., $\Pi = 1$), $w(\Gamma, \Pi) = 1$ and $\Delta(\Pi) = 1$. In turn the implied real wage is $w^d = \mu^{-1}$ while hours worked is given by a solution to

$$
\mu^{-1} = \frac{\sigma \gamma_1}{h^{-\gamma_1} + (\sigma - 1) \gamma_1/(1 + \phi)}.
$$

(15)

Thus when the rate of inflation is zero equilibrium hours worked is independent of productivity growth. By contrast, when trend inflation rate is positive ($\Pi > 1$), $w_T \equiv \partial w(\Gamma, \Pi)/\partial \Gamma > 0$. In this case since the left hand side of equation (14) is larger the higher is trend productivity growth, hours worked must be higher (as then the right hand side of the equation become larger). As remarked above the intuition is that a higher trend productivity growth implies a lower average price markup (i.e., a higher real marginal cost, which is equal to the real wage) since the real interest rate effect of growth dominates the aggregate demand effect. Higher real wage in turn implies higher hours supply, which is supported by higher labor demand, as higher wage income supports higher consumption and therefore higher output, which is demand determined. Moreover, higher trend inflation reinforces the effect of trend productivity growth on hours.

In the next section we extend the baseline model so that intermediate good firms adjust labor not only on the intensive margin (hours per worker) but also on the extensive margin (employment). As is shown below the resulting model introduces an additional channel whereby trend productivity growth affects hours worked. As a result, and unlike the model with neoclassical labor market, trend productivity growth matters for hours even when the trend rate of inflation is zero.

3 The model with labor market frictions

For better tractability we follow Blanchard and Gali (2010) and assume that labor market frictions are due to the presence of hiring costs. However, unlike Blanchard and Gali (2010) firms can adjust along the hours and employment margins.

Here the economy is composed of two sectors—an intermediate good sector and a final goods sector. Firms in the intermediate good sector face perfectly competitive output market and use labor as an input in production. Firms in the final goods sector are monopolistically competitive with a technology that transforms the intermediate good

9 The derivation is straightforward, as $\partial w(\Gamma, \Pi)/\partial \Gamma = (\sigma - 1)\mu^{-1} \beta \omega \Gamma^{-\gamma}(\Pi - 1)\Pi^{\sigma-1} p^*(\Pi)/(1 - \beta \omega \Gamma^{1-\sigma} \Pi^{\sigma-1})^2$.

into a (differentiated) final good. The final goods sector is subject nominal price rigidity, analogous to the model with neoclassical labor market.

3.1 Households

There is a representative household with a continuum of members. As is shown by Shimer (2010) the utility function of the household can be derived from the optimal allocation of consumption across its members.\(^{11}\)

The period utility of a household member is
\[ C_t^{1-\sigma} V(h_t) \]
when employed, where \( V(h_t) \) is as defined above, and
\[ C_t^{1-\sigma} \]
when unemployed. Then the household behaves as if it has a utility function of the form
\[ U(C_t, h_t, N_t) = C_t^{1-\sigma} V(h_t, N_t) \]
where \( N_t \) is the fraction of employed household members, \( C_t = N_t C_{e,t} + (1 - N_t) C_{u,t} \) is average consumption and \( V(h_t, N_t) \equiv (1 + (\sigma - 1) \gamma h_t^{1+\phi}/(1 + \phi)N_t)^\sigma \). The representative household maximizes
\[ E_t \sum U(C_t, h_t, N_t) \]
subject to the budget constraint
\[ P_t C_t + B_t = W_t h_t N_t + \zeta(1 - N_t) + R_{t-1} B_{t-1} + D_t, \]
where \( \zeta \) is the unemployment benefit of an unemployed member and \( D_t \) is now the per capita nominal profit income from the ownership of firms net of lump-sum taxes paid to finance unemployment benefits.

The consumption Euler equation and allocation of consumption across the differentiated goods take the same form as in the case of a neoclassical labor market except that the function \( V(h_t) \) is replaced by \( V(h_t, N_t) \).

3.2 Firms

3.2.1 Intermediate goods sector

The output of a representative intermediate good firm, \( Y_t \), is equal to \( A_t h_t N_t \), where \( h_t \) is hours per worker and \( N_t \) is number of workers.\(^{12}\) Employment evolves according to the dynamic equation
\[ N_t = (1 - \delta) N_{t-1} + H_t, \tag{16} \]

\(^{11}\)Monacelli, Perotti and Trigari (2010) also follow a similar approach to analyze fiscal multipliers but abstract from considerations of hours and trend productivity growth.

\(^{12}\)Some studies assume diminishing returns in hours per worker (see e.g., Trigari (2006) and Christoffel and Kuester (2008)). In our case the version of the model with diminishing returns in hours leads to similar qualitative results as the one with linear technology.
where $\delta$ represents an exogenous job separation rate and $H_t$ is hiring in period $t$. The size of the labor force is normalized to 1 so that the stock of unemployed workers in period $t$ before hiring takes place is given by $U_t = 1 - (1 - \delta)N_{t-1}$. As workers start working immediately after getting hired, the unemployment rate (after hiring takes place) is given by $u_t = 1 - N_t$.

As in Blanchard and Gali (2010), frictions in the labor market arise from the presence of hiring costs, which take the form

$$C^H_t = G_t H_t$$  \hspace{1cm} (17)

where $G_t \equiv B A_t x_t$ is the cost per hire, $B > 0$ and $x_t \equiv H_t/U_t$ is the job finding rate.\textsuperscript{14} Hiring costs are expressed in terms of the CES bundle of final goods.\textsuperscript{15} Since the model features balanced growth the presence of $A_t$ ensures that along the balanced growth path the cost per hire increases at the same rate as aggregate final output. For future reference the detrended version of equation (17) is

$$c^H_t = g_t H_t,$$ \hspace{1cm} (18)

where $g_t = B x_t$.

Intermediate good firms face perfectly competitive output market and sell output at the nominal price $P^I_t$. The presence of hiring costs makes the hiring decision intertemporal. To see this, a firm’s lifetime discounted profit is given by

$$E_t \sum_{i=0}^{\infty} Q_{t,t+i} \left( p^I_{t+i} A_{t+i} h_{t+i} N_{t+i} - w_{t+i} h_{t+i} N_{t+i} - G_{t+i} H_{t+i} \right),$$  \hspace{1cm} (19)

where $p^I_t \equiv P^I_t / P_t$ is the relative price of the intermediate good and $w_t \equiv W_t / P_t$ is the real wage. In any given period profits are equal to revenues net of the total wage bill

\textsuperscript{13}This section draws partly on Blanchard and Gali (2010). The assumption that firms can hire a worker instantaneously subject to paying hiring costs simplifies our analysis. Alternatively, one may assume vacancy posting costs as in the labor search and matching literature (see, e.g., Christoffel and Kuester (2008)). In the present paper, we do not need to track vacancies, which is necessary when one is interested, say, in the Beveridge curve (i.e., the relationship between vacancies and unemployment).

\textsuperscript{14}In this setup, a vacancy is filled instantaneously if the firm pays the hiring cost. As a matter of comparison, in the standard search and matching model the job-posting cost is constant for each posted vacancy. Assuming a matching function of the form $H_t = U_t^{\alpha_0} V_t^{1-\alpha_0}$, where $V_t$ is the number of posted vacancies, the hiring cost is proportional to the expected vacancy duration, which is equal to the inverse of the job-filling rate $H_t/V_t$. It can be shown that $V/H = x^\alpha$, where $\alpha \equiv \alpha_0/(1-\alpha_0)$. Our specification of the cost per hire assumes implicitly that $\alpha_0 = .5$, which is close to empirical estimates (see Blanchard and Gali (2010)).

\textsuperscript{15}We follow the common assumption that intermediate goods firms’ allocation of their hiring resources across the differentiated goods is analogous to that of households so that $C^H_{k,t} = (P^I_{k,t}/P_t)^{-\theta} C^H_t$. Furthermore, for simplicity we assume that hiring costs are rebated to households.
and the total hiring cost. Maximizing the sum of discounted profits (19) subject to the employment dynamics (16) leads to the first order condition for an optimum level of hiring,

\[ p_t^I A_t h_t = w_t h_t + G_t - (1 - \delta)E_t \{ Q_{t,t+1} G_{t+1} \}. \]  

(20)

The left hand side of equation (20) is the marginal revenue product of labor while the right hand side is the cost of the marginal worker, which includes the real wage and the hiring cost net of discounted savings in future hiring costs. Dividing through by \( A_t \) and slightly manipulating the resulting equation gives

\[
p_t^I h_t = w_t^d h_t + g_t - (1 - \delta)E_t \{ Q_{t,t+1} \Gamma g_{t+1} \}
\]

\[ = w_t^d h_t + g_t - (1 - \delta)\beta \Gamma^{1-\sigma} E_t \left\{ \frac{c_{t+1}^{1-\sigma} V(h_{t+1}, N_{t+1})}{c_t^{1-\sigma} V(h_t, N_t)} g_{t+1} \right\}, \]

(21)

where \( g_t \equiv G_t/A_t \) and the second equality follows from using equation (2) to substitute out \( Q_{t,t+1} \).

From the right hand side of equation (21) we see that there are two counteracting effects of higher trend productivity growth on the firm’s hiring policy for a given level of the relative price \( p_t^I \).\(^{16}\) On the one hand, higher trend productivity growth raises the returns from current hiring. With higher trend productivity growth, hiring today implies larger savings in future hiring costs while benefiting from faster labor productivity growth. On the other hand, higher trend productivity growth lowers the returns from current hiring because higher trend productivity growth raises the real interest rate, and in turn the present discounted value of surplus over the life of a job. Following Aghion and Howitt (1994) we call the former the “capitalization effect” and the latter the “interest rate effect.” The capitalization effect leads to higher job creation while the interest rate effect leads to lower job creation. Given the assumption that \( \sigma > 1 \), the interest rate effect dominates the capitalization effect, so that higher trend productivity growth leads to higher unemployment.

3.2.2 Wage and hours setting

Following much of the literature we assume Nash bargaining over wages and hours, whereby the resulting wage and hours maximize the joint surplus of the marginal firm-

\(^{16}\)As will be shown below, under non-zero inflation trend productivity growth also affects hiring through its influence on \( p_t^I \).
worker pair. Consider first the determination of the household’s surplus from an employment relationship. The value (expressed in terms of the CES bundle of final goods) to the household of one additional employed member is given by

\[ V_e^t = w_t h_t - \frac{\sigma \gamma_1^{h_{1+\phi}} C_t}{1 + (\sigma - 1) \gamma_1^{h_{1+\phi}} N_t} + E_t \left[ (1 - \delta(1 - x_{t+1})) V_{t+1}^e + \delta(1 - x_{t+1}) V_{t+1}^u \right]. \]  

(22)

The right side is the sum of the worker’s current gain from employment (the wage net of the marginal rate of substitution) and the continuation value.\(^{17}\) The corresponding value of one additional unemployed worker is given by

\[ V_u^t = \zeta_t + E_t \left[ Q_{t,t+1} \left( x_{t+1} V_{t+1}^e + (1 - x_{t+1}) V_{t+1}^u \right) \right], \]  

(23)

where the unemployment benefit \( \zeta_t \) is assumed to be proportional to be proportional to productivity growth \( \zeta_t = u_b A_t \), where \( u_b > 0 \).\(^{18}\) The equation takes into account that with probability \( x_{t+1} \) the unemployed finds a job in period \( t + 1 \).

The household’s surplus from an employment relationship \( S_h^t \) is the difference between equation (22) and equation (23),

\[ S_h^t = w_t h_t - \left( \frac{\sigma \gamma_1^{h_{1+\phi}} C_t}{1 + (\sigma - 1) \gamma_1^{h_{1+\phi}} N_t} + \zeta_t \right) + (1 - \delta) E_t \left( Q_{t,t+1} (1 - x_{t+1}) S_{t+1}^h \right). \]  

(24)

The firm’s surplus from an employment relationship is given by

\[ S_f^t = p_t^I A_t h_t - w_t h_t + (1 - \delta) E_t \left( Q_{t,t+1} S_{t+1}^f \right). \]  

(25)

Note that, equations (25) and (21) imply that the firm’s surplus from additional hire is equal to the cost per hire \( G_t \).

Next, under Nash bargaining maximization of a weighted average of the firm’s and household’s surpluses \((S_h^t)\eta (S_f^t)^{1-\eta}\) with respect to the real wage, where \( \eta \) denotes the bargaining power of the household

\(^{17}\)Here \( \delta(1 - x_{t+1}) \) is the probability that an employed worker is separated from his job at the end of period \( t \) and stays unemployed in period \( t + 1 \) while \( 1 - \delta(1 - x_{t+1}) \) is the probability that an employed worker keeps his current job in period \( t + 1 \) or he is separated from his current job at the end of period \( t \) but finds a job in period \( t + 1 \).

\(^{18}\)This is a reasonable assumption since it implies that along a balanced growth path \( \zeta_t \) grows at the same rate as the real wage.
The first order condition for wage setting is given by

$$\eta S^f_t = (1 - \eta)S^h_t,$$  \hspace{1cm} (26)

while that for hours setting is given by

$$(1 - \eta)S^h_t (mrs_t - w_t) = \eta S^f_t (mrp_t - w_t),$$  \hspace{1cm} (27)

where \(mrs_t = \sigma \gamma h_t^\phi C_t / (1 + (\sigma - 1)\gamma h_t^{1+\phi}/(1 + \phi)N_t)^2\) is the marginal rate of substitution between consumption and hours and \(mrp_t = p_t^f A_t\) is the marginal revenue product.

Equations (24), (25) and (26) determine the wage, given hours,

$$w_t h_t = \frac{\sigma \gamma_1 h_t^{1+\phi} C_t}{1 + (\sigma - 1)\gamma h_t^{1+\phi} N_t} + \zeta_t + \nu (G_t - (1 - \delta)E_t \{Q_{t,t+1}(1 - x_{t+1})G_{t+1}\}),$$  \hspace{1cm} (28)

where \(\nu \equiv \eta/(1 - \eta)\) is the relative bargaining power of the household and we made use of the equation \(S^f_t = G_t\) to substitute out \(S^f_t\). Dividing equation (28) through by \(A_t\) and slightly manipulating the resulting equation gives

$$w_t^d h_t = \frac{\sigma \gamma_1 h_t^{1+\phi} c_t}{1 + (\sigma - 1)\gamma h_t^{1+\phi} N_t} + u_b
+ \nu \left( g_t - (1 - \delta)\beta^{1-\sigma} E_t \left\{ c_t^{\sigma-1}V(h_{t+1}, N_{t+1})/c_t^{\sigma-1}V(h_t, N_t) (1 - x_{t+1})g_{t+1}\right\} \right).$$  \hspace{1cm} (29)

The implied wage rate is increasing in the current cost per hire \((g_t)\), as this raises the firm’s surplus from an existing relationship, and decreasing in the expected future cost per hire \((g_{t+1})\) and in the probability \((1 - x_{t+1})\) of not finding a job next period in the event that the worker separates from the firm, both of which raise the continuation value to currently employed worker and hence reduce the required wage.

Combining equations (26) and (27) and dividing through by \(A_t\),

$$p_t^f = \frac{\sigma \gamma_1 h_t^{\phi} c_t}{(1 + (\sigma - 1)\gamma h_t^{1+\phi}/(1 + \phi)N_t)^2}.$$  \hspace{1cm} (30)

Analogous to the neoclassical labor market, equilibrium hours are such that the marginal product of hours is equal to the marginal rate of substitution between consumption and
hours. But now, hours are indirectly affected by labor market frictions indirectly—through changes in employment $N$, which in turn affects the marginal rate of substitution between consumption and hours. Moreover, in contrast to the neoclassical labor market the wage rate does not equal the marginal product of labor or the marginal rate of substitution between consumption and hours. Rather it is a forward looking variable, since its determination takes account of future labor market conditions (see equation (29)).

### 3.2.3 Final goods sector

The pricing decision of monopolistically competitive firms in the final goods sector is similar to the one described above. Since the input of the final goods sector is the intermediate good, the real marginal cost is given by the relative price of the intermediate good $p^I$. Thus, the optimal relative price takes the same form as its counterpart under a neoclassical labor market. The steady state version is given by

$$p^* = \mu \frac{1 - \beta \omega \Gamma^{1-\sigma} \Pi^{\theta-1}}{1 - \beta \omega \Gamma^{1-\sigma} \Pi^{\theta-1}} p^I,$$

which is identical to equation (6) except that instead of $w^d$ we have $p^I$.

### 3.3 Steady state equilibrium and results

In steady state the aggregate price index is given by equation (10) while the steady state optimal relative price (31) can be rewritten as

$$p^I = \frac{p^*(\Pi) (1 - \beta \omega \Gamma^{1-\sigma} \Pi^{\theta})}{\mu (1 - \beta \omega \Gamma^{1-\sigma} \Pi^{\theta-1})} \equiv p^I(\Gamma, \Pi).$$

Equation (32) has similar properties as those of equation (12). In particular, under the special case of zero trend rate of inflation (i.e., $\Pi = 1$) $p^I(\Gamma, \Pi) = 1$ and $p^I = 1/\mu$ so that the relative intermediate good price is independent of productivity growth. By contrast, when trend inflation rate is positive ($\Pi > 1$), $p^I = \partial p^I(\Gamma, \Pi)/\partial \Gamma > 0.21$

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19 Note here that the marginal cost to the firm is given by the marginal disutility to the household of supplying hours.

20 Given hours and consumption, the lower is the employment rate the larger is the marginal rate of substitution between consumption and hours. To restore equality either hours or consumption must adjust accordingly.

21 The derivation is straightforward, as $\partial p^I(\Gamma, \Pi)/\partial \Gamma = (\sigma - 1)\mu^{-1}\beta \omega \Gamma^{-\sigma} (\Pi - 1)\Pi^{\theta-1}p^*(\Pi)/(1 - \beta \omega \Gamma^{1-\sigma} \Pi^{\theta-1})^2$. 

13
In steady state the flow into unemployment is equal to the flow out of unemployment so that $H = \delta N$. In turn the steady state job finding rate $x$ is positively related to employment $N$ by

$$x = \frac{\delta N}{1 - (1 - \delta)N}.$$ \hfill (33)

Next, using (32) in steady state the optimal hiring condition (21) we get

$$p^f(\Gamma, \Pi)h = w^d h + \left(1 - \beta \Gamma^{1-\sigma}(1 - \delta)\right) g,$$ \hfill (34)

noting that $g = Bx$. All else equal faster growth decreases steady state hiring by decreasing the discounted savings in future hiring costs. Similarly, in steady state the wage setting equation (29) becomes

$$w^d h = \frac{\sigma \gamma_1 \phi h^{\phi - 1}(1 + \phi)C}{1 + (\sigma - 1)\gamma_1 h^{\phi - 1}(1 + \phi)N} + u_b + \nu \left(1 - \beta \Gamma^{1-\sigma}(1 - \delta)(1 - x)\right) g,$$ \hfill (35)

which shows that all else equal faster growth increases the steady state real wage by decreasing the discounted continuation value to an employed worker.

$$p^f(\Gamma, \Pi) = \frac{\sigma \gamma_1 h^{\phi - 1}(1 + \phi)C}{(1 + (\sigma - 1)\gamma_1 h^{\phi - 1}(1 + \phi)N)^2}.$$ \hfill (36)

Combining the aggregate production function of the intermediate good sector ($y^I = hN$) and that of the final goods sector ($y = y^I / \Delta$) leads to $hN = \Delta y$. Finally, the aggregate resource constraint is given by $c = y$. The last two equations imply that $c = hN / \Delta(\Pi)$. Using this equation to substitute out $c$ in equation (36) the steady state determination of hours, given employment, is given by

$$\Delta(\Pi)p^f(\Gamma, \Pi) = \frac{\sigma \gamma_1 h^{\phi - 1}(1 + \phi)N}{(1 + (\sigma - 1)\gamma_1 h^{\phi - 1}(1 + \phi)N)^2}.$$ \hfill (37)

It is easy to check that a decrease (increase) in employment must be associated with an increase (decrease) in the optimal steady state hours.\textsuperscript{22}

Similarly, substituting out $c$ in equation (35) and using the resulting equation to substitute out the wage rate in equation (34)

$$p^f(\Gamma, \Pi)h = \frac{\sigma \gamma_1 h^{\phi - 1}(1 + \phi)N}{1 + (\sigma - 1)\gamma_1 h^{\phi - 1}(1 + \phi)N} + u_b + BxZ(\Gamma, x),$$ \hfill (38)

\textsuperscript{22}Note that the right hand side term is increasing in $N$ and in $h$ if and only if $(\sigma - 1)\gamma_1 h^{\phi - 1}(1 + \phi)N < 1.$
where \( Z(\Gamma, x) \equiv (1 - \beta \Gamma^{1-\sigma}(1-\delta)) + \nu (1 - \beta \Gamma^{1-\sigma}(1-\delta)(1-x)) \) and the cost per hire \( g \) has been substituted out using \( g = Bx \).

Equations (33), (37) and (38) jointly determine the equilibrium values of \( x, h \) and \( N \). Unlike the model with neoclassical labor market, there is no explicit reduced-form solution to these nonlinear system of equations. Therefore, we analyze the solution of the model numerically by calibrating it to U.S. economy. Most of the parameter values are within ranges that are commonly used in the business cycle literature. The calibration of some of the labor market parameters closely follows Blanchard and Gali (2010). In particular, the exogenous job separation rate \( \delta \) is set equal to 0.12 while the bargaining parameter \( \nu \) is set equal to one (implying \( \eta = 0.5 \)—symmetric bargaining). Somewhat in line with Shimer (2010) inverse of the elasticity of intertemporal substitution \( \sigma \) is set equal to 3. The value of \( \sigma \) is set such that the degree of consumption smoothing is at the lower end of reported estimates in the literature with nonseparable utility (see, e.g., Guerron-Quintana (2008)). The disutility of work parameter \( \gamma_1 \) and the scale parameter \( B \) in the hiring cost function are set such that in a zero inflation and zero trend productivity growth steady state the unemployment rate is 5 percent, the job finding rate is 0.7 (as in Blanchard and Gali (2010)) and hours worked is 1/3 (household members allocated 1/3 of their time to market production). These steady state targets are close to long-run averages in the data.

The other model parameters take values that are common in the New-Keynesian literature: \( \beta = 0.99, \theta = 11 \) (implying that firms choose a 10 percent price markup under flexible prices or when the inflation rate is zero) and \( \omega = 0.75 \) (prices are fixed on average for four quarters). Finally, as a baseline we assume 4 percent trend rate of inflation, somewhat consistent with the postwar average rate of inflation in the US. Further below we discuss the sensitivity of the baseline result to alternative assumptions about trend inflation and labor market parameters—in particular, changes in the degree of labor market rigidity.

For any variable \( z \), let \( \bar{z} \) represent the percentage deviation of \( z \) from its value under a zero trend productivity growth rate. The exception is the employment rate, in which case \( \bar{N} \) denotes absolute deviation of steady state employment rate from the zero trend productivity growth steady state. Figure 1 illustrates the results under the baseline calibration by plotting steady state employment rate \( \bar{N} \), hours per worker \( \bar{h} \), the real rate \( \bar{w} \), and the wage income \( \bar{T} \), which is the product of the real wage and hours per worker, as a function of the trend productivity growth rate, \( \gamma \), expressed in annualized terms and over the interval \([0\%, 4\%]\). As in the model with a neoclassical labor market steady state hours per worker and the real wage rise with trend productivity growth. As will be shown below, whereas the positive relationship between trend productivity growth and hours per
worker is robust to alternative calibrations this is not true for the wage rate. The effect of trend productivity growth on the real wage is sensitive to assumptions about trend inflation and is in contrast to the clear-cut result (namely, a positive effect of growth on the real wage) found under the neoclassical labor market.

Moreover, the steady state employment rate rises with trend productivity growth under the baseline calibration. The reason is that under the maintained assumption of trend inflation the markup effect of trend productivity growth is relatively strong and thus dominates the offsetting negative capitalization effect of trend productivity growth. For this reason higher trend productivity growth increases the joint surplus from an employment relationship, which in turn induces intermediate good firms to increase hiring and thereby employment. However, as with the wage rate this result is sensitive to the level of trend inflation and mimics qualitatively the result found in Tesfaselassie (2014), for a similar model but without the hours margin.

Figure 1: The effects of trend productivity growth on steady state employment, hours, wage income and the wage rate. Baseline calibration.
3.4 Sensitivity of baseline results

As remarked above, the interplay between the markup channel and the capitalization channel determines the net effect of trend productivity growth on the labor market. It is thus of interesting to examine how features of the model related to price rigidity as well as to labor market frictions matter in terms of the relative strength of the two channels and thereby the labor market effects of trend productivity growth. We focus on three such features of the model: the rate of inflation (which matters due to price rigidity), the cost per hire (capturing labor market rigidity) and the job destruction rate (capturing labor market turbulence).

Trend inflation. As discussed above the markup channel is stronger the higher is the rate of trend inflation while the capitalization channel is independent of inflation. It follows that if trend inflation is high enough, as was the case under our baseline calibration, the markup channel dominates the capitalization channel so that trend productivity growth raises the surplus from an employment relationship, the wage rate, hours per worker as well as employment. The opposite is true when trend inflation is low enough. For instance, in the special case of zero rate of inflation only the capitalization channel, which is independent of trend inflation, is operative. In this case, higher trend productivity growth leads to lower employment because of the resulting decline in joint surplus. The wage rate, which is chosen to split the joint surplus, also declines. The accompanying decline in the worker surplus acts like a negative wealth effect on households, who are in turn induced to consume less and spend less time on leisure (i.e., work more hours), much the same as in the neoclassical labor market. Because hours per worker is costless (for firms) to adjust, firms and workers choose to increase hours.\footnote{It is to be remembered that adjusting employment is costly for a firm (which makes hiring decisions unilaterally) while hours per worker is determined, joint with the real wage, as part of Nash bargaining.}

As an illustration Figure 2 shows the effect of trend productivity growth on the labor market for alternative values of steady state inflation (annualized), namely, 0\%, 4\% (baseline) and 6\%. Other model parameters are set equal to their baseline values. As can be seen, the positive effect of trend productivity growth on hours per worker is stronger under the 6\% inflation rate than under the baseline case of 4\% inflation rate (which replicates Figure 1). This result is similar to the case with a neoclassical labor market. The difference is that, in the presence of labor market frictions, trend productivity growth matters for hours, as well as the real wage and the employment rate, even if trend inflation rate is zero. In line with our intuition, Figure 1 also shows that under 0\% inflation rate higher trend productivity growth is associated with a lower employment rate and a lower real
wage. Moreover, the effect of growth on the wage rate dominates that on hours so that higher trend productivity growth implies lower wage income $\bar{T}$.

**Hiring costs.** Figure 3 shows the relationship between trend productivity growth and labor market variables when allowing for a lower value of $B$ while keeping other parameters at their baseline values. The solid line replicates the baseline case shown in Figure 1, while the dashed line shows the case with a value of $B$ that is half as large as the baseline value. We see that a lower cost per hire reinforces the rise, due to higher trend productivity growth, in employment and the wage rate while it weakens the rise in hours. The magnified rise in employment and the wage rate implies that the direct effect of $B$ on the cost per hire dominates the indirect effect (see also Tesfaselassie (2014)).

The intuition is as follows. From the definition of the cost per hire, $g = Bx$, a decrease in the scale parameter $B$ directly decreases $g$ but indirectly increases $g$ by increasing the equilibrium job finding rate $x$. The indirect effect arises since firms increase employment when hiring is less costly.\textsuperscript{24} To the extent that the direct effect of $B$ dominates the indirect effect via $x$, the cost per hire $g$ is smaller the smaller is $B$. By the optimal hiring condition \textsuperscript{24}By equation (33) lower employment rate implies a lower job finding rate.
the decline in future savings in hiring costs, due to higher trend productivity growth, is less pronounced the lower is the cost per hire (a weakening of the capitalization channel of trend productivity growth). In this case the markup channel is more likely to dominate the capitalization channel, implying a stronger rise in the employment rate and the wage rate. From equation (37), which is related to the optimal setting of hours, the implied rise in hours must be smaller.

Job destruction. Figure 4 shows the relationship between trend productivity growth and labor market variables when allowing for a higher job destruction rate $\delta$ while keeping other parameters at their baseline values. The solid line replicates the baseline case shown in Figure 1, while the dashed line shows the case with a value of $\delta$ that is twice as large as the baseline value. We see that a higher job destruction rate reinforces the rise in the employment rate and the wage rate while it weakens the rise in hours. The magnified rise in employment and wages implies that the direct effect of $B$ on the cost per hire dominates the indirect effect.

The intuition here is analogous to that under the assumed changes in $B$. The job destruction rate $\delta$ weakens the capitalization channel of trend productivity growth. The reason is that a higher job destruction rate shortens the expected life of a job and therefore de-
Figure 4: The effects of trend productivity growth on steady state employment, hours, wage income and the wage rate under alternative rates of trend inflation.

increases the savings in future hiring costs resulting from current hiring (see equation (38)). Thus at a higher job destruction rate the markup channel is more likely to dominate the capitalization channel and, as a result, the employment rate and the real wage rise more strongly with trend productivity growth while hours rise less strongly.

4 Concluding remarks

The paper analyzes the effects of disembodied technological progress on steady state hours worked in the workhorse New-Keynesian model, which features a neoclassical labor market, and its extension that allows for equilibrium unemployment. We show that both versions can rationalize the long-term trend decline in productivity growth and the average number of hours per person observed across major industrialized countries during the postwar period.

The advantage of the workhorse model is that it is analytically tractable leading to an exact result. It is shown that lower trend productivity growth is associated with lower aggregate hours due to a price markup channel. However, the model has a drawback
because it predicts no relationship between trend productivity growth and hours when the rate of inflation is zero. In order to overcome this drawback, the second framework introduces labor market frictions into the standard New-Keynesian model, thereby allowing for adjustments in employment—the so-called extensive margin. The presence of labor market frictions gives rise to a second channel—the capitalization channel—whereby trend productivity growth affects hours. Under standard assumptions about the determination of wages, hours and employment, the effect of trend productivity growth on hours is shown to be similar to the version of the model neoclassical labor market. Importantly, since the capitalization channel is independent of inflation, the extended model predicts a relationship between trend productivity growth and hours even when the rate of inflation is zero. The prediction of the extended model with respect to the effect of trend productivity growth on the (un)employment rate mimics that in Tesfaselassie (2014), in which only adjustments in employment (but not hours) are considered.

The model is kept as simple as possible so as to present our results in a transparent way. A possible extension of the model is to relax the assumption of an exogenous technological progress. The determination of hours under endogenous growth implies a feedback from hours (and employment) to growth, for example as in Aghion and Howitt (1994), who assume learning-by-doing, or as in Eriksson (1997), who assumes positive externality from aggregate capital accumulation. Furthermore, one can also study how growth and hours respond to structural parameters. Finally, while we use nominal price rigidity as a rationale for the real effects of inflation, as in much of the business cycle literature, it would be interesting to examine whether alternative frameworks (such the the flexible price model with efficiency wages considered in, for e.g., Vaona (2013)) can explain the observed relationship between trend productivity growth and hours.

References


