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## Monetary policy and the asset risk-taking channel

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# **Non-technical summary**

## **Research Question**

The recent financial crisis has marked the importance of understanding the different types of risk to which the financial sector, and ultimately the real economy, are exposed. In particular, monetary policy might influence financial sector risk through the so called risk-taking channel, i.e. the mechanism by which low levels of the risk-free interest rate induce financial institutions to make riskier investments. We explore the functioning and relevance of this channel using a quantitative macroeconomic model, and assess whether the monetary authority should take its influence on bank asset risk into account when setting the interest rate

## **Contribution**

Motivated by empirical VAR evidence for the US banking sector, we develop a quantitative monetary DSGE model where monetary policy influences bank asset risk, an aspect of financial sector risk that might have played an important role in the lead-up to the 2008 financial crisis. We furthermore estimate the model using US data and systematically explore how this novel channel affects the trade-off faced by the monetary policy authority.

## **Results**

In the model, an agency problem between depositors and equity providers, stemming from limited liability and the unobservability of risk taking, distorts the banks' incentives and leads them to choose excessively risky investments. A monetary policy expansion causes a decline in the real interest rate, which amplifies these distortions and increases excessive risk taking. This in turn lowers the expected return on investment, and dampens the positive output effects of the interest rate cut.

The connection between interest rates and asset risk raises the question of whether the monetary authority should take this channel into account when setting the interest rate. To answer the question we first estimate the model on US data using Bayesian techniques and find that the inclusion of this additional channel improves the in-sample fit of the model, yields impulse responses that are broadly in line with the results of our VAR analysis, and predicts a path of risk taking that matches US survey evidence. We then assess optimal monetary policy conduct in the estimated model, and find that, if the risk-taking channel is active, monetary policy should stabilize the real interest rate, in order to reduce the welfare detrimental volatility of the banks' risk choice.

# **Nichttechnische Zusammenfassung**

## **Fragestellung**

Die letzte Finanzkrise hat deutlich gemacht, wie wichtig das Verständnis der unterschiedlichen Risikoarten ist, denen der Finanzsektor und letztlich auch die Realwirtschaft ausgesetzt sind. Im Hinblick darauf ist es möglich, dass insbesondere die Geldpolitik das Risiko im Finanzsektor über den sogenannten Risikoneigungskanal beeinflusst. Durch diesen Mechanismus werden die Finanzinstitute bei einem niedrigen risikofreien Zins verleitet, sich in risikoreicheren Anlagen zu engagieren. Wir erforschen die Funktionsweise und Bedeutung dieses Kanals mithilfe eines quantitativen makroökonomischen Modells und nehmen eine Einschätzung vor, ob die Geldpolitik bei der Bestimmung des Leitzinses den von ihr ausgehenden Einfluss auf das Anlagerisikoverhalten der Banken berücksichtigen sollte.

## **Forschungsbeitrag**

Auf der Grundlage empirischer VAR-Evidenz für den US-Bankensektor entwickeln wir ein quantitatives monetäres DSGE-Modell, bei dem das Anlagerisiko der Banken durch die Geldpolitik beeinflusst wird – ein Aspekt des Finanzsektorrisikos, der im Vorfeld der Finanzkrise des Jahres 2008 eine wichtige Rolle gespielt haben könnte. Außerdem schätzen wir das Modell anhand von US-Daten und testen systematisch aus, wie sich dieser neue Kanal auf den Zielkonflikt auswirkt, mit dem es die Geldpolitik zu tun hat.

## **Ergebnisse**

In dem Modell führt ein Agency-Problem zwischen Einlegern und Eigenkapitalgebern, das von einer begrenzten Haftung und der Unbeobachtbarkeit des Risikoverhaltens rührt, zu Anreizverzerrungen bei den Banken, sodass diese zu übermäßig riskanten Anlageentscheidungen neigen. Eine expansivere Geldpolitik bedingt einen niedrigeren Realzins, der diese Verzerrungen vergrößert und somit exzessives Risikoverhalten verstärkt. Dies verringert wiederum die erwartete Anlagerendite und dämpft die positiven Effekte der Zinssenkung auf die Produktion.

Um zu sehen, ob die Geldpolitik auf diesen Kanal reagieren sollte, schätzen wir das Modell zunächst mit bayesianischen Methoden auf der Basis von US-Daten. Dabei gelangen wir zu dem Ergebnis, dass die Einbeziehung dieses zusätzlichen Kanals die Anpassungsgüte des Modells verbessert und Impuls-Antwort-Folgen generiert, die weitgehend im Einklang mit den Resultaten unserer VAR-Analyse stehen. Des Weiteren deckt sich der vom Modell prognostizierte Risikoneigungspfad, mit den Umfrageergebnissen von US Banken. Wir zeigen auch, dass die Geldpolitik auf den Risikoneigungskanal der Banken durch ein Stabilisieren des Realzinses reagieren sollte. Dadurch kann sie die wohlfahrtsschädliche Volatilität mindern, die mit der Risikoentscheidung der Banken einhergeht.

# Monetary policy and the asset risk-taking channel\*

Angela Abbate<sup>†</sup>

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## Abstract

Motivated by VAR evidence, we develop a monetary DSGE model where an agency problem between bank financiers, stemming from limited liability and unobservable risk taking, distorts banks' incentives leading them to choose excessively risky investments. A monetary policy expansion magnifies these distortions, increasing excessive risk taking and lowering the expected return on investment. We estimate the model on US data using Bayesian techniques and assess how this novel channel affects optimal monetary policy. Our results suggest that the monetary authority should stabilize the real interest rate, trading off more inflation volatility in exchange for less volatility in risk taking and output.

**J.E.L. Classification:** E12 E44 E58

**Keywords:** Bank Risk; Monetary policy; DSGE Models;

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# 1 Introduction

The recent financial crisis has marked the importance of monitoring the different types of risks to which the financial sector, and ultimately the real economy, are exposed. A relevant aspect is whether interest rates, and therefore monetary policy, can influence the risk-taking behavior of financial intermediaries. This transmission mechanism, known as the risk-taking channel of monetary policy,<sup>1</sup> could have contributed to the excessive levels of financial sector balance-sheet risk which lead to the 2008 financial crisis. In the aftermath of the crisis interest rates have fallen considerably in many countries, raising concerns on whether financial market participants might be once again induced to reallocate portfolios towards riskier investments, creating the risk of yet another crisis.<sup>2</sup>

This paper addresses these concerns from a theoretical point of view, motivated by structural VAR evidence showing that expansionary monetary policy shocks increase bank asset risk in the US. We build a monetary DSGE model, where investment in capital is intermediated by a banking sector and is furthermore risky. Building on Dell’Ariccia et al. [2014], we assume that banks can choose from a continuum of investment projects, each defined by different risk-return characteristics. Every project has a certain probability of yielding capital in the next period. The safer the project is, however, the lower is the return in case of success. Since depositors cannot observe the investment choice, and because bank owners are protected by limited liability, an agency problem emerges: banks are partially isolated from the downside risk of their investment and hence choose a risk level that exceeds what would be chosen if these frictions were absent. This problem could be mitigated if bankers held more equity. Yet, banks optimally rely on both types of funding and hence the agency problem persists, because equity is relatively more costly than deposits due to deposit insurance and a friction in the equity market. Since the importance of these distortions is proportional to the real rate of return, lower levels of the real risk-free rate induce banks to increase leverage and choose riskier investment projects. This implies that the investment intermediated by banks becomes less efficient, leading to a sizeable decline in the

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<sup>1</sup>The term was first coined by Borio and Zhu [2008].

<sup>2</sup>See for instance: (2015, 26 September). Repeat Prescription, *The Economist*.

capital stock both directly, as a lower fraction of capital projects is successful, and indirectly, as households will not save as much. Overall, a monetary policy expansion worsens the financial market distortions, which in turn attenuate the positive output effects of the interest rate cut.

This connection between interest rates and asset risk raises the question of whether the monetary authority should take this channel into account when setting the interest rate. Since the answer to this question is of quantitative nature, we embed the banking sector in a medium scale Smets and Wouters [2007]-type DSGE model, known to fit the data well along many dimensions, and estimate it on US data with Bayesian techniques. We find that the inclusion of this additional channel improves the in-sample fit of the model, yields impulse responses that are broadly in line with the results of our VAR analysis, and predicts a path of risk taking for the estimation period that matches survey evidence. We then analyze optimal monetary policy in the estimated model using simple rules and find that, if the risk-taking channel is active, monetary policy should be less responsive to inflation and output fluctuations. In this way, the monetary authority allows more inflation volatility in exchange for stabilizing the real interest rate, which in turn reduces the welfare detrimental volatility of the banks' risk choice. The welfare gains from taking the risk-taking channel into account are significant.

Our work relates to a small but growing theoretical literature that links monetary policy to financial sector risk in a general equilibrium framework. Most of the existing works focus on *funding* risk, associating risk with leverage, and build on the financial accelerator framework of Bernanke et al. [1999].<sup>3</sup> The mechanism in these models relies on the buffer role of equity and therefore leverage is found to be counter-cyclical with respect to the balance sheet size. Our model on the contrary gives rise to pro-cyclical leverage, which is in line with the empirical evidence reported in Adrian and Shin [2014] and Adrian et al. [2015]. Following a different strategy, Angeloni and Faia [2013] and Angeloni et al. [2015] augment the financial accelerator framework and construct a model where higher leverage induced by expansionary monetary policy does not just amplify other shocks but

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<sup>3</sup>For example in Gertler et al. [2012] and de Groot [2014] a monetary expansion increases banking sector leverage, which in turn amplifies the financial accelerator and strengthens the propagation of shocks to the real economy.

also translates into a higher fraction of inefficient bank runs.

In this paper by contrast we model *asset* risk, i.e. the riskiness of the assets on the banks' balance sheets, another aspect of financial sector risk that seems to have played an important role in the lead-up to the 2008 financial crisis. This type of risk has so far mainly been discussed in the literature on optimal regulation such as Christensen et al. [2011] and Collard et al. [2012]. In these papers however, either the depositors or the financial regulator ensure that risk is always chosen optimally, so monetary policy has no influence on risk taking.<sup>4</sup> In contrast to the previous two papers we provide microfoundations for the asset risk-taking channel and focus on monetary policy while abstracting from regulation.<sup>5</sup> Adding to the literature on both types of risk taking, we furthermore systematically explore how this novel channel affects optimal monetary policy in an estimated medium scale model, where the policy maker needs to trade off several conflicting frictions.

The lack of theoretical papers on the asset risk-taking channel is not mirrored by a lack of empirical evidence. Several studies find a causal link between monetary policy and risk taking. Most of the existing research relies on loan or bank level panel data and identification is based on the assumption that monetary policy is exogenous. Jimenez et al. [2014] use micro data of the Spanish Credit Register from 1984 to 2006 and find that lower interest rates induce banks to make relatively more loans to firms that qualify as risky *ex ante* (firms with a bad credit history at time of granting the loan) as well as *ex post* (firms that default on the granted loan). They argue that this effect is economically significant and particularly strong for thinly capitalized banks. These findings are confirmed for Bolivia using credit register data in Ioannidou et al. [2014], and for the US using confidential loan level data from the Terms of Business Lending Survey in Dell'Ariccia et al. [2013]. For the US, these findings are furthermore corroborated by evidence from aggregate time series data, where identification is obtained through restrictions on the dynamic responses. Angeloni et al. [2015] and Afanasyeva and Guentner [2015] find that monetary policy shocks increase asset risk, respectively proxied by the debt stock of households and non-financial corporations, and by the net percentage

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<sup>4</sup>Both papers feature ad-hoc extension that relate risk to the amount of lending and hence indirectly to monetary policy.

<sup>5</sup>One could reinterpret our model as applying to an economy where regulation is not able to fully control risk taking.



of banks reporting tighter lending standards in the Fed survey of business lending. These results are confirmed for small banks by the FAVAR analysis of Buch et al. [2014], who use a more direct measure of bank risk from the Terms of Business Lending Survey. We complement these results in the following section, where we show that further evidence on the risk-taking channel for the whole banking sector can be obtained using a more parsimonious setup. All these findings can be summarized by the stylized fact that interest rate cuts increase bank asset risk

A second stylized fact that motivates the theoretical model is found by Buch et al. [2014] and Ioannidou et al. [2014]. Both show that the increase in risk taking induced by low interest rates is not accompanied by an offsetting increase in the risk premium on loans, indicating that the additional risk might be priced inefficiently.

Motivated by this comprehensive empirical evidence and the VAR analysis in section 2, we develop in section 3 a DSGE model of asset risk-taking, where banks respond to low interest rates by inefficiently taking more risk. Section 4 presents the results from the estimation of the model and discusses the steady-state and dynamic implications of bank risk taking. Section 5 analyzes how monetary policy should be conducted if a risk-taking channel is present and section 6 concludes.

## 2 The asset risk-taking channel in the US

To motivate our subsequent theoretical analysis, we provide additional empirical evidence on the existence of the asset risk-taking channel in the US. We employ a classical small-scale VAR that includes inflation, output, a measure of bank risk-taking and the effective federal funds rate, taken as the monetary policy instrument. Output is measured by real GDP growth, while inflation is defined as the log change in the GDP deflator.

Measuring risk taking is less straightforward. There are many notions of asset risk. One can distinguish between ex-ante, ex-post and realized asset risk. The former is the risk perceived by the bank when making a loan or buying an asset. Banks can influence this class of risk directly, when making their investment decisions (the ex-ante risk choice). On the other hand, the ex-post risk of a bank's balance sheet is also affected by unforeseen changes in asset riskiness, that take

place after origination and are largely outside the banks' influence. Lastly, the payoff ultimately paid by an asset is a materialization of the former two types of uncertainty (realized asset risk). In this paper we focus on active risk taking, that is the level of ex-ante risk that intermediaries choose, which is however difficult to observe directly.<sup>6</sup> Therefore we follow Dell'Ariccia et al. (2014) and proxy bank risk-taking by the weighted average of the internal risk rating assigned by banks to newly issued loans, reported in the US Terms of Business Lending Survey. See appendix A for a plot of this risk index.<sup>7</sup>

We estimate a VAR over the period 1997:Q2, the start of the survey-based proxy for risk taking, to 2007:Q3<sup>8</sup>. The lag length is chosen to be 1, as indicated by the BIC information criterion. We identify an unexpected monetary policy shock by using a conventional set of sign restrictions that are robust across a variety of general-equilibrium models. In particular, we assume that an expansionary monetary policy shock decreases the nominal risk-free interest rate and increases inflation and output, both at the time of the shock and in the quarter immediately after. Risk is left unrestricted. Note that the response of inflation ensures that this shock is identified separately from a productivity shock, which increases output but decreases both the interest rate and inflation.

The response of bank asset risk to an expansionary monetary policy shock is shown in figure 1. An unexpected decrease in the nominal interest rate is followed by a moderate macroeconomic expansion: output growth increases for less than a year, while inflation displays a longer reaction of about two years. The results are compatible with the existence of a risk-taking channel in the US: a fall in the nominal interest rate leads in fact to a decrease in the ex-ante proxy for the safety of banks' assets, i.e. banks issue riskier loans. Interestingly, the implied responses

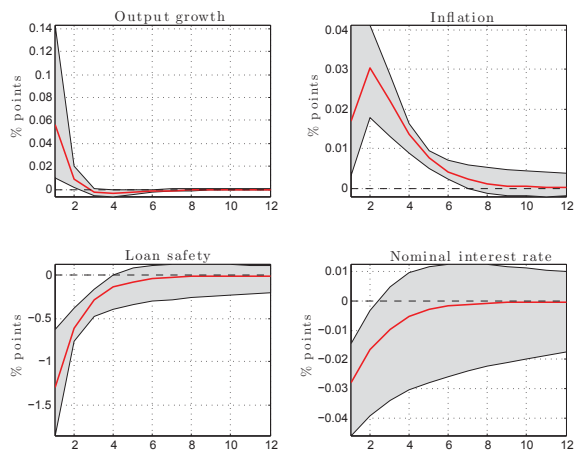
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<sup>6</sup>Inferring it from realized risk (e.g. loan losses) is hardly possible with aggregate data. Inference from the spread between bank funding costs and loan rates neglects the fact that this spread not only reflects default risk but also incorporates a liquidity premium and the markup, which are likely to be affected by the same variables that influence the risk choice.

<sup>7</sup>The average loan risk is a perfect measure for bank risk taking if we assume that the volume of loans is constant. Else, banks could also minimize their risk exposure by reducing the quantity of loans as their average quality goes down. While the correlation between risk and loan volume growth is slightly negative, it is not significant at a 10% significance level. For a more in-depth discussion of the data we refer to Buch et al. [2014].

<sup>8</sup>We have decided to cut the zero-lower bound period, but our results still hold when the latest available data are used.

Figure 1: **Monetary policy shock on bank risk-taking:** Impulse responses over a 3-year horizon, identified through the sign restriction scheme in Table 1. Error bands correspond to 90% confidence intervals reflecting rotation uncertainty. Loan safety is defined as the inverse of the average loan risk rating, standardized to take values between 0 and 100. The remaining variables are annualized. See text for further details.



of the nominal interest rate and the risk measure are approximately proportional. These results are robust to using a recursive identification scheme, as shown in appendix B.<sup>9</sup>

### 3 A Dynamic New Keynesian model with a bank risk-taking channel

In this section we build a general-equilibrium model where monetary policy can influence the risk-taking behavior of banks, thus providing an explanation for the risk-taking channel observed in the data. As a starting point we use a standard New Keynesian model with imperfect competition and price stickiness in the goods market, which implies a role for monetary policy. We augment this basic framework with an intermediation sector based on Dell’Ariccia et al. [2014]: competitive banks obtain funds from depositors and equity holders, which they invest into capital projects carried out by capital producers. Every bank chooses its investment from

<sup>9</sup>We tested two orderings (output, inflation, interest rate, risk) and (output, inflation, risk, interest rate).

a continuum of available capital production technologies, each defined by a given risk-return characteristic. The risk choice of the representative bank is affected by the level of the real interest rate, and can be shown to be suboptimal. This model reproduces two features found in the data: risk taking depends on the contemporaneous interest rate and is priced inefficiently.

While the aforementioned blocks are the necessary ingredients, in order to obtain a quantitatively more realistic model we add further elements, which are typically used in the DSGE literature. In particular we follow Smets and Wouters [2007] and allow for internal habits, investment adjustment costs and imperfect competition and wage stickiness in the labor market. Our model therefore features seven agents that are typical for DSGE models (households, unions, labor packers, capital producers, intermediate goods producers, final goods producers, and a central bank) and two agents that we introduce to model risk taking (banks, funds). Eight structural shocks hit the economy: these affect productivity, investment, time preferences, the equity premium, wage and price markups, as well as monetary and fiscal policy.

### 3.1 Households

The representative household chooses consumption  $c_t$ , working hours  $L_t$  and savings in order to maximize its discounted lifetime utility. Saving is possible through three instruments: government bonds  $s_t$ , deposit funds  $d_t$ , and bank equity funds  $e_t$ . The nominal return on government bonds is safe and equal to the nominal interest rate  $R_t$ . The two funds enable the representative household to invest into the banking sector, and pay an uncertain nominal return of  $R_{d,t+1}$  and  $R_{e,t+1}$ .<sup>10</sup> Households maximize their lifetime utility function:

$$\max_{d_t, e_t, s_t, c_t, L_t} E \left[ \sum_{t=0}^{\infty} \beta^t \varepsilon_t^B \frac{(c_t - \iota c_{t-1})^{1-\sigma_C}}{1-\sigma_C} \exp \left( \varphi L_t^{1+\sigma_L} \frac{\sigma_C - 1}{1 + \sigma_L} \right) \right], \quad (1)$$

subject to the per-period budget constraint in real terms:

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<sup>10</sup>Note that in our notation the time index refers to the period when a variable is determined.

$$c_t + d_t + e_t + s_t + T_t = L_t w_t + d_{t-1} \frac{R_{d,t}}{\pi_t} + e_{t-1} \frac{R_{e,t}}{\pi_t} + s_{t-1} \frac{R_{t-1}}{\pi_t} + \Pi_t \quad , \quad (2)$$

where  $\pi_t$  is the inflation rate, while  $T_t$  and  $\Pi_t$  are taxes and profits from firm ownership, expressed in real terms. We allow for habits in consumption ( $\iota$ ) and a time preference shock  $\varepsilon_t^B$ . This shock is assumed to be persistent with log-normal innovations, like all following shocks unless otherwise specified. The household's optimality conditions are given by the usual Euler equation and two no-arbitrage conditions:

$$\Lambda_t = \beta E_t \left[ \Lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right] \quad , \quad (3)$$

$$E_t \left[ \Lambda_{t+1} \frac{R_{d,t+1}}{\pi_{t+1}} \right] = E_t \left[ \Lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right] \quad , \quad (4)$$

$$E_t \left[ \Lambda_{t+1} \frac{R_{e,t+1}}{\pi_{t+1}} \right] = E_t \left[ \Lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right] \quad , \quad (5)$$

where  $\Lambda_t = \varepsilon_t^B (c_t - \iota c_{t-1})^{-\sigma_C} - \beta \iota E_t [\varepsilon_{t+1}^B (c_{t+1} - \iota c_t)^{-\sigma_C}]$  is the marginal utility of consumption.

### 3.2 Labor and goods sectors

The labor and goods sectors feature monopolistic competition and nominal rigidities, which allow for a role for monetary policy. Since the modeling of these sectors follows the canonical New Keynesian model, we discuss them only briefly and refer to Smets and Wouters [2007] and Adjemian et al. [2008] for further details. The corresponding equilibrium conditions are listed in appendix C.

Final good producers assemble different varieties of intermediate goods through a Kimball [1995] aggregator with elasticity of substitution  $\epsilon_p$  and Kimball parameter  $k_p$ , taking as given both the final good price and the prices of intermediate goods. Their optimization problem yields demand functions for each intermediate good variety as a function of its relative price.

A continuum of firms produces differentiated intermediate goods using capital

$K_{t-1}$  and “packed” labor  $l_t^d$  as inputs. The production function is Cobb-Douglas and is affected by a total factor productivity shock  $\varepsilon_t^A$ . Firms use their monopolistic power to set prices, taking as given their demand schedule. As in Calvo [1983], they can reset their prices in each period with probability  $\lambda^p$ , otherwise they index their prices to past inflation with degree  $\gamma^p$  and to steady state inflation with degree  $(1 - \gamma^p)$ . Furthermore they are subject to a time-varying revenue tax  $\varepsilon_t^p$  that is equivalent to a markup shock, up to a first-order approximation.

The labor market resembles the product market: Packed labor is produced by labor packers, who aggregate differentiated labor services using a Kimball [1995] aggregator with elasticity of substitution  $\epsilon_w$  and Kimball parameter  $k_w$ .

Differentiated labor services are produced by a continuum of unions from the households labor supply. They use their monopolistic power to set wages. Wages are reset with probability  $\lambda^w$ , otherwise they are indexed to past inflation (with degree  $\gamma^w$ ) and steady state inflation. Like intermediate firms, unions are subject to a stochastic wage tax  $\varepsilon_t^w$ .<sup>11</sup>

### 3.3 Equity and deposit funds

As we explain in detail below, there is a continuum of banks which intermediate the households’ savings using deposits and equity. Each bank is subject to a binary idiosyncratic shock which makes a bank fail with probability  $1 - q_{t-1}$ , in which case equity is wiped out completely and depositors receive partial compensation from the deposit insurance. We assume that households invest into bank equity and deposits through two funds. The function of the equity (deposit) fund is to eliminate the idiosyncratic bank default risk by buying a perfectly diversified portfolio of 1 period equity (deposits) of all banks.

The deposit fund works without frictions, and represents the depositors’ interests perfectly. The deposit fund raises money from the households and invests it into  $d_t$  units of deposits<sup>12</sup>. In the next period, the fund receives the nominal deposit rate  $r_{d,t}$  from each bank that does not fail. Deposits of failing banks are

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<sup>11</sup>Both  $\varepsilon_t^p$  and  $\varepsilon_t^w$  follow the standard shock process augmented by an moving average component, as in Smets and Wouters [2007].

<sup>12</sup>We use deposits to refer to both units of deposit funds and units of bank deposits since they are equal. We do the same for equity.

partially covered by deposit insurance. Most deposit insurance schemes around the world, including the US, guarantee all deposits up to a certain maximum amount per depositor<sup>13</sup>. We represent this capped insurance model by assuming that the deposit insurance guarantees deposits up to a fraction  $\psi$  of total bank liabilities  $l_t$ , which are the sum of deposits  $d_t$  and equity  $e_t$ . We assume that the deposit insurance cap is inflation adjusted, to avoid complicating the monetary policy trade-off by allowing an interdependence between monetary policy and deposit insurance. As we will show later, the deposit insurance cap is always binding in equilibrium, i.e. the bank's liabilities exceed the cap of the insurance  $r_{d,t}d_t > \psi(d_t + e_t)\pi_{t+1}$ . Defining the equity ratio  $k_t = \frac{e_t}{d_t + e_t}$ , the deposit fund therefore receives a real return of  $\psi/(1 - k_t)$  per unit of deposits from each defaulting bank at  $t$ . The deposit fund hence pays a nominal return of:

$$R_{d,t+1} \equiv q_t r_{d,t} + (1 - q_t) \frac{\psi}{1 - k_t} \pi_{t+1} . \quad (6)$$

Unlike the deposit fund, which is managed frictionlessly, the equity fund is subject to a simple agency problem. In particular, we assume that the fund manager faces two options. He can behave diligently and use the funds  $e_t$ , raised at  $t$ , to invest into  $e_t$  units of bank equity. A fraction  $q_t$  of banks will pay back a return of  $r_{e,t+1}$  next period while the defaulting banks pay back nothing. Alternatively the fund manager can abscond with the funds, in which case he consumes a fraction  $\xi_t$  of the funds in the subsequent period and the rest is lost. Since he is a small member in the big family of the representative household his utility from doing so is  $\Lambda_{t+1}\xi_t e_t$ . To prevent the fund manager from absconding funds, the equity providers promise to pay him a premium  $p_t$  at time  $t + 1$  conditional on not absconding. This premium is rebated to the household in a lump-sum fashion and the associated utility for the fund manager is  $\Lambda_{t+1}p_t$ . Equity providers pay the minimal premium that induces diligent behavior, i.e.  $p_t = \xi_t e_t$ . Once absconding is ruled out in equilibrium, the equity fund manager perfectly represents the interests of the equity providers. The nominal return on the bank equity portfolio

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<sup>13</sup>For a comprehensive documentation see, for instance, Demirgüç-Kunt et al. [2005].

is  $q_t r_{e,t+1}$  per unit of equity, hence one share of the equity fund pays:

$$R_{e,t+1} \equiv q_t r_{e,t+1} - \xi_t \pi_{t+1}. \quad (7)$$

We allow the equity premium  $\xi_t$  to vary over time.<sup>14</sup> Note that, since bank equity is the residual income claimant, the return of the equity fund is affected by all types of aggregate risk that influences the surviving banks' returns.

The two financial distortions introduced so far have important implications. The agency problem implies an equity premium, i.e. a premium of the risk-adjusted return on equity over the risk-free rate. Deposit insurance on the other hand acts as a subsidy on deposits, which implies a discount on the risk-adjusted return on deposits. As explained below, the difference in the costs of these two funding types induces a meaningful trade-off between bank equity and bank deposits under limited liability.

### 3.4 Capital producers

We assume that the capital production process is risky in a way that nests the standard capital production process in the New Keynesian model. In particular, capital is produced by a continuum of capital producers indexed by  $m$ . At period  $t$  they invest  $i_t^m$  units of final good into a capital project of size  $o_t^m$ . This project is successful with probability  $q_t^m$ , in which case the project yields  $(\omega_1 - \frac{\omega_2}{2} q_t^m) o_t^m$  units of capital at  $t + 1$ . Else, the project fails and only the liquidation value of  $\theta o_t^m$  units of capital can be recovered (where  $\theta \ll a - \frac{b}{2} q_t^m$ ). Each capital producer has access to a continuum of technologies with different risk-return characteristics indexed by  $q^m \in [0, 1]$ . Given a chosen technology  $q_t^m$ , the output of producer  $m$  therefore is:

$$K_t^m = \begin{cases} (\omega_1 - \frac{\omega_2}{2} q_t^m) o_t^m & \text{with probability } q_t^m \\ \theta o_t^m & \text{else} \end{cases}$$

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<sup>14</sup>This shock, driving a wedge between deposit and safe rates on one hand and equity rates on the other is similar to the risk premium shock often found in medium scale DSGE models (e.g. Smets and Wouters [2007]).



This implies that the safer the technology (higher  $q_t^m$ ), the lower is output in case of success.

The bank orders the capital projects and requires the capital producer to use a certain technology, but this choice cannot be observed by any third party. Given the technology choice  $q_t$  and assuming that the projects of individual producers are uncorrelated, we can exploit the law of large numbers to derive aggregate capital:

$$K_t = o_t \left( q_t \left( \omega_1 - \frac{\omega_2}{2} q_t \right) + (1 - q_t)\theta \right) . \quad (8)$$

Furthermore we assume that capital, which depreciates at rate  $\delta$ , becomes a project (of undefined  $q_t$ ) at the end of every period. That is, existing capital may be destroyed due to unsuccessful reuse, and it can be reused under a different technology than it was originally produced.<sup>15</sup>

The total supply of capital projects by the capital producers is the sum of the existing capital projects  $o_t^{old} = (1 - \delta)K_{t-1}$ , which they purchase from the owners (the banks) at price  $Q_t$ , and the newly created projects  $o_t^{new}$ , which are created by investing  $i_t$  units of the final good. We allow for investment adjustment costs and investment efficiency shocks, i.e. we assume that  $i_t$  units of investment yield  $\varepsilon_t^I (1 - S(i_t/i_{t-1}))$  units of project, where  $S = \kappa \left( \frac{i_t}{i_{t-1}} - 1 \right)^2$ . Hence  $o_t = o_t^{new} + o_t^{old}$  and  $o_t^{new} = \varepsilon_t^I \left( 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right) i_t$ . Capital producers maximize their expected discounted profits taking as given the price  $Q_t$  and the households stochastic discount factor<sup>16</sup>:

$$\max_{i_t, o_t^{old}} E_t \sum_0^{\infty} \beta^t \Lambda_t \left[ Q_t \varepsilon_t^I \left( 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right) i_t + Q_t o_t^{old} - i_t - Q_t o_t^{old} \right] .$$

While the old capital projects are always reused, the marginal capital project is

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<sup>15</sup>This assumption ensures that we do not have to keep track of the distribution of different project types. Think of a project as a machine that delivers capital services and that can be run at different speeds (levels of risk). In case it is run at higher speed, the probability of an accident that destroys the machine is higher. After each period the existing machines are overhauled by the capital producers and at this point the speed setting can be changed.

<sup>16</sup>Their out of steady state profits are rebated lump sum to the household.

always a new one.<sup>17</sup> Hence, the price of projects  $Q_t$  is determined by new projects according to the well known Tobin's q equation:

$$Q_t \varepsilon_t^I \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) - S' \left( \frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \right] - 1 = \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \varepsilon_{t+1}^I Q_{t+1} S' \left( \frac{i_{t+1}}{i_t} \right) \left( \frac{i_{t+1}}{i_t} \right)^2 \right]. \quad (9)$$

Note that our model of risky capital production boils down to the standard riskless setting of the New Keynesian model if we set  $q_t = 1$  and  $\omega_1 - \frac{\omega_2}{2} = 1$ .<sup>18</sup>

### 3.5 The Bank

The bank is the central agent of our model and is modeled similarly as in Dell'Ariccia et al. [2014]. Banks raise resources through deposits and equity and invest them into a risky project. Since depositors cannot observe the banks' risk choice and banks are protected from the downside risk of their investment by limited liability, an agency problem arises between them when the banks choose the risk level. The less equity a bank has, the higher the incentives for risk taking. Yet, since deposit insurance and the equity premium drive a wedge between the costs of deposits and equity, the banks' optimal capital structure comprises both equity and deposits, balancing the agency problem associated with deposits with the higher costs of equity. We will show that the equilibrium risk chosen by the banks is excessive, and that the interest rate influences the degree of its excessiveness.

We assume that there is a continuum of banks who behave competitively so that there is a representative bank (we therefore omit the bank's index in what follows). The bank is owned by the equity providers, and hence maximizes the expected discounted value of profits<sup>19</sup> using the household's stochastic discount factor. Every period the bank raises deposits  $d_t$  and equity  $e_t$  from the respective funds (optimally choosing its liability structure). These resources are then invested into  $o_t$  capital projects, purchased at price  $Q_t$ . When investing into capital projects, the bank chooses the risk characteristic  $q_t$  of the technology applied by

<sup>17</sup>We abstract from a non-negativity constraint on new projects.

<sup>18</sup>Or, more generally when taking into account the aggregation through banks and funds, if we set  $q_t (\omega_1 - \frac{\omega_2}{2} q_t) + (1 - q_t)\theta = 1$  and  $q_t = q$ .

<sup>19</sup>Profits in excess of the opportunity costs of equity.

the capital producer. This risk choice is not observable for depositors. Each bank can only invest into one project and hence faces investment risk<sup>20</sup>: with probability  $q_t$  the bank receives a high pay-off from the capital project; with probability  $1 - q_t$  the investment fails and yields only the liquidation value. Assuming a sufficiently low liquidation value  $\theta$ , a failed project implies the default of the bank. In this case, given limited liability, equity providers get nothing and depositors get the deposit insurance benefit. In case of success the bank can repay its investors: depositors receive their promised return  $r_{d,t}$  and equity providers get the state contingent return  $r_{e,t+1}$ .

It is useful to think of the bank's problem as a recursive two-stage problem. At the second stage, the bank chooses the optimal risk level  $q_t$  given a certain capital structure and a certain cost of deposits. At the first stage, the bank chooses the optimal capital structure, anticipating the implied solution for the second-stage problem. Note that not only the bank but also the bank's financiers anticipate the second-stage risk choice and price deposits and equity accordingly, which is understood by the bank. Below we derive the solution for this recursive problem.

Before we do so, we establish the bank's objective function. Per dollar of nominal funds raised (through deposits and equity) in period  $t$  the bank purchases  $Q_t/P_t$  units of the capital project from the capital producer, choosing a certain riskiness  $q_t$ . If the project is successful it turns into  $(\omega_1 - \frac{\omega_2}{2}q_t)/(Q_tP_t)$  capital goods. In the next period  $t + 1$ , the bank rents the capital to the firm, who pays the real rental rate  $r_{k,t+1}$  per unit of capital. Furthermore the bank receives the depreciated capital, which becomes a capital project again, with real value of  $(1 - \delta)Q_{t+1}$  per unit of capital. The bank's total nominal income, per dollar raised, conditional on success is therefore:

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<sup>20</sup>The assumption that the bank can only invest into one project and can not diversify the project risk might sound stark. Yet three clarifications are in place: First, our setup is isomorphic to a model where the bank invests into an *optimally* diversified portfolio of investments but is too small to *perfectly* diversify its portfolio. The binary payoff is then to be interpreted as the portfolio's expected payoff conditional on default or repayment respectively. Second, if the bank was able to perfectly diversify risk, then limited liability would become meaningless and we would have a model without financial frictions. Third, we don't allow the bank to buy the safe asset. Yet this assumption is innocuous: since the banks demand a higher return on investment than the households due to the equity premium, banks wouldn't purchase the safe asset even if they could.

$$\left(\omega_1 - \frac{\omega_2}{2}q_t\right) \frac{r_{k,t+1} + (1-\delta)Q_{t+1} \frac{P_{t+1}}{P_t}}{Q_t}.$$

At the same time, the bank has to repay the deposit and equity providers. Using the equity ratio  $k_t$ , the total nominal repayment per dollar of funds due in  $t + 1$  in case of success is  $r_{e,t+1}k_t + r_{d,t}(1 - k_t)$ .

The bank maximizes the expected discounted value of excess profits, i.e. revenues minus funding costs, using the stochastic discount factor of the equity holders, i.e. the household. Given the success probability of  $q_t$  and the fact that the equity providers receive nothing in case of default, the bank's objective function is:

$$\max_{q_t, k_t} \beta E \left[ \frac{\Lambda_{t+1}}{\pi_{t+1}} q_t \left( \left(\omega_1 - \frac{\omega_2}{2}q_t\right) \frac{r_{k,t+1} + (1-\delta)Q_{t+1}}{Q_t} \pi_{t+1} - r_{d,t}(1 - k_t) - r_{e,t+1}k_t \right) \right]. \quad (10)$$

Note that we did not multiply the per-unit profits by the quantity of investment. In doing so we anticipate the equilibrium condition that the bank, whose objective function is linear in the quantity of investment, needs to be indifferent about the quantity of investment. The quantity will be pinned down together with the return on capital by the bank's balance sheet equation  $e_t + d_t = Q_t o_t$ , the market clearing and zero profit conditions.

The bank's problem can be solved analytically, yet the expressions get fairly complex. Therefore we derive here the solution for  $\psi = \theta = 0$ , that is without deposit insurance and with a liquidation value of 0. This simplifies the expressions but the intuition remains the same. Allowing  $\psi$  and  $\theta$  to be nonzero on the other hand is necessary to bring the model closer to the data. The solution for the general case is discussed in section 3.5.5.

To make notation more tractable we rewrite the bank's objective function (10) in real variables expressed in marginal utility units<sup>21</sup>:

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<sup>21</sup>That is we use the following definitions:  $\tilde{r}_{l,t} = E_t \left[ \Lambda_{t+1} \left( \frac{r_{k,t+1} + (1-\delta)Q_{t+1}}{Q_t} \right) \right]$ ,  $\tilde{r}_{d,t} = E_t \left[ \Lambda_{t+1} \frac{r_{d,t}}{\pi_{t+1}} \right]$ ,  $\tilde{r}_{e,t} = E_t \left[ \Lambda_{t+1} \frac{r_{e,t+1}}{\pi_{t+1}} \right]$ ,  $\tilde{R}_t = E_t \left[ \Lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right]$ ,  $\tilde{\xi}_t = E_t [\Lambda_{t+1} \xi_t]$ .

$$\omega_1 q_t \tilde{r}_{l,t} - \frac{\omega_2}{2} q_t^2 \tilde{r}_{l,t} - q_t \tilde{r}_{d,t} (1 - k_t) - q_t \tilde{r}_{e,t} k_t, \quad (11)$$

For later use we rewrite the household's no-arbitrage conditions (3) and (5) combined with the definition of the funds' returns (6) and (7) as  $\tilde{r}_{d,t} = \frac{\tilde{R}_t}{q_t}$  and  $\tilde{r}_{e,t} = \frac{\tilde{R}_t + \tilde{\xi}_t}{q_t}$ . Let us now solve the bank's problem recursively.

### 3.5.1 Second-stage problem:

At the second stage, the bank has already raised  $e_t + d_t$  funds and now needs to choose the risk characteristic of the investment  $q_t$ , such that equity holders' utility is maximized. As already mentioned, we assume that the bank cannot write contracts conditional on  $q_t$  with the depositors at stage one, since  $q_t$  is not observable to them. Therefore at the second stage the bank takes the deposit rate as given. Furthermore, since the capital structure is already determined, maximizing the excess profit coincides with maximizing the profit of equity holders. The bank's objective function is therefore:

$$\max_{q_t} \omega_1 q_t \tilde{r}_{l,t} - \frac{\omega_2}{2} q_t^2 \tilde{r}_{l,t} - q_t \tilde{r}_{d,t} (1 - k_t). \quad (12)$$

Deriving problem (12) with respect to  $q_t$  yields the following first-order condition<sup>22</sup>:

$$q_t = \frac{\omega_1 \tilde{r}_{l,t} - \tilde{r}_{d,t} (1 - k_t)}{w_2 \tilde{r}_{l,t}}. \quad (13)$$

### 3.5.2 First-stage problem:

At the point of writing the deposit contract at stage one, depositors anticipate the bank's choices at stage two and therefore the depositors' no arbitrage condition  $\tilde{r}_{d,t} = \frac{\tilde{R}_t}{q_t}$  must hold in equilibrium.<sup>23</sup> Using this equation together with the previous first-order condition (13) we can derive the optimal  $q_t$  as a function of  $k_t$  and

<sup>22</sup>We focus on interior solutions and choose the larger of the two roots, which is the closest to the optimum, as we will see below.

<sup>23</sup>Note that the agency problem arises from the fact that the bank does not consider this as a constraint of its maximization problem.

$\tilde{r}_{l,t}$ :

$$\hat{q}_t \equiv q_t(k_t) = \frac{1}{2\omega_2\tilde{r}_{l,t}} \left( \omega_1\tilde{r}_{l,t} + \sqrt{(\omega_1\tilde{r}_{l,t})^2 - 4\omega_2\tilde{r}_{l,t}\tilde{R}_t(1-k_t)} \right) . \quad (14)$$

We can now solve the first-stage problem of the banker. The bank chooses the capital structure  $k_t$  to maximize her excess profits, anticipating the  $q_t(k_t)$  that will be chosen at the second stage:

$$\max_{k_t} \hat{q}_t\omega_1\tilde{r}_{l,t} - \frac{\omega_2}{2}\tilde{r}_{l,t}\hat{q}_t^2 - q_t\tilde{r}_{d,t}(1-k_t) - q_t\tilde{r}_{e,t}k_t , \quad (15)$$

subject to the no-arbitrage condition for depositors ( $\tilde{r}_{d,t} = \frac{\tilde{R}_t}{q_t}$ ) and for equity providers ( $\tilde{r}_{e,t} = \frac{\tilde{R}_t + \tilde{\xi}_t}{q_t}$ ). Plugging these in and deriving, we obtain the first-order condition for  $k_t$ :

$$\omega_1\tilde{r}_{l,t}\frac{\partial\hat{q}_t}{\partial k_t} - \tilde{\xi}_t - \frac{\omega_2}{2}\tilde{r}_{l,t}\frac{\partial\hat{q}_t^2}{\partial k_t} = 0 . \quad (16)$$

which (assuming an interior solution) can be solved for  $k_t$  as:

$$\hat{k}_t \equiv k_t(\tilde{r}_{l,t}) = 1 - \frac{\tilde{\xi}_t(\tilde{R}_t + \tilde{\xi}_t)(\omega_1\tilde{r}_{l,t})^2}{\omega_2\tilde{R}_t\tilde{r}_{l,t}(\tilde{R}_t + 2\tilde{\xi}_t^2)} . \quad (17)$$

### 3.5.3 Closing the bank model: the zero-profit condition

Since there is a continuum of identical banks, each bank behaves competitively and takes the return on investment  $\tilde{r}_{l,t}$  as given. Perfect competition and free entry imply that banks will enter until there are no expected excess profits to be made. In the presence of uncertainty it is natural to focus on the case that banks make no excess profit in any future state of the world:

$$\left( \omega_1 - \frac{\omega_2}{2}q_{t-1} \right) \left( \frac{r_{k,t} + (1-\delta)Q_t}{Q_{t-1}} \right) - \frac{r_{d,t-1}}{\pi_t}(1 - \hat{k}_{t-1}) - \frac{r_{e,t}\hat{k}_{t-1}}{\pi_t} = 0 . \quad (18)$$

Using the equity and deposit supply schedules and taking expectation over this equation we get:

$$\hat{q}_t\omega_1\tilde{r}_{l,t} - \frac{\omega_2}{2}\tilde{r}_{l,t}\hat{q}_t^2 - \hat{k}_t\tilde{\xi}_t - \tilde{R}_t . \quad (19)$$

Combining (19) with the optimality conditions (14) and (17), we can derive analytical expressions for the equity ratio  $k_t$ , riskiness choice  $q_t$  (the last term in each row is an approximation under certainty equivalence and  $R_t^r \equiv R_t/E[\pi_{t+1}]$ ):

$$k_t = \frac{\tilde{R}_t}{\tilde{R}_t + 2\tilde{\xi}_t} \approx \frac{R_t^r}{R_t^r + 2\xi_t} \quad (20)$$

$$q_t = \frac{\omega_1(\tilde{\xi}_t + \tilde{R}_t)}{\omega_2(2\tilde{\xi}_t + \tilde{R}_t)} \approx \frac{\omega_1(\xi_t + R_t^r)}{\omega_2(2\xi_t + R_t^r)} \quad (21)$$

### 3.5.4 Properties of the banking sector equilibrium

These results for the banking sector risk choice have five interesting implications that we first summarize in a proposition, before intuitively discussing them in turn.

**Proposition 1:** Be  $[\tilde{r}_{l,t}, q_t, k_t]$  an equilibrium in the banking sector with interior bank choices under perfect competition. Denote the expected return on investment in capital units by  $f(q_t) \equiv (\omega_1 - \frac{\omega_2}{2}q_t)q_t$ . Then:

- (1) Risk decreases in the real interest rate:  $\frac{\partial q_t}{\partial R_t} > 0$ .
- (2) The equity ratio increases in the real interest rate:  $\frac{\partial k_t}{\partial R_t} > 0$ .
- (3) Risk taking is excessive:  $q_t < \text{argmax } f(q_t)$ .
- (4) The expected return of an investment increases in the real interest rate:  $\frac{\partial f(q_t)}{\partial R_t} > 0$ .
- (5) The expected return of an investment is a concave function of the real interest rate  $\frac{\partial^2 f(q_t)}{\partial R_t^2} < 0$ .

The proof can be found in appendix D.

The first two results can be easily seen from equations (20) and (21). As the real risk-free rate  $R_t^r$  decreases, the equity ratio  $k_t$  falls as banks substitute equity with deposits and the riskiness of the bank increases ( $q_t$  falls).<sup>24</sup> The intuition

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<sup>24</sup>At least under certainty equivalence or up to a first order approximation, when the  $\Lambda_{t+1}$  terms contained in the tilde variables cancel out.

behind this result is as follows: On one hand, a lower risk-free rate decreases the rate of return on capital projects, reducing the benefits of safer investments, conditional on repayment. This induces the bank to adopt a riskier investment technology. On the other hand, the lower risk-free rate reduces the cost of funding, leaving more resources available to the bank's owners in case of repayment: this force contrasts the first one, making safer investments more attractive. There is a third force: a lower risk-free interest rate means that the equity premium becomes relatively more important. As a result the bank shifts from equity to deposits, internalizing less the consequences of the risk decision and choosing a higher level of risk. The first and third effects dominate, and overall a decline in the real interest rate induces banks to choose more risk. Notice that these two results depend on the assumption that the (discounted) equity premium is independent of the (discounted) real interest rate. If we allowed the equity premium to be a function  $\tilde{\xi}_t(\tilde{R}_t)$  of the real interest rate, the result would continue to hold under the condition that  $\tilde{\xi}_t(\tilde{R}_t) > \tilde{\xi}'_t(\tilde{R}_t)\tilde{R}_t$ , which rules out proportionality. This mechanism provides a rationalization of the empirical finding in section 2: that a decline in the nominal interest rate<sup>25</sup> causes an increase in bank risk taking behavior.

The third result implies that the bank's investment could have a higher expected return (in units of capital) if the bank chose a higher level of safety. In other words, risk taking is excessive, i.e. suboptimally high. This is due to the agency problem, which arises from limited liability and the lack of commitment/contractability of the banker regarding his risk choice. The importance of this friction can be assessed by comparing the solution of the imperfect markets bank model to the solution of the model without any frictions. The frictionless risk choice can be derived under any of the following alternative scenarios: Either both equity premium and deposit insurance are zero (which eliminates the cost disadvantage of equity and leads to 100% equity finance), or contracts are complete and deposit insurance is zero (which eliminates the agency problem and leads to 100% deposit finance), or liability is not limited and deposit insurance is zero (as before), or household invests directly into a diversified portfolio of capital projects

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<sup>25</sup>In a monetary model, a cut in the real interest rate, the standard monetary policy tool, is followed by a decline in the real interest rate due to price stickiness.



(which eliminates the financial sector all together). Since in a frictionless model  $q_t$  is chosen to maximize the consumption value of the expected return:

$$\max_{q_t} \tilde{r}_{l,t} (\omega_1 - \frac{\omega_2}{2} q_t) q_t ,$$

the optimal level of  $q_t$  trivially is  $q_t^o = \frac{\omega_1}{\omega_2}$ . Comparing the frictionless risk choice  $q_t^o$  and the choice given the friction  $q_t^f$

$$q_t^f = q_t^o \frac{\tilde{\xi}_t + \tilde{R}_t}{2\tilde{\xi}_t + \tilde{R}_t} \approx q_t^o \frac{\xi_t + R_t^r}{2\xi_t + R_t^r} ,$$

we observe that the agency friction drives a wedge between the frictionless and the actually chosen risk level. This wedge has two important features. First, it is smaller than one,<sup>26</sup> implying that under the agency problem the probability of repayment is too low, and hence banks choose excessive risk. Second, note that the wedge depends on  $R_t^r$  and that the derivative of the first order approximation of the wedge w.r.t.  $R_t^r$  is positive. This implies that the wedge increases, i.e. risk taking gets more excessive, as the real interest rate falls. As we move further away from the optimal level of risk the expected return on investments necessarily falls, which is the fourth result above.

Note that this feature of the model is consistent with the empirical finding of Ioannidou et al. [2014] and Buch et al. [2014] that the additional loan risk taking spurred by low interest rates is not fully compensated by a sufficient increase in the return on loans: As  $q_t$  decreases,  $(\omega_1 - \frac{\omega_2}{2} q_t)$  increases but not sufficient to avoid a drop in  $(\omega_1 - \frac{\omega_2}{2} q_t) q_t$ .

But not only the bank risk choice is suboptimal. Also the capital structure is chosen suboptimally, given the equity premium. If banks could commit to choose the optimal level of risk, they would not need any skin in the game. Hence they would avoid costly equity and would finance themselves fully by deposits:  $k_t^o = 0$ . Instead they choose  $k_t^f = \frac{\tilde{R}_t}{\tilde{R}_t + 2\tilde{\xi}_t}$ . The equity ratio resembles the two features of the risk taking. First, there is excessive use of equity funding. Second, the equity ratio is increasing in  $R_t^r$  up to a first order approximation.

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<sup>26</sup>This is true under certainty equivalence, i.e. up to first order approximation.

Both the risk and the capital structure choice have welfare implications. A marginal increase in  $q_t$  means a more efficient risk choice, i.e. a higher expected return, and hence should be welfare improving, *ceteris paribus*. At the same time a marginal increase in  $k_t$  implies, due to the equity premium, a higher markup in the intermediation process, which distorts the consumption savings choice and hence lowers welfare, *ceteris paribus*. Since both  $q_t$  and  $k_t$  are increasing functions of the real interest rate, this begs the question of whether an increase in the real rate alleviates or intensifies the misallocation due to the banking friction.<sup>27</sup> The answer to this question depends on the full set of general equilibrium conditions. Given the estimated model, we will later numerically verify that the positive first effect dominates, i.e. an increase in  $R_t^r$  has welfare improving consequences on the banking market.<sup>28</sup> The existence of these opposing welfare effects motivates our optimal policy experiments in section 5.

Finally, the last statement of the proposition implies that a mean preserving increase in the variance of the real interest rate decreases the mean of the expected return of the banks investment. This has implications for optimal monetary policy. As we discuss in detail later, the monetary authority cannot affect the nonstochastic steady state of the real rate, but it can influence its volatility. The policy maker therefore has an incentive to keep the real interest rate stable, at least as long as the opposing effect of the equity premium is negligible.

### 3.5.5 Full model with deposit insurance and liquidation value

The simplified version of the bank's problem presented so far is useful to explain the basic mechanism. Yet deposit insurance and a non zero liquidation value are important to improve the quantitative fit of our model to the data.

The assumptions made about deposit insurance and the liquidation value imply that depositors get the maximum of the amount covered by deposit insurance and

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<sup>27</sup>These two opposing forces are well known from the literature on bank capital regulation, where a raise in capital requirements hampers efficient intermediation but leads to a more stable banking sector.

<sup>28</sup>The dominance of the risk-taking effect is intuitive for two reasons: First, while risk taking entails a real cost, the equity premium just entails a wedge but no direct real costs. Second, as the real interest rate increases the equity premium becomes less important, so a more efficient allocation is intuitive.

the value of the capital recovered from a failed project. That means that their return in case of default is:

$$\min \left( \frac{r_{d,t}}{\pi_{t+1}}, \max \left( \frac{r_{k,t+1} + (1-\delta)Q_{t+1}}{Q_t(1-k_t)} \frac{\theta}{1-k_t}, \frac{\psi}{1-k_t} \right) \right).$$

To make deposit insurance meaningful we assume that the liquidation value  $\theta$  is small enough such that  $\frac{r_{k,t+1} + (1-\delta)Q_{t+1}}{Q_t(1-k_t)} \frac{\theta}{1-k_t} < \frac{\psi}{1-k_t}$ , which eliminates the inner maximum.<sup>29</sup> As the following lemma, proven in appendix D, states, the outer maximum is unambiguous in equilibrium.<sup>30</sup>

**Lemma:** There can be no equilibrium such that the insurance cap is not binding, i.e.  $\frac{r_{d,t}}{\pi_{t+1}} > \frac{\psi}{1-k_t}$ .

Deposits therefore pay  $\frac{\psi}{1-k_t}$  in case of default. Combining the nominal return on the the deposit funds (6) with the households no-arbitrage condition, and defining  $\tilde{\psi}_t = E[\Lambda_{t+1}] \psi$ , we can write the deposit supply schedule as:

$$q_t \tilde{r}_{d,t} + (1-q_t) \frac{\tilde{\psi}_t}{1-k_t} = \tilde{R}_t. \quad (22)$$

We assume that the deposit insurance scheme, which covers the gap between the insurance cap and the liquidation value for the depositors of failing banks, is financed through a variable tax on capital that is set ex post each period such that the insurance scheme breaks even. The return on loans  $\tilde{r}_{l,t}$  can then be rewritten as:

$$\tilde{r}_{l,t} \equiv E_t \left[ \frac{r_{k,t+1} + (1-\delta)Q_{t+1} - \tau_{t+1}}{Q_t} \right] \quad \text{where} \quad \tau_t = \frac{Q_{t-1} \frac{1-q_{t-1}}{q_{t-1}} \left( \psi - \theta \frac{r_{k,t} + (1-\delta)Q_t}{Q_{t-1}} \right)}{\omega_1 - \frac{\omega_2}{2} q_{t-1}}$$

<sup>29</sup>In principle the fact that the return on capital is determined only one period later implies that we could have cases where this inequality is satisfied for some states of the world and violated for others. Since we will later approximate our model locally around the steady state, which allows us to consider only small shocks, we abstract from this complication. Note that this simplification is quantitatively unimportant if shocks are small and the difference between the LHS and the RHS is big in steady state.

<sup>30</sup>For this result we again abstract from the effect of uncertainty. See the previous footnote.

This way the tax also perfectly offsets the distortion on the quantity of investment caused by the deposit insurance. Deposit insurance therefore influences only the funding decision of the bank and, through that, the risk choice. Hence, if  $q_t$  was chosen optimally (or was simply a parameter) the deposit insurance would not have any effect.

The same procedure as outlined above can be applied to obtain closed-form solutions<sup>31</sup> for the risk choice and the equity ratio. The solutions can be found in appendix C. As state below in proposition 2, the equilibrium characterizations in subsection 3.5.4 remain valid. In particular note that the deviation of the chosen risk (equity ratio) from the optimal level decreases (increases) in the real interest rate. Given our estimation, the risk effect dominates in terms of welfare implications. The intuition for the risk-taking channel is similar as before.

Deposit insurance makes deposits cheaper relative to equity: As a result, the bank will demand more deposits and choose a riskier investment portfolio. Deposit insurance furthermore strengthens the risk-taking channel, which is now affected not only by the importance of the equity premium relative to the real interest rate, but also by the importance the the deposit insurance cap relative to the real interest rate. On the other hand, the efficient risk level is not affected by the deposit insurance.

The liquidation value on the other hand is irrelevant for the banks' and investors' choice since it is assumed to be smaller than deposit insurance. Yet it eases the excessiveness of risk taking since it increases the optimal level of risk:

$$q_t^o = \frac{\omega_1 - \theta}{\omega_2}.$$

Finally, we would like to point out that none of the results in proposition 1 is due to the functional form that we have assumed for the risk return trade-off.

The statement holds even for a generic function  $f(q_t)$ <sup>32</sup> under relatively weak assumptions, some of which are sufficient but non necessary. For a proof and a discussion of these assumptions see appendix D.

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<sup>31</sup>In this case, one needs to apply the adjusted deposit supply schedule (22) and to make sensible assumptions about the relative size of parameters and about the root when solving the zero-profit equation.

<sup>32</sup>Given the recovery value  $f(q_t)$  now describes the expected return *conditional on success*.

**Proposition 2:** Consider proposition 1, but replace  $f(q_t)$  by the expected return taking into account the liquidation value of failed projects:  $f(q_t) + (1 - q_t)\theta$ .

- (1) Given this adjustment, all statements of proposition 1 hold for the full bank model with deposit insurance and a small enough liquidation value as well.
- (2) Given this adjustment, statements (1)-(4) of proposition 1 hold for a generic conditional expected return function  $f(q_t)$  with deposit insurance and a small enough liquidation value under the additional assumptions that  $f(q_t)$  satisfies  $f(q_t) \geq 0$ ,  $f''(q_t) < 0$ ,  $f'''(q_t) \leq 0$ ,  $f''''(q_t) \leq 0$ . Statement (5) holds if furthermore either the default probability is low relative to the parameters  $\frac{q_t}{(1-q_t)}\tilde{\xi}_t \geq \tilde{R}_t - \tilde{\psi}_t$  or there is no deposit insurance  $\tilde{\psi}_t = 0$ .

### 3.6 Monetary and fiscal policy

The central bank follows a nominal interest rate rule, targeting inflation and output deviations from the steady state:

$$R_t - \bar{R} = (1 - \rho) (\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t) + \rho (R_{t-1} - \bar{R}) + \varepsilon_t^R, \quad (23)$$

where  $\rho$  is a smoothing parameter, the hat symbol denotes percentage deviations from the steady state values,  $\bar{R} = \frac{\pi_{ss}}{\beta}$  is the steady state nominal interest rate, and  $\varepsilon_t^R$  is a monetary policy shock. In addition, the fiscal authority finances a stochastic expenditure stream  $g_y \bar{Y} \varepsilon_t^G$ :

$$\ln(\varepsilon_t^G) = \rho_g \ln(\varepsilon_{t-1}^G) + u_t^G + \rho_{GA} u_t^A,$$

where we are allowing for a correlation between exogenous spending and innovations to total factor productivity.<sup>33</sup> For simplicity we rule out government debt

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<sup>33</sup>This is a shortcut to take exports into account. Productivity innovations might rise exports in the data, and a way to capture it in a closed-economy model such as ours is to allow for  $\rho_{GA} \neq 0$  as in Smets and Wouters [2007].

( $s_t = 0$ ), implying that all expenditures are financed by lump sum taxes; i.e.  $g_y \bar{Y} \varepsilon_t^G = T_t$ .

## 4 Steady-state and dynamic implications the risk taking channel in the estimated model

We have embedded our risk-taking channel in a medium-scale model which closely resembles the non-linear version<sup>34</sup> of Smets and Wouters [2007] and we next estimate the model parameters using Bayesian techniques. This serves two purposes. First, in order to perform a sound monetary policy evaluation we need a quantitative model that is able to replicate key empirical moments of the data. Second, it helps to understand whether our channel is quantitatively important compared to other monetary and real frictions that affect the monetary policy trade-off.

In this section we first discuss the estimation, and then examine the steady-state and dynamic macroeconomic implications of the risk-taking channel, before we turn to optimal monetary policy in section 5.

### 4.1 Model estimation

We estimate a linearized version of the model with Bayesian techniques using eight US macroeconomic time series covering the period of the great moderation from 1984q1 to 2007q3. These include the seven series used by Smets and Wouters [2007], i.e. the federal funds rate, the log of hours worked, inflation and the growth rates in the real hourly wage and in per-capita real GDP, real consumption, and real investment. To identify the banking sector parameters we add a series of the banking sector equity ratio, which we construct from aggregate bank balance-sheet data provided by the FDIC. For a full description of the data we refer to appendix A and to the supplementary material of Smets and Wouters [2007]. The observation equations, linking the observed time series to the variables in the model, as well

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<sup>34</sup>Our model deviates from Smets and Wouters [2007] only to the extent that we abstract (for simplicity) from capital utilization, shown by the authors to be of secondary importance once wage stickiness is taken into account, and growth. Furthermore, since we use one additional time series we have added a time preference shock and reinterpreted it as an equity premium shock that affects only bank equity.

as the prior specifications can be found in appendix C. While the priors of the non-bank parameters follow Smets and Wouters [2007], the priors for the banking sector parameters are motivated by historical averages and external estimates for the US. Note that, instead of forming priors directly about  $\omega_2$  (risk return trade-off) and  $\psi$  (deposit insurance), we rewrite these parameters as functions of the steady state equity ratio  $\bar{k}$  and default rate  $\bar{q}$ . The prior mean of the steady state equity premium  $\xi$  is centered around an annualized value of 6%, in line with the empirical estimates of Mehra and Prescott [1985], while the prior distribution for  $\bar{k}$  is diffuse and centered around the historic mean of 12%. The prior for the liquidation value is set such that the prior value is contained between 0.3 and 0.7 with 95% probability, in line with the evidence provided by Altman et al. [2003]. The default rate  $\bar{q}$  is not well identified and is therefore fixed to 0.99, which implies an annual default rate of 4%, roughly in line with the historical average of delinquency rates on US business loans. Sensitivity tests have moreover shown that this parameter is only of small quantitative relevance.<sup>35</sup> Lastly, we normalize the units of capital versus final goods by setting  $\omega_1$  (return of most risky asset) such that one unit of final good is expected to produce one unit of capital good in steady state.

Table 1 summarizes the posterior parameter values, which are broadly in line with existing empirical estimates for the US. The steady state inflation rate is estimated to be about 2.5% on an annual basis, while the posterior mean of the discount factor implies an annual steady-state real interest rate of around 1.7%. Wages are slower moving than prices: wages are reoptimized every year and a half, while prices are reoptimized approximately every three quarters. The coefficient of relative risk aversion  $\sigma_c$  is estimated to be 1.7, above its prior mean. The posterior estimates of the Taylor rule parameters show a strong response to inflation (1.87), a small response to output (0.02), and a high degree of interest rate smoothing (0.84).

The key banking sector parameters that determine the importance of the risk-taking channel are well identified by the data. The steady state equity ratio has a

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<sup>35</sup>In particular, the implications for optimal monetary policy behavior are very robust to the value of the steady state default rate. What matters is the importance of the channel over the business cycle, determined by the liquidation value and the extent of deposit insurance.

tight posterior around 12%, the posterior mean of the equity premium is around an annualized value of 9%, and the liquidation value is about 74%.<sup>36</sup> For the following quantitative analysis we set the parameters to their posterior means.

## 4.2 Steady-state and dynamic implications

In Table 2 we compare the non-stochastic steady state of the model with banking frictions (henceforth bank model) with that of the model without banking frictions. In the latter model the capital structure is undetermined and risk is equal to the socially optimal level. For the given set of estimated values, the optimum is a corner solution:  $q^o = 1$ . In the bank model, the capital ratio is below one, implying that banks do not fully internalize the implications of their risk choice, and hence choose an excessive level of risk. This implies that the capital production technology is inefficient. Consequently, the bank economy is under-capitalized in the steady state, and output, consumption and welfare are inefficiently low.

To understand the dynamic effects of the risk-taking channel, we assess how the propagation mechanism of the model differs if a risk-taking channel is present. For illustration, we discuss an expansionary monetary policy shock. As we have just seen, the economy without financial frictions and the bank economy have different steady states. This makes dynamic comparisons of the two models difficult, since both the different behaviors of  $q_t$  and  $k_t$  as well as the different steady states imply different dynamics. In order not to mix the two effects, we focus on comparing models with the same steady state. For this purpose we alter the model without financial frictions by treating the risk choice  $q_t$  and the equity ratio  $k_t$  as parameters, which we set to the steady state values of the bank model. This model, henceforth benchmark model, not only has the same steady state as the bank model but also corresponds to a standard New Keynesian model with a small markup in capital markets.

In figure 2 we compare the dynamic responses in the bank model (solid red lines)

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<sup>36</sup>The implied mean value for deposit insurance cap  $\psi$  of about 88% implies that 99% of deposits are insured in steady state. Demirgüç-Kunt et al. [2005] report that the explicit deposit insurance scheme in the US is estimated to cover between 60% and 65% of deposits. The divergence can be interpreted as implicit deposit guarantees resulting from the expectation of bailouts. The implied mean values of  $\omega_1$  (1.13) and  $\omega_2$  (0.2561) yield a corner solution for  $q^{opt}$  at 1.



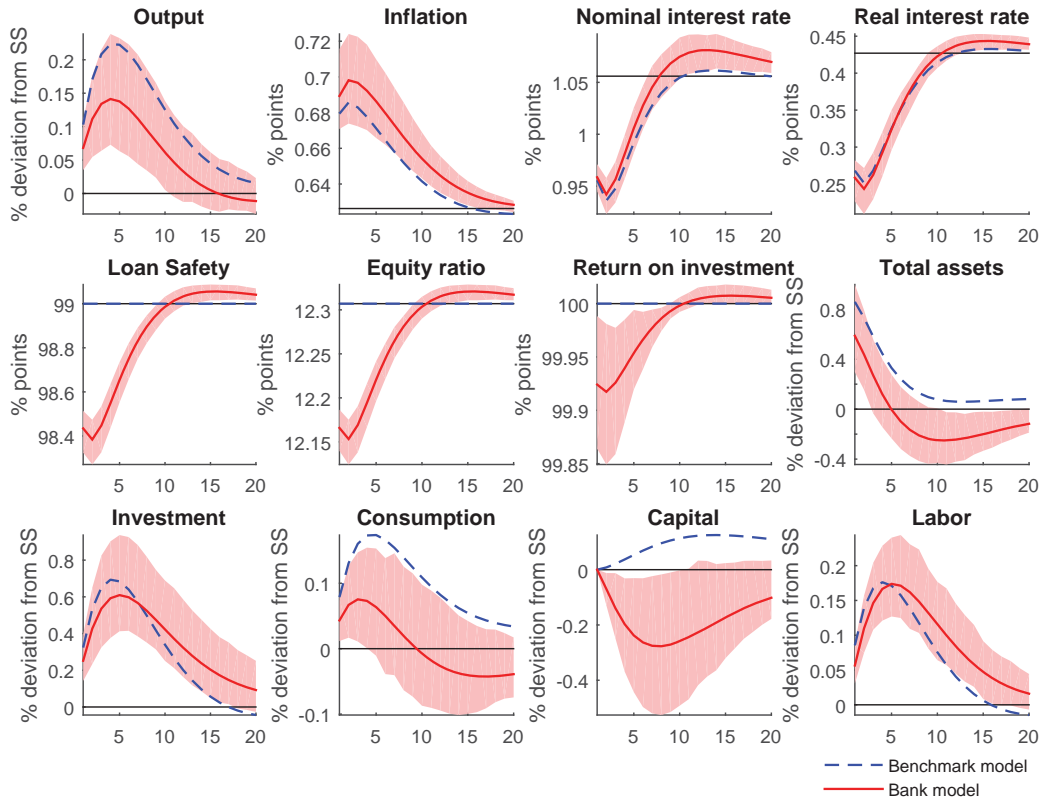
Table 1: Model estimation: prior and posterior values

	parameter	prior shape	prior mean	prior std	post. mean	90% HPD interval	
<b>structural parameters</b>							
$\mu_y$	trend growth	norm	0.4	0.1	0.4264	0.3908	0.4618
$\mu_l$	labor normalization	norm	0	2	-0.0938	-1.6569	1.4777
$\alpha$	output share	norm	0.3	0.05	0.2001	0.1602	0.2395
$100 \frac{1-\beta}{\beta}$	real rate in %	norm	0.25	0.1	0.427	0.2992	0.5485
$\bar{\varepsilon}^P$	price markup	norm	1.25	0.12	1.5068	1.3621	1.6523
$\bar{\pi}$	inflation in %	gamma	0.62	0.1	0.6263	0.4893	0.7616
$\phi_\pi$	TR weight on inflation	norm	1.5	0.25	1.8723	1.5489	2.2003
$\phi_y$	TR weight on output	norm	0.12	0.05	0.0198	-0.0348	0.0753
$\rho$	TR persistence	beta	0.75	0.1	0.8411	0.8057	0.8768
$\kappa$	investment adj. costs	norm	4	1.5	7.4584	5.5992	9.3376
$\iota$	habits	norm	0.7	0.1	0.7774	0.7042	0.8532
$\sigma_c$	risk aversion	gamma	1.5	0.375	1.7362	1.2809	2.1939
$\sigma_l$	disutility from labor	gamma	2	0.75	2.0183	0.9726	3.0566
$\lambda_p$	price calvo parameter	beta	0.5	0.1	0.6206	0.5429	0.701
$\lambda_w$	wage calvo parameter	beta	0.5	0.1	0.8476	0.8099	0.8864
$\gamma_p$	price indexation	beta	0.5	0.15	0.1533	0.0537	0.2479
$\gamma_w$	wage indexation	beta	0.5	0.15	0.448	0.2066	0.6829
$\xi$	equity premium	norm	0.015	0.01	0.0213	0.0054	0.0348
$\theta$	liquidation value	norm	0.5	0.1	0.7416	0.6425	0.8385
$\bar{k}$	equity ratio	norm	0.12	0.05	0.1231	0.1208	0.1254
<b>structural shock processes</b>							
$\sigma_A$	stdev TFP	unif	0	10	0.3665	0.3172	0.414
$\sigma_B$	stdev preference	unif	0	10	3.4696	2.2271	4.6946
$\sigma_G$	stdev govt. spending	unif	0	10	2.2678	1.984	2.5382
$\sigma_I$	stdev investment	unif	0	10	4.7269	3.0495	6.3757
$\sigma_P$	stdev price markup	unif	0	1	0.1332	0.109	0.1574
$\sigma_R$	stdev monetary	unif	0	1	0.1164	0.1009	0.1315
$\sigma_W$	stdev wage markup	unif	0	10	0.4742	0.4088	0.5389
$\sigma_\xi$	stdev equity premium	unif	0	10	0.5805	0.199	1.0255
$\rho_A$	persistence TFP	beta	0.5	0.2	0.4623	0.3496	0.5765
$\rho_B$	persistence preference	beta	0.5	0.2	0.9004	0.8549	0.9486
$\rho_G$	persistence gov. spending	beta	0.5	0.2	0.9009	0.8471	0.9556
$\rho_I$	persistence investment	beta	0.5	0.2	0.1924	0.0357	0.3396
$\rho_P$	persistence price markup	beta	0.5	0.2	0.9772	0.9625	0.9925
$\rho_R$	persistence monetary	beta	0.5	0.2	0.9585	0.918	0.9967
$\rho_W$	persistence wage markup	beta	0.5	0.2	0.7721	0.6706	0.8734
$\rho_\xi$	persistence equity premium	beta	0.5	0.2	0.8156	0.7623	0.8699
$\rho_{G,A}$	correlation gov. spending & TFP	beta	0.5	0.2	0.6513	0.3835	0.9394
$m_p$	MA component of price markup	beta	0.5	0.2	0.7765	0.6826	0.875
$m_w$	MA component of wage markup	beta	0.5	0.2	0.9741	0.9516	0.9972

Table 2: **Steady state comparison:** The model without banking sector frictions features an undetermined equity ratio and risk equal to the socially optimal level; i.e.  $\bar{q}^o = 1$ . Parameters are fixed to the posterior mean estimates of the bank model reported in table 1.

VARIABLE	MODEL WITH BANKING FRICTIONS	MODEL WITHOUT BANKING FRICTIONS
q loan safety	0.99	1
k equity ratio	0.1231	0
Y output	0.9484	0.9803
C consumption	0.6289	0.6376
I investment	0.1488	0.1662
K capital	5.9517	7.0227
L labor	1	1
$\pi$ inflation	0.0063	0.0063
R interest rate	0.0106	0.0106

Figure 2: **Monetary policy shock in the bank and benchmark models:** dynamic responses in the bank model (solid red lines) and in the benchmark model (dashed blue lines) to an expansionary monetary policy shock, at the mean of the posterior distribution. Shaded areas denote the highest posterior density interval at 90% for the bank model impulse responses and the black line the steady state level. Inflation and interest rates are quarter on quarter rates.



and in benchmark model (dashed blue lines) to an expansionary monetary policy shock. A monetary policy expansion triggers a set of standard reactions, which are evident in the benchmark model. An unexpected fall in the nominal risk-free rate causes a drop in the real interest rate, since prices are sticky. Consequently, consumption is shifted forward, firms that can adjust the price do so causing an increase in inflation, while the remaining firms increase production. The risk-taking channel adds two further elements as both the risk level and the capital structure chosen by the bank respond to the real interest rate movement. On impact, the drop in the real interest rate cause banks to substitute equity for deposits, since the relative cost advantage of deposits increases. Consequently, banks have less skin in the game and hence take more risk (lower loan safety). The risk choice therefore moves further away from the optimal level and the expected return on aggregate investment  $f(q_t) = q_t(\omega_1 - \omega_2/2q_t) + (1 - q_t)\theta$  drops.<sup>37</sup> To maintain the same path of capital as in the benchmark case, households would have to invest more and consume less. Yet this would not be optimal because of consumption smoothing and because of the lower expected return on investment. Therefore investment rises by less than what would be needed to compensate the loss in investment efficiency, which makes the capital stock decline considerably. Overall, agents are worse off (in terms of welfare) in the bank model than in the benchmark economy.

We conclude this section with a few remarks on the fit of the estimated model. Comparing our bank model to the benchmark Smets and Wouters [2007]-type model we find that the parameter estimates are rather similar and the fit of the two models is comparable.<sup>38</sup> The posterior odds ratio of  $\exp(2.86)$  favors the bank model, though it is close but not above the value of  $\exp(3)$ , which, according to Jeffreys [1961], can be interpreted as conclusive evidence.

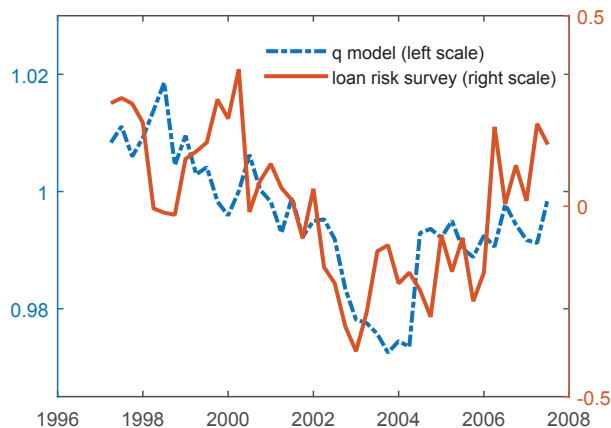
To evaluate the fit particularly with respect to the risk-taking channel we look

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<sup>37</sup>Note that the decline in the equity ratio diminishes the distortion due to the equity premium, which reduces the cost of capital. Yet this effect is tiny relative to the increase in the cost of capital due to lower investment efficiency.

<sup>38</sup>Recall that the Smets and Wouters [2007] model is obtained by turning off the banking sector frictions. Hence bank leverage is no longer defined. For the comparison we therefore estimate the two versions of the model (with and without the banking frictions) using only the seven macro aggregates used by Smets and Wouters [2007], and calibrate the banking parameters in the bank model to the posterior estimates in table 1.

Figure 3: **Risk taking in the model and in the data:** The figure compares the value of loan safety  $q_t$  implied by the estimated model (in particular we plot the mean of the series posterior distribution) with the risk index discussed in section (2)



at three statistics that were not targeted by the estimation. First, note that the responses shown in figures 2 are in accordance with the structural VAR results in section 2, in particular with the finding that the response of risk is proportional to that of the interest rate, even though our model displays a higher degree of persistence. Second, we compare the model-implied series for the risk variable  $q_t$  with the risk-taking index used in the VAR analysis. Figure 3 shows that the model implies a cyclical pattern of risk that is roughly in line with the survey measure (the correlation is 60%). Third, the responses in figure 2 also show that, conditional on the monetary policy shock, leverage (the inverse of the equity ratio  $k_t$ ) is procyclical with respect to the size of the bank balance sheet  $e_t + d_t$ . Conditional on the full set of shocks we find a correlation of 43% which is in line with the evidence for US data provided by Adrian and Shin [2014] and distinguishes our model from canonical financial accelerator models that build on Bernanke et al. [1999].<sup>39</sup>

Lastly, we find that the introduction of the banking frictions reduces the role of the investment efficiency shock. In particular, we find that the forecast variance of the output level drops by a third, while the variance decomposition share of the investment shock drops from around 49% (estimated benchmark model) to

<sup>39</sup>See, for instance, the discussion in Adrian et al. [2015].

34% (estimated bank model) for horizons between 3 and 8 quarters. This relates to the argument of Justiniano et al. [2011], who find that the large role of this shock in explaining GDP volatility in the canonical medium-scaled Smets and Wouters [2007] DSGE model could be a spurious result that captures unmodeled financial frictions. In reducing the importance of this shock, the risk-taking channel proves to be capable of capturing at least some of this missing mechanism. This is intuitive because both the investment shock  $\varepsilon_t^I$  and the expected return of the banks' investment  $q_t \left( \omega_1 - \frac{\omega_2}{2} q_t \right) + (1 - q_t)\theta$  enter the capital accumulation equation multiplicatively:

$$K_t = \left[ \varepsilon_t^I (1 - S(i_t/i_{t-1})) i_t + (1 - \delta)K_{t-1} \right] \left[ q_t \left( \omega_1 - \frac{\omega_2}{2} q_t \right) + (1 - q_t)\theta \right] ,$$

Yet they are not perfectly isomorphic, since the shock affects only net investment (new capital), while the expected return on investment affects gross investment (all capital). Moreover the path of  $\varepsilon_t^I$  backed from the estimated benchmark model is strongly correlated with the path of the return on investment in the estimated bank model.<sup>40</sup>

Overall, we interpret these findings as suggestive that the additional dynamics implied by the risk-taking channel are not rejected by the data and, on the contrary, help to reduce the mismatch between the benchmark model and the data.

## 5 Monetary policy with a risk-taking channel

We have seen that the risk-taking channel has both static and dynamic effects. While monetary policy does not affect the non-stochastic steady state, it can influence the dynamics of the economy. In particular, it can influence the real rate and hence affect bank risk taking. But are these additional mechanisms implied by the risk-taking channel actually quantitatively significant for monetary

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<sup>40</sup>For this exercise we use only the 7 nonfinancial series. Notice that the specification of our model is not exactly the same as Smets and Wouters [2007] and Justiniano et al. [2011] since we have abstracted from capital utilization. This means that the numbers are not directly comparable.

policy? To answer this question we determine the *optimal simple implementable* monetary policy rule in the risk-taking channel model. We then compare this policy to the optimal policy in the benchmark economy with the same steady state but without the risk-taking channel. This comparison has an interesting interpretation. Suppose that the actual economy features the risk-taking channel (the bank model), but that the central bank is unaware of this channel and believes that risk is an irrelevant constant from her point of view. The central bank would then implement optimal policy based on a wrong model (the benchmark model). Our comparison then answers the question of how important understanding the risk-taking channel is, in terms of optimal policy and welfare.

Notice that in this paper we consider a central bank that has no policy tools besides the interest rate. With a second instrument, such as capital regulation, the central bank could do better or even eliminate the friction. Exploring optimal macroprudential regulation is however beyond the scope of the present paper.<sup>41</sup>

In what follows, we first discuss the concept of the optimal simple implementable monetary policy rule, and then present our results.

## 5.1 The central bank problem

We follow Schmitt-Grohe and Uribe [2007] and characterize optimal monetary policy as the policy rule that maximizes welfare among the class of simple, implementable interest-rate feedback rules<sup>42</sup> given by:

$$R_t - \bar{R} = \phi_\pi \hat{\pi}_{t+s} + \phi_y \hat{y}_{t+s} + \phi_k \hat{k}_{t+s} + \rho (R_{t-1} - \bar{R}) . \quad (24)$$

where the hat symbol denotes percentage deviations from the steady state, and the index  $s$  allows for forward- or contemporaneous-looking rules (respectively by setting  $s = 1$  or  $s = 0$ ). The policy rule specification (24) is chosen for its generality, as it encompasses both standard Taylor-type rules (setting  $\phi_k = 0$ ),

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<sup>41</sup>For a thorough analysis of macroprudential policy in an economy with bank risk-taking see Collard et al. [2012].

<sup>42</sup>The implementability criterion requires uniqueness of the rational expectations equilibrium, while simplicity requires the interest rate to be a function of readily observable variables. For a complete discussion, see Schmitt-Grohe and Uribe [2007]. Notice that we drop their second requirement for implementability which is that an implementable rule must avoid regular zero lower bound violations.

and the possibility that the central bank reacts to banking sector leverage, the inverse of the equity ratio  $k$  ( $\phi_k \neq 0$ ). A fall in the equity ratio implies that banks increase their debt financing, i.e. they increase leverage. As a consequence banks internalize less the downside risk of their investments, and choose loans with a higher default probability. Hence, a fall in the equity ratio signals an increase in risk taking, to which the central bank may want to respond by increasing the interest rate. We choose not to let the interest rate depend on risk taking directly, because the latter is not a readily observable variable. We furthermore impose that the inertia parameter  $\rho$  has to be non-negative. Since we are interested in the effect of systematic monetary policy, we switch off the monetary policy shock for this experiment.

The welfare criterion that defines the optimal parameter combination for rule (24) is the household's conditional lifetime utility:

$$V \equiv E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^B u(c_t, L_t) . \quad (25)$$

This measure is commonly used in the literature and yields the expected lifetime utility of the representative household, conditional on the economy being at the deterministic steady state. To be able to make meaningful comparisons of welfare levels we furthermore define the measure  $\Omega$  as the fraction of the consumption stream that a household would need to receive as a transfer under the suboptimal rule to be equally well off as under the optimal rule. If  $o$  denotes the optimal and  $s$  another suboptimal rule, this fraction  $\Omega$  is implicitly defined by the equation:

$$V^o = E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^B u((1 + \Omega)c_t^s, L_t^s) .$$

## 5.2 Findings

Using the welfare criterion just described we numerically determine the coefficients of the optimal simple implementable rules in the benchmark and in the bank model using second order approximations around the non-stochastic steady state. The first 5 rows of table 3 report the optimal coefficients for 5 different specifications of

**Table 3: Optimal simple rules:** optimal parameters for policy rules of the class  $R_t - \bar{R} = \phi_\pi \hat{\pi}_{t+s} + \phi_y \hat{y}_{t+s} + \phi_k \hat{k}_{t+s} + \rho (R_{t-1} - \bar{R})$ . The hat symbol denotes percentage deviations from the steady state. The first column describes the timing of the policy rule. The second column describes the restrictions we enforced. The first row for example corresponds to the standard Taylor rule with no smoothing. The last row is somewhat different. It reproduces line 2 for the benchmark model and in the bank model shows the optimal rule given that the first three parameters are fixed to their benchmark values.  $V$  is the welfare level associated with each policy in the bank model.  $\Omega$  is the welfare cost (in % of the consumption stream) associated to implementing in the bank model the optimal policy rule of the benchmark model (given the same restrictions on parameters). For the benchmark model the restriction  $\phi_k = 0$  is irrelevant, since the equity ratio is a constant in the benchmark model. Entries in italics indicate restricted parameters.

$s$	rule	<i>benchmark model</i>			<i>bank model</i>					
		$\rho$	$\phi_{\pi_{t+s}}$	$\phi_{y_{t+s}}$	$\rho$	$\phi_{\pi_{t+s}}$	$\phi_{y_{t+s}}$	$\phi_{k_t}$	$V$	$\Omega$
0	$\phi_k, \rho = 0$	<i>0</i>	7.100	0.115	<i>0</i>	3.080	0.126	<i>0</i>	-185.321	0.476
0	$\phi_k = 0$	0.000	7.100	0.115	1.059	0.510	0.005	<i>0</i>	-184.750	0.898
0	$\rho = 0$	<i>0</i>	7.100	0.115	<i>0</i>	2.637	0.097	0.027	-185.314	0.481
1	$\phi_k, \rho = 0$	<i>0</i>	17.222	0.148	<i>0</i>	4.294	0.172	<i>0</i>	-185.209	0.687
1	$\phi_k = 0$	0.236	12.084	0.124	1.114	0.072	0.074	<i>0</i>	-184.656	0.813
0	choose $\phi_k$	0.000	7.100	0.115	<i>0</i>	<i>7.100</i>	<i>0.115</i>	-0.177	-185.438	0.389

the monetary policy rule: contemporaneous and forward-looking, without inertia and with optimal inertia, without and with reaction to current leverage. The coefficients of the optimal rules generally vary greatly between the two models. A set of results, which are robust across policy rule and estimation<sup>43</sup> specifications, are worth noticing.

First, the optimal coefficients on inflation deviations are smaller in the bank model compared to the benchmark model. For any given change in inflation, the nominal interest rate should move less if a risk-taking channel is present. Furthermore, if the central bank can optimize over its smoothing parameter, then full interest rate smoothing is optimal in the bank model. Given that the optimal output coefficient is close to zero, the optimal rule is closer to a stable real interest rate rule in the bank model than in the benchmark model. In doing so, the central bank limits fluctuations in the real interest rate and hence in risk taking and

<sup>43</sup>We have experimented with different estimation samples and calibrated parameter values: while the optimized parameters and transfers slightly change, the qualitative results discussed in the text are very robust.



Table 4: **Differences in moments associated to the optimal simple rules in the benchmark and in the bank model:** This table shows the % differences in the mean and standard deviation associated to applying the different optimal rules in the bank model. The first entry, for example, indicates that under the optimal bank policy rule average risk would be 0.15% lower than if the rule optimal for the benchmark model had been applied.

$s$	rule	<i>mean</i>					<i>standard deviation</i>				
		$q$	$R^r$	$\pi$	$y$	$c$	$q$	$R^r$	$\pi$	$y$	$c$
0	$\phi_k, \rho = 0$	0.151	0.002	-0.051	0.311	0.499	-43.880	-47.975	52.470	-0.843	-4.108
0	$\phi_k = 0$	0.214	0.007	-0.038	0.439	0.701	-67.949	-77.760	64.393	-9.545	-9.566
0	$\rho = 0$	0.152	0.003	-0.015	0.323	0.506	-41.666	-47.248	53.194	-0.773	-3.800
1	$\phi_k, \rho = 0$	0.194	0.011	-0.037	0.413	0.652	-50.536	-55.417	57.719	-2.781	-6.906
1	$\phi_k = 0$	0.195	0.004	-0.054	0.458	0.724	-65.839	-76.3112	71.906	-10.373	-11.737
0	choose $\phi_k$	0.130	0.001	-0.070	0.244	0.417	-41.691	-41.948	31.838	3.323	-0.345

slightly raises the average level of  $q$  towards the efficient value, as it can be seen in table 4. At the same time inflation is significantly more volatile under the optimal rule. If a risk-taking channel is present, the central bank should accept higher inflation fluctuations in order to reduce the distortion stemming from risk taking.

This is because monetary policy cannot affect the deterministic steady state, but it can control the real interest rate and therefore the fluctuations in excessive risk. Upward movements of the real interest rate are welfare enhancing since they lower the level of risk taking towards the efficient level, whereas downward changes of the real interest rate lead to even more excessive risk taking. But this does not mean that movements in the interest rate are irrelevant. Since the expected return on investments  $q_t (\omega_1 - \frac{\omega_2}{2} q_t) + (1 - q_t)\theta$  is concave in the real interest rate, as we have shown above, a mean preserving increase in the volatility of the real rate reduces the average expected return of investments. Therefore the risk-taking channel provides a motive for keeping the real interest rate constant. This adds a third dimension to the central bank problem: besides trading off inflation versus output stabilization the central bank would now also like to stabilize the real interest rate. As a result, the optimal policy is tilted away from inflation stabilization.

To understand *how different* the equilibria associated to the two optimal rules are, and therefore how important it is for the central bank to take the risk-taking

channel into account, we compute the cost  $\Omega$  of applying the rule that is optimal for the benchmark model in the bank model. These costs, expressed in % of the lifetime consumption stream, are reported in the last column of table 3. Though the costs vary a lot across policy specifications, they are always non significant. For the best performing policy (fifth row of table 3), the costs of applying the benchmark policy in the bank model is around 0.81% of the lifetime consumption stream. Hence, internalizing the feedback effect that the nominal interest rate has on bank risk taking pays off in terms of welfare.

Second, including an explicit reaction to banking sector leverage, in addition to inflation and output, improves welfare only marginally (compare the last column of the first and third row of table 3). Recall that leverage depends on both the nominal interest rate and expected inflation. By setting the nominal rate optimally as a function of current inflation, the central bank can already steer risk taking, to the extent that current and expected future inflation are highly correlated. The fact that this correlation is not perfect, and that our approximation allows for nonlinearities, accounts for the small improvement in welfare obtained by allowing a response to leverage to the policy function. To further illustrate this point, in the first row we fix the coefficients of current inflation and output to the values optimal in the benchmark economy, and allow the central bank to respond optimally only to leverage. In this case, it is optimal to strongly raise the interest rate in response to higher leverage (lower equity ratio  $k$ ). Thereby the central bank again stabilizes the real interest rate and does not much worse in terms of welfare than when the responses to inflation and output are chosen optimally (compare the last column of the third and sixth row of tables 3 and 4).

## 6 Conclusion

The recent financial crisis has highlighted the importance of monitoring the level of risk to which the financial sector is exposed. In this paper we focus on one aspect of financial sector risk, ex-ante bank asset risk, and on how the latter can be influenced by monetary policy.

First, we provide new empirical evidence of the impact of monetary policy on bank risk taking. We document that unexpected monetary policy shocks, identi-

fied through sign restrictions in a classical VAR framework, increase a measure for ex-ante bank risk taking in the US. This conclusion, robust to using a recursive identification scheme, is compatible with the monetary policy transmission mechanism in the theoretical model that we build to explain the effects of monetary policy on risk taking.

For this purpose, we extend the work of Dell’Ariccia et al. [2014] and build a dynamic general-equilibrium model where low levels of the risk-free interest rate induce banks to make riskier investments. At the core of this mechanism is an agency problem between depositors and equity providers: the latter choose the level of risk but are protected by limited liability. In general equilibrium, this friction leads to a steady state with excessive risk taking, and inefficiently low levels of capital, output and consumption. Furthermore, risk taking alters the dynamic response of the economy to shocks. In particular, an expansionary monetary policy shock has non-standard consequences: because banks choose a riskier and less efficient investment strategy, the growth of capital, output and consumption will be lower than in the model without the risk-taking channel.

In order to assess the importance of the risk-taking channel and to study optimal monetary policy, we estimate the model on US data using Bayesian methods. Including this additional channel improves the in-sample fit, yields a path for risk taking that matches survey evidence for the US and implies a pro-cyclical behavior of leverage with respect to total assets which is in line with US evidence documented by Adrian and Shin [2014]. Our policy experiments using optimal simple rules suggest that, if a risk-taking channel is present and the interest rate is the only instrument available to the monetary authority, the optimal rule should stabilize the path of the real interest rate more than without the risk-taking channel. This implies that the central bank should tolerate higher inflation volatility in order to reduce welfare detrimental fluctuations in risk taking. The welfare gains of taking the channel into account are found to be significant. Nevertheless, these results do not rule out that an alternative instrument could perform better at maximizing consumer welfare, an issue that deserves to be investigated in future work.

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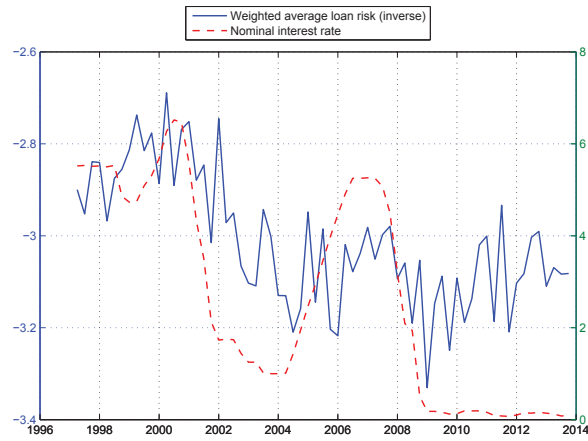
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## Appendix A: Data description

Table 5: **Data description:** All level variables are expressed in per-capita terms (divided by  $N$ ). Hours are measured as  $H_1 \cdot H_2/N$  where  $H_1$  is converted into an index. The nominal wage  $W$  is deflated by the GDP deflator. We define equity capital as equity plus reserves plus subordinated debt, and total liabilities as equity plus deposits. In doing so we net out two types of liabilities, since they are typically overcollateralized: federal funds purchased & repurchase agreements and federal home loan bank advances. Furthermore we omit a few categories of debt that match neither of our concepts of insured deposits and equity, or that are simply not well enough characterized: other borrowed money, uncategorized liabilities, trading book liabilities, banks liability on acceptances. All of these balance sheet positions are minor. Over the observation period, the first group accounts for roughly 11% of the balance sheet, the second for about 9%. All indexes are adjusted such that 2009 = 100. The estimation sample spans from 1984Q1 to 2007Q3 for the DSGE and from 1997Q2 to 2007Q3 for the VAR.

SYMBOL	SERIES	MNEMONIC	UNIT	SOURCE
$Y$	REAL GROSS DOMESTIC PRODUCT	GDPC96	BN. USD	FRED / BEA
$P$	GDP DEFLATOR	GDPDEF	INDEX	FRED / BEA
$R$	EFFECTIVE FEDERAL FUNDS RATE	FEDFUNDS	%	FRED / BOARD OF GOVERNORS
$C$	PERSONAL CONSUMPTION EXPENDITURE	PCEC	BN. USD	FRED / BEA
$I$	FIXED PRIVATE INVESTMENT	FPI	BN. USD	FRED / BEA
$H_1$	CIVILIAN EMPLOYMENT	CE16OV	THOUSANDS	FRED / BLS
$H_2$	NONFARM BUSINESS (..) HOURS	PRS85006023	INDEX	DEPARTMENT OF LABOR
$W$	NONFARM BUSINESS (..) HOURLY COMPENSATION	PRS85006103	INDEX	DEPARTMENT OF LABOR
$N$	CIVILIAN POPULATION	LNS1000000	0CE16OV	BLS
$q$	AVERAGE WEIGHTED LOAN RISK	own calculation	%	BOARD OF GOVERNORS
$E$	EQUITY CAPITAL OVER LIABILITIES	own calculation	%	FDIC

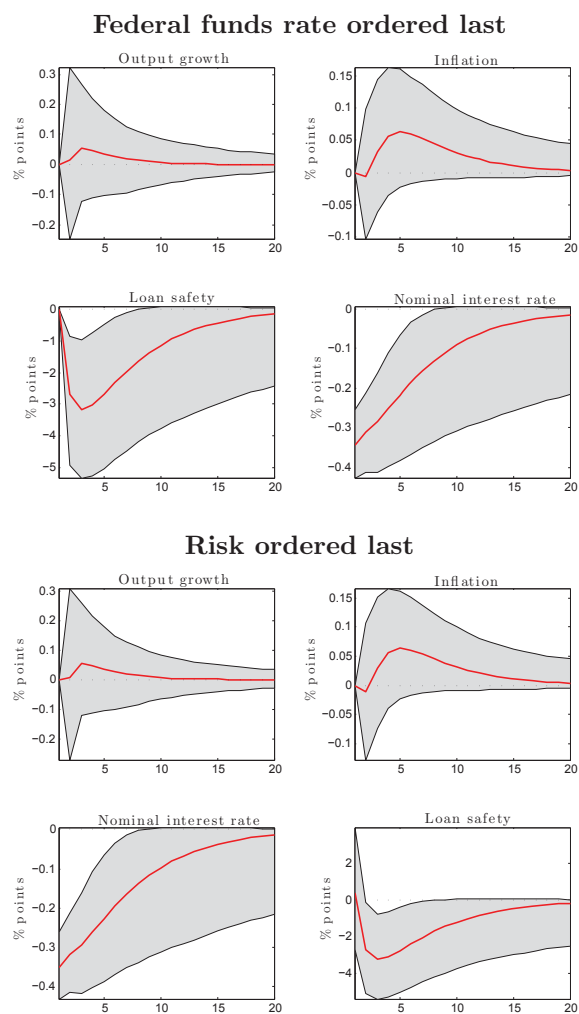
Figure 4: **Bank risk taking and nominal interest rate:** The risk measure (solid blue line, left axis) is redefined such that a decrease is associated with higher risk-taking of the banking sector, matching the definition in the theoretical model discussed later. The nominal interest rate (dashed line, right axis) is the effective federal funds rate.





## Appendix B: Empirical motivation - recursive identification scheme

Figure 5: **An expansionary monetary policy shock - Recursive identification scheme.** Error bands correspond to 90% confidence intervals obtained by bootstrap. Loan safety is defined as the inverse of the average loan risk rating, standardized to take values between 0 and 100. The remaining variables are annualized. See text for further details.



## Appendix C: The full model - Equilibrium and estimation details

**Model summary:** We report here the equations that enter the non-linear model, grouped by sector. Note that following Smets and Wouters [2007] we assume that different varieties of intermediate goods and of labor are assembled through a Kimball [1995] aggregator, rather than a Dixit-Stiglitz one. This latter assumption is introduced in order to obtain estimates of price and wage rigidity that are closer to micro estimates, but we do not derive the recursive formulation here (see e.g. Adjemian et al. [2008]). Also note that the variables regarding the equity and deposit funds  $R^d$  and  $R^e$  have been substituted out.

**Competitive equilibrium:** The competitive equilibrium is a path of 41 variables ( $\Lambda, K, L, y, l, c, q, k, d, e, \pi, r_k, r_d, r_e, R, W, mc, o^{new}, o, \pi^*, Z_{p1}, Z_{p2}, Z_{p3}, Z_{w1}, Z_{w2}, Z_{w3}, \Delta_{p1}, \Delta_{p2}, \Delta_{p3}, \Delta_{p4}, \Delta_{w1}, \Delta_{w2}, \Delta_{w3}, \Delta_{p4}, W^*, i, \tilde{R}, \xi, \tilde{\xi}, \tilde{\psi}, \tau$ ) that satisfy the following 41 equations at each point in time given initial conditions and the exogenous shock processes  $\varepsilon^A, \varepsilon^B, \varepsilon^G, \varepsilon^I, \varepsilon^P, \varepsilon^R, \varepsilon^W, \varepsilon^\xi$ .

### Household

$$\Lambda_t = \varepsilon_t^B (c_t - \iota c_{t-1})^{-\sigma_C} - \beta \iota E_t [\varepsilon_{t+1}^B (c_{t+1} - \iota c_t)^{-\sigma_C}] \quad (26)$$

$$E_t \left[ \Lambda_{t+1} \frac{q_t r_{d,t+1} + (1 - q_t) \frac{\psi}{1 - k_t} \pi_{t+1}}{\pi_{t+1}} \right] = E_t \left[ \Lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right] \quad (27)$$

$$E_t \left[ \Lambda_{t+1} \frac{q_t r_{e,t+1} - \xi_t \pi_{t+1}}{\pi_{t+1}} \right] = E_t \left[ \Lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right] \quad (28)$$

$$\Lambda_t = \beta E_t \left[ \Lambda_{t+1} \frac{R_t}{\pi_{t+1}} \right] \quad (29)$$

$$y_t = c_t + i_t + g_y \bar{Y} \varepsilon_t^G \quad (30)$$

Goods sector

$$\frac{L_t^d}{K_{t-1}} \frac{\alpha}{1-\alpha} = \frac{r_{k,t}}{w_t} \quad (31)$$

$$mc_t = \frac{1}{A_t} \alpha^{-\alpha} r_{kt}^\alpha w_t^{1-\alpha} (1-\alpha)^{\alpha-1} \quad (32)$$

$$\pi_t^* = \frac{\epsilon_p (1+k_p)}{\epsilon_p (1+k_p) - 1} \frac{Z_{p1,t}}{Z_{p2,t}} + \frac{k_p}{\epsilon_p - 1} (\pi_t^*)^{1+\epsilon_p(1+k_p)} \frac{Z_{p3,t}}{Z_{p2,t}} \quad (33)$$

$$Z_{p1,t} = (1 - \tau_{p,t}) \Lambda_t mc_t y_t \Delta_{p1,t}^{\epsilon_p(1+k_p)/(1-\epsilon_p(1+k_p))} + \beta \lambda_p E_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_p} \bar{\pi}^{1-\gamma_p}} \right)^{\epsilon_p(1+k_p)} Z_{p1,t+1} \right] \quad (34)$$

$$Z_{p2,t} = (1 - \tau_{p,t}) \Lambda_t y_t \Delta_{p1,t}^{\epsilon_p(1+k_p)/(1-\epsilon_p(1+k_p))} + \beta \lambda_p E_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_p} \bar{\pi}^{1-\gamma_p}} \right)^{\epsilon_p(1+k_p)-1} Z_{p2,t+1} \right] \quad (35)$$

$$Z_{p3,t} = \Lambda_t y_t + \beta \lambda_p E_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_p} \bar{\pi}^{1-\gamma_p}} \right)^{-1} Z_{p3,t+1} \right] \quad (36)$$

$$\Delta_{p1,t} = (1 - \lambda_p) (\pi_t^*)^{1-\epsilon_p(1+k_p)} + \lambda_p \Delta_{p1,t-1} \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_p} \bar{\pi}^{1-\gamma_p}} \right)^{\epsilon_p(1+k_p)-1} \quad (37)$$

$$1 = \frac{1}{1+k_p} \Delta_{p1,t}^{1/(1-\epsilon_p(1+k_p))} + \frac{k_p}{1+k_p} \Delta_{p2,t} \quad (38)$$

$$\Delta_{p2,t} = (1 - \lambda_p) \pi_t^* + \lambda_p \Delta_{p2,t-1} \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_p} \bar{\pi}^{1-\gamma_p}} \right)^{-1} \quad (39)$$

$$\Delta_{p3,t} = \frac{1}{1+k_p} \Delta_{p1,t}^{\epsilon_p(1+k_p)/(1-\epsilon_p(1+k_p))} \Delta_{p4,t} + \frac{k_p}{1+k_p} \quad (40)$$

$$\Delta_{p4,t} = (1 - \lambda_p) (\pi_t^*)^{-\epsilon_p(1+k_p)} + \lambda_p \Delta_{p4,t-1} \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_p} \bar{\pi}^{1-\gamma_p}} \right)^{\epsilon_p(1+k_p)} \quad (41)$$

$$A_t K_{t-1}^\alpha \left( \frac{L_t}{\Delta_{p3,t}} \right)^{1-\alpha} = \Delta_{p3,t} y_t \quad (42)$$

### Labor sector

$$w_t^* = \frac{\epsilon_w (1 + k_w)}{\epsilon_w (1 + k_w) - 1} \frac{Z_{w1,t}}{Z_{w2,t}} + \frac{k_w}{\epsilon_w - 1} (w_t^*)^{1+\epsilon_p(1+k_p)} \frac{Z_{w3,t}}{Z_{w2,t}} \quad (43)$$

$$Z_{w1,t} = \varepsilon_t^B \bar{L} L_t^{1+\sigma_L} w_t^{\epsilon_w(1+k_w)} (C_t - \iota C_{t-1})^{1-\sigma_c} \exp \left( \bar{L} \frac{\sigma_c - 1}{1 + \sigma_l} L_t^{1+\sigma_L} \right) \quad (44)$$

$$\cdot \Delta_{w1,t}^{\epsilon_w(1+k_w)/(1-\epsilon_w(1+k_w))} + \beta \lambda_w E_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_w} \bar{\pi}^{1-\gamma_w}} \right)^{\epsilon_w(1+k_w)} Z_{w1,t} \right]$$

$$Z_{w2,t} = (1 - \tau_{w,t}) \Lambda_t L_t \left[ w \Delta_{w1,t}^{1/(1-\epsilon_w(1+k_w))} \right]^{\epsilon_w(1+k_w)} + \beta \lambda_w E_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_w} \bar{\pi}^{1-\gamma_w}} \right)^{\epsilon_w(1+k_w)-1} Z_{w2,t+1} \right] \quad (45)$$

$$Z_{w3,t} = (1 - \tau_{w,t}) \Lambda_t L_t + \beta \lambda_w E_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_p} \bar{\pi}^{1-\gamma_p}} \right)^{-1} Z_{w3,t+1} \right] \quad (46)$$

$$\Delta_{w1,t} = (1 - \lambda_w) \left( \frac{w_t^*}{w_t} \right)^{1-\epsilon_w(1+k_w)} + \lambda_w \Delta_{w1,t-1} \left( \frac{w_{t-1}}{w_t} \right)^{1-\epsilon_w(1+k_w)} \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_w} \bar{\pi}^{1-\gamma_w}} \right)^{\epsilon_w(1+k_w)-1} \quad (47)$$

$$1 = \frac{1}{1 + k_w} \Delta_{w1,t}^{1/(1-\epsilon_w(1+k_w))} + \frac{k_w}{1 + k_w} \Delta_{w2,t} \quad (48)$$

$$\Delta_{w2,t} = (1 - \lambda_w) \left( \frac{w_t^*}{w_t} \right) + \lambda_w \Delta_{w2,t-1} \left( \frac{w_t}{w_{t-1}} \frac{\pi_{t+1}}{\pi_t^{\gamma_w} \bar{\pi}^{1-\gamma_w}} \right)^{-1} \quad (49)$$

$$\Delta_{w3,t} = \frac{1}{1 + k_w} \Delta_{w1,t}^{\epsilon_w(1+k_w)/(1-\epsilon_w(1+k_w))} \Delta_{w4,t} + \frac{k_w}{1 + k_w} \quad (50)$$

$$\Delta_{w4,t} = (1 - \lambda_w) \left( \frac{w_t^*}{w_t} \right)^{-\epsilon_w(1+k_w)} + \lambda_w \Delta_{w4,t-1} \left( \frac{w_t}{w_{t-1}} \frac{\pi_{t+1}}{\pi_t^{\gamma_w} \bar{\pi}^{1-\gamma_w}} \right)^{\epsilon_w(1+k_w)} \quad (51)$$

## Government

$$R_t - \bar{R} = \phi_\pi \frac{\pi_{t+s}}{\bar{\pi}} + \phi_y \frac{y_{t+s}}{\bar{y}} + \phi_k \frac{k_{t+s}}{\bar{k}} + \rho (R_{t-1} - \bar{R}) \quad (52)$$

## Capital producer

$$K_t = q_t \left( \omega_1 - \frac{\omega_2}{2} q_t \right) o_t + (1 - q_t) \theta \quad (53)$$

$$o_t = o_t^{new} + (1 - \delta) K_{t-1}$$

$$o_t^{new} = \varepsilon_T^I i_t \left( 1 - \frac{\kappa}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \right) \quad (55)$$

$$Q_t \varepsilon_t^I \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) - S' \left( \frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \right] - 1 = \beta E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \varepsilon_{t+1}^I Q_{t+1} S' \left( \frac{i_{t+1}}{i_t} \right) \left( \frac{i_{t+1}}{i_t} \right)^2 \right]. \quad (56)$$

## Bank

$$q_t = 1 - \frac{\tilde{R}}{\tilde{\psi}_t} + \frac{\sqrt{\omega_2 (\tilde{R}_t - \tilde{\psi}_t) (\tilde{R}_t + 2\tilde{\xi}_t) \left( 2\omega_1 \tilde{\psi}_t (\tilde{R}_t + \tilde{\xi}_t) + \omega_2 (\tilde{R}_t - \tilde{\psi}_t) (\tilde{R}_t + 2\tilde{\xi}_t) \right)}}{\omega_2 \tilde{\psi}_t (\tilde{R}_t + 2\tilde{\xi}_t)} \quad (57)$$

$$k_t = \frac{\tilde{R}_t - \tilde{\psi}_t}{\tilde{R}_t + 2\tilde{\xi}_t} \quad (58)$$

$$\left( \omega_1 - \frac{\omega_2}{2} q_{t-1} \right) \frac{r_{k,t} + (1 - \delta) Q_t - \tau_{:t}}{Q_{t-1}} - \frac{r_{d,t}}{\pi_{t+1}} (1 - k_t) - \frac{r_{e,t+1}}{\pi_{t+1}} k_t = 0 \quad (59)$$

$$\tau_t = \frac{Q_{t-1} \frac{1 - q_{t-1}}{q_{t-1}} \left( \psi - \theta \frac{r_{k,t} + (1 - \delta) Q_t}{Q_{t-1}} \right)}{\omega_1 - \frac{\omega_2}{2} q_{t-1}} \quad (60)$$

$$\tilde{\xi}_t = \xi_t E_t [A_{t+1}] \quad (61)$$

$$\xi_t = \xi \varepsilon_t^\xi \quad (62)$$

$$\tilde{R}_t = E_t \left[ A_{t+1} \frac{R_t}{\pi_{t+1}} \right] \quad (63)$$

$$\tilde{\psi}_t = \psi E_t [A_{t+1}] \quad (64)$$

$$o_t Q_t = e_t + d_t \quad (65)$$

$$k_t = e_t / (e_t + d_t) \quad (66)$$

### Shock processes

$$\log (\varepsilon_t^B) = \rho_P \log (\varepsilon_{t-1}^B) + \sigma^B u_t^B \quad (67)$$

$$\log (\varepsilon_t^Q) = \rho_I \log (\varepsilon_{t-1}^Q) + \sigma^Q u_t^Q \quad (68)$$

$$\log (\varepsilon_t^\xi) = \rho_\xi \log (\varepsilon_{t-1}^\xi) + \sigma^\xi u_t^\xi \quad (69)$$

$$\log (\varepsilon_t^P) = (1 - \rho_P) \log (\bar{\varepsilon}_{t-1}^P) + \rho_P \log (\varepsilon_{t-1}^P) + \sigma^P (u_t^P + m_p u_{t-1}^P) \quad (70)$$

$$\log (\varepsilon_t^W) = (1 - \rho_W) \log (\bar{\varepsilon}_{t-1}^W) + \rho_W \log (\varepsilon_{t-1}^W) + \sigma^W (u_t^W + m_W u_{t-1}^W) \quad (71)$$

$$\log (\varepsilon_t^A) = \rho_A \log (\varepsilon_{t-1}^A) + \sigma^A u_t^A \quad (72)$$

$$\log (\varepsilon_t^R) = \rho_R \log (\varepsilon_{t-1}^R) + \sigma^R u_t^R \quad (73)$$

$$\log(\varepsilon_t^G) = \rho_G \log(\varepsilon_{t-1}^G) + \sigma^G u_t^G + \rho_{GA} \sigma^A u_t^A \quad (74)$$

**Observational equations:** The observation equations, linking the observed time series (left hand-side) to the variables in the non-linear model (right hand-side) are the following:

$$100\Delta \log\left(\frac{Y_t}{Y_{t-1}}\right) = 100\Delta \log\left(\frac{y_t}{y_{t-1}}\right) + 100\mu_y$$

$$100\Delta \log\left(\frac{C_t}{C_{t-1}}\right) = 100\Delta \log\left(\frac{c_t}{c_{t-1}}\right) + 100\mu_y$$

$$100\Delta \log\left(\frac{I_t}{I_{t-1}}\right) = 100\Delta \log\left(\frac{i_t}{i_{t-1}}\right) + 100\mu_y$$

$$100\Delta \log\left(\frac{W_t}{W_{t-1}}\right) = 100\Delta \log\left(\frac{w_t}{w_{t-1}}\right) + 100\mu_y$$

$$100\Delta \log\left(\frac{P_t}{P_{t-1}}\right) = 100\pi_t$$

$$100 \log\left(\frac{H_t}{\bar{H}}\right) = 100 \log\left(\frac{L_t}{\bar{L}}\right) + 100\mu_l$$

$$\left(\frac{R_t}{4}\right) = 100R$$

$$\tilde{E}_t = 100k_t$$

where  $\bar{H}$  are hours worked in 2009 and  $\mu_l$  is a shift parameter. Since there is no growth in the model, we estimate the mean growth rate in the data  $\mu_y$ . The equity ratio in the data  $\tilde{E}_t$  is transformed by taking deviations from its linear trend and adding back the mean.

**Prior specifications:** We fix parameters that are not identified to values commonly used in the literature. In particular, we choose a depreciation rate  $\delta$  of

0.025, a steady-state wage markup  $\bar{\varepsilon}^W$  of 1.05, a steady-state spending to GDP ratio  $g_y$  of 18%, a weight of labor in the utility function  $\bar{L}$  such that steady-state hours are equal to 1, and curvatures of the Kimball aggregator for goods and labor varieties of 10.

For all structural shocks, we employ a non-informative uniform distribution. The persistences of the shock processes are assumed to have a beta prior distribution centered at 0.5, and with standard deviation of 0.2. Following Smets and Wouters [2007], we further assume that the two markup shows have a moving average component.

The priors of the Taylor rule parameters are centered around very common values: the smoothing parameter has a Beta distribution with a mean of 0.75, while the responses to inflation and output are assumed to follow a Normal distribution with a mean of 1.5 and of  $0.5/4 = 0.125$ .

Since we use level data of the inflation rate and of the nominal interest rate, we choose the priors for the steady state of the inflation rate  $\bar{\pi}$  and the real interest rate  $1/\beta - 1$  to match the mean in the data, i.e. we assumed they follow a gamma distribution respectively centered around annualized values of 2.5% and 0.9.

The parameters affecting price and wage stickiness have a beta distribution centered at 0.5 with standard deviation of 0.1. Our prior is that prices and wages are reoptimized on average every 6 months, and that the degree of indexation to past inflation is only up to 50%. The steady-state price markup is assumed to be centered around 1.25, slightly above the steady-state wage markup.

We employ very common priors for all the parameters of the utility function. Habits are centered around 0.7, the intertemporal elasticity of substitution  $\sigma_c$  has a prior mean of 1.5, while the elasticity of labor supply  $\sigma_l$  has a prior mean of 2. The capital share in production has a prior mean of 0.3 while the investment adjustment costs parameter has a loose prior around 4.

For the discussion on the priors for the banking sector parameters, we refer to section 4.1 in the main text.



## Appendix D: Proofs

### The risk-taking channel for a generic expected return function

Consider the bank problem discussed in section 3 with deposit insurance partial recovery but replace the expression for the expected return conditional on success  $q_t (\omega_1 - \omega_2/2q_t)$  by the generic function  $f(q_t)$ .

Assume there exists an equilibrium  $[\tilde{r}_{l,t}, q_t, k_t]$  under perfect competition that satisfies the following conditions: (1) the bank's choices are interior, i.e.  $[k_t, q_t] \in [0, 1]^2$ , (2a) the default probability is low relative to the parameters  $\frac{q_t}{(1-q_t)}\tilde{\xi}_t \geq \tilde{R}_t - \tilde{\psi}_t$  or (2b) there is no deposit insurance  $\tilde{\psi}_t = 0$ , the conditional expected return function  $f(q_t)$  satisfies (3)  $f(q_t) \geq 0$ ,  $f''(q_t) < 0$  and (4)  $f'''(q_t) \leq 0$ ,  $f''''(q_t) \leq 0$ .

Notice that assumption 2a), which is sufficient but by no means necessary and only needed for claim (e), is weak if we consider the empirically relevant section of the parameter space with a low equity premium (around 0.0x), a real rate just above 1 (1.0x) and high deposit insurance (0.x) and high repayment probabilities (0.9x). Assumption 3 is straightforward as it guarantees a meaningful risk turn trade-off with an interior solution. Assumption 4 is another a sufficient but non necessary condition.

We prove that, if such a solution exists, then:

- (a) risk taking is excessive:  $q_t < \operatorname{argmax} f(q_t)$ ,
- (b) the safety of assets  $q_t$  is a positive function of  $\tilde{R}_t$ :  $\frac{\partial q_t}{\partial \tilde{R}_t} > 0$ ,
- (c) the equity ratio  $k_t$  is a positive function of  $\tilde{R}_t$ :  $\frac{\partial k_t}{\partial \tilde{R}_t} > 0$ ,
- (d) the expected return of an investment is a positive function of  $\tilde{R}_t$ :  $\frac{\partial f(q_t(\tilde{R}_t)) + (1-q_t(\tilde{R}_t))^\theta}{\partial \tilde{R}_t} > 0$ ,
- (e) the expected return of an investment is a concave function of  $\tilde{R}_t$ :  $\frac{\partial^2 f(q_t(\tilde{R}_t)) + (1-q_t(\tilde{R}_t))^\theta}{\partial \tilde{R}_t^2} < 0$ .

For a generic return function  $f(q_t)$  the bank's objective function at the second stage is:

$$\max_{q_t} f(q_t) \tilde{r}_{l,t} - q_t \tilde{r}_{d,t} (1 - k_t)$$

Deriving this problem with respect to  $q_t$  yields the following first-order condition, which by concavity is necessary and sufficient:

$$f'(q_t) \tilde{r}_{l,t} = \tilde{r}_{d,t} (1 - k_t) \quad (75)$$

Notice that this condition implies  $f'(q_t) > 0$  ( $k_t \in (0, 1]$  by assumption,  $\tilde{r}_{d,t} > 0$  by the deposit supply schedule, and  $\tilde{r}_{l,t} > 0$  by the zero profit condition). Notice further that in a frictionless world, e.g. without limited liability, the banks risk choice would satisfy  $q_t^{opt} = \operatorname{argmax} f(q_t) + (1 - q_t)\theta$ , i.e.  $f'(q_t^{opt}) = \theta$ . Since we have assumed above that the recovery value is smaller than the deposit insurance cap, which in turn is smaller than the cost of deposits by lemma 1, we have:  $\tilde{r}_{l,t}\theta < \tilde{\psi}_t < \tilde{r}_{d,t}(1 - k_t)$ . Combining this with equation (75) and the frictionless optimality condition and rearranging, we obtain  $f'(q_t) > f'(q_t^{opt})$ . Given  $f''(q_t) < 0$  this implies excessive risk taking, i.e.  $q_t < q_t^{opt}$  (claim(a)).

Since the deposit supply schedule must hold in equilibrium, we can rewrite this condition as

$$f'(q_t) \tilde{r}_{l,t} - \frac{\tilde{R}_t(1 - k_t) + (1 - q_t)\tilde{\psi}_t}{q_t} = 0 \quad (76)$$

Equation (76) implicitly defines  $\hat{q}_t(k_t)$ . Using the implicit function theorem we find that,  $f'(q_t^{opt}) = \theta < \tilde{r}_{d,t}(1 - k_t) = f'(q_t) \tilde{r}_{l,t}$

$$\frac{\partial q_t}{\partial k_t} = \frac{-q_t \tilde{R}_t}{(1 - k_t) \tilde{R}_t - \tilde{\psi}_t + q_t^2 \tilde{r}_{l,t} f''(q_t)}$$

At the first stage the maximization problem is

$$\max_{k_t} f(q_t) \tilde{r}_{l,t} - q_t \tilde{r}_{d,t} (1 - k_t) - q_t k_t \tilde{r}_{e,t}.$$

which, using the deposit and equity supply schedules  $\tilde{r}_{d,t} = \frac{\tilde{R}_t - \frac{1 - q_t}{1 - k_t} \tilde{\psi}_t}{q_t}$   $\tilde{r}_{e,t} = \frac{\tilde{R}_t + \tilde{\xi}_t}{q_t}$ ,

can be written as

$$\max_{k_t} f(\hat{q}_t) \tilde{r}_{l,t} + (1 - q_t) \tilde{\psi}_t - k_t \tilde{\xi}_t - \tilde{R}_t .$$

The corresponding FOC is

$$\left( f'(\hat{q}_t) \tilde{r}_{l,t} - \tilde{\psi}_t \right) \frac{\partial q_t}{\partial k_t} - \tilde{\xi}_t . \quad (77)$$

Finally, the zero profit condition can in expectations be written as

$$f(\hat{q}_t) \tilde{r}_{l,t} + (1 - q_t) \tilde{\psi}_t - k_t \tilde{\xi}_t - \tilde{R}_t . \quad (78)$$

Equations (76), (77), (78) implicitly define  $q_t$ ,  $k_t$  and  $\tilde{r}_{l,t}$ . Solving the latter two equations for  $k_t$  and  $\tilde{r}_{l,t}$  we obtain:

$$k_t = \frac{\left( -\tilde{\xi}_t \tilde{R}_t + q_t \tilde{R}_t \tilde{\psi}_t + \tilde{\xi}_t \tilde{\psi}_t \right) f(q_t) - q_t \left( \tilde{R}_t - (1 - q_t) \tilde{\psi}_t \right) \left( \tilde{R}_t f'(q_t) + q_t \tilde{\xi}_t f''(q_t) \right)}{-\tilde{\xi}_t \left( \tilde{R}_t f(q_t) - q_t \left( \tilde{R}_t f'(q_t) + q_t \tilde{\xi}_t f''(q_t) \right) \right)} \quad (79)$$

$$\tilde{r}_{l,t} = \frac{\left( \tilde{R}_t + \tilde{\xi}_t \right) \left( \tilde{R}_t - \tilde{\psi}_t \right)}{\tilde{R}_t f(q_t) - q_t \left( \tilde{R}_t f'(q_t) + q_t \tilde{\xi}_t f''(q_t) \right)} \quad (80)$$

Plugging these equations into (76) and rearranging we obtain the following equation, which implicitly defines  $q_t$

$$\left( \tilde{R}_t + \tilde{\xi}_t \right) \frac{\tilde{R}_t \tilde{\psi}_t f(q_t) - \left( \tilde{R}_t \left( \tilde{R}_t + \tilde{\xi}_t \right) - \left( (1 - q_t) \tilde{R}_t + \tilde{\xi}_t \right) \tilde{\psi}_t \right) f'(q_t) - q_t \tilde{\xi}_t \left( \tilde{R}_t - (1 - q_t) \tilde{\psi}_t \right) f''(q_t)}{-\tilde{\xi}_t \left( \tilde{R}_t f(q_t) - q_t \left( \tilde{R}_t f'(q_t) + q_t \tilde{\xi}_t f''(q_t) \right) \right)} = 0$$

We can simplify this condition further by multiplying with the denominator and dividing by  $\left( \tilde{R}_t + \tilde{\xi}_t \right) \tilde{R}_t$

$$F(q_t, R_t) \equiv \frac{\tilde{R}_t \tilde{\psi}_t f(q_t) - \left( \tilde{R}_t \left( \tilde{R}_t + \tilde{\xi}_t \right) - \left( (1 - q_t) \tilde{R}_t + \tilde{\xi}_t \right) \tilde{\psi}_t \right) f'(q_t) - q_t \tilde{\xi}_t \left( \tilde{R}_t - (1 - q_t) \tilde{\psi}_t \right) f''(q_t)}{\tilde{R}_t} = 0 \quad (81)$$

Using the implicit function theorem on equation (81) we find that

$$\frac{\partial q_t}{\partial R_t} = -\frac{\frac{\partial F}{\partial R_t}}{\frac{\partial F}{\partial q_t}} \quad (82)$$

where

$$\begin{aligned} \frac{\partial F}{\partial R_t} &= \frac{\left(\tilde{R}_t^2 + \tilde{\xi}_t \tilde{\psi}_t\right) f'(q_t) + (1 - q_t) q_t \tilde{\xi}_t \tilde{\psi}_t f''(q_t)}{-\tilde{R}_t^2} \\ \frac{\partial F}{\partial q_t} &= \frac{\left(\tilde{R}_t - (1 - q_t) \tilde{\psi}_t\right) \left(\left(\tilde{R}_t + 2\tilde{\xi}_t\right) f''(q_t) + q_t \tilde{\xi}_t f'''(q_t)\right)}{-\tilde{R}_t} \end{aligned}$$

Using our assumptions on  $f$ , the parameters and assuming an interior solution it is obvious that  $\frac{\partial F}{\partial q_t} > 0$ . How about the  $\frac{\partial F}{\partial R_t}$ ?

To get at the sign of  $\frac{\partial F}{\partial R_t}$ , we solve 81 for  $f(q_t)$

$$f(q_t) = \frac{\left(\tilde{R}_t \left(\tilde{R}_t + \tilde{\xi}_t\right) - \left((1 - q_t) \tilde{R}_t + \tilde{\xi}_t\right) \tilde{\psi}_t\right) f'(q_t) + q_t \tilde{\xi}_t \left(\tilde{R}_t - (1 - q_t) \tilde{\psi}_t\right) f''(q_t)}{\tilde{R}_t \tilde{\psi}_t}$$

and plug this expression into the equations (79) and (80) for  $k_t$  and  $\tilde{r}_{l,t}$ :

$$k_t = \frac{f'(q_t)(\tilde{R}_t + \tilde{\xi}_t)(\tilde{R}_t - \tilde{\psi}_t) + f''(q_t) q_t \tilde{\xi}_t (\tilde{R}_t - (1 - q_t) \tilde{\psi}_t)}{\tilde{R}_t \left(\left(\tilde{R}_t + \tilde{\xi}_t\right) f'(q_t) + q_t \tilde{\xi}_t f''(q_t)\right)^2} \quad (83)$$

$$\tilde{r}_{l,t} = \frac{\left(\tilde{R}_t + \tilde{\xi}_t\right) \tilde{\psi}_t}{\left(\tilde{R}_t + \tilde{\xi}_t\right) f'(q_t) + q_t \tilde{\xi}_t f''(q_t)} \quad (84)$$

Since in equilibrium  $\tilde{r}_{l,t} > 0$  and since the numerator of  $\tilde{r}_{l,t}$  is obviously positive it must hold that its denominator is also positive:

$$\left(\tilde{R}_t + \tilde{\xi}_t\right) f'(q_t) + q_t \tilde{\xi}_t f''(q_t) > 0 \quad (85)$$

Similarly, since  $k_t > 0$  and since the denominator of  $k_t$  is obviously positive, the numerator must be positive too:

$$f'(q_t)(\tilde{R}_t + \tilde{\xi}_t)(\tilde{R}_t - \tilde{\psi}_t) + f''(q_t)q_t\tilde{\xi}_t(\tilde{R}_t - (1 - q_t)\tilde{\psi}_t) > 0 \quad (86)$$

Since  $f' > 0$  and  $f'' < 0$  we can conclude from the previous inequality that for any  $[x_1, x_2] \in \mathbb{R}^2$  it must hold that  $f'(q_t)x_1 + f''(q_t)x_2 > 0$  if

$$\frac{x_1}{x_2} \geq \frac{(\tilde{R}_t + \tilde{\xi}_t)(\tilde{R}_t - \tilde{\psi}_t)}{q_t\tilde{\xi}_t(\tilde{R}_t - (1 - q_t)\tilde{\psi}_t)} \quad (87)$$

We now test this condition for the numerator of  $\frac{\partial F}{\partial \tilde{R}_t}$

$$\frac{\tilde{R}_t^2 + \tilde{\xi}_t\tilde{\psi}_t}{(1 - q_t)q_t\tilde{\xi}_t\tilde{\psi}_t} \leq \frac{(\tilde{R}_t + \tilde{\xi}_t)(\tilde{R}_t - \tilde{\psi}_t)}{q_t\tilde{\xi}_t(\tilde{R}_t - (1 - q_t)\tilde{\psi}_t)}$$

Rearranging, multiplying only with positive values, yields

$$0 \leq -\tilde{R}_t(\tilde{R}_t - (1 - q_t)\tilde{\psi}_t) - q_t\tilde{\psi}_t\tilde{\xi}_t - \left(\tilde{R}_t(1 - q_t)\tilde{\psi}_t + (1 - q_t)\tilde{\psi}_t\tilde{\xi}_t\right) \frac{q_t\tilde{\psi}_t}{(\tilde{R}_t - (1 - q_t)\tilde{\psi}_t)}$$

The RHS is obviously negative since from the proposition that every deposit insurance cap will be exceeded it follows that  $\tilde{R}_t > \tilde{\psi}_t$ . Hence the condition  $\frac{\tilde{R}_t^2 + \tilde{\xi}_t\tilde{\psi}_t}{(1 - q_t)q_t\tilde{\xi}_t\tilde{\psi}_t} \geq \frac{\tilde{R}_t + \tilde{\xi}_t}{q_t\tilde{\xi}_t}$  is satisfied and we can conclude the the numerator of  $\frac{\partial F}{\partial \tilde{R}_t}$  is positive. Hence  $\frac{\partial F}{\partial \tilde{R}_t} < 0$  and therefore  $\frac{\partial q_t}{\partial \tilde{R}_t} > 0$  (claim (b)).

Equation (83) defines  $k_t = \mathfrak{K}(q_t, \tilde{R}_t)$ . Its derivative is given by

$$\frac{\partial k_t}{\partial \tilde{R}_t} = \frac{\partial \mathfrak{K}}{\partial \tilde{R}_t} + \frac{\partial \mathfrak{K}}{\partial q_t} \frac{\partial q_t}{\partial \tilde{R}_t}$$

where

$$\frac{\partial \mathfrak{K}}{\partial \tilde{R}_t} = \frac{\left((f'(q_t))^2 \tilde{R}_t^2 + 2f'(q_t)(f'(q_t) + f''(q_t)(1 - q_t)q_t)\tilde{R}_t\tilde{\xi}_t + (f'(q_t) + f''(q_t)q_t)(f'(q_t) - f''(q_t)(1 - q_t)q_t)\tilde{\xi}_t^2\right)\tilde{\psi}_t}{\tilde{R}_t^2 \left(f''q\tilde{\xi}_t + f'(\tilde{R}_t + \tilde{\xi}_t)\right)^2}$$

$$\frac{\partial \bar{\kappa}}{\partial q_t} = \frac{q_t \tilde{\xi}_t f''(q_t) \tilde{\psi}_t \left( 2 \left( \tilde{R}_t + \tilde{\xi}_t \right) f'(q_t) + q_t \tilde{\xi}_t f''(q_t) \right)}{\tilde{R}_t \left( f'' q \tilde{\xi}_t + f'(\tilde{R}_t + \tilde{\xi}_t) \right)^2}$$

From (87) it is immediately obvious that the numerator of  $\frac{\partial \bar{\kappa}}{\partial q_t}$  is negative, hence  $\frac{\partial \bar{\kappa}}{\partial q_t} < 0$ . After division by  $\tilde{\psi}_t$ , the numerator of  $\frac{\partial \bar{\kappa}}{\partial \tilde{R}_t}$  can be rewritten as

$$\left( \left( \tilde{R}_t + \tilde{\xi}_t \right) f'(q_t) + q_t \tilde{\xi}_t f''(q_t) \right)^2 - (f''(q_t))^2 q_t^3 \tilde{\xi}_t^2 - f''(q_t) f'(q_t) q_t^2 \tilde{\xi}_t (2\tilde{R}_t + \tilde{\xi}_t)$$

Since the first term is positive and bigger then the absolute value of the second term we can see that  $\frac{\partial \bar{\kappa}}{\partial \tilde{R}_t} > 0$ . Hence we have shown that  $\frac{\partial k_t}{\partial \tilde{R}_t} > 0$  (claim (c)).

Applying the implicit function theorem a second time on equation (81) we can find the following expression for the second derivative of  $q_t$

$$\frac{\partial^2 q_t}{\partial \tilde{R}_t^2} = \frac{\left( \frac{\partial^2 F}{\partial R_t \partial q_t} + \frac{\partial^2 F}{\partial q_t^2} \frac{\partial q_t}{\partial R_t} \right) \frac{\partial F}{\partial R_t} - \left( \frac{\partial^2 F}{\partial R_t \partial q_t} \frac{\partial q_t}{\partial R_t} + \frac{\partial^2 F}{\partial R_t^2} \right) \frac{\partial F}{\partial q_t}}{\left( \frac{\partial F}{\partial q_t} \right)^2} \quad (88)$$

where

$$\frac{\partial^2 F}{\partial \tilde{R}_t \partial q_t} = \frac{\left( \tilde{R}_t^2 + 2(1 - q_t) \tilde{\xi}_t \tilde{\psi}_t \right) f''(q_t) + q_t (1 - q_t) \tilde{\xi}_t \tilde{\psi}_t f'''(q_t)}{-\tilde{R}_t}$$

$$\frac{\partial^2 F}{\partial q_t^2} = \frac{\tilde{\psi}_t \left( q_t \tilde{\xi}_t f'''(q_t) + \left( \tilde{R}_t + 2\tilde{\xi}_t \right) f''(q_t) \right) + \left( f''''(q_t) q_t \tilde{\xi}_t + f'''(q_t) \left( \tilde{R}_t + 3\tilde{\xi}_t \right) \right) \left( \tilde{R}_t - (1 - q_t) \tilde{\psi}_t \right)}{-\tilde{R}_t}$$

$$\frac{\partial^2 F}{\partial \tilde{R}_t^2} = \frac{2 \left( f'(q_t) + f''(q_t) (1 - q) q \right) \tilde{\xi}_t \tilde{\psi}_t}{\tilde{R}_t^3}$$

since  $f'' < 0$  and  $f''' \leq 0$   $f'''' \leq 0$  and all parameters are positive it is obvious that  $\frac{\partial^2 F}{\partial R_t \partial q_t} > 0$  and  $\frac{\partial^2 F}{\partial q_t^2} > 0$ . The term  $\frac{\partial^2 F}{\partial \tilde{R}_t^2}$  is less straight forward. A sufficient condition for  $\frac{\partial^2 F}{\partial \tilde{R}_t^2} > 0$  can be found using again condition (87)

$$\frac{1}{(1-q_t)q_t} \geq \frac{(\tilde{R}_t + \tilde{\xi}_t)(\tilde{R}_t - \tilde{\psi}_t)}{q_t \tilde{\xi}_t (\tilde{R}_t - (1-q_t)\tilde{\psi}_t)}$$

Which simplifies to

$$\frac{q_t}{(1-q_t)} \tilde{\xi}_t \geq \tilde{R}_t - \tilde{\psi}_t$$

Given the signs of the terms in (88) we have finally verified that

$$\frac{\partial^2 q_t}{\partial \tilde{R}_t^2} = \frac{((+) + (+)(+))(-) - ((+)(+) + (+))(+)}{(+) } < 0$$

Under alternative assumption (2b) the expression for  $\frac{\partial^2 q_t}{\partial \tilde{R}_t^2}$  simplifies to

$$\frac{\partial^2 q_t}{\partial \tilde{R}_t^2} = - \frac{f'(q_t) \left( -2f''(q_t)f'''(q_t)q_t\tilde{\xi}_t - 2(f''(q_t))^2 (\tilde{R}_t + 2\tilde{\xi}_t) + f'(q_t) \left( f''''(q_t)q_t\tilde{\xi}_t + f'''(q_t) (\tilde{R}_t + 3\tilde{\xi}_t) \right) \right)}{\left( f'''(q_t)\tilde{\xi}_t + f''(q_t) (\tilde{R}_t + 2\tilde{\xi}_t) \right)^3}$$

which is negative without further conditions.

Using the signs of the derivatives of  $q_t$  and the fact that  $f'(q_t) > f'(q_t^{opt}) = \theta$ , we can finally determine the slope and curvature of the expected return of the bank's investment.

$$\begin{aligned} \frac{\partial [f(q_t) + (1 - (q_t))\theta]}{\partial \tilde{R}_t} &= \underbrace{(f'(q_t) - \theta)}_{+} \underbrace{\frac{\partial q_t}{\partial \tilde{R}_t}}_{+} > 0 \\ \frac{\partial^2 [f(q_t) + (1 - (q_t))\theta]}{\partial \tilde{R}_t^2} &= \underbrace{(f'(q_t) - \theta)}_{+} \underbrace{\frac{\partial^2 q_t}{\partial \tilde{R}_t^2}}_{+} + \underbrace{f''(q_t)}_{-} \underbrace{\frac{\partial q_t}{\partial \tilde{R}_t}}_{+} < 0 \end{aligned}$$

This completes the proof of claims (d) and (e).

Notice that the quadratic functional form we assumed for  $f(q_t)$  in the model section satisfies assumptions (3) and (4) and we focussed on interior solutions (assumption (1)). Therefore claims (1), (2), (3) and (4) in propositions 1 and 2 hold. Furthermore, claim (5) in proposition 1 holds since assumption (2a) is satisfied. Finally, to see that claim (5) in proposition 2.1 holds independent of

assumption (2a) and (2b), consider the solution for  $q_t$

$$q_t = 1 - \frac{\tilde{R}}{\tilde{\psi}_t} + \frac{\sqrt{\omega_2 (\tilde{R}_t - \tilde{\psi}_t) (\tilde{R}_t + 2\tilde{\xi}_t) \left( 2\omega_1 \tilde{\psi}_t (\tilde{R}_t + \tilde{\xi}_t) + \omega_2 (\tilde{R}_t - \tilde{\psi}_t) (\tilde{R}_t + 2\tilde{\xi}_t) \right)}}{\omega_2 \tilde{\psi}_t (\tilde{R}_t + 2\tilde{\xi}_t)}$$

The second derivative of this expression is

$$\frac{\partial^2 q}{\partial \tilde{R}_t^2} = - \frac{\left( \begin{aligned} & \omega_1 \omega_2 \left\{ 2\omega_2 (\tilde{R}_t - \tilde{\psi}_t)^3 \tilde{\xi}_t (\tilde{R}_t + 2\tilde{\xi}_t) + \omega_1 \tilde{\psi}_t \dots \right. \\ & \left. \dots \left[ \tilde{R}_t^4 + 2\tilde{R}_t (4\tilde{R}_t^2 - 3\tilde{R}_t \tilde{\psi}_t + 2\tilde{\psi}_t^2 \tilde{\xi}_t) + (12\tilde{R}_t - 4\tilde{R}_t \tilde{\psi}_t + 5\tilde{\psi}_t^2) \tilde{\xi}_t^2 + 4(2\tilde{R}_t + \tilde{\psi}_t \tilde{\xi}_t^3 + 4\tilde{\xi}_t^4) \right] \right\} \right)}{\left( \tilde{R}_t + 2\tilde{\xi}_t \right) \left\{ \omega_2 (\tilde{R}_t - \tilde{\psi}_t) (\tilde{R}_t + 2\tilde{\xi}_t) \left[ 2\omega_1 \tilde{\psi}_t (\tilde{R}_t + \tilde{\xi}_t) + b (\tilde{R}_t - \tilde{\psi}_t) (\tilde{R}_t + 2\tilde{\xi}_t) \right] \right\}^{2/3}} \end{aligned} \right)$$

Both numerator and denominator are obviously positive, so  $\frac{\partial^2 q}{\partial \tilde{R}_t^2} < 0$ . Hence

$$\frac{\partial^2 [f(q_t) + (1 - q_t)\theta]}{\partial \tilde{R}_t^2} < 0. \quad \blacksquare$$

## Deposits in excess of insurance

The proof is by contradiction: Assume that there exists an equilibrium with no excess profits where the bank would issue so little deposits that the promised repayment  $r_{d,t}$  would be lower than the cap on deposit insurance  $\psi / (1 - k_t) \pi_{t+1}$ .<sup>44</sup> In this case the deposit rate  $r_{d,t}$  would be equal to the risk free rate  $R_t$ .

The second stage maximization problem of the bank would then be

$$\max_{q_t \in [0,1]} f(q_t) - q_t \tilde{R}_t (1 - k_t)$$

and its solution  $\hat{q}_t$  is implied by

$$f'(\hat{q}_t) = \tilde{R}_t (1 - k_t)$$

The first stage maximization problem would be

$$\max_{k_t \in [0,1]} V(k) = f(\hat{q}_t) - \hat{q}_t \tilde{R}_t (1 - k_t) - (\tilde{\xi}_t + \tilde{R}_t) k_t$$

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<sup>44</sup>For simplicity, we abstract from the possibility that the cap is binding for some states of the future but not for others, which would be possible due to the inconsistency between the timing of inflation and the nominal deposit rate. Note that this distinction disappears under certainty equivalence or first order approximation.



$\hat{q}_t$  can either be a corner or an interior solution. If  $\hat{q}_t$  is a corner solution, the first stage objective function of the bank is obviously decreasing in  $k_t$ , hence  $k_t = 0$  is optimal. If  $\hat{q}_t$  is an interior solution, the first derivative of the first stage objective function is

$$\tilde{R}_t - \tilde{\xi}_t - \tilde{R}_t(1 - \hat{q}_t)$$

Since  $\hat{q}_t \in [0, 1]$  this derivative is negative for all  $k_t \in [0, 1]$ , i.e. the objective function again is decreasing in  $k$ . Hence the solution to the first stage problem is  $k_t = 0$ . Optimality with full insurance therefore requires that the bank uses only deposits. This contradicts our initial assumption. This result implies that any insurance cap smaller than 100% would be exceeded by the deposit liabilities in case of default. Depositors are therefore never fully insured.

Notice that for a cap to be effective in the sense of ruling out full insurance equilibria, the cap has to be low enough. Formally speaking, it needs to hold that  $r_{d,t}(1 - k_t) > \tilde{\psi}_t$  even under full insurance, i.e.  $\tilde{R}_t > \tilde{\psi}_t$ . ■