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WIDER Working Paper 2015/016

On the dynamics of multidimensional chronic poverty

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Abstract: Understanding chronic poverty and its evolution is complex given the amount of information involved. This paper proposes a new approach to analysing the evolution of chronic poverty in a multivariate setting using a Shapley decomposition of a multidimensional chronic poverty measure proposed by Alkire and colleagues. This makes it possible to assess a vast array of information to find the drivers of change in chronic poverty, and has proved to be a valuable tool in public policy programmes. We present an empirical application of the changes in chronic poverty in Argentina during the period 2004-12 using the Permanent Household Survey.

Keywords: chronic poverty, multidimensional poverty, Shapley decomposition. **JEL classification:** I32, I38

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1 Introduction

Poverty eradication is a long-term project in which many developing countries are engaged. In order to deepen our understanding of why poverty occurs, and significantly improve the effectiveness of poverty reduction strategies, attention has been paid to its determinants and evolution over time. It is well recognized in the literature that measures of living standards at one point in time may provide limited information regarding the evolution of poverty across time and its persistence (Aliber 2003; Hoy and Zheng 2011; Hoy et al. 2012; McKay and Lawson 2003). Hulme and Shepherd (2003) point out that precious resources are wasted if a distinction is not made between the chronically poor and episodically poor. The chronically poor are those most likely to remain in poverty in the absence of effective assistance and this is characterized in terms of policy discussions as the type of poverty that does not easily resolve itself. Persistent conditions of poverty have a long-lasting effect, since there is evidence that households in chronic poverty have a higher risk of passing on the same living standards to the next generation, therefore perpetuating poverty (Hulme and Shepherd 2003).

Recognizing poverty dynamics is relevant, yet underpinning its determinants is a complex task. Chronic poverty has mostly considered the monetary dimension of poverty, partly because it is the indicator that can fluctuate the most in a short time (Baulch and Hoddinott, 2000; McKay and Lawson 2003). Yet, there is evidence that chronic poverty is more incisive in dimensions of poverty other than income. As pointed out by Hulme (2003), the chronically poor are likely to be neglected given the multiple factors that constrain their prospects, which makes it necessary to move efforts to measure poverty dynamics beyond income and consumption to multidimensional concepts and definitions of poverty. Recent advances in the conceptualization and measurement of multidimensional chronic poverty offer the possibility of analysing the determinants that can affect poverty over time (Alkire et al. 2013; Apablaza and Yalonetzky 2012). With such an array of information, it is important to disaggregate the poor in order to refine the understanding of the causes of multidimensional chronic poverty and create the knowledge that is needed to design effective policy interventions.

We propose a method to analyse the factors that are driving change in multidimensional chronic poverty. In order to do so, we build on Alkire et al. (2013) to apply the Shapley (1953) decomposition approach to isolate the marginal contributions of each well-being source in the analysis. We follow Ravallion and Huppi's (1991) decomposition of poverty changes by using two factors in the analysis: (1) changes due to the within-group chronic poverty effect associated with changes in the headcount of the chronically poor and (2) changes due to demographic or between-group effects characterized by the average measure of the intensity of chronic poverty over time. Since there is no natural order of elimination of factors to isolate their marginal contribution to overall chronic poverty, we average the overall possible sequence of eliminations of these impacts. Thus, in order to assess the marginal contribution of a given factor to overall chronic poverty we apply the before-and-after concept to the set of all possible combinations of factors and take the average of its contributions.

The determinants of changes in poverty have been well established in the literature. A widely used decomposition analysis in applied studies is the change in poverty in terms of growth and inequality components (some important contributions, among others, are Datt and Ravallion 1992; Ravallion and Chen 1997; Tsui 1996). These decompositions allow us to understand the interrelation between growth, inequality, and poverty, and, in particular, how increasing trends in inequality may offset the benefits of economic growth. Another dynamic decomposition looks for a series of determinants in some of the important demographic and sectoral characteristics of households.

Ravallion and Huppi (1991) follow a similar decomposition but instead focus on quantifying the changes in aggregate poverty in terms of factors relating to sectors in the economy and according to distributional parameters. Despite being widely used, these decompositions have some limitations in the interpretation of contributing factors since they are not always interpreted intuitively and they are path-dependent with regard to the initial income in the analysis (Shorrocks 2013). These limitations become more significant as we try to identify relevant contributions in a multivariate and dynamic setting. The Shapley method, as suggested by Shorrocks (2013), overcomes these limitations by generating a path-independent and exact additive decomposition of changes in poverty into factors. A similar study to ours is Roche (2013), which used the Shapley decomposition for changes in multidimensional poverty in order to assess the overall progress of child poverty reduction in Bangladesh.

Nevertheless, the changes in poverty analysed so far refer to one point in time. Multidimensional chronic poverty, as mentioned above, is a specific conceptualization of poverty that focuses on its multidimensionality and considers its monotonicity and time persistence, as suggested by Foster (2009). The framework proposed here allows us to focus on the permanent rather than the episodic components of poverty. In the long term, the effect of the episodic components averages out, while the effect of the permanent ones persists. This allows us to elucidate the driving forces of some well-being sources that have a strong resistance to change in the standard of living of the chronically poor and could potentially help in the design of better anti-poverty policy strategies.

An empirical section estimates the Shapley decomposition of multidimensional chronic poverty applied to panel data from Argentina's Permanent Household Survey (Encuesta Permanente de Hogares [EPH]) conducted by Instituto Nacional de Estadísticas y Censos (INDEC), covering 32,772 households for two periods over 18 months – January 2004 to June 2005 and January 2012 to June 2013. We found that chronic multidimensional poverty decreased from 2.7 per cent in 2004 to 0.84 per cent in 2012. The vast majority of this change was driven by a change in the incidence of poverty rather than in the intensity of poverty, which maintained relatively the same level throughout the period of analysis. Furthermore, the households with children but without older adults accounted for 77 per cent of the total change in chronic multidimensional poverty. The subgroup of households with older adults presents the lowest improvements in incidence and a strong persistence in the level of chronic poverty. Regarding the importance of the indicators, the change in income poverty was the main driver of the improvement in chronic multidimensional poverty, whereas the variables of unemployment and availability of proper shelter were the indicators that worsened the most. Nevertheless, in terms of public policy decisions, we find that the relative importance of household demographic characteristics is more informative about the evolution of chronic poverty than the relative importance of each indicator. The methodology proposed allows us to systematically assess a vast array of information on chronic poverty defined in several dimensions. Considering all the dimensions and time variations, we found that changes in chronic poverty are sometimes small in relation to changes in income poverty; nevertheless, it is possible to disentangle and quantify the impacts of various causal factors that play a role in the gestation of chronic poverty.

The structure of this paper is as follows. The chronic multidimensional framework used in the analysis is supplied in Section 2. In Section 3 we define our decomposition analysis using the Shapley rule to analyse the determinants of the dynamics of poverty over time. Section 4 contains the application to the case of Argentina. Section 5 offers the conclusions.

2 Multidimensional chronic poverty

This section lays down the conceptualization of multidimensional chronic poverty. Before introducing the definitions, an explanation of our notation is in order.

We suppose a population of size n, person i possesses a d-row vector of attributes in time $t, x_{i.}^{t} \in R_{+}^{d}$ where R_{+}^{d} is the non-negative orthant of the Euclidean d-space R^{d} . The vector $x_{i.}^{t}$ is the row of an $n \times a$ matrix and $x^{t} \in M^{n}$ is the set of all $n \times a$ matrices in time t, whose entries are non-negative reals. The x_{ij}^{t} denotes the quantity of attribute j possessed by person i in time t. Therefore, x_{ij}^{t} the jth column of x^{t} , gives a distribution of attribute j among n persons in time t. The median of each of the attributes in time t is denoted by μ_{j}^{t} . With regard to the problem identification in time t, a threshold for each dimension is determined to represent the minimum level of basic needs and $z_{j}^{t} \in z^{t}$ to be a vector of thresholds for different dimensions, where z^{t} is a non-empty subset of R_{++}^{d} .

In what follows, it is convenient to re-express the original matrix of achievements in time t, x^t , in terms of deprivations. To this end, from the original matrix of attributes we can generate an associated matrix of deprivations. For a given x^t , let $g^t(0)$ denote a matrix of deprivations associated with x^t , whose typical element of the matrix is $g_{ij}^t(0) = 1$ if $x_{ij}^t < z_j^t$, while $g_{ij}^t(0) = 0$ if $x_{ij}^t \ge z_j^t$. The matrix $g^t(0)$ is the size $n \times a$, and elements are either zero or one – zero when the individual is non-poor and one when the individual is poor. We now generate a matrix of normalized poverty gaps or shortfalls that allow us to evaluate different aspects of poverty. Let g^1 be the matrix of normalized gaps of size $n \times a$, where a typical element of the matrix is defined by $g_{ij}^t(1) = g_{ij}^0(z_j^t - x_{ij}^t)/z_j^t$. The poverty gap measures the depth of poverty by weighting for the difference between the attribute and its poverty line. We can generalize the associated matrix to analyse different aspects of poverty and, for this purpose, we can define an associated matrix $g_{ij}^t(\alpha)$, whose typical entry is $g_{ij}^t(\alpha) = (1 - \frac{x_{ij}^t}{z_j^t})^{\alpha}$, where $\alpha \ge 0$.

We follow the two-stage process of Alkire et al. (2013) to identify multidimensional chronic poverty. First, the identification procedure consists of a series of transformations of the original matrix, x^t , in relation to its dimensions and time. Then, the aggregation step takes the set of multidimensional chronically poor as given and combines information on both the number of deprivations and their level across periods; information on poverty depth and distribution can also be incorporated. The resulting functional relationship, M^c , is called an index, or measure of multidimensional chronic poverty.

2.1 Identification of the multidimensionally chronically poor

The identification of the multidimensionally poor is well recognized in the literature (Bourguignon and Chakravarty 2003; Duclos et al. 2006; Tsui 2002). A natural starting point is to consider all those who are deprived in at least one dimension to be poor, the so-called union approach. However, we might consider more demanding criteria and deem an individual to be poor if he/she is deprived in all dimensions, defined as the intersection approach. Alkire and Foster (2011) generalize these two positions by defining an intermediate cut-off, k, which is the number of dimensions required for someone to be considered poor. The identification cut-off ranges from k = 1, corresponding to the union approach, to k = d, corresponding to the intersection approach. This approach also makes it possible to assign different positive weights to the attributes according

to their importance, for which we define a vector of attribute weights, $w = [w_1, w_2, ..., w_d]$ where $\sum_{j=1}^{d} w_j = d$.

We proceed to identify the multidimensionally poor in time t, for which we need to count the number of deprivations suffered by a person i in time t, denoted as c_i^t . Then, the deprivation count in time t is an n-dimensional vector given by, $c^t = g^t(0)w$, where a typical element of the vector is given by $c_i^t = \sum_{j=1}^d w_j g_{ij}^t(0)$. Using the deprivation count vector in time, c^t , we now identify the multidimensionally poor through an identification vector in time t, $I^t(k)$, so that a typical element is given by $\rho_i^t(k) = I(c_i^t < k)$. The value of these identification vector elements is one if $c_i^t < k$, or zero otherwise.

In order to identify multidimensional poverty across time, we use the duration cut-off τ that specifies the minimum fraction of time that must be spent in poverty in order for a person to be considered chronically poor. In each period, t = 1, 2, ..., T, households' poverty status is determined by the identification vector $I^t(k)$, previously defined, thus we define an $n \times T$ matrix in which each of the t column vectors is the identification vector $I^t(k)$. With that information, we now proceed to define the chronic counting vector $c = I(k)1_T$, where I_T is a T-dimensional column vector of ones. The chronic counting vector is an n-dimensional vector, whose typical element is given by $c_i = \sum_{t=1}^T \rho_i^k(k)$. Finally we identify the chronically poor by an n-dimensional vector $\rho^c(k, \tau)$, in which a typical element, $\rho_i^c(k, \tau)$, is given by: $\rho_i^c(k, \tau) = I(c_i > k)$. As before, the value of the identification vector elements is zero when $c_i > k$ and zero otherwise.

2.2 Multidimensional chronic poverty aggregation

The aggregation step takes the identification function $\rho^c(k,\tau)$ and its associated matrix of achievements, $x = (x^1, x^2, ..., x^T)$, the attributes' cut-off vector, $z = (z^1, z^2, ..., z^T)$, the weights of the attributes, w, the number of dimensions cut-off, k and the duration period cut-off τ . The resulting functional relationship $M: x \times R^d_{++} \to R$ is an index called a multidimensional chronic poverty index.

The multidimensional headcount is the simplest version of a multidimensional index. Calculating the fraction of the population deprived in k or more dimensions and during at least τ fraction of time is straightforward. Formally, this can be expressed as:

$$H_{c}(x; z, k, \tau) = \frac{1}{n} \sum_{i=1}^{n} \rho_{i}(k; \tau) = \frac{q}{n}$$
(1)

That is, the number of the poor identified using the dual cut-off approach and the duration approach (q) over the total population (n). The headcount has some important shortcomings. One limitation is that the multidimensional account is not sensitive to the number of deprivations and the number of periods that the multidimensionally poor experience. That is, the index violates dimensional monotonicity (Alkire and Foster 2011) and time monotonicity (Alkire et al. 2013).

Given the k value, if an individual is identified as poor and becomes deprived in an additional dimension or for another period of time, the multidimensional headcount does not change. Another important shortcoming of the multidimensional headcount is that it ignores all the information about the extent of poverty. In this sense, a multidimensional poverty measure should show that poverty becomes more severe at an increasing rate as successive decrements of achievements and longer periods of poverty are considered.

In order to overcome the limitations of the multidimensional headcount measure we need to include more information on the number of deprived dimensions and the number of periods of poverty experienced by the poor. Alkire et al. (2013) proposed the dimension- and time-adjusted FGT measure, or M_c^{α} , family of measures, defined as:

$$M_{c}^{\alpha}(x; z, w, k, \tau) = \frac{1}{ndT} \sum_{i=1}^{n} \rho_{i}(k; \tau) \sum_{t=1}^{T} \sum_{j=1}^{d} w_{j} g_{ij}^{t}(\alpha) = H_{c} A_{c}$$
(2)

where H_c is as in Equation (1) and $A_c = \frac{1}{qdT} \sum_{i=1}^n \rho_i^c(k; \tau) \sum_{t=1}^T \sum_j^d w_j g_{ij}^t(\alpha) = \sum_{j=1}^d A_{cj}$. The partial index A_c represents the average deprivation share across the chronically poor. It is important to notice that the simple product of the two partial indices H_c and A_c generates a weighting system in Equation (2) that is affected this time by the frequency, the number of deprived dimensions and the period of time in deprivation. When $\alpha = 0$ is the adjusted headcount ratio, this time the multidimensional poverty measure clearly satisfies dimensional and time monotonicity. When $\alpha = 1$, the measure is the adjusted chronic poverty gap which is the sum of the normalized chronic poverty gaps of the poor. If the deprivation of a poor person deepens in any dimension or duration, then the index will rise. When $\alpha = 2$, we obtain the squared poverty gaps. In this case the index provides information on the average severity of deprivations in dimensions and time that people experience.

We follow the same framework to characterize transient multidimensional poverty, M_{tr}^{α} , which identifies individuals who are multidimensionally poor during at least one period, but are not chronically poor. We identify the transient poor by an *n*-dimensional vector $\rho^{tr}(k, \tau)$, in which a typical element, $\rho_i^{tr}(k, \tau)$, is given by: $\rho_i^{tr}(k, \tau) = I(0 < c_i < \tau)$. The value of this identification function is one if the condition is reached and zero otherwise. Then, multidimensional transient poverty is

$$M_{tr}^{\alpha}(x; z, w, k, t) = \frac{1}{ndT} \sum_{i=1}^{n} \rho_i^{tr}(k; \tau) \sum_{t=1}^{T} \sum_{j=1}^{d} w_j g_{ij}^t(\alpha) = H_{tr} A_{tr}$$
(3)

3 The Shapley decomposition of chronic poverty

In order to disaggregate the effect some household characteristics have on chronic poverty, we follow Roche's (2013) two-stage disaggregation procedure. In the first stage, it is important to disaggregate the way in which the household's characteristics determine chronic poverty. For that purpose, we partitioned the population into m sub-groups of households differentiated by characteristics l. Let θ_l be sub-group l's population share, that is, the number of households in sub-group l, divided by the total number of households. In order to identify the sub-groups' contributions to poverty changes over time, if θ_l^t and M_l^c represent the population share and chronic poverty level of sub-group $l \in m$, at time t(t = 1, 2) then Equation (2) yields:

$$\Delta M_{c} = \sum_{l \in I}^{m} \left[\theta_{l}^{2} M_{cl}^{2} - \theta_{l}^{1} M_{cl}^{1}\right] = \sum_{l \in I}^{m} \left[\theta_{l}^{2} H_{cl}^{2} A_{cl}^{2} - \theta_{l}^{1} H_{cl}^{1} A_{cl}^{1}\right]$$
(4)

Equation (4) represents the overall change in chronic poverty, ΔM_c , in terms of changes in chronic poverty within groups, $\Delta M_{cl} = M_{cl}^2 - M_{cl}^1, l \in m$, and the population shifts between groups $\Delta \theta_l = \theta_l^2 - \theta_l^1, l \in m$. The second part of the equality in Equation (4) re-expresses changes in chronic poverty in terms of its incidence and intensity components. Using the first part of the equality in Equation (4) we apply the Shapley decomposition proposed by Shorrocks (2013) to changes in decomposable poverty indices, for which we obtain:

$$\Delta M_c = \sum_{l=1}^m \frac{\theta_l^2 + \theta_l^1}{2} \left(M_{cl}^2 - M_{cl}^1 \right) + \sum_{l=1}^m \frac{M_l^2 + M_l^1}{2} \left(\theta_l^2 - \theta_l^1 \right)$$
(5)

The first term in Equation (5) represents the Shapley contribution associated with the changes in chronic poverty within population subgroups l and the second term represents the Shapley contribution to demographic shift factors.

The second stage involves a decomposition of changes in chronic poverty in terms of its incidence and intensity components. For this, we use the second part of the equality in Equation (4) for sub-population $l \in m$ and apply the Shapley decomposition again. It follows that:

$$\Delta M_{cl} = \frac{A_{cl}^2 - A_{cl}^1}{2} \left(H_{cl}^2 - H_{cl}^1 \right) + \frac{H_{cl}^2 - H_{cl}^1}{2} \left(A_{cl}^2 - A_{cl}^1 \right)$$
(6)

where the first component is the Shapley contribution associated with the incidence of chronic poverty and the second component is the Shapley contribution associated with the intensity of chronic poverty. Combining Equations (5) and (6) we obtain the overall decomposition of changes in poverty:

$$\Delta M_{c} = \sum_{l=1}^{m} \left(\frac{M_{l}^{2} + M_{l}^{1}}{2} \right) \left(\theta_{l}^{2} - \theta_{l}^{1} \right) + \sum_{l=1}^{m} \left(\frac{\theta_{l}^{2} + \theta_{l}^{1}}{2} \right) \left(\frac{A_{cl}^{2} + A_{cl}^{1}}{2} \right) \left(H_{cl}^{2} - H_{cl}^{1} \right) + \sum_{l=1}^{m} \left(\frac{\theta_{l}^{2} + \theta_{l}^{1}}{2} \right) \left(\frac{H_{cl}^{2} + H_{cl}^{1}}{2} \right) \sum_{j=1}^{d} \frac{W_{j}}{d} \left(A_{cjl}^{2} - A_{cjl}^{-1} \right).$$
(7)

The first term in the equation refers to the population shift effect that shows how changes in the distributions of the population across sub-groups contributed to the change in aggregate multidimensional chronic poverty, ΔM_c . The second and third terms account for changes in multidimensional chronic poverty within sub-groups, which is further decomposed in terms of the incidence and intensity effect in relation to each indicator in the analysis.

The Shapley decomposition is very useful in understanding the driving forces behind multidimensional chronic poverty, yet it is not free of limitations. Its main limitation is that the decomposition measure that results from eliminating each factor in succession lacks equilibrium consistency (Azevedo et al. 2013). That is, when eliminating a factor to analyse its marginal effect, the outcome is not the result of an economic equilibrium, but rather the result of a statistical exercise in which we assume, *ceteris paribus*, that we can change one factor at a time. This would not be a problem if the factors were independent of each other. In our case, when applying the Shapley decomposition to M_c as denoted in Equation (2), we note that A_c will depend on H_c by construction. Therefore, assuming a constant level of A_c when H_c changes may lead to an underestimation of the effect of A_c . In order to assess the implication of this possible bias we performed a sensitivity analysis to observe the change in the effect of A_c under different circumstances. A low variability in the effect of A_c would avoid the underestimation of A_c . Nevertheless, this exercise is specific to this data set and no generalizations can be made.

4 Application to chronic multidimensional poverty in Argentina

In this section we study the presence of chronic multidimensional poverty in Argentina and present its Shapley decomposition for the period from 2004 to 2012. We use the rotating panel EPH, which uses the 2-2-2 sampling format, that is, the survey follows a household for two consecutive periods, retrieves those households for the following two, and finally resamples them in the subsequent two. With this format, the survey allows us to follow a household at four points in time in a span of one and a half years. We use this data for illustration purposes, since the EPH database presents a wide range of variables that allow us to construct a multidimensional poverty measure. Moreover, given that the questionnaire has not changed since 2003, it allows us to keep track of the development of the chosen variables over a long span of time.

In this section we will first discuss the selection of the well-being indicators, followed by a crosssection description of the deprivation indicators at a point in time. Subsequently, we report various measures of chronic multidimensional poverty for different dimensions and time cut-offs. Finally, the Shapley decomposition of multidimensional chronic poverty is used to analyse the driving forces of chronic poverty according to a series of demographic characteristics and indicators.

Following Alkire et al. (2013), we used three dimensions: education, housing, and employment/income. We followed their selection of variables because they also considered a chronic multidimensional measure, and because they applied their research to the case of Chile, a country that has similar characteristics to Argentina. Table 1 offers a description of the indicators used and their corresponding cut-offs. For the estimation of chronic multidimensional poverty all our indicators will be equally weighted, $w_i = 1/9$.

For the education dimension we use the indicators of educational achievement, school attendance, and illiteracy. For the indicators related to housing, we employ a measure of overcrowding, a measure of shelter deprivation, and a dummy variable for the availability of a toilet in the household. Finally, when considering the income/employment measures we use the indicators of income poverty, unemployment, and quality of employment. In Table 1 we also report the raw headcounts of each variable for 2003 and 2012, and their respective change in percentage points. All variables improved with the exception of educational achievement and unemployment. The most striking change is that of income poverty, with a drop of 40 percentage points, which, in turn, could have been driven either by an increase in the per capita income or by an underestimation of the general level of prices.

Two of the great advantages of the Alkire and Foster (2011) methodology are their dimension and sub-group decomposability. We will distinguish four different household types, classified depending on the presence of children or older adults in the household. Specifically, the groups are: households with children and older adults (HH1), households with children but no older adults (HH2), households with older adults but no children (HH3), and households without children or older adults (HH4).¹

¹A household with children is defined as a household with at least one member under 12 years old. Similarly, a household with older adults is defined as a household with at least one member over 65 years old.

Dimension	Variable	Deprivation out off	% in deprivation		۸ demmi
Dimension	Variable	Deprivation cut-off	2004	2012	Δ depriv.
Education	Educational achievement	No member fulfilling the number of years of education as prescribed by law	8.55%	8.87%	0.32%
	School attendance	At least one individual in school age (6-17 years) not attending school or evidence of more than three years of educational gap	7.81%	6.56%	-1.25%
	Illiteracy	At least one member older than 17 is illiterate	4.23%	2.79%	-1.44%
Housing	Overcrowding	More than 2.5 persons per habitable bedroom	31.35%	27.15%	-4.19%
	Shelter	At least one insufficient housing material (indicators for walls, floor and roof considered)	12.66%	9.33%	-3.34%
	Toilet	If there is no toilet inside the household	10.06%	5.55%	-4.50%
Income / employment	Income	The household per capita income falls below the official poverty line	45.81%	4.97%	-40.84%
	Unemployment	No member older than 17 is employed	10.97%	12.48%	1.52%
	Quality of employment	No member employed has any kind of benefit in his or her job	43.29%	37.82%	-5.47%
	Ν	-	32772	38812	

Source: Authors' calculations using the EPH.

Before describing in further detail the household dynamics by group, we discuss the results of the entire population. In Table 2 we show the results of chronic poverty $M_c^{\alpha}(x; w, k, \tau = 4)$ as defined in Equation (2), with $\tau = 4$ and for different dimensional cut-offs. If we follow the union approach for the dimensional cut-offs, we see that, in 2004, 63.73 per cent of the population was chronically poor in at least one dimension. That percentage decreased to 51.79 per cent in 2012. Since in both years the intensity of poverty was low, around 7 per cent and 5.5 per cent for the

respective years, the censored matrix is as low as 4.49 per cent and 2.87 per cent when we consider the union approach, and 1.09 and 0.22 per cent when we consider the intersection approach.

	Pover	ty cut-off	Pover	ty cut-off	Poverty	/ cut-off	Pover	ty cut-off
	k	= 1	k	= 2	k =	= 3	k	= 4
	2004	2012	2004	2012	2004	2012	2004	2012
н	63.73%	51.79%	36.70%	22.72%	19.54%	8.53%	8.26%	1.67%
А	7.04%	5.54%	9.13%	7.80%	11.08%	9.88%	13.23%	13.28%
М	4.49%	2.87%	3.35%	1.77%	2.17%	0.84%	1.09%	0.22%

Table 2: Headcount ratio by poverty cut-off ($\tau = 4$)

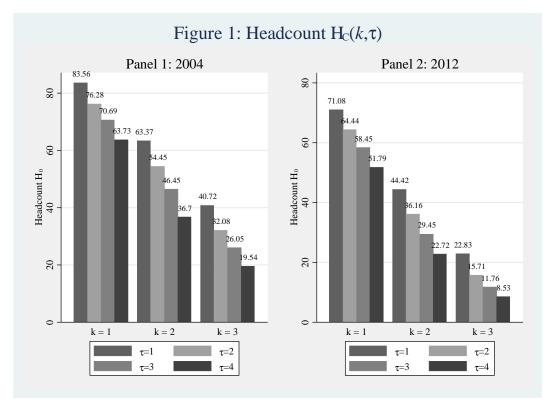
Note: H – Headcount; A – Intensity; M – Multidimensional chronic poverty Source: Author's calculations using the EPH.

A more complete image can be seen when we vary both the dimensional and the time cut-offs. Figure 1, panels 1 and 2, shows the different headcounts (not censored) for each combination of $\tau = 1,2,3,4$ and k = 1,2,3 for the years 2004 and 2012, respectively.

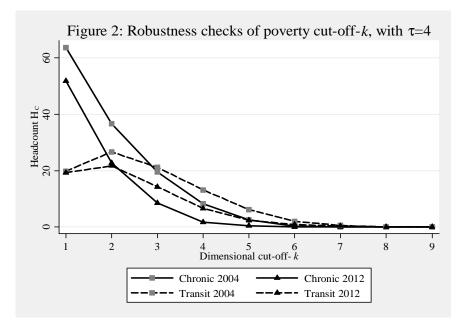
In Figure 1, panel 1, we see that, when considering the union approach, 83.56 per cent of the population in 2004 is poor in at least one dimension and at least at one point in time during the span of one and a half years. That same percentage decreases to 71.08 per cent for 2012, as seen in panel 2. At the other extreme, when considering the intersection approach for the time cut-off and using the dimensional cut-off of k = 3, we see percentages of 19.54 and 8.53, respectively, for 2004 and 2012. Note that if we fix the dimensional cut-off, the headcount does not decrease rapidly. For example, in 2012 when we have k = 1, 71.08 per cent of the population is poor with $\tau = 1$, 64.44 per cent when $\tau = 2$, 58.45 per cent when $\tau = 3$, and 51.79 per cent when $\tau = 4$. Each of these changes is around six percentage points, which is very different from the drops we see when we fix the time cut-off and vary the dimensional cut-off. These last drops are around the order of 30 percentage points. This applies for both 2004 and 2012 and for every dimensional cut-off, suggesting that given a dimensional cut-off we can see persistence in chronic poverty.

In Figure 2 we check for the robustness of the measure, considering the headcount of both chronic and transient poverty for each dimensional cut-off.

There are two important facts to point out from Figure 2. The first is that the curves for 2004 monotonically dominate the curves in 2012, that is, for each dimensional cut-off, both chronic and transient poverty were higher in 2004 than in 2012. The second is that when we have low dimensional cut-offs of one to three dimensions, the level of chronic poverty is higher or equal to the level of transient poverty. This last observation highlights the importance of considering both chronic and transient poverty measures since significant shares of the population lie in one of these forms of poverty.



Source: Author's calculations using the EPH.



Source: Author's calculations using the EPH.

It is important to mention that this counting approach does not take into consideration whether or not the household was chronically poor in the same indicator; the household could change deprivations from one quarter to another and it would be counted as chronically poor as long as it presented a greater number of deprivations than the dimensional cut-off chosen. What this measure addresses is the extent to which a household persistently experiences deprivations in the indicators mentioned. Given the results in Figure 2, from here onwards we will use a dimensional cut-off of k = 3, as this seems to be reasonable given that it represents a crossing point between chronic and transient multidimensional poverty.

4.1 Shapley decomposition

In this section we apply the decomposition described in Equation (7) in order to study the drivers of the change in multidimensional chronic poverty M_c . This decomposition will allow us to determine whether the change in M_c was due to a change in the incidence of poverty (H_c) or to a change in its intensity (A_c) . Furthermore, it will allow us to separate the marginal effect of each group and the marginal effects of each indicator.

Table 3 presents the main results in the paper, a set of drivers of the change in chronic poverty at different levels of aggregation. There are three sections on the table. First, we report the levels of poverty for each sub-group and for the total population. In the second section we describe the share of poverty by household sub-group. Finally, we describe the decomposition in Equation (7).

The columns correspond to each sub-group in the analysis and the last column to the total population. Note that in the first section the percentages represent the share of poor people from the total of each sub-group. In contrast, in the second and third sections, the percentages report the relative weight of each sub-group of the total population studied.

We describe the analysis of the first section and we observe the aggregate changes. As can be seen, the overall level of multidimensional chronic poverty M_c decreased. When comparing the headcount H_c and the intensity of poverty A_c , we see that the intensity of poverty remains almost the same, reducing 1.2 percentage points, whereas the overall headcount reduced 11.01 percentage points. Nevertheless, this varies for each household group. Although all the groups improved, the third household group experienced poverty improvements relative to other household groups.

In the second section we see the share of poverty by household sub-group. In 2004, households with older adults represented 16.27 per cent of the population, but represented 19.11 per cent of the poor. For 2012, their share in the population did not change significantly (0.65 per cent), but their share in the poor population almost doubled to 39.61 per cent. On the other hand, sub-group two, that is, households with children only, experienced the biggest improvement. Their headcount reduced 16.53 percentage points and their share of the total poor population decreased 17.51 percentage points (from 65.46 to 47.95 per cent). It is still the group that presents the highest incidence of poverty, but it did improve significantly in the period studied.

In the third part of the table we see the results of the Shapley decomposition, which, as mentioned before, has the virtue of being an exact additive decomposition of changes in poverty. This decomposition corresponds to Equation (7). For its description we will partition the analysis by the level of disaggregation, as presented in the equation. We will first consider the overall contribution of each sub-group and then separate this effect into the demographic and within-group effect. Third, we will decompose the within-group effect in the incidence and intensity effect and, finally, we will analyse the relative importance of each indicator in the incidence effect.

Table 3: Shapley decomposition

Decomposition variation in multidimensional poverty (2004 and 2012)					
	HH1	HH2	ННЗ	HH4	Total
Multidimensional chronic poverty					
2004					
Headcount (H)	13.35%	25.07%	22.96%	8.27%	19.54%
Intensity (A)	11.38%	11.39%	9.81%	11.33%	11.08%
Mult. chronic poverty (M)	1.52%	2.85%	2.25%	0.94%	2.17%
2012					
Headcount (H)	2.99%	8.54%	19.98%	3.03%	8.53%
Intensity (A)	9.93%	10.48%	8.98%	10.59%	9.88%
Mult. chronic poverty (M)	0.30%	0.89%	1.79%	0.32%	0.84%
% share					
2004					
Population	6.13%	51.03%	16.27%	26.57%	100.00%
Mult. headcount ratio (H)	4.18%	65.46%	19.11%	11.24%	99.99%
2012					
Population	6.91%	47.93%	16.92%	28.24%	100.00%
Mult. headcount ratio (H)	2.42%	47.95%	39.61%	10.02%	100.00%
Decomposition					
Total % contribution	5.49%	77.77%	4.75%	11.98%	100.00%
- Demographic effect	-0.53%	4.39%	-1.00%	-0.79%	2.07%
- Within group effect	6.03%	73.38%	5.75%	12.77%	97.93%
- Incidence	5.44%	67.67%	3.52%	11.91%	88.54%
- Intensity	0.58%	5.71%	2.24%	0.86%	9.40%
- Educ. achievement	0.00%	0.00%	-1.05%	-0.02%	-1.07%
- School attendance	0.24%	-0.47%	0.05%	-0.02%	-0.20%
- Illiteracy	-0.11%	0.57%	0.32%	-0.36%	0.42%
- Overcrowding	-0.02%	0.62%	0.86%	-0.17%	1.29%
- Shelter	0.03%	-2.96%	0.28%	0.06%	-2.60%
- Toilet	-0.22%	-0.99%	0.28%	-0.12%	-1.04%
- Income	0.89%	11.96%	3.06%	2.33%	18.23%
- Unemployment	-0.22%	-2.42%	-1.26%	-0.73%	-4.63%
- Quality of employment	-0.01%	-0.59%	-0.31%	-0.10%	-1.01%

HH1 = households with children and older adults

HH2 = households with children but without older adults

HH3 = households without children but with older adults

HH4 = households without children and without older adults

Source: Authors' calculations using the EPH.

In the third part of the table we see the results of the Shapley decomposition, which, as mentioned before, has the virtue of being an exact additive decomposition of changes in poverty. This decomposition corresponds to Equation (7). For its description we will partition the analysis by the level of disaggregation, as presented in the equation. We will first consider the overall contribution of each sub-group and then separate this effect into the demographic and within-group effect. Third, we will decompose the within-group effect in the incidence and intensity effect and, finally, we will analyse the relative importance of each indicator in the incidence effect.

When observing the contribution of each sub-group to the change in poverty, we see that households with children but without older adults (HH2) fare the best, as this sub-group contributes 77 per cent of the total change in multidimensional chronic poverty, a much higher percentage than its population share. In contrast, the rest of the households contributed less than their population shares.

We further decompose the total contribution of each group in their demographic and within-group effects. The demographic group effect reflects the changes in poverty due to changes in population shares in each household group, holding the poverty level within a household sub-group constant.

The within-group effect reflects the changes in poverty that would have occurred if the population shares of the household sub-groups had not changed. We can observe that most of the changes in chronic poverty are driven by the within-group effect as it accounts for 98 per cent of the total contributions. For households with children and older adults (HH1), households with older adults and no children (HH3), and households with neither older adults nor children, the demographic effect barely decreased the change in poverty, whereas for households with children but without older adults (HH2) it contributed 4 per cent to the overall change in poverty.

On a second level of disaggregation, following Equation (6) by sub-group, we can assess whether the change in the within-group effect was due to a change in the incidence of poverty or if it was due to a change in the intensity of poverty. The biggest change was due to the incidence of poverty, the overall effect of which accounts for 88.54 per cent of the change in M_c . In all the household sub-groups the changes in incidence are much higher than the intensity effect, with the exception of households with older adults and no children (HH3) for which the incidence and intensity effects are almost the same. This household sub-group presents the lowest improvement in incidence, especially when considering its population share of 16 per cent.

Finally, we disaggregate the intensity effect to study the marginal effect of each indicator. The main indicator that is driving the improvement in the intensity effect is income. The rest of the variables contributed marginally to an increase in the intensity of chronic poverty or did not have any effect at all. The unemployment indicator fared worst, followed by a worsening in the shelter conditions. This worsening of the employment condition may be the reason why the households with older adults performed relatively poorly. Again, the leading sub-group is the households with children but without older adults (HH2). This consistent pattern shows the difference in mobility between these household sub-groups, where the households that have children but not older adults are moving upward at a faster rate than the rest of the household sub-groups.

As mentioned before, the results in Table 3 must be interpreted with caution since there might be a risk of underestimating the effect of A_c in the analysis. We performed a series of sensitivity analyses on the magnitudes of A_c in order to assess the impact of this bias. The results are presented in Appendix Table A.1. Departing from the fact that we are not able to observe, specifically, the level of A_c for the individuals who left chronic poverty from 2004 to 2012, we proceed to measure the level of A_c for those individuals who were more likely to leave chronic poverty by comparing the level of A_c for alternative cut-offs. The headcount levels are also shown in parenthesis. We can see that the magnitude of A_c does not change much for alternative cut-off levels, while the headcount levels drop significantly. This may imply that the bulk of the chronically poor are already near the poverty cut-offs, thus, the individuals who leave poverty seem to have the same level of A_c as those who remain in it.

Additionally we also present in Appendix Table A.2 a bootstrap analysis of the variability of the effect of A_c . Having fixed H_c and A_c in period t = 2, we resampled individuals in period t = 1 and implemented the decomposition as in equation (6) and report the mean and confidence intervals of the bootstraps at different percentage levels of sampled individuals. Again, we find a low variability of the intensity effect, A_c , with confidence intervals that are not wide. This provides additional support for the decomposition analysis we implemented.

4.2 Shapley decomposition over time

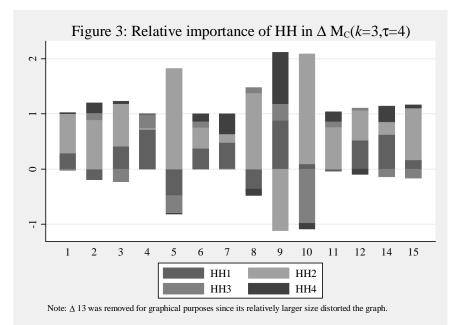
As described earlier, the EPH is a rotating panel, which means that it contains superimposed panels over time. This characteristic allows us to derive the Shapley decomposition for different periods and, therefore, we can observe how the drivers of chronic multidimensional poverty change over time.

Thus, in Figure 3 we follow the relative importance of each household group in the change of multidimensional chronic poverty. Notice that each bar adds up to one. On the vertical axis we measure the contribution of each factor to total change in M_c , and on the horizontal axis we arrange the different periods for which we applied the decomposition. The first observation represents the decomposition when we consider the first and second panel of the EPH, that is, comparing the panel that began in January 2004 and finished in June 2005 with the panel that began in July 2004 and finished in December 2005. The second observation refers to the subsequent comparison.

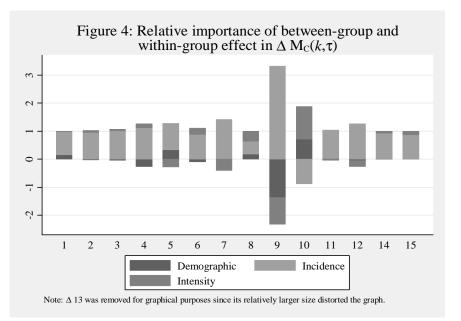
Figure 3 demonstrates that the relative importance of each household sub-group presents some patterns. On the one hand, households with children but without older adults (HH2) constantly drove the decrease in chronic poverty, but with various degrees of influence, and households with children and older adults (HH1) were the second in importance in this respect. On the other hand, households without children and older adults (HH4) had little weight until the first half of the period analysed, and increased in significance toward the second half. Finally, households with older adults (HH3) were the least influential throughout the whole period.

In each period (with the exception of periods 3 and 13), chronic multidimensional poverty decreased, so the fact that in period 3 household sub-group two (HH2) presents positive values actually reflects that this group increased the level of chronic multidimensional poverty.

In Figure 4 we consider the relative importance of the demographic effect and within-group effect, and we further decompose the within-group effect in the incidence and intensity effect. As can be seen, the incidence effect dominates over the others, and its relative importance is around 90 per cent of the total change in multidimensional chronic poverty. We can see that the incidence effect varies more in relation to the intensity effect, which shows a more stable pattern. This may suggest that the incidence of poverty is more volatile, probably due to the fact that it is more sensitive to shocks in the economy. The results found here help to support the validity of the results we discussed in Table 3, which assumes a constant level of intensity when analysing the changes in incidence effect.



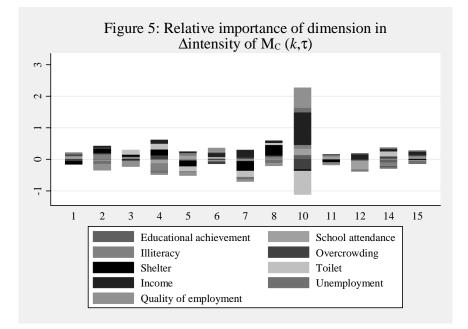
Source: Authors' calculations using the EPH.



Source: Authors' calculations using the EPH.

Finally, in Figure 5 we decompose the intensity effect to study the relative importance of each indicator. Notice that the data in the graph represent the relative importance over the change in multidimensional chronic poverty and also that periods 9 and 13 were not included for illustration purposes. The message of this figure is that the relative importance of the level of intensity of each indicator varies greatly across time, and it is not possible to point to a single indicator or a couple of indicators that may be leading to the change in A_c and subsequently in M_c . This reflects the complexity of the problem of chronic multidimensional poverty. Nevertheless, it is important to remember that we are analysing the relative importance for each group of a small absolute change. We expect this graph to be much more informative when the absolute change in poverty is greater. When considering the same graph by household group (not shown), the lack of patterns remains, and the weight of the indicators still varies greatly over time.

Therefore, at least in analysing the change in poverty, the relative importance of household groups is more informative than the relative importance of each indicator. This may denote that the focalization of public policy programmes for chronic poverty should prioritize the household subgroup, and then target the most relevant indicator. In the case of Argentina, households with older adults were the most vulnerable, and they performed poorly in the unemployment indicators.



Source: Authors' calculations using the EPH.

5 Conclusions

The central purpose of this paper has been to propose a coherent framework that allows us to analyse the factors that are driving the change of multidimensional chronic poverty. For that, we build on Alkire et al. (2013) to apply the Shapley (1953) decomposition approach to isolate the contribution of different dimensions and sub-group populations. The advantage of the Shapley decomposition is that it allows us to identify relevant contributions in a vast array of factors and dimensions while having an exact additive sum. The decomposition obtained is also path independent, in the sense that the values of the contributions are independent in the order in which the factors appear in the sequence. We distinguish between changes due to the within-group chronic poverty effect associated with changes in the headcount of the chronically poor and changes due to between-group effects characterized by the average measure of the intensity of chronic poverty over time.

We have applied the methodology to panel data from Argentina's EPH during October 2004 and May 2012. We found that the general level of multidimensional chronic poverty decreased from 2.17 to 0.84 per cent. We find that although the level of multidimensional chronic poverty decreased, the overall picture is not so optimistic when analysing all the variables. We observe that the incidence effect decreased significantly, while the intensity effect remained relatively constant. With regard to intensity, the income variable drove the change, but not all of the other variables fared well. The variables of unemployment, shelter and educational achievement performed worst in regard to intensity of poverty.

The proposed decomposition reveals that households with children but without older adults performed best and drove the change in multidimensional chronic poverty. In contrast, households with older adults performed the worst, and were the least influential in the change of poverty, showing great persistence of poverty in time. If we follow the decomposition over time, the relative importance of the indicators' intensity varies greatly, while the relative importance of the household sub-groups presents some patterns. The resultant characterization of the evolution of chronic poverty suggests that focalization by household sub-groups rather than by indicators may prove to be more effective in policy programmes concerned with the reduction of chronic poverty.

Appendix

A and H levels	2004	2012
K = 2, t = 4	9.13 (36.7%)	7.8 (22.7%)
K = 3, t = 3	8.03 (26.05%)	8.13 (11.7%)
K = 3, t = 4	11.08 (19.5%)	9.88 (8.5%)
K = 4, t = 4	13.22 (8.2%)	13.28 (1.6%)
K = 5, t = 4	15.17 (2.5%)	15.8 (.3%)

Appendix Table A.1: The intensity effect, Ac*

Note: * Headcount levels are shown in parenthesis. Source: Author's calculations using the EPH.

Appendix Table A.2. Bootstrapping: intensity effect

% sample	Ν	Mean	CI 95%	
60	300	7.894	7.853	7.934
65	300	7.888	7.848	7.928
70	300	7.898	7.864	7.931
75	300	7.868	7.835	7.901
80	300	7.876	7.849	7.903
85	300	7.889	7.867	7.911
90	300	7.876	7.859	7.893
100		7.876		

Source: Author's calculations using the EPH.

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