

Huber, Florian; Kaufmann, Daniel

Working Paper

Trend fundamentals and exchange rate dynamics

KOF Working Papers, No. 393

Provided in Cooperation with:

KOF Swiss Economic Institute, ETH Zurich

Suggested Citation: Huber, Florian; Kaufmann, Daniel (2015) : Trend fundamentals and exchange rate dynamics, KOF Working Papers, No. 393, ETH Zurich, KOF Swiss Economic Institute, Zurich,
<https://doi.org/10.3929/ethz-a-010513063>

This Version is available at:

<http://hdl.handle.net/10419/126241>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

KOF Working Papers

Trend Fundamentals and Exchange Rate Dynamics

Florian Huber and Daniel Kaufmann



KOF

KOF Swiss Economic Institute
ETH Zurich
LEE G 116
Leonhardstrasse 21
8092 Zurich
Switzerland

Phone +41 44 632 42 39
Fax +41 44 632 12 18
www.kof.ethz.ch
kof@kof.ethz.ch

Trend Fundamentals and Exchange Rate Dynamics*

Florian Huber[†] and Daniel Kaufmann[‡]

8 September, 2015

Abstract

We estimate a multivariate unobserved components-stochastic volatility model to explain the dynamics of a panel of six exchange rates against the US Dollar. The empirical model is based on the assumption that both countries' monetary policy strategies may be well described by Taylor rules with a time-varying inflation target, a time-varying natural rate of unemployment, and interest rate smoothing. The estimates closely track major movements along with important time-series properties of the real and nominal exchange rates across all currencies considered. The model generally outperforms a simple benchmark model that does not account for changes in trend inflation and trend unemployment.

JEL classification: F31, E52, F41, C5, E31.

Keywords: Exchange rate models, trend inflation, natural rate of unemployment, Taylor rule, unobserved components-stochastic volatility model.

*The authors thank Stefan Neuwirth, Samad Sarferaz and participants at the KOF Brown Bag Seminar for helpful comments and suggestions. Any views expressed in this paper represent those of the authors only and not necessarily of the Oesterreichische Nationalbank or the Eurosystem.

[†]Oesterreichische Nationalbank (OeNB), Otto-Wagner Platz 1, AT-1090 Vienna, Austria. florian.huber@oenb.at

[‡]KOF Swiss Economic Institute, ETH Zurich, Leonhardstrasse 21, CH-8092 Zurich, Switzerland. kaufmann@kof.ethz.ch

1 Introduction

To what extent do economic fundamentals explain exchange rate movements? Following the seminal work by Meese and Rogoff (1983), a wealth of studies has aimed to answer this question by comparing the out-of-sample predictive ability of economic exchange rate models to random-walk forecasts, with mixed success (see Rossi 2013, for an overview). However, as Engel and West (2005) show, the random walk property of the exchange rate does not imply that economic fundamentals are irrelevant for exchange rate movements. In fact, they stress that the current exchange rate depends on future expected fundamentals. If some fundamentals are non-stationary, and the discount factor associated with these expectations is large, unpredictable shocks to the non-stationary fundamentals will dominate and lead to a persistent process for the exchange rate, which is almost indistinguishable from a random walk in finite samples.

In this paper, we build on this insight and show that changes in non-stationary trend inflation rates play a relevant role for explaining bilateral exchange rate dynamics. To do so, we derive a partial equilibrium expression for the bilateral real exchange rate assuming that each countries' central bank targets short-term interest rates according to a Taylor rule with a time-varying inflation target and a time-varying natural rate of unemployment. Combining these Taylor rules with a no-arbitrage condition reveals that the current real exchange rate is determined by future expected trend inflation, inflation gaps, unemployment gaps, and short-term interest rates. To estimate these trends and gaps and to derive the corresponding expectations, we use an unobserved components-stochastic volatility (UC-SV) model, similar to Stock and Watson (2007), in which trend inflation and trend unemployment follow non-stationary processes with stochastic volatility. This choice is common in a recent literature estimating trend inflation over different monetary policy regimes (see Ascari and Sbordone 2014, and references therein).

Our findings can be summarized as follows. First, the UC-SV model captures the major up- and downturns of the bilateral real exchange rate against the US Dollar for a panel comprising of six economies during the post-Bretton Woods era. In fact, the correlations between the model-based predictions and the actual real exchange rates are as high as

0.56. A benchmark model, which is estimated on the same information set but does not discriminate between trend and gap components, yields significantly lower correlations comparable to existing studies (see Engel and West 2006, Mark 2009). Second, the UC-SV model is capable of reproducing all major long-run trends of the nominal exchange rates over the last 40 years. Third, the model successfully mimics the actual exchange rates with respect to several key time series properties. More specifically, we accurately reproduce the persistence of the real exchange rates and the correlations with other macroeconomic variables.

In what follows, Section 2 motivates the UC-SV model by deriving a partial equilibrium expression for the real exchange rate in terms of future expected fundamentals. Then, Section 3 outlines the empirical strategy adopted along with the corresponding prior specification. Finally, Section 4 presents the empirical results and the last section concludes.

2 Theoretical framework

Following Engel and West (2006) we derive an expression for the real exchange rate in terms of future expected fundamentals if monetary policy in two countries is characterized by Taylor rules. All equations are shown in log-linearized terms. Let the short-term policy interest rate i_t in the home economy be determined as

$$i_t = i_{t-1} + \gamma_\pi E_t \hat{\pi}_{t+1} + \gamma_u E_t \hat{u}_{t+1} + \gamma_q q_t + \varepsilon_t. \quad (1)$$

The central bank in the home economy targets the short-term interest rate as a function of deviations of expected inflation from the target ($E_t \hat{\pi}_{t+1}$), deviations of the expected unemployment rate from its natural level ($E_t \hat{u}_{t+1}$) and of the lagged interest rate, whereas ε_t is a monetary policy innovation.¹ The inflation and unemployment gaps are defined as $\hat{\pi}_t = \pi_t - \bar{\pi}_t$ and $\hat{u}_t = u_t - \bar{u}_t$, respectively. Therefore, the inflation target ($\bar{\pi}_t$) as well as the natural rate of unemployment (\bar{u}_t) change over time. As is standard in the literature $\gamma_\pi > 0$, $\gamma_u < 0$ such that the central bank increases its policy interest rate in response to

¹This specification reflects studies that find movements in trend inflation over time (Ascari and Sbordone 2014), changes in the non-accelerating inflation rate of unemployment over time (Gordon 1998) and relevant interest-rate smoothing behavior of central banks (Coibion and Gorodnichenko 2012).

a higher inflation gap or a lower unemployment gap.

We follow Engel and West (2006) and assume that the central bank responds to the real exchange rate defined as $q_t = e_t - p_t + p_t^*$. The nominal exchange rate (e_t) is expressed as the price of one unit of the foreign currency in terms of domestic currency such that a rise in the exchange rate implies a depreciation of the home currency. Furthermore, p_t and p_t^* denote the domestic and foreign price levels, respectively. We assume that $\gamma_q > 0$, thus, the central bank lowers the interest rate when the exchange rate is appreciated in real terms.

The central bank in the foreign economy targets the short-term interest rate using an analogous rule, except that it does not respond to the real exchange rate, where foreign variables are labeled by an asterisk:²

$$i_t^* = i_{t-1}^* + \gamma_\pi E_t \hat{\pi}_{t+1}^* + \gamma_u E_t \hat{u}_{t+1}^* + \varepsilon_t^*. \quad (2)$$

Furthermore, we assume that an uncovered interest parity relationship holds period-by-period:³

$$i_t - i_t^* = E_t[\Delta q_{t+1} + \pi_{t+1} - \pi_{t+1}^*]. \quad (3)$$

Replacing the interest rate differential by the two policy rules and rearranging terms we obtain:

$$\begin{aligned} q_t &= \rho E_t q_{t+1} + (1 - \gamma_\pi) \rho E_t (\hat{\pi}_{t+1} - \hat{\pi}_{t+1}^*) + \rho E_t (\bar{\pi}_{t+1} - \bar{\pi}_{t+1}^*) \\ &\quad - \gamma_u \rho E_t (\hat{u}_{t+1} - \hat{u}_{t+1}^*) - \rho (i_{t-1} - i_{t-1}^*) - \rho (\varepsilon_t - \varepsilon_t^*). \end{aligned} \quad (4)$$

with $\rho = \frac{1}{1+\gamma_q}$. Solving the equation forward allows to express the real exchange rate in terms of future expected fundamentals. This expression is of the present-value solution of Engel and West (2005) taking into account that trend inflation may have different time-series properties than the inflation gap:

²The Taylor rule parameters in the home and foreign economy are homogeneous for ease of exposition. In the empirical application we relax this restriction.

³A risk premium term would be straightforward to incorporate, see Engel and West (2006).

$$\begin{aligned}
q_t = & \rho^{J+1} E_t q_{t+J+1} + (1 - \gamma_\pi) E_t \sum_{j=0}^J \rho^{j+1} (\hat{\pi}_{t+j+1} - \hat{\pi}_{t+j+1}^*) \\
& + E_t \sum_{j=0}^J \rho^{j+1} (\bar{\pi}_{t+j+1} - \bar{\pi}_{t+j+1}^*) - E_t \sum_{j=0}^{J-1} \rho^{j+1} (i_{t+j} - i_{t+j}^*) \\
& - \gamma_u E_t \sum_{j=0}^J \rho^{j+1} (\hat{u}_{t+j+1} - \hat{u}_{t+j+1}^*) - \rho(i_{t-1} - i_{t-1}^*) - \rho(\varepsilon_t - \varepsilon_t^*).
\end{aligned} \tag{5}$$

Despite the partial equilibrium nature of the analysis some interesting insights emerge. First, Engel et al. (2008) emphasize that, if the Taylor principle holds in a Taylor rule without interest rate smoothing, an increase in the expected inflation gap at home relative to the foreign economy implies a real appreciation. However, our specification implies that an increase in the expected home inflation trend relative to the foreign economy results in a real depreciation, everything else being equal. Second, as trend inflation is usually modeled as a non-stationary process, the near-random walk behaviour of the real exchange rate is more likely driven by the inflation trend than the inflation gap. The high persistence in combination with a high discount factor implies that a change in the trend inflation rate has a larger effect on the current exchange rate than a change in the inflation gap.

To map the theoretical equation to empirical data, we need to form expectations about nominal short-term rates, the inflation and unemployment gaps, as well as future trend inflation. In what follows we outline the empirical strategy to model the future expected evolution of these measures.

3 Empirical strategy

We propose a simple multivariate unobserved components-stochastic volatility (UC-SV) model to describe the dynamics of the fundamentals. The model may be viewed as an open economy variant of earlier UC-SV specifications that aim to model inflation and unemployment dynamics by decomposing the respective variables in non-stationary trend and stationary gap components (see e.g. Gordon 1998, Stock and Watson 2007, Stella and Stock 2012).

3.1 Unobserved components-stochastic volatility model

Let us store the observed inflation and unemployment series measured at time $t = 1, \dots, T$ in a 4×1 vector $\mathbf{x}_t = (\pi_t, \pi_t^*, u_t, u_t^*)'$. We assume \mathbf{x}_t may be decomposed as follows

$$\mathbf{x}_t = \bar{\mathbf{f}}_t + \hat{\mathbf{f}}_t + \boldsymbol{\varepsilon}_t, \quad (6)$$

$$\bar{\mathbf{f}}_t = \bar{\mathbf{f}}_{t-1} + \bar{\boldsymbol{\eta}}_t, \quad (7)$$

$$\hat{\mathbf{f}}_t = \boldsymbol{\Phi} \hat{\mathbf{f}}_{t-1} + \hat{\boldsymbol{\eta}}_t, \quad (8)$$

with $\bar{\mathbf{f}}_t = (\bar{\pi}_t, \bar{\pi}_t^*, \bar{u}_t, \bar{u}_t^*)$ being a 4×1 vector of latent trend components of inflation and unemployment at home and abroad. Likewise, $\hat{\mathbf{f}}_t = (\hat{\pi}_t, \hat{\pi}_t^*, \hat{u}_t, \hat{u}_t^*)$ denotes a 4×1 vector of (stationary) latent gap components of inflation and unemployment. We assume that $\boldsymbol{\Phi} = \text{diag}(\phi_\pi, \phi_\pi^*, \phi_u, \phi_u^*)$ is a 4×4 matrix of autoregressive coefficients with absolute value below unity. This ensures that $\hat{\mathbf{f}}_t$ is mean reverting and thus permits us to interpret $\hat{\mathbf{f}}_t$ as a vector containing the inflation and unemployment gap, respectively. Finally, $\boldsymbol{\zeta}_t = (\boldsymbol{\varepsilon}'_t, \bar{\boldsymbol{\eta}}'_t, \hat{\boldsymbol{\eta}}'_t)'$ is a vector white noise error with time-varying variance-covariance $\boldsymbol{\Sigma}_t = \text{diag}(\mathbf{V}_t, \bar{\mathbf{V}}_t, \hat{\mathbf{V}}_t)$ being a diagonal matrix with typical element denoted by σ_{it}^2 ($i = 1, \dots, 9$).⁴ Following Kastner and Frühwirth-Schnatter (2014) we assume that the logarithm of σ_{it}^2 , $h_{it} = \log(\sigma_{it}^2)$, evolves according to

$$h_{it} = \mu_i + \rho_i(h_{it-1} - \mu_i) + \sqrt{\vartheta_i}v_{it}, \quad (9)$$

where μ_i is the level of the log-volatility, $\rho_i \in (-1, 1)$ denotes the autoregressive parameter and ϑ_i denotes the variance of the log-volatility. This choice ensures that the volatility is bounded in the limit and rules out odd behavior related to random walk state equations for log-volatilities.

The UC-SV model explicitly discriminates between components that are non-stationary, capturing trends in the respective macroeconomic variable, and stationary processes that capture the high frequency behavior. Moreover, to improve the fit of the model we also assume that all components are allowed to follow distinct stochastic-volatility processes.

⁴This implies that $\mathbf{V}_t, \bar{\mathbf{V}}_t, \hat{\mathbf{V}}_t$ are also diagonal matrices and the errors in the observation and state equations are uncorrelated.

The specification described by Eqs. (6) – (8) is closely related to the model put forward by Stella and Stock (2012). However, while they assume that the inflation gap is proportional to the unemployment gap we allow for more flexibility by assuming that the inflation gap is allowed to have its own dynamics, independent from the unemployment gap.

3.2 Relation to the real exchange rate

We can derive an approximation that maps the model described in subsection 3.1 to the theoretical exchange rate model. If we let $\rho \rightarrow 1$ and under the expectations hypothesis, $E_t \sum_{j=0}^{J-1} (i_{t+j} - i_{t+j}^*)$ is J times the interest rate differential for J -period bonds which we denote as $J(b_{J,t} - b_{J,t}^*)$. Furthermore, if we assume that the real exchange rate, although potentially very persistent, is stationary we have for large J that $E_t \rho^{J+1} q_{t+J+1} \approx \bar{q}$ even for $\rho \rightarrow 1$. Finally, under the structure of the UC-SV model we have that expectations of the gap components are formed as $E_t \hat{\pi}_{t+j} = \phi_\pi^j \hat{\pi}_t$, $E_t \hat{\pi}_{t+j}^* = \phi_\pi^{*j} \hat{\pi}_t^*$, $E_t \hat{u}_{t+j} = \phi_u^j \hat{u}_t$ and $E_t \hat{u}_{t+j}^* = \phi_u^{*j} \hat{u}_t^*$. Since the trend components follow a random walk process the expectations are given by $E_t \bar{\pi}_{t+j} = \bar{\pi}_t$, $E_t \bar{\pi}_{t+j}^* = \bar{\pi}_t^*$, $E_t \bar{u}_{t+j} = \bar{u}_t$ and $E_t \bar{u}_{t+j}^* = \bar{u}_t^*$.

It follows that, for large J we can approximate the exchange rate relationship in Eq. (5) as:

$$\begin{aligned} q_t \approx & \bar{q} + \frac{1 - \gamma_\pi}{1 - \phi_\pi} (\hat{\pi}_t - \hat{\pi}_t^*) - \frac{\gamma_u}{1 - \phi_u} (\hat{u}_t - \hat{u}_t^*) \\ & + (J + 1)(\bar{\pi}_t - \bar{\pi}_t^*) - J(b_{J,t} - b_{J,t}^*) - (i_{t-1} - i_{t-1}^*) - (\varepsilon_t - \varepsilon_t^*). \end{aligned} \quad (10)$$

The terms involving the gap components are exact for $J \rightarrow \infty$ and $\rho \rightarrow 1$. However, it is worth noting that these approximating assumptions are accurate even for finite J and relatively persistent processes.⁵ In the empirical specification, we relax the assumption of parameter homogeneity across both countries' Taylor rules. The empirical model that relates the system described in the previous subsection to Eq. (10) is therefore given by

⁵For an AR(1) process with autoregressive parameter $\rho = 0.97$ and forecast horizon $J = 120$, implying that $b_{J,t} - b_{J,t}^*$ is the difference in a ten-year government bond yield, the approximation error for the gap components amounts to 2.6% in terms of the correct finite-horizon expectation.

$$q_t = \mathbf{X}_t \boldsymbol{\beta} + \nu_t, \quad (11)$$

with

$$\begin{aligned} \mathbf{X}_t = & [1, i_{t-1}, i_{t-1}^*, \frac{\hat{\pi}_t}{1 - \phi_\pi}, \frac{\hat{\pi}_t^*}{1 - \phi_\pi^*}, \frac{\hat{u}_t}{1 - \phi_u}, \frac{\hat{u}_t^*}{1 - \phi_u^*}, \\ & (J+1)\bar{\pi}_t, (J+1)\bar{\pi}_t^*, Jb_{Jt}, Jb_{Jt}^*], \end{aligned} \quad (12)$$

and $\nu_t \sim \mathcal{N}(0, \sigma_\nu^2)$ being a homoscedastic white noise error term. While it would be straightforward to allow for stochastic volatility in Eq. (11) we leave this possibility aside because we are mainly interested in capturing the dynamics of the exchange rate related to the first moment of the corresponding predictive density.

3.3 Prior setup and posterior simulation

The approach to estimation and inference is Bayesian. Thus we have to specify suitable prior distributions for all coefficients of the UC-SV model.

Point of departure is a normally distributed prior for the initial value of $\mathbf{f}_t = (\bar{\mathbf{f}}_t', \hat{\mathbf{f}}_t')'$,

$$\mathbf{f}_1 \sim \mathcal{N}(\mathbf{0}, \mathbf{V}_f). \quad (13)$$

Here \mathbf{V}_f is a diagonal prior variance-covariance matrix where we set the diagonal elements equal to ten, implying that we are relatively uninformative about the specific value of the initial state of the system.

For the diagonal elements of $\boldsymbol{\Phi}$ we also impose a normally distributed prior. More specifically, we set

$$\phi_{ii} \sim \mathcal{N}(\underline{\phi}_{ii}, v_{\phi ii}) \text{ for } i = 1, \dots, 4, \quad (14)$$

with $\underline{\phi}_{ii}$ and $v_{\phi ii}$ being prior mean and variance, respectively. We center the prior means associated with the inflation gap to 0.75 and the corresponding prior variance to 0.001.⁶ In addition, we set the prior mean related to the unemployment gap to 0.99, with prior variance set equal to 0.001. This tight prior implies that the inflation gap is less

⁶This is broadly consistent with findings on the persistence of the inflation gap for the US before the Great Moderation (see Cogley and Sbordone 2008, Cogley et al. 2010).

persistent than the unemployment gap. A prior setup that is relatively uninformative on the autoregressive coefficients of the gap components yields results that are qualitatively similar. However, inspection of the posterior draws reveals that the likelihood is relatively uninformative on the persistence, and we thus experimented with different values of the parameters for the US to match the results presented in Stella and Stock (2012).

For the priors on the level of the log-volatility μ_i we impose a normal prior with mean $\underline{\mu}_i$ and variance \underline{v}_{μ_i} ,

$$\mu_i \sim \mathcal{N}(\underline{\mu}_i, \underline{v}_{\mu_i}). \quad (15)$$

We set $\underline{\mu}_i = 0$ and $\underline{v}_{\mu_i} = 10^2$ for $i = 1, \dots, 9$ to render this prior effectively uninformative. In addition, we impose a Beta prior on the persistence parameter ρ_i

$$\frac{\rho_i + 1}{2} \sim \mathcal{B}(b_0, b_1), \quad (16)$$

where we set $b_0 = 25$ and $b_1 = 5$ for all i leading to a prior mean of 0.83 with prior standard deviation of 0.07, thus placing considerable prior mass on high persistence regions of ρ_i . Note that this choice proves to be quite influential in practice since the likelihood typically carries little information about the persistence of the log-volatility.

Following Kastner and Frühwirth-Schnatter (2014) we use a non-conjugate Gamma prior on the variance of the log-volatility,

$$\vartheta_i \sim \mathcal{G}(1/2, \frac{1}{2B_{\vartheta}}). \quad (17)$$

The hyperparameter B_{ϑ} controls the tightness of the prior, where it is possible to show this prior implies

$$\pm\sqrt{\vartheta_i} \sim \mathcal{N}(0, B_{\vartheta}). \quad (18)$$

In the empirical application we set B_{ϑ} equal to unity. After experimenting with different values of B_{ϑ} the specific choice of this hyperparameter proves to be rather unimportant in the present application. This prior setup has been motivated in Frühwirth-Schnatter and Wagner (2010) and provides several convenient properties. For instance, the Gamma prior does not bound ϑ_i away from zero and thus induces more shrinkage as the typical

conjugate inverted Gamma prior.

For the elements of β , denoted as β_i , we use a normal prior with mean $\underline{\beta}_i$ and variance \underline{v}_{β_i} ,

$$\beta_i \sim \mathcal{N}(\underline{\beta}_i, \underline{v}_{\beta_i}). \quad (19)$$

We center the prior on the values analysed by Giannoni (2014) for a quasi-optimal Taylor rule with interest-rate smoothing ($\gamma_\pi = \gamma_\pi^* = 0.64$, $\gamma_u = \gamma_u^* = -0.33$). For the remaining coefficients, we center the prior on the values implied by Eq. (10). However, because this is only a partial equilibrium expression for the real exchange rate we set the prior variance equal to ten to be rather uninformative.

Finally, we use a inverted Gamma prior for σ_ν^2 ,

$$\sigma_\nu^2 \sim \mathcal{IG}(c_0, c_1), \quad (20)$$

where c_0 and c_1 are equal to 0.001 to render this prior effectively non-influential.

The Markov chain Monte Carlo algorithm iterates between the following steps:

- Simulate the full history of \mathbf{f}_t , denoted as $\mathbf{f}^T = (\mathbf{f}_1, \dots, \mathbf{f}_T)'$ conditional on all other parameters and the data using the well-known algorithm developed by Carter and Kohn (1994) and Frühwirth-Schnatter (1994).
- The parameters of the log-volatility in Eq. (9) and the full history of log-volatilities $h_i^T = (h_{i1}, \dots, h_{iT})'$ are simulated by means of the algorithm spelled out in Kastner and Frühwirth-Schnatter (2014), which proves to be an efficient alternative to other popular algorithms.⁷
- The autoregressive parameters of the state equations in Eq. (7) and Eq. (8) are sampled through Gibbs steps, sampling from Gaussian distributions. To ensure stationarity we impose the constraint that all draws have to be smaller than unity in absolute values.
- Similarly, given the conjugacy of the prior setup employed, β is simulated from a normal distribution with well-known posterior mean and variance.
- Finally, σ_ν^2 is sampled with a Gibbs step by noting that the conditional posterior is of a well-known form, namely a inverted Gamma distribution.

⁷This step is implemented using the R package *stochvol* (Kastner 2015a,b).

In the empirical application we repeat this algorithm 30,000 times and discard the first 15,000 iterations as burn-in. Moreover we impose the restriction that the variance of the unemployment gap at home and abroad equals a 0.3. We fix the variance of \hat{u}_t and \hat{u}_t^* since allowing for stochastic volatility in the measurement error and the gap components separately typically leads to empirical problems. Again, setting the variance equal to 0.3 is predicated by calibrating the model to match the trend unemployment rate and unemployment gap estimated by previous studies for the US.

4 Results

We estimate the model for the US Dollar against the currencies of a panel of six economies: Germany, UK, Japan, Canada, Sweden and Switzerland (see Appendix A for a description of the data). For the DEM/USD exchange rate, the series is linked with the EUR/USD exchange rate after the introduction of the Euro.⁸ The real exchange rate is calculated using consumer price indices. We use 10-year government bond yields to approximate the sum of future expected short-term interest rates and thus set $J = 120$ months.

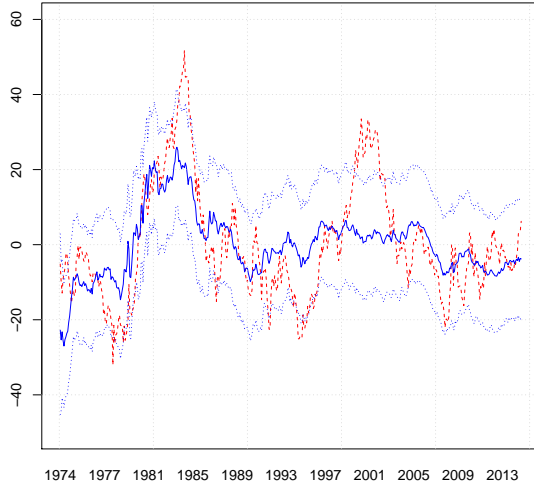
Figures 1 and 2 show the actual real and nominal exchange rates along with the mean and the 5th and 95th percentiles of the posterior distribution from the UC-SV model. The posterior distribution reflects uncertainty associated with the estimation of the UC-SV model as well as due to the estimation of the linear relationship of the exchange rate equation. For all countries, the posterior mean tracks major exchange rate movements well. Most turning points of the real exchange rate are reflected in the estimates. Moreover, we match the appreciation trends of the nominal exchange rate, in particular, for Japan and Switzerland well.

In what follows, we discuss the rare episodes when the actual exchange rate moves outside of the 5th and 95th percentiles. In the mid-1980s the real exchange rate briefly leaves the credible bands for all countries except Canada. Similar problems of matching the strong US Dollar during this period are reported by Engel and West (2006); they suggest that this period has frequently been labeled a US Dollar “bubble”. This is in line with the idea that the fundamentals included in the extended model do not explain the

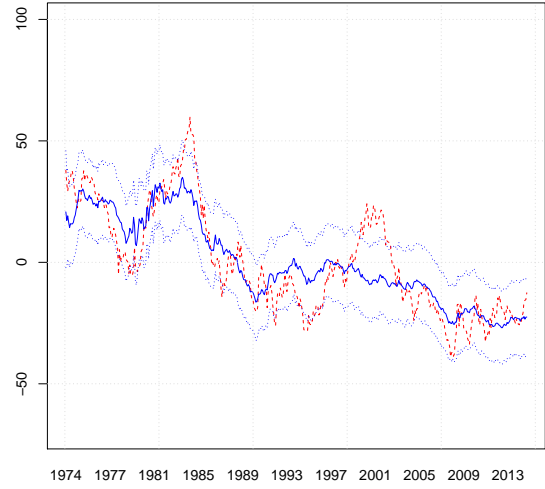
⁸We experimented with linking all data with euro area aggregates after the euro changeover and the results prove robust to this alternative.

FIGURE 1 — MODEL PREDICTIONS FOR LARGE ECONOMIES

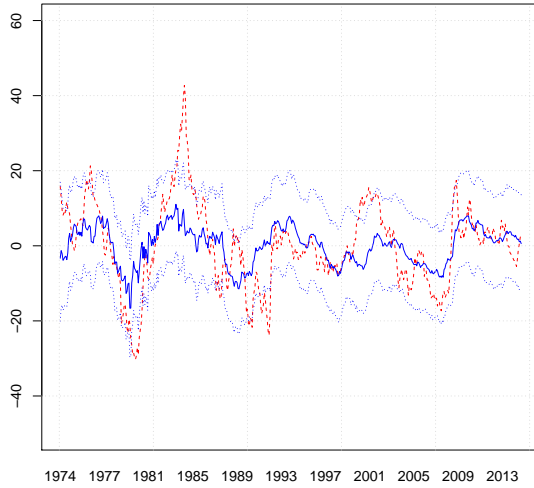
(A) REAL DEM/USD



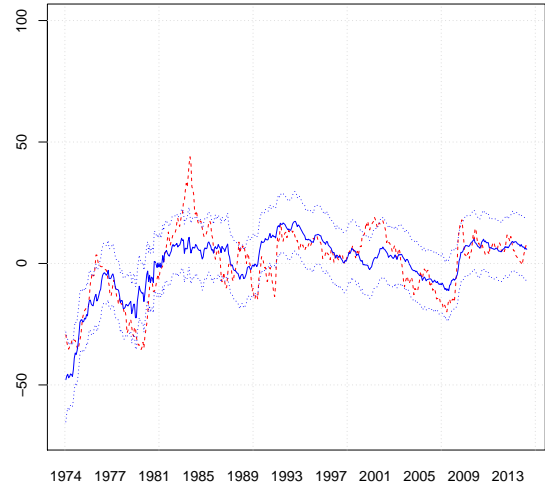
(B) NOMINAL DEM/USD



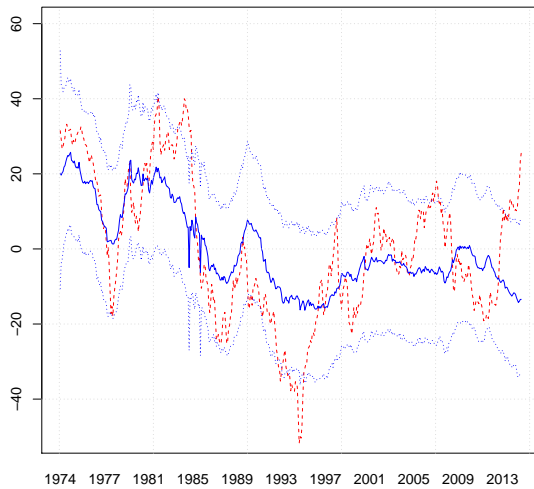
(C) REAL GBP/USD



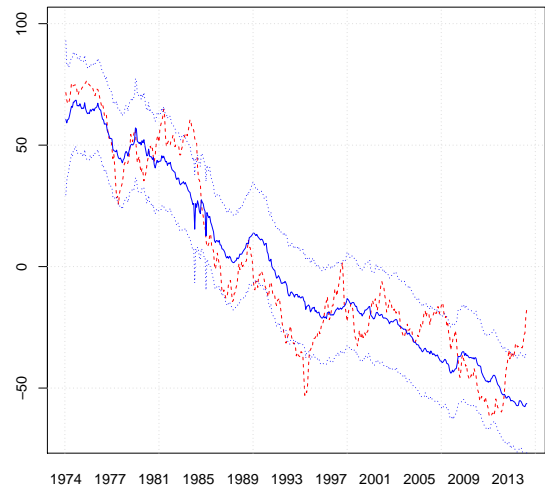
(D) NOMINAL GBP/USD



(E) REAL JPY/USD

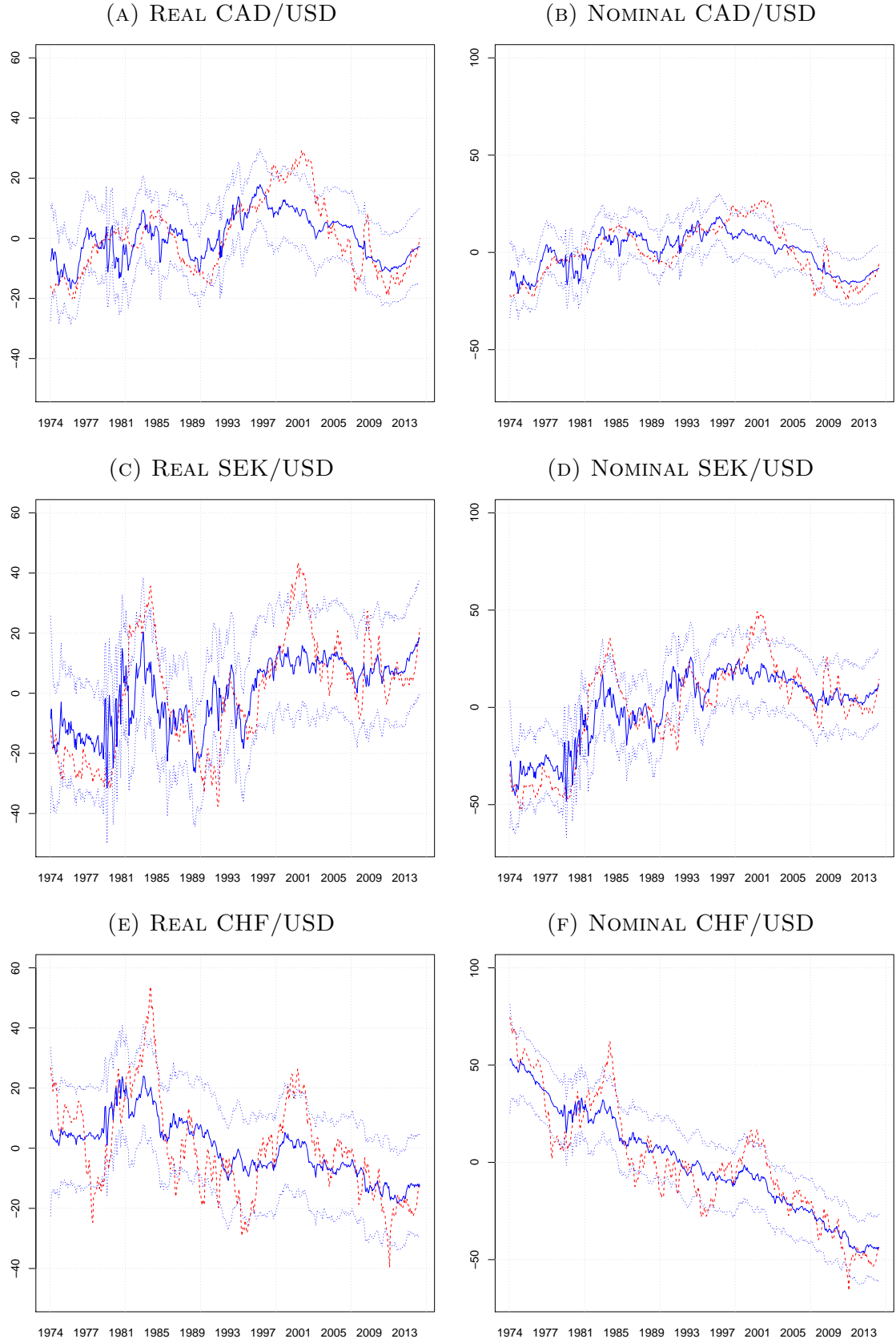


(F) NOMINAL JPY/USD



Notes: Actual real and nominal US Dollar exchange rates are given as dashed red lines (in logarithms times 100, centered around 0). The posterior median is given by the solid blue lines and the dashed blue lines correspond to 5th and 95th percentiles. The results are based on 15,000 posterior draws.

FIGURE 2 — MODEL PREDICTIONS FOR SMALL ECONOMIES



Notes: Actual real and nominal US Dollar exchange rates in dashed red lines (in logarithms times 100, centered around 0). The posterior median is given by the solid blue lines and the dashed blue lines correspond to 5th and 95th percentiles. The results are based on 15,000 posterior draws.

strong Dollar.

Starting in 1998 the US Dollar appreciated and rose outside of the 95th percentile for most currencies under consideration. We conjecture that this is closely related with several major economic crises that forced investors to reduce their non-USD exposure (“flight to safety”). More specifically, the Asian financial crisis of 1997 and 1998 was closely followed by the sovereign default of Russia and the default of Long-Term Capital Management. Beside these developments in Asia, increased uncertainty surrounding the Argentinean crisis between 1998 and 2002 presumably contributed to the upward pressure on the US Dollar. Such save-haven considerations are probably not well captured in the factors affecting short-term interest rates via the Taylor rule.

Generally speaking, significant deviations from the model predictions occur when the Taylor rule is a poor approximation to monetary policy, for example, at the effective lower bound on short-term interest rates and during unconventional monetary policy actions. In 1978 and 2011, the real exchange rate leaves the credible bands for Switzerland when the short-term interest rate was constrained by the effective lower bound. Bäumle and Kaufmann (2014) argue that a currency is likely to appreciate strongly at the effective lower bound in response to modestly deflationary risk premium shocks because of increasing instead of declining real interest rates. In the late 1970s as well as in 2011, the SNB counteracted the appreciation by introducing a minimum exchange rate against the German Mark and the Euro, respectively. Also for Japan, we observe a substantial deviation from the prediction in 1995 when short-term interest rates fell to very low levels (to 0.4% in September 1995).

Similarly, the UC-SV approach may not fully include unconventional monetary policy actions and sharp and sudden changes in inflation expectations. For Japan, the real and nominal exchange rates leave the percentiles in 2014. But the posterior mean moves into the opposite direction of the actual exchange rate already since 2012. This episode was governed by exceptional policy actions due to Abenomics which may not be appropriately reflected in the empirical UC-SV model: a higher inflation target, quantitative easing and an expansionary fiscal policy stance.

Using the posterior distribution of the exchange rate prediction, we may investigate the model fit more formally by calculating the posterior distribution of the correlation

TABLE 1 — CORRELATION WITH ACTUAL EXCHANGE RATE

		(A) Real		(B) Nominal	
		Log-level	Log-change	Log-level	Log-change
DEM/USD	Benchmark	0.35	0.01	0.66	0.02
		[0.26, 0.43]	[−0.09, 0.12]	[0.61, 0.80]	[−0.09, 0.12]
	UC-SV	0.47	0.02	0.75	0.02
		[0.38, 0.59]	[−0.05, 0.09]	[0.71, 0.80]	[−0.05, 0.10]
GBP/USD	Benchmark	0.22	0.02	0.55	0.03
		[0.12, 0.31]	[−0.08, 0.10]	[0.49, 0.69]	[−0.06, 0.11]
	UC-SV	0.33	0.02	0.64	0.03
		[0.24, 0.45]	[−0.04, 0.08]	[0.59, 0.69]	[−0.04, 0.09]
JPY/USD	Benchmark	0.36	0.00	0.85	0.02
		[0.27, 0.44]	[−0.08, 0.09]	[0.83, 0.92]	[−0.07, 0.10]
	UC-SV	0.50	0.01	0.90	0.01
		[0.38, 0.64]	[−0.05, 0.07]	[0.87, 0.92]	[−0.05, 0.07]
CAD/USD	Benchmark	0.48	0.03	0.59	0.04
		[0.40, 0.55]	[−0.07, 0.12]	[0.52, 0.75]	[−0.05, 0.13]
	UC-SV	0.56	0.03	0.65	0.04
		[0.40, 0.68]	[−0.03, 0.10]	[0.52, 0.75]	[−0.03, 0.10]
SEK/USD	Benchmark	0.44	0.03	0.65	0.04
		[0.36, 0.51]	[−0.05, 0.11]	[0.60, 0.78]	[−0.04, 0.12]
	UC-SV	0.52	0.02	0.72	0.02
		[0.43, 0.63]	[−0.04, 0.07]	[0.66, 0.78]	[−0.04, 0.08]
CHF/USD	Benchmark	0.39	0.02	0.82	0.03
		[0.31, 0.47]	[−0.06, 0.11]	[0.79, 0.89]	[−0.06, 0.12]
	UC-SV	0.48	0.02	0.86	0.03
		[0.40, 0.60]	[−0.04, 0.09]	[0.84, 0.89]	[−0.04, 0.09]

Notes: Posterior mean correlation with actual US Dollar exchange rate. 5th and 95th percentiles in brackets. The benchmark model does not take into account changes in the inflation and unemployment trends.

with the actual exchange rate. The model predictions match the dynamics of the level of the exchange rate well, however, they do not explain exchange rate changes. Table 1 shows the posterior mean and percentiles for the correlation with the actual real and nominal exchange rates for each country. The first line is a benchmark model where we do not control for the fact that trend inflation and trend unemployment may change over time. This specifications includes the same information set as the UC-SV model, however, without decomposing inflation and unemployment into trends and cycles. The second line gives the UC-SV model specification with the decomposition. Using the benchmark model

we obtain correlations between 0.22 for the UK and 0.48 for Canada. The correlation for Germany at 0.35 is close to existing estimates by Engel and West (2006) and Mark (2009).

If we include trend inflation rates, the inflation gaps and the unemployment gaps separately, the correlation rises to 0.33 for the UK and even to 0.56 for Canada. For Germany, the posterior mean correlation amounts to 0.47. The model thus improves existing predictions for the real exchange rate and this can be traced back to accounting for changes in trend inflation and the trend unemployment rate. For the nominal exchange rate, the correlation is generally higher reflecting that we match the trends for Japan and Switzerland particularly well. But also, the correlation is substantial for Canada where the nominal exchange rate does not exhibit a strong secular trend.

TABLE 2 — AUTOCORRELATION REAL EXCHANGE RATE

		Log-level			Log-change		
		1st	2nd	3rd	1st	2nd	3rd
DEM/USD	Actual	0.98	0.96	0.93	0.01	0.04	0.04
	Benchmark	0.94	0.90	0.88	-0.14	-0.24	-0.02
	UC-SV	0.98	0.96	0.94	0.13	-0.17	0.05
GBP/USD	Actual	0.97	0.94	0.90	0.33	0.02	0.05
	Benchmark	0.90	0.82	0.78	-0.12	-0.17	-0.07
	UC-SV	0.96	0.93	0.89	0.02	-0.06	-0.09
JPY/USD	Actual	0.99	0.96	0.94	0.33	0.08	0.05
	Benchmark	0.90	0.89	0.87	-0.41	-0.02	-0.02
	UC-SV	0.99	0.98	0.97	-0.17	0.09	-0.04
CAD/USD	Actual	0.99	0.98	0.97	0.21	0.05	0.03
	Benchmark	0.95	0.91	0.89	-0.17	-0.11	-0.09
	UC-SV	0.98	0.95	0.92	0.10	-0.03	-0.13
SEK/USD	Actual	0.99	0.97	0.95	0.36	0.04	0.05
	Benchmark	0.86	0.76	0.73	-0.12	-0.26	-0.10
	UC-SV	0.95	0.89	0.86	0.04	-0.24	-0.19
CHF/USD	Actual	0.98	0.96	0.93	0.27	0.03	0.02
	Benchmark	0.94	0.91	0.90	-0.17	-0.26	-0.03
	UC-SV	0.99	0.97	0.96	0.11	-0.21	-0.08

Notes: Sample autocorrelation function for the actual real US Dollar exchange rate and sample autocorrelation function for the posterior mean of the model predictions up to 3rd order. The benchmark model does not take into account changes in the inflation and unemployment trends.

For exchange rate changes the model does not outperform the benchmark. While the posterior mean correlation is usually higher for the UC-SV model when compared with the benchmark, in fact, the percentiles always include zero for both specifications. This suggests that we mainly capture the major exchange rate movements while month-to-month movements are not well captured.

An important aspect for an exchange rate model to match is the high persistence or near-random walk properties of the real exchange rate. Table 2 shows the sample autocorrelation up to the third order for the actual real exchange rate along with the autocorrelation of the posterior means of the predictions. The benchmark model already implies a highly persistent real exchange rate. Nevertheless, the persistence is lower than of the actual real exchange rate for all countries and all lags. The UC-SV model can explain the higher persistence of the real exchange rate and matches the actual persistence closely. The only countries where the model does not quite match the persistence are Canada and Sweden. Nevertheless, the model still outperforms the benchmark.

Similarly, we also make progress of matching the persistence of exchange rate changes. In the actual data the first order autocorrelation is larger than zero for all countries. By contrast, the benchmark model implies a negative first order autocorrelation for exchange rate changes. Although the UC-SV model does not exactly reproduce the pattern in the data, first order autocorrelation are mostly positive (except for Japan and UK) and second order correlations are closer to the actual values.

As a further check whether the exchange rate model predicts a real exchange rate with reasonable properties we compare correlations of the real exchange rate with the fundamentals. Table 3 shows that the model matches the correlation between the actual exchange rate and the fundamentals closely. The posterior mean is close to the actual correlation and the 5th and 95th percentiles mostly include the actual value. The actual value of the correlation lies outside of the percentiles for inflation in Japan, Canada and Switzerland and long-term bonds in Germany and Switzerland. However, even for these correlations the sign of the posterior mean is consistent with the sign of the actual correlation.

Finally, we performed robustness checks with respect to the model specification. In particular, we experimented with simpler detrending methods to construct a measure of

TABLE 3 — CORRELATION OF REAL EXCHANGE RATE WITH FUNDAMENTALS

		$\pi_t - \pi_t^*$	$u_t - u_t^*$	$i_t - i_t^*$	$b_t - b_t^*$
DEM/USD	Actual	0.15	0.19	-0.20	-0.44
	UC-SV	0.08	0.21	-0.24	-0.53
		[0.01, 0.15]	[0.12, 0.31]	[-0.30, -0.17]	[-0.58, -0.47]
GBP/USD	Actual	0.11	0.04	-0.06	-0.02
	UC-SV	0.04	0.02	-0.07	-0.02
		[-0.04, 0.12]	[-0.10, 0.13]	[-0.15, 0.00]	[-0.10, 0.06]
JPY/USD	Actual	0.17	-0.39	-0.07	0.20
	UC-SV	0.07	-0.47	-0.09	0.24
		[0.01, 0.14]	[-0.58, -0.36]	[-0.15, -0.02]	[0.18, 0.30]
CAD/USD	Actual	-0.03	0.54	-0.38	-0.18
	UC-SV	-0.11	0.54	-0.44	-0.21
		[-0.17, -0.05]	[0.36, 0.66]	[-0.50, -0.39]	[-0.28, -0.15]
SEK/USD	Actual	-0.06	0.38	-0.07	-0.43
	UC-SV	-0.14	0.42	-0.08	-0.50
		[-0.20, -0.08]	[0.31, 0.51]	[-0.14, -0.01]	[-0.56, -0.44]
CHF/USD	Actual	0.08	-0.32	-0.30	-0.47
	UC-SV	-0.03	-0.37	-0.36	-0.56
		[-0.10, 0.05]	[-0.49, -0.26]	[-0.42, -0.29]	[-0.61, -0.50]

Notes: Correlations of the actual and predicted real US Dollar exchange rate with differences in the fundamentals. The 5th and 95th percentiles are given in brackets. The benchmark model does not take into account changes in the inflation and unemployment trends.

trend inflation and the unemployment gap. Applying a Hodrick and Prescott (1997)-filter and performing the corresponding linear regression yields similar correlations between the actual and predicted real exchange rate as in the UC-SV model. Moreover, in versions of the regression where we include the differences in domestic and US variables directly reveals that the coefficient on trend inflation differential is always significantly positive, except for Canada. Although we refrain from reading too much into the actual coefficients due to the reduced-form nature of the regression equation, we take this feature as evidence that expectations about trend inflation rate are key to explaining the current real exchange rate.

5 Closing remarks

Recent research has documented that trend inflation changes over time. We add an international dimension to this line of research and highlight that changes in trend inflation explain important aspects of exchange rate dynamics. We develop a multivariate UC-SV model that is theoretically motivated by assuming that both countries' central banks follow Taylor rules.

The UC-SV model succeeds in capturing major up- and downturns of the real US Dollar exchange rate against the currencies of six economies. In fact, the correlations of the model predictions with the actual real exchange rates are higher than in existing studies. While a benchmark model performs comparatively well, the improvements obtained by explicitly discriminating between non-stationary trend and stationary gap components are significant for all currencies under consideration. Looking at nominal exchange rates reveals that we are able to accurately reproduce major exchange rate trends observed over the last 40 years. Finally, the model successfully captures several key time series characteristics commonly found for real exchange rates. More specifically, we accurately reproduce the persistence of the real exchange rate and its correlation with other macroeconomic variables.

Our discussion shows that, although the model explains a larger share of exchange rate fluctuations than previous studies, it fails during episodes when the Taylor rule is unlikely to be an accurate description of the central banks' conduct of monetary policy. Improving the model predictions by accounting for unconventional monetary policy actions and constraints on the operational targets of central banks might be a promising avenue for future research.

References

- ASCARI, G. AND A. M. SBORDONE (2014): “The macroeconomics of trend inflation,” *Journal of Economic Literature*, 52, 679–739.
- BÄURLE, G. AND D. KAUFMANN (2014): “Exchange rate and price dynamics in a small open economy – the role of the zero lower bound and monetary policy regimes,” Working Paper 2014-10, Swiss National Bank.
- CARTER, C. K. AND R. KOHN (1994): “On Gibbs sampling for state space models,” *Biometrika*, 81, 541–553.
- COGLEY, T., G. E. PRIMICERI, AND T. J. SARGENT (2010): “Inflation-gap persistence in the US,” *American Economic Journal: Macroeconomics*, 2, 43–69.
- COGLEY, T. AND A. M. SBORDONE (2008): “Trend inflation, indexation, and inflation persistence in the New Keynesian Phillips Curve,” *American Economic Review*, 98, 2101–2126.
- COIBION, O. AND Y. GORODNICHENKO (2012): “Why are target interest rate changes so persistent?” *American Economic Journal: Macroeconomics*, 4, 126–62.
- ENGEL, C., N. C. MARK, AND K. D. WEST (2008): “Exchange rate models are not as bad as you think,” in *NBER Macroeconomics Annual 2007*, National Bureau of Economic Research, vol. 22 of *NBER Chapters*, 381–441.
- ENGEL, C. AND K. D. WEST (2005): “Exchange rates and fundamentals,” *Journal of Political Economy*, 113, 485–517.
- (2006): “Taylor rules and the Deutschmark-Dollar real exchange rate,” *Journal of Money, Credit and Banking*, 38, 1175–1194.
- FRÜHWIRTH-SCHNATTER, S. (1994): “Data augmentation and dynamic linear models,” *Journal of Time Series Analysis*, 15, 183–202.
- FRÜHWIRTH-SCHNATTER, S. AND H. WAGNER (2010): “Stochastic model specification search for Gaussian and partial non-Gaussian state space models,” *Journal of Econometrics*, 154, 85–100.
- GIANNONI, M. P. (2014): “Optimal interest-rate rules and inflation stabilization versus price-level stabilization,” *Journal of Economic Dynamics and Control*, 41, 110–129.
- GORDON, R. J. (1998): “Foundations of the Goldilocks Economy: Supply shocks and the time-varying NAIRU,” *Brookings Papers on Economic Activity*, 29, 297–346.
- HODRICK, R. J. AND E. C. PRESCOTT (1997): “Postwar U.S. business cycles: An empirical investigation,” *Journal of Money, Credit and Banking*, 29, 1–16.
- KASTNER, G. (2015a): “Dealing with stochastic volatility in time series using the R package *stochvol*,” *Journal of Statistical Software*, forthcoming.
- (2015b): *stochvol: Efficient Bayesian inference for stochastic volatility (SV) models*, R package version 1.1.1.

- KASTNER, G. AND S. FRÜHWIRTH-SCHNATTER (2014): “Ancillarity-sufficiency interweaving strategy (ASIS) for boosting MCMC estimation of stochastic volatility models,” *Computational Statistics & Data Analysis*, 76, 408–423.
- MARK, N. C. (2009): “Changing monetary policy rules, learning, and real exchange rate dynamics,” *Journal of Money, Credit and Banking*, 41, 1047–1070.
- MEESE, R. A. AND K. ROGOFF (1983): “Empirical exchange rate models of the seventies: Do they fit out of sample?” *Journal of International Economics*, 14, 3–24.
- ROSSI, B. (2013): “Exchange rate predictability,” *Journal of Economic Literature*, 51, 1063–1119.
- STELLA, A. AND J. H. STOCK (2012): “A state-dependent model for inflation forecasting,” International Finance Discussion Paper 1062, Board of Governors of the Federal Reserve System.
- STOCK, J. H. AND M. W. WATSON (2007): “Why has U.S. inflation become harder to forecast?” *Journal of Money, Credit and Banking*, 39, 3–33.

Appendix A Data

TABLE 4 — DATA, SOURCES, TRANSFORMATIONS

	Country	FRED identifier	Source	Comments
Exchange rates	CA	EXCAUS	FRB	
	JP	EXJPUS	FRB	
	SE	EXSDUS	FRB	
	CH	EXSZUS	FRB	
	UK	EXUSUK	FRB	Inverted
	DE	CCUSSP01DEM650N	MEI	Inverted, EUR/USD after euro changeover
CPI	CA	CANCPIALLMINMEI	MEI	Census X13 seas. adj.
	JP	JPNCPIALLMINMEI	MEI	Census X13 seas. adj.
	SE	SWECPIALLMINMEI	MEI	Census X13 seas. adj.
	CH	CHECPIALLMINMEI	MEI	Census X13 seas. adj.
	UK	GBRCPIALLMINMEI	MEI	Census X13 seas. adj.
	US	CPIAUCSL	BLS	
	DE	DEUCPIALLMINMEI	MEI	Census X13 seas. adj.
Unemployment rates	CA	LRUNTTTTTCAM156S	MEI	
	JP	LRUN24TTJPM156N	MEI	Census X13 seas. adj.
	SE	LRHUTTTTTSEM156S, SWEURHARMMDSM	MEI	Sources linked in 1983
	CH	LMUNRRRTTCHM156N	MEI	Census X13 seas. adj.
	UK	LMUNRRRTTGBM156S	MEI	
	US	UNRATE	BLS	
	DE		BA	Downloaded from Datastream
Short rates	CA	IR3TIB01CAM156N	MEI	Interbank rate
	JP	INTGSTJPM193N	IFS	T-Bill rate
	SE	IR3TIB01SEM156N	MEI	Linked with Riksbank data (see notes)
	CH	IR3TIB01CHM156N	MEI	Interbank rate
	UK	IR3TTS01GBM156N	MEI	T-Bill rate
	US	IR3TIB01USM156N	MEI	Interbank rate
	DE	IR3TIB01DEM156N	MEI	Interbank rate
Long rates	CA	IRLTLT01CAM156N	MEI	
	JP	INTGSBJPM193N	IFS	
	SE	IRLTLT01SEM156N	MEI	Linked with Riksbank data (see notes)
	CH	IRLTLT01CHM156N	MEI	
	UK	IRLTLT01GBM156N	MEI	
	US	IRLTLT01USM156N	MEI	
	DE	IRLTLT01DEM156N	MEI	

Notes: All data, unless otherwise indicated, was retrieved from FRED, Federal Reserve Bank of St. Louis <https://research.stlouisfed.org/fred2/>. Data for short-term and long-term interest rates for Sweden was downloaded from <http://www.riksbank.se/en/The-Riksbank/Research/Historical-Monetary-Statistics-/Interest-and-stock-returns/>.