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Monetary Policy Evaluation using a Rational Expectations Model: the UK case

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Abstract

This study follows Rotemberg and Woodford (1998) and estimates a three-equation model of output, interest rate and inflation, in order to evaluate alternative rules by which the bank of England may decide on setting the main interest rate. As in the original paper, the model setup is a rational-expectations setup, augmented with nominal price-setting frictions a la Calvo (1983). The model-generated impulse responses match quite well the estimated responses to a monetary shock. In addition, when additional shocks are added, the theoretical model can account for the fluctuations in the UK data as well as an unrestricted VAR(1) does.

Key Words: VAR, Taylor rule, monetary shock.

JEL Classification: E37

1 Introduction

In this paper Rotemberg and Woodford's (1998) small structural econometric model will be used to evaluate quantitatively monetary policy rules for the case of UK monetary authority. The micro-founded New Keynesian model, based on inter-temporal optimization of both monopolistic suppliers and atomistic consumers, and augmented with nominal frictions in output prices. Since the model incorporates forward-looking specifications of some of the structural relations and agents have rational expectations, it responds to the Lucas (1976) critique of econometric policy evaluation.¹

The methodology used in the paper follows the four steps described in Rotemberg Woodford (2008). In the first step, a vector-auto-regression model of order one (VAR(1)) will be estimated, in order to capture the joint dynamics of interest rates, inflation and output. The estimated VAR(1) will serve a two-fold purpose: on the one hand, it will be used to identify the monetary policy rule used by the Bank of England (BoE). Following Taylor (1993), the policy rule used by BoE will be assumed to be an interest rate one, which is set up to offset negative effects of shocks on inflation and real activity (proxied by output). More specifically, the interest rate will be a function of weighted inflation and output. From the VAR, we can quantify the way the variables of interest (output, inflation and interest rate) respond when subjected to stochastic disturbances to the Taylor rule. The produced response functions then would give us an idea about the nature of the propagation mechanism in the the economy when the current interest rate rule is in place.

In the second step, the setup laid in Rotemberg and Woodford (1998) will be presented in a stylized manner. The main source of nominal frictions in the model with optimizing households and monopolistically-competitive firms is the Calvo-type (1983) price adjustment. More specifically, an individual firm may decide to keeps its price fixed, since changing prices is costly due to some menu costs. As a result, prices will be re-set only when a stochastic shock is big enough to justify adjustment. Model parameters will be calibrated to minimize the distance between the impulse responses from the VAR and the simulated ones.

¹In addition, since the econometric framework is derived from individual inter-temporal optimization, and thus the "observational equivalence" problem is solved.

In the third step, the model parameters estimated from the earlier step will be combined with the theoretical model and the VAR in order to identify the shocks in the structural equations. Those VAR residuals are interpreted as shocks to technology, preferences, and monetary disturbances. From the estimated residuals the underlying stochastic processes followed by monetary policy innovations, technology progress and shocks to tastes can be obtained. Once the constructed shocks are incorporated, the theoretical model provides as good fit as the unrestricted VAR. The good fit of the micro-founded model is good news for the applicability of the benchmark Rotemberg and Woodford (1998) model for monetary policy analysis.

Lastly, the calibrated quantitative model will be used to simulate the effect of hypothetical monetary policy rules. In that way, using the estimated historical shock series, alternative paths for the UK economy will be simulated. The rest of the paper is organized as follows: Section 2 describes the econometric specification. Section 3 presents Rotemberg and Woodford's (1998) model setup. Section 4 estimates model parameters. Section 5 describes the identification of the shock processes. Section 6 simulates the model under alternative monetary policy rules, and Section 7 concludes.

2 Econometric Specification and VAR estimation

This section provides the econometric characterization of the monetary policy regime and the estimated quantitative effect of monetary shocks under the Taylor rule, which is assumed to represent the reaction function of the UK monetary authority:

$$r_t = r^* + \sum_{k=1}^{n_r} \mu_k [r_{t-k} - r^*] + \sum_{k=0}^{n_\pi} \phi_k [\pi_{t-k} - \pi^*] + \sum_{k=0}^{n_y} \theta_k [y_{t-k} - y^*] + \epsilon_t \quad (2.1)$$

where r_t is the interest rate on 3-month Treasury bill rate, π_t is the rate of inflation between $t-1$ and t , y_t is the output gap, measured as percentage deviation of real GDP from trend, y^* is the long-run value of y_t ("potential output"). The smoothing parameters μ_k denote the weights attached to lagged interest rates, and ϕ_k and θ_k are the weights attached to inflation rate and the output gap. Lastly, r^* and π^* are the long-run target interest and inflation rate, respectively. The $\{\epsilon_t\}$ series are the exogenous *i.i.d* disturbances to the monetary policy that

are hitting the economy.²

In order to identify those monetary policy shocks and to estimate the coefficients of the regression equation (2.1) above, an important assumption will be imposed, namely that shock at time t has no effect on output or inflation during the same period, *i.e* monetary policy comes into effect with a lag. This will allow Ordinary least Squares (OLS) method to be used to produce coefficient estimates, which are consistent.

Next, we estimate the Taylor rule equation as a part of a three-variable exactly identified VAR model, where the variables included are the interest rate, the inflation rate, and detrended real GDP. From the VAR, it will be easy to obtain the impulse response functions to the monetary shock under the followed Taylor rule. The sample period for the VAR is 1980:1- 1995:2, thus the time series are long enough to provide a sensible estimation. The particular starting point was chosen to get rid of the oil price shock effect in the 70s. As explained above, a recursive VAR (which is a type of a structural VAR), will be employed as an estimation strategy with a state vector

$$Z_t = [r_t, \pi_{t+1}, y_{t+1}]'. \quad (2.2)$$

As explained above, interest rate is the first one in the causal chain. The estimated system is of the form

$$T\bar{Z}_t = A\bar{Z}_{t-1} + \bar{e}_t, \quad (2.3)$$

where

$$T\bar{Z}_t = [Z'_t, Z'_{t-1}, Z'_{t-2}], \quad (2.4)$$

and as in Woodford and Rotemberg (1998), T is a lower triangular matrix with ones on the diagonal and non-zero off-diagonal elements only in the first three rows. Next, matrix A is one whose first three rows contain the VAR estimates. Lastly, the first three rows of \bar{e}_t contain the VAR residuals and the other elements are equal to zero.³ The VAR coefficients

²As in Rotemberg and Woodford (1998), those are assumed to be serially uncorrelated.

³Note that the first element of the residual vector e_t represents the identification of the monetary shock ϵ_t .

are displayed in Table 1. Given the research question, the most interesting equation from the system is the first one because it describes the interest rate rule:

$$r_t - r^* = 0.621(r_{t-1} - r^*) + 0.184(r_{t-2} - r^*) - 0.009(r_{t-3} - r^*) + 0.276(\pi_t - \pi^*) + 0.4(\pi_t - \pi^*) + 0.158(\pi_t - \pi^*) + 0.002(y_t - y^*) - 0.001(y_{t-1} - y^*) - 0.003(y_{t-2} - y^*), (2.5)$$

where the coefficients for y are multiplied by 4 to adjust for annual data. This specification of the monetary policy rule indicates a significant interest-rate smoothing, as the μ_k coefficients are all significant and sum to 0.81. Indeed, the third coefficient is negative, but the overall dynamics of the real interest rate is dominated by the persistence in the first and second lag of interest rate.

Next, the responses of interest rate, inflation and output to a a one-standard-deviation shock that raises the interest rate (unexpectedly) by about eight-tenths of a percent, as done in Rotemberg-Woodford (1998), and plotted in Figure 1. In each panel, the central dashed line indicates the point estimate of the impulse response function (IRF), while the two other indicate the confidence band.⁴ The empirical impulse responses are in line with the conventional wisdom on the subject. First, interest rates are increase only temporarily: according to the VAR estimates, for 4 quarters. Second, output does not decline until two quarters later, and three quarters later, returns to normal. Lastly, inflation also declines with a lag, with the greatest decline occurring 2 quarters later than the shock, and returns to normal another four lags after the drop. The estimated effects featured by the IFRs are important, as they give us some indication about possible features that a micro-founded model should possess in order to generate predictions that match the data. That is, we need our model to produce temporarily lowers both π and y , and these negative effects occur with a lag, in case the monetary authority implements a surprise tightening by raising the interest rate.

⁴The confidence intervals are constructed using analytical derivatives of the responses with respect to the parameters and the parameters' covariance matrix.

3 A Simple Model of Output and Inflation Determination

In this section we present Rotemberg and Woodford's (1998) general equilibrium model is presented. Its setup features a continuum of infinitely-lived households that are distributed uniformly on the $[0, 1]$ interval. Each household consumes the whole range of varieties produced in the economy, while at the same time each household i is also a producer of a single differentiated good. Financial markets are complete. Household i 's objective is to

$$\max_{\{C_t^i, y_t^i\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [u(C_t^i; \xi_t) - v(y_t^i; \xi_t)] \quad (3.1)$$

s.t

$$E_t \sum_{T=t}^{\infty} \delta_{t,T} S_t^i \leq E_t \sum_{T=t}^{\infty} \delta_{t,T} [p_t^i y_t^i - T_t] + A_t^i, \quad (3.2)$$

where the instantaneous felicity function of consumption $u(\cdot; \xi_t)$ is an increasing, concave function, and the instantaneous utility function of labor $v(\cdot; \xi_t)$ is an increasing, convex function, $\forall \xi_t$. In addition, β is a discount factor, y_t^i is the output of the good produced by household i at time t and ξ_t is a vector of random disturbances. A_t^i denotes the nominal value of the households's financial assets at the beginning of period t , T_t denotes net nominal tax payments at date t , and $\delta_{t,T}$ is the stochastic discount factor that defines the nominal present value at t of nominal income in any given state at date $T \geq t$.

The equilibrium condition that follows from the assumption of complete markets is that

$$R_t = E_t \frac{1}{\delta_{t,t+1}}, \quad (3.3)$$

where R_t is the nominal interest rate on a risk-less one-period bond purchased in period t . Households choose C_t^i at date $t - 2$. This assumption may seem strange at first sight, but it is consistent with the internal logic of the model: consumption includes both consumption and investment goods. Therefore, Rotemberg and Woodford appeal to the "time-to-build" argument, as in Kydland and Prescott (1982).

Next, C_t^i is a Dixit-Stiglitz (1977) aggregate of the household's consumption of differentiated goods, given by

$$C_t^i = [\int_0^1 c_t^i(z)^{\frac{\theta-1}{\theta}} dz]^{\frac{\theta}{1-\theta}}, \quad (3.4)$$

and the constant elasticity of substitution is denoted by $\theta > 1$.

next, total nominal spending at t across all the differentiated goods is $S_t^i = P_t C_t^i$, where

$$P_t = [\int_0^1 p_t^i(z)^{1-\theta} dz]^{\frac{1}{1-\theta}} \quad (3.5)$$

is the Dixit-Stiglitz price aggregator. The optimal demand of an individual variety is then

$$c_t^i(z) = C_t^i \left(\frac{p_t^i}{P_t} \right)^{-\theta}, \forall z, \quad (3.6)$$

which is a standard result in the literature.

Solving for the equilibrium, the following FOCs are obtained:

$$E_t u_c(C_{t+2}^i; \xi_{t+2}) = E_t \lambda_{t+2}^i P_{t+2} \lambda_t^i \delta_{t,T} = \beta^{T-t} \lambda_T^i, T \geq t. \quad (3.7)$$

Therefore

$$\lambda_t = \beta E_t [R_t \lambda_{t+1}]. \quad (3.8)$$

Log-linearizing the equation around the steady-state yields:

$$\hat{\lambda}_t = E_t [\hat{R}_t - \pi_{t+1} + \hat{\lambda}_{t+1}], \quad (3.9)$$

where $\hat{\lambda}_t$, \hat{R}_t denote percentage deviations from $\lambda^* P^*$ and R^* , and $\pi_t \equiv \log P_t / \log P_{t-1}$ is the rate of inflation. Note that the approximation is performed for a target inflation rate of zero, $\pi^* = 0$.

Next, iterating the equation forward produces

$$\lambda_t = \hat{r}_t^l \equiv \sum_{T=t}^{\infty} E_t [\hat{R}_T - \pi_{T+1}] \quad (3.10)$$

where \hat{r}_t^l denotes the percentage deviation in the long-run real rate of return.

Log-linearizing the optimality condition for consumption yields

$$-\tilde{\sigma} E_t[\hat{C}_{t+2} - \bar{C}_{t+2}] = E_t \hat{r}_{t+2}^2 \quad (3.11)$$

where

$$\tilde{\sigma} = -u_{CC}C/u_C, \quad (3.12)$$

and $\bar{C}(\xi_t)$ is an exogenous disturbance, function of the preference shock ξ_t .

Output is divided between consumption and autonomous government spending as follows

$$Y_t = C_t + G_t, \quad (3.13)$$

where

$$G_t = \left[\int_0^1 g_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}. \quad (3.14)$$

That is, individual purchases, $g_t(z)$, are chosen to maximize autonomous expenditure aggregate.

Next, we log-linearize the market clearing equation to obtain

$$\hat{Y}_t = s_C \hat{C}_t + \tilde{G}_t \quad (3.15)$$

Substituting into the first-order condition for consumption, we obtain the model's IS equation

$$\hat{Y}_t = -\sigma^{-1} E_{t-2} \hat{r}_t^l + \hat{G}_t, \quad (3.16)$$

where $\sigma \equiv \tilde{\sigma}/s_C$, and $\hat{G}_t = \tilde{G}_t + s_C E_{t-2} \bar{C}_t$ is some composite exogenous disturbance term. Thus the aggregate demand (AD) consists of the monetary policy rule, the term-structure of the interest rate, and the IS equation.

Next, we describe the price-setting rule and specify the aggregate supply function. The aggregate demand faced by an individual satisfies

$$y_t^i = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta}, \quad (3.17)$$

and individual producers are assumed to be too small to affect either Y_t or P_t .

Monetary policy in this model with monopolistic producers has a real effect because of the Calvo (1983) pricing. Namely, a fraction $1 - \alpha$ of sellers will choose a new price at the beginning of any period. The rest will not change their prices. Of those who can change the price, a fraction γ start charging the new price during that period, while $1 - \gamma$ share will wait until next period to charge a new price due to the presence of menu costs.

Let p_t^1 be the price set by sellers that decide at $t - 1$ upon a new price to take effect at date t , and p_t^2 denote the price set by sellers that decide at date $t - 2$ upon a new price to take effect 2 periods later. These prices are chosen to maximize expected utility of sales revenues less the disutility of output, at each future date and each future state in which the price commitment still applies. More specifically, p_t^1 solves

$$\max_{p_t^1} \Phi_{t-1}(p) \equiv \max_{p_t^1} E_{t-1} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [\lambda_t(1 - \tau)pY_T(\frac{p}{P_T})^{-\theta} - v(Y_T(\frac{p}{P_T})^{-\theta}; \xi_T)]. \quad (3.18)$$

Note that the factor α^{T-t} appears as the probability that the price that is charged in the beginning of period t is still in effect in period $T \geq t$, and revenues are taxed at a rate τ each period. Log-linearizing the FOC above around the steady-state yields

$$E_{t-1} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(\lambda_T + \hat{Y}_T - (\theta - 1)p_{t,T}^1) - \omega(\hat{Y}_T - \theta p_{t,T}^1 - \bar{Y}_T) - (\hat{Y}_T - \theta p_{t,T}^1)] = 0, \quad (3.19)$$

where

$$\omega \equiv v_{yy}\bar{Y}/v_y \quad (3.20)$$

and

$$p_{t,T}^1 \equiv \log(p_t^1/P_T). \quad (3.21)$$

Let

$$\hat{X}_t \equiv \frac{1 - \alpha}{\alpha} \log(p_t^1/P_T), \quad (3.22)$$

so that

$$p_t^1 = \frac{1-\alpha}{\alpha} \frac{1-\alpha\beta}{1+\omega\theta} [E_{t-1} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [-\hat{\lambda}_T + \omega(\hat{T}_T - \bar{Y}_T) + (1+\omega\theta) \sum_{s=t+1}^T \pi_s]] \quad (3.23)$$

be the optimizing choice of the relative price in period t of goods with new prices chosen just the period before.

Note that

$$E_{t-2} \hat{\lambda}_t = -\sigma E_{t-2} [\hat{Y}_t - \hat{G}_t] \quad (3.24)$$

and after some manipulations

$$E_{t-1} \hat{\lambda}_t = \phi_{t-1} - \sigma E_{t-1} [\hat{Y}_t - \hat{G}_t], \quad (3.25)$$

where

$$\phi_t = E_{t-1} \sum_{T=t}^{\infty} (\hat{R}_t - \pi_{T+1}) - E_{t-2} \sum_{T=t}^{\infty} (\hat{R}_t - \pi_{T+1}). \quad (3.26)$$

Substituting those into the expression for \hat{X}_t and after some algebra we obtain

$$\hat{X}_t = \frac{1-\alpha}{\alpha} \frac{1-\alpha\beta}{1+\omega\theta} [E_{t-1} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} [(\sigma + \omega)(\hat{Y}_t - \hat{Y}_t^s) + (1+\omega\theta) \frac{\alpha\beta}{1-\alpha\beta} \pi_{T+1}] - \phi_{t-1}] \quad (3.27)$$

where

$$\hat{Y}_t^s \equiv \frac{\omega}{\omega + \sigma} E_{t-1} \bar{Y}_t + \frac{\sigma}{\omega + \sigma} \hat{G}_t \quad (3.28)$$

is a composite exogenous disturbance representing variation in the natural level of output.

Next, the price-setting decision of sellers that choose a new price p_t^2 at $t-2$ to apply beginning in period t has to be taken care of as well. Since such a price is expected to apply in periods $t+j$ with the same probabilities, the objective function to maximize over p_t^1 is $E_{t-2} \Phi_{t-1}(p)$, which results in the FOC given by $E_{t-2} \Phi'_{t-1}(p)$. This implies that $\log p_t^2 = E_{t-2} \log p_t^1$. The price aggregator then evolves according to

$$P_t = [\alpha P_{t-1}^{1-\theta} + (1-\alpha)\gamma(p_t^1)^\theta + (1-\alpha)(1-\gamma)(p_t^2)^{1-\theta}]^{\frac{1}{1-\theta}}. \quad (3.29)$$

Dividing through by P_T , log-linearizing and substituting out p_t^2 yields

$$\pi_t = \gamma \hat{X}_t + (1 - \gamma)[E_{t-2}\hat{X}_t - \frac{1 - \alpha}{\alpha}(\pi_t - E_{t-2}\pi_t)]. \quad (3.30)$$

Observing that

$$E_{t-2}\pi_t = E_{t-2}\hat{X}_t, \quad (3.31)$$

the following relation can be derived:

$$\pi_t = \psi \hat{X}_t + (1 - \psi)E_{t-2}\hat{X}_t, \quad (3.32)$$

with

$$\psi \equiv \gamma\alpha/[1 - \gamma(1 - \alpha)]. \quad (3.33)$$

This equation shows how aggregate inflation results from the incentives of individual price-setters to choose a higher relative price. Solving the equation forward produces

$$\hat{X}_t = \kappa(\hat{Y}_t - \hat{Y}_t^s) + \beta E_{t-1}\hat{X}_{t+1} - \frac{\kappa}{\sigma + \omega}\phi_{t-1}, \quad (3.34)$$

where

$$\kappa \equiv (1 - \alpha)(1 - \alpha\beta)(\omega + \sigma)/\alpha(1 + \omega\theta). \quad (3.35)$$

Solving forward yields

$$\hat{X}_t = \kappa E_{t-1}(\sum_{T=t}^{\infty} \beta^{(T-t)}(\hat{Y}_t - \hat{Y}_t^s)) - \frac{\kappa}{\sigma + \omega}\phi_{t-1}. \quad (3.36)$$

Substituting this into (3.32) yields

$$\begin{aligned} \pi_t = & (1 - \psi)E_{t-2}\pi_t + \psi[\kappa E_{t-1}(\sum_{T=t}^{\infty} \beta^{(T-t)}(\hat{Y}_t - \hat{Y}_t^s)) \\ & - \frac{\kappa}{\sigma + \omega}(E_{t-1} \sum_{T=t}^{\infty} \beta^{(T-t)}(\hat{R}_t - \pi_{T+1}) - E_{t-1} \sum_{T=t}^{\infty} \beta^{(T-t)}(\hat{R}_t - \pi_{T+1}))]. \end{aligned} \quad (3.37)$$

This is nothing else but the aggregate supply equation, which relates variation in inflation to the deviations of output from the potential one. Since prices are set in advance, expectations of output level higher than the potential one also increase prices. Inflation declines when

when the long-term interest rate at t exceeds expected level, with expectations formed in $t - 1$. Such revisions in upward direction raise the returns households can expect to earn from their revenues at t . They are inclined to do so by cutting prices.

Next, when conditional expectations of (3.32) are taken at date $t - 2$, "the New Keynesian Phillips Curve" is obtained:

$$E_{t-2}\pi_t = \kappa E_{t-2}(\hat{Y}_t - \hat{Y}_t^s) + \beta E_{t-2}\pi_{t+1}. \quad (3.38)$$

If conditional expectations of (3.32) and (3.26) are taken at date $t - 2$ that yields

$$E_{t-2}[\hat{R}_t - \pi_{t+1}] = \sigma E_{t-2}[(\hat{Y}_{t+1} - \hat{Y}_t) + (\hat{G}_{t+1} - \hat{G}_t)]. \quad (3.39)$$

The aggregate demand not only specifies a relationship between real interest rates and output fluctuations that can be forecasted far into the future, but also allows for a more flexible short-term relationship.

4 Estimation of Model Parameters

This section considers the estimation and calibration of the structural parameters $\alpha, \beta, \gamma, \sigma, \theta$ and ω . Our aim is to pick plausible estimates while at the same time matching the variances of the three VAR disturbance terms and the IRFs of the three variables of interest to each of the three innovations already mentioned. In order to obtain the monetary rule, the estimates in the first column of Table 1 will be used, with the output coefficient divided by four, so that the estimates will be identical to those that would have been obtained if interest and inflation rate were not annualized.

It is the only the IRFs that provide information about the structural parameters. Thus the parameters will be estimated with the ultimate aim of matching the empirical IRFs. Note, however, that not all of them are identified. Starting with θ , we observe it only matters to determine κ , so it suffices to estimate the latter. In addition, γ and α only show up in the composite term $\psi = \frac{\gamma\alpha}{1-\gamma(1-\alpha)}$. Therefore, the set of identified parameters is $\beta, \kappa, \sigma, \omega, \psi$.

The first parameter, β will be calibrated from first moments in the data. Its inverse in the model corresponds to the gross real rate of return. Thus β will be set to 0.99 because that corresponds to 1 percent real quarterly return from UK data. Next, only a single function of ω and ψ is identified, rather than either one of those be identified independently. The parameters ψ and ω only matter for determining the response of π at $t + 1$, but the response of just one variable at $t + 1$ cannot separately identify two parameters. In the case of ψ , the impulse responses can only be used to put a lower bound on it, after the whole range of variation in ω is considered. To ensure that ψ is strictly positive, there will be a subset of producers that determines price changes two quarters in advance. This corresponds to the $\gamma < 1$ case.

In order to calibrate α , which determines how frequently, on average, a producer changes the price. Note that the mean time a set price remains in effect is $\frac{1}{1-\alpha}$. For a plausible value, will be considered. Blinder's (1998) estimate from microeconomic studies will be used, $\alpha = 0.66$, because his study covered a broad range of industries. His finding was that on average a price is set for $\frac{1}{1-\alpha} = 3$ quarters.

Using labor costs data, ω parameter can also be pinned down from elasticity of marginal cost with respect to output.⁵ That would require some additional structure to be imposed in the model. More specifically, output will be assumed to be produced via the production function

$$Y = f(H), \tag{4.1}$$

where H is hours worked. If the representative household has a disutility of working $g(H)$, then

$$v(Y) = g(f^{-1}(Y)). \tag{4.2}$$

That further implies that

$$v' = g'/f', \tag{4.3}$$

⁵In the model ω parameter measures the elasticity of the marginal disutility of producing output with respect to the increase in output.

hence

$$\omega \equiv \frac{v''Y}{v'} = \left(\frac{g''H}{g'} - \frac{f''H}{f'} \right) \frac{f}{f'H}. \quad (4.4)$$

Next, from the consumer maximization problem, the marginal rate of substitution between consumption and hours is

$$w^i = g'(H^i)/u'(C). \quad (4.5)$$

Log-linearizing the equation around the steady-state, and aggregating across labor markets produces

$$\frac{g''H}{g'} \frac{f}{f'H} = \epsilon_{wY} - \sigma, \quad (4.6)$$

where

$$\frac{g''H}{g'} \quad (4.7)$$

is the inverse elasticity of labor supply with respect to the real wage, and ϵ_{wY} is the elasticity of average real wage with respect to the volatility in output that is not due to preference or technology shocks. As in Rotemberg and Woodford(1998), we set $\epsilon_{wY} = 0.3$.

One final assumption added will be that the production function is iso-elastic. Denoting elasticity of output with respect to hours worked by η , one obtains that

$$-f''f/(f')^2 = (1 - \eta)/\eta. \quad (4.8)$$

From the firm maximization problem it follows that

$$w^i = f'(H^i). \quad (4.9)$$

Thus, the share of wages in output equals η/μ^i , where μ^i is the markup of price over marginal cost. As in Rotemberg and Woodford (1998), we set $\eta = 0.75$.⁶ Therefore,

$$-f''f/(f')^2 = 0.33. \quad (4.10)$$

Thus $\omega = 0.63 - \sigma$. Lastly, using Woodford's values, $\kappa = 0.024$, $\sigma = 0.16$, and $\psi = 0.88$, which in turn imply $\gamma = 0.63$, $\omega = 0.47$, and $\theta = 7.88$.

⁶That is, we assume that markups are modest in size.

In addition to displaying the empirical IRFs together with the confidence intervals, Fig. 1 also gives the theoretical responses that correspond to the estimated parameter value. In each panel, the theoretical response is a solid line, while the estimated response is the middle dashed line. As the figure indicates, the theoretical responses of output, inflation and interest rate match very closely the estimated responses. The model accounts well for both the magnitude and the persistence of the monetary shock effect on output.

The positive shock to interest rate lowers inflation, and, as a result, raises the nominal interest rate. From the IS equation, output falls. The increase in real interest rate relative to the fall in output is relatively small, therefore a value of σ that justifies such relative movements is also small. Note that inflation reverts more quickly to its mean in the theoretical response compared to the estimated response from the unrestricted VAR.

5 Identification of the Shock Processes

This section describes how the time series for the three stochastic disturbances, ϵ_t , \hat{G}_t and \hat{Y}_t^s , are constructed. It will be also shown how the VAR can be used to infer the stochastic process generating these variables. Then responses of the three endogenous variables to the shocks above are constructed for any given monetary rule.

The VAR system (2.3) is first pre-multiplied by T^{-1} to obtain

$$\bar{Z}_t = B\bar{Z}_{t-1} + U\bar{e}_t, \quad (5.1)$$

where $B = T^{-1}A$ and the matrix U consists of zeros except for its upper 3×3 block, which is a lower triangular matrix with ones on the diagonal. Therefore, the historical time series for the monetary shock ϵ_t can be derived from the following expression

$$\epsilon_t = i_1'(\bar{Z}_t - B\bar{Z}_{t-1}), \quad (5.2)$$

where i_n is a vector whose n -th coordinate is unity and all others are zeros. \tilde{Z}_t will be the vector whose elements are the model's theoretical predictions concerning the elements of

\bar{Z}_t , the vector of historical time series. The new notation is introduced, as there is need to distinguish between the *historical* law of motion, and the *theoretical* law of motion. After some algebraic manipulations, the main structural equations of the previous section can be written in terms of \tilde{Z}_t as

$$M' \tilde{Z}_t - N' \sum_{j=1}^{\infty} E_{t-1} \tilde{Z}_{t+j} = \hat{G}_{t+1} \quad (5.3)$$

$$P' E_{t-1} \tilde{Z}_t + R' E_t \tilde{Z}_{t+1} = \hat{Y}_{t+1}^s + \frac{\sigma}{\sigma + \omega} E_t (\hat{G}_{t+2} - \hat{G}_{t+1}). \quad (5.4)$$

Note that the time subscript is increased by 1 in both equations, so that the right-hand sides represent exogenous shocks in period t .

Under the assumption that the VAR correctly captures the stochastic process followed by the variables in Z_t , the time series for \hat{G}_t and \hat{Y}_t^s can be reconstructed, conditional on the assumption that agents' expectations coincide with the VAR forecasts. This implies that under the policy regime that generates the historical data, agents' forecasts

$$E_{t-1} \tilde{Z}_t = B \bar{Z}_t. \quad (5.5)$$

Thus, historical time series for \hat{G}_t and \hat{Y}_t^s can be reconstructed using the following

$$s_t \equiv [\hat{G}_t \hat{Y}_t^s]' = C \bar{Z}_{t-1} + D \bar{e}_t \quad (5.6)$$

$$C = \left(\begin{array}{c} M' - N' B (I - B)^{-1} \\ P' + R' B - \frac{\sigma}{\sigma + \omega} (N' B - M' (I - B)) \end{array} \right) B, \quad (5.7)$$

$$D = \left(\begin{array}{c} M' \\ R' B + \frac{\sigma}{\sigma + \omega} (M' (I - B) + N' B^2 (I - B)^{-1}) \end{array} \right) U. \quad (5.8)$$

Since the model incorporates forward-looking behavior, the simulations will also require that agents' beliefs about the stochastic processes generating the shock series be specified. In this case, agents regard the vector of shocks s_t , from $t = 1$, together with the law of motion for the stochastic process \bar{Z}_t , given a specified initial condition \bar{Z}_0 and the distribution from which the *i.i.d* innovations \bar{e}_t are drawn every period.

A complete simulation model consists of the Taylor rule, the distribution of the monetary shock, the structural equations in this section, the law of motion of the real disturbances, and the distribution of the shocks \bar{e}_t . The model will determine the evolution of $(\tilde{Z}_t, \bar{Z}_t, s_t)$

given initial conditions $(\tilde{Z}_0, E_0\tilde{Z}_1, \bar{Z}_0)$, and the white noise shocks (ϵ_t, \bar{e}_t) . By supplying the historical shock series computed above, counterfactual history can be simulated. Suitable initial conditions would be $\tilde{Z}_0 = \bar{Z}_0$, and $E_0\tilde{Z}_1 = \bar{Z}_0$, where \bar{Z}_0 is the historical data the period before simulations begin.

In the case when the assumed monetary rule is the estimated historical one, and the constructed historical shocks are feeded in, the predicted series (\tilde{Z}_t) will coincide with the historical data series (\bar{Z}_t) . This means that the identification of the shocks methodology allows a complete reconstruction of the historical data as the unraveling of a stationary rational expectations equilibrium.

For purposes of counterfactual historical simulations, Rotemberg and Woodford (1998) do not construct the historical shock series by substituting the historical data \bar{Z}_{t-1} and VAR residuals \bar{e}_t into model equations. Instead, they use historical VAR residuals \bar{e}_{2t} and \bar{e}_{3t} to construct the series e_t^\dagger , and then simulate the model, starting from an initial condition $Z_0^\dagger = \bar{Z}_0$, to generate the series (Z_t^\dagger) . The series (e_t^\dagger) and (Z_t^\dagger) are then used to construct the historical sequence of real shocks. Therefore, as pointed out in Rotemberg and Woodford (1998), "the method applied does not use the residuals from the structural equations, but rather the component of those residuals that is orthogonal to the monetary shock and all its lags." Therefore, as a result of the modification, the simulated paths produced when the estimated monetary rule is used no longer exactly equal the actual paths. This is important, as the degree to which the simulated data track the actual data then becomes measure of overall fit and an accuracy test for the structural model. As shown in the next section, the estimated model with historical shocks as described above feeded in the framework accounts well for the variation in real GDP, inflation, and nominal interest rate.

Another way of assessing the degree of correspondence between the Rotemberg and Woodford (1998) theoretical model and the UK data is to compare the empirical auto- and cross-correlation functions (ACFs and CCFs) for the three series of interest with the simulated ones from the micro-founded model, where the stochastic processes for the shocks are specified following the methodology described in this section. The advantage of comparing the

ACFs and CCFs is that those (second moment statistics) are invariant to the ordering of the variables in the VAR. The comparison is shown in Figure 2, where in each of the nine panels, the solid line indicates the theoretical cross-correlation function, and the dashed line the empirical cross-correlation function that is implied by the unrestricted VAR analysis of the UK data. It is clear that Rotemberg and Woodford's (1998) model accounts for the second moments of the UK data as well as the unrestricted VAR does. Moreover, the model is able to reproduce the degree of persistence in inflation observed in data. Lastly, it is able to match the negative correlation of output with the lagged nominal interest rate.

6 Simulation of Alternative Monetary Policy Rules

Thus section illustrates how the simulation model built in the previous two sections can be applied to predict the effects of an alternative monetary rule. Fig. 3 provides the results in a scenario, where a rule with the coefficients of the estimated historical rule was followed. In each panel of this figure, the dashed line represents the actual data of the series, while the solid one represents the simulation of the model under the assumption that the historical sequence of monetary policy shocks as well as the historical series for the real shocks took place. The dash-dot line represents a simulation in which the historical Taylor rule for the interest rate was followed, but with all monetary shocks set to zero.

The plot of output shows the two paths are identical. That is an indication that, when formulating monetary policy, the UK Monetary authority attaches a significant weight on output. The plots for inflation and interest rate are different, and there is an important reason why: In the absence of monetary shocks, simulation path for inflation is a straight line, because the best policy is to keep inflation rate constant, and adjust interest rate to smooth the fluctuations in output. In the case when the economy is hit by monetary shocks, however, it is best for the monetary authority to fix the interest rate. That is depicted by a horizontal LM curve, and an upward-sloping IS curve shifting around in the $IS - LM$ framework. When there is a positive monetary shock, e.g an increase in the demand for real balances, Bank of England increases the money supply so that the new equilibrium is at the same level of interest rate. However, output is now higher, thus IS shifts to the

right. Similarly, in the case with negative shock money supply decreases, and output falls. Thus, unexpected tight money has led to recession in the UK, and unexpected loose money stimulated the economy. Therefore, it can be safely concluded that monetary shocks have played an important role in driving output fluctuations.

7 Conclusion

This paper applied a rational expectations model for evaluating monetary policy that fits the UK data as well as a recursive VAR. The model is simple, and is supposed to be understood as a benchmark for more complicated and elaborated models. Its tractability allows for easy applicability. In addition, since this unsophisticated model fits data closely, that is an indicator that the model is not a bad description of reality. Models with explicit modeling of the investment and capital accumulation processes, as well as the exclusion of labor supply decision are left for future research.

It should be admitted that all the conclusions drawn from this study are have to be taken with some caution because the generated responses of output, inflation and interest rates to the stochastic disturbances may be sensitive to the particular specification of the model chosen. It is possible that conclusions change if elastic labor supply or the structure or producers' markups change. The important message is, however, that models that specify explicitly the channels of propagation of disturbances and their effects on real activity via the use of monetary policy are useful because they go hand in hand with theory when deriving econometric specifications.

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Table 1
The Vector Autoregression
Sample: 1980:1-1995:2

	R_t	π_{t+1}	Y_{t+1}
Indep. Variables	-	-	0.001715 (0.000346)
R_t	-	0.308024 (0.09418)	3.742599 (8.886678)
R_{t-1}	0.6205 (0.0001)	0.0620 (0.0000)	-39.9682 (0.0059)
R_{t-2}	0.1842 (0.0000)	0.0434 (0.0000)	9.3121 (0.0060)
R_{t-3}	-0.0093 (0.0059)	0.0006 (0.0060)	1.1155 (25.5020)
π_t	0.2761 (0.0001)	0.1151 (0.0000)	26.9448 (0.0079)
π_{t-1}	0.4100 (0.0000)	0.2430 (0.0000)	7.9641 (0.0057)
π_{t-2}	0.1581 (0.0046)	-0.0003 (0.0059)	0.0448 (25.1097)
Y_t	0.0006 (0.0000)	-0.1535 (0.0000)	-24.3564 (0.0098)
Y_{t-1}	-0.00014 (0.0000)	0.0350 (0.0000)	10.8676 (0.0053)
Y_{t-2}	-0.0006 (0.0039)	-0.0002 (0.0056)	-0.1991 (24.5338)
R^2	0.76	0.41	0.95

Standard Errors below estimates

Figure 1: Estimated and Theoretical Responses to a Monetary Shock

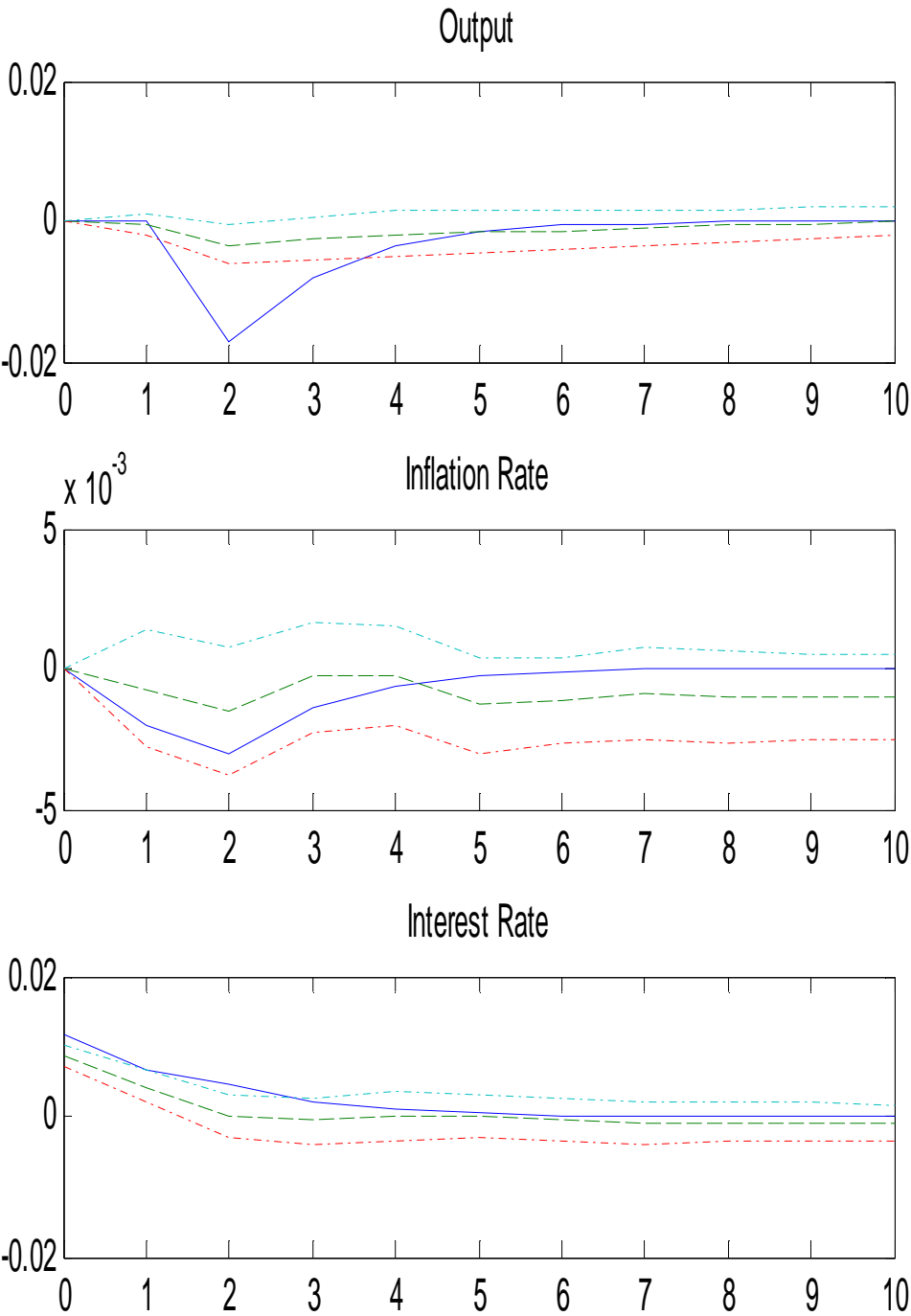


Figure 2 Empirical and Theoretical Auto and Cross-Correlation

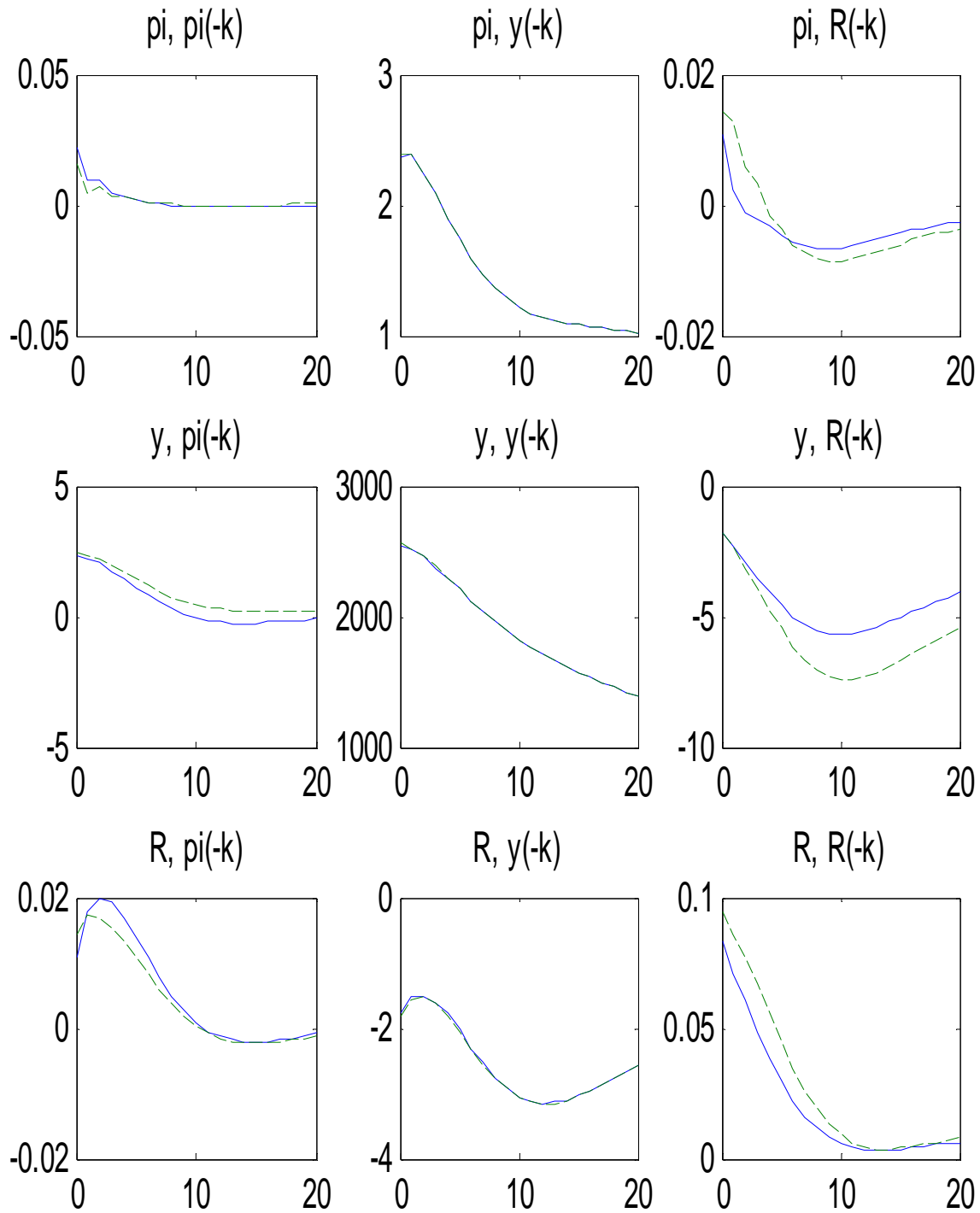


Figure 3: Actual and Simulates Paths

