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An incomplete markets explanation of the UIP puzzle

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ABSTRACT

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Keywords: Uncovered Interest Rate Parity, Incomplete Markets Precautionary Savings, Time-Varying Risk Premium

JEL-Codes: F31, F41, G12, G15

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An Incomplete Markets Explanation of the UIP Puzzle ^{*}

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November 6, 2015

Abstract

A large literature attributes failure of uncovered interest rate parity (UIP) to the existence of a time-varying risk premium. This paper presents a mechanism in a simple two-country two-good endowment economy with incomplete markets that generates sizeable deviations from UIP. In a parameterization where international wealth effects are important, liquidity constraints on an internationally traded bond and agents' strong resulting precautionary motives successfully generates a time-varying risk premium: countries that have accumulated large outstanding external positions have, being closer to the constraints, stronger precautionary motives and their asset carries a risk premium.

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1 Introduction

Large deviations from uncovered interest parity (UIP) are a strong regularity in the data. This paper addresses this puzzling empirical fact on international relative prices in one of the simplest models in international macroeconomics – a two-country two-good endowment model with standard preferences. The novel aspect is the precise incompleteness of financial markets introduced: countries face borrowing constraints on an internationally traded bond, the presence of which generates a time-varying wedge – a risk-premium – in the foreign exchange market. The risk premium, and resulting deviations from UIP, become quantitatively important when the model is parameterized with a low trade elasticity, which implies that the international transmission mechanism is characterized by large equilibrium wealth effects, as in Corsetti et al. (2008b). The model’s risk premium is bigger the larger a country’s outstanding external position, because of increasingly stronger precautionary motives of avoiding binding liquidity constraints. The main mechanism of the theoretical model is therefore consistent with recent empirical findings of Della Corte et al. (2014), who find that a currency becomes risky when it exposes investors to the risk of excessive debt, as reflected in large external positions.

The paper connects to a long literature, both empirical and theoretical. Uncovered interest rate parity, which states that the (nominal) interest rate differential should be equated to the expected change in the exchange rate, is a central feature of virtually all general equilibrium open economy models. While it would appear logical that investors would demand higher interest rates on currencies that are expected to fall in value, empirical evidence suggests that currency prices for high interest rates tend, instead, to appreciate. This departure from uncovered interest rate parity, also known as the forward premium anomaly, has been extensively documented. Following a seminal paper of Fama (1984), one strand of the literature argues that the failure of this interest parity relationship can be attributed to a time-varying risk premium on foreign exchange.¹

Empirically, the violation of the uncovered interest rate parity condition is usually tested by a simple regression of (actual) exchange rate variations on the nominal interest rate differential. Under the assumption of rational expectations and risk neutral agents, the forward exchange rate is an unbiased estimator of the future spot exchange rate and therefore, since covered interest rate parity holds, the UIP regression should theoretically deliver a regression coefficient of 1. This is severely violated in the data, where, for advanced economies, the UIP coefficient is generally found to be much lower than 1, typically even negative.

Standard models of the international economy have a hard time generating time-varying risk premia, at least when conventional preferences such as of the constant relative risk aversion (CRRA) form and temporary autoregressive shocks are employed, such as typically used in the literature on macroeconomic fluctuations.² I start from the simple and standard modeling framework of a two-country two-good endowment economy, generalizing Cole and Obstfeld (1991), or equivalent to the introductory example of Corsetti et al. (2008b). This paper asks

¹Other explanations include the ‘peso problem’ (i.e. that agents need to learn about structural changes of the economy over time and that during this transitional learning period, market participants make systematic prediction errors) or ‘noise trading’ (i.e. that agents are actually irrational because they believe the value of an asset depends on information other than economic fundamentals).

²A number of recent contributions have addressed the failure of UIP based on a combination of non-CRRA preferences (Epstein and Zin (1989) or Campbell and Cochrane (1999)) and particular shock dynamics, see, e.g. Bansal and Shaliastovich (2013), Colacito and Croce (2013) and Verdelhan (2010).

the question under which conditions a risk-premium can be generated in such a setting, that disturbs the otherwise tight link between the interest rate differential and subsequent exchange rates. These are twofold: one, a parameterization of the model with a low trade elasticity – as shown by Corsetti et al. (2008b), in this case international business cycle models are able to generate high volatility of international relative prices (terms of trade and the real exchange rate) and strong wealth effects that result in low risk sharing when international financial markets are incomplete. I develop intuition for why the model is able to generate deviations from UIP in this region of a low trade elasticity, from a version of the model economy with complete markets (CM), financial autarky (FA), and an incomplete markets bond-economy without constraints (IM-unconstrained). I examine elasticities that imply what Corsetti et al. (2008b) call 'positive international transmission' – an increase in domestic supply that leads to a real exchange rate depreciation, benefitting the foreign economy – as well as 'negative international transmission' – an increase in domestic supply, where the trade elasticity is so low that to increase world demand for domestic goods, the real exchange rate needs to appreciate, hurting the foreign economy.³ In the FA setting the interest rate differential and the expected exchange rate depreciation display a negative co-movement in the low trade elasticity region, while under CM, and even in the case of IM-unconstrained, they always co-move (almost) one-to-one.

Two, countries are assumed to be able to trade an international bond, but are subject to borrowing constraints, which state that a country cannot borrow more than a constant fraction K of its output. Typically, when there are no frictions in international borrowing and lending, a country that experiences a shock that lowers the value of its income, would like to access international financial markets in order to achieve a smooth consumption path. On the other hand, when borrowing constraints are present, there is a possibility that these constraints become binding at any point in time. As a result, that country might not be able to make use of the international financial market for its consumption smoothing purposes, and instead will want to save for bad times, to prevent becoming constrained in the future. Consequently, close to the constraints, when precautionary motives become large, consumption risk is increasingly less shared across countries and the behavior of the model becomes akin to the FA model. The strength of this precautionary motive on interest rates, the intertemporal prices of consumption, varies over the cycle. While the effect is strongest close to the constraint, it also has a non-negligible effect when bond holdings are away from (either country's) constraint, introducing a wedge between the interest rate differential and the expected change in exchange rates. This wedge, which varies with the strength of precautionary effects, can thus be interpreted as a time-varying risk premium on foreign exchange. In the baseline model, a regression of exchange rate variations on the interest rate differential from simulated model data delivers a UIP regression coefficient of -0.300 (-0.301) for the case of positive (negative) international transmission. This shows that the risk premium generated by the proposed mechanism can go a long way in realigning UIP regression coefficient produced by model simulated data with those found in the data.

Finally, it should be noted that the mechanism that generates the time-varying risk premium is not connected to nominal risks in the economy. In fact, the model presented in section 2 is an entirely real model. It can be shown, however, that the exchange rate premium can be decomposed into a term relating to real risk, a term relating to nominal (inflation) risks,

³As discussed in section 2.2, the empirical evidence for the prevalence of positive or negative transmission is mixed (see also, Corsetti et al. (2006, 2014)).

and the interaction of the two. Hollifield and Yaron (2001) perform such a decomposition and argue that empirically the inflation risk and the interaction terms seem to be of minor importance, and that models of exchange rate risk premia should focus on real risk, as opposed to nominal risk. Engel (2011) provides estimates, based on monthly data, from both standard nominal UIP regressions and equivalent real UIP regressions of the real exchange rate on the real interest rate differential. He finds that, for advanced economies, the interest parity puzzle is similar in real terms as in nominal terms: the resulting UIP slope coefficient from the regression in real terms is, like for the traditional regression in nominal terms, strictly below one, and generally negative.

The literature dedicated to the uncovered interest rate parity puzzle is enormous and cannot possibly be covered here. A number of surveys, by Froot and Thaler (1990), Lewis (1995), and Engel (1996) provide a good overview of the literature until then. Engel (1999) examines the properties of the foreign exchange risk premium in sticky price general equilibrium models, finding that, while such models are capable of producing large enough risk premia, the implied premia are typically constant⁴. More recently, Alvarez et al. (2009) stress the importance of time-varying risk premia resulting from endogenous market segmentation, Bacchetta and van Wincoop (2010) find costs of actively managing foreign exchange portfolios responsible for the failure of UIP, and Burnside et al. (2009) suggest a microstructure approach in addressing the puzzle. Verdelhan (2010), Bansal and Shaliastovich (2013), and Colacito and Croce (2013) address the UIP puzzle in frameworks of non-standard preferences combined with particular shock dynamics, typically in a setting of complete markets. To my knowledge, this paper is one of the first to theoretically address the UIP puzzle through a link of the currency risk premium with a country's external position (another recent contribution is Gabaix and Maggiori (2015)) – which is of empirical relevance, as recently documented in Della Corte et al. (2014). In a related paper, Leduc (2002) explores the implications of borrowing constraints in the two-country monetary model of Lucas (1982), with aggregate and idiosyncratic uncertainty. The mechanism, however, is quite different, as in his setup, no bond trade takes place in equilibrium. In the present paper, agents are allowed to borrow up to the borrowing constraint, which becomes binding only occasionally, and varying debt holdings are at the core of the time-varying risk premium.

The rest of the paper is structured as follows. Section 2 lays out the model setup, of the baseline economy with international liquidity constraints (IM-constrained), as well as the comparison cases: complete international financial markets (CM), financial autarky (FA), and the standard incomplete-markets bond-economy without borrowing constraints (IM-unconstrained). Section 3 develops intuition from these comparison economies for the co-movement properties of international interest differential with expected exchange rate changes. Section 4 discusses parameterization and briefly comments on the solution technique. Section 5 presents results for the baseline model with international liquidity constraints. I present business cycle stylized facts and results from a UIP regression of simulated model data, together with sample simulated time paths, some selected policy functions of interest, and the stationary distribution of bond holdings. I also present some sensitivity analysis on the UIP regression results. Finally, section 6 concludes.

⁴At least, as long as the exogenous forcing process has constant variance.

2 The model

2.1 Model setup

The world economy consists of two countries, Home and Foreign, each of which specializes in production of one type of (traded) good. All idiosyncratic risk is assumed to be perfectly insured among residents of a country, i.e. within-country financial markets are complete. Foreign variables are denoted with an asterisk. We can therefore think of a representative consumer in each country that maximizes the expected sum of future discounted utilities from consumption, C_t :

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t), \quad (1)$$

where β is the discount factor. The utility function $u(C_t)$ is taken to be of the constant relative risk aversion form, $u(C_t) = (1/(1-\sigma)) [C_t^{1-\sigma} - 1]$, where σ is the coefficient of relative risk aversion. Aggregate consumption is a constant elasticity of substitution (CES) basket over domestic good and foreign good consumption:

$$C_t = \left[\gamma^{\frac{1}{\omega}} C_{H,t}^{\frac{\omega-1}{\omega}} + (1-\gamma)^{\frac{1}{\omega}} C_{F,t}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \quad (2)$$

where $C_{H,t}$ and $C_{F,t}$ are the home country's consumption of home and foreign goods. Parameter γ is the degree of home bias in consumption, parameter ω is the intratemporal elasticity of substitution between domestic and foreign consumption goods, which in this model corresponds to the trade elasticity. The foreign representative agent faces an equivalent problem, where foreign variables are denoted with an asterisk. Agents of each country receive an exogenous country-specific (and therefore good-specific) endowment Y_t or Y_t^* respectively in every period t . I abstract from modeling a production side, and assume instead that outputs arrive exogenously each period, following a bivariate autoregressive process of order 1:

$$\begin{pmatrix} y_t \\ y_t^* \end{pmatrix} = \begin{pmatrix} \rho & \psi \\ \psi & \rho \end{pmatrix} \begin{pmatrix} y_{t-1} \\ y_{t-1}^* \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^* \end{pmatrix}, \quad (3)$$

where ρ and ψ are coefficients describing the autocorrelation and spillover properties of the process, and ε_t and ε_t^* are normally distributed mean-zero shocks with variance σ_ε and correlation ρ_ε . Throughout the remainder of the paper I denote with lowercase variables the logarithm of original variables, that is, $y_t = \log(Y_t)$. Mean income, \bar{Y} and \bar{Y}^* , is normalized to one.

Asset markets are incomplete: I assume that there is a one-period risk-free bond, $B_{H,t}$, that is only traded within the domestic economy, and a one-period risk-free bond, $B_{F,t}^*$ that is only traded within the foreign economy. $B_{H,t}$ and $B_{F,t}^*$, promise, one unit of domestic, and one unit of the foreign (final) consumption good, respectively, in the next period and trade at prices $\frac{1}{R_t}$ and $\frac{1}{R_t^*}$, where R_t and R_t^* are the gross domestic and foreign real interest rates.⁵ In addition, I assume that there is a (one-period risk-free) bond that can be traded internationally, between the two countries, denoted B_t . I assume that the internationally traded bond B_t pays in units of the domestic consumption good, and trades at price $\frac{1}{R_t^w}$

⁵Since agents within an economy are identical (i.e. there is a representative agent within each country), there will be no trade in bonds $B_{H,t}$ and $B_{F,t}^*$ in the aggregate. Nevertheless, they are modeled here explicitly to derive (optimal) domestic and foreign interest rates.

(superscript w stands for the world interest rate).⁶ Furthermore, define the real exchange rate, Q_t , as the relative price of the foreign final consumption good to the domestic final consumption good, and $p_{H,t}$ ($p_{F,t}$) as the domestic consumer's relative price of the domestic (foreign) consumption good to the price of domestic final consumption.⁷ We can then write the home country's and the foreign country's budget constraints as:

$$\frac{B_{H,t}}{R_t} + \frac{B_t}{R_t^w} = B_{H,t-1} + B_{t-1} + p_{H,t}Y_t - C_t, \quad (4)$$

$$\frac{B_{F,t}^*}{R_t^*} + \frac{B_t^*}{Q_t R_t^w} = B_{F,t-1}^* + \frac{B_{t-1}^*}{Q_t} + p_{F,t}^* Y_t^* - C_t^*. \quad (5)$$

Even though agents are assumed to be able to trade in the risk-free bond B_t in order to smooth their consumption, they cannot do so unrestrictedly. In particular, I assume that the home country's debt level cannot exceed some fraction K of the level of its steady state output (which is normalized to one):⁸

$$B_t \geq -K \quad (6)$$

The foreign country's borrowing constraint is an equivalent version (6). The borrowing limit for the foreign country is therefore given by $B_t^*/Q_t \geq -K^*$.⁹

The domestic household's intratemporal optimization problem of choosing optimal consumptions of the domestic and foreign good give the following intratemporal optimality conditions:

$$C_{H,t} = \gamma p_{H,t}^{-\omega} C_t \quad (7)$$

$$C_{F,t} = (1 - \gamma) p_{F,t}^{-\omega} C_t \quad (8)$$

Denote with λ_t the Lagrange multiplier on the domestic household's budget constraint, and with μ_t the multiplier on the borrowing constraint. The representative household's first order conditions to the intertemporal optimization problem can then be stated as:

$$C_t^{-\sigma} = \lambda_t \quad (9)$$

$$C_t^{-\sigma} = \beta R_t E_t [C_{t+1}^{-\sigma}] \quad (10)$$

$$C_t^{-\sigma} = \beta R_t^w E_t [C_{t+1}^{-\sigma}] + \mu_t R_t^w \quad (11)$$

⁶In order to avoid having to solve a portfolio problem, I assume that only there is no foreign-good bond traded internationally.

⁷The final consumption price is the usual CES-based price aggregator, $P_t = [\gamma P_{H,t}^{1-\omega} + (1 - \gamma) P_{F,t}^{1-\omega}]^{\frac{1}{1-\omega}}$. Lowercase variables $p_{H,t}$ and $p_{F,t}$ are thus defined as $p_{H,t} \equiv \frac{P_{H,t}}{P_t}$ and $p_{F,t} \equiv \frac{P_{F,t}}{P_t}$.

⁸In principle, there is also a 'natural debt' limit as in Aiyagari (1994) according to which both countries will not borrow more than the minimum value that the endowment can take at period $t+1$ discounted to period t prices. To compute the natural debt limit in a two-country model, where the interest rate is endogenous, is more difficult than in a partial equilibrium model where the interest rate is exogenous. In addition if one of the constraint binds for one of the economies the interest rate generally differs for each agent (for a detailed discussion see Anagnostopoulos (2006)). However, the debt limits we impose here are generally stricter than the natural debt limit.

⁹Note that, should the trade elasticity fall below a threshold $\tilde{\omega} = 1 - \frac{1}{2\gamma}$, the borrowing constraints just described need to be replaced by lending constraints (a discussion of this point follows in section 4).

$$\frac{B_{H,t}}{R_t} + \frac{B_t}{R_t^w} = B_{H,t-1} + B_{t-1} + p_{H,t}Y_t - C_t, \quad (12)$$

$$B_t \geq 0, \mu_t \geq 0, 0 = \mu_t [B_t + K] \quad (13)$$

Equation (9) relates Lagrange multiplier λ_t to the marginal utility of consumption. Equation (10) is the Home country's Euler equation w.r.t. the nationally traded bond paying in domestic goods, which is obtained from the combination of the first order conditions for C_t and $B_{H,t}$ and states that the marginal benefit from using debt to increase consumption at time t must be greater than or equal to the expected marginal loss at time $t+1$ arising from the additional debt. Equation (11) is the corresponding Euler equation w.r.t. the internationally traded bond (that pays in units of domestic goods). It should be noted that it follows from equations (10) and (11) that, unless the borrowing constraint becomes binding, there always will be interest rate equalization between R_t and R_t^w . Equation (12) is the Home country's budget constraint, stating that current consumption and outstanding debt have to be financed either from current output or by issuing new debt. Equation (13) is the complementary slackness condition with μ_t being the multiplier on inequality constraint (6). The foreign country faces an equivalent problem, which results in a set of optimality conditions similar to equations (7) to (13).

Note that all relative prices, the real exchange rate, Q_t , and the domestic (foreign) relative prices of domestic and foreign goods to the domestic (foreign) CPI, $p_{H,t}$ and $p_{F,t}$ (or $p_{H,t}^*$ and $p_{F,t}^*$ for the foreign economy), can be written as functions of the terms of trade, τ_t , which is defined as the relative price of domestic versus foreign goods:

$$\tau_t \equiv \frac{p_{F,t}}{p_{H,t}} = \frac{p_{F,t}^*}{p_{H,t}^*}. \quad (14)$$

The equilibrium of this economy is defined as a path of prices $\{R_t, R_t^*, R_t^w, \tau_t\}_{t=0}^{\infty}$ together with consumption plans $\{C_{H,t}, C_{F,t}, C_t\}_{t=0}^{\infty}$ and $\{C_{H,t}^*, C_{F,t}^*, C_t^*\}_{t=0}^{\infty}$ and debt plans $\{B_t\}_{t=0}^{\infty}$ and $\{B_t^*\}_{t=0}^{\infty}$ such that:

1. $C_{H,t}$ and $C_{F,t}$ minimize the expenditure needed to buy one unit on domestic consumption good, C_t , given by (2). Similarly, $C_{H,t}^*$ and $C_{F,t}^*$ minimize the expenditure needed to buy one unit on foreign consumption good, C_t^* .
2. C_t and $B_{H,t}$ and B_t maximize (1) subject to (4)-(6), for all t , and $B_{H,0}$, B_0 given,
3. C_t^* , $B_{F,t}^*$ and B_t^* maximize the foreign version of (1) s.t. the foreign versions of (4)-(6), for all t , and $B_{F,0}^*$ and B_0^* given,
4. the real interest rates clear the bond markets, $B_{H,t} = 0$, $B_{F,t}^* = 0$, and $B_t + B_t^* = 0$, for all t ,
5. the goods markets also clear, that is $C_{H,t} + C_{H,t}^* = Y_t$, $C_{F,t} + C_{F,t}^* = Y_t^*$, and $\frac{B_t}{R_t^w} = B_{t-1} + p_{H,t}Y_t - C_t$ for all t .

The complete list of equilibrium conditions is summarized in Appendix A.

2.2 Comparison model economies

For easier interpretation of the workings of the financial market assumption of the baseline model with borrowing constraints (abbreviated as 'IM-constrained' hereafter), and to put the results into perspective, it is of interest to compare the model of section 2.1 to a number of benchmark cases studied previously in the literature (see, e.g. Cole and Obstfeld (1991), Corsetti et al. (2008b), Baxter and Crucini (1995), Heathcote and Perri (2002)), which are briefly summarized in this section.

The first of these benchmark cases is one of complete international financial markets (CM). In this model economy there exists a complete set of state-contingent assets for each possible state of the world tomorrow. It is well known, that this financial market structure leads to full international risk sharing, that is, to an equalization of the the ratio of marginal utilities across countries to the real exchange rate:

$$Q_t = \frac{C_t^{*-\sigma}}{C_t^{-\sigma}}. \quad (15)$$

The second benchmark case is the exact opposite of the above, the case in which no assets can be traded internationally and the two economies are in financial autarky (FA). As a result, there is no room for intertemporal trade and the current consumption expenditure has to be fully financed from current income in both economies:

$$p_{H,t}Y_t = C_t. \quad (16)$$

Finally, a third interesting reference case is the incomplete-markets bond-economy without the presence of additionally imposed ad-hoc borrowing constraints (IM-unconstrained). In order to avoid a well-known non-stationarity problem I assume that agents' discount factor are endogenous, with $\beta'(C_t) < 0$, such as in Uzawa (1968) or Schmitt-Grohé and Uribe (2003).¹⁰ The resulting Euler equation under this financial market assumption is given by:

$$1 = \beta(C_t^*) R_t E_t \left[\frac{C_{t+1}^{*-\sigma}}{C_t^{*-\sigma}} \frac{Q_t}{Q_{t+1}} \right] \quad (17)$$

3 Building up intuition: the international transmission mechanism in the comparison economies

Before turning to the baseline model of incomplete markets with borrowing constraints, I use the comparison models, in particular, the economies under CM and FA to develop intuition, drawing heavily from Corsetti et al. (2008b). These authors emphasize that the value of the trade elasticity is crucial in shaping the international transmission mechanism, because of its

¹⁰Schmitt-Grohé and Uribe (2003) document the problem of non-stationarity of the first-order approximated incomplete-markets bond-economy model, and propose several technical devices to introduce stationarity – out of which the endogenous discount factor assumed here is one. In addition, Bodenstein (2011) documents that for certain parameter constellations of the trade elasticity, (in particular, in the low region implying negative transmission) only the endogenous discount factor imposes stationarity. I choose the functional form, $\beta(C_t) = \beta C_t^{-\eta}$, and set parameter $\eta = 0.005$.

large influence on how international price movements impinge on the relative wealth of both countries. Because of the relevance for understanding the mechanism of the paper, I review part of their analysis here, supporting and complementing it by figure 1. The figure plots, for several variables of interest, the first period – impact – response to a one percent domestic output increase as a function of the trade elasticity ω , which is on the horizontal axis.^{11,12}

Complete markets: Corsetti et al. (2008b) show, that under CM, an increase in domestic output (relative to Foreign) unambiguously leads to a depreciation in the Home terms of trade and real exchange rate, as shown in panel 1A. The international transmission of the shock is thus positive under CM, as foreign consumers benefit from more favorable international relative prices (panel 2A). Because the exchange rate depreciates on impact of the shock, and is then, as the shock fades out, expected to slowly appreciate back to its steady state level, $E_t \Delta q_{t+1}$ drops on impact of the shock. The response of the interest rate differential under CM, can be understood as follows: in response to the shock consumptions in both countries increase, but more strongly so in the domestic economy. As a result the domestic interest rate falls more strongly than the foreign, leading to a drop of the interest rate differential. As can be seen the drop in the expected exchange rate change and the interest rate differential under CM is one-to-one (panel 3A). The qualitative behavior of these variables does not change depending on the trade elasticity ω .

Financial autarky: The international transmission mechanism is markedly different in the case of the economy under financial autarky, and strongly dependent on the value of the trade elasticity. To understand the responses in figure 1, it is useful to reflect on size and interplay of substitution and income effects of a shock. As documented in detail by Corsetti et al. (2008b), an increase in the terms of trade (a lower relative price of domestic to foreign goods) leads to a positive substitution effect in Home, increasing the domestic demand for domestic good, C_H . However, since it decreases the value of domestic output it also, simultaneously, imposes a negative income effect. The overall effect on C_H is thus, generally ambiguous. Foreign demand of domestic goods increases, as the increase in the terms of trade induces, for foreign country, both a positive substitution effect (home goods are now cheaper), but also a positive income effect (the value of foreign output increases). Panel 1B of figure 1 shows that, in response to the domestic output increase, the terms of trade and the real exchange rate depreciate under FA as long as the trade elasticity is above a crucial threshold of $\tilde{\omega} = 1 - 1/(2\gamma)$. Since foreign consumption increases in response to the domestic output increase, Corsetti et al. (2008b) talk of 'positive international transmission' in the region of $\omega > \tilde{\omega}$. If ω is instead low enough, $\omega < \tilde{\omega}$, the negative income effect in the home country becomes so strong, that the increase in domestic supply cannot be matched by an increase in world demand at lower prices: in this case, the Home terms of trade need to appreciate, as seen from panel 1B to the left of the transmission threshold, $\tilde{\omega}$ (depicted as a horizontal line in the figure). Since, with a terms of trade and real exchange rate appreciation, the Foreign substitution effect and income effect both lead to a fall in foreign consumption,

¹¹For convenience, the model impulse responses of figures 1 are derived from a log-linear solution of the model – in response to small (and temporary) shocks, model variables do not stray far away from their respective steady states. All results in the following sections will, however, be based on the true, nonlinear solution (also for the comparison models).

¹²See table 1, for the parameters used for the generation of figure 1. Note that in particular, the economy features home bias in preferences, $\gamma = 1 - \gamma^* = 0.78$.

Corsetti et al. (2008b) label the international transmission 'negative' in the region where $\omega < \tilde{\omega}$.

Which one is the empirically relevant transmission mechanism – positive or negative? To answer this question, Corsetti et al. (2014) use a VAR with sign restrictions to study the (conditional) effects of a US productivity shock, finding evidence for the case of negative transmission: a US productivity increase leads to a real exchange rate appreciation. In an earlier paper, Corsetti et al. (2006) perform a similar exercise in a VAR, identified with long-run restrictions, for a sample of five G7 countries. The evidence here is mixed: while a productivity increase leads to a real exchange rate appreciation for the US and Japan, it leads to a depreciation in the cases of the UK and Italy, with inconclusive results for Germany. Given that the empirical evidence is mixed, the calibration strategy I follow will be to allow for both cases, presenting model results for the case of both positive and negative transmission.¹³

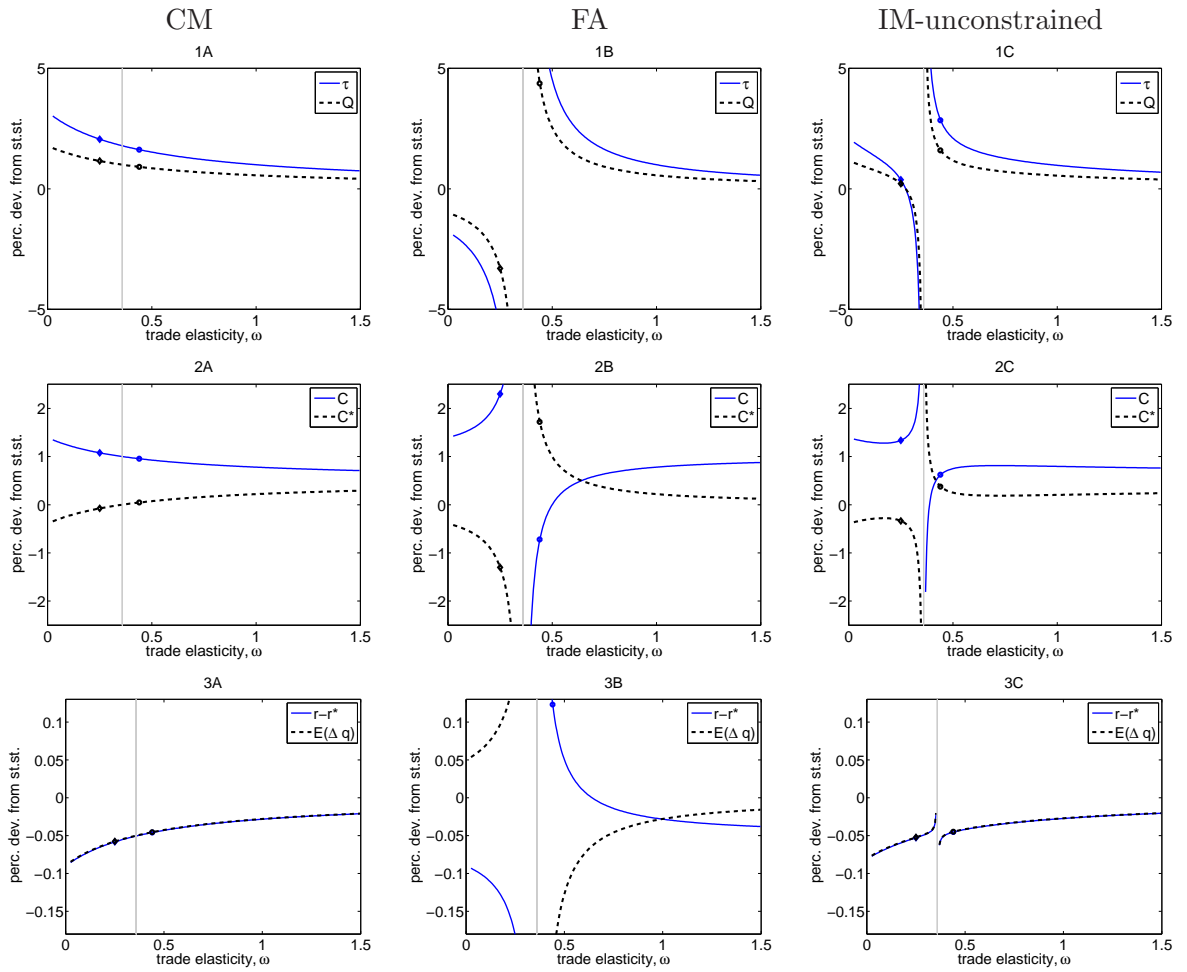
Panel 3B shows that, under FA, the model is able to decouple the interest rate differential from expected exchange rate changes, and produce a negative co-movement in these variables. This is possible for low values of the trade elasticity, both in the region of positive and negative transmission. The underlying mechanism differs however: in the case of positive transmission, the exchange rate, having depreciated in response to the domestic output increase, is expected to appreciate back to its steady state as the shock dissipates, thus $E_t \Delta q_{t+1}$ drops. The interest differential, at the same time, rises instead of falling, as would be the case in a model with complete markets. This reflects the fact that foreign consumption rises more than home consumption due to the large positive income effect in Foreign, coming from a foreign real exchange rate appreciation. In the case of negative transmission, the appreciation of the real exchange rate implies an expected depreciation, i.e. an increase in $E_t \Delta q_{t+1}$. At the same time, because consumption at Home expands strongly and consumption abroad contracts, the interest rate differential is negative.

Incomplete markets bond economy – unconstrained: Finally, for sake of completeness, I also present the responses in the IM bond economy, in the case where no borrowing constraints are present (IM-unconstrained). An interesting finding is that, despite market incompleteness, the responses of interest rate differential and expected exchange rate changes are almost identical to the ones obtained under CM, unless one considers a very narrow region of the trade elasticity around the transmission threshold, $\tilde{\omega}$.

While figure 1 suggests that the model under financial autarky can potentially realign model predictions with the UIP regression results obtained from the data, international financial autarky hardly seems a reasonable model framework. Nevertheless, what it suggests is, that, in order to realign the empirical finding of deviations from uncovered interest parity in a model of the international economy with conventional preferences and shock processes two ingredients are crucial, that have been employed in the paper: financial markets need to be sufficiently incomplete, and the incompleteness needs to matter a great deal. The latter is achieved through the parameterization of a low trade elasticity, which leads to large wealth transfers through international relative price movements. The former results from the introduction of borrowing constraints to a bond economy setup, as presented in the baseline model, results of which will be presented in section 5.

¹³It should be noted that, in the positive transmission region close to the transmission threshold, also implies that domestic consumption may rise less than foreign consumption in response to the domestic output increase (in the case of FA domestic consumption even falls) – this behavior is clearly counterfactual and makes the negative transmission mechanism empirically more appealing.

Figure 1: Impact impulse response to a domestic output increase under various international financial market assumptions



This figure presents first-period (impact) impulse responses to a one percent domestic output increase as a function of the trade elasticity (on the horizontal axis). The figure distinguishes three forms of the international financial market: panels 1A-3A shows the case for complete markets (CM); panel 1A plots the responses of the terms of trade and the real exchange rate, panel 2A the consumption responses and panel 3A the interest rate differential and expected exchange rate change; panels 1B-3B show the equivalent responses under financial autarky (FA); panels 1C-3C show impact responses under a bond-economy without borrowing constraints (IM-unconstrained). In each plot, 'circles' indicate the impact responses of the respective variable under the value of the trade elasticity of $\omega = 0.44$ ('positive transmission'), 'diamonds' the responses under $\omega = 0.25$ ('negative transmission').

4 Parameterization and solution technique

In this section, I outline the baseline parameterization of the model, which is also summarized in Table 1. Both countries are symmetric in all structural parameters. The discount factor, β is set to 0.99, corresponding to an annualized interest rate of about 4 percent. The degree of home bias in consumption, that is the weight of domestic goods in the consumption basket, γ , is set to 0.78, a value that matches the average import share (as a percent of GDP) for G7 economies of 22.1 percent over the period of 1979-2013.¹⁴

Table 1: Baseline parameterization

Discount factor	β	0.99
Coeff. of relative risk aversion	σ	1
Degree of home bias	γ	0.78
Trade elasticity	ω	0.44, 0.25
Autocorrelation of AR(1)	ρ	0.95
Standard deviation of AR(1)	σ_ϵ	0.01
Borrowing constraint	K, K^*	0.5

In line with most of the international business cycle literature, I assume a rather persistent exogenous forcing process, taking the coefficient of autocorrelation of the endowment process, ρ , to be 0.95 and a standard deviation of $\sigma_\epsilon = 0.01$. I abstract from spillovers or cross-country correlation of the shocks. The trade elasticity ω , that is, the elasticity of substitution between domestic and foreign good, is a crucial parameter in the class of open economy macro models, and its value varies largely in the literature, both on the theory side and in empirics. Empirical estimates for trade elasticities vary depending on the level of data aggregation and whether they reflect short-term or long-term elasticities. E.g., based on aggregate data, Whalley (1984) reports a trade elasticity of 1.5. Hooper et al. (1998) find a short-run trade elasticity of 0.6 for the US and values in the range of 0 and 0.6 for the remaining G7 countries. Taylor (1993) reports a short-run trade elasticity of 0.22. At lower levels of aggregation, based on various country pairs of traded goods, Broda and Weinstein (2006) report mean estimates for the elasticity of substitution in the range of 4 and 6. In many multi-good models of the international macroeconomy, the trade elasticity has been calibrated at values which imply goods being slight substitutes, at values around or slightly above 1 (see, e.g., Backus et al. (1995) or Chari et al. (2002)). However, there also exists a macro literature that suggests substantially lower values, especially when stylized facts on international relative prices are matched (see, e.g. Heathcote and Perri (2002), Corsetti et al. (2008b), Enders and Mueller (2009), Thoenissen (2010)).

Movements in the real exchange rate and the terms of trade will be larger in an economy where the trade elasticity is lower. Since the present paper's aim is to address exchange rate

¹⁴Average import shares are calculated using WDI data. Considering the later period of 2000-2013, the average import share of G7 economies stands at 25.6 percent, reflecting the increase in global trade. There also is considerable heterogeneity among G7 economies. Average import shares over the 1979-2013 period (over the 2000-2013 period) stand at 30.1 (33.2) percent for Canada, 24.1 (27.6) percent for France, 27.6 (34.3) percent for Germany, 22.2 (25.7) percent for Italy, 11.2 (13.6) percent for Japan, 26.8 (29.5) percent for the UK, and 12.5 (15.3) percent for the US.

related puzzles, I follow the macro literature in choosing a low trade elasticity. As outlined previously, in this class of two-country two-good incomplete markets models, there are two such values of the trade elasticity, one associated with positive international transmission (to the right of the transmission threshold $\tilde{\omega}$) and one associated with negative international transmission (to the left of $\tilde{\omega}$). Corsetti et al. (2008a) report a relative volatility of the (trade-weighted) real exchange rate to GDP for the U.S. of 3.25%, based on quarterly data over the period 1931Q1-2007Q2. To generate similarly volatile real exchange rates in the model, ω is taken to be 0.44 (0.25), implying that international transmission of shocks is positive (negative). This value also implies that wealth effects from any underlying shock are large and the incompleteness of financial markets is of great importance.

Parameter K governs the tightness of borrowing constraints, which I set to 50% of steady state output in the baseline setting. This choice implies that in a model simulation the borrowing constraint binds only 0.22 (0.73) percent of the time in the case of positive (negative) transmission. This choice guarantees that results are not driven by international financial markets coming to an actual halt. Section 5 also presents sensitivity analysis for alternative values of parameter K .

Finally, I want to comment briefly on the model solution method. To evaluate the mechanism of the paper, local approximation techniques like log-linearization around the non-stochastic steady state cannot be used. Instead, a global solution technique needs to be used, that can explicitly account for the influence of second moments and occasionally binding inequality constraints on agent's policy functions. Further details about the solution technique are provided in the appendix.

With the intuition about international transmission developed in section 3, I now want to comment on an important issue, briefly mentioned in section 2 in the model description. While in the region $\omega > \tilde{\omega}$, a country is constrained in not borrowing more than a threshold value K , the logic is, in fact, opposite in the case of negative international transmission, $\omega < \tilde{\omega}$, in which the countries need to be constrained from lending. In the region $\omega < \tilde{\omega}$, domestic consumption responds very strongly to an increase in domestic supply, as the *TOT* and *RER* appreciate. As described previously in section 3, this is because no fall in domestic prices (relative to foreign) can match the increased world supply of the *H*-good with an increase in world demand for the *H*-good. In fact, inspecting again figure 1, we can observe that domestic consumption (panels 2A-2C) in the incomplete markets economies (FA, IM-unconstrained, and similarly in the baseline model with liquidity constraints) expands inefficiently much compared to CM. This large (overproportional) increase in domestic consumption in response to the increase in domestic supply, can only be achieved if the domestic country is allowed to borrow heavily from abroad, thus borrowing constraints need to be extremely loose, or, more precisely, lifted entirely. But even this is not enough: the loosest possible borrowing constraint is only enough to allow the amount of borrowing that enables the domestic agent to come close to the CM allocation; any borrowing constraint tighter than that would make the baseline 'liquidity constraints model' deviate from CM, but would move it, not closer, but further away from how consumption would behave under FA. To induce the domestic agent to borrow enough to achieve that overproportional consumption expansion, borrowing needs to be incentivized, over and above the amount that would take place when borrowing constraints are lifted – this can only be achieved by disincentivizing lending. Thus, in the region of a trade elasticity that is associated with negative transmission, $\omega < \tilde{\omega}$, lending constraints are required instead of borrowing constraints.

Lending constraints can be interpreted, for example, as constraints on a country’s capital outflows (see Prades and Rabitsch (2012)). Schindler (2009) and Binici et al. (2010) construct and use a capital control index based on legal restrictions (a *de jure* index) that distinguishes between inflow and outflow controls; they document not only the existence of both inflow *and* outflow constraints for many economies, but also show that such distinction may matter for both the volume and the composition of capital flows.

5 Results from the baseline model economy

This section illustrates the workings of the model with borrowing constraints (IM-constrained), contrasting it throughout to the comparison cases of complete financial markets (CM), financial autarky (FA) and the case where a single bond is internationally traded, but trade is not restricted through ad-hoc borrowing constraints (IM-unconstrained). I start by presenting some business cycle stylized facts, and sample time paths from a model simulation. I then turn to the main results, presenting the implied UIP coefficients from simulated model data, as well as some additional model moments documenting the properties of exchange rate variations, interest rate differentials, and the risk premium. Finally, I present a comparison of the bond-economy models with and without constraints in terms of policy functions of interest, and the implied stationary distribution. Finally, I discuss sensitivity of UIP regression coefficients.

5.1 Business cycle statistics and sample time paths

Table 2 presents a summary of second moments of model simulated data, both for the case of a trade elasticity implying positive international transmission ($\omega = 0.44 > \tilde{\omega}$) and negative international transmission ($\omega = 0.25 < \tilde{\omega}$). In case of positive transmission, the baseline model with international liquidity constraints matches the data well, generating a consumption volatility that is somewhat lower than that of GDP. While the degree of exchange rate volatility was matched by calibrating the trade elasticity correctly, it is noteworthy that the model is successful at addressing also another long-standing puzzle in international economics. A large literature documents that exchange rate data display a negative correlation with cross-country consumption ratios, that is, that there is an apparent lack of efficient risk sharing (see Backus and Smith (1993)). The presence of international borrowing constraints increases the incompleteness of the financial markets which decouples consumption paths from their relative price and thus help realign the consumption-real exchange rate anomaly with respect to the data. For the case of the low trade elasticity, implying negative transmission, I find that the volatility of consumption generated by the model is too high relative to the data, an unwanted side-effect of the large income effects that come with it. Also, the relative volatility of the real exchange rate to GDP is too low, despite the fact that it is very volatile in absolute terms.¹⁵ I attribute some of these shortcomings to the (overly) simplistic model setup of the international economy. As shown by Corsetti et al. (2008b), a richer macroeconomic

¹⁵The relative volatility of the real exchange rate to output, $\sigma(q_t)/\sigma(y)_t$, is 3.06 in this parameterization, but gross domestic product, $GDP_t = p_{Ht}Y_t$, is itself very volatile because of the volatility of p_{Ht} .

Table 2: Business Cycle Statistics

	data	IM-constrained model (baseline)	IM-unconstrained model	CM model	FA model
case of positive transmission, $\omega > \tilde{\omega}$					
σ_c/σ_{gdp}	0.94	0.91	1.02	1.30	1.00
σ_q/σ_{gdp}	3.90	3.32	2.98	1.75	3.24
$\rho_{c,gdp}$	0.86	0.94	0.88	0.90	1.00
$\rho_{q,gdp-gdp^*}$	-0.19	-0.95	-0.93	1.00	-1.00
$\rho_{\Delta(c-c^*),\Delta q}$	-0.70	-0.56	0.94	1.00	-1.00
case of negative transmission, $\omega < \tilde{\omega}$					
σ_c/σ_{gdp}	0.94	1.23	1.50	1.52	0.66
σ_q/σ_{gdp}	3.90	1.09	0.47	2.29	0.56
$\rho_{c,gdp}$	0.86	0.98	0.94	0.72	0.69
$\rho_{q,gdp-gdp^*}$	-0.19	-0.91	0.75	1.00	-0.89
$\rho_{\Delta(c-c^*),\Delta q}$	-0.70	-0.85	0.95	1.00	-0.39

This table presents a summary of key second moments from model simulated data, contrasting them to their empirical counterpart. Column 'data' reproduces the stylized facts from Corsetti et al. (2008), which are based on HP-filtered data of the United States taken as the Home country and a trade-weighted aggregate of the OECD comprising the European Union, Japan, and Canada as the Foreign country, for the period 1970:2001. Column 'IM-constrained' refers to the baseline economy: a bond economy with international liquidity constraints. Column 'IM-unconstrained' refers to a bond economy without such borrowing constraints. Column 'CM' refers to the model version with complete international financial markets. Column 'FA' refers to the model version under financial autarky.

model, calibrated with a value of the trade elasticity below the threshold $\tilde{\omega}$, performs well at matching business cycle stylized facts, both within-country and in terms of international aspects, such as international relative prices.

Figure 2 portrays the behavior of the economy in response to a random sequence of shocks, for the baseline model with borrowing constraints and the three comparison model economies. To save space, I here only present sample time paths for the economy parameterized to imply positive international transmission. Panels 1A-1D of figure 2 display the time paths of the terms of trade, τ , and the real exchange rate, Q ; panels 2A-2D the time paths of the countries' GDP, panels 3A-3D turn to the implied consumption paths. Panels 4A-4D turn to the cyclical behavior of the variables of main interest, displaying the behavior of the (log) interest rate differential and its decomposition into (log) expected exchange rate variations and a (log) risk premium. This decomposition follows Fama (1984), and, stated for real variables, is given by

$$r_t - r_t^* = E_t[q_{t+1} - q_t] + p_t. \quad (18)$$

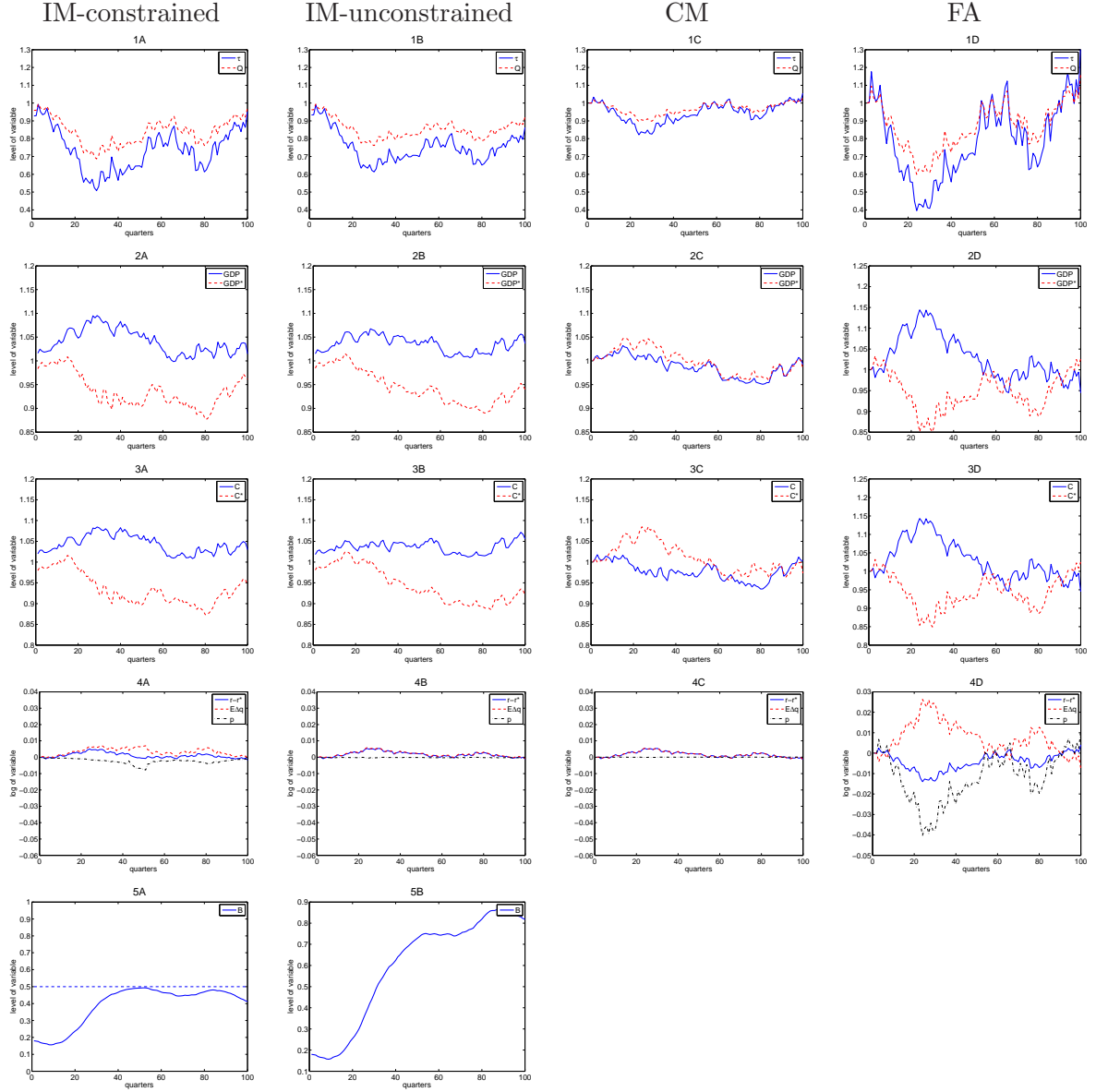
In the case of *complete international financial markets* (CM), the presence of state contingent assets implies that the economies are sheltered from large negative income effects and that the consumption paths are largely unaffected by them. E.g., when, as a consequence of a series of shocks more favorable to Foreign, consumption in the foreign country is higher than in the domestic (as happens around period 20), the interest rate differential (panel 4C) is positive, as r^* tends to fall more strongly than r . At the same time, the real exchange rate is relatively appreciated, as foreign goods are relatively inexpensive (panel 1C).

In contrast, in the other extreme case of *financial autarky* (FA), no assets can be traded internationally, which means consumption paths are dominated by strong income effects: a positive output shock that lowers the price of its good actually hurts the country, since it lowers the value of its resources from which to finance current consumption. As a result the (log) interest rate differential, $r - r^*$, behaves exactly opposite to the case of CM and drops (panel 4D). The cases of incomplete financial markets where a bond can be traded to borrow and lend internationally can be understood as intermediate cases between CM and FA, where some of the negative income effect can – not be averted as in the case of state-contingent payments, but – be alleviated by borrowing internationally. As can be seen in Figure 2, initially, in the first 20 periods, when bond holdings are not too different from zero (panel 5A), the consumption paths in the economy with borrowing constraints (panel 3A) do not behave too differently from the behavior under IM-unconstrained (or even from the behavior under CM), and similarly for the interest rate differential. Panel 5A shows that, once bond holdings move closer to one of the countries’ constraint (in this case Foreign’s – depicted as the dashed horizontal line), this country’s precautionary motives increase drastically. As a result, the consumption paths and interest rate differential start to diverge from their respective equivalent paths under CM and behave somewhat closer like under financial autarky. It is important to emphasize that this does happen not only once a borrowing constraint becomes actually binding, but before, as a result of the fear of becoming constrained in the future and the increasingly strong precautionary motives.

As seen by the panels 4A-4D, the interest rate differential and expected exchange rate change in the case of CM always move together almost one-to-one, and the implied risk premium is (virtually) zero. On the other hand, there are very large deviations of interest rate differential and expected exchange rate variations under financial autarky (panel 4D) – a finding that is not surprising as the UIP condition is not an equilibrium condition of the FA model, and the exchange rate is entirely determined in the goods market. Turning to the *baseline model with borrowing constraints* (IM-constrained), there are also substantial deviations from the UIP relation. When bond holdings are centered around zero (panel 5A) and precautionary motives are relatively weak for both countries, the implied consumption paths behave similar to the case of the standard IM model. In such case, also in the model with borrowing constraints, interest rate differential and expected exchange rate changes move closely together, and the implied risk premium is close to zero. This changes markedly when a state is reached in which one of the countries has accumulated a substantial amount of debt. In such case, the indebted country increasingly has the incentive to reduce its borrowing somewhat, thereby trying to avoid becoming constrained in the future and accepting that it has to reduce consumption already somewhat earlier. As a result, interest rate differential and expected exchange rate start behave somewhat closer like under financial autarky and a wedge between the two appears. The asset of the country that has a stronger precautionary motive – that is, the country that has become a debtor because of a history of relatively worse income situations – has to carry a risk premium.

The mechanism generating the time-varying risk premium matches very well the empirical findings of Della Corte et al. (2014). These authors investigate the role of large outstanding external asset and liability positions in generating a time-varying risk premium that can account for failure of UIP. To do so, they sort countries’ currencies into groups according to a country’s net foreign asset position and its propensity to issue external liabilities in domestic currency, and use these groups to construct an external imbalance risk factor. Intuitively, the global imbalance factor captures (the excess return obtained between) the group of creditor

Figure 2: Sample simulated time paths, case of positive transmission, $\omega > \tilde{\omega}$



This figure presents simulated model time series of interest, in the case of a parameterization of the model that implies positive international transmission, i.e., where the trade elasticity $\omega = 0.44 > \tilde{\omega}$. Row 1 depicts the behavior of the terms of trade and the real exchange rate, row 2 the behavior of GDP, row 3 the consumption paths, row 4 the interest rate differential and its decomposition into expected exchange rate changes and a risk-premium, and row 5 the behavior of bond holdings. These simulated time series are shown for the baseline model with borrowing constraints (IM-constrained), the bond-economy without borrowing constraints (IM-unconstrained), complete markets (CM) and financial autarky (FA).

countries with liabilities mainly in domestic currency, and debtor countries with liabilities primarily in foreign currency. They find that this external imbalance factor explains the majority of average currency excess returns in a standard asset pricing model. The economic intuition for this risk factor is simply that net debtor countries offer a currency risk premium to compensate investors willing to finance negative external imbalances.

5.2 UIP regression results

Table 3 reports the results from the uncovered interest parity regression. There is a large empirical literature, on the UIP coefficients found from (nominal) UIP regressions – based on a regression of nominal exchange rate variations on the nominal interest rate differential. As has been argued in the literature, it would seem intuitive to expect the currency of a country that currently pays a higher interest rate to depreciate, such that no excess returns could be made in expectations. Equating the nominal interest rate differential to the expected change in the exchange rate would imply a UIP slope coefficient of one. The empirical evidence for this type of regression for advanced economies is in stark contrast to this. Coefficients very different from one have been found empirically, for advanced economies these are typically even negative, suggesting that currency prices for high interest rates tend, instead, to appreciate. Backus et al. (2001) report values of $\hat{\beta}$ between -1.84 and -0.74 , based on exchange rates of the US with the United Kingdom, Germany, and Japan. Using monthly data for 1979:6-2009:10 Engel (2011) provides estimates for the UIP slope coefficient from both standard nominal UIP regressions and equivalent real UIP regressions of the real exchange rate on the real interest rate differential. He finds that, for advanced economies, the interest parity puzzle is similar in real terms as in nominal terms: the slope coefficient of a nominal UIP regression using data of the U.S. versus an average of six other advanced economies¹⁶, is found to be -1.467 ; the slope coefficient of the equivalent real regression is found to be -0.933 or -0.914 respectively (depending on the model used for obtaining estimates of real interest rates). Like for nominal variables, the coefficient from the regression in real terms is thus found to be strictly below one, and generally negative.¹⁷

As argued originally by Fama (1984), the empirical findings of large deviations from interest rate parity can be reconciled if one accounted for the possible presence of an unobserved variable, a time-varying risk premium, which macroeconomic models have a hard time generating. In a theoretical model, the risk premium can be understood to capture the conditional covariance of the stochastic discount factor (the intertemporal marginal rate of substitution) with future exchange rates variations, which in principle disturbs the co-movement of the interest rate differential with expected exchange rate changes. However, there are two reasons why exchange rate risk premia in theoretical models are generally small. One concerns the solution methods applied – often macro models are solved with (log) linear approximation methods, that impose certainty equivalence and thereby ignore the influence of a covariance term between future marginal utility and future exchange rates. This problem is explicitly accounted for in the present paper, as the model simulations, on which the UIP regression

¹⁶The countries considered are Canada, France, Germany, Italy, Japan and the United Kingdom.

¹⁷As pointed out by Engel (2011), this may not be surprising, as for the currencies of low-inflation high-income countries nominal and real exchange rates are highly correlated. Iwata and Tanamee (2009) run UIP regressions in real terms for both developed (low inflation) and developing (high inflation) countries. Based on quarterly data, they find slope coefficients of 0.22 and 0.13, respectively.

Table 3: UIP regression coefficients

	IM-constrained model (baseline)	IM-unconstrained model	CM model	FA model
case of positive transmission, $\omega > \tilde{\omega}$				
α	0.000	0.000	0.000	0.000
β	-0.300	0.973	0.994	-1.800
case of negative transmission, $\omega < \tilde{\omega}$				
α	0.000	0.000	0.000	-0.047
β	-0.307	1.036	0.997	-0.187

This table presents UIP regression results based on model simulated data, for the baseline model with borrowing constraints (IM-constrained), and the comparison economies of the bond economy without borrowing constraints (IM-unconstrained), complete markets (CM) and financial autarky (FA). It shows regression results for both the case of a parameterization that implies positive international transmission, $\omega = 0.44 > \tilde{\omega}$, and for the case of a parameterization that implies negative international transmission, $\omega = 0.25 < \tilde{\omega}$.

results are based, are derived from global solutions (this is true also for comparison model economies). Nevertheless, in a model that features standard CRRA preferences and autoregressive (business cycle) shocks as typically used in macroeconomics, the covariance term of future marginal utility and future exchange rates typically is of minor quantitative importance, implying a strongly positive, almost one-to-one relationship between the interest rate differential and expected exchange rate changes. Columns 2 and 3 of Table 3 show that, in the case of complete financial markets (CM) and the incomplete-markets bond economy without borrowing constraints (IM-unconstrained) this fact is manifested in regression coefficients of (close to) 1. Figure 2 has shown that the implied dynamics of the model with borrowing constraints differ markedly from the case of complete markets and the unconstrained incomplete-markets bond-economy scenario. The presence of the time-varying risk premium drives a wedge between interest rate differential and expected exchange rate changes, and lowers the UIP regression coefficient, $\hat{\beta}$, to -0.300 in the case of positive international transmission, and to -0.307 in the case of negative international transmission. The channel of international liquidity constraints therefore goes a long way towards resolving the UIP puzzle quantitatively.

To understand the sources of the regression results in Table 3 it is useful to consider again the decomposition of the (log) interest rate differential into (log) expected exchange rate variation plus the risk premium. As pointed out by Fama (1984) from the decomposition in equation (18) it can be seen that in a regression of actual changes in the (log) exchange rate on the (log) interest rate rate differential, $\Delta q_{t+1} = \alpha + \beta(r_t - r_t^*) + u_{t+1}$, the sign and size of the regression coefficient $\hat{\beta}$ depends on the variance and covariance of the variables of equation (18). In particular, to replicate the negative UIP coefficient $\hat{\beta} = \frac{Cov\{E_t\Delta q_{t+1}, E_t\Delta q_{t+1} + p_t\}}{Var(E_t\Delta q_{t+1} + p_t)} = \frac{Cov(E_t\Delta q_{t+1}, p_t) + Var(E_t\Delta q_{t+1})}{Var(E_t\Delta q_{t+1} + p_t)}$, p_t and $E_t\Delta q_{t+1}$ need to have a negative covariance, and p_t needs to have greater variance than $E_t\Delta q_{t+1}$. Table 4 summarizes the volatility and comovement properties of the interest differential, expected exchange rate changes and the risk-premium term. In both cases of positive and negative international transmission, Fama's two conditions are satisfied in the baseline model with borrowing constraints, thus giving rise to the negative UIP regression coefficient found.

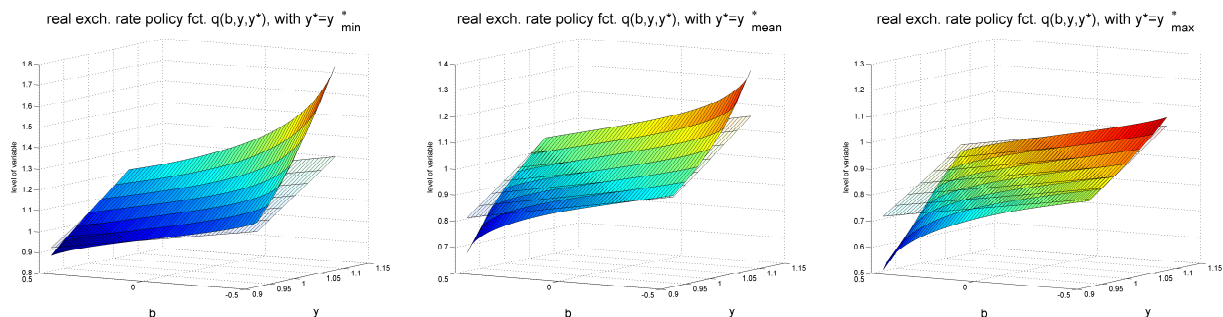
Table 4: Properties of interest rate differential, expected exchange rate variations and risk premium

		IM-constrained model (baseline)	IM-unconstrained model	CM model	FA model
case of positive transmission, $\omega > \tilde{\omega}$					
$cov(E\Delta q), p$	$1e-2 \times$	-0.0018	0.0000	-0.0000	-0.0143
$cov(E\Delta q, r - r^*)$	$1e-2 \times$	-0.0001	0.0005	0.0004	-0.0052
$var(r - r^*)$	$1e-2 \times$	0.0005	0.0005	0.0004	0.0030
$var(E\Delta q)$	$1e-2 \times$	0.0017	0.0005	0.0004	0.0091
$var(p)$	$1e-2 \times$	0.0025	0.0000	0.0000	0.0224
case of negative transmission, $\omega < \tilde{\omega}$					
$cov(E\Delta q), p$	$1e-2 \times$	-0.0023	-0.0000	-0.0000	-4.5793
$cov(E\Delta q, r - r^*)$	$1e-2 \times$	0.0009	0.0004	0.0007	-0.2022
$var(r - r^*)$	$1e-2 \times$	0.0020	0.0004	0.0007	25.7831
$var(E\Delta q)$	$1e-2 \times$	0.0014	0.0005	0.0007	4.4895
$var(p)$	$1e-2 \times$	0.0051	0.0000	0.0000	30.4523

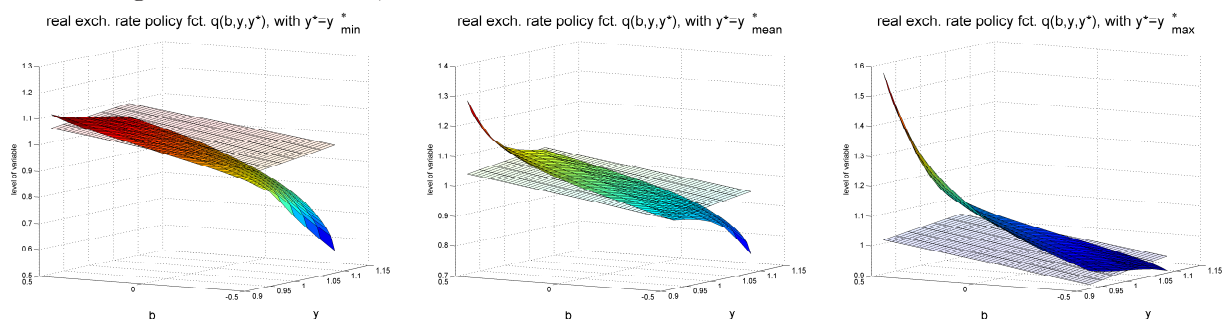
This table presents the components of a Fama-decomposition of the UIP regression coefficient, for the baseline model with borrowing constraints (IM-constrained), and the comparison economies of the bond economy without borrowing constraints (IM-unconstrained), complete markets (CM) and financial autarky (FA). This decomposition is shown for both the case of a parameterization that implies positive international transmission, $\omega = 0.44 > \tilde{\omega}$, and for the case of a parameterization that implies negative international transmission, $\omega = 0.25 < \tilde{\omega}$. The two conditions to generate a negative UIP regression coefficient outlined by Fama are satisfied in the baseline model with borrowing constraints.

Figure 3: Policy functions of the real exchange rate

case of positive transmission, $\omega > \tilde{\omega}$



case of negative transmission, $\omega < \tilde{\omega}$



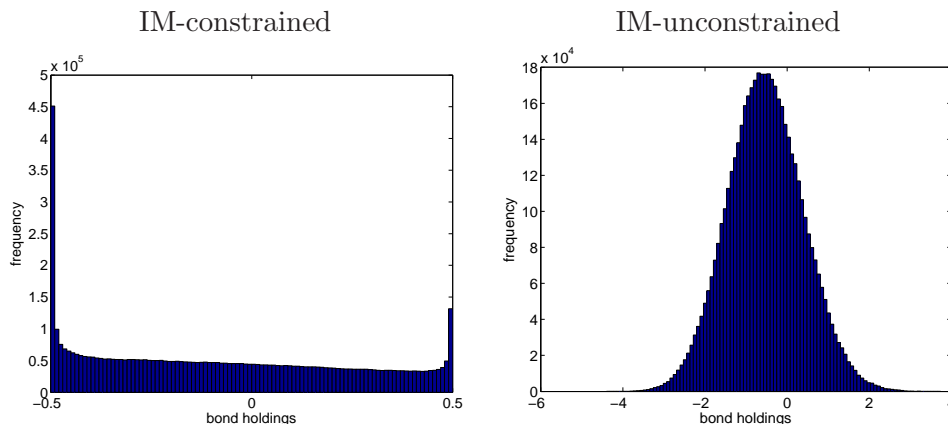
This table shows policy functions of the real exchange rate for the baseline model of the bond-economy with borrowing constraints (IM-constrained) and the comparison bond-economy without such constraints (IM-unconstrained), for the case of positive international transmission, $\omega = 0.44 > \tilde{\omega}$, and negative international transmission, $\omega = 0.25 < \tilde{\omega}$.

5.3 Nonlinearity of policy functions, and stationary distribution of bond holdings

The effect of the presence of the international borrowing constraints and strength of precautionary motive can also be seen by directly inspecting the policy functions. As an example, I present, in figure 3, the model's policy functions for the real exchange rate, as functions of bond holdings and the domestic output endowment, keeping the level of the foreign output endowment at a constant (low, average and high) level. I also plot, as comparison, the equivalent policy function from the standard incomplete-markets bond-economy case without borrowing constraints, which are included as the transparent surfaces in figure 3. As can be seen, in the region of bond holdings close to zero the policy functions of IM-constrained and IM-unconstrained are still similar, but the presence of the borrowing constraints introduces strong non-linearities in regions of bond holdings further away from zero.

Moreover, these nonlinearities influence and shape the model dynamics in a significant way, because bond holdings travel frequently over regions close to constraint. This can be seen from figure 4, which plots the stationary distribution of bond holdings in its unconstrained and constrained version (for simplicity only for the case of positive transmission – the other case looks similar). In the case of IM-unconstrained, the presence of the endogenous discount factor introduces a bell-shaped bond holdings distribution. In the baseline model with borrowing constraints, however, no such mechanism tilting bond holdings towards the

Figure 4: Stationary distribution of bond holdings, case of positive transmission, $\omega > \tilde{\omega}$



This table shows the stationary distribution of bond holdings for the baseline model of the bond-economy with borrowing constraints (IM-constrained) and the comparison bond-economy without such constraints (IM-unconstrained), for the case of both positive international transmission, $\omega = 0.44 > \tilde{\omega}$. The stationary distributions under the alternative of negative international transmission, $\omega = 0.25 < \tilde{\omega}$, is similar.

center is present, and bond holdings travel more equally across the state space. In addition, there is an increasing mass close to the constraints, because of the increasing precautionary motives to avoid the actual constraints from binding. As has been argued, the presence of borrowing constraints makes the incompleteness of the standard bond-economy case worse: as can be expected, the co-movement behavior of interest rate differential and exchange rate changes to be somewhat closer to the case of financial autarky.¹⁸ It is crucial, however, to note that the wedge in the interest rate parity relationships generally does not stem from international trade in the bond actually coming to a halt because a borrowing constraint is hit, which happens only 0.22 (0.73) percent of the times for the parameterizations implying positive (negative) international transmission. As long as bond holdings are often enough close to either country’s borrowing constraint, they will have a significant influence on the behavior of the model economy’s variables.

5.4 Sensitivity analysis

This section presents some sensitivity analysis for the main results of UIP regression coefficients for the model with borrowing constraints. In particular, table 5 presents results for the UIP regression coefficient, $\hat{\beta}$, for several parameter variations. Row 1 of table 5 documents that, when borrowing constraints become tighter, the UIP regression coefficient decreases. E.g., for $K = 0.25$, $\hat{\beta}$ drops to -0.956 (-0.552) in the case of positive (negative) transmission, and rises to 0.459 (0.068) for $K = 1$. Similarly, more volatile (row 2) or more persistent (row 3) output shocks imply that bond holdings more frequently travel to regions where precautionary motives are strong, leading to a larger risk-premium and translating into larger deviations of the UIP regression coefficient from 1. Row 4 presents sensitivity analysis with respect to changes in the degree of home bias in preferences, γ . While in the baseline parameterization γ was chosen to match the average import share of G7 economies, the variations of

¹⁸In fact, one can interpret the scenario of financial autarky as the limit case of the model presented in section 2, as constraints K, K^* approach zero.

Table 5: Sensitivity analysis for UIP regression coefficient $\hat{\beta}$

	$K = 0.25$	$K = 0.5$	$K = 1$
positive transmission, $\omega > \tilde{\omega}$	-0.956	-0.300	0.459
negative transmission, $\omega < \tilde{\omega}$	-0.552	-0.307	0.068
	$\sigma_\varepsilon = 0.005$	$\sigma_\varepsilon = 0.01$	$\sigma_\varepsilon = 0.015$
positive transmission, $\omega > \tilde{\omega}$	0.295	-0.300	-0.606
negative transmission, $\omega < \tilde{\omega}$	0.024	-0.307	-0.462
	$\rho = 0.9$	$\rho = 0.95$	$\rho = 0.98$
positive transmission, $\omega > \tilde{\omega}$	0.681	-0.300	-1.247
negative transmission, $\omega < \tilde{\omega}$	0.335	-0.307	-0.715
	$\gamma = 0.85$	$\gamma = 0.78$	$\gamma = 0.70$
positive transmission, $\omega > \tilde{\omega}$	0.855	-0.300	-0.569
negative transmission, $\omega < \tilde{\omega}$	-0.435	-0.307	-0.399

This table presents sensitivity analysis for the main results of the UIP regression coefficient for the baseline model economy with borrowing constraints. It does so for variations in the tightness of borrowing constraint parameter K (row 1), for variations in the size of shocks (row 2), for variations in the persistence of shocks (row 3), and for variations in the degree of trade openness, γ , (row 4).

γ allow to evaluate the significance of the paper’s mechanism for more or less open economies (see section 4).¹⁹ E.g., $\gamma = 0.85$ reflects a parameterization of countries that are relatively closed in (goods) trade like the United States or Japan. The UIP regression coefficients obtained in this case are 0.855 (−0.435) for the case of positive (negative) transmission. The empirical evidence of Corsetti et al. (2006, 2014) for these relatively closed countries documents an exchange rate appreciation in response to positive supply shocks, thus suggesting that the result under the negative transmission mechanism appears more applicable for these countries. The parameter variation $\gamma = 0.70$ reflects a parameterization to countries that are more open in trade, such as Germany or France, countries for which Corsetti et al. (2006) suggest a positive international transmission. For this setting the UIP regressions obtained equal −0.569 (−0.399) for the case of positive (negative) transmission.

6 Conclusions

This paper presented a stylized two-country model in which precautionary motives help generate a time-varying exchange rate risk premium. It has long been argued that a time-varying risk premium on foreign exchange may help explain the empirical failure of uncovered interest rate parity, yet few theoretical models have had success in generating such a premium, at least under conventional CRRA preferences and shock processes. Two ingredients are crucial in giving rise to a time-varying risk-premium: one, the model is parameterized in the region of low trade elasticities, which implies large equilibrium wealth effects. Two, international financial markets cannot be accessed unrestrictedly to insure against or alleviate these large wealth effects, because of the presence of international liquidity constraints. The generated

¹⁹Note that to keep the implied volatility of international relative prices comparable to that of the baseline setting, the trade elasticity needs to be recalibrated in accordance with the change in the degree of home bias parameter γ .

premium drives a wedge between the interest rate differential and expected exchange rate changes, thereby significantly lowering the coefficient in a UIP regression. The paper thus documents the existence of a novel and, as documented in Della Corte et al. (2014), empirically relevant channel in explaining deviations from UIP. Also, despite the model's reliance on standard preferences and shock processes, it manages to go a long way in rationalizing the observed negative UIP regression coefficients found in the data.

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A Appendix: The Model's Equilibrium Conditions

The model's equilibrium equations can be listed as follows:

$$Q_t = \frac{[\gamma^* + (1 - \gamma^*) \tau_t^{1-\omega}]^{\frac{1}{1-\omega}}}{[\gamma + (1 - \gamma) \tau_t^{1-\omega}]^{\frac{1}{1-\omega}}} \quad (19)$$

$$C_t = \left[\gamma^{\frac{1}{\omega}} C_{H,t}^{\frac{\omega-1}{\omega}} + (1 - \gamma)^{\frac{1}{\omega}} C_{F,t}^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (20)$$

$$C_t^* = \left[\gamma^{*\frac{1}{\omega}} C_{H,t}^{*\frac{\omega-1}{\omega}} + (1 - \gamma^*)^{\frac{1}{\omega}} C_{F,t}^{*\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \quad (21)$$

$$\frac{1 - \gamma}{\gamma} \frac{C_{H,t}}{C_{F,t}} = \tau_t^\omega \quad (22)$$

$$\frac{1 - \gamma^*}{\gamma^*} \frac{C_{H,t}^*}{C_{F,t}^*} = \tau_t^\omega \quad (23)$$

$$C_t^{-\sigma} = \beta R_t E_t [C_{t+1}^{-\sigma}] \quad (24)$$

$$C_t^{-\sigma} = \beta R_t^w E_t [C_{t+1}^{-\sigma}] + \mu_t R_t^w \quad (25)$$

$$C_t^{*-\sigma} = \beta R_t^* E_t [C_{t+1}^{*-\sigma}] \quad (26)$$

$$\frac{C_t^{*-\sigma}}{Q_t} = \beta R_t^w E_t \left[\frac{C_{t+1}^{*-\sigma}}{Q_{t+1}} \right] + \mu_t^* \frac{R_t^w}{Q_t} \quad (27)$$

$$\frac{B_t}{R_t^w} = B_{t-1} + p_{H,t} Y_t - C_t \quad (28)$$

$$Y_t = C_{H,t} + C_{H,t}^* \quad (29)$$

$$Y_t^* = C_{F,t} + C_{F,t}^* \quad (30)$$

$$0 = \mu_t [B_t + K] \quad (31)$$

$$0 = \mu_t^* \left[\frac{B_t^*}{Q_t} - K \right] \quad (32)$$

In case of a trade elasticity falling in the negative transmission region, $\omega < \tilde{\omega}$, the last two equations are, instead, replaced by:

$$\begin{aligned} 0 &= \mu_t [B_t - K] \\ 0 &= \mu_t^* \left[\frac{B_t^*}{Q_t} + K \right] \end{aligned}$$

B Appendix: Solution Technique

The model is solved by an iterative algorithm to find the conditional expectations of the model's equilibrium conditions. Below I briefly outline the steps of the algorithm used:

- In the following, denote $t + 1$ variables with a prime, e.g. $B = B_{t-1}$, $B' = B_t$, and accordingly, $B'' = B_{t+1}$. I construct a 3-dimensional grid over the model's state variables at time t , that is, over $B, y = \log Y, y^* = \log Y^*$ consisting of $n_b n_y n_{y^*}$ grid points. The grid in dimension of b ranges from $[-K, K]$. The gridpoints in dimensions of y, y^*

are obtained by discretizing the continuous AR processes by following the method of Rouwenhorst (1995). As a recent contribution by Kopecky and Suen (2010) shows, the Rouwenhorst discretization has proven to lead to substantially better approximations than more conventional discretizations, e.g. Tauchen and Hussey (1991), particularly when the underlying process is very persistent, as is the case here. The number of gridpoints was chosen to be $n_b = 51$ and $n_y = n_{y^*} = 7$.

- Set counter equal to 1. I make initial guesses on the model's conditional expectations by using the log-linear solution as starting point. In the endowment economy guesses are made for the conditional expectations of the bond Euler equations, $CE_B(B; y, y^*) \equiv E\{u_{C'}\}$, and $CE_{B^*}(B; y, y^*) \equiv E\left\{\frac{u_{C^*}}{Q'}\right\}$, at each gridpoint $(B; y, y^*)$.
- Using the guesses for the conditional expectations, the endogenous variables $B', C_H, C_F, C, C_H^*, C_F^*, C^*, R, R^*, \mu, \mu^*, \tau$, and Q can be computed at each gridpoint, by solving the set of equilibrium conditions outlined in Appendix A, equations (19)-(28). This is done, by initially assuming that the borrowing constraints do not bind, i.e. $\mu = 0$ and $\mu^* = 0$. In case that at any gridpoint $(B; y, y^*)$ one of the borrowing constraints is hit, the bond holding is set to the constraint and system (19)-(28) is solved again for the respective (positive) value of the multiplier on the constraint. Having in hand the guesses of the conditional expectations as functions of $(B; y, y^*)$, I use interpolation methods to find CE'_B and CE'_{B^*} , as functions of $(B'; Ey', Ey^{*'})$. CE'_B and CE'_{B^*} can in turn be used to obtain $B'', C'_H, C'_F, C', C'^*_H, C'^*_F, C'^*, R', R'^*, \mu', \mu'^*, \tau'$, and Q' for any $(B; y, y^*)$.
- Having in hand solutions for C', C'^* , and Q' , together with the discretized states and transition matrix, π , for the exogenous processes, the guesses of the conditional expectations can be updated by computing:

$$CE_{BH}(B; y, y^*) \equiv E_t [C_{t+1}^{-\sigma}] = \sum \pi(y', y^{*'} | y, y^*) C' (B'; y', y^{*'})^{-\sigma}$$

$$CE_{BH^*}(B; y, y^*) \equiv E_t [C_{t+1}^{*-\sigma} / Q_{t+1}] = \sum \pi(y', y^{*'} | y, y^*) [C^{*'} (B'; y', y^{*'})^{-\sigma} / Q' (B'; y', y^{*'})]$$

- The above steps are repeated until convergence is achieved.