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Cross-border or online: Tax competition with mobile consumers under destination and origin principle

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CROSS-BORDER OR ONLINE – TAX
COMPETITION WITH MOBILE
CONSUMERS UNDER DESTINATION
AND ORIGIN PRINCIPLE

Laura Birg
Cross-border or Online – Tax Competition with Mobile Consumers under Destination and Origin Principle

Laura Birg*

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Abstract

This paper studies the effect of an online retailer on spatial tax competition with mobile consumers. If taxation for online purchases follows the destination principle, the entry of the online retailer mitigates tax competition; if taxation for online purchases follows the origin principle, the entry of the online retailer enhances tax competition. Lower cost of online shopping enhances the effect of the online retailer on tax competition, limited internet access decreases the effect of the online retailer on tax competition. For sufficiently low cost of online shopping, welfare in the country where the online retailer is located is higher under the origin principle and welfare in the other country is higher under the destination principle. Total welfare is higher under the destination principle.

JEL Classification: F12, H20, L13
Keywords: tax competition, cross-border shopping, online retailer, destination principle, origin principle

1 Introduction

This paper studies the effect of an online retailer on spatial tax competition with mobile consumers.

Tax differentials may provide an important incentive for cross-border shopping, if the tax savings offset costs of traveling from one country to another (Leal, Lopez-Laborda & Rodrigo, 2010). If governments are interested in maximizing their tax revenue, attracting cross-border shoppers may also become a goal in tax policy. At the same time, consumers

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purchase goods online at an increasing rate. As globalization reduces transaction cost, the importance of both cross-border and online shopping increases (Leal, Lopez-Laborda & Rodrigo, 2010).

In the European Union, autonomous decisions of member states on tax policy may give rise to tax competition. The European Single market and free movement of goods, capital, services, and persons weaken the importance of national borders. Thus cross border shopping is a frequent phenomenon in the European Union. The Digital Agenda for Europe seeks to create a digital single market. As policy targets, it aims at getting 50% of all EU citizens to buy online and 20% to engage in online cross-border transactions until 2015 (European Commission, 2010).

In 2008, only 25% of consumers in the European Union have purchased purchase goods or service in other member states (Eurostat, 2009). Geographical location and the close proximity of neighboring countries determine the extent of cross-border shopping, as 68% of consumers in Luxembourg have purchased purchase goods or service in other member states. In contrast, in countries at the European periphery the prevalence of cross-border shopping is much lower, e.g. 10% of consumers in Greece and 9% of consumers in Portugal and Bulgaria have purchased goods abroad (Eurostat, 2009).

With the growth of internet use, online shopping has become more important. In 2014, 50% of citizens in the European Union have made purchases online (European Commission, 2015). Consumers show a substantial degree of home bias for online shopping: In 2014, 44% of consumers purchased online nationally, only 15% bought from an online retailer from another EU country (European Commission, 2015). Cowgill, Dorobantu & Martens (2013) estimate from Google e-commerce data that over the period 2008-2011, online consumers in the EU were up to 55 times more likely to buy in their own country than in another EU country. Consumers from smaller countries are more likely to purchase from retailers from other member states, e.g. 42% of consumers in Malta have purchased from an online retailer from another country vs. 11% who purchased from a domestic online seller (Flash Eurobarometer 358, 2012). With improved access to online shops and reduced cost of buying online, the importance of online retailers is expected to increase further.

In the European Union, member states are free to set value added tax rates, with a minimum standard rate of 15% (Directive 2006/112/EG). Tax rates vary between high tax countries such as Hungary (27%), Denmark (25%), Sweden (25%), and Romania (24%) and low tax countries such as Luxembourg (15%), Cyprus (18%), Malta (18%), and Germany (19%). Currently, the origin principle applies for online purchases, unless the recipient is a private household. Then the destination principle applies. If sales are
below a threshold of 100,000 Euros, the origin principle applies. This is, for the majority of online purchases by private households, the destination principle applies.

Previous literature on tax competition and cross-border shopping has emphasized the importances of differences between countries (see e.g. Kanbur & Keen, 1993; Nielsen, 2001). In these papers, the smaller country undercuts the tax rate of the larger country. In their seminal paper, Kanbur & Keen (1993) study revenue-maximizing governments in an open economy with two countries differing in population size. In the non-cooperative Nash equilibrium, the tax rate of the smaller country is lower than the tax rate in the larger country, with the tax revenue being higher in the larger country. Subsequent studies have also focused on differences between countries, in population size (Trandel 1994; Wang 1999) or geographical size (Ohsawa 1999; Nielsen 2001, 2002)\(^1\). Aiura & Ogawa (2013) examine the choice between ad valorem tax and unit tax. They show that governments endogenously choose the ad valorem tax method to compete for mobile consumers, which leads to stronger tax competition with lower tax rates and smaller tax revenue.

The literature on tax competition when consumers are able to shop cross-border has regularly assumed that taxation follows the origin principle, e.g. Mintz & Tulkens (1986), Kanbur & Keen (1993), and Nielsen (2001). Keen & Lahiri (1998) compare commodity taxation under the destination and origin principle under imperfect competition. They find that for non-cooperatively set tax rates, the Cournot equilibrium under the origin principle is potentially Pareto-superior to the equilibrium under the destination principle. Comparing destination and origin principle in a framework with possible tax spillovers, Lockwood (2001) finds that welfare is higher under the destination principle, unless there are producer price or rent spillovers.

Several empirical studies have analyzed the effect of taxes on the decision of consumers to buy in brick-and-mortar stores or online. Goolsbee (2000) finds that consumers in high sales tax locations are more likely to buy online. A 1% increase in the sales tax increases the probability of of buying online by 0.5%. Goolsbee (2000) estimates an elasticity of online buying with respect to the tax price (one plus the tax rate) of between 2 and 4. Alm and Melnik (2005) find a much lower elasticity of online purchases with respect to the tax price of approximately 0.5. Ballard and Lee (2007) show that consumers shop online to avoid sales taxes. They also find that consumers who live close to counties with lower sales tax rates are less likely to shop online.\(^2\) Using

\(^1\)See Leal, Lopez-Laborda & Rodrigo (2010) for a survey on theoretical and empirical studies on cross-border shopping.

\(^2\)Leal, Lopez-Laborda & Rodrigo (2010) interpret these findings as cross-border shopping and Internet
eBay data, Einav et al. (2014) estimate the impact of sales taxes on online. They find that a one percentage point increase in a state’s sales tax increases online purchases by state residents by approximately 2 percent, but decreases their online purchases from home-state retailers by 3–4 percent. Using data from a retailer that sells through the internet and catalogs, Hu & Tang (2014) study the effect of sales tax changes, finding that a tax cut by 4 percentage points has decreased remote sales by about 15%. To my knowledge, the effect of an online retailer on tax competition has not been considered in the literature.

Introducing an online retailer a la Lijesen (2013) into the Kanbur & Keen (1993)-framework of spatial tax competition, this paper studies the effect of an online retailer on spatial tax competition with mobile consumers. If taxation for online purchases follows the destination principle, the entry of the online retailer mitigates tax competition; if taxation for online purchases follows the origin principle, the entry of the online retailer enhances tax competition. Lower cost of online shopping enhances the effect of the online retailer on tax competition, limited internet access decreases the effect of the online retailer on tax competition. For sufficiently low cost of online shopping, welfare in the country where the online retailer is located is higher under the origin principle and welfare in the other country is higher under the destination principle. Total welfare is higher under the destination principle.

The rest of the paper is organized as follows. In the next section, the model is presented and the effect of the entry of the online retailer on tax competition under the destination and origin principle is studied. Section 3 analyzes welfare. Section 4 and section 5 study the effect of a decrease in the cost of online shopping and limited internet access. Section 6 and section 7 analyze tax setting for cooperation between governments and benevolent governments. Section 8 studies the effect of asymmetric population size. Section 9 discusses assumptions and extensions. Section 10 concludes.

2 The Model

Consider a Hotelling economy with two countries, \( j = H, F \) (home, foreign) on the line segment \([0, 1]\). An open border at \( b = \frac{1}{2} \) separates the line segment into two countries, country \( H \) extends to the interval \([0, b]\), country \( F \) to the interval \([b, 1]\). In each country, there is a brick-and-mortar shop \( i = H, F \) located at \( x_H = 0, x_F = 1 \). An online shop \( i = 0 \) is located in country \( H \). Firms sell a single homogeneous product at price \( p_i \). Shopping being substitutes.
A unit mass of consumers is uniformly distributed on the line segment. Consumers differ in location $y \in [0, 1]$. The utility of a consumer located at $y$ and buying from the store in country $H$ is given by $U = v - t (y - x_H) - p_H$, where $v$ denotes the value of the product and $t$ is the traveling cost. Assume that $v$ is sufficiently large so that the market is covered. Similarly, the utility of a consumer located at $y$ and buying from the store in country $F$ is given by $U = v - t (x_F - y) - p_F$. The utility of a consumer buying online is given by $U = v - \theta - p_0$, where $\theta$ denotes fixed cost of buying online. This can be interpreted as the cost of going online, delivery cost, inconvenience of waiting for the parcel service or the opportunity cost of non-immediate availability of the good purchased online.

In each country, there is a single revenue-maximizing government, imposing unit taxes at rate $\tau_j$.

The structure of the model can be summarized by the following two-stage game: In the first stage, governments set tax rates, in the second stage firms compete in prices.

### 2.1 Offline Equilibrium

Consider first the case without the online retailer, when consumers buy from brick-and-mortar stores only. An asterisk is used to denote variables associated with the offline equilibrium.

The location of the consumer indifferent between buying at $x_H$ in country $H$ and at $x_F$ in country $F$ is given by $y_{HF}^* = \frac{1}{2} + \frac{p_F - p_H}{2t}$. Demand is $q_{HF}^* = y_{HF}^*$ and $q_{F}^* = 1 - y_{HF}^*$. Firm profits are $\pi_H^* = (p_H^* - \tau_H^*) q_{HF}^*$ and $\pi_F^* = (p_F^* - \tau_F^*) q_{F}^*$. Tax revenue is $R_H^* = \tau_H^* q_{HF}^*$ and $R_F^* = \tau_F^* q_{F}^*$.

Equilibrium prices are $p_H^* = p_F^* = 4t$. Quantities are $q_{HF}^* = q_{F}^* = \frac{1}{2}$. Tax rates and revenues, respectively, are $\tau_H^* = \tau_F^* = 3t$ and $R_H^* = R_F^* = \frac{3}{2}t$. Tax rates and revenues increase in traveling cost $t$. The tax differential is zero ($\Delta \tau^* = \tau_F^* - \tau_H^* = 0$).

### 2.2 Online Equilibrium under Destination Principle

Consider now the case with the online retailer. Assume first that taxation for online purchases follows the destination principle. This is equivalent to the online retailer selling to private households and having revenues above the threshold of 100,000 Euros. Taxation for purchases at the brick-and-mortar stores follows the origin principle. For low cost of buying online, i.e. $\theta < t$, the online retailer is selling in both countries.

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3Country-specific fixed cost, as for example shipping cost depending on the consumer’s place of residence, would introduce an asymmetry between countries, but would yield qualitatively similar results.
Cross-border shopping does not take place then. For $\theta > t$, the online retailer is not active.

The location of the consumer indifferent between buying from $x_H$ in country $H$ and buying online is given by $y_{H0} = \frac{\theta + p_0 - p_H}{t}$; this consumer is located in country $H$. The location of the consumer indifferent between buying from $x_F$ in country $F$ and buying online is given by $y_{0F} = \frac{1 - \theta - p_0 + p_F}{t}$; this consumer is located in country $F$. Demand is given as $q_H = y_{H0}, q_F = (1 - y_{0F})$, and $q_0 = y_{0F} - y_{H0}$. Firm profits are given by $\pi_H^d = (p_H - \tau_H) q_H, \pi_F^d = (p_F - \tau_F) q_F$, and $\pi_0^d = (p_0 - \tau_H) \left(\frac{1}{2} - y_{H0}\right) + (p_0 - \tau_F) \left(y_{0F} - \frac{1}{2}\right)$.

Tax revenue is given by $R_H^d = \tau_H \left(q_H + \left(\frac{1}{2} - y_{H0}\right)\right)$ and $R_F^d = \tau_F \left(q_F + \left(y_{0F} - \frac{1}{2}\right)\right)$.

Equilibrium prices are $p_H^d = p_F^d = v - \frac{1}{5}t - \frac{1}{2}\theta$ and $p_0^d = v - \theta$. Quantities are $q_H^d = q_F^d = \frac{1 + 2\theta}{5}$ and $q_0^d = \frac{2(1-\theta)}{5}$. Tax rates and revenues, respectively are given as $\tau_H^d = \tau_F^d = v - \frac{1}{5}t - \frac{1}{2}\theta$ and $R_H^d = R_F^d = \frac{1}{2} \left(v - \frac{1}{5}t - \frac{1}{2}\theta\right)$. In this online equilibrium, tax rates and revenues are higher than in the offline equilibrium ($\tau_H^d = \tau_F^d > \tau_H^* = \tau_F^*$, $R_H^d = R_F^d > R_H^* = R_F^*$). The tax differential is zero ($\Delta \tau^d = \tau_F^d - \tau_H^d = \Delta \tau^* = 0$). Due to the online retailer selling in both countries and taxation following the destination principle, governments cannot increase tax revenue by lowering the tax rate and the border at $b$ becomes relevant for tax revenue. Consumers may choose where to buy, but not where to pay taxes. Hence, they cannot avoid high taxes. This equilibrium is qualitatively equivalent to the closed borders-equilibrium in Kanbur & Keen (1993). Accordingly, the entry of the online retailer mitigates tax competition.

Proposition 1 summarizes the effect of the entry of the online retailer on tax rates for symmetric countries and the destination principle.

**Proposition 1** Assume that taxation for online purchases follows the destination principle. a) For low fixed cost of online shopping ($\theta < t$), the online retailer is active in both markets, tax rates are higher in the online equilibrium than in the offline equilibrium, and the tax differential is zero. b) For high fixed cost of online shopping ($\theta > t$), the online retailer is not active.

### 2.3 Online Equilibrium Under Origin Principle

Assume now that taxation for online purchases follows the origin principle. This is equivalent the online retailer selling to private households and having sales revenues below the threshold of 100.000 Euros. For low cost of buying online, i.e. $\theta < \frac{1}{7}t$, the online retailer is selling in both countries. Then cross-border shopping does not take place anymore. For intermediate cost of buying online, i.e. $\frac{1}{7}t < \theta < \frac{11}{23}t$, the online
retailer is selling only in country $H$. Then cross-border shopping takes place. For $\theta > \frac{11}{23} t$, the online retailer is not active.

The consumer indifferent between buying from $x_H$ in country $H$ and buying online, $y_{H0}$, is located in country $H$. The consumer indifferent between buying from $x_F$ in country $F$ and buying online, $y_{0F}$, is located in country $F$. Demand is given as $q_H = y_{H0}$, $q_F = (1 - y_{0F})$, and $q_0 = y_{0F} - y_{H0}$. Firm profits are given by $\pi_H^o = (p_H - \tau_H) q_H$, $\pi_F^o = (p_F - \tau_F) q_F$, and $\pi_0^o = (p_0 - \tau_H) q_0$. Tax revenue is given by $R_H^o = \tau_H (q_H + q_0)$ and $R_F^o = \tau_F (q_F)$.

Equilibrium prices are $p_H^o = \frac{143(t+29)}{90}$, $p_F^o = \frac{119(t+29)}{90}$, $q_0^o = \frac{77(t+29)}{45}$. Quantities are $q_H^o = \frac{11(t+29)}{90}$, $q_F^o = \frac{7(t+29)}{15}$, and $q_0^o = \frac{2(11t-230)}{45}$. Tax rates and revenues, respectively, are given as $\tau_H^o = \frac{22(t+29)}{15}$, $\tau_F^o = \frac{14(t+29)}{15}$, $R_H^o = \frac{11(11t-149)(t+29)}{135t}$, and $R_F^o = \frac{49(t+29)^2}{135t}$. Tax rates and revenue increase in gross valuation $v$ and decrease in traveling cost $t$ and fixed cost of buying online $\theta$. In this online equilibrium, tax rates and revenues are lower than in the offline equilibrium ($\tau_H^o < \tau_H^d$, $\tau_F^o < \tau_F^d$, $R_H^o < R_H^d$, $R_F^o < R_F^d$). The tax differential is negative ($\Delta \tau^o = \tau_F^o - \tau_H^o < 0 = \Delta \tau^d$).

Proposition 2 summarizes the effect of the entry of the online retailer on tax rates for symmetric countries and the origin principle.

**Proposition 2** Assume that taxation for online purchases follows the origin principle. 

a) For low fixed cost of online shopping ($\theta < t$), i) the online retailer is active in both markets, ii) tax rates are lower in the online equilibrium than in the offline equilibrium, and iii) the tax differential is negative. b) For intermediate fixed cost of online shopping ($\frac{11}{23} t < \theta < 11 t$), i) the online retailer is active only in country $H$, ii) tax rates are lower in the online equilibrium than in the offline equilibrium, and iii) the tax differential is negative. c) For high fixed cost of online shopping ($\theta > \frac{11}{23} t$), the online retailer is not active.

3 Welfare Analysis

This subsection studies welfare in the two taxation regimes.

In country $H$, the brick-and-mortar stores’s profit is higher under the destination principle ($\pi_H^d < \pi_H^o$). In country $F$, the brick-and-mortar stores’s profit is higher under the origin principle ($\pi_F^o > \pi_F^d$).

The online retailer’s profit is higher under the destination principle ($\pi_0^d < \pi_0^o$).

In country $H$, tax revenues are higher under the destination principle if the gross
valuation is sufficiently high \((R_H^o < R_H^d \text{ if } v > \tilde{v}^4)\). In country \(F\), tax revenues are higher under the destination principle \((R_F^o < R_F^d)\).

Consumer surplus in country \(H\) is higher under the origin principle if the gross valuation is sufficiently high \((CS_H^o - CS_H^d < 0 \text{ if } v > \tilde{v}^5)\). Consumer surplus in country \(F\) is higher under the origin principle \((CS_F^o > CS_F^d)\).

This is, in country \(H\), the brick-and-mortar store and the online retailer would prefer the destination principle, while tax authorities and consumers would prefer the origin principle, if gross valuation is sufficiently high, and they would prefer the origin principle otherwise. In country \(F\), the brick-and-mortar store and consumers would prefer the origin principle, while tax authorities would prefer the destination principle.

For \(\theta < \frac{173}{2084} t\), welfare in country \(H\), given as the sum of consumer surplus, firm’s profits, and tax revenue, is higher under the origin principle, while welfare in country \(F\) is higher under the destination principle. For \(\frac{173}{2084} t < \theta < \frac{19}{148} t\), in both countries, welfare is higher under the destination principle. For \(\frac{19}{148} t < \theta\), welfare in country \(H\) is higher under the destination principle, while welfare in country \(F\) is higher under the origin principle \((W_H^o > W_H^d \text{ if } \theta < \tilde{\theta}^6, W_F^o < W_F^d, \text{if } \theta < \tilde{\theta}^7)\). Total welfare is higher under the destination principle \((W^o < W^d)\).

Proposition 3 summarizes the welfare effect of the two taxation principles.

**Proposition 3** For sufficiently low cost of online shopping, welfare in country \(H\) is higher under the origin principle and welfare in country \(F\) is higher under the destination principle. Total welfare is higher under the destination principle.

### 4 Decrease in the Cost of Online Shopping

This section studies the effect of a decrease in the cost of online shopping \(\theta\), e.g. due to improved logistics with faster delivery.

Under the destination principle, lower cost of online shopping increases prices of the brick-and-mortar stores as well as the price of the online retailer \((\frac{\partial p_H}{\partial \theta} = \frac{\partial p_F}{\partial \theta} < 0, \frac{\partial q_H}{\partial \theta} < 0, \frac{\partial q_F}{\partial \theta} > 0)\). It shifts market shares from the brick-and-mortar stores to the online retailer \((\frac{\partial q_H}{\partial \theta} = \frac{\partial q_F}{\partial \theta} > 0, \frac{\partial q_H^d}{\partial \theta} < 0, \frac{\partial q_F^d}{\partial \theta} < 0, \frac{\partial q_F^d}{\partial \theta} > 0)\). Lower cost of online shopping increases tax rates and tax revenues \((\frac{\partial t_H}{\partial \theta} = \frac{\partial t_F}{\partial \theta} < 0, \frac{\partial R_H}{\partial \theta} = \frac{\partial R_F}{\partial \theta} < 0)\). This is, under the destination principle,

\[\begin{align*}
\tilde{v}^4 &= \frac{287t^2 + 2069t - 616}{2084} \\
\tilde{v}^5 &= \frac{134915 + 528}{148} t \\
\tilde{\theta}^6 &= \frac{173}{2084} t \\
\tilde{\theta}^7 &= \frac{19}{148} t
\end{align*}\]
lower cost of online shopping enhances the anti-competitive effect, which results from the entry of the online retailer.

Under the origin principle, lower cost of online shopping decreases prices at the brick-and-mortar stores as well as the price of the online retailer \( \frac{\partial p_H}{\partial t} > 0 \), \( \frac{\partial p_F}{\partial t} > 0 \), \( \frac{\partial p_0}{\partial t} > 0 \). It shifts market shares from the brick-and-mortar stores to the online retailer \( \frac{\partial q_H}{\partial t} < 0 \), \( \frac{\partial q_F}{\partial t} < 0 \). Lower cost of online shopping decreases tax rates \( \frac{\partial t_H}{\partial t} > 0 \), \( \frac{\partial t_F}{\partial t} > 0 \), \( \frac{\partial t_0}{\partial t} > 0 \).

In country \( H \), lower cost of online shopping decreases tax revenue if \( \theta < \frac{1}{2} t \), i.e. the online retailer is selling in both countries, and increases tax revenues if \( \theta > \frac{1}{2} t \), i.e. the online retailer is selling in country \( H \) only \( \frac{\partial R_H}{\partial t} > 0 \), if \( \theta < \frac{1}{2} t \), \( \frac{\partial R_F}{\partial t} < 0 \), if \( \theta > \frac{1}{2} t \). In country \( H \), lower cost of online shopping decreases tax revenue \( \frac{\partial R_F}{\partial t} < 0 \).

This is, under the destination principle, lower cost of online shopping enhances the pro-competitive effect, which is caused by the entry of the online retailer.

Proposition 4 summarizes the effect of a decrease in the cost of online shopping.

**Proposition 4** Under the destination (origin) principle, lower cost of online shopping enhances the anti-competitive (pro-competitive) effect of the online retailer.

5 Limited Internet Access

Assume that only a fraction \( \lambda \in (0, 1) \) of consumers has access to the internet and to the online retailer. The remaining fraction \( 1 - \lambda \) buys from brick-and-mortar stores only.

5.1 Offline Equilibrium

The offline equilibrium is equivalent to the equilibrium described in 2.1.

5.2 Online Equilibrium under Destination Principle

Consider first the case of taxation for online purchases following the destination principle. Other than in the case of full internet access, cross-border shopping takes place, as consumers without internet access cannot buy online, but can still travel abroad for shopping. For this equilibrium assume low cost of online shopping, i.e. \( \theta < t \frac{\lambda + 3}{2(\lambda + 1)} \).

The indifferent consumer \( y_{H0} \) is located in country \( H \), the indifferent consumer \( y_{0F} \) is located in country \( F \). Demand is given as \( q_H^d = (1 - \lambda) (y_{HF}) + \lambda y_{H0} \), \( q_F^d = (1 - \lambda) (1 - y_{HF}) + \lambda (1 - y_{0F}) \), and \( q_0^d = \lambda (y_{0F} - y_{H0}) \). Firm profits are given by \( \pi_H^d = (p_H - \tau_H) q_H \), \( \pi_F^d = (p_F - \tau_F) q_F \), and \( \pi_0^d = (p_0 - \tau_F) \lambda \left( \frac{1}{2} - y_{H0} \right) + (p_0 - \tau_F) \lambda \left( y_{0F} - \frac{1}{2} \right) \). Tax revenue is given by \( R_H^d = \tau_H \left( q_H + \lambda \left( \frac{1}{2} - y_{H0} \right) \right) \) and \( R_F^d = \tau_F \left( q_F + \lambda \left( y_{0F} - \frac{1}{2} \right) \right) \).
Equilibrium prices are \( p_H^d = p_F^d = \frac{t(13\lambda + 2\lambda^2 + \lambda^3 + 8) + 29\lambda(1 - \lambda^2)}{2(2\lambda + 1)(1 - \lambda^2)} \) and 
\( p_0^d = \frac{t(\lambda + 3)(8\lambda - \lambda^2 + 5) - 29(1 - \lambda)(\lambda + 1)^2}{4(2\lambda + 1)(1 - \lambda^2)} \). Quantities are \( q_H^d = q_F^d = (\lambda + 1) \frac{t(2 - \lambda) + 29\lambda}{4t(2\lambda + 1)} \) and \( q_0^d = \lambda \frac{t(\lambda + 3) - 29(\lambda + 1)}{2t(2\lambda + 1)} \). Tax rates and revenues, respectively are given as \( \tau_H^d = \tau_F^d = \frac{t \lambda + 3}{1 - \lambda^2} \) and \( R_H^d = R_F^d = \frac{t \lambda + 3}{2(1 - \lambda^2)} \). Tax rates and revenues are higher than in the offline equilibrium \( (\tau_H > \tau_F > \tau_H^* = \tau_F^* = R_H^* = R_F^*) \), but lower than in the equivalent equilibrium under full internet access, see 2.2 \( (\tau_H^d - \tau_H^* < \tau_F^d - \tau_F^* \), 
\( R_H^d - R_H^* < R_F^d - R_F^* \)). Limited internet access decreases the effect of the online retailer in mitigating tax competition.

5.3 Online Equilibrium under Origin Principle

Consider now the case of taxation for online purchases following the origin principle. Assume \( \theta < \frac{t \lambda + 3}{2(2\lambda + 1)} \).

The indifferent consumer \( y_H0 \) is located in country \( H \), the indifferent consumer \( y_0F \) is located in country \( F \).

Demand is given as \( q_H^{o,\lambda} = (1 - \lambda)(y_{HF}) + \lambda y_{H0} \), \( q_F^{o,\lambda} = (1 - \lambda)(1 - y_{HF}) + \lambda(1 - y_{0F}) \), and 
\( q_0^{o,\lambda} = \lambda(y_{0F} - y_{H0}) \). Firm profits are given by \( \pi_H^{o,\lambda} = (p_H - \tau_H)q_H \), \( \pi_F^{o,\lambda} = (p_F - \tau_F)q_F \), and \( \pi_0^{o,\lambda} = (p_0 - \tau_H)q_0 \). Tax revenue is given by \( R_H^{o,\lambda} = \tau_H(q_H + q_0) \) and 
\( R_F^{o,\lambda} = \tau_Fq_H \).

Equilibrium prices are 
\( p_H^{o,\lambda} = \frac{t(15\lambda + 3\lambda^2 + 8)(15\lambda + \lambda^2 + 6) + 29\lambda(\lambda + 1)(-3\lambda^2 + 9\lambda + 4)}{6(2\lambda + 1)(\lambda + 1)(7\lambda + \lambda^2 + 2)} \), 
\( p_F^{o,\lambda} = \frac{(21\lambda + 5\lambda^2 + 8)(t(9\lambda - \lambda^2 + 6) + 29\lambda(\lambda + 1))}{6(2\lambda + 1)(\lambda + 1)(7\lambda + \lambda^2 + 2)} \), and 
\( p_0^{o,\lambda} = \frac{t(9\lambda + 5)(\lambda + 3)(15\lambda + \lambda^2 + 6) - 29(\lambda + 1)(33\lambda + 44\lambda^2 + 9\lambda^3 + 6)}{12(2\lambda + 1)(\lambda + 1)(7\lambda + \lambda^2 + 2)} \).

Quantities are 
\( q_H^{o,\lambda} = \frac{(\lambda + 1)(t(2 - \lambda)(15\lambda + \lambda^2 + 6) + 29\lambda(23\lambda + \lambda^2 + 10))}{12t(2\lambda + 1)(7\lambda + \lambda^2 + 2)} \), 
\( q_F^{o,\lambda} = \frac{t(9\lambda - \lambda^2 + 6) + 29\lambda(\lambda + 1)}{12t(2\lambda + 1)} \), and 
\( q_0^{o,\lambda} = \frac{\lambda(15\lambda + \lambda^2 + 6)(t(3\lambda + 3) - 29(\lambda + 1))}{6(2\lambda + 1)(\lambda + 1)(7\lambda + \lambda^2 + 2)} \). Tax rates and revenues, respectively are given as 
\( \tau_H^{o,\lambda} = \frac{(15\lambda + \lambda^2 + 6) - 29\lambda(\lambda + 1)}{3(\lambda + 1)(7\lambda + \lambda^2 + 2)} \) and 
\( \tau_F^{o,\lambda} = \frac{(9\lambda - \lambda^2 + 6) + 29\lambda(\lambda + 1)}{3t(\lambda + 1)(7\lambda + \lambda^2 + 2)} \), 
\( R_H^{o,\lambda} = \frac{(\lambda + 3)(t(15\lambda + \lambda^2 + 6) - 29\lambda(\lambda + 1))^2}{36t(2\lambda + 1)(\lambda + 1)(7\lambda + \lambda^2 + 2)} \), 
\( R_F^{o,\lambda} = \frac{(\lambda + 3)(t(9\lambda - \lambda^2 + 6) + 29\lambda(\lambda + 1))^2}{36t(\lambda + 1)(2\lambda + 1)(7\lambda + \lambda^2 + 2)} \). Tax rates and revenues are lower than in the offline equilibrium \( (\tau_H^{o,\lambda} < \tau_H^*, \tau_F^{o,\lambda} < \tau_F^*, R_H^{o,\lambda} < R_H^*, R_F^{o,\lambda} < R_F^*) \). In country \( H \), tax rates and revenues are higher than in the equivalent equilibrium under full internet access, see 2.3 \( (\tau_H^{o,\lambda} > \tau_H^*, R_H^{o,\lambda} > R_H^* \). In country \( F \), tax rates and revenues are higher than in the equivalent equilibrium under full internet.
access, if the cost of online shopping is sufficiently low ($\tau_F^0 > \tau_F^0$ if $\theta < \bar{\theta}$, $R_F^0 > R_F^0$ if $\theta < \bar{\theta})$.

Proposition 5 summarizes the effect of limited internet access.

**Proposition 5** Under the destination (origin) principle, limited internet access decreases the anti-competitive (pro-competitive) effect of the online retailer.

6 Cooperative Governments

Consider the case of governments cooperating in setting tax rates and maximizing joint revenue.

6.1 Offline Equilibrium

In the offline equilibrium, cooperatively set tax rates are given as $\tau_H^C = \tau_F^C = v - \frac{3}{2}t$, which are higher than under no cooperation ($\tau_F^C = \tau_F^C > \tau_H^* = \tau_F^*$). Equilibrium prices are $p_H^C = p_F^C = v - \frac{1}{2}t$. Quantities are $q_H^C = q_F^C = \frac{1}{2}$. Tax revenue is $R_H^C = R_F^C = \frac{1}{2} (v - \frac{3}{2}t)$.

6.2 Online Equilibrium under Destination Principle

The equilibrium under the destination principle and cooperation is equivalent to the equilibrium under no cooperation described in 2.2.

6.3 Online Equilibrium under Origin Principle

Under the origin principle, cooperatively set tax rates are given as $\tau_H^o = \tau_F^o = v - \frac{1}{3}t - \frac{3}{2}\theta$, which are higher than under no cooperation ($\tau_F^o = \tau_F^o > \tau_H^o = \tau_F^o$). Equilibrium prices are $p_H^o = p_F^o = v - \frac{1}{6}t - \frac{1}{3} \theta$ and $p_0^o = v - \theta$. Quantities are $q_H^o = q_F^o = \frac{t + 2\theta}{6t}$ and $q_0^o = \frac{2(t - \theta)}{3}$. Tax revenue is $R_H^o = R_F^o = \frac{1}{2} (v - \frac{1}{2}t - \frac{3}{2} \theta)$.

Proposition 6 summarizes the effect of the cooperation between governments.

**Proposition 6** In the offline equilibrium and under the origin principle, cooperatively set tax rates are higher than non-cooperatively set tax rates. Under the destination principle, cooperatively set tax rates are equivalent to non-cooperatively set tax rates.

\[
\tilde{s} = \frac{(1-\lambda)(103\lambda+19\lambda^2+62)}{2(1+1)(83\lambda^2+9\lambda^2+26)} \\
\tilde{\theta} = \frac{(1-\lambda)(-15\lambda^3+602\lambda^2+3709\lambda^2+4080\lambda+1228)}{4(1+1)(302\lambda+392\lambda+1886\lambda+2156\lambda+2445\lambda^2+302\lambda^2+151\lambda^3+2895\lambda^2+3329\lambda^2+3329\lambda^3-156\lambda^4)}
\]
7 Benevolent Governments

Consider now the case of governments setting tax rates to maximize welfare, given as the sum of consumer surplus, firm’s profit, and tax revenue.

7.1 Offline Equilibrium

In the offline equilibrium, welfare in country $H$ and country $F$, respectively, is given as $W^*_H = \frac{(3t+\tau_F-\tau_H)(12t-3t+\tau_F+\tau_H)}{12t}$ and $W^*_F = \frac{(3t-\tau_F+\tau_H)(12t-3t+\tau_F-\tau_H)}{12t}$. Welfare in country $j$ is decreasing in the tax rate of country $j$ and increasing in the tax rate in the tax of the other country ($\frac{\partial W_j}{\partial \tau_j} < 0$, $\frac{\partial W_j}{\partial \tau_{-j}} > 0$). This is, increasing the tax rate decreases welfare in the respective country and increases welfare in the other country. For the equilibrium $\tau^W_H = \tau^W_F = 0$, equilibrium prices are $p^W_H = p^W_F = t$, quantities are $q^W_H = q^W_F = \frac{1}{2}$.

7.2 Online Equilibrium under Destination Principle

Under the destination principle, welfare in country $H$ and country $F$, respectively, is given as

$$W^d_H = \frac{144t+4(-44t^2+28t^2+7t^2)-9(t-\tau_H)(4t-8t+5\tau_F-5\tau_H)}{96t} \quad \text{and} \quad W^d_F = \frac{48t-4(3t-29)(t+29)+3(\tau_F-\tau_H)(4t-8t+3\tau_F-3\tau_H)}{96t}.$$  

Welfare in country $j$ is decreasing in the tax rate of country $j$ and increasing in the tax rate in the tax of the other country ($\frac{\partial W^d_j}{\partial \tau_j} < 0$, $\frac{\partial W^d_j}{\partial \tau_{-j}} > 0$). For the equilibrium $\tau^W_H = \tau^W_F = 0$, equilibrium prices are

$$p^d_H = p^d_F = \frac{1}{6}t + \frac{1}{4}\theta \quad \text{and} \quad p_0 = \frac{1}{2}t - \frac{1}{3}\theta,$$

quantities are $q^d_H = q^d_F = \frac{(t+29)}{6t}$ and $q^d_0 = \frac{1}{2(2-t)}$.

7.3 Online Equilibrium under Origin Principle

Under the origin principle, welfare in country $H$ and country $F$, respectively, is given as

$$W^o_H = \frac{144t+4(-44t^2+28t^2+7t^2)+52t-88t+44t+19t^2-101t^2-40\tau_F-56\tau_F-82\tau_F-5\tau_H+82\tau_F+\tau_H}{96t} \quad \text{and} \quad W^o_F = \frac{48t-4(3t-29)(t+29)-12t-20\tau_F-80t+3\tau_F+15t^2-10\tau_F+25\tau_H}{96t}.$$  

Equilibrium non-negative tax rates are $\tau^W_H = \frac{2(111t-149)}{101}$ and $\tau^W_F = 0$, which are lower than under revenue-maximizing governments ($\tau^W_H < \tau^W_F$, $\tau^W_F < \tau^W_F$). Equilibrium prices are $p^W_H = \frac{(3t+80)}{101}$, $p^W_F = \frac{(2t+119)}{101}$, and $p_0 = \frac{(52t-579)}{101}$. Quantities are $q^W_H = \frac{3(5t+129)}{101}$, $q^W_F = \frac{2(13t+119)}{101}$, and $q^W_0 = \frac{2(30t-29)}{101}$.

Proposition 7 summarizes the effect of benevolent governments.

**Proposition 7** Tax rates set by benevolent governments are lower than tax rates set by revenue-maximizing governments.
8 Asymmetric Countries

Consider now that both countries differ in population density $\mu_j$, with $\mu_H = \mu < \mu_F = 1$. This is, country $H$ is smaller in terms of population size.

8.1 Offline Equilibrium

Consider first the case without the online retailer, when consumers buy from brick-and-mortar stores only.

The indifferent consumer $y_{HF}^*$ is located in country $F$. Demand is $q_H^* = \mu_F + \left( y_H^* - \frac{1}{2} \right)$ and $q_F^* = 1 - y_H^*$. Firm profits are $\pi_H^* = (p_H^* - \tau_H^*) \cdot q_H^*$ and $\pi_F^* = (p_F^* - \tau_F^*) \cdot q_F^*$. Tax revenue is $R_H^* = \tau_H^* \cdot q_H^*$ and $R_F^* = \tau_F^* \cdot q_F^*$. Equilibrium prices are $p_H^* = \frac{4(5\mu+4)}{9}$ and $p_F^* = \frac{(4\mu+5)}{18}$. Quantities are $q_H^* = \frac{(5\mu+4)}{18}$ and $q_F^* = \frac{(4\mu+5)}{18}$. Tax rates are $\tau_H^* = \frac{(4\mu+5)}{3}$ and $\tau_F^* = \frac{(5\mu+4)}{3}$. Tax revenues are $R_H^* = \frac{(5\mu+4)(3\mu-4)}{54}$ and $R_F^* = \frac{(4\mu+5)^2}{54}$. The tax differential is positive ($\Delta \tau^* = \tau_F^* - \tau_H^* > 0$). This is, the smaller country sets the lower tax rate. This equilibrium is equivalent to the open borders-case in Kanbur & Keen (1993).

8.2 Online Equilibrium under Destination Principle

Consider now the case with the online retailer. For low cost of buying online, i.e. $t < t_{27+17}^{44}$, the online retailer is selling only in country $F$. For intermediate cost of buying online, i.e. $\theta < t$, the online retailer is selling in both countries. Then cross-border shopping does not take place anymore. For high cost of buying online, i.e. $\theta > t$, the online retailer is not active.

8.2.1 Online Retailer Active in One Country

Consider now the case of the online retailer selling in country $F$ only under low cost of online shopping, i.e. $t_{5(17\mu-9)}^{44} < \theta < t_{2(7+17\mu)}^{44}$. Then cross-border shopping takes place. Variables associated with the equilibrium with the online retailer active in both countries are denoted by the (additional) superscript $F$.

Both indifferent consumers $y_{H0}, y_{0F}$ are located in $F$. Demand is given as $q_{H}^{d,F} = \mu b + y_{H0} - b$, $q_{F}^{d,F} = (1 - y_{0F})$, and $q_{0}^{d,F} = y_{0F} - y_{H0}$. Firm profits are given by $\pi_{H}^{d,F} = (p_{H} - \tau_{H}) \cdot q_{H}$, $\pi_{F}^{d,F} = (p_{F} - \tau_{F}) \cdot q_{F}$, and $\pi_{0}^{d,F} = (p_{0} - \tau_{F}) \cdot (y_{0F} - y_{H0})$. Tax revenue is given by $R_{H}^{d,F} = \tau_{H} \cdot q_{H}$, $R_{F}^{d,F} = \tau_{F} \cdot (q_{F} + q_{0})$.

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10This is, for $t_{5(17\mu-9)}^{44} < \theta < t_{2(7+17\mu)}^{44}$, two equilibria are possible, for $t_{2(7+17\mu)}^{44} < \theta < t_{2(7+17\mu)}^{44}$, only one equilibrium is possible.
Equilibrium prices are $p^d_F = \frac{17t(9+19\mu)+80}{900}$, $p^0_F = \frac{1(351+221\mu)+409}{900}$, and $p^d_0 = \frac{((189+19\mu)-929)}{900}$. Quantities are $q^d_H = \frac{((9+19\mu)+80}{900}$, $q^d_F = \frac{((27+17\mu)+1360)}{360}$, and $q^d_0 = \frac{((9+19\mu)+80}{900}$. Tax rates are $\tau^d_H = \frac{((9+19\mu)+80}{900}$ and $\tau^d_F = \frac{((27+17\mu)-86)}{360}$; tax revenues are $R^d_H = \frac{((9+19\mu)+80}{2160}$ and $R^d_F = \frac{((27+17\mu)-86)}{2160}$. In this online equilibrium, tax rates and revenues are lower than in the offline equilibrium ($\tau^d_H < \tau^*_H$, $\tau^d_F < \tau^*_F$, $R^d_H < R^*_H$, $R^d_F < R^*_F$). The tax differential is positive ($\Delta \tau^d_H = \tau^d_F - \tau^d_H > 0$).

### 8.2.2 Online Retailer Active in Both Countries

Consider first the case of the online retailer selling in both countries under intermediate cost of online shopping, i.e. $\theta < t$. Then cross-border shopping does not take place anymore. Variables associated with the equilibrium with the online retailer active in both countries are denoted by the (additional) superscript $HF$.

The indissuasive consumer $y_{0H}$ is located in country $H$, the indissuasive consumer $y_{0F}$ is located in country $F$. Demand is given as $q^d_H = \mu y_{0H}$, $q^d_F = (1 - y_{0F})$, and $q^d_0 = y_{0F} - b + \mu (b - y_{0H})$. Firm profits are given by $\pi^d_H = (p_H - \tau_H)q_H$, $\pi^d_F = (p_F - \tau_F)q_F$, and $\pi^d_0 = (p_0 - \tau_H)\mu \left(\frac{1}{2} - y_{0H}\right) + (p_0 - \tau_F)\left(\frac{1}{2} - y_{0F}\right) - \frac{1}{2}$. Tax revenue is given by $R^d_H = \tau_H (q_H + \left(\frac{1}{2} - y_{0H}\right))$ and $R^d_F = \tau_F (q_F + \left(\frac{1}{2} - y_{0F}\right))$.

Equilibrium prices are $p^d_H = p^d_F = \frac{17t(9+19\mu)+80}{900}$, $p^d_0 = \frac{1(351+221\mu)+409}{900}$, and $p^d_H = \frac{((189+19\mu)-929)}{900}$. Quantities are $q^d_H = \frac{((9+19\mu)+80}{900}$, $q^d_F = \frac{((27+17\mu)+1360)}{360}$, and $q^d_0 = \frac{((9+19\mu)+80}{900}$. Tax rates and revenues, respectively are given as $\tau^d_H = \tau^d_F = \frac{((9+19\mu)+80}{900}$ and $\tau^d_0 = \frac{((9+19\mu)+80}{900}$. Tax rates and revenues are higher than in the offline equilibrium ($\tau^d_H > \tau^*_H$, $\tau^d_F > \tau^*_F$, $R^d_H > R^*_H$, $R^d_F > R^*_F$). The tax differential is zero ($\Delta \tau^d = 0 < \Delta \tau^*$). The entry of the online retailer mitigates tax competition. This equilibrium is qualitatively equivalent to the closed borders-equilibrium in Kanbur & Keen (1993).

Proposition 8 summarizes the effect of the entry of the online retailer on tax rates for asymmetric countries and the destination principle.

**Proposition 8** Assume that taxation for online purchases follows the destination principle. 

1. For low fixed cost of online shopping ($t^{17\mu} < \theta < t^{27+17\mu}/44$), the online retailer is active in country $F$ only, tax rates are lower in the online equilibrium than in the offline equilibrium, and the tax differential is positive. 
2. For intermediate cost of online shopping ($\theta < t$), the online retailer is active in both markets, tax rates are higher in the online equilibrium than in the offline equilibrium, and the tax differential is zero. 
3. For high fixed cost of online shopping ($\theta > t$), the online retailer is not active.
8.3 Online Equilibrium under the Origin Principle

Consider now the case with the online retailer. For \( \theta < t \frac{21 \mu + 13}{2(9\mu + 8)} \), the online retailer is selling in both countries. Then cross-border shopping does not take place anymore. For \( t \frac{(5\mu + 3)}{8} < \theta < t \frac{(39 + 5\mu)}{44} \), the online retailer is selling only in country \( F \). For \( \mu < \frac{26}{45} \) and \( \theta > t \frac{21 \mu + 13}{2(9\mu + 8)} \) for \( \mu > \frac{26}{45} \), the online retailer is not active.

8.4 Online Retailer Active in Both Countries

Consider first the case of the online retailer selling in both countries for \( \theta < t \frac{21 \mu + 13}{2(9\mu + 8)} \). Then cross-border shopping does not take place anymore. Variables associated with the equilibrium with the online retailer active in both countries are denoted by the (additional) superscript \( HF \).

The indifferent consumer \( y_{H0} \) is located in country \( H \), the indifferent consumer \( y_{0F} \) is located in country \( F \). Demand is given as \( q_0^{o, HF} = \mu y_{H0} \), \( q_F^{o, HF} = (1 - y_{0F}) \), and \( q_0^{o, HF} = y_{0F} - b + \mu (b - y_{H0}) \). Firm profits are given by \( \pi^{o, HF}_H = (p_H - \tau_H) q_H \), \( \pi^{o, HF}_F = (p_F - \tau_F) q_F \), and \( \pi_0^{o, HF} = (p_0 - \tau_H) q_0 \). Tax revenue is given by \( R^{o, HF}_H = \tau_H (q_H + q_0) \) and \( R^{o, HF}_F = \tau_F q_F \).

Equilibrium prices are \( p^{o, HF}_H = \frac{(3\mu + 4)(6\mu + 5) - \theta(15\mu + 8)}{9(3\mu + 2)} \) and \( p^{o, HF}_F = \frac{(3\mu + 4)(6\mu + 5) - \theta(9\mu + 2)}{18(3\mu + 2)} \). Quantities are \( q^{o, HF}_H = \mu \frac{3(3\mu + 4) - \theta(3\mu + 2)}{18(3\mu + 2)} \) and \( q^{o, HF}_F = \frac{(3\mu + 4)(6\mu + 5) - \theta(9\mu + 2)}{18(3\mu + 2)} \). Tax rates are \( \tau^{o, HF}_H = \frac{(3\mu + 4)(6\mu + 5) - \theta(15\mu + 8)}{9(3\mu + 2)} \) and \( \tau^{o, HF}_F = \frac{(3\mu + 4)(6\mu + 5) - \theta(9\mu + 2)}{9(3\mu + 2)} \).

In this online equilibrium, tax rates are lower than in the offline equilibrium (\( \tau^{o, HF}_H < \tau^*_H \), \( \tau^{o, HF}_F < \tau^*_F \)). In country \( H \), tax revenue is lower if the cost of online shopping is sufficiently low, in country \( F \), tax revenue is lower than in the offline equilibrium (\( R^{o, HF}_H < R^*_H \), if \( \theta < \hat{\theta}^{12} \), \( R^{o, HF}_F < R^*_F \)). The tax differential is negative, if the cost of online shopping is sufficiently low (\( \Delta \tau^{o, HF} = \tau^{o, HF}_F - \tau^{o, HF}_H < 0 \) if \( \theta < \hat{\theta}^{13} \)).

\[ \hat{\theta}^{12} = \left( \frac{(t2\mu + 13)\mu^2 + 272\mu^3 - t\sqrt{1514\mu + 2735\mu^2 - 480\mu^3 - 4383\mu^4 - 1863\mu^5 + 729\mu^6 + 209}}{22\mu + 28\mu^2 + 4} \right) \]

\[ \hat{\theta}^{13} = \left( \frac{(3\mu + 1)}{4} \right) \]
8.5 Online Retailer Active in One Country

Consider now the case of the online retailer selling in country $F$ only for $t\frac{(5\mu+3)}{8} < \theta < t\frac{(3\mu+5\nu)}{8}$. Then cross-border shopping takes place. Variables associated with the equilibrium with the online retailer active in both countries are denoted by the (additional) superscript $F$.

The indifferent consumers $y_{H0}$ and $y_{0F}$ are located in country $F$. Demand is given as $q_{H}^{o,F} = \mu b + y_{H0} - b$, $q_{F}^{o,F} = 1 - y_{0F}$, and $q_{0}^{o,F} = y_{0F} - y_{H0}$. Firm profits are given by $\pi_{H}^{o,F} = (p_{H} - \tau_{H}) q_{H}$, $\pi_{F}^{o,F} = (p_{F} - \tau_{F}) q_{F}$, and $\pi_{0}^{o,F} = (p_{0} - \tau_{H}) q_{0}$. Tax revenue is given by $R_{H}^{o,F} = \tau_{H} (q_{H} + q_{0})$ and $R_{F}^{o,F} = \tau_{F} (q_{F})$.

Equilibrium prices are $p_{H}^{o,F} = \frac{t(201+371\mu)+40\theta}{360}$, $p_{F}^{o,F} = \frac{17(t(15+13\mu)+8\theta)}{360}$, and $p_{0}^{o,F} = \frac{t(165+143\mu)-92\theta}{90}$. Quantities are $q_{H}^{o,F} = \frac{t(21+23\mu)-92\theta}{360}$, $q_{F}^{o,F} = \frac{t(15+13\mu)+8\theta}{360}$, and $q_{0}^{o,F} = \frac{t(39+5\mu)-44\theta}{90}$. Tax rates are $\tau_{H}^{o,F} = \frac{t(21+23\mu)-8\theta}{30}$ and $\tau_{F}^{o,F} = \frac{t(15+13\mu)+8\theta}{30}$; tax revenues are $R_{H}^{o,F} = \frac{(21t+8+23\mu)^2}{2160}$ and $R_{F}^{o,F} = \frac{(15t+8+13\mu)^2}{2160}$.

In this online equilibrium, tax rates are lower than in the offline equilibrium ($\tau_{H}^{o,F} < \tau_{H}$, $\tau_{F}^{o,F} < \tau_{F}$). In country $H$, tax revenue is higher if the cost of online shopping is sufficiently low, in country $F$, tax revenue is lower than in the offline equilibrium ($R_{H}^{o,F} > R_{H} > 0$, if $\theta < \tilde{\theta}_{14}$, $R_{F}^{o,HF} < R_{F}^{o}$). The tax differential is negative, if the cost of online shopping is sufficiently low ($\tau_{F}^{o,HF} - \tau_{H}^{o,HF} = -\frac{(3t-8\theta+5\mu\nu)}{15} < 0$, if $\theta < \tilde{\theta}_{15}$).

Proposition 9 summarizes the effect of the entry of the online retailer on tax rates for asymmetric countries and the origin principle.

**Proposition 9** Assume that taxation for online purchases follows the origin principle.  
\begin{itemize}
  \item[a)] For $\theta < t\frac{21\mu+13}{2(9\mu+8)}$, the online retailer is active in both markets, tax rates are lower in the online equilibrium than in the offline equilibrium, and the tax differential is negative, if the cost of online shopping is sufficiently low. 
  \item[b)] For $t\frac{(5\mu+3)}{8} < \theta < t\frac{(3\mu+5\nu)}{8}$, the online retailer is active in country $F$ only, tax rates are lower in the online equilibrium than in the offline equilibrium, and the tax differential is negative, if the cost of online shopping is sufficiently low. 
  \item[c)] For $\theta > t\frac{(39+5\nu)}{44}$ for $\mu < \frac{26}{45}$ and $\theta > t\frac{21\mu+13}{2(9\mu+8)}$ for $\mu > \frac{26}{45}$ the online retailer is not active.
\end{itemize}

9 Discussion

This section addresses assumptions of the model and their implications for the analysis.

\footnotesize
\begin{itemize}
  \item[$\tilde{\theta}_{14}$] $\theta = t\frac{23\mu-2\sqrt{3\mu(3\mu+6\mu^2-16+21)}}{2(9\mu+8)}$ \\
  \item[$\tilde{\theta}_{15}$] $\theta = t\frac{(5\mu+3)}{8}$
\end{itemize}

9.1 Competition

So far, the model has assumed that the brick-and-mortar stores are local monopolies and that all three retailers have pricing power. Assuming the opposite extreme, perfect competition would result in marginal cost pricing and effective consumer prices amounting to the (respective) tax rate. Under destination based taxation, this would imply that the online retailer charges an average price of the two tax rates or sets country-specific (and tax rate-specific) prices.

In the offline equilibrium, assuming perfect competition among brick-and-mortar stores yields tax rates $\tau_{H}^{\text{comp}} = \tau_{F}^{\text{comp}} = t$, which are lower than under market power. In the online equilibrium under the destination principle, assuming perfect competition among online retailers and/or brick-and-mortar stores yields the same tax rates, i.e. $\tau_{H}^{\text{d,comp}} = \tau_{F}^{\text{d,comp}} = \tau_{H}^{d} = \tau_{F}^{d} = v - \frac{1}{3}t - \frac{2}{3}\theta$. In the online equilibrium under the origin principle, assuming perfect competition among online retailers and/or brick-and-mortar stores yields lower tax rates than under market power, i.e. $\tau_{H}^{\text{o,comp}, 0} = \frac{4t-\theta}{3}$, $\tau_{F}^{\text{o,comp}, 0} = \frac{2t - \frac{1}{3}\theta}{3}$ for competition among online retailers and $\tau_{H}^{\text{o,comp}, HF0} = \frac{2}{3}t - \frac{1}{3}\theta$, $\tau_{F}^{\text{o,comp}, HF0} = \frac{1}{3}t + \frac{1}{3}\theta$ for competition among online retailers and brick-and-mortar stores. This is, assuming perfect competition yields qualitatively similar results. Under the destination principle, the entry of the online retailer mitigates tax competition and results in higher tax rates in the online equilibrium ($\tau_{H}^{\text{d,comp}} > \tau_{H}^{\text{comp}} = \tau_{F}^{\text{comp}} > \tau_{F}^{\text{d,comp}}$). Under the origin principle, the entry of the online retailer enforces tax competition and results in lower tax rates in the online equilibrium ($\tau_{H}^{\text{o,comp}, HF0} < \tau_{H}^{\text{comp}}$, $\tau_{F}^{\text{o,comp}, HF0} < \tau_{F}^{\text{comp}}$).

9.2 Tax Policy

As in Kanbur & Keen (1993), consider two tax policies, tax harmonization and the implementation of a minimum tax rate. The latter can also be found in Directive 2006/112/EG.

In the offline equilibrium, tax rates are symmetric. This is, there is actually no scope for tax harmonization. If governments choose a common tax rate, they choose $\tau_{H}^{\text{h}} = \tau_{F}^{\text{h}} = \tau^{\text{h}} = v - \frac{1}{3}t$, which is higher than the tax rates in the offline equilibrium without tax harmonization ($\tau^{\text{h}} > \tau_{H}^{\text{h}} = \tau_{F}^{\text{h}}$). In the online equilibrium under the destination principle, the harmonized tax rate is $\tau_{H}^{\text{d,h}} = \tau_{F}^{\text{d,h}} = v - \frac{1}{3}t - \frac{2}{3}\theta$, which is equivalent to the tax rates without tax harmonization ($\tau^{\text{d,h}} = \tau_{H}^{\text{d,h}} = \tau_{F}^{\text{d,h}}$). In the online equilibrium under the origin principle, the harmonized tax rate is $\tau_{H}^{\text{o,h}} = \tau_{F}^{\text{o,h}} = v - \frac{1}{3}t - \frac{2}{3}\theta$, which is higher than the tax rates without tax harmonization ($\tau^{\text{o,h}} = \tau_{H}^{\text{o,h}} = \tau_{F}^{\text{o,h}}$).

A (binding) minimum tax rate in the offline equilibrium would increase tax rates.
It would raise prices, but not affect market shares. In the online equilibrium under the destination principle, a higher tax rate would increase tax rates and result in incomplete market coverage. In the equilibrium without the minimum tax rate the marginal consumer has a consumer surplus of zero and an increase in the tax rate and accordingly, a higher price would result in the marginal consumer not buying. In the online equilibrium under the origin principle, a minimum tax rate would raise both tax rates, no matter whether it is binding one tax rate or both tax rates, since tax rates are strategic substitutes. A minimum tax rate would decrease the tax differential, with a higher minimum tax rate resulting in a lower tax differential.

10 Conclusion

This paper has studied the effect of an online retailer on spatial tax competition with mobile consumers.

If taxation for online purchases follows the destination principle, the entry of the online retailer mitigates tax competition; if taxation for online purchases follows the origin principle, the entry of the online retailer enhances tax competition. This is, the choice of the taxation principle may shape the effect of the online retailer on tax competition. In the European Union, the destination principle applies to online retailers with sales to private households and with sales above the threshold of 100,000 Euros in the European Union, suggesting that the entry of online retailers has mitigated tax competition.

Lower cost of online shopping enhances the effect of the online retailer on tax competition, limited internet access decreases the effect of the online retailer on tax competition. This suggests that with increasing access to the internet in the European Union and decreasing cost of online shopping due to improved technology – and as a result of the Digital Agenda for Europe – one can expect that the tax competition-mitigating impact of online retailers increases over time.

For sufficiently low cost of online shopping, welfare in the country where the online retailer is located is higher under the origin principle and welfare in the other country is higher under the destination principle. Total welfare is higher under the destination principle. This result implies that the choice of a taxation regime on a European level to maximize (total) welfare, with side payments being feasible, may result in a regime with decreased tax competition.

This model has considered commodity tax competition so far. An issue of raising relevance in the European Union is the taxation of profit’s of online retailers, especially
with respect to the question which member states may tax online retailers. This question is left for further research.
References


Appendix

Symmetric Countries

Offline Equilibrium

\[ \pi^*_H = \pi^*_F = \frac{1-6t}{4} \]
\[ C_{S,H}^* = \int_0^{y^*_{H,F}} (v - p^*_H - tx) \, dx = \]
\[ C_{S,F}^* = \int_{y^*_{H,F}}^{1} (v - p^*_F - t(1-x)) \, dx = \frac{1}{2}v - \frac{17}{8}t \]

Online Equilibrium under Destination Principle

\[ \pi^d_H = \pi^d_F = \frac{(t+2\theta)^2}{36t} \]
\[ \pi^d_0 = \frac{2(t-\theta)^2}{9t} \]
\[ C_{S,H}^d = \int_0^{y^*_{H_0}} (v - p_H - tx) \, dx + \int_{y^*_{H_0}}^{\frac{1}{2}} (v - p_0 - \theta) \, dx = C_{S,F}^d = \int_{\frac{1}{2}}^{1} (v - p_F - t(1-x)) \, dx = \frac{1}{72t} (t + 2\theta)^2 \]
\[ W_H^d = \frac{36vt - 44\theta^2 + 28\theta^2 + 7t^2}{72t} \]
\[ W_F^d = \frac{124vt - 3\theta^2 - 4\theta^2 + 4t^2}{24t} \]
\[ W_0^d = \frac{36vt - t^2 - 28\theta + 200t^2}{36t} \]
\[ p_H^d - p_H^* = p_F^d - p_F^* = v - \frac{25}{6}t - \frac{1}{3} \theta > 0 \]
\[ q_H^d - q_H^* = q_F^d - q_F^* = -\frac{(t-\theta)}{3t} < 0 \]
\[ \tau_H^d - \tau_H^* = \tau_F^d - \tau_F^* = v - \frac{10}{3}t - \frac{2}{3} \theta > 0 \]
\[ R_H^d - R_H^* = R_F^d - R_F^* = \frac{1}{2}v - \frac{5}{3}t - \frac{1}{3} \theta > 0 \]

Online Equilibrium under Origin Principle

\[ \pi^o_H = \frac{121(t+2\theta)^2}{8100t} \]
\[ \pi^o_F = \frac{49(t+2\theta)^2}{343t} \]
\[ \pi^o_0 = \frac{2(11t-23\theta)^2}{2025t} \]
\[ C_{S,H}^o = \int_0^{y^*_{H_0}} (v - p_H - tx) \, dx + \int_{y^*_{H_0}}^{\frac{1}{2}} (v - p_0 - \theta) \, dx = \frac{8100vt - 11(1249t - 229)(t+2\theta)}{1800\theta t} \]
\[ C_{S,F}^o = \int_{\frac{1}{2}}^{1} (v - p_F - t(1-x)) \, dx = \frac{8100vt - 7(3611 - 70\theta)(t+2\theta)}{3240t} \]
\[ W_H^o = \frac{8100vt + 2959\theta^2 - 238040\theta - 270446}{1600\theta t} \]
\[
W^o_F = \frac{540\theta - 287t^2 + 700\theta^2 + 2548\theta}{100t} \\
W^o_H = \frac{8100\theta - 673t^2 - 6652\theta + 5588\theta^2}{1000t}
\]
\[
\hat{p}_H^* - p_H^* = -\frac{(217t - 2866)}{91} < 0 \\
\hat{p}_F^* - p_F^* = -\frac{(241t - 2386)}{90} < 0 \\
\hat{q}_H^* - q_H^* = -\frac{(177t - 116)}{365} < 0 \\
\hat{q}_F^* - q_F^* = -\frac{(t - 70)}{15} < 0 if \theta < \frac{1}{7}t \\
\hat{\tau}_H^* - \tau_H^* = -\frac{(99t - 449)}{15} < 0 \\
\hat{\tau}_F^* - \tau_F^* = -\frac{(31t - 289)}{15} < 0 \\
R^o_H - R^*_H = -\frac{(163t^2 - 1769\theta + 6166^2)}{270t} < 0 \\
R^o_F - R^*_F = -\frac{(307t^2 + 392\theta + 392\theta^2)}{270t} < 0
\]

**Welfare Analysis**

\[
\hat{\pi}_H^* - \hat{\pi}_H^d = -\frac{26(t+29)^2}{2025t} < 0 \\
\hat{\pi}_F^* - \pi_F^d = -\frac{10(t+29)^2}{90t} > 0 \\
\hat{\pi}_0^* - \pi_0^d = -\frac{18(13t - 199)(t+29)}{2025t} < 0 \\
R^o_H - R^*_H = -\frac{7(41t^2 + 38\theta - 880^2)}{270t} - \frac{1}{2}v < 0 if v > \tilde{v}_H = \frac{287t^2 + 266\theta - 6166^2}{135t} \\
R^o_F - R^*_F = -\frac{(143t^2 + 482\theta + 392\theta^2)}{270t} - \frac{1}{2}v < 0
\]

market coverage under origin principle if \( U = v - t(y - x_H) - p_H > 0 \) if \( v > \tilde{v} = \frac{90t}{143t + 376\theta} \)

\[
\tilde{v} - \tilde{v} = -\frac{596t\theta - 12392^2 + 145t^2}{2025t} > 0 if \theta < \frac{3}{615} \sqrt{7429 - \frac{149}{615} t} \\
CS^o_H - CS^d_H = \frac{2025t(t - (349t + 529)(t+29)}{4050t} > 0 if \theta > \tilde{v} = \frac{(349t + 529)(t+29)}{2025t} \\
\tilde{v} - \tilde{v} = -\frac{547t^2 + 2852\theta + 2086^2}{270t} > 0, if \theta < \frac{173}{289} - \frac{3240}{289} \sqrt{237}t \\
CS^o_F - CS^d_F = \frac{4050t(t - 7(361t + 706)(t+29)}{3240t} > 0 if \theta < \tilde{v} = \frac{405t(t - (643t + 1009)(t+29)}{810t} > 0 \\
W^o_H - W^d_H = \frac{173t^2 - 1738\theta - 4168^2}{2025t} > 0 if \theta < \tilde{\theta} = \frac{173}{2084} t \\
W^o_F - W^d_F = -\frac{19t^2 - 1109\theta - 2966^2}{135t} < 0 if \theta < \tilde{\theta} = \frac{19}{148} t \\
W^o - W^d = -\frac{8(14t^2 + 11\theta - 340^2)}{2025t} < 0
\]

**Decrease in the Cost of Online Shopping**

\[
\frac{\partial p_H^*}{\partial \theta} = \frac{\partial p_F^*}{\partial \theta} = -\frac{1}{3} < 0 \\
\frac{\partial \pi_H^*}{\partial \theta} = -1 < 0 \\
\frac{\partial \pi_F^*}{\partial \theta} = \frac{1}{3t} > 0 \\
\frac{\partial \tau_H^*}{\partial \theta} = \frac{2}{3} < 0 \\
\frac{\partial \tau_F^*}{\partial \theta} = -\frac{2}{3} < 0 \\
\frac{\partial R_H^o}{\partial \theta} = \frac{1}{3t} < 0
\]
\[
\frac{\partial \rho_H}{\partial \theta} = \frac{143}{45} > 0 \\
\frac{\partial \rho_F}{\partial \theta} = \frac{119}{45} > 0 \\
\frac{\partial \rho_0}{\partial \theta} = \frac{109}{45} > 0 \\
\frac{\partial \rho_H}{\partial \theta} = \frac{11}{45} > 0 \\
\frac{\partial \rho_F}{\partial \theta} = \frac{7}{9} > 0 \\
\frac{\partial \rho_0}{\partial \theta} = \frac{-46}{45} < 0 \\
\frac{\partial \rho_H}{\partial \tau} = \frac{44}{45} > 0 \\
\frac{\partial \rho_F}{\partial \tau} = \frac{28}{15} > 0 \\
\frac{\partial \rho_0}{\partial \tau} = \frac{88(1-\theta)}{135 \theta} > 0, \text{ if } \theta < \frac{1}{7}t \\
\frac{\partial \rho_H}{\partial \theta} = \frac{49(4t+8\theta)}{135 \theta} > 0
\]

**Imperfect Internet Access**

\[
\tau_H^d - \tau_H^r = \tau_F^d - \tau_F^r = t(3\lambda+1) > 0 \\
R_H^d - R_H^r = R_F^d - R_F^r = 1 - t \lambda \rho_H > 0 \\
p_H^d - p_H^r = p_F^d - p_F^r = -6 \lambda(1-\lambda^2)(2\lambda+1)(\lambda+3)(\lambda+2) > 0 \\
p_0^d - p_0^r = \frac{4(1-\lambda^2)(2\lambda+1)(\lambda+3)(\lambda+2)}{6(1-\lambda^2)(2\lambda+1)} > 0 \\
q_H^d - q_H^r = q_F^d - q_F^r = \frac{1}{12t(2\lambda+1)} > 0 \\
q_0^d - q_0^r = \frac{1}{6t(2\lambda+1)} > 0 \\
\tau_H^d - \tau_F^d = \frac{t(3\lambda+1)-(2\lambda+1)(\lambda+3)(\lambda+2)}{3(1-\lambda^2)} > 0 \\
R_H^d - R_F^d = R_H^r - R_F^r = \frac{t(3\lambda+1)-(2\lambda+1)(\lambda+3)(\lambda+2)}{3(1-\lambda^2)} > 0 \\
\tau_H^d - \tau_H^r = -\frac{2(1) \lambda(2\lambda+4) \rho_0(2\lambda+1)}{3(1-\lambda^2)(1+(\lambda+3)(\lambda+1))} < 0 \\
\tau_F^d - \tau_F^r = -\frac{2(1) \lambda(2\lambda+4) \rho_0(2\lambda+1)}{3(1-\lambda^2)(1+(\lambda+3)(\lambda+1))} < 0 \\
R_H^d - R_H^r = -\frac{t(1-\lambda)(1725\lambda+1772\lambda^2+241\lambda^3+434)-29(\lambda+1)(1513\lambda+2010\lambda^2+331\lambda^3+286)}{90(2\lambda+1)(\lambda+1)(7\lambda+\lambda^2+2)} > 0, \text{ if } \theta < \frac{1}{7}t \\
R_F^d - R_F^r = -\frac{t(1-\lambda)(1905\lambda+1976\lambda^2+313\lambda^3+482)-29(\lambda+1)(1189\lambda+1470\lambda^2+163\lambda^3+238)}{90(2\lambda+1)(\lambda+1)(7\lambda+\lambda^2+2)} > 0, \text{ if } \theta < \frac{1}{7}t \\
\bar{\theta} = \frac{(1-\lambda)(1725\lambda+1772\lambda^2+241\lambda^3+434)}{2(\lambda+1)(1513\lambda+2010\lambda^2+331\lambda^3+286)} \\
\bar{\rho}_F^d - \bar{\rho}_F^r = \frac{(1-\lambda)(1905\lambda+1976\lambda^2+313\lambda^3+482)-29(\lambda+1)(1189\lambda+1470\lambda^2+163\lambda^3+238)}{90(2\lambda+1)(\lambda+1)(7\lambda+\lambda^2+2)} > 0, \text{ if } \theta < \frac{1}{7}t
\[ p_{0}^{\theta} - p_{0}^{\theta} = \frac{t(1-\lambda)(2985\lambda+3212\lambda^2+481\lambda^3+734) - 26(\lambda+1)(2893\lambda+3930\lambda^2+571\lambda^3+526)}{180(2\lambda+1)(\lambda+1)(7\lambda+\lambda^2+2)} > 0, \text{ if } \theta < \]

\[ \tilde{\theta} = \frac{t(1-\lambda)(2985\lambda+3212\lambda^2+481\lambda^3+734)}{2(\lambda+1)(2893\lambda+3930\lambda^2+571\lambda^3+526)} > 0 \]

\[ q_{H}^{\theta} - q_{F}^{\theta} = \frac{t(1-\lambda)^3(\lambda+4)(83\lambda+5\lambda^2+34) - 26(3\lambda+2)(-5\lambda^3+102\lambda^2+13\lambda+22)}{180(2\lambda+1)(7\lambda+\lambda^2+2)} > 0 \]

\[ q_{F}^{\theta} - q_{F}^{\theta} = \frac{t(3\lambda+4)\lambda(1-\lambda) - 26(-3\lambda+25\lambda+14)}{360(2\lambda+1)} > 0, \text{ if } \theta < \tilde{\theta} = \frac{t(3\lambda+4)(1-\lambda)}{2(-3\lambda+25\lambda+14)} \]

\[ q_{0}^{\theta} - q_{0}^{\theta} = \frac{t(1-\lambda)^3(\lambda+4)(59\lambda+5\lambda^2+22) - 26(-15\lambda^4+148\lambda^3+375\lambda^2+461\lambda+92)}{90(2\lambda+1)(7\lambda+\lambda^2+2)} < 0, \text{ if } \theta < \tilde{\theta} = \]

\[ \frac{t(1-\lambda)(3\lambda+4)(59\lambda+5\lambda^2+22)}{2(-15\lambda^4-148\lambda^3+375\lambda^2+461\lambda+92)} \]

\[ \tau_{H}^{\theta} - \tau_{H}^{\theta} = \frac{t(1-\lambda)(10\lambda+17\lambda^2+46) - 26(\lambda+1)(169\lambda+27\lambda^2+44)}{15(\lambda+1)(7\lambda+\lambda^2+2)} > 0 \]

\[ \tau_{F}^{\theta} - \tau_{F}^{\theta} = \frac{t(1-\lambda)(10\lambda+19\lambda^2+62) - 26(\lambda+1)(83\lambda+9\lambda^2+28)}{15(\lambda+1)(7\lambda+\lambda^2+2)} > 0, \text{ if } \theta < \tilde{\theta} = \frac{t(1-\lambda)(10\lambda+19\lambda^2+62)}{2(\lambda+1)(83\lambda+9\lambda^2+28)} \]

\[ R_{H}^{\lambda} - R_{H}^{\lambda} = \frac{t^2(1-\lambda)(-15\lambda^4+458\lambda^3+3781\lambda^2+300\lambda+652)}{540(2\lambda+1)(\lambda+1)(7\lambda+\lambda^2+2)} \]

\[ -49(\lambda+1)(176\lambda-6169+1289\lambda-33889\lambda+20851\lambda^2+446\lambda^3+15\lambda^4-465\lambda^5-676\lambda^3-15\lambda^4) > 0 \]

\[ R_{F}^{\lambda} - R_{F}^{\lambda} = \frac{t^2(1-\lambda)(-15\lambda^4+602\lambda^3+3709\lambda^2+4070\lambda+1228)}{540(2\lambda+1)(\lambda+1)(7\lambda+\lambda^2+2)} \]

\[ -49(\lambda+1)(392\lambda+392\lambda+1886\lambda+2156\lambda^2+2445\lambda^3+302\lambda^4+15\lambda^4+2895\lambda^5+332\lambda^3-15\lambda^4) > 0, \text{ if } \theta < \]

\[ \tilde{\theta} = \frac{t^2(1-\lambda)(-15\lambda^4+602\lambda^3+3709\lambda^2+4070\lambda+1228)(392\lambda+392\lambda+1886\lambda+2156\lambda^2+2445\lambda^3+302\lambda^4+15\lambda^4+2895\lambda^5+332\lambda^3-15\lambda^4)}{4(\lambda+1)(392\lambda+392\lambda+1886\lambda+2156\lambda^2+2445\lambda^3+302\lambda^4+15\lambda^4+2895\lambda^5+332\lambda^3-15\lambda^4)} \]

Benevolent Governments

\[ \tau_{H}^{W} - \tau_{H}^{\theta} = \frac{-4(473\lambda+12169)}{1515} < 0 \]

Asymmetric Countries

\[ \tau_{H}^{d, HF} = \frac{t(t+2\theta)^2}{36t} \]

\[ \tau_{F}^{d, HF} = \tau_{d, HF} = \frac{(t+2\theta)^2}{36(t-\theta)(t-4)} \]

\[ p_{H}^{d, HF} - p_{F}^{d, HF} = \frac{18t}{18v-(35t+60+40\mu)} > 0 \]

\[ q_{H}^{d, HF} - q_{F}^{d, HF} = \frac{18t}{(2t+4\mu-36\theta)} < 0 \]

\[ \tau_{H} = \tau_{F} = \frac{3t}{5} > 0 \]

\[ R_{H}^{d, HF} - R_{F}^{d, HF} = \frac{16t-41\mu+27v\mu-186\mu-65\mu^2}{54} > 0 \]

\[ R_{F}^{d, HF} - R_{F}^{d, HF} = \frac{-16t(t-40\mu+34t+27v-186)}{54} > 0 \]
\[ \rho_{\text{H}}^{d,F} = \frac{(9t+8\theta+19t_{\mu})(45t-886+95t_{\mu})}{255920t} \]

\[ \pi_{\text{H}}^{d,F} = \frac{(27t+136\theta+17t_{\mu})}{643040t} \]

\[ \pi_{\text{H}}^0 = \frac{(27t-44\theta+17t_{\mu})(27t+136\theta+17t_{\mu})}{643040t} \]

\[ p_{\text{H}}^{d,F} - p_{\text{H}}^* = \frac{(487t-89+477t_{\mu})}{360} < 0 \]

\[ p_{\text{F}}^{d,F} - p_{\text{F}}^* = \frac{(449-499+419t_{\mu})}{360} < 0 \]

\[ q_{\text{H}}^{d,F} - q_{\text{H}}^* = \frac{(71t-80+1\mu)}{72t} < 0 \]

\[ q_{\text{F}}^{d,F} - q_{\text{F}}^* = \frac{(73t-136\theta+63t_{\mu})}{360} < 0 \]

\[ r_{\text{H}}^{d,F} - r_{\text{H}}^* = \frac{(31t-80+31t_{\mu})}{30} < 0 \]

\[ r_{\text{F}}^{d,F} - r_{\text{F}}^* = \frac{(23t+89+23t_{\mu})}{30} < 0 \]

\[ R_{\text{H}}^{d,F} - R_{\text{H}}^* = - \frac{(2111t^2\mu^2+362t^2\mu-1369t^2+272\theta+432\theta-64\theta^2)}{2160t} < 0 \]

\[ R_{\text{F}}^{d,F} - R_{\text{F}}^* = - \frac{(351t^2\mu^2+682t^2\mu+271t^2+272\theta+432\theta-64\theta^2)}{2160t} < 0 \]

\[ \pi_{\text{H}}^{o,H} = \frac{(5t+16\theta+6t_{\mu}+189t_{\mu})^2}{2916t^2\mu^2+3888\mu+1209t_{\mu}} \]

\[ \pi_{\text{F}}^{o,H} = \frac{(5t-28+6t_{\mu}+99t_{\mu})(5t-28+6t_{\mu}+99t_{\mu})}{2916t^2\mu^2+3888\mu+1209t_{\mu}} \]

\[ \pi_{\text{H}}^0 = \frac{(27t-28+6t_{\mu}+99t_{\mu})(5t-28+6t_{\mu}+99t_{\mu})}{2916t^2\mu^2+3888\mu+1209t_{\mu}} \]

\[ p_{\text{H}}^{o,H} - p_{\text{H}}^* = \frac{29t-49+104t_{\mu}-69\mu^2+84t_{\mu}^2}{18(5t^2+2)} < 0 \]

\[ p_{\text{F}}^{o,H} - p_{\text{F}}^* = \frac{(48t-169+124t_{\mu}-189t_{\mu}^2+69t_{\mu}^2)}{34(t^2+3t_{\mu}+36)} < 0 \]

\[ q_{\text{H}}^{o,H} - q_{\text{H}}^* = \frac{(9t+8)(t+5)t_{\mu}^2-20t_{\mu}^2}{18t(3t_{\mu}+2)} < 0 \]

\[ q_{\text{F}}^{o,H} - q_{\text{F}}^* = \frac{(40+t_{\mu}+229t_{\mu}^2)}{18t(3t_{\mu}+2)} < 0 \]

\[ r_{\text{H}}^{o,H} - r_{\text{H}}^* = \frac{(3t_{\mu}+29+11t_{\mu}+29t_{\mu}^2+9t_{\mu}^2)}{9t_{\mu}+6} < 0 \]

\[ r_{\text{F}}^{o,H} - r_{\text{F}}^* = \frac{(6t_{\mu}+29+16t_{\mu}-29t_{\mu}^2+9t_{\mu}^2)}{9t_{\mu}+6} < 0 \]

\[ R_{\text{H}}^{o,H} - R_{\text{H}}^* = \frac{t^4(3t_{\mu}+2)^2(27t^3+670t^2+124t^2\mu^2+765t^3+495t^4-104)(5t^2+48t+19t_{\mu}+27t_{\mu}^2)}{153(3t_{\mu}+2)^2} < 0 \]

if \( \theta < \hat{\theta} = \frac{(t+22t_{\mu}+148t_{\mu}^2+27t_{\mu}^3-\sqrt{1514t^2+2735t^2+480t^2+4383t^2+1863t^2+229t_{\mu}^2+209)}{22t_{\mu}+18t_{\mu}^2+4} \)

\[ R_{\text{F}}^{o,H} - R_{\text{F}}^* = \frac{t^2(302t^2+537t^2\mu^2+402t^2\mu^2+104t^2\mu^4+60t_{\mu}+1)^2(5t^2-2t+40t_{\mu}+139t_{\mu}+186t_{\mu}+27t_{\mu}^2)}{54t(3t_{\mu}+2)^2} < 0 \]

\[ \Delta_{\text{H}}^{o,H} = r_{\text{F}}^{o,H} - r_{\text{H}}^* = \frac{(\mu+1)(t-40+3t_{\mu})}{54(3t_{\mu}+2)^2} < 0 \text{ if } \theta < \hat{\theta} = \frac{t(3t_{\mu}+1)}{4} \]

\[ \pi_{\text{H}}^{o,F} = \frac{(15t+86+13t_{\mu})^2}{5184t} \]

\[ \pi_{\text{F}}^{o,F} = \frac{(39t-449+5t_{\mu})^2}{16200t} \]

\[ p_{\text{H}}^{o,F} - p_{\text{H}}^* = \frac{(439t-406+429t_{\mu})}{360} < 0 \]

\[ p_{\text{F}}^{o,F} - p_{\text{F}}^* = \frac{(545t-1366+419t_{\mu})}{360} < 0 \]

\[ q_{\text{H}}^{o,F} - q_{\text{H}}^* = \frac{(131t-136\theta+51t_{\mu})}{360} < 0 \]

\[ q_{\text{F}}^{o,F} - q_{\text{F}}^* = \frac{(5t-89+3t_{\mu})}{72} < 0 \]

\[ r_{\text{H}}^{o,F} - r_{\text{H}}^* = \frac{(194t+86+27t_{\mu})}{30} < 0 \]
\[
\tau^{o,F}_F - \tau^{H}_F = -\frac{(35t+8\theta+27\mu)}{3t} < 0 \\
R^{o,F}_H - R^{H}_H = \frac{t^2 \left( -207t\mu^2 - 3144\mu + 108 \right) - 169(21t-4\theta+23\mu)}{2160} > 0, \text{ if } \theta < \tilde{\theta} = t \frac{23\mu - 2\sqrt{15}\sqrt{32\mu + 65\mu^2 - 16 + 21}}{8} \\
R^{o,F}_F - R^{*}_F = -\frac{t^2 \left( 1210\mu + 471\mu^2 + 775 \right) - 169(15t+4\theta+13\mu)}{16} < 0 \\
\Delta \tau^{o,HF} = \tau^{o,HF}_F - \tau^{o,HF}_H = -\frac{2160}{15} < 0, \text{ if } \theta < \tilde{\theta} = t \frac{5\mu+3}{8} \\
\]

**Discussion**

\[
\tau^{d,comp}_H - \tau^{*,comp}_H = \tau^{d,comp}_F - \tau^{*,comp}_F = \frac{1}{3} (3v - 4t - 2\theta) > 0 \\
\tau^{o,comp,HF0}_H - \tau^{*,comp}_H = -\frac{1}{3} (t + \theta) < 0 \\
\tau^{o,comp,HF0}_F - \tau^{*,comp}_F = -\frac{1}{3} (2t - \theta) < 0 \\
\]