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Straight-time and Overtime: A Sequential Lottery Approach

Technical Appendix

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Sequential Lotteries and Insurance markets in a General Equilibrium Model with Straight-time and Over-time employment

A competitive Equilibrium with sequential Lotteries for this economy is a list $\{c^u(i), c^{e,f}(i), c^{e,o}(i), q(i), \lambda(i), \bar{h}, h^o, w, w^o, \pi\}$ s.t.

(1) Consumers maximize - taking prices w, w^o, π as given, for each i , $c^u(i), c^{e,f}(i), c^{e,o}(i), q(i), \lambda(i), \bar{h}, h^o$ solve¹

$$\max_{\{c^u(i), c^{e,f}(i), c^{e,o}(i), q(i), \lambda(i)\}} q(i) \left\{ \lambda(i) \left[\ln(c^{e,f}) + \alpha \ln(1 - \bar{h}) \right] + [1 - \lambda(i)] \left[\ln(c^{e,o}) + \alpha \ln(1 - \bar{h} - h^o) \right] \right\} + [1 - q(i)] \ln(c^u) \quad (1)$$

s.t

$$q(i) \left[\lambda(i) c^{e,f} + (1 - \lambda(i)) c^{e,o} \right] + (1 - q(i)) c^u = q w \bar{h} + q [1 - \lambda(i)] w^o h^o + \pi, \quad (2)$$

$$c^{e,f}, c^{e,o}, c^u \geq 0, 0 < q(i), \lambda(i) < 1 \quad (3)$$

(2) Firm max - taking prices w, w^o, π as given,

$$\max_{\bar{H}, H^o} F(\bar{H}, H^o) - w \bar{H} - w^o H^o \quad \text{s.t.} \quad \bar{H} \geq 0, H^o \geq 0 \quad (4)$$

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¹Note that now when households trade lotteries the outcome is no longer deterministic. Consumers now maximize expected utility, i.e. if the unemployed type is not selected to work with probability $1 - q$, that individual will get expected income $(1 - \lambda)w\bar{h}$.

(3) Market clearing

$$\int_i q(i) \bar{h} di = \bar{H} \quad (5)$$

$$\int_i q(i) (1 - \lambda(i)) h^o di = H^o \quad (6)$$

$$\int_i \left\{ q(i) \left[\lambda(i) c^{e,f} + [1 - \lambda(i)] c^{e,o} \right] + (1 - q(i)) c^u \right\} di = F(\bar{K}, H^o) \quad (7)$$

Characterizing the CE:

Household Problem:

$$\begin{aligned} \mathcal{L} = & \max_{\{c^u(i), c^{e,f}(i), c^{e,o}(i), q(i), \lambda(i)\}} q(i) \left\{ \lambda(i) \left[\ln(c^{e,f}) + \alpha \ln(1 - \bar{h}) \right] + [1 - \lambda(i)] \left[\ln(c^{e,o}) + \alpha \ln(1 - \bar{h} - h^o) \right] \right\} \\ & + [1 - q(i)] \ln(c^u) - \mu \left[q(i) \left[\lambda(i) c^{e,f} + [1 - \lambda(i)] c^{e,o} \right] + (1 - q(i)) c^u - q(i) w \bar{h} - q(i) [1 - \lambda(i)] w^o h^o - \pi \right], \quad (8) \end{aligned}$$

where μ is the Lagrangian multiplier in front of the budget constraint.

FOCs:

$$c^u : \frac{1 - q(i)}{c^u} = \mu [1 - q(i)] \quad (9)$$

$$c^{e,f} : \frac{q(i) \lambda(i)}{c^{e,f}} = \mu q(i) \lambda(i) \quad (10)$$

$$c^{e,o} : \frac{q(i) [1 - \lambda(i)]}{c^{e,o}} = \mu q(i) [1 - \lambda(i)] \quad (11)$$

It follows that $c^u = c^{e,f} = c^{e,o} = 1/\mu$. We simplify the Lagrangian by suppressing all consumption superscripts and i notation in the derivations to follow

$$q(i) : \lambda \alpha \ln(1 - \bar{h}) + (1 - \lambda) \alpha \ln(1 - \bar{h} - h^o) = -\mu [w \bar{h} + (1 - \lambda) w^o h^o] \quad (12)$$

This equation implicitly defines optimal q .

$$\lambda(i) : q \alpha [\ln(1 - \bar{h}) - \ln(1 - \bar{h} - h^o)] = \mu q w^o h^o \quad (13)$$

or

$$\frac{[\ln(1 - \bar{h}) - \ln(1 - \bar{h} - h^o)]}{c h^o} = w^o \quad (14)$$

This equation is a discrete version of the marginal product of labor equals the marginal rate of substitution. It implicitly characterizes optimal λ .

Note that it is optimal from the benevolent planner/government point of view to choose randomly q and λ and to introduce uncertainty. With randomization, choice sets are convexified, and thus market completeness

is achieved. A household is exposed to risk: first, it can be chosen to work with some probability; second, conditional on being chosen to work, it can be picked to provide overtime labor services. Given the risk in the economic environment, it would be optimal to have insurance. The government sells employment lotteries, and individuals will buy insurance to cover any risk exposure. With insurance, the employer pays wage to individuals only if they work. Now we extend the commodity space a little bit to include insurance markets explicitly.

A competitive Equilibrium with sequential Lotteries and insurance markets for this economy is a list $\{c^u(i), c^{e,f}(i), c^{e,o}(i), q(i), \lambda(i), \bar{h}, h^o, b^q, b^\lambda, w, w^o, p^q, p^\lambda, \pi\}$ s.t.

(1) Consumers maximize - taking prices w, w^o, π as given, for each i , $c^u(i), c^{e,f}(i), c^{e,o}(i), q(i), \lambda(i), \bar{h}, h^o$ solve

$$\max_{\{c^u(i), c^{e,f}(i), c^{e,o}(i), q(i), \lambda(i)\}} q(i) \left\{ \lambda(i) \left[\ln(c^{e,f}) + \alpha \ln(1 - \bar{h}) \right] + [1 - \lambda(i)] \left[\ln(c^{e,o}) + \alpha \ln(1 - \bar{h} - h^o) \right] \right\} + [1 - q(i)] \ln(c^u) \quad (15)$$

s.t

$$c^{e,f} + b^q p^q = b^\lambda + w \bar{h} + \pi \quad (16)$$

$$c^{e,o} + b^q p^q + b^\lambda p^\lambda = w \bar{h} + w^o h^o + \pi \quad (17)$$

$$c^u = b^q + \pi \quad (18)$$

$$c^{e,f}, c^{e,o}, c^u \geq 0, 0 < q(i), \lambda(i) < 1 \quad (19)$$

The interpretation of the constraints is as follows: In the first stage, workers buy unemployment insurance, while unemployed households will receive the payout (unemployment benefits, denoted by b^q). Then, conditional on being employed, overtime workers will buy non-overtime insurance (in case they are not chosen to work overtime), while full-time workers will receive the payout b^λ . Thus, overtime workers need to buy two types of insurance.

Also, in equilibrium, it must be that $b^q = q w \bar{h}$, and $b^\lambda = (1 - \lambda) w^o h^o$.

Before optimizing, let's first simplify the constraint set. First, substitute out b^q from the budget constraint in the state the household is unemployed. We obtain

$$b^q = c^u - \pi. \quad (20)$$

Next, plug the obtained expression in the budget constraint in the state when the household is employed full-time. We obtain

$$c^{e,f} + p^q (c^u - \pi) = b^\lambda + w \bar{h} + \pi. \quad (21)$$

Now substitute out b^λ from the budget constraint in the state the household is employed full-time only. We obtain

$$b^\lambda = c^{e,f} + p^q (c^u - \pi) - w \bar{h} - \pi. \quad (22)$$

plug the obtained expression in the budget constraint in the state when the household is employed overtime. We obtain

$$c^{e,o} + p^q(c^u - \pi) + p^\lambda[c^{e,f} + p^q(c^u - \pi) - w\bar{h} - \pi] = w^o h^o + w\bar{h} + \pi, \quad (23)$$

or

$$c^{e,o} + p^q(1 + p^\lambda)c^u + p^\lambda c^{e,f} = w^o h^o + (1 + p^\lambda)w\bar{h} + (1 + p^q + p^\lambda p^q)\pi. \quad (24)$$

Thus the problem simplifies to

$$\begin{aligned} \max_{\{c^u(i), c^{e,f}(i), c^{e,o}(i), q(i), \lambda(i)\}} q(i) & \left\{ \lambda(i) \left[\ln(c^{e,f}) + \alpha \ln(1 - \bar{h}) \right] + [1 - \lambda(i)] \left[\ln(c^{e,o}) + \alpha \ln(1 - \bar{h} - h^o) \right] \right\} \\ & + [1 - q(i)] \ln(c^u) \end{aligned} \quad (25)$$

s.t.

$$c^{e,o} + p^q(1 + p^\lambda)c^u + p^\lambda c^{e,f} = w^o h^o + (1 + p^\lambda)w\bar{h} + (1 + p^q + p^\lambda p^q)\pi. \quad (26)$$

FOCs:

$$c^{e,f} : \frac{q\lambda}{c^{e,f}} = p^\lambda \mu \quad (27)$$

$$c^{e,o} : \frac{q(1 - \lambda)}{c^{e,o}} = \mu \quad (28)$$

$$c^u : \frac{1 - \lambda}{c^u} = \mu p^q(1 + p^\lambda) \quad (29)$$

Since we already established that consumption will be equalized across all states, we can obtain (divide FOCs for full-time and unemployed)

$$p^\lambda = \frac{\lambda}{1 - \lambda} \quad (30)$$

price equals the odds ratio of being chosen to work.

$$q : \lambda[\ln(c) + \alpha \ln(1 - \bar{h})] + (1 - \lambda)[\ln(c) + \alpha \ln(1 - \bar{h} - h^o)] - \ln(c) = 0 \quad (31)$$

Hence,

$$\lambda = \frac{\ln(1 - \bar{h} - h^o)}{\ln(1 - \bar{h}) - \ln(1 - \bar{h} - h^o)} \in (0, 1) \quad (32)$$

With the obtained value for λ we can solve for p^λ . Divide FOCs for overtime employed and unemployed to obtain

$$\frac{q(1 - \lambda)}{1 - q} = \frac{1}{p^q(1 + p^\lambda)} \quad (33)$$

since

$$1 + p^\lambda = \frac{1}{1 - \lambda} \quad (34)$$

it follows

$$\frac{q(1-\lambda)}{1-q} = \frac{1-\lambda}{p^q} \quad (35)$$

or

$$\frac{q}{1-q} = \frac{1}{p^q}, \quad (36)$$

which (the odds ratio being chosen to work) characterizes optimal q .

(2) Firm max - taking prices w, w^o, π as given,

$$\max_{\bar{H}, H^o} F(\bar{K}, H^o) - w\bar{H} - w^o H^o \quad \text{s.t.} \quad \bar{H} \geq 0, H^o \geq 0 \quad (37)$$

(3) Insurance market: Insurance company maximizes profit.

The insurance is sequential: In stage 1, by taking $p^q(i)$ as given, $b^q(i)$ solves

$$\max_{b^q} qp^q(i)b^q - (1-q)b^q \quad (38)$$

i.e. the revenue if individual is working minus payment is s/he is not, or the proportion of people working and contributing towards the unemployment benefits pool and the distribution of benefits to the unemployed.

In the second stage, a separate insurance scheme is run among those that are selected for employment.

Taking $p^\lambda(i)$ as given, $b^\lambda(i)$ solves

$$\max_{b^\lambda} q[(1-\lambda)p^\lambda(i)b^\lambda - \lambda b^\lambda] \quad (39)$$

i.e. the revenue if, conditional on being employed, an individual is working overtime $(1-\lambda)$ minus payment is s/he is not, or the proportion of people working overtime and contributing towards the benefits pool for those who are not selected for extra hours and the distribution of benefits to the full-time employees. In a way, the extra gains (compensate) from the overtime wage premium are shared with the workers who do not do overtime.

This implicitly clears the insurance market for each individual.

In equilibrium, the price of insurance depends on the probability of the event the household is insuring against. We cannot enforce $p^q(i) = p^q$ and $p^\lambda(i) = p^\lambda$ although ex post (in equilibrium) that would indeed be the case. For the insurance firm, the profits are linear in p^q and p^λ . This implies that profits cannot be positive or negative in equilibrium, but have to be zero.

Zero profits means that $p^\lambda = \frac{\lambda}{1-\lambda}$, and $p^q = \frac{1-q}{q}$. A common interpretation is that for insurance companies the price of the insurance is the odds ratio, or the ratio of probabilities of the two events. q is the

same for all households, and λ is the same for all employed households.

(4) Market clearing

$$\int_i q(i) \bar{h} di = \bar{H} \quad (40)$$

$$\int_i q(i) (1 - \lambda(i)) h^o di = H^o \quad (41)$$

$$\int_i \left\{ q(i) \left[\lambda(i) c^{e,f} + [1 - \lambda(i)] c^{e,o} \right] + (1 - q(i)) c^u \right\} di = F(\bar{K}, H^o) \quad (42)$$

Whether consumers want equal consumption depends on the price of insurance. Buy full insurance to smooth consumption perfectly.

Again, the consumers will buy full insurance to equalize consumption in all states (employed, unemployed), (full-time employed—employed), (overtime work—employed). In particular, when income is stochastic, i.e., it is uncertain whether the individual will be employed, we need the insurance market to offer conditional insurance. This is because of the sequential non-convexity of the labor choice set, which is similar to having incomplete markets. Lotteries then can be introduced to achieve market completeness. Therefore, randomization may be optimal in a non-convex environment even though there is no aggregate uncertainty.

Note that with those employment lotteries and insurance, every household enjoys the same level of consumption. The introduction of lotteries in the model achieves perfect consumption smoothing, by breaking the link between the labor choice and the affordable consumption. In equilibrium, not everyone will work, and not everyone will work overtime, but everyone will consume the same. This consumption equalization though the sale and purchase of employment lotteries.