

Biller, Dan; Andres, Luis; Cuberes, David

Conference Paper

A Dynamic Spatial Model of Rural-Urban Transformation with Public Goods

55th Congress of the European Regional Science Association: "World Renaissance: Changing roles for people and places", 25-28 August 2015, Lisbon, Portugal

Provided in Cooperation with:

European Regional Science Association (ERSA)

Suggested Citation: Biller, Dan; Andres, Luis; Cuberes, David (2015) : A Dynamic Spatial Model of Rural-Urban Transformation with Public Goods, 55th Congress of the European Regional Science Association: "World Renaissance: Changing roles for people and places", 25-28 August 2015, Lisbon, Portugal, European Regional Science Association (ERSA), Louvain-la-Neuve

This Version is available at:

<https://hdl.handle.net/10419/124823>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

A Dynamic Spatial Model of Rural-Urban Transformation with Public Goods

Dan Biller, Luis Andres, David Cuberes*

June 30, 2014

Abstract

In this paper we develop a dynamic model that explains the pattern of population and production allocation in an economy with an urban location and a rural one. Agglomeration economies make urban dwellers benefit from a larger population living in the city and urban firms become more productive when they operate in locations with a larger labor force. However, congestion costs associated with a too large population size limit the process of urban-rural transformation. Firms in the urban location also benefit from a public good that enhances their productivity. The model predicts that in the competitive equilibrium the urban location is inefficiently small because households fail to internalize the agglomeration economies and the positive effect of public goods in urban production.

1 Introduction

Urbanization - defined as the percentage of a country's population that lives in urban areas- among developing countries exhibits significant variation across different world regions and countries. The Middle East and North Africa region is the most urbanized region, with around 70% of its population living in cities. By contrast, South Asia has a surprisingly low urbanization, around 28%. Differences in income per capita cannot account for this variation. Figure 1 shows the level of urbanization and annual GDP per capita in PPP terms in selected developing countries around the world in 1960.¹ South Asia and East Asia are

*Dan Biller and Luis Andres are respectively Sector Manager and Lead Economist at the World Bank Group. David Cuberes is a Lecturer at the University of Sheffield, UK. The findings, interpretations, and conclusions expressed in the reports and outputs under this initiative are those of the authors and do not necessarily reflect the views of the Executive Directors of The World Bank, the governments they represent, or the counterparts that were consulted or engaged with during the informal study process. Any factual errors are the responsibility of the team. Contact author: d.cuberes@sheffield.ac.uk

¹This figure is similar to the one presented in Henderson (2009) which shows the variation in urbanization rates at different levels of income for different countries. He emphasizes that China's urbanization rate is also too low at its stage of development.

clustered together, while Latin American economies are slightly detached given its higher GDP per capita PPP and larger urbanization levels. In this figure the size of the bubbles represent countries with their population size – the larger the bubble the larger the population. Figure 2 shows that in half a century East Asia followed the Latin American path with some countries like South Korea and Malaysia even surpassing Latin American economies both on urbanization levels and on GDP per capita PPP, but South Asia lagged well behind. The eight countries of South Asia (Afghanistan, Bangladesh, Bhutan, India, The Maldives, Nepal, Pakistan, and Sri Lanka) display very low urbanization given their income levels.²

FIGURES 1,2 HERE

This paper develops a model to understand the dynamic process of rural-urban transformation in a developing country where urbanization is far from being completed. We first solve for the competitive equilibrium of the model and then we find the optimal solution of a benevolent social planner. In both cases we focus on the analysis of the steady-state of the model. The study of both positive and normative aspects of urbanization make this paper of interest from an academic point of view but also for policymakers wishing to address specific problems related to the process of urbanization.³

Probably the best known model of urbanization is the so-called Harris-Todaro model (Harris and Todaro, 1970). In their paper they develop a simple static theory in which there are two sectors in the economy, a modern one and an agricultural one, with declining marginal productivity in both of them. This implies that the higher the wage is, the lower is the demand for workers in both sectors. The Harris-Todaro model has clear predictions on labor migration, but it ignores technological differences between the rural and urban areas as well as the dynamics of the urbanization process. Chan and Yu (2010), Neary (1981, 1988) and Riadh (1998) have added capital goods to this model and solved for its dynamics, although their emphasis is again on labor migration between two technologically identical regions.

Another strand of the literature has sprung as a result of the interest in the field of urban economics in understanding the process of city formation. There exist several papers that study urban processes in the presence of capital goods (Anas, 1978, 1992; Kanemoto, 1980; Henderson and Ioannides, 1981;

²These figures are constructed grouping countries by region according to the World Bank classification. See http://www.worldbank.org/depweb/beyond/beyondco/beg_ce.pdf

³Some papers have studied the effect of different policies on the process of urbanization. Henderson and Kuncoro (1996), for example, show the effects of favoritism for certain regions in Indonesia. Other cases are discussed in Henderson (1988), Lee and Choe (1990), and Jefferson and Singhe (1999).

Miyao, 1981; Fujita, 1982; Ioannides, 1994; Palivos and Wang, 1996). One important limitation of these models is that they assume free mobility of all factors of production. A direct consequence of this assumption is that these models predict large and rapid swings in the population of cities that reach a critical level and that, when new cities form, their population jumps instantly to some arbitrarily large size. This leads to counterfactual predictions - the existing data on cities' population exhibit smooth fluctuations as countries urbanize. By contrast, in our model the population of rural and urban areas changes smoothly over time because of the assumption that investment in capital goods is irreversible. To our knowledge, only the papers by Henderson and Venables (2009) and Cuberes (2009) explicitly assume irreversibility in capital investment, hence solving the problem of sudden changes in cities' population. The former present a model in which cities form in sequential order as a consequence of the presence of increasing returns in production and congestion costs.

The model by Cuberes (2009) is closely related to the one analyzed here but our framework has several important differences. First, Cuberes (2009) analyzes city formation only and does not consider rural areas. Second, agglomeration benefits only firms, not consumers directly. Third, there are no public goods in the analysis. While it is also the case that in his model the competitive equilibrium is inefficient, the present model is richer since the inefficiency comes both from preferences and production and because it discusses normative implications of the model. The main goal of Cuberes (2009) is to rationalize a pattern of sequential growth between existing cities, whereas the model in the present paper introduces sufficient structure to explain the process of migration from a rural location to an urban one.

The new economic geography models presented in Fujita et al (2001) are also related to our analysis. In their benchmark core-periphery model there are sectors, a manufacturing one, where production takes place under monopolistic competition, and an agricultural one, where goods are produced under perfect competition. Their model also includes transporting goods across the regions in the form of iceberg costs so that a fraction proportional to the distance between the locations is lost in the trading process. In these models, urbanization is the result of technological progress or productivity differentials across regions. Our model significantly simplifies the analysis by assuming a single homogeneous good - as opposed to an agricultural good and a manufactured one - and no transport costs. These two simplifications allow us to introduce a capital good in the model which, in turn, generates more realistic dynamics than the ones in standard economic geography models. A second advantage of our model is that we explicitly model the fact that private agents do not internalize the benefits associated with agglomeration economies.

In terms of policy implications, the literature is more scarce. As stated in Fujita et al. (2001) this is probably due to the fact that there is a need for better empirical studies to pin down the exact external economies and diseconomies

associated with urban agglomerations. Along these lines, Au and Henderson (2006) estimate a theoretical model of city formation using Chinese data and conclude that most cities in China are undersized. However, their model does not study the process of urbanization per se and they do not analyze market failures and the related policy implications. The main aim of their paper is to empirically estimate the inverse U-shape pattern predicted by theirs and many other models in the literature (Henderson, 1974; Helsley and Strange, 1990; Black and Henderson, 1999; Fujita et al., 2001; Duranton and Puga, 2001) by using measures of urban economies but also urban diseconomies.⁴ The theoretical model of Henderson and Wang (2005) studies the process of rural-urban spatial transformation as a country urbanizes. The model is based on a system of cities and it focuses on explaining how the number of cities and their size evolves in a context of positive population growth. Our model does not have population growth and consequently it does not consider city creation. However, these simplifications allow us to compare the decentralized and efficient equilibrium in more detail. Finally, Henderson and Wang (2007) analyze, from an empirical point of view, how urbanization is accommodated by increases in numbers and sizes of cities.⁵

The rest of the paper is organized as follows. Section 2 presents the decentralized model and solves its steady-state equilibrium. Section 3 does the same for the problem of a benevolent social planner. Section 4 presets a numerical example that allows us to compare the predictions of the two problems. Finally, Section 4 concludes the paper.

2 A Spatial Equilibrium Model of Rural-Urban Transformation

2.1 Setup

Our model is based on Cuberes (2009). The economy is closed and populated by a large number N of agents who work and live in one of two possible locations: an urban one (U) and a rural one (R).

2.2 The Decentralized Problem

2.2.1 Households

The maximization problem of a representative agent is contingent to the location where she lives.⁶

⁴Rosenthal and Strange (2004) and Moretti (2004) offer reviews of these papers.

⁵Richardson (1987) presents a cost-benefit analysis of the urbanization process in four different countries and proposes some policies to address different policy issues.

⁶For simplicity, the model assumes that households live and work in the same location.

Location U There are N_t^U agents living in location U at period t . Utility in location U is given by $V(c_t^U, \Phi(N_t^U))$, where $c_t^U \equiv \frac{C_t}{N_t^U}$ denotes consumption of the private good in per capita terms in location U . The function $\Phi()$ is increasing and it reflects the existence of an originated from network effects, knowledge spillovers, information sharing, companionship, safety, among others. The function V is increasing and concave in the two inputs.

An agent in location U has two sources of income: her wage earnings and the returns to her investment in the only asset in the economy, namely physical capital in location U . Let's denote \dot{z}_t the investment in asset z . The agent U 's budget constraint is then:

$$\dot{z}_t = w_t^U + r_t z_t - c_t^U$$

where w_t^U is the wage rate in U and r_t is the return on the private asset z_t .⁷ The intertemporal problem of a representative agent is given by:

$$\begin{aligned} \max_{c_t^U, z_t} \int_0^\infty e^{-\rho t} V(c_t^U, \Phi(N_t^U)) dt \\ \dot{z}_t = w_t^U + r_t z_t - c_t^U \\ z_0 \text{ given} \end{aligned}$$

Note that agents located in U obtain direct utility from the total population of the city, N_t^U . However, because agents are atomistic, they do not have control over the cumulative location decisions represented by N_t^U . The Hamiltonian of this problem is

$$\Lambda = e^{-\rho t} V(c_t^U, \Phi(N_t^U)) + \varsigma_t (w_t^U + r_t z_t - c_t^U)$$

and the first-order conditions of the problem are:⁸

$$\frac{\partial \Lambda}{\partial c_t^U} = 0 \Leftrightarrow e^{-\rho t} V_c - \varsigma_t = 0 \quad (1)$$

$$\frac{\partial \Lambda}{\partial z_t} = -\dot{\varsigma}_t \Leftrightarrow \varsigma_t r_t = -\dot{\varsigma}_t \quad (2)$$

Taking logs in (1):

$$-\rho t + \ln V_c = \ln \varsigma_t$$

⁷We assume that the investment in the physical good must be positive. This assumption is clarified in the firm's problem. In the agent's optimization problem we further assume that this non-negativity constraint on investment is not binding, that is household invest a positive amount in every period.

⁸The transversality condition is $\lim_{t \rightarrow \infty} \varsigma_t z_t^U = 0$

and differentiating with respect to time we have

$$-\rho + \frac{\frac{\partial V_c}{\partial t}}{V_c} = \frac{\dot{\varsigma}_t}{\varsigma_t}$$

But note that

$$\frac{\partial V_c(c_t^U, \Phi(N_t^U))}{\partial t} = V_{cc}\dot{c}_t^U + V_{c\Phi}\Phi'\dot{N}_t^U$$

so

$$-\rho + \frac{V_{cc}\dot{c}_t^U + V_{c\Phi}\Phi'\dot{N}_t^U}{V_c} = \frac{\dot{\varsigma}_t}{\varsigma_t}$$

Rearranging we have the following expression for the growth rate of consumption in location U :

$$\frac{\dot{c}_t^U}{c_t^U} = \frac{V_c}{V_{cc}c_t^U} \left[\rho - r_t - \frac{V_{c\Phi}\Phi'}{V_c}\dot{N}_t^U \right] \quad (3)$$

Location R Utility in location R is given by $V(c_t^R, H_t)$, where $c_t^R \equiv \frac{C_t}{N_t^R}$ denote consumption of the private good in per capita terms in location R . H_t is a public good that comes from nature i.e. it is not under the control of the government from which agents derive utility (for example, open space, clean air, etc...). Agents in location R cannot save via the accumulation of physical capital so they consume all their income - which comes entirely from wages - every period. We believe this is a reasonable assumption, especially in countries that still have low levels of urbanization. Therefore

$$c_t^R = w_t^R$$

2.2.2 Firms

Location U A representative firm in location U uses a constant-returns-to-scale technology but is subject to an external effect from the total population - or labor force- in the city where it operates. So firm i in location U produces output according to:

$$y_t^{iU} = f(k_t^i)q \left(N_t^{iU}, \sum_{m \neq i}^M N_t^{mU}, G_t \right)$$

where k_t^i is the amount of capital used by firm i , N_t^{iU} is labor employed by firm i in location U , and $\sum_{m \neq i}^M N_t^{mU}$ is the total number of workers (excluding those employed by firm i) employed by the $M - 1$ firms that operate in location U .⁹ operating in U . The function $f()$ is increasing and concave. The function

⁹Note that because there are constant returns to scale from the point of view of the firm, the number of firms is indeterminate.

$q()$ is increasing in N_t^{iU} . Moreover, firms are subject to an external positive agglomeration effect such that a firm's productivity increases as the number of workers in the location where the firm operates increases. This may be due to labor pooling so that search costs are lower or an increase in the quality of workers if drawn from a larger pool.¹⁰ However, if the total number of workers in location U becomes too large, output decreases as a result of congestion costs. In particular, the function $q()$ satisfies the following properties $q_2 \begin{cases} >0 & \text{if } \Upsilon \leq \hat{\Upsilon} \\ <0 & \text{if } \Upsilon > \hat{\Upsilon} \end{cases}$

where $\Upsilon \equiv \sum_{m \neq i}^M N_t^{mU}$ and $\hat{\Upsilon}$ is a critical value above which congestion costs dominate the agglomeration effects. One rationale for such congestion costs is offered in Becker and Murphy (1992) where there exist coordination costs among workers.¹¹ Finally, we assume that the government can invest in infrastructure that reduces congestion via the public good G_t , i.e. $q_2 > 0$.¹²

Assuming that all firms are identical we have

$$y_t^U = f(k_t)q\left(N_t^U, (M-1)\tilde{N}_t^U, G_t\right)$$

where we use the notation \tilde{N}_t^U to indicate that firm i does not take this term into account when optimizing. Normalizing the price of the final good to one profits are:

$$\pi_t^{iU} = f(k_t^i)q\left(N_t^{iU}, (M-1)\tilde{N}_t^U, G_t\right) - (r_t + \delta)k_t^i - w_t^U N_t^{iU}$$

We assume that investment is irreversible i.e. it is not possible for firms to have negative investment. This is reasonable if one assumes that a significant fraction of firms' physical capital takes the form of infrastructure. This assumption is common in recent papers like Henderson and Venables (2009) and Cuberes (2009). In the absence of this assumption the model would predict discrete jumps in population between the two locations, which is clearly at odds with the data. The first-order conditions for this firm are:

$$\frac{\partial \pi_t^{iU}}{\partial k_t^i} = 0 \Leftrightarrow f'q = r_t + \delta \quad (4)$$

$$\frac{\partial \pi_t^{iU}}{\partial N_t^{iU}} = 0 \Leftrightarrow f q_1 = w_t^U \quad (5)$$

Since there are constant returns to scale from the point of view of the firm, profits are exhausted from input remuneration so

¹⁰Rosenthal and Strange (2003) provide empirical evidence supporting the view that productivity of firms depends positively on nearby employment.

¹¹Arnott (2007) argues that, in many cities, a high density of population results in congestion externalities.

¹²See Venables (2007) who underlines the relevance of indirect benefits of policies to reduce congestion.

$$w_t^U = fq - f'qk_t^i \quad (6)$$

Location R Production in R is simply given by the constant-returns to scale production function

$$Y_t^R = N_t^R$$

or, in per capita terms

$$y_t^R = 1$$

Workers in R receive the competitive wage

$$w_t^R = \frac{\partial Y_t^R}{\partial N_t^R} = 1$$

2.2.3 Equilibrium

Since the economy is closed, at any period, the assets held by agents in location U must be equal to the stock of capital in this location:

$$Z_t \equiv N_t^U z_t = K_t$$

and so in per-capita terms

$$z_t = k_t$$

The agent's flow budget constraint then determines the change in the per capita stock of capital in location U :

$$\dot{k}_t = \dot{z}_t$$

From the agent's problem we have

$$\dot{k}_t = w_t^U + r_t k_t - c_t^U \quad (7)$$

From the firm's first-order condition (4) in location U we have

$$r_t = f'q - \delta$$

So

$$\dot{k}_t = w_t^U + (f'q - \delta)k_t - c_t^U$$

Using (6) we then have the first crucial law of motion of the model.

$$\dot{k}_t = fq - \delta k_t - c_t^U \quad (8)$$

From the household's problem in location U (equation (3)) the growth rate of consumption in this location is

$$\frac{\dot{c}_t^U}{c_t^U} = \frac{V_c}{V_{cc}c_t^U} \left[\rho - f'q + \delta - \frac{V_{c\Phi}\Phi'}{V_c} \dot{N}_t^U \right] \quad (9)$$

which is the second key law of motion in this model.

2.3 Steady-State Spatial Equilibrium

In this section we analyze the steady-state equilibrium of this model. The two laws of motions of the model are given by (8) and (9). In steady-state $\dot{k}_t = \dot{c}_t^U = 0$. Moreover, spatial indifference between locations implies $\dot{N}_t^U = 0$. This is the case since agents must be indifferent between the two locations and hence rural-urban migration (or urban-rural) must be nonexistent.

$$\dot{k}_t = f'q - \delta k_t - c_t^U = 0 \quad (10)$$

$$\frac{\dot{c}_t^U}{c_t^U} = \frac{V_c}{V_{cc}c_t^U} [\rho - f'q + \delta] = 0 \quad (11)$$

Moreover, in a spatial equilibrium, agents must be indifferent between the two locations:

$$V(c_t^U, \Phi(N_t^U)) = V(c_t^R, H_t)$$

Finally, the labor market must clear, i.e.

$$N_t^U + N_t^R = N_t \quad (12)$$

The steady-state equilibrium is given by equations (13)-(16):

$$c^{U*} = f(k)q \left(N^{U*}, (M-1)\tilde{N}^{U*}, G^* \right) - \delta k^* \quad (13)$$

$$\rho + \delta = f'(k^*)q \left(N^{U*}, (M-1)\tilde{N}^{U*}, G^* \right) \quad (14)$$

$$V(c^{U*}, \Phi(N^{U*})) = V(c^{R*}, H^*) \quad (15)$$

$$N^{U*} + N^{R*} = N \quad (16)$$

3 The Planner's problem

In this section we consider a benevolent social planner who gives a weight λ to urban dwellers and $1 - \lambda$ to rural ones, where $\lambda \in (0, 1)$. This problem can be written as

$$\begin{aligned} & \max_{c_t^U, c_t^R, N_t^U, N_t^R, I_t, I_{Gt}, K_t, G_t} \int_0^\infty e^{-\rho t} [\lambda V(c_t^U, \Phi(N_t^U)) + (1 - \lambda)V(c_t^R, H_t)] dt \\ & I_t + I_{Gt} + N_t^U c_t^U + N_t^R c_t^R = Y_t^U + Y_t^R \\ & Y_t^U = F(K_t)Q(N_t^U, G_t) \\ & Y_t^R = N_t^R \\ & \sum_{j=U,R} N_t^j = N \\ & \dot{K}_t = I_t - \delta K_t \\ & \dot{G}_t = I_{Gt} - \xi G_t \\ & \dot{H}_t = g_{Ht} - \mu H_t \\ & I_t, I_{Gt} \geq 0 \\ & K_0, G_0, H_0 \text{ given} \end{aligned}$$

where

$$Y_t^U = F(K_t)Q(N_t^U, G_t)$$

is an aggregation of

$$y_t^U = f(k_t)q(N_t^U, (M - 1)\tilde{N}_t^U, G_t)$$

and so it satisfies $F' > 0, F'' < 0, Q_1 \begin{cases} > 0 & \text{if } \Psi < \hat{\Psi} \\ < 0 & \text{if } \Psi > \hat{\Psi} \end{cases}$ where $\Psi \equiv \sum_{m=1}^M N^{mU}$ and $Q_2 > 0$.

The public good H_t in location R can be thought of as a natural resource which grows exogenously at a rate $g_{Ht} > 0$ and it depreciates at a rate $\mu \in (0, 1)$. For simplicity, we assume that the planner has no control over it.

The Hamiltonian of this problem is

$$\begin{aligned} J &= e^{-\rho t} [\lambda V(c_t^U, \Phi(N_t^U)) + (1 - \lambda)V(c_t^R, H_t)] + \\ & \vartheta_1 [F(K_t)Q(N_t^U, G_t) + N_t^R - I_t - I_{Gt} - N_t^U c_t^U - N_t^R c_t^R] + \\ & \vartheta_2 [N_t - N_t^R - N_t^U] + \vartheta_3 [I_t - \delta K_t] + \vartheta_4 [I_{Gt} - \xi G_t] \end{aligned}$$

The first-order conditions are:

$$\frac{\partial J}{\partial c_t^U} = 0 \Leftrightarrow e^{-\rho t} \lambda V_c - \vartheta_1 N_t^U = 0 \quad (17)$$

$$\frac{\partial J}{\partial c_t^R} = 0 \Leftrightarrow e^{-\rho t} (1 - \lambda) V_c - \vartheta_1 N_t^R = 0 \quad (18)$$

$$\frac{\partial J}{\partial N_t^U} = 0 \Leftrightarrow e^{-\rho t} \lambda V_\Phi \Phi' + \vartheta_1 [F(K_t) Q_N(N_t^U, G_t) - c_t^U] - \vartheta_2 = 0 \quad (19)$$

$$\frac{\partial J}{\partial N_t^R} = 0 \Leftrightarrow \vartheta_1 [1 - c_t^R] - \vartheta_2 = 0 \quad (20)$$

$$\frac{\partial J}{\partial I_t} \leq 0 \Leftrightarrow -\vartheta_1 + \vartheta_3 \leq 0 \quad (21)$$

$$\frac{\partial J}{\partial I_{Gt}} \leq 0 \Leftrightarrow -\vartheta_1 + \vartheta_4 \leq 0 \quad (22)$$

$$\frac{\partial J}{\partial K_t} = -\dot{\vartheta}_3 \Leftrightarrow \vartheta_1 F'(K_t) Q(N_t^U, G_t) - \vartheta_3 \delta = -\dot{\vartheta}_3 \quad (23)$$

$$\frac{\partial J}{\partial G_t} = -\dot{\vartheta}_4 \Leftrightarrow \vartheta_1 F(K_t) Q_G(N_t^U, G_t) - \vartheta_4 \xi = -\dot{\vartheta}_4 \quad (24)$$

Assuming the inequalities are binding we have

$$\vartheta_3 = \vartheta_1 = \vartheta_4$$

so

$$\dot{\vartheta}_3 = \dot{\vartheta}_1 = \dot{\vartheta}_4$$

and then we have from (3)

$$\frac{\dot{\vartheta}_1}{\vartheta_1} = \delta - F'(K_t) Q(N_t^U, G_t) \quad (25)$$

From (17) we have

$$e^{-\rho t} \lambda V_c = \vartheta_1 N_t^U$$

Taking logs and differentiating over time we have

$$\frac{\dot{c}_t^U}{c_t^U} = \frac{V_c}{V_{cc} c_t^U} \left[\rho + \delta - F'(K_t) Q(N_t^U, G_t) + \frac{\dot{N}_t^U}{N_t^U} \left(1 - \frac{V_{c\Phi} \Phi' N_t^U}{V_c} \right) \right] \quad (26)$$

where we use the fact that

$$\frac{\partial V_c(c_t^U, \Phi(N_t^U))}{\partial t} = V_{cc} \dot{c}_t^U + V_{c\Phi} \Phi' \dot{N}_t^U$$

Equation (26) represents the growth rate of consumption per capita in location U in the planner's problem.

Similarly, taking logs in (18) and differentiating with respect to time we obtain the growth rate of consumption in location R :

$$\frac{\dot{c}_t^R}{c_t^R} = \frac{V_c}{V_{cc}c_t^R} \left[\rho + \delta - F'(K_t)Q(N_t^U, G_t) + \frac{\dot{N}_t^R}{N_t^R} - \frac{V_{cH}}{V_c} \dot{H}_t \right] \quad (27)$$

From the population FOC in U (equation (19)) we have

$$e^{-\rho t} \lambda V_\Phi \Phi' + \vartheta_1 [F(K_t)Q_N(N_t^U, G_t) - c_t^U] - \vartheta_2 = 0$$

Using $\vartheta_1 = \vartheta_2$

$$e^{-\rho t} \lambda V_\Phi \Phi' = \vartheta_1 [1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U]$$

In the Appendix we show that taking logs and differentiating with respect to time gives us the law of motion of population in U :

$$\frac{\dot{N}_t^U}{N_t^U} = \frac{1}{\Omega} \left[\rho - \frac{V_{\Phi c} \dot{c}_t^U}{V_\Phi} + \delta - F'(K_t)Q(N_t^U, G_t) - \frac{F'(K_t)K_t Q_N(N_t^U, G_t)}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} - \frac{\dot{c}_t^U}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} \right. \\ \left. - \frac{F(K_t)Q_{NG}(N_t^U, G_t)\dot{G}_t}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} \right] \quad (28)$$

$$\text{where } \Omega \equiv \frac{V_{\Phi \Phi} \Phi' N_t^U}{V_\Phi} + \frac{\Phi'' N_t^U}{\Phi'} + \frac{F(K_t)Q_{NN}(N_t^U, G_t)N_t^U}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U}.$$

From the FOC with respect to G_t (equation 25) we have

$$\frac{\partial J}{\partial G_t} = -\dot{\vartheta}_4 \Leftrightarrow \vartheta_1 F(K_t)Q_G(N_t^U, G_t) - \vartheta_4 \xi = -\dot{\vartheta}_4$$

But since

$$\vartheta_1 = \vartheta_4$$

we have

$$\vartheta_1 F(K_t)Q_G(N_t^U, G_t) - \vartheta_1 \xi = -\dot{\vartheta}_1$$

Dividing both sides by ϑ_1 and using (25) we get

$$F(K_t)Q_G(N_t^U, G_t) - \xi = F'(K_t)Q(N_t^U) - \delta \quad (29)$$

This condition states that the planner sets the net marginal benefit of the public good G_t equal to the net opportunity cost of investment not used in the capital good. It is important to notice that this condition was not present in the decentralized equilibrium.

3.1 Steady-State Spatial Equilibrium

In this section we analyze the steady-state equilibrium of this model. The laws of motions of the model are given by (26)-(28) as well as those associated with K_t, G_t and H_t . In steady-state $\dot{c}_t^U = \dot{c}_t^R = \dot{N}_t^U = \dot{K}_t = \dot{G}_t = \dot{H}_t = 0$ so:

$$\frac{\dot{c}_t^U}{c_t^U} = \frac{V_c}{V_{cc}c_t^U} \left[\rho + \delta - F'(K_t)Q(N_t^U, G_t) + \frac{\dot{N}_t^U}{N_t^U} \left(1 - \frac{V_{c\Phi}\Phi'N_t^U}{V_c} \right) \right] = 0 \quad (30)$$

$$\frac{\dot{c}_t^R}{c_t^R} = \frac{V_c}{V_{cc}c_t^R} \left[\rho + \delta - F'(K_t)Q(N_t^U, G_t) + \frac{\dot{N}_t^R}{N_t^R} - \frac{V_{cH}}{V_c} \dot{H}_t \right] = 0 \quad (31)$$

$$\frac{\dot{N}_t^U}{N_t^U} = \frac{1}{\Omega} \left[\rho - \frac{V_{\Phi c}\dot{c}_t^U}{V_{\Phi}} + \delta - F'(K_t)Q(N_t^U, G_t) - \frac{F'(K_t)\dot{K}_t Q_N(N_t^U, G_t)}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} - \frac{\dot{c}_t^U}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} - \frac{F(K_t)Q_{NG}(N_t^U, G_t)\dot{G}_t}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} \right] = 0 \quad (32)$$

$$F(K_t)Q_G(N_t^U, G_t) - F'(K_t)Q(N_t^U) = \xi - \delta \quad (33)$$

$$\dot{K}_t = I_t - \delta K_t = 0$$

$$\dot{G}_t = I_{Gt} - \xi G_t = 0$$

$$\dot{H}_t = I_{Ht} - \mu H_t = 0$$

Next we eliminate investment from this system. From the planner's problem (omitting time subscripts) we have

$$I = Y^U + Y^R - I_G - N^U c^U - N^R c^R$$

Using the law of motion for K

$$\dot{K} = Y^U + Y^R - I_G - N^U c^U - N^R c^R - \delta K$$

Using the laws of motions of the remaining capital good under the control of the planner G (remember that the "natural" public good H is given by nature) we have

$$\dot{K} = Y^U + Y^R - \dot{G} - \xi G - N^U c^U - N^R c^R - \delta K$$

so this equation no longer has investment in it. In steady-state $\dot{K} = \dot{G} = 0$ and so

$$0 = Y^U + Y^R - \xi G - N^U c^U - N^R c^R - \delta K \quad (34)$$

The first three conditions simplify to:

$$\rho + \delta - F'(K^*)Q(N^{U*}) = 0 \quad (35)$$

And we also have from (34)

$$F(K^*)Q_G(N^{U*}, G^*) - F'(K^*)Q(N^{U*}) = \xi - \delta \quad (36)$$

Plus the spatial condition

$$V(c^{U*}, \Phi(N^{U*})) = V(c^{R*}, H^*) \quad (37)$$

and the labor market-clearing condition

$$N^{U*} + N^{R*} = N \quad (38)$$

4 A Numerical Example

In this section we use functional forms to solve for the unknowns $c^{U*}, c^{R*}, k, N^{U*}, G^*$, and H^* in the competitive and the planner's problem. For the competitive equilibrium we use:

$$\begin{aligned} f(k) &= k^\alpha, \alpha \in (0, 1) \\ q(N_t^U, G) &= (N^U)^\beta G^\nu [100N_t^U - (N_t^U)^2] \\ V^U &= a \ln c^U + b \ln G + d \ln((N_t^U)^\sigma) \end{aligned}$$

where $a, b, d, \sigma > 0$

For simplicity we normalize the amount of the "natural" public good H to one¹³ so we have

$$V^R = a \ln c^R$$

With these functional forms we have the following steady-state conditions. In the competitive equilibrium (omitting stars to simplify notation):

$$c^U = k^\alpha (N^U)^\beta G^\nu [100N_t^U - (N_t^U)^2] - \delta k \quad (39)$$

$$\rho + \delta = \alpha k^{\alpha-1} (N^U)^\beta G^\nu [100N_t^U - (N_t^U)^2] \quad (40)$$

In the competitive solution $c^R = y^R = 1$ because there is no saving in this location. Therefore $V^R = 0$ and we have

¹³This is irrelevant in our model since neither consumers nor the government choose H .

$$a \ln c^U + b \ln G + d \ln((N^U)^\sigma) = 0 \quad (41)$$

Moreover, we have the labor market-clearing condition

$$N^U + N^R = N \quad (42)$$

Note that there are four equations and four unknowns: c^U, k, N^U, N^R since the public good G is treated as a parameter in the competitive problem.

In the efficient equilibrium we have:

$$\rho + \delta - \alpha k^{\alpha-1} (N^U)^\beta G^\nu [100N_t^U - (N_t^U)^2] = 0 \quad (43)$$

$$k^\alpha Q_G(N^U, G) - \alpha k^{\alpha-1} (N^U)^\beta G^\nu [100N_t^U - (N_t^U)^2] = \xi - \delta$$

where

$$Q_G(N^U, G) = \nu G^{\nu-1} (N^U)^\beta [100N_t^U - (N_t^U)^2]$$

or

$$(N^U)^\beta [100N_t^U - (N_t^U)^2] \{k^\alpha \nu G^{\nu-1} - \alpha k^{\alpha-1} G^\nu\} = \xi - \delta \quad (44)$$

Moreover

$$0 = k^\alpha (N^U)^\beta G^\nu [100N_t^U - (N_t^U)^2] + N^R - \xi G - N^U c^U - N^R c^R - \delta k \quad (45)$$

$$a \ln c^U + b \ln G + d \ln(N^U)^\sigma = a \ln c^R$$

where, as before, we assumed $H = 1$. Finally,

$$N^{U*} + N^{R*} = N \quad (46)$$

Note that there are five equations and six unknowns: k, N^U, G, N^R, c^U, c^R . The source of this indeterminacy is that there is no optimal condition to choose c^R . In order to solve our numerical example we therefore consider an equilibrium where the planner chooses the same consumption level in R as the market, i.e. $c^R = 1$.

Table 1 gives the parameter values that we use in the simulation of our example.

Table 1

parameter	α	β	δ	ρ	v	a	b	d	σ	ξ	N
value	0.5	0.5	0.2	0.5	0.3	0.5	0.5	0.5	0.5	0.2	100

Table 2 shows the results of our simulation. Using the proposed functional forms and parameters we observe that population in the urban area N^U is much larger in the planner’s problem than in the competitive one. Indeed, in this example, almost the whole population moves to location U in the planner’s problem, whereas in the competitive one, the country remains practically rural, with 99% of its population in location R . This was to be expected since, as stated above, the planner takes into account the two positive external effects that population has in the city: it increases agent’s utility through network effects or other agglomeration economies in their utility function and it makes workers more productive in firms. It is also interesting to note that the consumption level in U is higher in the competitive equilibrium than in the efficient one. The reason is that, by providing the public good, the planner understands that it is profitable to accumulate more physical capital in location U because the two inputs are complements in the production function. This, in turn, increases wages in U and hence attracts population there. The increase in population generates additional benefits to agents living in location U . However, in a spatial equilibrium, it must be the case that utility in this location must decrease to compensate for this effect. The planner achieves this by substantially reducing consumption in U .

Table 2

	<i>competitive</i>	<i>efficient</i>
c	1.79	0.04
k	1.49	3.93
N^U	0.06	99.99
N^R	99.94	0.002

5 Conclusions

This paper presents a theoretical model that helps understanding the process of rural-urban migration and how the provision of a public good that affects output in the urban location can affect this process. The model consists of a rural location and urban one, with the latter exhibiting agglomeration economies from preferences and production. We solve for the competitive equilibrium and the solution of a benevolent social planner. Neither agents nor firms take into account the positive effect of their decisions on others’ utility functions or in aggregate urban output. Moreover, the social planner understands how the

provision of a public good in the urban location interacts with these agglomerations. For these two reasons, the decentralized equilibrium in this model is inefficient.

The model can be applied to interpret the evolution of urbanization in different world regions, in particular South Asia, where the percentage of population living in urban areas is puzzlingly low given the region's income level. We show that, from a theoretical point of view, the large rural population in these countries can be rationalized by policies that give incentives for workers to stay in rural areas. For example, in the model, a lack of investment in the urban public good discourages urban production and so it delays urbanization. Finally, although this is not the focus of the paper, our model can also be used to study the evolution of rural-urban consumption and income gaps over time and how they may be affected by specific policies. While we agree with Fujita et al. (2001)'s view that more empirical work is needed to guide policy makers on how to deal with urbanization, we believe this research needs to go hand-in-hand with sound theoretical frameworks that allows us to interpret and organize the data in a clear way. The present paper can be read as a contribution towards generating more dynamics to understand the process of rural-urban transformation and how different policies affect it.

References

- [1] Anas, A., 1978. Dynamics of urban residential growth. *Journal of Urban Economics*, 5, 66 -87.
- [2] Arnott, R. 2007. Congestion tolling with agglomeration externalities. *Journal of Urban Economics*, 62:2, 187-203.
- [3] Au, C-C., Henderson, J.V., 2006. Are Chinese cities too small? *Review of Economic Studies* 73(3), 549-576.
- [4] Becker, G.S. and Murphy, K.M., 1992. The division of labor, coordination costs, and knowledge. *Quarterly Journal of Economics* 11-37.
- [5] Black, D, Henderson J.V. 1999. The theory of urban growth. *Journal of Political Economy* 107, 252-284
- [6] Chan, J-Y., Yu, E.S.H., 2010. Imperfect capital mobility: A general approach to the two-sector Harris-Todaro model. *Review of International Economics* 18(1), 81-94.
- [7] Cuberes, D., 2009. A model of sequential city growth. *The B.E. Journal of Macroeconomics (Contributions)*, March, 9(1), Article 18.
- [8] Duranton, G., Puga, D. 2001. Nursery cities: urban diversity, process innovation, and the life cycle of products. *American Economic Review*, 91(5), December, 1454-1477.
- [9] Fujita, M., 1982. Spatial patterns of residential development. *Journal of Urban Economics* 12, 22-52.
- [10] Fujita, M., Krugman, P.R., Venables, A.J., 2001. *The Spatial Economy. Cities, Regions, and International Trade*. The MIT Press.
- [11] Gillespie, F., 1983. Comprehending the slow pace of urbanization in Paraguay. *Economic Development and Cultural Change* 31, 355-375.
- [12] Harris, J., Todaro, M., 1970. Migration, unemployment and development: A two-sector analysis. *American Economic Review* 60, 126-142.
- [13] Helsley, R.W., Strange, W.C., 1990. City formation with commitment. *Regional Science and Urban Economics* 24, 373-390.
- [14] Henderson, J.V., 1974. The sizes and types of cities. *American Economic Review*, LXIV, 640-656.
- [15] Henderson, J.V., 1988. *Urban Development: Theory, Fact and Illusion*, Oxford: Oxford University Press.
- [16] Henderson, J.V., Ioannides, Y.M., 1981. Aspects of growth in a system of cities. *Journal of Urban Economics* 10, 117-139.

- [17] Henderson, J.V., Kuncoro, A., 1996. Industrial centralization in Indonesia. *World Bank Economic Review*, 10, 513-540.
- [18] Henderson, J.V., Venables, A.J., 2009. Dynamics of city formation. *Review of Economic Dynamics* 12(2), April, 233-254.
- [19] Henderson, J.V., Wang, H.G., 2005. Aspects of the rural-urban transformation of countries. *Journal of Economic Geography* 5(1), 23-42.
- [20] Henderson, J.V., Wang, H.G., 2007. Urbanization and city growth: The role of institutions. *Regional Science and Urban Economics* 37(3), May, 283-313.
- [21] Ioannides, Y., 1994. Product differentiation and economic growth in a system of cities. *Regional Science and Urban Economics* 24, 461-484.
- [22] Jefferson, G., Singhe, I., 1999. *Enterprise Reform in China: Ownership Transition and Performance*. New York: Oxford University Press.
- [23] Kanemoto, Y., 1980. Externality, migration, and urban crises. *Journal of Urban Economics*, 8(2), September, 150-164.
- [24] Lee, S.-K., Choe S.-C., 1990. Changing location patterns of industry and urban decentralization policies in Korea. in J.K. Kwon (ed.), *Korean Economic Development*. Santa Barbara, California: Greenwood Press.
- [25] Miyao, T., 1981 *Dynamic analysis of the urban economy*. New York: Academic Press.
- [26] Moretti, E., 2004. Human Capital Externalities in Cities, *Handbook of Urban and Regional Economics*, North Holland-Elsevier.
- [27] Neary, J.P, 1981. On the Harris-Todaro model with intersectoral capital mobility. *Economica* 48, August, 219-234.
- [28] Neary, J.P, 1988. Stability of the mobile-capital Harris-Todaro model: Some further results. *Economica* 55:217, February, 123-127.
- [29] Palivos, T., Wang, P., 1996. Spatial agglomeration and endogenous growth. *Regional Science and Urban Economics* 26(6), December, 645-669.
- [30] Riadh, B.J., 1998. Rural-urban migration: On the Harris-Todaro model." Working paper.
- [31] Richardson, H. (1987), "The Costs of Urbanization: A Four Country Comparison." *Economic Development and Cultural Change*, 33, 561-580.
- [32] Rosenthal, S. S. and W. C. Strange (2003), "Geography, Industrial Organization, and Agglomeration," *Review of Economics and Statistics*, 85:2, 377-393

- [33] Venables, A. J. (2007), "Evaluating Urban Transport Improvements," *Journal of Transport Economics and Policy*, 41:2, pp. 173-188

Figure 1: 1960

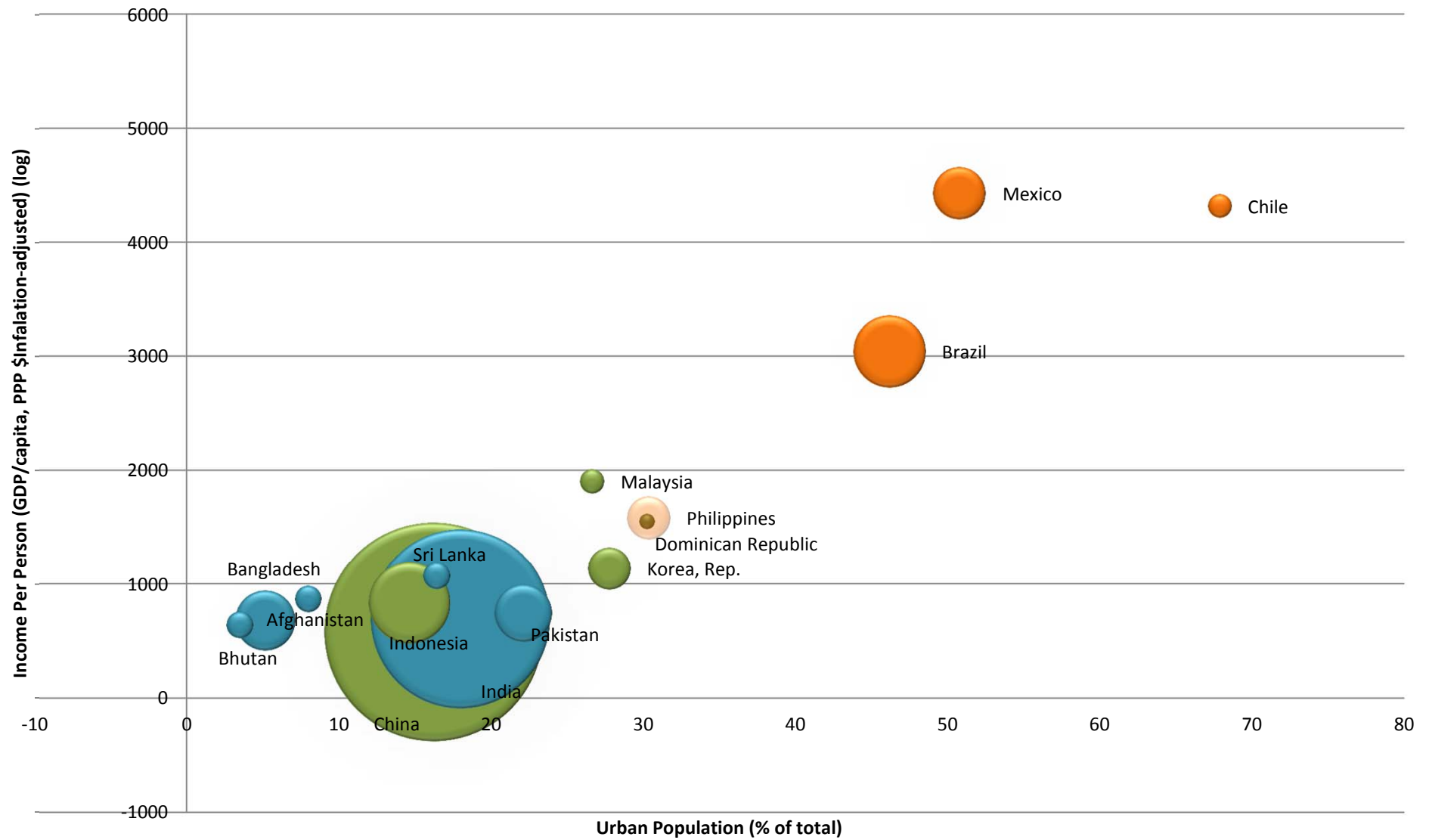
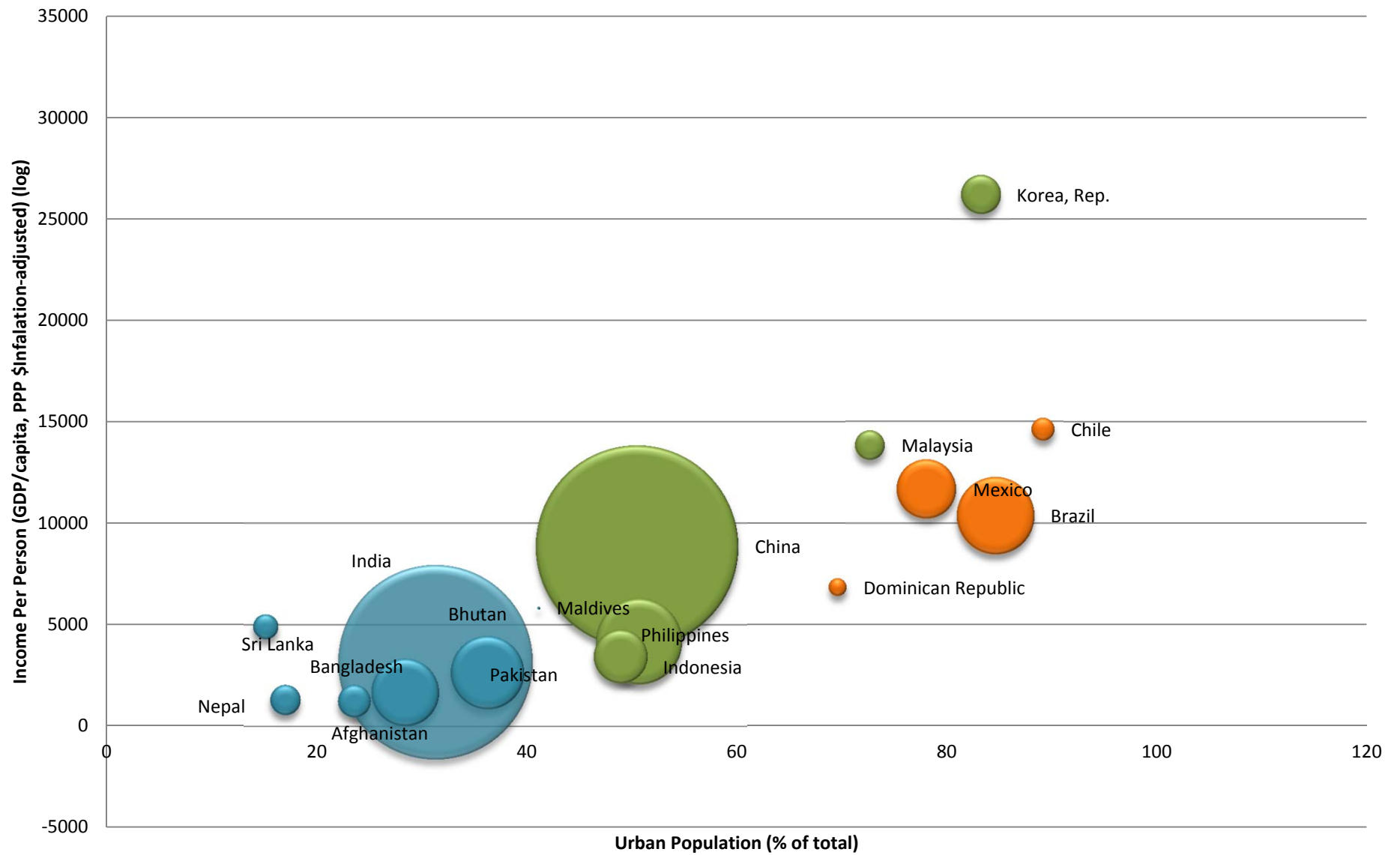


Figure 2: 2011



Appendix

Law of motion of population in location U

From the population FOC in U (equation (19)) we have

$$e^{-\rho t} \lambda V_{\Phi} \Phi' + \vartheta_1 [F(K_t) Q_N(N_t^U, G_t) - c_t^U] - \vartheta_2 = 0$$

Using $\vartheta_1 = \vartheta_2$

$$e^{-\rho t} \lambda V_{\Phi} \Phi' = \vartheta_1 [1 - F(K_t) Q_N(N_t^U, G_t) + c_t^U]$$

Taking logs

$$-\rho t + \ln \lambda + \ln V_{\Phi} + \ln \Phi' = \ln \vartheta_1 + \ln [1 - F(K_t) Q_N(N_t^U, G_t) + c_t^U]$$

and differentiating with respect to time

$$-\rho + \frac{\frac{\partial V_{\Phi}}{\partial t}}{V_{\Phi}} + \frac{\frac{\partial \Phi'}{\partial t}}{\Phi'} = \frac{\dot{\vartheta}_1}{\vartheta_1} + \frac{\frac{\partial [1 - F(K_t) Q_N(N_t^U, G_t) + c_t^U]}{\partial t}}{1 - F(K_t) Q_N(N_t^U, G_t) + c_t^U}$$

Now note that

$$\frac{\partial V_{\Phi}(c_t^U, \Phi(N_t^U))}{\partial t} = V_{\Phi c} \dot{c}_t^U + V_{\Phi \Phi} \Phi' \dot{N}_t^U$$

Moreover

$$\begin{aligned} \frac{\partial [1 - F(K_t) Q_N(N_t^U, G_t) + c_t^U]}{\partial t} &= -F'(K_t) \dot{K}_t Q_N(N_t^U, G_t) - F(K_t) Q_{NN}(N_t^U, G_t) \dot{N}_t^U \\ &\quad - F(K_t) Q_{NG}(N_t^U, G_t) \dot{G}_t + \dot{c}_t^U \end{aligned}$$

So using (25) again we end up with

$$\begin{aligned} -\rho + \frac{V_{\Phi c} \dot{c}_t^U + V_{\Phi \Phi} \Phi' \dot{N}_t^U}{V_{\Phi}} + \\ \frac{\Phi'' \dot{N}_t^U}{\Phi'} &= \\ \delta - F'(K_t) Q(N_t^U, G_t) + \\ &\quad - \frac{F'(K_t) \dot{K}_t Q_N(N_t^U, G_t)}{1 - F(K_t) Q_N(N_t^U, G_t) + c_t^U} \\ &\quad - \frac{F(K_t) Q_{NN}(N_t^U, G_t) \dot{N}_t^U}{1 - F(K_t) Q_N(N_t^U, G_t) + c_t^U} \\ &\quad - \frac{F(K_t) Q_{NG}(N_t^U, G_t) \dot{G}_t + \dot{c}_t^U}{1 - F(K_t) Q_N(N_t^U, G_t) + c_t^U} \end{aligned}$$

Rearranging we can obtain an optimal law of motion for N_t^U . Divide the relevant terms by N_t^U

$$\begin{aligned}
& -\rho + \frac{V_{\Phi\Phi}\Phi'N_t^U}{V_{\Phi}} \frac{\dot{N}_t^U}{N_t^U} + \frac{V_{\Phi c}\dot{c}_t^U}{V_{\Phi}} \\
& + \frac{\Phi''N_t^U}{\Phi'} \frac{\dot{N}_t^U}{N_t^U} = \\
& \delta - F'(K_t)Q(N_t^U, G_t) + \\
& - \frac{F'(K_t)\dot{K}_tQ_N(N_t^U, G_t)}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} \\
& - \frac{F(K_t)Q_{NN}(N_t^U, G_t)N_t^U}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} \frac{\dot{N}_t^U}{N_t^U} \\
& - \frac{\dot{c}_t^U}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} \\
& - \frac{F(K_t)Q_{NG}(N_t^U, G_t)\dot{G}_t}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} \\
& \frac{\dot{N}_t^U}{N_t^U} \left[\frac{V_{\Phi\Phi}\Phi'N_t^U}{V_{\Phi}} + \frac{\Phi''N_t^U}{\Phi'} + \frac{F(K_t)Q_{NN}(N_t^U, G_t)N_t^U}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} \right] = \rho \\
& - \frac{V_{\Phi c}\dot{c}_t^U}{V_{\Phi}} + \delta - F'(K_t)Q(N_t^U, G_t) + \\
& - \frac{F'(K_t)\dot{K}_tQ_N(N_t^U, G_t)}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} \\
& - \frac{\dot{c}_t^U}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} \\
& - \frac{F(K_t)Q_{NG}(N_t^U, G_t)\dot{G}_t}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U}
\end{aligned}$$

Let $\Omega \equiv \frac{V_{\Phi\Phi}\Phi'N_t^U}{V_{\Phi}} + \frac{\Phi''N_t^U}{\Phi'} + \frac{F(K_t)Q_{NN}(N_t^U, G_t)N_t^U}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U}$. Then

$$\frac{\dot{N}_t^U}{N_t^U} = \frac{1}{\Omega} \left[\begin{aligned} & \rho - \frac{V_{\Phi c}\dot{c}_t^U}{V_{\Phi}} + \delta - F'(K_t)Q(N_t^U, G_t) - \frac{F'(K_t)\dot{K}_tQ_N(N_t^U, G_t)}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} \\ & - \frac{\dot{c}_t^U}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} \\ & - \frac{F(K_t)Q_{NG}(N_t^U, G_t)\dot{G}_t}{1 - F(K_t)Q_N(N_t^U, G_t) + c_t^U} \end{aligned} \right] \quad (47)$$