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Multi-objective optimization models for urban readjustment in Turkey

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MULTI-OBJECTIVE OPTIMIZATION MODELS FOR URBAN READJUSTMENT PRACTICES IN TURKEY¹

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Abstract

The *Dough Rule*, in Turkish Land Law terminology, is a technical approach to readjust land resources in urban and agricultural areas. The Regulation on the 18th Article of the Land and Building Development Law defines the procedures of land readjustment. The procedure is as follows: First obtain cadastral land resources and ownership records and then overlay graphic information on top of zoning plans, later allocate the land resources to the relevant owners after proportional deduction for public uses, such as schools, roads, and parks. In many cases, in the allocation process, relevant parties compete for higher-value urban lots, and the resulting allocation scheme is taken to court. In this study, the primary goal is to develop techniques to handle the difficulties in land readjustment and reallocation practices in Turkey. The developed techniques contain three authentic features: The **first feature** is the original mathematical models based on *operation research* techniques. In this feature, there are two *Linear Programming* (LP) and one *Mixed Integer Programming* (MIP) models reallocating cadastral parcel areas into standard high quality urban subdivision lot(s) after proportional deduction of public land uses from all landowners under the frame of the Regulation on the 18th Article. The **second feature** is the *evaluation criteria* which are developed to present superiority of the optimization models over the existing expert based subjective practices in land reallocation. Those criteria are defined in the form of equations utilizing the allocation results (objective and subjective). The **third feature** is the graphical presentation of allocation results in a system of rays from cadastral parcel centroids to urban subdivision lot centroids. This provides a succinct way of visual comparison among alternative allocation results. Over a case study area, first, the optimization techniques are applied, then, the obtained results are evaluated via the evaluation criteria, and later visually presented by the ray based allocation/transfer diagrams. Finally superiority of the developed techniques are discussed for further sophistications.

Keywords: Subdivision Regulations, Urban Land Readjustment, Optimization, Linear and Mixed Integer Programming

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I. INTRODUCTION

Since the Second World War, especially, during the 1960-2000 period, rapid urbanization in Turkey resulted in various problems in urban spaces such as squatter houses, substandard subdivisions, and low-quality urban environment. In order to cope with these problems, the government has enacted various laws and regulations and made amendments in legal frameworks as deemed necessary. The most prominent of these regulations is the one called "Regulation on Land Readjustment based on the 18th Article of the Land and Building Development Law" (hereinafter referred to as 'the Regulation') (Web 1). One of the major goals of aforementioned law and the associated Regulation (the Regulation on the 18th Article, Web 2) is to prepare standard high quality subdivision lots with regards to zoning plans. However, there are a series of legal and practical difficulties, and legal dilemmas in allocating the properties among various land owners: **First**, because, neither local nor the central government has the financial capability to confiscate private properties for public land uses, the government proportionally (up to a maximum of 40%) confiscates properties without paying the cost of expropriation. Implicitly, the current law assumes that land owners take the monetary advantage of being in the well-designed planned subdivision and having standard lot(s). **Second**, allocation from cadastral properties to the standard lots neither provides each and every one of the owners a unique private lot(s) in the planned subdivision, and nor guarantees that every land owner acquires a piece of land in the same location (spot) as before the readjustment process. Although rules for reallocation in the Regulation are fairly specific, the reallocations are open to harsh disputes and often end up in court cases. This study develops a new approach to reallocate properties while minimizing the adversaries and maximizing the property owners' desires. The developed optimization techniques, *first*, consider the main principles of the regulation, *second*, find the best allocation solution for relevant property owners. To illustrate, an allocation problem is revisited and the differences between the models and the actual case are reviewed. The obtained results are evaluated by a set of *evaluation criteria* developed in this research and presented by a *system of rays* from cadastral parcel centroids to urban subdivision lot centroids.

II. LITERATURE

Land Readjustment (LR) is described as i) an effective planning tool for not only transforming agricultural lands into usable urban lots, but also redevelopment technique for the blight areas of urban lands, ii) an effective plan implementation technique transforming useless parcels into economically usable lots, iii) an efficient tool to provide basic public lands improving the quality of environment and quality of life for the residents of land readjustment area. According to Sorensen (2000a) "Land readjustment is a method whereby the ownership of scattered and irregular plots of agricultural land is pooled, roads and main infrastructure are built, and the land is then subdivided into urban plots." It is a technique that involved stakeholders (not only land owners but also planning and municipal authorities) are most likely to participate in the LR process; "The attractiveness of the method for landowners is based on the fact that substantial increases in the value of land may be achieved by the process, so that the value of the individual land holdings can be greatly increased, even though the remaining area is smaller. The attraction for planning authorities is that projects provide land for public facilities, rationalize patterns of property division, and build needed urban infrastructure." (Sorensen 2000a). It is a widely used planning and plan implementation technique in various parts of the world such as in Germany, France, Italy, Sweden in Europe, in India, Nepal, Thailand, Malaysia, Indonesia, South Korea, Japan, Western Australia in Asia-Pacific, Canada in America (Sorensen 2000a; Sorensen 2000b; Larson 1997). The use of LR varies from country to country. While in one country effectiveness of land owners is observed, in another state-public institutions' roles are heavily seen (Türk, 2009). Without exception, in every country, from a total of LR area, public land uses are subtracted and the remaining land is reallocated to all land owners by their proportional shares (Türk, 2009).

Land Readjustment (LR) is also described as an effective plan implementation technique transforming useless parcels into economically usable lots (Chou ve Shen, 1982; Doebele, 1986; Doebele, 1982; Yomraliođlu, 1992). The most important characteristics of the technique are the practical uses in very extensive areas and the shortening of plan implementation period (Doebele, 1982; Turk, 2007). The LR provides various advantages to both land owners and public, such as, creation of i) standard accessible lots with needed public land uses (road, park), ii) higher standard, and stable land values which is easily traded, iii) clear and identified ownership structure, and minimized property sharing (Turk, 2009). According to Seele (1982, 1994), the main beneficiaries of LR are municipalities; municipalities save from expropriation cost by a proportional taking from land owners in order to provide public land uses and services.

Turkey has been one of the earliest countries applying a form of LR technique dating back to the Ottoman period especially after fires destroying the wooden urban fabric. In this respect, LR, as a technique used in the western world for a long time, has been seen for nearly 150 years in this country (Turk, 2009). During the first 50 years of the Republic of Turkey, there have been several LR regulations in Laws (the Building Regulations of 1848; Road and Building Regulations of 1864; the Building Law of 1882; Law dated 1925, No. 642; Law dated 1930, No. 1663; Building and Roads Law dated 1933, No. 2290; the Reconstruction Law dated 1956, No. 6785 [Tekeli (1994) and Ersoy (2000) in eds. of Türk (2007)]); however, none of these were satisfactory to handle the post Second World War rapid urbanization. The most effective applications of LR technique have been seen in the last quarter of 20th century while hassling with rapid urbanization. Especially, the 18th Article (Dough Rule) of the Land and Building Development Law (İmar Kanunu - 3194) (Web 1) and the associated Regulation (Web 2) enacted in 1985 can be considered as a major step to solve rapid urbanization problems in this respect.

The current literature in Turkey focuses mainly on the actualization of the techniques in terms of multiparty participation, financing, and administration. However, there are a limited number of papers focusing on technical details of reallocation and on problems arising during the process of reallocation. In Turkey, every year many land reallocation applications are taken to courts by unhappy participants. Most of the complaints arise from unbalanced and inequitable (in economic and spatial sense) relocation of readjusted lands among stakeholders. Çete and Eđerciođlu (2013), Köktürk and Köktürk (2005) and Köktürk and Köktürk (2009) and Yomraliođlu (1993), Yomraliođlu et al. (2007), in their researches, focus on the techniques to reallocate the land resources among land owners in the practices of the Turkish LR. They propose value based land reallocations rather than area based land reallocations after proportional deduction of public land uses. While Çete and Eđerciođlu (2013) dwell on value based evaluations, Köktürk and Köktürk (2009) propose equivalency criteria for pre and post LR with a minimum deviation principle. Nişancı (2005) and Yomraliođlu et al. (2007) develop a pixel (raster) based land valuation and reallocation system minimizing subjectivity by means of Geographic Information System (GIS).

The technical steps of the LR practice in Turkey, which is an adapted form of German LR practices (*Umlegung*),³ are presented in Figure 1 (derived from Türk 2005). The **first step (I)** is the identification of land and building resources and associated ownership structure, in

³ "Initially records and maps detailing owners, parcels and buildings are stated. From the parcel areas the total readjustment area is calculated and then reduced by a common share for streets, green places and other public areas. In the reduced area every owner gets a share in proportion to either the area or the value of his included land. Further land may be taken over by the municipality as contributions to the costs. In principle an owner cannot claim a higher value back — calculated under the new conditions — than he has left — calculated according to the old situation. Based on everyone's share, a new parcel plan is worked out, adapted to the approved building plan. The scheme, including related economic questions, is then discussed with the individual owners. After revisions, if any, preliminary boundaries are fixed for the ground and records are made of new parcels, inclusive of encumbrances and

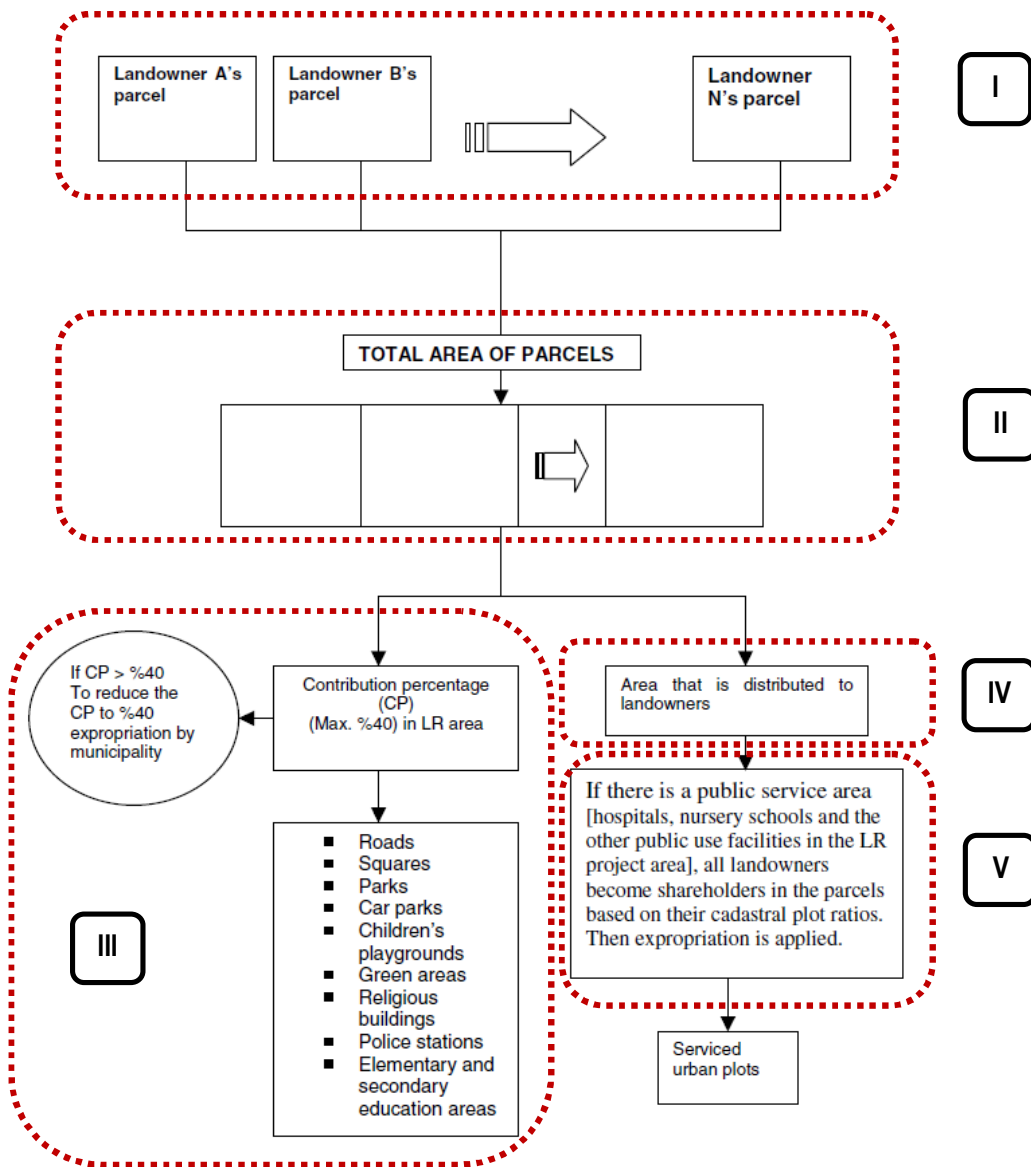


Figure 1: The steps in the land readjustment (LR) practice in Turkey (derived from Türk, 2004)

which there are cadastral maps, existing building stocks, and land titles. In the **second step (II)**, the total land resources are overlaid with the current Development Plans (scale: 1/1000) showing land use decisions for public and private land uses. In the **third step (III)**, the public land uses are confiscated without any monetary compensation, which might reach 40% of all the LR area. In the **fifth step (V)**, if the size of public land uses is more than the 40%, the exceeding portion is required to be expropriated via just monetary compensation, considering the shares in total LR area. The **fourth step (IV)**, **which is the focus of this research**, is a critical step in that the prepared urban parcels (lots) are assigned to the landowners in a way that parties are required/expected to be satisfied with the allocation. In most cases, there are unhappy

compensations incurred. This is the latest a new building plan must be approved by. The parcel plan is then displayed in the locality detailing the period of time for appeal. The legal process is then finished. The building of streets and other constructions is not included in the proceedings, they are the responsibility of the municipality and are paid for according to conventional charges." (Larson 1997)

participants who do not like the location and share of the lot(s) provided to them. The main reason for that is the fact that lots on the main street have more value than the lots on the minor streets. This research mainly focuses on the **fourth step (IV)** and introduces a transport model to allocate the urban lands to the relevant parties regarding the principles outlined in Article 10⁴ (about the rules for creating parcels and allocating them to the involved parties) of the Regulation (Dough Rule). These principles are i) land owners are entitled to the closest post-allocation parcel to their own pre-allocation cadastral parcel, and ii) The created parcel(s) may be owned by more than one owner with varying shares, iii) existing buildings and their owners are primarily considered in the process of assignment of the lots and reallocation, and iv) scattered land shares owned by single owner or family groups are amalgamated in less number of and larger lots.⁵

III. METHODOLOGY

III.1. Optimization Models

A new form of optimization based LR technique is the content and contribution of this research. This technique is basically on the reassignment of land resources from cadastral to urban lots after deducting public land use shares. The developed approach for the Land Readjustment (LR) problems methodologically is a modified form of classical transportation problem (Dantzig, 1963). The general characteristic of a transportation problem is based on the cost minimization in the process of delivering goods from supply nodes to demand nodes. At the end, the optimal solution provides the least cost of delivery. **In this study, instead of delivering goods, square meter land areas are delivered from given cadastral parcels to urban subdivision parcels.** The total square meter land getting into the development site is required to be equal to that of those getting out of the development site. The mathematical structure of the model is presented in the coming sections. Initially, the basic transportation model is introduced, then, its modified form, called as ***the Land Readjustment Models via Transport Problem (LRMTP)***, is presented.

Transportation Model: The transportation problem is a widely known simple and basic form of the linear programming problems. It handles commodity flows from sources to destinations. The objective is to determine the amount of commodity to be transported from each source to each destination in a way that the total transportation cost is minimum. The transportation problem, no matter how large the case, carries a simple algebraic structure. In the transportation problem, the supplies at several plants and the demands at several markets for a single commodity, and the unit costs of shipping the commodity from plants to markets are provided. Then the question is that how much the shipment there should be between each plant and each market so as to minimize total transport cost [for an illustration, please refer *A GAMS Tutorial* by Rosenthal (2008)].

The Land Readjustment Models via Transport Problem (LRMTP): As an adjusted form of the transportation problem, the land readjustment process delivers the square meter land area from one land owner to the same land owner after deducting proportional share of public land uses (road, school, government school, park, car park, public square, police station etc.). These square meter land area transfers take place from the centroids of cadastral parcels to the centroids of urban subdivision lots/parcels within the limit of shares and lot sizes (after deductions) in a way that total transport cost is minimum (i.e. allocations to the nearest lots). The developed models are three sequential adopted and improved transportation models for urban land readjustment problems, therefore can be named as the **Land Readjustment Models via Transport Problem**

⁴ It specifies the rules for creating urban parcels and allocating cadastral shares into them.

⁵ Although there is no any written code, the fourth principle is widely considered in current practices.

(LRMTP). The principles of the models are based on the 4 codes⁶ (listed above) in the 10th Article of the Regulation. The *first* and the *second* rules are directly adopted into the classical **transportation problems** with minor changes (Transportation Problem: Dantzig, 1963; GAMS⁷ in Model Library). The *third* rule requires additional direct assignment constraints for the available buildings. The *fourth* rule, which is not written as an official code but used in practice, requires some changes in objective function and in a single constraint or a set of constraints providing unification/amalgamation privilege for scattered property shares (for the same owner and/or group of family members) in the land readjustment area.

Base Mathematical Structure of the LRMTP

The classical transportation model by Dantzig (1963) delivers a homogenous product from supply nodes to demand nodes by minimizing total transport cost. In the LRMTP, instead of homogenous product, uniform square meter land areas are delivered (x_{ij}) from cadastral properties (i) to the urban lots/parcels (j) by *minimizing the total transport cost*, which is the sum of the product of transferred area and distances between parcel and lot centroids ($\sum_{ij} x_{ij} \cdot c_{ij}$). The distances between cadastral and urban lots/parcels centroids are considered as *costs parameter* (c_{ij}) in the LRMTP. The LRMTP results provide unique optimal nearest distance allocation solution in matrix form (x_{ij}^{OPT}).

The LRMTP consists of three separate optimization models. The first two of the models are based on *linear programming* and the last one is based on *mixed integer programming*. Because models use mainly common equation systems, the mathematical forms of both objective functions and constraints are listed altogether sequentially in Equations (1)-(7-K). The associated variables, parameters, and indices follow afterward.

For objective functions; while the *first* and *second* models (Model I and II) use only Equation (1a), the third model (Model III) uses Equations (1b). **For constraints;** while the Model I uses only Equations (2)-(5), the Model II adds Equation (6), the Model III enlarges the system by adding Equations (7-1)-(7-K) and uses the Equation (1b) instead of Equation (1a). Equation (1b) and Equations (7-1)-(7-K) also contain binary variables (0 or 1) and transform the model into a form of Mixed Integer Programming (MIP). Later, the details of each model and associated equations are provided in details.

Objective functions to be minimized

$$Z = \sum_{i=1}^m \sum_{j=1}^n x_{ij} \cdot c_{ij} \quad (1a)$$

⁶ The 10th article of Regulation of the 18th Article (Dough Rule) of the Land and Building Development Law (*İmar Kanunu*) (Web 2) requires that:

Rule 1: Land owners take the closest zoning based subdivision parcels to their own cadastral parcel.

Rule 2: The created parcel(s) may be owned by more than one land owners with varying shares.

Rule 3: For those buildings complying with the current codes and plans can be retained and they are assigned into a single lot without shared ownership title. Due to small size cadastral land ownership structure, in a case that a single lot and ownership solutions are not technically possible, condominium solutions can be considered.

Rule 4 (not written but conventionally used): Scattered land shares owned by a single owner or family/friend groups are collected in less number and larger lots.

<http://www.mevzuat.gov.tr/>

$$\begin{aligned}
Z = & \left(\sum_{i=1}^m \sum_{j=1}^n x_{ij} \cdot c_{ij} \right) \\
& + \left(w_1 \cdot \sum_j BIN1_j \cdot BS \right) \\
& + \dots \dots \dots \\
& + \left(w_k \cdot \sum_j BIN2_j \cdot BS \right) \\
& + \dots \dots \dots \\
& + \left(w_K \cdot \sum_j BINK_j \cdot BS \right)
\end{aligned} \tag{1b}$$

Subject to the constraints

$$\sum_{j=1}^n x_{ij} \leq a_i \quad \forall i \tag{2}$$

$$\sum_{i=1}^m x_{ij} \geq b_j \quad \forall j \tag{3}$$

$$\sum_{i=1}^m \sum_{j=1}^n x_{ij} = \sum_i a_i \tag{4}$$

$$x_{ij} \geq 0 \quad \forall i \ \& \ j \tag{5}$$

$$x_{ij} \geq MCBR_j \quad \forall i \ \& \ j \tag{6}$$

$$\sum_{i \in hb1} x_{ij} \leq BIN1_j \cdot BS \quad \forall j \tag{7-1}$$

.....

$$\sum_{i \in hbk} x_{ij} \leq BIN2_j \cdot BS \quad \forall j \tag{7-k}$$

.....

$$\sum_{i \in hbK} x_{ij} \leq BINK_j \cdot BS \quad \forall j \tag{7-K}$$

Sets

- i*: cadastral parcel and/or shareholder indices
- j*: urban parcel/lot indices
- k*: shareholder group indices for land amalgamation/unification
- n*: the total number of cadastral parcels and/or the number of shareholders
- m*: the total number of urban lots/parcels

hb1: shareholder group 1 for scattered land amalgamation/unification (a subset of *i*)

.....
hbk: shareholder group *k* for scattered land amalgamation/unification (a subset of *i*)

.....
hbK: shareholder group *K* for scattered land amalgamation/unification (a subset of *i*)

Variables

- Z*: objective value; the total delivery cost (total of distance and area product)
- x_{ij}*: amount of land area to be delivered from cadastral parcel *i* to urban lot/parcel *j*
- BIN1_j*: binary variable for the 1st shareholder group; in urban lot/parcel *j*, if there is an amalgamation/unification, it is 1 otherwise 0

.....
BINK_j: binary variable for the *k*th shareholder group; in urban lot/parcel *j*, if there is an amalgamation/unification, it is 1 otherwise 0

$BINK_j$: binary variable for the K^{th} shareholder group; in urban lot/parcel j , if there is an amalgamation/unification, it is 1 otherwise 0

Parameters

a_i : land area for reallocation after public uses deduction in base cadastral parcel i
 b_i : land area of lot/parcel j after subdivision regulation,
 c_{ij} : transportation cost: distance between centroids of cadastral parcels i and subdivision lots/parcels j
 BS : Very large number (1 000 000 000)
 w_k : weights for every k^{th} shareholder group
 p_i^x : x coordinate values for cadastral parcel i
 p_i^y : y coordinate values for cadastral parcel i
 p_j^x : x coordinate values for subdivision lot/parcel j
 p_j^y : y coordinate values for subdivision lot/parcel j
 $MCBR_{ij}$: amount of land area required to be delivered from cadastral parcel i to urban lot/parcel j

As one of the main component of objective functions, c_{ij} is a distance matrix presenting Euclidean distances between centroids of cadastral and subdivision parcels, and it is computed by the following Equation (8).

$$c_{ij} = \sqrt{(p_i^x - p_j^x)^2 + (p_i^y - p_j^y)^2} \quad (8)$$

where, p_i^x and p_i^y are the centroid coordinate (x and y value) of cadastral parcel i ; in the same way, p_j^x and p_j^y are the centroid coordinate (x and y value) of subdivision parcel j . The derived distance matrix is one of the main components of the optimization model. Later, the same matrix is used in the evaluation of the results.

The obtained optimization results are self-contained and sufficient to pursue a land reallocation application. Besides, the LRMTTP results also can be used for post evaluation of any actualized land allocation applications.

In the coming sections, first, three different optimization models are structurally introduced, and then an actual land reallocation application is evaluated in order to present the strength of proposed models.

• The Structure of Model I (Base Model)

Model I:

Minimize: Equation (1a)
 Subject to: Equations (2), (3), (4), (5)

Model I is designed to satisfy the Clause (a) of the 10th Article of the Regulation. This clause requires that land owners are entitle to the zoning based subdivision parcels, which are at the closest distance to their own cadastral parcel, and the created parcel(s) may be owned by more than one land owners with varying shares.

Model I minimizes the Equation (1a) containing total transport cost of transferred areas (m^2) from cadastral parcels to subdivision lots/parcels subject to Equations (2)-(5) providing quantities of demands (urban parcel, i , areas) and supplies (cadastral parcel areas, j , after deductions). In Equation (1a), the c_{ij} and x_{ij} are respectively the *distance matrix* and *delivered areas (m^2)* between the cadastral parcel and urban lot/parcel centroids. The sum of their products is the total costs of allocated land areas (m^2). The Z value in Equation (1a) is the total

transport cost ($\sum_{ij} x_{ij} \cdot c_{ij}$). The c_{ij} also later will be used to compute total transport cost of various allocation scenarios including subjective allocation practices.

Equation (2) states that sum of delivered areas (m^2) to urban lot/parcel i is required to be less than or equal to the lot sizes a_i . Similarly, Equation (3) states that the sum of delivered areas (m^2) from cadastral parcel (after public use deductions) j is required to be greater or equal to the lot sizes b_j . Then Equation (4) together with Equation (3) guarantees in and out balance, in other words, fulfills system equality ($\sum_i a_i = \sum_j b_j$). And finally, Equation (5) is the non-negativity constraint that it makes sure all delivered quantities be either zero or in positive quantities.

• The Structure of Model II

Model II:

Minimize: Equation (1a)
 Subject to: Equations (2), (3), (4), (5), (6)

Model II has a similar structure to Model I. The only difference comes from Equation (6) that is designed to fulfill the 8th Article, and Clause (b) of 10th Article of the Regulation. In this clause, those buildings complying with the current codes and plans can be retained and they are assigned into a single lot without shared ownership title. Due to small size cadastral land ownership structure, in case that single ownership solutions are not technically possible, condominium solutions also can be considered.

Equation (6) integrates predefined required allocations ($MCBR_{ij}$) into the model for the existing buildings, and automatically prioritizes them in allocations. $MCBR_{ij}$ is the allocation direction matrix mostly full of zeros except the required allocations. Equation (6) makes x_{ij} greater or equal to $MCBR_{ij}$, which is an initial assignments based on available buildings and associated subdivision lots in the land readjustment area. The remaining land resources optimally reallocated regarding the principles outlined in Model I in the body of Model II.

The total cost (Z) obtained from the optimization result is higher for Model II than the Model I due to required pre-assignments deviating from the optimization results of Model I.

• The Structure of Model III

Model III:

Minimize: Equation (1b)
 Subject to: Equation (2), (3), (4), (5), (6), (7-1), ..., (7-K)

Model III is different from both Model I and II in terms of programming technique; instead of Linear Programming (LP), Mixed Integer Programming (MIP) is the used optimization technique. The reason for that is the amalgamation of scattered properties in LR area. Although there is no written rule in the body of the Regulation, practitioners and local governments prefer amalgamation of scattered properties into less number and larger lots for single owner or family groups.

Model III have a different objective function and additional constraints using binary variables. Equation (1b) is a modified form of Equation (1a). In Equation (1b), every additional statement, such as $+(w_k \cdot \sum_j BIN_{2j} \cdot BS)$, has a corresponding equation in constraints [in Equations (7-1), ..., (7-K)]. Each equation (Equation (7-k)) presents a group of properties (hbk) desired to be amalgamated.

$BINK_j$ is a binary variable for each subdivision lot, which can be potentially selected as a target for amalgamation; If selected, it is 1, otherwise 0. In Equation (7-k) if sum of x_{ij} for shareholder

group k (hb_k) is greater than zero ($\sum_{i \in hb_k} x_{ij} > 0$) for any j , then the associated $BINK_j$ becomes 1. Those binary variables ($BINK_j$) are added in the minimized objective function with a product of a very large number (BS). The more scattered allocation from cadastral property i to subdivision lot j , the larger the objective value as an outcome of product of integer value and large number ($\sum_j BINK_j \cdot BS$). Because the problem is minimization problem, model automatically forces the binary variable $BINK_j$ to zero (0). However, instead of all $BINK_j$ are zero, due to Equation (2)-(6), at least one of the $BINK_j$ is required to be 1 or can be more regarding the sizes of available or designed subdivision lots. In other words, because total area transfers from cadastral parcels are equal to the total area of subdivision lots ($\sum_i a_i = \sum_j b_j$), there exist at least one $BINK_j$ for each group which needs to be amalgamated. Besides, minimization forces the integer variables, $BINK_j$, be minimally different from zero.

Furthermore, w_k is a weight parameter for amalgamation group k . The more weight for a group, the more privilege attributed to that group in assigning nearer-larger subdivision lots in the amalgamation process. In the selected case studies, for the simplicity, all weights are set to be equal.

Though the allocation results (x_{ij}^{OPT}) obtained from Model III are comparable with Model I and II, the total cost (Z) figures are naturally affected from the used large numbers (BS). In order to obtain comparable total cost (Z) results, optimization based allocation results, x_{ij}^{OPT} , are separately multiplied by the transport cost figures, c_{ij} , and their totals ($\sum_{ij} x_{ij}^{OPT} \cdot c_{ij}$) are obtained externally. Similarly, if there is an actual land readjustment case based on expert judgment rather than optimization, the total cost figures are computed externally ($\sum_{ij} x_{ij}^{SBJ} \cdot c_{ij}$) and used for comparison purposes. The comparable results are presented for the selected case study area.

III.2. Evaluation Measures

In order to present advantages and superiority of the developed *optimization* models over the *subjective* land reallocation practices, two different form of evaluation measures are developed: These area i) *Transfer Cost Measures* and ii) *Allocation Link Measures*. The *first* one is based on the various transfer cost ($x_{ij} \cdot c_{ij}$) measures, and the *second* one is based on the number of allocation links (count of positive x_{ij}).

a) Transfer Cost Measures

A series of *transfer cost measures* are developed and presented in Eqs. (9)-(13). All measures are based on **optimization** result and **subjective** allocation presenting the area (m^2) transfers (x_{ij}) and cost parameter measuring the unit cost of transferring the same area (c_{ij}). Equation (9) is for the minimized optimal total transfer cost - (TC^{OPT}); Equation (10) is for the total transfer cost from the *subjective* land reallocation solution - (TC^{SBJ}); Equation (11) measures the *difference* of optimal total transfer cost and subjective total transfer cost - (F); Equation (12) assesses total Absolute Value deviation (DV) between optimal land reallocation transfer matrix and subjective reallocation transfer matrix, element by element in absolute value; and Equation (13) calculates the ratio of total deviation (DV) to the total land readjustment area for transfer - (RA^{DV}). All equations are used as summary statistics in order to present the differences of Model I, II, and III from actual cases. They can be used in various scientific studies in order to evaluate characteristics of large number of land readjustment application.

$$TC^{OPT} = \sum_{i=1}^m \sum_{j=1}^n x_{ij}^{OPT} c_{ij} \quad (9)$$

$$TC^{SBJ} = \sum_{i=1}^m \sum_{j=1}^n x_{ij}^{SBJ} c_{ij} \quad (10)$$

$$F = TC^{OPT} - TC^{SBJ} \quad (11)$$

$$DV = \sum_{i=1}^m \sum_{j=1}^n |x_{ij}^{OPT} - x_{ij}^{SBJ}| \quad (12)$$

$$RA^{DV} = DV / AREA \quad (13)$$

b) Allocation Links Measures

A series of *allocation link measures* are developed and presented in Equations (14)-(18). Allocation link measures are based on the counts of links from cadastral properties to any subdivision lots. For any positive value in the allocation matrix, x_{ij} , the value is 1, otherwise 0. All following measures are based on a form of counts of transferred positive areas (m^2) ($x_{ij}>0$) of optimization and subjective allocation results. The Equation (14) is for the count of optimal transfer links - ($TLNK^{OPT}$); The Equation (15) is for the count of subjective transfer links - ($TLNK^{SBJ}$); Equation (16) measures the count difference of optimal and subjective transfer link numbers - ($FLNK$); Equation (17) transforms element by element deviation matrix between optimal land reallocation transfer matrix (x_{ij}^{OPT}) and subjective reallocation transfer matrix (x_{ij}^{SBJ}) into a total deviation link count (DV^{LNK}). All values different from 0 is converted into -1 and +1 via *signum function* and then they are converted into positive count values via *absolute value function* to attain an overall total afterward; and Equation (18) calculates the ratio of total deviation link count (DV^{LNK}) to the total reallocation matrix elements ($m \cdot n$) in percent - (RMX^{DV}).

$$TLNK^{OPT} = \sum_{i=1}^m \sum_{j=1}^n \text{sgn } x_{ij}^{OPT} \quad (14)$$

$$TLNK^{SBJ} = \sum_{i=1}^m \sum_{j=1}^n \text{sgn } x_{ij}^{SBJ} \quad (15)$$

$$FLNK = TLNK^{OPT} - TLNK^{SBJ} \quad (16)$$

$$DV^{LNK} = \sum_{i=1}^m \sum_{j=1}^n |\text{sgn}(x_{ij}^{OPT} - x_{ij}^{SBJ})| \quad (17)$$

$$RMX^{DV} = 100 \cdot DV^{LNK} / (m \cdot n) \quad (18)$$

In order to present the compactness of Model III results as compared to Model II results, Equation (19) is developed. If a share group *hbk* is amalgamated into a *j*, for that *j* there exist a 1 for count. If that share group *hbk* is amalgamated into two different *j*, there exist two 1 for count. If an allocation (optimal or subjective) is compact, total of counts (*TIP*) is expected to be lower. *Signum function* makes the total of number of rays getting into a single *j* equal to 1. The count of the amalgamating parcels are used for comparison of Model II and III. The lower the number the higher the compactness. In the same way, all models, including subjective one, can be compared mutually but especially to Model III.

$$TIP = \sum_j \left[\text{sgn} \left(\sum_{i \in hb1} x_{ij}^{opt} \right) \right] + \dots + \sum_j \left[\text{sgn} \left(\sum_{i \in hbK} x_{ij}^{opt} \right) \right] \quad (19)$$

IV. MODEL APLICATIONS

The developed models are applied to an actual land readjustment case in Turkey. In the case, the total LR area is 23829.88 m^2 and the final amount entered optimization models for reallocation is 16666.06 m^2 which is the area after 30.06% public land use deductions, 7163.82 m^2 . The total number of cadastral parcels and/or shareholders is 55, and the total number of urban parcels for allocation is 68. While Figure 2 and 3 presenting cadastral parcels and urban subdivision lots of the LR area, respectively, Table 1 provides associated necessary inputs for Model I, II, III applications, which are cadastral parcels and/or shareholder *id* numbers⁷ (*i*) and subdivision lot *id* numbers (*j*), and associated parcel centroid coordinates. These centroid coordinates will later be used to compute (via Equation 8) distance matrix which is used as cost parameters in matrix form (c_{ij}) for optimization operations. In the same table, there are also square meter land

⁷ If there is more than one shareholder in a cadastral parcels, each shareholder has a unique *id* number (*i*)

resources which are assigned for each shareholder and subdivision lots. In Table 1, the red segment of the table refers the cadastral parcels having more than one shareholders needs to be considered. The grey based shareholders' properties are selected to be amalgamated, and there are 4 different shareholder groups in this LR case.

• **Model I Results (Base Model)**

Model I is the basic model which transfers cadastral land resources to the standard urban parcels after proportional deduction of public land uses without considering existing buildings and required assignments. Amalgamation of scattered properties is not included into Model I, either. Model I allocation results (x_{ij}^{OPT}) are presented in Table 2. In order to be able to compare with the expert judgment based subjective LR, subjective allocations (x_{ij}^{SBJ}) are also presented in matrix form in Table 3. The *Transfer Cost Measures* and *Allocation Link Measures* are computed and later added in the result Column (1) of Table 6 for an overall evaluation and conclusion. For a clear and comprehensive presentation, Table 2 and 3 are presented in Figure 4 and 5 with rays from cadastral centroids to urban lot centroids, respectively. From these figures it is clear that optimization based allocations provides more compact, less scattered, and closer reallocations from cadastral to urban parcels as compared to subjective land reallocation case. Some of the differences can be explained by the built structures in the LR area. Those special cases are modeled in the coming section.

• **Model II Results (Required Allocations)**

Model II considers the available buildings and investments of land owners in the cadastral parcels. During the optimization process, those parcels having buildings on are directly assigned to the associated owners with designed and designated urban parcels, and the remaining cadastral properties optimally allocated to the remaining urban parcels. For the case study, the required allocation matrix ($MCBR_{ij}$) is presented in Table 4, and Model II optimization results (x_{ij}^{OPT}) considering the required allocation matrix ($MCBR_{ij}$) are presented in Table 5 with highlights. Table 5 later is graphically presented in Figure 6 for a comparative purpose with the Model I results. From Figure 6, it is clearly seen that required allocations are primarily considered (magenta lines) and the remainder are optimally allocated. The green colored lines in Figure 6 are a part of the optimal results, but the reason for presenting them in green is to distinguish Model III results with Model II later in the coming section. Those green colored lines are going to be redirected to less number of parcels (j) for amalgamations in Model III.

In sum, the required allocations can be considered as deviations from Model I. Due to forced allocations, *Total Transport Cost* (TC^{OPT}) increases as compared to the one in Model I (Table 6). The *Transfer Cost Measures* and *Allocation Link Measures* are aggregated in Table 6 for an overall evaluation and conclusion.

• **Model III Results (Scattered Land Amalgamation)**

Model III considers the scattered land areas owned by a single owner and/or a family group, and brings them together by an amalgamation. In the case area, this process is optimally actualized given the size limitations of urban parcels. Because additional constraints are added into Model II, it is seen that the *Total Transfer Cost* (TC^{OPT}) is the highest in all three models. Figure 7 is clearly presents the amalgamation when compared to Figure 6. Amalgamation is visually observed with the decreased number of green lines and directed less number of parcels (j) in Figure 7 (after) as compared to Figure 6 (before).

The *Transfer Cost Measures* and *Allocation Link Measures* from Model III applications are again added in Table 6 for an overall evaluation and conclusion.

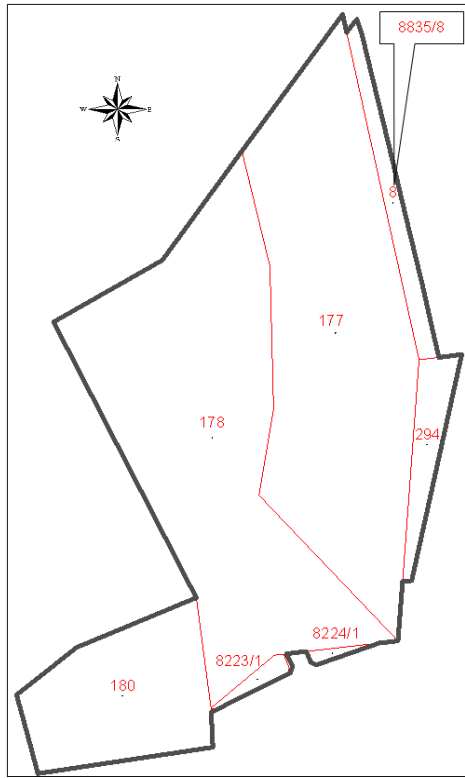


Figure 2: Land Readjustment area with cadastral parcel id numbers and centroids

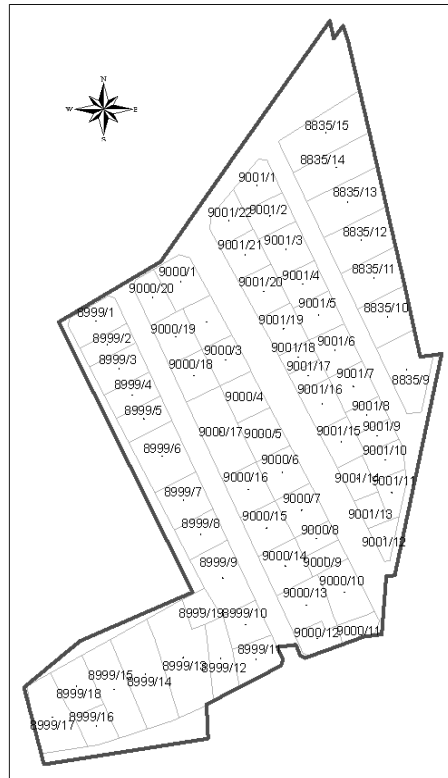


Figure 3: Land Readjustment area with urban subdivision lot id numbers and centroids

Table 1: Input values#

Cadastral Parcel	Area (m ²)	p ^x	p ^y	Urban Parcel	Area (m ²)	p ^x	p ^y
177-1	210	436333.82	4539921.51	8835-9	381.8	436366.06	4539907.6
177-2	193	436333.82	4539921.51	8835-10	300.15	436358.62	4539927.46
177-3	213	436333.82	4539921.51	8835-11	320.86	436353.45	4539942.18
177-4	161	436333.82	4539921.51	8835-12	376.47	436348.45	4539956.27
177-5**	1388	436333.82	4539921.51	8835-13	431.12	436343.38	4539970.55
.....	8835-14	392.27	436339.03	4539983.69
177-12***	87	436333.82	4539921.51	8835-15	499.13	436334.83	4539996.64
177-13	196	436333.82	4539921.51	8999-1	246.87	436246.49	4539925.99
177-14***	210	436333.82	4539921.51	8999-2	160.79	436250.88	4539917
177-15	140	436333.82	4539921.51	8999-3	239.16	436255.43	4539908.04
177-16****	102	436333.82	4539921.51	8999-4	190.46	436260.37	4539898.18
.....	8999-5	192.3	436264.87	4539889.16
178-8**	1073	436289.49	4539883.01	8999-6	384.04	436271.97	4539875.17
178-9	213	436289.49	4539883.01	8999-7	269.88	436280.25	4539858.61
.....	8999-8	213.1	436286.77	4539845.91
178-16***	135	436289.49	4539883.01	8999-9	384.04	436295.2	4539829.35
178-17****	111	436289.49	4539883.01	8999-10	205.1	436304.04	4539811.76
.....	8999-11	229.48	436308.04	4539798.94
294-2*	147	436366.87	4539880.47	8999-12	259.9	436294.42	4539799.12
294-3*	147	436366.87	4539880.47	8999-13	663.28	436279.98	4539799.5
294-4*	72	436366.87	4539880.47	8999-14	555.48	436265.45	4539793.34
294-5*	50	436366.87	4539880.47
294-6*	50	436366.87	4539880.47	9001-19	208.47	436318.66	4539923.01
8223-1	115	436305.64	4539794.09	9001-20	225.42	436311.04	4539937.21
8224-1	52	436332.81	4539803.76	9001-21	208.47	436303.8	4539950.85
8835-8	555	436354.62	4539969.34	9001-22	201.17	436297.66	4539963.67
Total	16673			Total	16666		

Due to paper size limit, some of the table is provided.

* Shareholders group 1 (hb1) for amalgamation in Model III.

** Shareholders group 2 (hb2) for amalgamation in Model III.

*** Shareholders group 3 (hb3) for amalgamation in Model III.

**** Shareholders group 4 (hb4) for amalgamation in Model III.

Table 2: Model I - Allocation matrix as a result of optimization (x_{ij}^{OPT})#

x_{ij}^{OPT}	8835-9	8835-10	8835-11	8835-12	8835-13	...	9001-21	9001-22	Total
177-1	210	0	0	0	0	...	0	0	210
177-2	0	0	0	0	0	...	0	0	193
177-3	0	0	0	0	57	...	0	0	213
177-4	0	0	0	0	161	...	0	0	161
177-5**	0	0	54	0	0	...	0	0	1388
177-6	0	0	0	264	0	...	0	0	264
177-7	0	0	0	0	0	...	0	201	213
177-8	0	0	267	112	0	...	0	0	400
177-9	2	0	0	0	0	...	0	0	140
177-10	4	0	0	0	0	...	0	0	210
177-11	0	300	0	0	1	...	0	0	301
177-12***	0	0	0	0	0	...	87	0	87
177-13	0	0	0	0	0	...	0	0	196
177-14***	0	0	0	0	0	...	0	0	210
177-15	0	0	0	0	0	...	0	0	140
177-16****	0	0	0	0	0	...	0	0	102
177-17	0	0	0	0	0	...	0	0	175
177-18	166	0	0	0	0	...	0	0	258
177-19	0	0	0	0	0	...	0	0	332
177-20	0	0	0	0	0	...	121	0	765
177-21	0	0	0	0	0	...	0	0	210
177-22	0	0	0	0	213	...	0	0	213
178-1	0	0	0	0	0	...	0	0	392
178-2	0	0	0	0	0	...	0	0	120
178-3	0	0	0	0	0	...	0	0	213
178-4	0	0	0	0	0	...	0	0	213
178-5	0	0	0	0	0	...	0	0	168
178-6	0	0	0	0	0	...	0	0	196
178-7	0	0	0	0	0	...	0	0	196
178-8**	0	0	0	0	0	...	0	0	1073
178-9	0	0	0	0	0	...	0	0	213
178-10	0	0	0	0	0	...	0	0	140
178-11	0	0	0	0	0	...	0	0	213
178-12	0	0	0	0	0	...	0	0	392
178-13	0	0	0	0	0	...	0	0	252
178-14	0	0	0	0	0	...	0	0	140
178-15	0	0	0	0	0	...	0	0	213
178-16***	0	0	0	0	0	...	0	0	135
178-17****	0	0	0	0	0	...	0	0	111
178-18	0	0	0	0	0	...	0	0	213
178-19	0	0	0	0	0	...	0	0	1056
178-20	0	0	0	0	0	...	0	0	425
178-21	0	0	0	0	0	...	0	0	213
178-22	0	0	0	0	0	...	0	0	182
178-23	0	0	0	0	0	...	0	0	425
180	0	0	0	0	0	...	0	0	2147
294-1	0	0	0	0	0	...	0	0	63
294-2*	0	0	0	0	0	...	0	0	147
294-3*	0	0	0	0	0	...	0	0	147
294-4*	0	0	0	0	0	...	0	0	72
294-5*	0	0	0	0	0	...	0	0	50
294-6*	0	0	0	0	0	...	0	0	50
8223-1	0	0	0	0	0	...	0	0	115
8224-1	0	0	0	0	0	...	0	0	52
8835-8	0	0	0	0	0	...	0	0	555
Total	382	300	321	376	431	...	208	201	16666

Due to paper size limit, some of the table is provided.

* Shareholders group 1 (*hb1*) for amalgamation in Model III.

** Shareholders group 2 (*hb2*) for amalgamation in Model III.

*** Shareholders group 3 (*hb3*) for amalgamation in Model III.

**** Shareholders group 4 (*hb4*) for amalgamation in Model III.

Table 3: Actual subjective land reallocation matrix via expert judgment (x_{ij}^{SBJ})#

X_{ij}^{SBJ}	8835-9	8835-10	8835-11	8835-12	8835-13	...	9001-21	9001-22	Toplam
177-1	0	0	0	0	0	...	0	0	206
177-2	0	0	0	0	0	...	0	0	189
177-3	0	0	0	0	0	...	0	0	208
177-4	158	0	0	0	0	...	0	0	158
177-5**	0	0	0	0	0	...	0	0	1361
177-6	0	259	0	0	0	...	0	0	259
177-7	0	0	0	0	0	...	0	0	208
177-8	0	0	0	0	0	...	0	0	392
177-9	0	0	0	0	0	...	0	0	137
177-10	0	0	0	0	0	...	0	0	206
177-11	0	0	0	295	0	...	0	0	295
177-12***	0	0	0	0	0	...	0	0	86
177-13	0	0	0	0	0	...	0	0	192
177-14***	0	0	0	0	0	...	0	0	206
177-15	0	0	0	0	0	...	0	0	137
177-16****	0	0	0	0	0	...	0	0	100
177-17	162	0	0	0	0	...	0	0	171
177-18	0	0	253	0	0	...	0	0	253
177-19	0	0	0	0	325	...	0	0	325
177-20	0	0	0	0	0	...	0	0	750
177-21	0	0	0	0	0	...	0	0	206
177-22	0	0	0	0	0	...	0	0	208
178-1	0	0	0	0	0	...	0	0	384
178-2	0	0	0	0	0	...	0	0	117
178-3	0	0	0	0	0	...	0	0	208
178-4	0	0	0	0	0	...	0	0	208
178-5	0	0	0	0	0	...	0	0	165
178-6	0	0	0	0	0	...	0	0	192
178-7	0	0	0	0	0	...	0	0	192
178-8**	0	0	0	0	0	...	0	0	1052
178-9	0	0	0	0	0	...	0	0	208
178-10	0	0	0	0	0	...	0	0	137
178-11	0	0	0	0	0	...	0	0	208
178-12	0	0	0	0	0	...	0	0	384
178-13	0	0	0	0	0	...	0	0	247
178-14	0	0	0	0	0	...	0	0	137
178-15	0	0	0	0	0	...	0	0	208
178-16***	0	0	0	0	0	...	0	0	132
178-17****	0	0	0	0	0	...	0	0	108
178-18	0	0	0	0	0	...	0	0	208
178-19	0	0	0	0	0	...	0	23	1036
178-20	0	0	0	0	0	...	0	0	417
178-21	0	0	0	0	0	...	208	0	208
178-22	0	0	0	0	0	...	0	178	178
178-23	0	0	0	0	0	...	0	0	417
180	0	0	0	0	0	...	0	0	2105
294-1	0	0	0	0	0	...	0	0	62
294-2*	0	0	0	0	0	...	0	0	145
294-3*	0	0	0	0	0	...	0	0	145
294-4*	0	0	0	0	0	...	0	0	71
294-5*	0	0	0	0	0	...	0	0	49
294-6*	0	0	0	0	0	...	0	0	49
8223-1	0	0	0	0	0	...	0	0	164
8224-1	0	0	0	0	0	...	0	0	74
8835-8	62	41	68	82	106	...	0	0	793
Toplam	382	300	321	376	431	...	208	201	16666

Due to paper size limit, some of the table is provided.

* Shareholders group 1 (*hb1*) for amalgamation in Model III.

** Shareholders group 2 (*hb2*) for amalgamation in Model III.

*** Shareholders group 3 (*hb3*) for amalgamation in Model III.

**** Shareholders group 4 (*hb4*) for amalgamation in Model III.

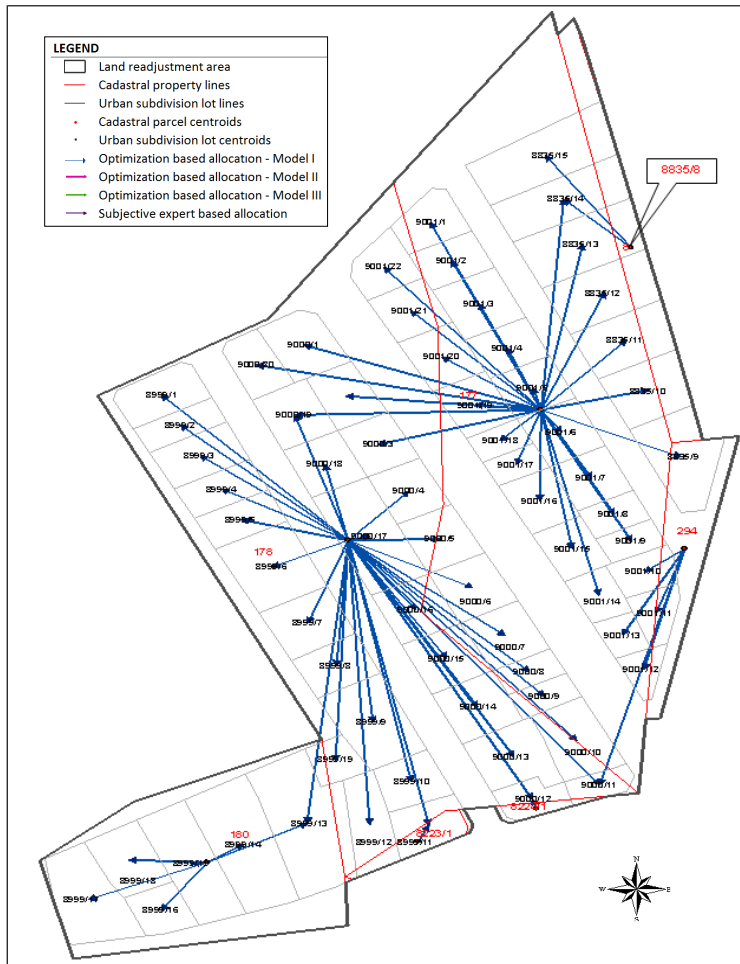


Figure 4: Objective (x_{ij}^{OPT}) - Model I based allocation presented by rays from cadastral to urban lots (Source: Model I - Table 2)

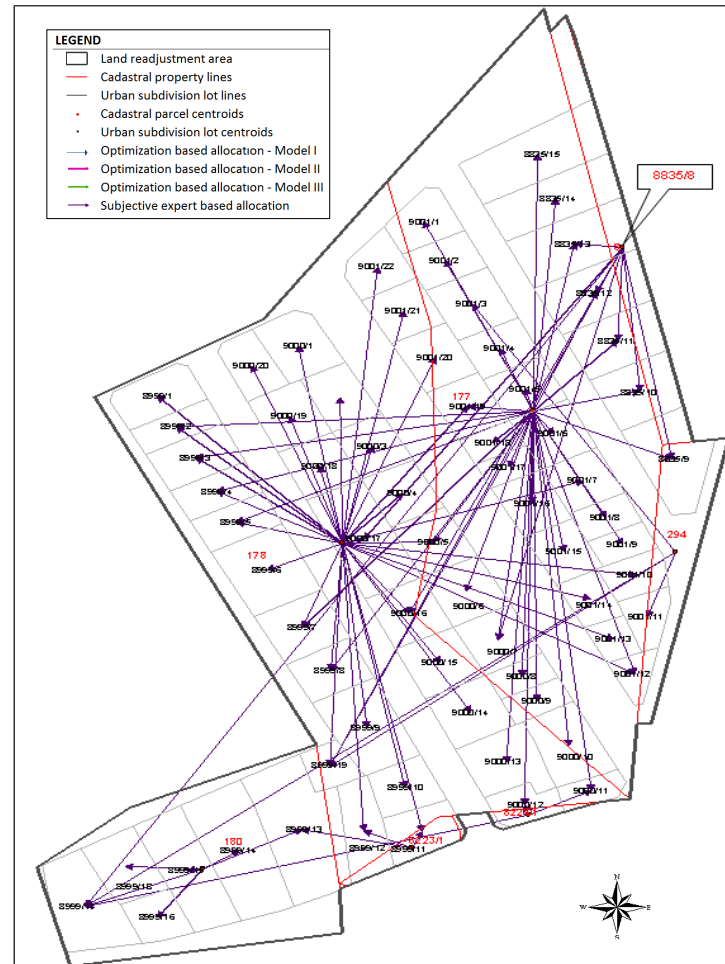


Figure 5: Subjective (x_{ij}^{SBJ}) - Actual land reallocation matrix via expert judgment presented by rays from cadastral to urban lots by rays (Source: Table 3)

Table 4: Required allocations matrix ($MCBR_{ij}$)#

MCBR _{ij}	8835-9	8835-10	8835-11	8835-12	8835-13	...	9001-21	9001-22	Toplam
177-1	0	0	0	0	0	...	0	0	210
177-2	0	0	0	0	0	...	0	0	193
177-3	0	0	0	0	0	...	0	0	213
177-4	0	0	0	0	0	...	0	0	161
177-5**	0	0	0	0	0	...	0	0	1388
177-6	0	264	0	0	0	...	0	0	264
177-7	0	0	0	0	0	...	0	0	213
177-8	0	0	0	0	0	...	0	0	400
177-9	0	0	0	0	0	...	0	0	140
177-10	0	0	0	0	0	...	0	0	210
177-11	0	0	0	0	0	...	0	0	301
177-12***	0	0	0	0	0	...	0	0	87
177-13	0	0	0	0	0	...	0	0	196
177-14***	0	0	0	0	0	...	0	0	210
177-15	0	0	0	0	0	...	0	0	140
177-16****	0	0	0	0	0	...	0	0	102
177-17	175	0	0	0	0	...	0	0	175
177-18	0	0	0	0	0	...	0	0	258
177-19	0	0	0	0	332	...	0	0	332
177-20	0	0	0	0	0	...	0	0	765
177-21	0	0	0	0	0	...	0	0	210
177-22	0	0	0	0	0	...	0	0	213
178-1	0	0	0	0	0	...	0	0	392
178-2	0	0	0	0	0	...	0	0	120
178-3	0	0	0	0	0	...	0	0	213
178-4	0	0	0	0	0	...	0	0	213
178-5	0	0	0	0	0	...	0	0	168
178-6	0	0	0	0	0	...	0	0	196
178-7	0	0	0	0	0	...	0	0	196
178-8**	0	0	0	0	0	...	0	0	1073
178-9	0	0	0	0	0	...	0	0	213
178-10	0	0	0	0	0	...	0	0	140
178-11	0	0	0	0	0	...	0	0	213
178-12	0	0	0	0	0	...	0	0	392
178-13	0	0	0	0	0	...	0	0	252
178-14	0	0	0	0	0	...	0	0	140
178-15	0	0	0	0	0	...	0	0	213
178-16***	0	0	0	0	0	...	0	0	135
178-17****	0	0	0	0	0	...	0	0	111
178-18	0	0	0	0	0	...	0	0	213
178-19	0	0	0	0	0	...	0	0	1056
178-20	0	0	0	0	0	...	0	0	425
178-21	0	0	0	0	0	...	208	0	213
178-22	0	0	0	0	0	...	0	0	182
178-23	0	0	0	0	0	...	0	0	425
180	0	0	0	0	0	...	0	0	2147
294-1	0	0	0	0	0	...	0	0	63
294-2*	0	0	0	0	0	...	0	0	147
294-3*	0	0	0	0	0	...	0	0	147
294-4*	0	0	0	0	0	...	0	0	72
294-5*	0	0	0	0	0	...	0	0	50
294-6*	0	0	0	0	0	...	0	0	50
8223-1	0	0	0	0	0	...	0	0	115
8224-1	0	0	0	0	0	...	0	0	52
8835-8	0	0	0	0	0	...	0	0	555
Toplam	175	265	0	0	332	...	208	0	16666

Due to paper size limit, some of the table is provided.

* Shareholders group 1 (*hb1*) for amalgamation in Model III.

** Shareholders group 2 (*hb2*) for amalgamation in Model III.

*** Shareholders group 3 (*hb3*) for amalgamation in Model III.

**** Shareholders group 4 (*hb4*) for amalgamation in Model III.

Table 5: Model II - Optimization results (x_{ij}^{OPT}) with required allocations ($MCBR_{ij}$)#

x_{ij}^{OPT}	8835-9	8835-10	8835-11	8835-12	8835-13	...	9001-21	9001-22	Toplam
177-1	0	0	0	0	0	...	0	0	210
177-2	0	0	0	0	0	...	0	0	193
177-3	0	0	0	0	0	...	0	0	213
177-4	0	0	0	0	93	...	0	0	161
177-5**	0	0	40	0	0	...	0	0	1388
177-6	0	264	0	0	0	...	0	0	264
177-7	0	0	0	0	0	...	0	0	213
177-8	0	0	32	368	0	...	0	0	400
177-9	0	0	0	0	0	...	0	0	140
177-10	17	0	0	0	0	...	0	0	210
177-11	0	36	249	0	0	...	0	0	301
177-12***	0	0	0	0	0	...	0	0	87
177-13	0	0	0	0	0	...	0	0	196
177-14***	0	0	0	0	0	...	0	0	210
177-15	0	0	0	8	0	...	0	0	140
177-16****	0	0	0	0	0	...	0	0	102
177-17	175	0	0	0	0	...	0	0	175
177-18	190	0	0	0	0	...	0	0	258
177-19	0	0	0	0	332	...	0	0	332
177-20	0	0	0	0	0	...	0	0	765
177-21	0	0	0	0	0	...	0	0	210
177-22	0	0	0	0	7	...	0	0	213
178-1	0	0	0	0	0	...	0	0	392
178-2	0	0	0	0	0	...	0	0	120
178-3	0	0	0	0	0	...	0	0	213
178-4	0	0	0	0	0	...	0	0	213
178-5	0	0	0	0	0	...	0	0	168
178-6	0	0	0	0	0	...	0	0	196
178-7	0	0	0	0	0	...	0	0	196
178-8**	0	0	0	0	0	...	0	0	1073
178-9	0	0	0	0	0	...	0	0	213
178-10	0	0	0	0	0	...	0	0	140
178-11	0	0	0	0	0	...	0	0	213
178-12	0	0	0	0	0	...	0	0	392
178-13	0	0	0	0	0	...	0	0	252
178-14	0	0	0	0	0	...	0	0	140
178-15	0	0	0	0	0	...	0	0	213
178-16***	0	0	0	0	0	...	0	0	135
178-17****	0	0	0	0	0	...	0	0	111
178-18	0	0	0	0	0	...	0	0	213
178-19	0	0	0	0	0	...	0	0	1056
178-20	0	0	0	0	0	...	0	0	425
178-21	0	0	0	0	0	...	208	0	213
178-22	0	0	0	0	0	...	0	0	182
178-23	0	0	0	0	0	...	0	0	425
180	0	0	0	0	0	...	0	0	2147
294-1	0	0	0	0	0	...	0	0	63
294-2*	0	0	0	0	0	...	0	0	147
294-3*	0	0	0	0	0	...	0	0	147
294-4*	0	0	0	0	0	...	0	0	72
294-5*	0	0	0	0	0	...	0	0	50
294-6*	0	0	0	0	0	...	0	0	50
8223-1	0	0	0	0	0	...	0	0	115
8224-1	0	0	0	0	0	...	0	0	52
8835-8	0	0	0	0	0	...	0	0	555
Toplam	382	300	321	376	431	...	208	0	16666

Due to paper size limit, some of the table is provided.

* Shareholders group 1 (*hb1*) for amalgamation in Model III.

** Shareholders group 2 (*hb2*) for amalgamation in Model III.

*** Shareholders group 3 (*hb3*) for amalgamation in Model III.

**** Shareholders group 4 (*hb4*) for amalgamation in Model III.

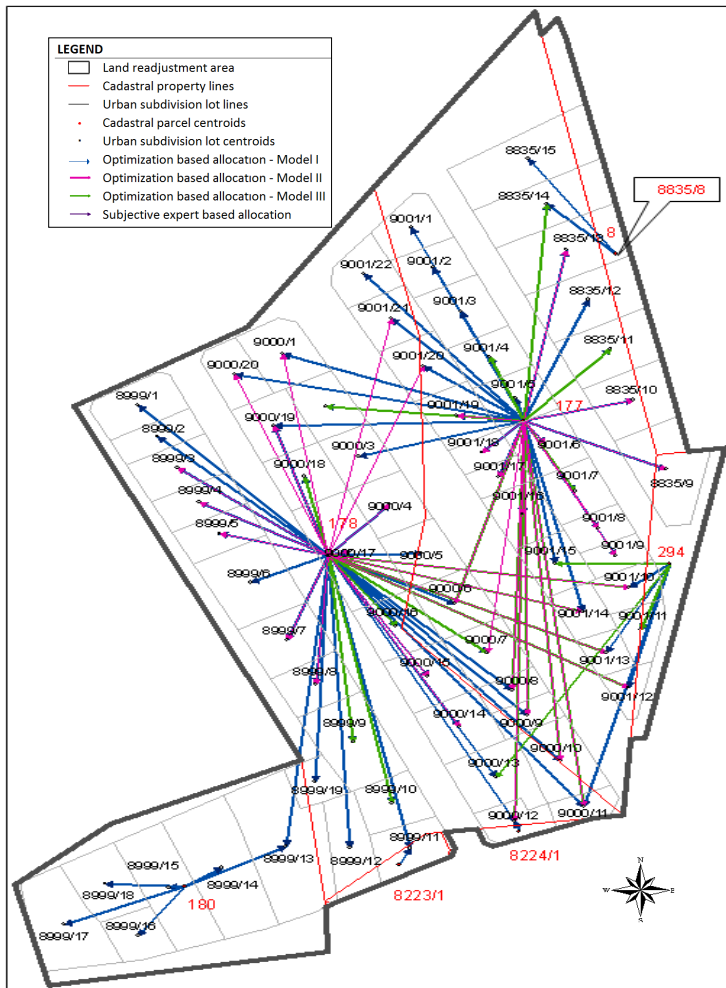


Figure 6: Model II results - Inclusion of required allocation into optimization (magenta lines) (x_{ij}^{OPT}) (Source: Table 5) (Green lines are the shareholder groups will be amalgamated in Model III)

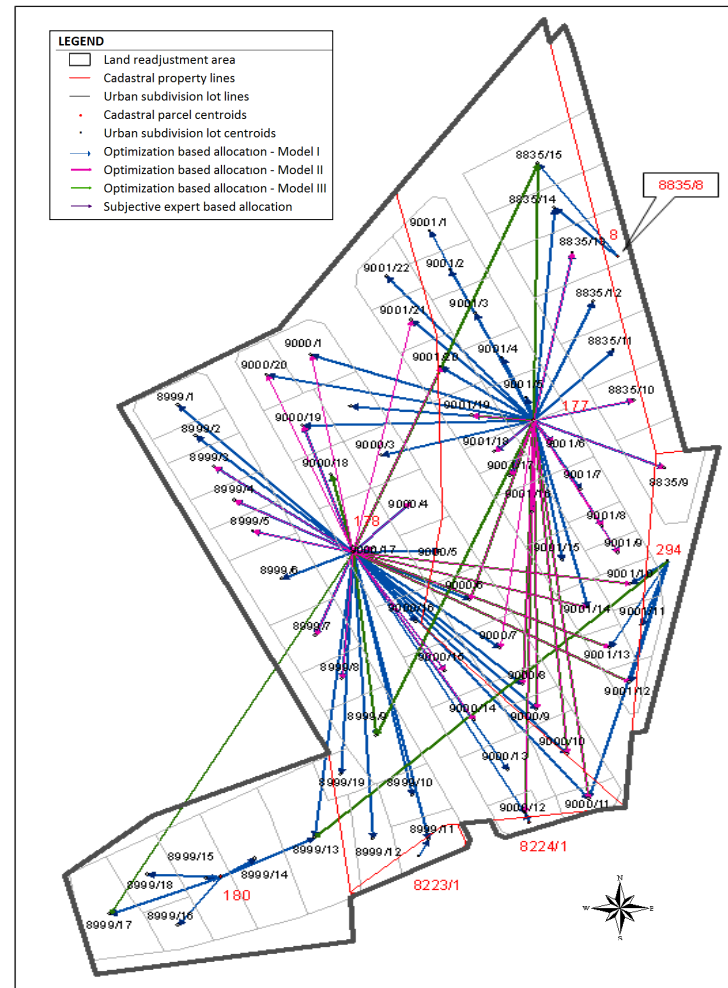


Figure 7: Model III results - Amalgamation of shareholder groups together with required allocations (x_{ij}^{OPT}) (Green lines are the shareholder groups are amalgamated)

- **Overall Evaluation**

Basic LR area input summary figures, the *Transfer Cost Measures* and *Allocation Link Measures* obtained from Model I, II, and III applications are aggregated in Table 6. While Model I, II, III results are presented in separate columns (1,2,3), the *Transfer Cost Measures* are presented in the first horizontal segment (via Equations 9-13), and the *Allocation Links Measures* are presented in the second horizontal segment (via Equations 14-19) of Table 6. All single measures are identified by the associated equation numbers. Table 6 not only comparatively presents the developed model applications (I,II,III) but also enables the comparison of optimization results to the actual subjective LR. The derived results from Table 6 and Figures 2-7 are as follows:

- Model results show the advantage of models regarding the Clause (a) of the 10th Article of the Regulation. This clause requires that *land owners take the zoning based subdivision parcels from the closest distance to their own cadastral parcel, and the created parcel(s) may be owned by more than one land owners with varying shares*. Regarding the values computed by Equation (9), Model I provides the lowest optimal *Total Transfer Cost* (TC^{OPT}), 629604. This value is followed by Model II (682450) and then Model III (746852). The reason for that is adding additional constraints requiring direct assignments which is a deviation from Model I, and in the same way, Model III adds binary variables and constraints to Model I and II. The *total transfer cost* of the actual subjective case (TC^{SBJ}) is computed as 815978, by means of Equation (10). This value is higher than the cost obtained from Model I, II, and III results. This shows the land reallocation capability of optimization models as compared to subjective judgment based land reallocation. Equation (11) presents the numeric difference between the subjective actual case and the optimization results. The differences get smaller when Model II and then III is applied. This implies that the optimization based allocations gets similar characteristics with subjective allocations. This is important for the inclusion of specific characteristics of LR area into the models.

While Figure 5 is prepared for the subjective actual allocations, Figure 4, 6, and 7 are drawn for Model I, II, and III optimization results, respectively. From the graphs, compactness of model results as compared to the subjective allocation, and slight deviations of Model II and III from Model I are clearly seen.

- Equation (12) is based on pairwise comparison of optimal (x_{ij}^{OPT}) and subjective (x_{ij}^{SBJ}) allocations. For each model application, the value differences are summed up to a total deviation and its ratio to the total LR area is computed. 1.68, 0.98, and 1.01 are the values for Model I, II, and III, respectively. The deviation is higher for Model I and lowest for Model II.
- The *Allocation Link Measures* calculated by Equations (14)-(16) do not provide very interesting results in terms of total link numbers. The values by Equation (17) shows significant decline in the deviation based total link numbers (DV^{LNK}) between optimization based models and the subjective allocations. The link deviation (DV^{LNK}) is the highest for Model I, but they are significantly lower for Model II and III. Because, we know that there are a large number of required transfers in the LR area, and they are considered in both Model II and III. Hence, the subjective approach and required links overlaps, and so deviation count decreases significantly as compare to Model I. Equation (18) normalizes the deviation link counts to the total matrix elements and the results are not very interesting.
- Equation (19) is prepared to monitor the effectiveness of Model III as an outcome of adding binary constraints to Model II. From the values, 26 scattered allocations are reduced to 17 scattered allocation by means of Model III. Because there is limited number of large parcels, the reduction is limited (only 9 links) as compared to the totals in Model II. In short, Model III successfully reduces the number of scattered allocations and so the links. This reduction can be seen in Figure 7 via decreased number of green lines as compared to Figure 6. For a larger LR area with multiple parties and parcels results are expected to be more prominent.

Table 6: Actual case and model results by means of the Land Readjustment Models

Equations		Descriptive Information and Computed Measures	Model I:	Model II:	Model III: Scattered
			Base Model (1)	Required Allocations (2)	Share Amalgamation (3)
		Total LR project area (m ²)	23829.88	23829.88	23829.88
		Area for public uses (m ²)	7163.82	7163.82	7163.82
		Actual area for LR allocation (m ²)	16666.06	16666.06	16666.06
		The total number of cadastral Properties and/or associated shareholders (m)	55	55	55
		The total number of urban parcels and/or subdivision lots (n)	68	68	68
i)	Equation (9)	$TC^{OPT} = \sum_{i=1}^m \sum_{j=1}^n x_{ij}^{OPT} c_{ij}$	629604	682450	746852
	Equation (10)	$TC^{SBJ} = \sum_{i=1}^m \sum_{j=1}^n x_{ij}^{SBJ} c_{ij}$	815978	815978	815978
	Equation (11)	$F = TC^{OPT} - TC^{SBJ}$	-186375	-133529	-69127
	Equation (12)	$DV = \sum_{i=1}^m \sum_{j=1}^n x_{ij}^{OPT} - x_{ij}^{SBJ} $	28037	16363	16782
	Equation (13)	$RA^{DV} = DV / AREA$	1.68	0.98	1.01
ii)	Equation (14)	$TLNK^{OPT} = \sum_{i=1}^m \sum_{j=1}^n \text{sgn } x_{ij}^{OPT}$	122	123	123
	Equation (15)	$TLNK^{SBJ} = \sum_{i=1}^m \sum_{j=1}^n \text{sgn } x_{ij}^{SBJ}$	117	117	117
	Equation (16)	$FLNK = TLNK^{OPT} - TLNK^{SBJ}$	5	6	6
	Equation (17)	$DV^{LNK} = \sum_{i=1}^m \sum_{j=1}^n \text{sgn}(x_{ij}^{OPT} - x_{ij}^{SBJ}) $	218	171	172
	Equation (18)	$RMX^{DV} = 100 \cdot DV^{LNK} / (m \cdot n)$	5.83	4.57	4.6
	Equation (19)	$TIP = \sum_j [\text{sgn}(\sum_{i \in hb1} x_{ij}^{opt})] + \dots + \sum_j [\text{sgn}(\sum_{i \in hbK} x_{ij}^{opt})]$		26 = (3+15+6+2)	17 = (1+12+2+2)

- It is noteworthy that varying weights (w_k) and large numbers (BS) are the parameters which may alter the reallocation for Model III. Their varying values can be source for various sensitivity analyses, which are beyond the scope of this study.

Finally, the optimization models satisfy the Clause (a) of the 10th Article of the Regulation. Transport problem based optimization models (Model I, II, III) technically work fine and provides a valuable quantitative approach in this area of study.

V. CONCLUSION

In this study, the developed techniques contain three authentic features:

The **first** feature is the original mathematical models based on operation research techniques. In this feature, there are two Linear Programming (LP) and one Mixed Integer Programming (MIP) models reallocating cadastral parcel areas into standard high quality urban subdivision lot(s) after proportional deduction of public land uses from all landowners under the frame of the Regulation. Within optimization feature, there are three different optimization models for different objectives: The *first* one fulfills the nearest distance reallocation principle under the 10th Rule in the Regulation. Within the same framework of the same 10th Rule, the *second* model additionally brings existing built structures and their owners together via incorporating compulsory reallocation equation. The *third* model incorporates equations providing privileges to the substandard scattered land areas of the same owner. How near and how big the amalgamation is a multi-objective programming issue that requires a series of optimization executions.

The **second** feature is about the evaluation criteria which are developed to present superiority of the optimization models over the existing expert based subjective practices in land reallocation. Those criteria are defined in the form of equations utilizing the allocation results (objective and subjective).

The **third** feature is the graphical presentation of allocation results in a system of rays from cadastral parcel centroids to urban subdivision lot centroids. This provides a succinct way of visual comparison among alternative (objective and subjective) allocation results.

Over the selected case study area the advantage and practicality of i) optimization models, ii) evaluation measures, and iii) graphical ways of presentation are presented. Several results need to be mentioned:

- Models carry very extensive application potentials not only in engineering, but also management of LR applications in social context. Model applications are not limited to urban lands, but also can easily applied in agricultural land readjustments.
- Models can be very effectively used not only in municipal operations but also in various court cases. Once models are applied in municipal and engineering context, a large number of court cases can proactively be prevented. LR applications via optimization models can be mathematically and visually introduced and proven to be almost ideal solutions to public. Any deviations from the optimization results can be brought to public attention with necessary justifications and explanations.
- The Models require different software platforms, these platforms later can be merged and used under a form of CAD. In this way, the models and the associated measures and graphical techniques have a potential of being a routine procedure in this area. Being a source of efficiency and effectiveness, this procedure can be used not only locally but also internationally..

And finally, this approach finds a quantitative rational solution to an area which is potentially prone to misuses and mistakes. The developed models and associated presentation and evaluation techniques are very valuable in this respect.

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