

Morita, Tadashi; Ago, Takanori; Tabuchi, Takatoshi; Yamamoto, Kazuhiro

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Endogenous Labor Supply and Trade

Takanori Ago* Tadashi Morita†

Takatoshi Tabuchi‡ Kazuhiro Yamamoto§

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Abstract

It is assumed in new trade theory and new economic geography that labor supply is perfectly inelastic, which is not true in real labor markets. We develop a model of new trade theory by incorporating elastic labor supply and study the effect of technology progress on working hours and economic welfare. We first show that the labor supply curve is backward bending. We then show that working hours in developed countries are longer in the early stages of development, while they are shorter in the late stages of development.

1 Introduction

The working hours are subject to important time-series and cross-sectional variations. The working hours have decreased in developed countries in 19th and 20th centuries after the Industrial Revolution. Maddison (1991) showed that annual working hours per person have been decreasing in the long term in developed countries. For example, the annual working hours per person were 2,984 in 1870 and 1,552 in 1989 in U.K., while 2,964 in 1870 and 1,604 in 1989 in the U.S.A. On the other hand, Maddison (1991) reported that the labor productivity has been increasing monotonically during the same periods. For example, the GDP per man-hour in 1985 U.S.A. prices was 2.15 in 1870 and 18.55 in 1989 in U.K., while 2.06 in 1870 and 23.87 in 1989 in the U.S.A. Thus, in accordance with the increase in labor productivity, the working hours have been decreased over time in these developed countries.

Blanchard (1994) documented that the total working hours in Europe were very short before the Industrial Revolution as compared to the recent years. They worked only 100-150 days a year. With the reduction in population and

*School of Commerce, Senshu University, E-mail address: ago@isc.senshu-u.ac.jp

†Faculty of Economics, Kindai University, E-mail address: t-morita@kindai.ac.jp

‡Faculty of Economics, University of Tokyo, E-mail address: ttabuchi@e.u-tokyo.ac.jp

§Graduate School of Economics, Osaka University. E-mail address: yamamoto@econ.osaka-u.ac.jp

a rise in wages in 15th-century England and the Netherlands, however, they worked only 80-100 days. The modern pattern of labor and leisure emerged only with the Industrial Revolution. The new norm was set about 10 hours a day and 300 days a year. In the process of Industrial Revolution, the working hours started to increase. Voth (2003) said: "During the Industrial Revolution, Europeans began to work longer-much longer. The age of the "dark satanic mills" saw adults toiling more than 3,200 hours per year, and child labor and women's work were common."

Voth (1998, 2003) reported that the number of annual working hours has risen sharply between during the second half of the 18th century: that the annual working hours per person in England was 2,763 in 1750, and 3,501 in 1800. Voth (2003) further showed that the working hours exhibit an inverted U-shaped curve: increasing in the 18th century and then decreasing in the 19th and 20th centuries in England. Ngai and Pissarides (2008) showed that before the 20th century, the working hours were, at least for a while, on an upward trend in the U.S.A.¹ Figure 1 displays the decreasing trends in working hours in many countries.

Then, we can say that after the beginning of the Industrial Revolution, the working hours first increased, and then decreased. In this paper, we derive such an inverted U-shaped movement of working hours in accordance with technological progress.

The working hours vary across countries. For example, the average working hours per person in 2012 were 1,393 in Germany, 1,430 in Denmark, 1,654 in U.K., and 1,790 in the U.S.A. In these countries, working hours are relatively short. On the other hand, the average working hour per person in 2012 was 2,029 in Chile, 2,034 in Greece, 2163 in Korea, 2,226 in Mexico. In these countries, working hours are relatively long. We will show in Figure 3 that working hours and labor productivity are negatively related. That is, the working hours are short in developed countries with high labor productivity, while the working hours are long in developing countries with low labor productivity. We will show this stylized fact and present a mechanism behind the relationship between working hours and labor productivity.

We construct a model in which the utility of a representative agent decreases with labor supply because each agent has a fixed time, which is allocated for labor and leisure. Each agent receives a wage by supplying labor to produce a variety of a differentiated good, which are exchanged and consumed by all agents. This is their incentive to work. On the other hand, if they work too much, they lose time for leisure, which decreases their utility. Therefore, there is a trade-off between labor and leisure.

When the labor productivity is enhanced due to technological progress, the nominal wage rate increases. This increase in the wage has two effects on labor supply: the substitution effect and the income effect. On the one hand, if the

¹de Vries (1994) called the increase in working hours in the 18th century in U.K. an "Industrious Revolution". He argued that since a variety of consumption goods has increased during this period, workers worked more in order to get more income for a variety of consumption goods.

wage is raised, the opportunity cost of leisure time is also raised. This is the substitution effect which lowers the labor supply of an agent. On the other hand, if the wage increases, the income of an agent grows, which raises the demand for leisure. This is the income effect, which raises the labor supply of an agent. When the wage rate is low, the substitution effect dominates the income effect. However, when the wage rate is sufficiently high, the income effect overwhelms the substitution effect. This is the mechanism of the so-called backward-bending labor supply (Robbins, 1930).

In the autarky economy, we study the effect of improvement in the labor productivity on labor supply. According to the above mechanism, technological progress enhances labor supply in the early stage of development, whereas it reduces labor supply in the late stage of development. This result is consistent with the stylized fact of the time-series variations in working hours: working hours first increased in the period of the Industrial Revolution, and then decreased after the Industrial Revolution. We then show that improvements in technological progress raises the welfare of agents monotonically.

We also study the effect of the population growth. The population growth expands the market size, which enlarges the number of varieties of the consumption good. When the level of production technology is low, the nominal wage is low and the number of varieties is small. In this stage, technological progress increases the number of varieties, and hence raises the labor supply, since substitution effect dominates the income effect. On the other hand, when the level of production technology is high, the wage rate becomes high and the number of varieties becomes large. In this stage, because the income effect is stronger than the substitution effect, the population growth decreases the labor supply.

We extend the autarky economy to the open economy with two countries of equal population size. We first characterize the symmetric equilibrium with trade. We then study the case of different labor productivities. We show that the labor supply in developed countries is larger in the early stage of development, while it is smaller in the late stage of development. This is consistent with the cross-sectional variations of the working hours in recent years, where the working hours are shorter in more developed countries.

There are some literatures on endogenous labor supply. King, Plosser and Rebelo (1988) and Rebelo (1991) considered endogenous labor supply in order to study the business cycles and endogenous growth. Turnovsky (2000) studied the effects of government policies under endogenous labor supply. Duranton (2001) constructed an overlapping generation model with endogenous labor supply. These literatures argued that labor supply is elastic with respect to the wage rate and the utility of agents decreases in the working hours (increases with leisure time). Following the established tradition of the literatures, we assume that the utility is decreasing in the working hours and investigate the effects of technological progress on the working hours etc. We derive that the labor supply curve is backward-bending. The backward-bending labor supply curve is often observed in the labor economics literatures, such as Blundell et al. (1992), Borjas (2010), and Cahuc and Zylberberg (2004).

The organization of this paper is as follows. In the next section, we propose

a benchmark model of one-country economy and solve the number of firms, working hours, and the economic welfare of each agent. Open economy of two countries with costly trade is presented in section 3 and the symmetric equilibrium is investigated. We then analyze the international differentials in the working hours and in the wage rate in section 4. Section 5 concludes.

2 One-country economy

We first construct a model with one country with a mass of population, L . A representative agent gets a utility from the consumption of varieties of a differentiated good and a disutility from the amount of labor supply. The utility function of each agent is given by

$$U = \alpha \int_0^n x(i) di - \frac{\beta}{2} \int_0^n x(i)^2 di - \frac{\gamma}{2} \left[\int_0^n x(i) di \right]^2 - l, \quad (1)$$

where $x(i)$ is the consumption of a variety indexed $i \in [0, n]$ of the good, n is the number of varieties, l is the amount of labor supply, $\alpha > 0$, $\beta > 0$, and $\gamma > 0$. The first three terms of (1) are the utility from consumption of varieties, while the last term is the disutility from supply of labor. Each agent controls the amount of labor supply as well as consumption of the varieties, and receives the wage by supplying labor. The budget constraint of each agent is given by

$$wl = \int_0^n p(i)x(i) di, \quad (2)$$

where w is the nominal wage per unit of labor and $p(i)$ is the price of variety i .

Substituting l of (2) into (1) and differentiating it with respect to $x(i)$, we get the first-order conditions for utility maximization²

$$\alpha - \beta x(i) - \gamma \int_0^n x(i) di = \frac{p(i)}{w}. \quad (3)$$

Integrating (3) yields

$$\alpha n - \beta \int_0^n x(i) di - \gamma n \int_0^n x(i) di = \frac{P}{w}, \quad (4)$$

where $P \equiv \int_0^n p(i) di$ is the price index. Solving (4) for $\int_0^n x(i) di$ and plugging it into (3), we have the demand function for variety i

$$x(i) = \frac{\alpha\beta w + \gamma P}{\beta(\beta + \gamma)n w} - \frac{1}{\beta w} p(i). \quad (5)$$

Since the demand (5) involves the wage w , there exists the income effect in this utility function unlike the quasilinear utility developed by Ottaviano et al.

²The second-order conditions are also satisfied.

(2002). Differentiating $x(i)$ with respect to w leads to

$$\frac{\partial x(i)}{\partial w} = \frac{1}{\beta(\beta + \gamma n)w^2} [(\beta + \gamma n)p(i) - \gamma P].$$

When price of all varieties are the same ($p(i) = p(j)$, $\forall i, j$), the income effect is positive $\partial x(i)/\partial w > 0$.³

Substituting (5) into the budget constraint (2) yields the supply of labor:

$$l^S = \frac{\alpha}{(\beta + \gamma n)} \frac{P}{w} - \frac{\beta(S + 1)/n + \gamma S}{\beta(\beta + \gamma n)} \frac{P^2}{w^2}, \quad (6)$$

where

$$S \equiv \frac{n \int_0^n p(i)^2 di}{[\int_0^n p(i) di]^2} - 1 = \frac{1}{2P^2} \int_0^n \int_0^n [p(i) - p(j)]^2 didj \geq 0.$$

Observe that the first term of (6) is positive, the second term is negative. This suggests that the labor supply function is *backward-bending*: it is first increasing, and then decreasing in w . Differentiating l^S of (6) with respect to w leads to

$$\frac{\partial l^S}{\partial w} \begin{matrix} \geq 0 \\ < 0 \end{matrix} \Leftrightarrow \frac{w}{p} \begin{matrix} \leq \\ \geq \end{matrix} \frac{2(\beta + \beta S + \gamma n S)}{\alpha \beta}.$$

An increase in the nominal wage w or the real wage w/p has two effects on the supply of labor, which are called the *substitution effect* and the *income effect*. On the one hand, when the wage goes up, consumers increase labor supply in order to purchase more quantities of the varieties. Since w is the opportunity cost of leisure, rising w lowers the leisure time, which turns to increase the labor supply. This is the substitution effect because the leisure is substituted by the varieties of the good. On the other hand, when the wage goes up, the nominal income also goes up, and thus, consumers may be able to increase consumption of both the varieties and leisure. This is the income effect. It can be shown that when the real wage w/p is small, the substitution effect is stronger than the income effect so that $\partial l^S/\partial w > 0$ holds. When w/p is large, the income effect outweighs the substitution effect so that $\partial l^S/\partial w < 0$ holds.

Turning to the production side, in order to produce $x(i)$ units of a differentiated good $mx(i) + f$ units of labor are needed. The marginal labor requirement m may be regarded as an inverse measure of production technology so that falling the marginal labor requirement m implies technological progress in production. The profit of a manufacturing firm producing variety i is the revenue $R(i)$ minus the wage bill $wl^D L/n$:

$$\begin{aligned} \pi(i) &= R(i) - wl^D L/n \\ &= p(i)x(i)L - w[mx(i)L + f]. \end{aligned} \quad (7)$$

³However, if $p(i)$ is low relative to the average price P/n , there is a possibility that $\partial x(i)/\partial w < 0$, implying that rising income shrinks consumption of variety i .

where l^D is the labor demand per capita and $l^D L/n$ is the labor demand per firm. Solving the first-order condition yields the profit-maximizing price as

$$p(i) = \frac{\alpha\beta w + \gamma P}{2(\beta + \gamma n)} + \frac{mw}{2}. \quad (8)$$

Since each firm sets the same price under the same production technology, we drop i hereafter. Solving $P = np$ and (8) simultaneously, we obtain the equilibrium price

$$p = \frac{\alpha\beta + m(\beta + \gamma n)}{2\beta + \gamma n} w \quad (9)$$

given the number of firms n . Substituting it into (5), we have

$$x = \frac{\alpha - m}{2\beta + \gamma n}, \quad (10)$$

from which we require $\alpha > m$ in order to guarantee positive demand. From (9), we readily have $\partial p/\partial n < 0$ under $\alpha > m$, which verifies existence of the *procompetitive effect*: more entries of firms make competition keener and the price lower. We also know from (9) and (10) that $\partial(w/p)/\partial m < 0$ and $\partial x/\partial m < 0$, implying that the technological progress enhances the nominal wage, real wage and the consumption of each variety.

Because the budget constraint (2) is $wl^S = np x$, the revenue of a firm can be rewritten as $R = p x L = w l^S L/n$. Hence, the profit of a firm is

$$\pi(i) = \frac{wL}{n} (l^S - l^D),$$

which expresses that the profit of a firm is positive under excess supply of labor while it is negative under excess demand for labor.

Plugging (9) and (10) into the profit (7) can be rewritten as

$$\pi = \frac{(\alpha - m)^2 \beta L - f(2\beta + \gamma n)^2}{(2\beta + \gamma n)^2} w.$$

Solving $\pi = 0$ yields the equilibrium number of firms

$$n^* = \frac{(\alpha - m)\sqrt{\beta f L} - 2\beta f}{\gamma f} \quad \text{for } m \in (0, \bar{m}), \quad (11)$$

where $\bar{m} \equiv \alpha - 2\sqrt{\beta f/L}$. The equilibrium number n^* of firms is zero for $m \geq \bar{m}$. Assuming the following ad hoc dynamics

$$\dot{n} = \pi,$$

the above equilibrium n^* is unique and stable because $d\pi/dn < 0$ holds for all n .

When m exceeds $\alpha - 2\sqrt{\beta f/L}$, no good is produced because the opportunity cost of labor as the benefit of leisure is larger than the benefit of consuming

varieties of the differentiated good. When m falls to \bar{m} , production begins. Then, falling m owing to technological progress raises the equilibrium number of firms. The number of firms and varieties is also decreasing in the fixed labor requirement f , which is also due to technological progress. Hence, we can conclude that *in accordance with technological progress, the number of firms and varieties increases*.

Since the labor demand is $(mx + f)n$, plugging (10) and (11) into (6), we have the equilibrium labor supply

$$l^* = \frac{1}{\gamma} \left(m + \sqrt{\beta f / L} \right) \left(\alpha - m - 2\sqrt{\beta f / L} \right) \quad \text{for } m \in (0, \bar{m}).$$

The equilibrium amount of labor l^* is zero for $m \geq \bar{m}$. This is in accord with the equilibrium number n^* of firms shown in the above. Therefore, if production technology is low enough such that $m \geq \bar{m}$, then no one has an incentive to work and no firm enters the market.

The equilibrium amount of labor l^* is inverted U-shaped in m if $\alpha > 3\sqrt{\beta f / L}$ as follows:

$$\frac{\partial l^*}{\partial m} \geq 0 \Leftrightarrow m \leq \frac{\alpha - 3\sqrt{\beta f / L}}{2}. \quad (12)$$

That is, falling m due to technological progress first increases the labor supply and then decreases it. However, if $\alpha \leq 3\sqrt{\beta f / L}$, the phase of $\partial l^* / \partial m \geq 0$ does not appear, implying that the technological progress always decreases the supply of labor. Since this does not account for the increasing labor supply during the Industrial Revolution, we assume $\alpha > 3\sqrt{\beta f / L}$ hereafter. When this inequality is satisfied, the demand for the manufacturing good is large, and hence, many firms enter the market ($\partial n^* / \partial \alpha > 0$ from (11)). This leads to the large real wage w/p , which ensures dominance of the income effect over the substitution effect, which is explained in the above.

Given large demand for the manufacturing good, there exists an inverted U-shaped relationship between the technological progress and the labor supply as follows.

Proposition 1 *Technological progress raises the labor supply at the early stage of development $m \in \left((\alpha - 3\sqrt{\beta f / L}) / 2, \bar{m} \right)$, whereas it reduces the labor supply at the late stage of development $m \in \left(0, (\alpha - 3\sqrt{\beta f / L}) / 2 \right)$.*

There are two effects of technological progress on the supply of labor. The first effect is that enhancing labor productivity increases the number of varieties, which raises the incentive to work. The second effect is that enhancing labor productivity decreases the prices of varieties, which raises the value of leisure, and thus reduces the incentive to work. Proposition 1 implies that when m is large, the first effect is dominant so that the labor increases according to technological progress as in the Industrial Revolution. However, when m is small, the second effect is dominant as observed after the Industrial revolution.

So far we have been focusing on m as the inverse measure of the technological progress. It is also important to consider the size of population, L , from a historical point of view. In order to study the effect of population growth on the working hours, we differentiate l^* with respect to L as follows

$$\frac{\partial l^*}{\partial L} = \frac{\sqrt{\beta f/L}}{2\gamma L} (3m - \alpha + 4\sqrt{\beta f/L}).$$

It depends on the level of production technology m :

$$\frac{\partial l^*}{\partial L} \geq 0 \Leftrightarrow m \geq \hat{m} \equiv \frac{\alpha - 4\sqrt{\beta f/L}}{3}.$$

Because $\alpha > 2\sqrt{\beta f/L}$, we have $\hat{m} = (\alpha - 4\sqrt{\beta f/L})/3 < \alpha - 2\sqrt{\beta f/L} = \bar{m}$. Hence, if $\alpha > 4\sqrt{\beta f/L}$, then there exists \bar{m} in the interval of $(0, \bar{m})$. The population growth increases the variety of the consumption good, since the market size expands with population size. When m is high, the nominal wage is low and the number of varieties is also low. In this case, the increase in consumption variety raises the labor supply, since substitution effect overcomes the income effect. On the other hand, when m is low, the wage rate is high and the variety of consumption goods is large. In this case, since the income effect is stronger than the substitution effect, population growth decreases the labor supply.

Proposition 2 (i) If $\alpha \leq 4\sqrt{\beta f/L}$, population growth always raises the labor supply

(ii) If $\alpha > 4\sqrt{\beta f/L}$, population growth raises the labor supply at the early stage of development $m \in (\hat{m}, \bar{m})$, whereas it reduces the labor supply at the late stage of development $m \in (0, \hat{m})$.

Proposition 2 is consistent with the historical experience of population growth and working hours. The population started to grow during the period of the Industrial Revolution, where the production technology is low (m large). In this phase, this population growth increased the working hours. However, the Industrial Revolution brought about the technological progress, which lowered the marginal labor requirement m , while the population kept growing. Thus, the population growth reduced working hours in the late stage of development.

2.1 Welfare

The indirect utility function is derived as follows:

$$V = \frac{1}{2\gamma} (\alpha - m - \sqrt{\beta f/L}) (\alpha - m - 2\sqrt{\beta f/L}).$$

Since $m \in (0, \bar{m})$, we can easily show that $\partial V/\partial L > 0$ and $\partial V/\partial m < 0$.

Hence, we can state the following.

Proposition 3 *The welfare rises in accordance with population increase and technological progress.*

The intuitions behind the proposition are straightforward. The increase in population implies expansion of the market. Because the number of varieties increases, consumers enjoy a wide array of varieties of the good. As to the technological progress, falling m or f encourages entry of firms and raises the real wage w/p . This would broaden the consumption possibility frontier, which always benefits consumers. To be more precise, rising labor productivity increases the number of varieties and decreases the prices of varieties, both of which contribute to the welfare gain.

3 Open economy with two countries

Thus far, we have been focusing on one country. In order to examine the impacts of international trade, we consider two countries 1 and 2. Extension to an arbitrary number of countries is straightforward if each country is symmetrically treated. While firms can enter, exit, and move between the countries, consumers are immobile following the established tradition of new trade theory a la Krugman (1980). The profit of a firm in country $r (= 1, 2)$ is now rewritten as

$$\begin{aligned} \max_{p_{rr}, p_{rs}} \pi_r &= R_{rr} + R_{rs} - w_r l_r^D \\ &= p_{rr} x_{rr} L_r + p_{rs} x_{rs} L_s - w_r [m(x_{rr} L_r + \tau x_{rs} L_s) + f], \end{aligned} \quad (13)$$

where $\tau \geq 1$ is the iceberg transport cost: τ units have to be shipped for one unit to reach another country. The population is equal between countries in order to examine the symmetric equilibrium and is normalized to $L_r = L_s = 1$.

The budget constraint of each agent is given by

$$w_r l_r^S = n_r p_{rr} x_{rr} + n_s p_{sr} x_{sr}$$

Because the GNP $n_r (R_{rr} + R_{rs})$ is equal to the gross national expenditure $w_r l_r^S L_r$ in country r , we get

$$n_r (p_{rr} x_{rr} L_r + p_{rs} x_{rs} L_s) = (n_r p_{rr} x_{rr} + n_s p_{sr} x_{sr}) L_r.$$

This is simplified as

$$n_r p_{rs} x_{rs} L_s = n_s p_{sr} x_{sr} L_r, \quad (15)$$

which shows the trade balance: the export equals the import.

Using (15), the profit (13) of a firm in country r can be rewritten as

$$\begin{aligned} \pi_r &= p_{rr} x_{rr} L_r + \frac{n_s p_{sr} x_{sr} L_r}{n_r} - w_r l_r^D \frac{L_r}{n_r} \\ &= \frac{w_r L_r}{n_r} (l_r^S - l_r^D) \end{aligned}$$

and that in country s is

$$\pi_s = \frac{w_s L_s}{n_s} (l_s^S - l_s^D).$$

As in the one-country economy, the profit is positive under excess supply of labor and is negative under excess demand for labor.

The demand (5) for the differentiated good is given by

$$x_{rs} = \frac{\alpha\beta w_s + \gamma P_s}{\beta[\beta + \gamma(n_r + n_s)]w_s} - \frac{1}{\beta w_s} p_{rs}. \quad (16)$$

Trade takes place only if the demand x_{rs} given by (16) is positive. Otherwise, it is zero.

Solving the first-order conditions $\partial\pi_r/\partial p_{rr} = \partial\pi_r/\partial p_{rs} = \partial\pi_s/\partial p_{sr} = \partial\pi_s/\partial p_{ss} = 0$ together with

$$P_r = n_r p_{rr} + n_s p_{sr}, \quad s \neq r,$$

we obtain the equilibrium prices

$$\begin{aligned} p_{rr}^* &= \frac{[2\beta(\alpha + m) + \gamma m(2n_r + n_s)]w_r + \tau\gamma m n_s w_s}{2[2\beta + \gamma(n_r + n_s)]}, \\ p_{sr}^* &= p_{rr}^* + \frac{m}{2}(\tau w_s - w_r), \quad \text{for } s \neq r. \end{aligned} \quad (17)$$

Substituting (16) and (17) into (13), we have the two free entry conditions:

$$\pi_r^*(w_r, w_s, n_r, n_s) = 0, \quad (18)$$

which are the *spatial equilibrium conditions*. Setting $w_s = 1$ and $w_r = w$, there are three unknowns are n_1 , n_2 and w , which are to be determined by the three equilibrium conditions (15) and (18) for $r = 1, 2$.

As in the one-country economy, firms enter the market if the profit is positive and exit the market if the profit is negative. An ad hoc dynamics may be expressed as

$$\dot{n}_r = \pi_r \quad (19)$$

for $r = 1, 2$.

3.1 Symmetric equilibrium

Because the population is the same between the two countries, the obvious symmetric equilibrium is defined by

$$\mathbf{sym} = \{n_r = n_s = n^*, w_r = w_s = 1\}.$$

Substituting this into (18) yields the equilibrium number of firms

$$n^*|_{\mathbf{sym}} = \frac{\beta}{\gamma A} [2\alpha - (\tau + 1)m - A],$$

where $A \equiv \sqrt{8\beta f - (\tau - 1)^2 m^2}$.

We assume $n^*|_{\mathbf{sym}} > 0$ for positive production under symmetry. Therefore, we assume that

$$2\alpha - (\tau + 1)m - A > 0. \quad (20)$$

The curve $n^*|_{\mathbf{sym}} = 0$ is drawn in Figure 2. The demand for the differentiated good should also be positive for trade to take place. Plugging (17) into (16) with \mathbf{sym} , we have

$$x_{rs}^*|_{\mathbf{sym}} > 0 \Leftrightarrow m < \frac{2\sqrt{\beta f}}{\tau - 1}. \quad (21)$$

The curve $x_{rs}^*|_{\mathbf{sym}} > 0$ is depicted in Figure 2. Hence, the two conditions (20) and (21) should be met for the symmetric equilibrium with trade to exist.

Unlike the autarkic equilibrium n^* , the symmetric equilibrium with trade $n^*|_{\mathbf{sym}}$ depends on the trade cost τ . The sign of

$$\frac{\partial n^*|_{\mathbf{sym}}}{\partial \tau} = \frac{2\beta m}{\gamma A^{\frac{3}{2}}} [(\alpha - m)m(\tau - 1) - 4\beta f]$$

is positive for large τ and negative for small τ . That is, falling the trade cost first reduces the number of firms with trade because of keen international competition. However, falling the trade cost then raises the number of firms so that consumers enjoy wide varieties of the differentiated good.

The equilibrium amount of labor is computed as

$$l^* = \frac{1}{8\gamma A} [2\alpha - m(\tau + 1) - A] \left[A^2 + 2m(\tau + 1)A - m^2(\tau - 1)^2 \right].$$

This is shown to be positive insofar as (21) is satisfied. As is inferred from the one country case, the amount of labor l^* is also inverted U-shaped in m . That is, Proposition 1 also holds in the case of two countries with trade.

Likewise, the indirect utility is shown to be decreasing in marginal labor requirement m as in the case of one-country economy (Proposition 3). Furthermore, the indirect utility is shown to be decreasing in the trade cost τ . That is,

Proposition 4 *The welfare rises in accordance with technological progress in production. It also rises as the trade cost declines.*

The stability of symmetric equilibrium can be checked as follows. Totally differentiating the RHS of (19) with respect to n_1 and evaluating it at \mathbf{sym} , we obtain the Jacobian:

$$\begin{aligned} \left. \frac{d\pi_r}{dn_s} \right|_{\mathbf{sym}} &= \left. \frac{\partial \pi_r}{\partial n_s} + \frac{\partial \pi_r}{\partial w_r} \frac{\partial w_r(n_r, n_s)}{\partial n_s} \right|_{\mathbf{sym}} \\ &= \left. \frac{\partial \pi_r}{\partial n_s} + \frac{\partial \pi_r}{\partial w_r} \left(-\frac{\partial B / \partial n_s}{\partial B / \partial w_r} \right) \right|_{\mathbf{sym}}, \end{aligned}$$

where $w_r(n_r, n_s) = 0$ is the implicit function of (15). Computing the eigenvalues, we can show that *the symmetric equilibrium is always asymptotically stable*. On the other hand, we can show that agglomerated configuration is not an equilibrium, which is in a sharp contrast with new trade theory a la Krugman (1980).

4 International differentials

In the case of symmetric setting of countries, there is no international difference in symmetric equilibrium. However, in the case of asymmetric setting, the equilibrium number of firms, wages, working hours, and welfare are different across countries. In order to investigate such differentials, we conduct comparative statics in the vicinity of symmetric equilibrium.

Assume that country 1 is more developed in the sense that the marginal labor requirement is less: $m_1 < m_{22}$, where m_r is the marginal labor requirement in country r . We are interested in the international differences in the working hours $\Delta l \equiv l_1 - l_2$ and the nominal wage $\Delta w \equiv w_1 - w_2$.

4.1 Differential in working hours

In order to investigate the differential in the working hours Δl analytically, we focus on derivatives in the neighborhood of the symmetric equilibrium as follows. The marginal change of m_2 about the symmetry $\mathbf{sym} = (n_1, w, m_1, m_2) = (n^*|_{\mathbf{sym}}, 1, m, m)$ can be computed as

$$\left. \frac{d\Delta l}{dm_2} \right|_{\mathbf{sym}} = \frac{\partial \Delta l}{\partial m_2} + \frac{\partial \Delta l}{\partial n_1} \frac{\partial n_1}{\partial m_2} + \frac{\partial \Delta l}{\partial w} \frac{\partial w}{\partial m_2} \Big|_{\mathbf{sym}}.$$

There are three endogenous variables n_1 , n_2 and w , and there are three equilibrium conditions: (15) and (18) for $r = 1, 2$. Subtracting (18) for $r = 1$ from (18) for $r = 2$ yields a linear function of n_2 . Solving it for n_2 and substituting it into (15) and (18) for $r = 1$, the three equilibrium conditions can be reduced to two equations $E_1(n_1, w) = 0$ and $E_2(n_1, w) = 0$ with two endogenous variables n_1 and w . From the standard comparative statics,

$$\begin{pmatrix} \frac{\partial n_1}{\partial m_2} \\ \frac{\partial w}{\partial m_2} \end{pmatrix} = - \begin{pmatrix} \frac{\partial E_1}{\partial n_1} & \frac{\partial E_1}{\partial w} \\ \frac{\partial E_2}{\partial n_1} & \frac{\partial E_2}{\partial w} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial E_1}{\partial m_2} \\ \frac{\partial E_2}{\partial m_2} \end{pmatrix}$$

holds. After some tedious calculations, we can express $\left. \frac{d\Delta l}{dm_2} \right|_{\mathbf{sym}}$ by the exogenous parameters $\alpha, \beta, \gamma, \tau, f$, and m . The curve $\left. \frac{d\Delta l}{dm_2} \right|_{\mathbf{sym}} = 0$ is drawn in Figure 2. Since its expression is very complicated, we deal with the two extreme cases of free trade $\tau \approx 1$ and autarky $x_{rs}^*|_{\mathbf{sym}} \approx 0$.

(I) Near free trade $\tau \approx 1$.

The unique solution of $\frac{d\Delta l}{dm_2}\Big|_{\mathbf{sym}, \tau=1} = 0$ is given by m_a and the unique solution of $n^*\Big|_{\mathbf{sym}, \tau=1} = 0$ is computed as $m_b \equiv \alpha - 2\sqrt{\beta f}$. It can be shown that $0 < m_a < m_b$. The proof is contained in Appendix A.

Consider a thought experiment of steady falling m , i.e., gradual technological progress. There are three stages of development in the neighborhood of free trade.

- i. $m \in [m_b, \infty)$. No firm enters and no good is produced.
- ii. $m \in (m_a, m_b)$. Production begins. The labor supply is larger in more developed countries.
- iii. $m \in (0, m_a)$. The labor supply smaller in more developed countries.

After production takes place, the labor supply first increases and then decreases in both countries. It is initially larger in developed countries (stage ii), and then smaller in developed countries (stage iii).

(II) Near autarky $x_{rs}^*\Big|_{\mathbf{sym}} \approx 0$.

Likewise, the unique solution of $\frac{d\Delta l}{dm_2}\Big|_{\mathbf{sym}, x_{rs}^*\Big|_{\mathbf{sym}}=0} = 0$ is computed as $\tilde{m}_a = (\alpha - 3\sqrt{\beta f})/2$ and the unique solution of $n^*\Big|_{\mathbf{sym}, x_{rs}^*\Big|_{\mathbf{sym}}=0} = 0$ is computed as $\tilde{m}_b = \alpha - 2\sqrt{\beta f}$. Since $0 < \tilde{m}_a < \tilde{m}_b$ holds, steadily falling m due to technological progress yields the three stages of development, which is similar to the free trade case.

By continuity near free trade and autarky, we can state the following.

Proposition 5 *In the vicinity of free trade and autarky, the working hours in more developed countries are longer at the early stage of development, while they are shorter at the late stage of development.*

The proof is contained in Appendix B.

4.2 Wage differential

Similarly, we can conduct comparative statics on the wage differential Δw in the neighborhood of the symmetric equilibrium. The marginal change of m_2 about the symmetry can be computed as

$$\frac{d\Delta w}{dm_2}\Big|_{\mathbf{sym}} = \frac{\partial w}{\partial m_2}\Big|_{\mathbf{sym}}.$$

For any admissible range of the parameter values, we can show the following.

Proposition 6 *The nominal wage rate in more developed countries is always higher.*

The proof is contained in Appendix C. Putting Propositions 5 and 6 together, we can say that the working hours l is positively related to the nominal wage rate w in the early stage of development, while negatively related in the late stage of development.

Figure 3 displays the working hours l and the labor productivity of 34 countries in 2012. This figure shows that the working hours l and the wage rate w are significantly negatively correlated $r = -0.806$. That is, the labor supply is smaller and the wage rate is higher in more developed countries, which corresponds to the late stages of development.⁴ We may therefore say that the marginal labor requirement m is already small in 2012.

5 Conclusion

We have extended a model of new trade theory by incorporating elastic labor supply and analyzed the impacts of technological progress on equilibrium outcomes of working hours and economic welfare. We have shown the following. First, there exists an inverted U-shaped relationship between the technological progress and the labor supply. Second, individual welfare rises as the production cost and trade cost decline and as the population grows. Third, population growth increases the labor supply at the early stage of development, while it decreases the labor supply at the late stage of development. Finally, the working hours in more developed countries are longer at the early stage of development, while they are shorter at the early stage of development.

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⁴Using Japanese 47 prefectural data, the correlation coefficient between the total working hours and the wage rate is -0.360 in 2009. The labor supply is smaller and the wage is higher in more developed prefectures

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Appendix A: Proof of $0 < m_a < m_b$ for $\tau = 1$

Define

$$i \equiv \frac{m}{\alpha - 2\sqrt{\beta f}}, \quad j \equiv \frac{m(\tau - 1)}{\sqrt{\beta f}}, \quad k \equiv \frac{\alpha}{\sqrt{\beta f}},$$

where $0 < i < 1$, $0 < j < 2$, $k > 3$.

Note that $i < 1$ corresponds to positive production $n^*|_{\text{sym}} > 0$, which is (20), $j < 2$ corresponds to positive trade $x_{rs}^*|_{\text{sym}} > 0$, which is (21), and $k < 3$ corresponds to U-shaped labor l^* , which is (12) or (21).

We have

$$\left. \frac{d\Delta l}{dm_2} \right|_{\text{sym}, \tau=1} = \frac{\sqrt{\beta f} \gamma (k-2) i}{[\sqrt{2}(k-2)i+1]^2 [(1-i)k+2i]} G(i)$$

where

$$G(i) \equiv -4(k-2)^3 i^3 + 2(3k-4\sqrt{2})(k-2)^2 i^2 - (2k^2-7\sqrt{2}+10)(k-2)i - 2\sqrt{2}.$$

Since $G'''(i) < 0$, $G''(i)$ is decreasing. Since $G''(0) > 0 > G''(1)$, $G'(i)$ is first increasing then decreasing. We also know that $G(0) < 0 < G(1)$. It can be shown that there are three cases: (1) $G'(0) > 0$ and $G(1) > 0$; (2) $G'(0) < 0 < G(1)$; and (3) $G'(0) < 0$ and $G(1) < 0$. In all cases, we can readily show that there exists a unique $i = i_a$ such that $G(i) < 0$ for all $0 \leq i < i_b$, $G(i_b) = 0$, and $G(i) < 0$ for all $i_a < i \leq 1$. Because $i = 0, i_a, 1$ correspond to $m = 0, m_a, m_b$, we have shown that $0 < m_a < m_b$.

Appendix B: Proof of Propositions 5

When $\tau = 1$, we have shown in Appendix A that

$$\left. \frac{d\Delta l}{dm_2} \right|_{\text{sym}, \tau=1} \begin{matrix} \geq 0 \\ \leq 0 \end{matrix} \Leftrightarrow m \begin{matrix} \geq \\ \leq \end{matrix} m_a. \quad (22)$$

When $x_{rs}^*|_{\text{sym}} = 0$, inequality (21) becomes equality so that

$$\tau = \frac{2\sqrt{\beta f}}{m} + 1.$$

Then,

$$\left. \frac{d\Delta l}{dm_2} \right|_{\text{sym}, \tau=\frac{2\sqrt{\beta f}}{m}+1} = \frac{2}{\gamma} (m - \tilde{m}_a),$$

where $\tilde{m}_a < m_b$. Thus,

$$\left. \frac{d\Delta l}{dm_2} \right|_{\text{sym}, \tau=\frac{2\sqrt{\beta f}}{m}+1} \begin{matrix} \geq 0 \\ \leq 0 \end{matrix} \Leftrightarrow m \begin{matrix} \geq \\ \leq \end{matrix} \tilde{m}_a. \quad (23)$$

From (22) and (23), we can say that when $\tau = 1, \frac{2\sqrt{\beta f}}{m} + 1$, the working hours in developed countries is longer for large $m \in (\max\{m_a, \tilde{m}_a\}, m_b)$ and while it is shorter for small $m \in (0, \min\{m_a, \tilde{m}_a\})$.

Appendix C: Proof of Propositions 6

We have

$$\left. \frac{d\Delta w}{dm_2} \right|_{\text{sym}} = \frac{I(i)}{\sqrt{\beta f}(k-2)iJ(i)}$$

where

$$\begin{aligned} I(i) &\equiv 64(k-2)^3 i^3 - 4(j^3 - 40j - 4\sqrt{8-j^2} + 16k)(k-2)^2 i^2 \\ &\quad - 4 \left[j^4 + j^3 \sqrt{8-j^2} - j^2(32 + \sqrt{8-j^2}k) - 8j(\sqrt{8-j^2} - 3k) + 4\sqrt{8-j^2}k \right] (k-2)i \\ &\quad - 2j \left[j^3(\sqrt{8-j^2} + k) - 2j^2(\sqrt{8-j^2}k + 6) - 8j(\sqrt{8-j^2} - 2k) + 4\sqrt{8-j^2}k \right] \\ J(i) &\equiv 64(k-2)^3 i^3 - 4(j^3 + j^2\sqrt{8-j^2} - 48j - 8\sqrt{8-j^2} + 16k)(k-2)^2 i^2 \\ &\quad - 4 \left[j^4 + j^3(2\sqrt{8-j^2} + k) - j^2(40 + \sqrt{8-j^2}k) - 6j(3\sqrt{8-j^2} - 4k) - 8 + 8\sqrt{8-j^2}k \right] (k-2)i \\ &\quad - j \left[j^3(3\sqrt{8-j^2} - 4k) - 4j^2(\sqrt{8-j^2}k + 7) - 4j(9\sqrt{8-j^2} - 8k) + 24\sqrt{8-j^2}k - 48 \right] - 32k. \end{aligned}$$

We first show that $J(i) < 0$. Since $J'''(i) > 0$ and $J''(1) > 0$, $J'(i)$ is (1) increasing or (2) decreasing and then increasing. Since $J'(0) < 0$, $J(i)$ is (a) decreasing or (2) decreasing and then increasing. However, because $J(0) < 0$ and $J(i) < 0$, it must be that $J(i) < 0$ for all $i \in (0, 1)$, $j \in (0, 2)$, and $k \in (3, \infty)$.

Exactly the same argument applies for $I(i)$, and so we can show $I(i) < 0$. Hence, $\left. \frac{d\Delta w}{dm_2} \right|_{\mathbf{sym}} > 0$ always holds for all $i \in (0, 1)$, $j \in (0, 2)$, and $k \in (3, \infty)$.

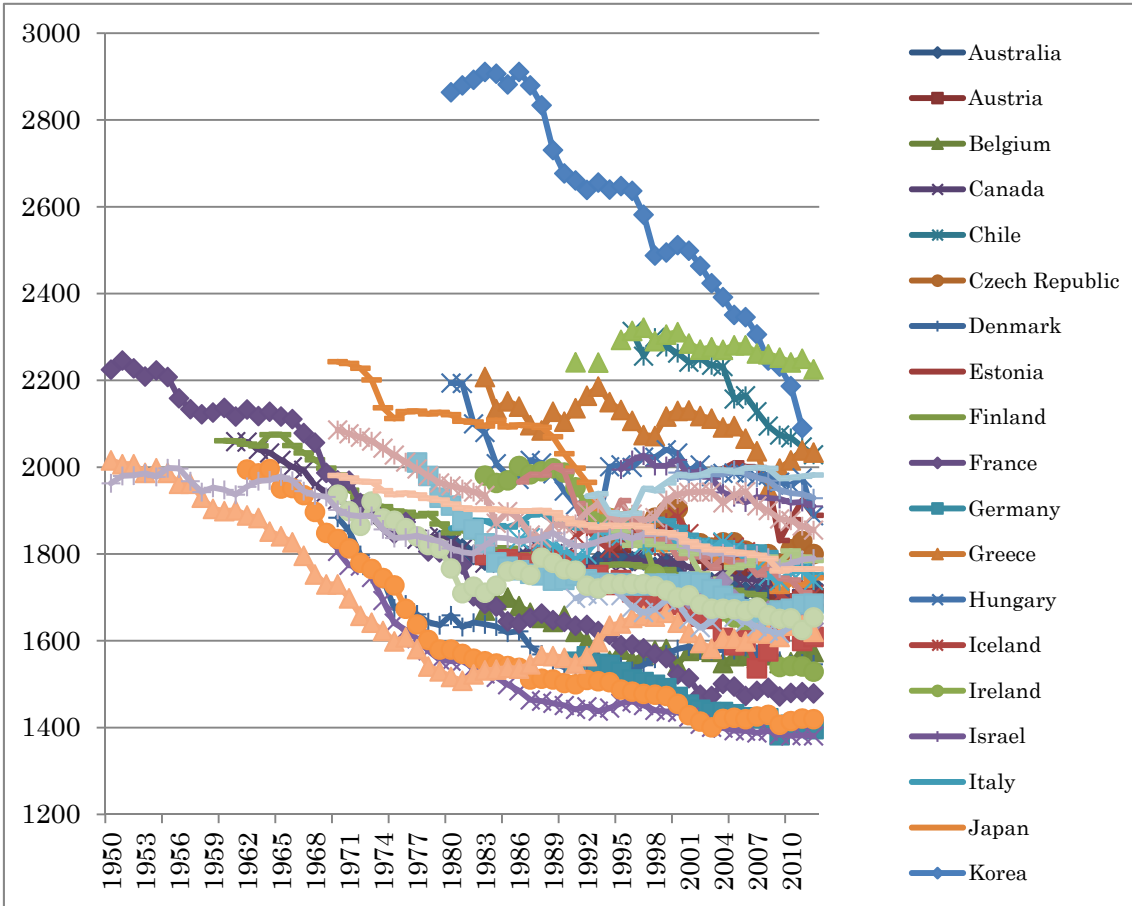


Figure 1: Working hours per year for 1950-2012

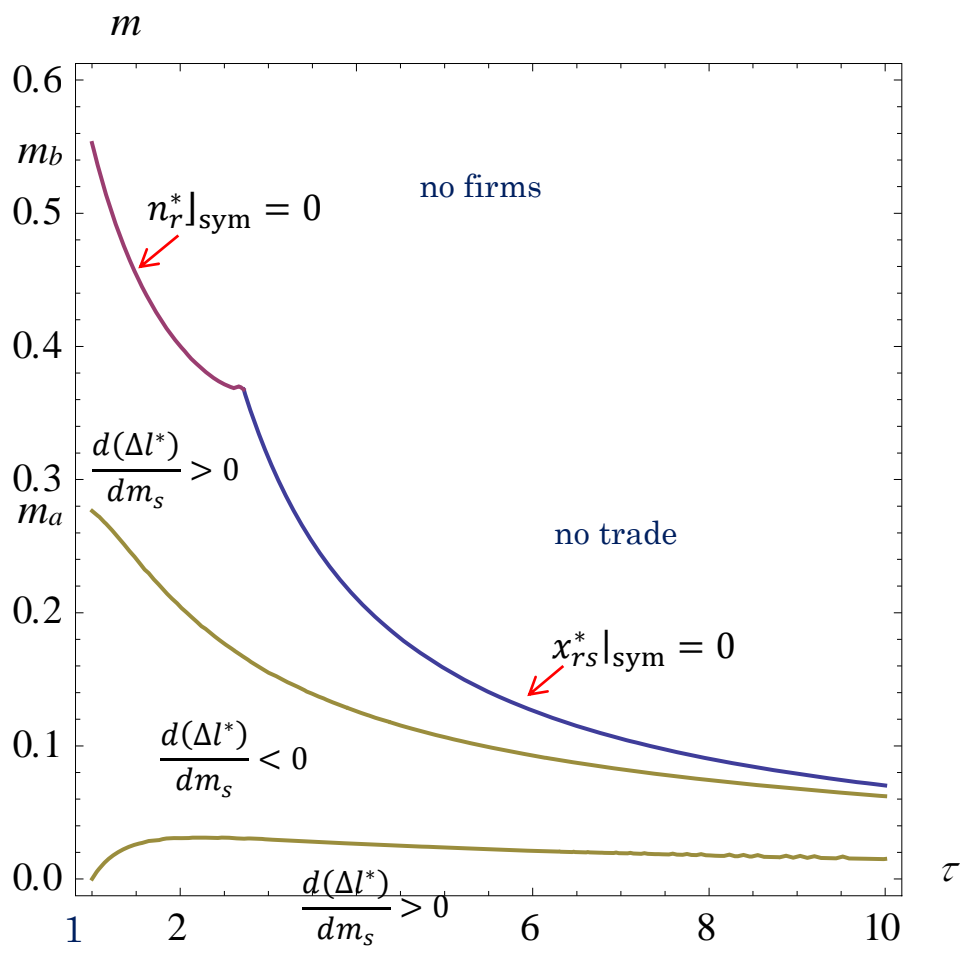


Figure 2: Differential in labor supply with $\alpha=\beta=\gamma=1$ and $f=1/10$

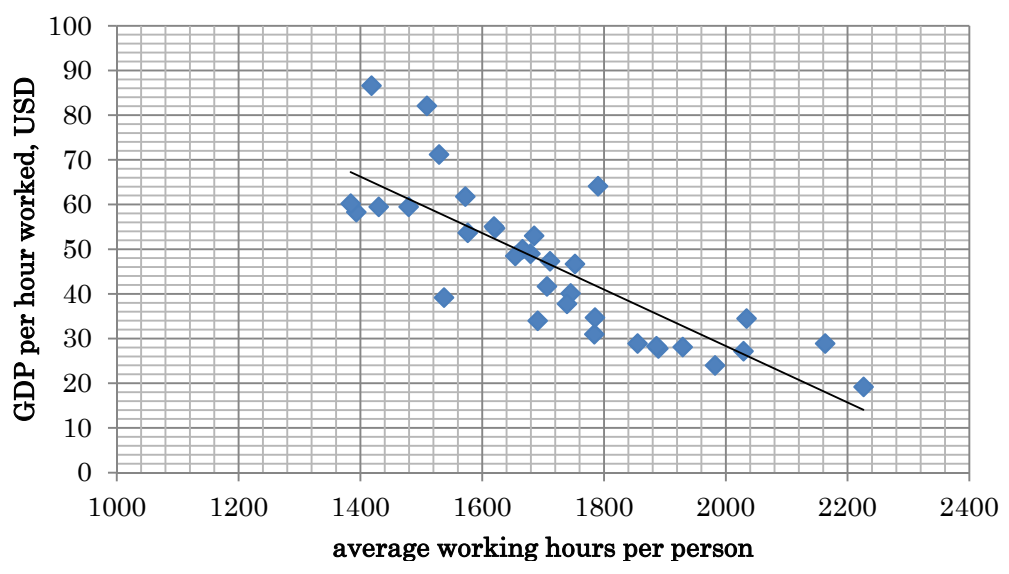


Figure 3: Working hours and GDP per hour in 2012