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Elhorst, Paul; da Silva, Diego Firmino Costa; da Mota Silveira Neto, Raul

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A Spatial Economic Model and Spatial Econometric Analysis of Population Dynamics in Brazilian MCAs

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A Spatial Economic Model and Spatial Econometric Analysis of Population Dynamics in Brazilian MCAs*

Diego Firmino Costa da Silva

Federal University of Pernambuco and University of Groningen. Address: Rua Dr. Odilon Lima, 44/02, 50850-010, Afogados, Recife, PE, Brazil. E-mail: diegofirmino@gmail.com, Phone: +55 (81) 32711196

J. Paul Elhorst

University of Groningen. Address: P.O. Box 800, 9700 AV, Groningen, the Netherlands. E-mail: j.p.elhorst@rug.nl, Phone: +31503633893

Raul da Mota Silveira Neto

Federal University of Pernambuco. Address: Av. Prof. Moraes Rego, 1235, Recife, PE, 50670-901, Brazil. E-mail: rau.silveira@uol.com.br, Phones: +55 (81) 21268381 r. 224, +55 (81) 91381357

Abstract

To extend existing population growth models, this article proposes a theoretical setting including spatial interaction effects. Using data pertaining to 3659 Brazilian Minimum Comparable Areas (MCA) over the period 1970-2010, this extension is tested by estimating a dynamic spatial panel model. The authors also compare the performance of a wide range of potential neighborhood matrices using Bayesian posterior model probabilities. Six of the thirteen determinants of population growth considered produce significant spatial interaction effects. Moreover, five produce significant long-term spatial spillover effects; a mathematical analysis reveals the strength of this result, which requires more than a single parameter. Treating areas as independent entities, as many previous population growth studies have done, underestimates the impact of various policy measures.

JEL Classification: C23, R23

Keywords: Population growth, regions, spatial interaction, dynamic spatial panel models, spillover effects

1. Introduction

Brazilian urbanization represents a highly significant, robust social phenomenon, such that through the vast economic and social changes of the past four decades, the percentage of people living in urban centers in Brazil has grown steadily. According to the last Brazilian Demographic Census (2010), it increased from 55.9% in 1970 to 84.4% in 2010 (IBGE, 2011). This process resulted largely from improved economic and social prospects in cities (Da Mata et al., 2007; Henderson, 1988; Yap, 1976), though Ramalho and Silveira-Neto (2012) also note that the most important migration of people in Brazil since the 1990s has occurred among cities due to specific urban characteristics.

Yet we know relatively little about how these specific factors condition this population growth. Henderson (1988) shows that the population growth of Brazilian cities between 1960 and 1970 related positively to initial increases in levels of education. Reviewing growth between 1970 and 2000, Da Mata et al. (2007) reveal that favorable supply and demand conditions, including market potential variables, better schooling, and limited opportunities in the agricultural sector, favored the growth of Brazilian cities. However, similar studies have not extended to more recent periods, which feature some notable shifts. After a decade of high inflation, the early 2000s offered a period of price stability, as well as income convergence among Brazilian states (Silveira-Neto and Azzoni, 2012). Substantial increases in the production of commodities and agricultural goods during this period also had positive impacts on opportunities available in towns further distant from large urban centers. In addition, Da Mata et al. (2007) focus on municipalities with more than 75,000 inhabitants, or about 75% of Brazil's urban population. They do not model or test for any spatial dependence of population growth dynamics among Brazilian cities. Yet spatial dependence is particularly severe for small spatial units, such as municipalities (Boarnet et al., 2005). In analyzing income dynamics at different levels of spatial aggregation, Resende (2013) confirms the importance of spatial dependence for Brazilian minimum comparable areas (MCA).¹

The Brazilian Constitution of 1988 states that the municipality is the third and lowest political administrative unit of the country, with autonomy to collect taxes on service activities (ISS: Imposto sobre Serviços) and urban real estate property (IPTU: Imposto Predial e Territorial Urbano), as well as to legislate land use. These policies potentially affect the location of firms and people, such that they might generate reactions from or affect neighboring cities (Brueckner, 2008). Furthermore, spatial technological spillovers (Ertur and Koch, 2007) may be more prevalent among small, urban, neighboring centers than among large ones. By considering spatial urban centers on a lower, fuller scale, we consider the possibility that common or shared amenities affect the population's welfare.

Because we assert that all of these factors might induce spatial dependence on the population growth dynamics of Brazilian cities and its determinants, we seek to model spatial dependence among spatial units explicitly. Our central objective is to present the population growth dynamics of Brazilian MCAs and thereby assess the determinants of the population

¹ A MCA is a municipality or aggregation of municipalities necessary to enable consistent spatial analyses over time; we provide more details in Section 4.2.

growth of these units between 1970 and 2010, as well as examine the existence and magnitude of spatial interaction and spatial spillover effects associated with these determinants. To model the population growth dynamics of Brazilian cities (MCA), we construct an economic–theoretical model that includes spatial interaction effects, then estimate this model using dynamic spatial panel techniques, with controls for spatial and time-specific effects. Accordingly, we can determine the magnitude and significance levels of spatial spillover effects, and guarantee that any support for these effects is not simply an artifact of ignoring time-specific effects that areas have in common. Almost half of our explanatory variables reveal significant spatial interaction and spatial spillover effects, namely, rural population size, population density, and birth, literacy, and homicide rates. These significant spatial interaction and spillover effects indicate that the impact of policy measures that act on these variables tends to be considerably underestimated (by up to 60%) by non-spatial approaches that treat MCAs as independent entities.

In the next section, we motivate our investigation by presenting a spatial extension of the city population growth model developed by Glaeser et al. (1995), which accounts for spatial interaction effects among productivity and city amenities and also implies an empirical specification for population growth dynamics that consists of spatial interaction effects in the dependent and independent variables. In Section 3, we present the econometric methodology underlying our empirical investigation. We also introduce spatial interaction effects and define spatial spillover effects. After we detail our data set in Section 4, we present the results of the empirical analysis in Section 5. Finally, we summarize our main findings and draw conclusions.

2. A Spatial Extension of Glaeser's Population Growth Model

Our theoretical framework of population growth across Brazil builds on previous work by Glaeser et al. (1995), which is taken as point of departure, and by Brueckner (2003) and Ertur and Koch (2007), which are used to extend the model. In the urban growth model developed by Glaeser et al. (1995),² cities are independent economies that share common pools of labor and capital and differ in their level of productivity (A_{it}) and quality of life (Q_{it}). The total output of an economy depends on the productivity level, which can be modeled as a Cobb-Douglas production function that depends on population size. The total welfare of a potential migrant to this economy equals wages multiplied by the quality of life, which decreases with population size. The net result is a population growth regression that contains several factors that determine quality of life, including crime, housing prices, traffic congestion, population size, and productivity growth.

An objection to this theoretical framework is that it ignores spatial interaction effects among economies, especially between a locality and its surroundings. To address this problem, we could increase the scale of the geographical units in the empirical analysis, with the assumption that interaction effects at this higher scale no longer exist (Glaeser et al.

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² A more sophisticated approach that also includes the housing market is available in Glaeser (2008).

1995). Another, more prevalent option is to model spatial interaction effects explicitly. Suppose the total output of an economy is given by

$$Y_{it} = A_{it} P_{it}^{\beta} K_{it}^{\gamma} \overline{Z}_i^{1-\beta-\gamma}, \tag{1}$$

where P_{it} represents the population size in economy i at time t, K_{it} denotes traded capital, and \overline{Z}_i is fixed non-traded capital. Then, the first extension includes productivity interaction effects among economies. Ertur and Koch (2007) argue that knowledge accumulated in one economy depends on knowledge accumulated in other economies, though with diminished intensity due to frictions caused by socio-economic and institutional dissimilarities, which in turn can be captured by geographical distance or border effects. More formally,

$$A_{it} = a_{it} \prod_{j \neq i}^{N} a_{jt}^{\rho w_{ij}} , \qquad (2)$$

where the productivity level of an economy A_{it} depends on urban differences in the productivity of labor related to social, technological, and political sources in the own economy (i) a_{it} , as well as those in neighboring economies $(j\neq i)$ a_{jt} ; in addition, N is the number of economies. The parameter ρ reflects the degree of interdependence among economies, with $0 < \rho < 1$. Although this parameter is assumed to be identical for all economies, the impact of the interaction effects on economy i depends on its relative location, reflecting the effect of being located closer to or further away from other economies. This relative location can be represented by the exogenous term w_{ij} , which is assumed to be nonnegative, non-stochastic, and finite, establishing an $N \times N$ neighborhood matrix W in which $0 \le w_{ij} \le 1$ and $w_{ij} = 0$ if i = j. By substituting Equation (2) into Equation (1), we find that the total output of an economy is given by

$$Y_{it} = \left(a_{it} \prod_{j \neq i}^{N} a_{jt}^{\rho w_{ij}}\right) P_{it}^{\beta} K_{it}^{\gamma} \overline{Z}_{i}^{1-\beta-\gamma} . \tag{3}$$

The first-order conditions for capital and labor, that is, capital income (normalized price = 1) and labor income (denoted S_{it}) are equal to their marginal products, yield the following labor demand equation, after the optimal solution for capital is substituted in the condition for labor:

$$S_{it} = \beta \gamma^{\frac{\gamma}{1-\gamma}} \left(a_{it} \prod_{j \neq i}^{N} a_{jt}^{\rho w_{ij}} \right)^{\frac{1}{1-\gamma}} P_{it}^{\frac{\beta+\gamma-1}{1-\gamma}} \overline{Z}_{i}^{\frac{1-\beta-\gamma}{1-\gamma}} . \tag{4}$$

As this labor demand equation shows, higher wages reflect higher productivity and fewer workers.

Consumers also have Cobb-Douglas utility functions for tradable goods and housing, denoted by C_{it} and H_{it} , respectively. We can assume that welfare is due, at least partly, to the (dis)amenities of the local economy Θ_{it} . The locality's (dis)amenities directly affect the welfare of people who live there; they might interfere negatively or positively with a resident's utility, and they can be either natural (e.g., climate, beaches, vegetation) or generated by humans (e.g., violence, entertainment, traffic, pollution). Formally,

$$U_{i,t} = C_{i,t}^{1-\alpha} H_{i,t}^{\alpha} \Theta_{i,t}, \tag{5}$$

where α is a constant. The price of tradable goods is normalized to 1; the housing price is p_{Hit} . Consumers maximize their utility, subject to a budget constraint,

$$C_{it} + p_{Hit}H_{it} = S_{it}, (6)$$

by choosing C_{it} and H_{it} .

Our second extension includes amenity interaction effects across economies. Some (dis)amenities may (dis)benefit persons living in other economies (Brueckner, 2003). For example, people might use facilities in other localities; the violence in a particular neighborhood can generate feelings of insecurity in adjacent neighborhoods; and water pollution can cause health damages downstream. In mathematical terms,

$$\Theta_{it} = \left(\theta_{it} \prod_{i \neq i}^{N} \theta_{jt}^{\eta w_{ij}}\right),\tag{7}$$

where the overall amenities of an economy Θ_{it} depend on local amenities θ_{it} and those in neighboring economies θ_{jt} , and the impact of the latter decreases with geographical distance. The parameter η measures the degree of interdependence among economies, with $0 < \eta < 1$. According to Glaeser et al. (1995), many potential (dis)amenities can be reflected by the level of population and the population growth rate; the greater the size of a city, the lower the quality of life. The costs of migration rise with the number of immigrants, and if the population size increases rapidly, expansions in public goods, infrastructure, and housing might not be able to keep pace. Therefore, residents of quickly growing cities suffer in terms of quality of life, yielding the utility function

$$U_{it} = C_{it}^{1-\alpha} H_{it}^{\alpha} \left(\theta_{it} \prod_{j \neq i}^{N} \theta_{jt}^{\eta w_{ij}} \right) P_{it}^{-\varphi} \left(\frac{P_{it}}{P_{it-1}} \right)^{-\tau}, \tag{8}$$

where $\varphi > 0$ and $\tau > 0$. In addition, total city demand for housing is given by

$$H_{it} = P_{it} \frac{\alpha S_{it}}{p_{Hit}} \,. \tag{9}$$

According to Glaeser and Gottlieb (2009), the spatial equilibrium condition is a primary theoretical tool for urban economists, as exemplified in pioneering work by Mills (1967), Rosen (1979), and Roback (1982) on population changes within a country. This condition states that utility equalizes across space, provided that labor is mobile; higher wages in urban areas get offset by negative urban attributes, such as higher prices and negative amenities. If the common utility level at a particular point in time is denoted by \overline{V}_t , application of the spatial equilibrium condition produces the following results when we substitute the demand equation for housing derived in Equation (9) into Equation (8), such that it yields the indirect utility function in Equation (10), equal to \overline{V}_t :

$$V(S_{it}, p_{Hit}) = \alpha \left(1 - \alpha\right)^{1 - \alpha} \left(\theta_{it} \prod_{j \neq i}^{N} \theta_{jt}^{\eta w_{ij}}\right) S_{it} p_{Hit}^{-\alpha} P_{it}^{-\varphi} \left(\frac{P_{it}}{P_{it-1}}\right)^{-\tau} = \overline{V}_{t}.$$

$$(10)$$

Following Glaeser (2008), housing floor space is produced competitively, either by land (L) or by height (h). A fixed quantity of land at a particular location (\overline{L}) determines an endogenous price for land (p_L) and housing (p_H), and the cost of producing hL units of structure on top of L units of land is given by $c_0h^{\delta}L$, where $\delta > 1$. The developer then maximizes profits,

$$\pi = p_{Hit}hL - c_0h^{\delta}L - p_LL. \tag{11}$$

The first-order condition of this maximization problem for height, $h = (p_H/\mathcal{E}_0)^{\frac{1}{\delta-1}}$, implies the total housing supply equation:

$$h\overline{L} = (p_{Hit}/\delta c_0)^{\frac{1}{\delta-1}}\overline{L} . \tag{12}$$

By comparing housing demand in Equation (9) with housing supply in Equation (12), we can obtain the housing price equation:

$$p_{Hit} = \left(\frac{P_{it}\alpha S_{it}}{\overline{L}}\right)^{\frac{\delta-1}{\delta}} \left(\delta c_0\right)^{\frac{1}{\delta}}.$$
 (13)

Labor demand in Equation (4), indirect utility in Equation (10), and housing prices in Equation (13) then form a system, with three unknown variables (P_{it} , S_{it} , and p_{Hit}). Solving this system for the population P_{it} yields

$$\log P_{it} = D_N + \psi \left\{ \log \theta_{it} + \left(\eta \sum_{j \neq i}^N w_{ij} \log \theta_{jt} \right) + \left(\frac{\delta - \alpha \delta - \alpha}{\delta} \right) \left(\log a_{it} + \rho \sum_{j \neq i}^N w_{ij} \log a_{jt} \right) + \tau \log P_{it-1} + \log \overline{V}_t \right\},$$
(14a)

where

$$\psi = \frac{\delta(1-\gamma)}{(1-\gamma)(\alpha(\delta-1)+\delta\varphi+\delta\tau)-(\delta-\alpha\delta+\alpha)(\beta+\gamma-1)}, \text{ and}$$
 (14b)

$$D_{N} = \psi \left(\alpha \log \alpha + (1 - \alpha) \log(1 - \alpha) + \left(\frac{\delta - \alpha \delta - \alpha}{\delta} \right) \left(\log \beta + \left(\frac{\gamma}{1 - \gamma} \right) \log \gamma + \left(\frac{1 - \beta - \gamma}{1 - \gamma} \right) \log \overline{Z} \right) - \frac{\alpha(\delta - 1)}{\delta} - \frac{\alpha}{\delta} \log(\delta c_{0}) + \left(\frac{\alpha(\delta - 1)}{\delta} \right) \log \overline{L} \right).$$

$$(14c)$$

According to Glaeser and Gottlieb (2009), the spatial equilibrium condition means that in a dynamic model, only lifetime utility levels get equalized across space. However, as long as housing prices or rents can change quickly, or to a reasonable extent within the observation periods being considered— which is 10 years for the present study³—a price adjustment is enough to maintain the spatial equilibrium. Then the change in utility between times t and t+1 is the same across space, $\overline{V}_{t+1}/\overline{V}_{t}$, and Equation (14a) can be rewritten as

$$\log\left(\frac{P_{it+1}}{P_{it}}\right) = \psi\left(\log\left(\frac{\theta_{it+1}}{\theta_{it}}\right) + \left(\eta \sum_{j \neq i}^{N} w_{ij} \log\left(\frac{\theta_{jt+1}}{\theta_{jt}}\right)\right) + \left(\frac{\delta - \alpha\delta - \alpha}{\delta}\right) \left(\log\left(\frac{a_{it+1}}{a_{it}}\right) + \rho \sum_{j \neq i}^{N} w_{ij} \log\left(\frac{a_{jt+1}}{a_{jt}}\right)\right) + \tau \log\left(\frac{P_{it}}{P_{it-1}}\right) + \log\left(\frac{\overline{V}_{t+1}}{\overline{V}_{t}}\right)\right). \tag{15}$$

Following Glaeser et al. (1995), we also assume that X_{it} is a vector of city characteristics at time t that determine both the growth of city-specific productivity denoted by a and city-specific amenity growth denoted by θ :

$$\log\left(\frac{a_{i+1}}{a_{i}}\right) = X_{i}'\lambda_a + \xi_{i+1} , \text{ and}$$
 (16a)

$$\log\left(\frac{\theta_{i+1}}{\theta_{it}}\right) = X_{it}'\lambda_{\theta} + \varepsilon_{i+1}. \tag{16b}$$

Combining Equations (14) and (16), we derive the dynamic spatial population growth equation:

$$\log\left(\frac{P_{it+1}}{P_{it}}\right) = \psi\left(\tau\log\left(\frac{P_{it}}{P_{it-1}}\right) + \left(1 + \frac{(\delta - \alpha\delta + \alpha)}{\delta}\right)X'_{it}(\lambda_a + \lambda_\theta)\right) + \left(\eta + \rho\frac{(\delta - \alpha\delta + \alpha)}{\delta}\right)\sum_{j \neq i}^{N} w_{ij}X'_{jt}(\lambda_a + \lambda_\theta)\right) + \psi\left(\log\left(\frac{\overline{V}_{t+1}}{\overline{V}_t}\right) + \eta\sum_{j \neq i}^{N} w_{ij}\varepsilon_{jt+1} + \rho\frac{(\delta - \alpha\delta + \alpha)}{\delta}\sum_{j \neq i}^{N} w_{ij}\xi_{jt+1} + \varepsilon_{it+1} + \xi_{it+1}\right),$$

$$(17)$$

³ Duranton and Puga (2013) cite cyclical behavior and sluggish adjustment as reasons to measure population growth over periods of five or ten years.

which contains spatial interaction effects among both the explanatory variables and the error terms. In spatial econometrics literature, such a model specification would be known as the spatial Durbin error model (SDEM; see LeSage and Pace, 2009). The right-hand side of this model also contains the dependent variable, lagged one period, so it also could be labeled a dynamic SDEM model.

The utility function specified in Equation (8) assumes that the quality of life for potential migrants declines with both the level and the growth rate of the population. Just as knowledge and amenities in one economy interact with knowledge and amenities in others, so might the level and growth rate of population depend on these values in neighboring economies. If residents of quickly growing cities suffer in terms of quality of life, they might move to neighboring areas. Therefore, assuming individual utility correlates negatively with the level of population (population size) and the population growth rate of neighbors, the utility function takes the form

$$U_{it} = C_{it}^{1-\alpha} H_{it}^{\alpha} \left(\theta_{it} \prod_{j \neq i}^{N} \theta_{jt}^{\eta w_{ij}} \right) P_{it}^{-\varphi} \left(\frac{P_{it}}{P_{it-1}} \right)^{-\tau} \left(\prod_{j \neq i}^{N} P_{jt}^{-w_{ij}} \right) \left(\prod_{j \neq i}^{N} \left(\frac{P_{jt}}{P_{jt-1}} \right)^{-\sigma w_{ij}} \right), \tag{18}$$

where v > 0 and $\sigma > 0$. Solving the system for the population P_{it} with this alternative specification of the utility function yields a population growth equation, as follows:

$$\log\left(\frac{P_{it+1}}{P_{it}}\right) = \psi\left(-(\nu+\sigma)\left(\sum_{j\neq i}^{N}w_{ij}\log\frac{P_{jt+1}}{P_{jt}}\right) + \tau\log\left(\frac{P_{it}}{P_{it-1}}\right) + \sigma\left(\sum_{j\neq i}^{N}w_{ij}\log\frac{P_{jt}}{P_{jt-1}}\right) + \left(1 + \frac{(\delta-\alpha\delta+\alpha)}{\delta}\right)X'_{it}(\lambda_{a}+\lambda_{\theta}) + \left(\eta+\rho\frac{(\delta-\alpha\delta+\alpha)}{\delta}\right)\sum_{j\neq i}^{N}w_{ij}X_{jt}(\lambda_{a}+\lambda_{\theta})\right) + \psi\left(\log\left(\frac{\overline{V}_{t+1}}{\overline{V}_{t}}\right) + \eta\sum_{j\neq i}^{N}w_{ij}\varepsilon_{jt+1} + \rho\frac{(\delta-\alpha\delta+\alpha)}{\delta}\sum_{j\neq i}^{N}w_{ij}\xi_{jt+1} + \varepsilon_{it+1} + \xi_{it+1}\right),$$

$$(19)$$

where ψ is defined as in Equation (14b). In addition to spatial interaction effects among the explanatory variables and the error terms, this model specification contains spatial interaction effects for the dependent variable. Thus it represents a general nesting spatial (GNS) model (Elhorst, 2014), and if we account for the dependent variable lagged one period, it is a dynamic GNS model.

Apart from dynamic effects in both space and time, the population growth rate depends on factors determining its productivity and amenities and that of its neighbors. In section 4 we introduce the variables used to capture both sets of factors and estimate the parameters of the model for the specific case of Brazilian municipalities. In the next section we first present the econometric strategy used in this paper to discriminate between the spatial population growth equations in (17) and (19) and some technical issues that arise when estimating the parameters of the model using panel data.

3. Econometric Methodology

The econometric counterpart of the dynamic spatial GNS model, which is the final equation

implied by the theoretical model presented in the previous section, reads, in vector form, as

$$Y_t = \tau Y_{t-1} + \delta W Y_t + \eta W Y_{t-1} + X_t \beta + W X_t \theta + \mu + \lambda_t I_N + \nu_t, \ \nu_t = \lambda W \nu_t + \varepsilon_t, \tag{20}$$

where Y_t denotes an $N \times 1$ vector that consists of one observation of the dependent variable for every economy (i = 1, ..., N) in the sample at time t (t = 1, ..., T), which for this study is the population growth rate, $\log(P_{it+1}/P_{it})$; and X_t is an $N \times K$ matrix of exogenous or predetermined explanatory variables, observed at the start of each observation period and associated to the determinants of local productivity and amenities. Table 1 provides a detailed description between the theoretical and econometric model equations. Although we tried to maintain consistent symbols, the limited supply of Greek letters mandated that many of the parameters in the econometric model relied on a different interpretation than those used in the theoretical model, as set out in the previous section. A vector or matrix with subscript t-1 in Equation (20) denotes its time lagged value, whereas a vector or matrix premultiplied by W denotes the spatially lagged value. The $N \times N$ matrix W is a non-negative matrix of known constants that describe the spatial arrangement of the economies in the sample, as introduced in the previous section. The parameters τ , δ , and η are the response parameters of, respectively, the dependent variable lagged in time Y_{t-1} , the dependent variable lagged in space WY_t , and the dependent variable lagged in both space and time WY_{t-1} . The symbols β and θ represent $K \times 1$ vectors of the response parameters of the exogenous explanatory variables. The error term specification consists of different components: the vector \mathbf{v}_t that is assumed to be spatially correlated with autocorrelation coefficient λ ; the $N \times 1$ vector $\varepsilon_t = (\varepsilon_{It}, \varepsilon_{It})$..., ε_{Nt})^T that consists of i.i.d. disturbance terms, which have zero mean and finite variance σ^2 . The $N \times 1$ vector $\boldsymbol{\mu} = (\mu_1, ..., \mu_N)^T$ that contains spatial specific effects μ_i and is meant to control for all spatial-specific, time-invariant variables whose omission could bias the estimates in a typical cross-sectional study (Baltagi, 2005); and time-specific effects λ_t (t = 1, ..., T), where ι_N is a $N \times 1$ vector of ones, meant to control for all time-specific, unit-invariant variables whose omission could bias the estimates in a typical time-series study.

<< Table 1 around here >>

Such spatial- and time period–specific effects generally can be treated as fixed or random effects, but as Elhorst (2014) notes, a random effects model in the cross-sectional domain might not be an appropriate specification if observations of adjacent units in an unbroken study area are being used and the whole population is sampled. In contrast, a random effects model would make sense if a limited number of MCAs were being drawn randomly from Brazil, but in that case the elements of the neighborhood matrix could not be defined, and the impact of spatial interaction effects could not be estimated consistently. Only when neighboring units are part of the sample is it possible to measure the impact of neighboring units. Therefore, this study is distinct from urban studies that seek to explain

economic growth in cities, such as those by Glaeser et al. (1995) and Da Mata et al. (2007).⁴ We include both urban and rural regions, to cover the whole country and model the interactions, whereas previous studies ignore the potential interaction effects with surroundings and treat cities as independent entities.

Direct interpretation of the coefficients in the dynamic GNS model is difficult, because they do not represent true partial derivatives (LeSage and Pace, 2009). Debarsy et al. (2011) and Elhorst (2012a) show that the matrix of (true) partial derivatives of the expected value of the dependent variable with respect to the k^{th} independent variable for i = 1, ..., N in year t for the short term is given by

$$\left[\frac{\partial E(\mathbf{Y})}{\partial x_{1k}} \cdots \frac{\partial E(\mathbf{Y})}{\partial x_{Nk}}\right]_{t} = (\mathbf{I} - \delta \mathbf{W})^{-1} [\beta_{k} \mathbf{I}_{N} + \theta_{k} \mathbf{W}].$$
(21)

In addition, LeSage and Pace (2009) advocate for decomposing the marginal effects into direct (own-economy) and indirect (spillover) effects. The direct effects stem from the own-partial derivatives along the diagonals in Equation (21); they reflect the effects on the dependent variable that result from a change in the k^{th} regressor x_k in economy i in the short term. The off-diagonal elements represent short-term spillover effects. Because the direct and spillover effects differ across units in the sample, LeSage and Pace (2009) propose one summary indicator for the direct effects, measured by the average of the diagonal elements, and another summary indicator for the spillover effects, measured by the average of the column sums of the non-diagonal elements of that matrix. The matrix reflecting the right-hand side of Equation (21) is independent of the time index t, so these calculations are equivalent to those presented by LeSage and Pace (2009) for a cross-sectional setting. From Elhorst (2012a), it follows that the long-term marginal effects are given by

$$\left[\frac{\partial E(\mathbf{Y})}{\partial x_{1k}} \cdots \frac{\partial E(\mathbf{Y})}{\partial x_{Nk}}\right] = \left[(1-\tau)\mathbf{I} - (\delta+\eta)\mathbf{W}\right]^{-1} \left[\beta_k \mathbf{I}_N + \theta_k \mathbf{W}\right]. \tag{22}$$

Thus, by examining the relationship between population growth and its determinants using the decompositions in Equations (21) and (22), we can disentangle direct and spillover effects into short- and long-term effects.

One problem with the dynamic GNS model is that its parameters are not identified, as acknowledged by Anselin et al. (2008), Gibbons and Overman (2012), and Elhorst (2014). The interaction effects among the dependent variable and the error terms cannot be distinguished formally, as long as the interaction effects among the explanatory variables are also included. Therefore, one of the two spatial interaction effects should be excluded, if we hope to obtain consistent parameter estimates. If the spatial interaction effects for the dependent variable are excluded, the dynamic SDEM specification results, consistent with the

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⁴ A recent overview is available in Duranton and Puga (2013).

utility function specified in Equation (8). Then $\delta = \eta = 0$, and the spatial multiplier matrix $(I - \delta W)^{-1}$ in Equation (21) reduces to the identity matrix, while the spatial multiplier matrix $[(1 - \tau)I - (\delta + \eta)W]^{-1}$ in Equation (22) reduces to the matrix $1/(1 - \tau)I$.

On the other hand, if we leave aside the spatial interaction effects among the error terms, a dynamic spatial Durbin model (SDM) results. This model specification is consistent with the utility function specified in Equation (18). Although the specification does not account for interaction effects among the error terms, which reduces the efficiency of the parameter estimates, it does not affect the consistency of the parameter estimates. Furthermore, imposing the restriction $\lambda = 0$ also does not influence the direct or spillover effects derived in Equations (21) and (22). As LeSage and Pace (2009) note, the cost of ignoring spatial dependence in the dependent and/or independent variables is relatively high, because if one or more relevant explanatory variable gets omitted from a regression equation, the estimator of the coefficients for the remaining variables is inherently biased and inconsistent (Greene, 2005). In contrast, ignoring spatial dependence in the disturbances, if present, only leads to a loss of efficiency.

Another important difference between the SDEM and SDM specifications is that the spillover effects in the first model are local, whereas in the second model, they are global in nature. Local spillovers occur at other locations only if they according to W are connected to each other, whereas global spillovers gets transmitted to all other locations even if the two locations are unconnected according to W. This requires that $\delta \neq 0$.

To choose between SDM and SDEM specifications of the population growth rate, and thus respectively between a global or local spillover model and the utility functions specified in Equations (8) or (18), as well as to choose between different potential specifications of the neighborhood matrix W, we apply a Bayesian comparison approach. This approach determines the Bayesian posterior model probabilities of the SDM and SDEM specifications given a particular neighborhood matrix, as well as the Bayesian posterior model probabilities of different neighborhood matrices given a particular model specification. These probabilities are based on the log marginal likelihood of a model obtained by integrating out all parameters of the model over the entire parameter space on which they are defined. If the log marginal likelihood value of one model or of one W is higher than that of another model or another W, the Bayesian posterior model probability is also higher. It should be stressed that the model parameters are not estimated and so cannot be reported when applying the Bayesian comparison approach. Whereas the popular likelihood ratio, Wald and/or Lagrange multiplier statistics compare the performance of one model against another model based on specific parameter estimates within the parameter space, the Bayesian approach compares the performance of one model against another model, in this case SDM against SDEM, on their entire parameter space. This is the main strength of this approach. Inferences drawn on the log marginal likelihood function values for the SDM and SDEM model are further justified because they have the same set of explanatory variables, X_t and WX_t , and are based on the same uniform prior for δ and λ . This prior takes the form $p(\delta)=p(\lambda)=1/D$, where $D=1/\omega_{max}$ $1/\omega_{min}$ and ω_{max} and ω_{min} represent respectively the largest and the smallest (negative) eigenvalue of the neighborhood matrix W. This prior requires no subjective information

on the part of the practitioner as it relies on the parameter space $(1/\omega_{min}, 1/\omega_{max})$ on which δ and λ are defined, where ω_{max} =1 if W is row-normalized. Full details regarding the choice of model can be found in LeSage (2014) and regarding the choice of W in LeSage and Pace (2009, Chs. 5 and 6).

Depending on the outcomes of these tests, we finally estimate either the SDM or the SDEM specification, using maximum likelihood (ML). If the test points to the dynamic SDM, it can be estimated using the bias-corrected ML estimator developed by Yu et al. (2008) and Lee and Yu (2010), depending on whether time period fixed are included. If it points to the dynamic SDEM, the model can be estimated with the bias-corrected ML estimator developed by Elhorst (2005).

4. Data Implementation

Our data come from the Brazilian Demographic Census for the years 1970, 1980, 1991, 2000, and 2010, as conducted by the Brazilian Institute of Geography and Statistics (IBGE), complemented by data collected by the Brazilian Institute for Applied Economic Research (IPEA).

The municipality constitutes the lowest administrative level in Brazil for which economic and demographic data are available. During 1970–2000, the number of municipalities increased from 3,952 to 5,565. Such ongoing changes in the number, area, and borders of municipalities mean that a consistent comparison over time is possible only if we aggregate the municipalities into broader geographical areas, or MCAs. Using the aggregation of municipalities developed by IPEA (Reis et al., 2010), we obtain a spatial panel of 3,659 MCAs during 1970–2010 (see also Da Mata et al., 2007). Therefore, we refer consistently to MCAs hereafter; Figure 1 maps the geographical delineation of all areas in our sample.

With Equations (17), (19), and (20), we measure the dependent variable of our empirical analysis Y_{it} by the rate of population growth in one particular MCA over a decade (t-1, t), where i runs from 1 to 3,659, t in Equation (20) spans from 1980 to 2010, and the number 1 represents a decade. This population growth rate depends on the population growth rate in the previous decade; when we use the dynamic spatial Durbin model, it also depends on the population growth rate in neighboring units in contemporaneous and previous decades. Based on our theoretical model (Section 2) and data availability, we consider the influences of 13 explanatory variables associated with local productivity and amenities. This selection reflected mainly the recent review by Duranton and Puga (2013) and previous studies by Glaeser et al. (1995), Da Mata et al. (2007), Gleaser (2008), and Chi and Voss (2010). Table 2 provides a systematic overview of the explanatory variables and their data sources.

In particular, Duranton and Puga (2013) discuss key theories from urban growth

research and their implications in terms of population, surface area, and income per person. They provide empirical evidence of the main drivers of city growth, drawn primarily from the Unites States and other developed countries. Although Brazil is an emerging economy, and we consider population growth in both urban and rural areas to be able to model spatial interaction effects, the explanations put forward in their overview remain helpful for selecting explanatory variables for the present study. However, the variables selected must be revised for the different context. For example, whereas Duranton and Puga (2013) observe a tendency to measure human capital by the share of university graduates, we focus on the share of people aged 25 years and over who are literate, a measure that is more meaningful in Brazil and that increased from 48% in 1970 to 82% in 2010. We also integrate the contributions of Glaeser et al. (1995) and Gleaser (2008), considering that their work provided the theoretical basis for our spatial extension in Section 2. We value Da Mata et al. (2007) for its empirical focus on population growth in Brazil, though it includes only 123 Brazilian agglomerations and does not span the whole country. Both Glaeser et al. and Da Mata et al. ignore spatial interaction effects, such as those between an agglomeration and its surroundings or between a city and its suburbs within an agglomeration. Finally, we rely on Chi and Voss (2011), because it estimates a dynamic spatial panel data model, though without providing a theoretical motivation for this model specification.

For the set of variables associated with local productivity, we consider a measurement of local human capital, measures of market potential and agglomeration gains, and characteristics of local productive structure. The positive influence of human capital on urban dynamics for U.S. cities is well documented (Gleaser et al. 1995; Gleaser and Shapiro, 2004; Shapiro, 2006; Gleaser, 2008); as for Brazilian cities, Da Mata et al. (2007) also found a positive association between human capital and population growth. This kind of association is commonly interpreted as the effect of local skills on productivity growth through knowledge or information spillovers (Lucas, 1988; Black and Henderson, 1999). As stated above, we focus specifically on the share of people aged 25 years and over who are literate, a measure that is more meaningful in Brazil.

According to Duranton and Puga (2013), the availability of road infrastructure is another important component, and agglomeration effects drive city growth. Da Mata et al. (2007) argue that city growth depends on demand and supply factors, which they summarize as the incomes a city can pay out and the incomes people demand to live in a city, respectively. To measure these factors, they use market potential variables, which depend partly on road transport networks. The demand market potential of a spatial unit in turn is defined as the product of per capita income and population size, divided by transport costs, summed over all spatial units in the sample. To describe the supply side, they consider two gravity measures for each spatial unit: the sum of per capita rural income divided by transport costs over all other spatial units in the sample, and the sum of the rural population divided by transport costs over all other spatial units in the sample. Both constitute market potential measures, in that they measure the potential population supply to a city from nearby rural areas.

From a spatial econometric view, market potential variables can be interpreted as spatially lagged explanatory variables or exogenous interaction effects, because they measure the impact of X variables in one spatial unit on the dependent variable in another spatial unit. The disadvantage of using market potential variables is that a certain neighborhood matrix structure is imposed on these X variables, without testing the structure first. According to Da Mata et al. (2007), the market potential variables relate inversely with transport costs, which in turn relate linearly to distance, though they do not test the extent of this degree of distance decay. Beyond imposing a particular neighborhood matrix structure in advance, market potential variables also suffer from inflexibility. For example, consider the non-dynamic spatial panel data model, $\tau = \eta = 0$ from Equation (20), in which the population growth rate $Y = \log(P_{t+1}/P_t)$ can be explained by rural GDP per capita (rGDPc) in the own spatial unit and those observed in neighboring units:

$$\ln \mathbf{Y}_{t+1} = \delta \mathbf{W} \ln \mathbf{Y}_t + \beta \ln(\mathbf{rGDPc}_t) + \theta \ln(\mathbf{W}^* \mathbf{rGDPc}_t) + \mathbf{R}, \qquad (23)$$

where R is a rest term containing the other explanatory variables, the fixed effects, and the error term. We take the logs of all variables, similar to Da Mata et al. (2007). Because Equation (23) is in vector form, W^*rGDPc offers another expression for market potential, provided that W is specified as an inverse distance matrix. Both variables take log forms, so the matrix of partial derivatives of the expected value of the dependent variable with respect to $rGDPc_{it}$ for i = 1, ..., N in year t is given by

$$\left[\frac{\partial E(\ln \mathbf{Y})}{\partial \ln(rGDPc_1)} \quad \cdots \quad \frac{\partial E(\ln \mathbf{Y})}{\partial \ln(rGDPc_N)} \right]_{l} = (\mathbf{I} - \delta \mathbf{W})^{-1} [\beta \mathbf{I}_N + \delta \mathbf{S}], \tag{24}$$

where S is an $N \times N$ matrix whose elements are defined by $s_{ij} = w_{ij}GDP_j/(w_{i1}GDP_1 + \cdots + w_{iN}GDP_N)$. Because s_{ij} measures the rural GDP per capita of one MCA relative to all other MCAs, the rows of S always sum to unity, independent of the relative location of a particular MCA—that is, whether it is located on the periphery or in the core of Brazil. In other words, irrespective of how W is specified, the structure of the partial derivatives in Equation (24) is exactly the same as in Equation (21), because the rows of both S and S sum to unity. In contrast, by replacing $\ln(W^*rGDPc)$ with $M^*\ln(rGDPc)$ and decomposing the composite variables, such as market potential, into their underlying components (which leads to partial derivatives similar to those in Equation (21)), we introduce more empirical flexibility, including the opportunity to test how S should be specified and thus to what extent MCAs likely interact. Furthermore, the magnitudes of the direct and spillover effect estimates can vary with different explanatory variables that determine the market potential. Just as in Da Mata et al. (2007), we assume that the population growth rate depends on GDP per capita, rural GDP per capita, rural population size, and their spatially lagged values, but as separate variables, so that we can test for agglomeration effects.

We also include other demand and supply factors that can affect productivity and city

growth: the industry mix and the size of the workforce. Glaeser et al. (1995) control for the share of employment in manufacturing and find that it has an adverse effect on population growth. Da Mata et al. (2007) control for the ratio between the share of employment in manufacturing to that of services, to account for local adjustments relative to changes in national output composition, and find a positive effect. Chi and Voss (2011) include the share of employment in agriculture, though they removed this variable from the model when it emerged as insignificant. Yet we still consider the share of employment in agriculture, because we focus on population growth in Brazil, with its vast agricultural areas; especially at the beginning of our observation period, the number of women in the labor force was relatively high, mostly as unpaid and low productivity workers on family farms who combined their agricultural work with childcare. When income levels started to rise, such as through the expansion of the manufacturing sector and the introduction of new technologies, women's labor force participation rates tended to fall (Goldin 1995; Mammen and Paxson, 2000). Men moved into new blue-collar jobs that increased family-level income, such that unearned income effects reduced women's participation. In addition, a high share of employment in agriculture means relatively less employment in urban occupations, which may affect local agglomeration gains from diversification (Gleaser et al., 1992). In summary, we expect that the share of employment in agriculture has a negative effect on population growth, due to the implied low productivity and the reduction in economic opportunities, especially for women, and to its potentially reduced gains from diversification. Similar to Da Mata et al. (2007), we also consider the ratio between the share of employment in manufacturing and that of services as a potential factor affecting local productivity dynamics.

Glaeser et al. (1995) also control for unemployment, because potential migrants do not move to areas with high unemployment rates. They find an adverse effect on U.S. population growth after 1970; we thus use the percentage of the economically active population that is occupied as an additional supply side factor. This variable offers an opposite of the unemployment rate.

Although GDP per capita measures often appear in empirical growth studies, they ignore a critical dimension of population dynamics, namely, populations' evolving age structure. Each age group in a population behaves differently, and the distribution across age groups changes over time, so economic opportunities can be boosted or slow down temporarily. Whereas prime-age adults supply labor and savings, the young require education, and the aged need health care and retirement income. Economic opportunities then should increase when the number of working-age adults is large, relative to the dependent population, but decrease when a population rapidly ages. By translating an economic model formulated as GDP per capita growth into a comparable model of GDP per worker, which largely determines demand and supply conditions in a city and its surroundings, different studies have shown that demographic variables might be important (Bloom and Williamson, 1998; Choudhry and Elhorst, 2010; Kelley and Schmidt, 2005). Such variables include the mean age of the population and the birth rate. If the population of a particular region is relatively immobile, differences in population growth across areas within that region likely

are due mainly to differences in fertility (Glaeser et al., 1995), which is an important reason to consider the birth rate.

From our theoretical model, the other influence on city growth arises from factors associated with local amenities that affect individuals' welfare. Here we consider local characteristics that are directly associated to local quality of life and city infrastructure. In fact, two of the key drivers of city growth that Duranton and Puga (2013) identify are infrastructure and housing supply. The first follows from the monocentric city model; the second determines how cities react to positive or negative shocks. If the supply of houses is limited by geographical constraints or land-use regulations, a positive shock leads to higher housing prices. If the supply of houses is elastic, housing prices might increase to some extent, but the local inhabitants and incoming migrants react by choosing to live in smaller dwellings. Important infrastructural components of the housing supply in Brazil are the proportions of people with access to public water or public sewers; insufficient supply may cause higher prices and deter potential migrants. These two variables also appear in Chi and Voss's (2010) study, whereas Glaeser et al. (1995) consider expenditures on sanitation. These variables increased from, respectively, 14% and 5% in 1970 to 71% and 37% in 2010. In addition, population density can control for the second factor. Many cities could take in more people, though at the expense of the quality of the living conditions, which might deter prospective migrants. Because it takes time to build public goods, infrastructure, or housing, the residents of quickly growing cities may suffer more in terms of their quality of life.

Brazilian cities are also characterized by very high levels of urban violence, a factor that directly affects urban life quality (Scorzafave and Soares, 2009; Menezes et al., 2013). Da Mata et al. (2007) explicitly mention local crime and violence, measured by the homicide rate or number of homicides per one million inhabitants and this variable exerts a negative and significant effect on population growth in their study. Duranton and Puga (2013) also highlight the importance of (dis)amenities for urban growth. So, we include the homicide rate in the present study as well.

Finally, although our theoretical model does not explicitly consider any kind of urban hierarchy conditioning the influence of the variables on urban dynamic, we will also estimate a heterogeneous version of equation (20) considering potentially different influences of the variables for metropolitan versus non-metropolitan cities. As we deal with all Brazilian municipalities, which include very different sizes of cities, the idea is to explore structural differences in population growth dynamics across cities that belong to and do not belong to a metropolitan region. The biggest municipalities in Brazil generally present a broader set of services (including federal government activities), specific kinds of manufacturing activities (with different degree of returns to scale), and higher levels of human capital and are located in metropolitan regions. The approach, thus, explores the possibility of different direct and spillover effects associated with the proximity to these big Brazilian cities.⁵

⁵ The numbers of MCAs that belonged to a metropolitan region in Brazil were 115 in 1980, 120 in 1991 and 285 in 2000. From an universe of 3,659 MCAs, these MCAs were big cities or municipalities influenced by big cities.

5. Empirical Analysis

The estimation results of the parameters of equation (20) are in Table 1. The first column reports the estimation results of a standard linear panel data model, extended to include spatial and time-period fixed effects, but without any spatial interaction effects. We discuss the results of several specification tests before turning to the estimation results in the second column of Table 3.

<< Table 3 around here >>

5.1 Model specification: fixed effect and spatial dependence

To investigate the (null) hypothesis that the spatial fixed effects are jointly insignificant, we performed a likelihood ratio (LR) test. The results (8674.34, with 3658 degrees of freedom [df], p < 0.01) reject this hypothesis. Similarly, we can reject the hypothesis that the time-period fixed effects are jointly insignificant (789.06, 3 df, p < 0.01). These results justify the extension of the model with spatial and time period fixed effects. Table 4 reports the correlation coefficients for the explanatory variables, which indicate that multicollinearity is not a problem.

<< Table 4 around here >>

To test whether the non-spatial model with spatial and time period fixed effects should be extended with spatial interaction effects for the dependent variable (SAR specification) or for the error terms (SEM specification), we used LM tests, applied to a first-order, binary, contiguity neighborhood matrix that we row-normalized to ensure row sums equal to 1. These LM tests follow a chi-squared distribution with one degree of freedom and reach a critical value of 3.84 at 5% significance or 2.71 at 10% significance. In classic LM tests, the hypotheses of both no spatially lagged dependent variable and no spatially autocorrelated error term must be rejected. With robust tests, we can reject the hypothesis of no spatially lagged dependent variable, but the hypothesis of no spatially autocorrelated error term cannot be rejected, at 10% significance. These test results suggest extending the non-spatial model with a spatially lagged dependent variable. However, if the robust LM tests reject a nonspatial model, in favor of the spatial lag or spatial error models, we must carefully endorse one of these models. LeSage and Pace (2009) and Elhorst (2012b) also recommend considering the spatial Durbin model and testing whether it can be simplified to the spatial lag or spatial error model. For this study, we take a broader view and apply the Bayesian approach from Section 3. First, we calculate the Bayesian posterior model probabilities of the SDM and SDEM specifications, as well as the simpler SAR and SEM specifications, to identify which model specification best describes the data. Second, we repeat this analysis for several specifications of the neighborhood matrix, to find the specification of W that best describes the data. In total, we consider 11 matrices: p-order binary contiguity matrices for p = 1–3, an inverse distance matrix, and q-nearest neighbors matrices for q = 5-10 and 20.

The results in Table 5 show that the SAR and SEM models are always outperformed by either the SDM or SDEM specifications. Therefore, spatially lagged explanatory variables (WX) are important and should be included in the model. The worst performing spatial neighborhood matrix is the inverse distance matrix, which corroborates our point that decomposing market potential variables into their underlying components and considering the spatially lagged values of these components creates a much greater degree of empirical flexibility. If the neighborhood matrix is specified as a first-order binary contiguity matrix or as a 5-nearest neighbors matrix, the Bayesian posterior model probabilities point to the SDM specification. The average number of neighbors in the sample amounts to 4.98, so these two neighborhood matrices are not substantially different. If we let the number of neighbors increase, by considering either higher-order binary contiguity matrices or nearest neighbors matrices with more neighbors, the Bayesian posterior model probabilities provide further evidence in favor of the SDEM specification. However, by also considering the log-marginal values of the different specifications of the neighborhood matrix, we note that the first-order binary contiguity matrix and the SDM specification achieve the best performance of all 44 combinations, in line with the initial robust LM test statistics for the non-spatial panel data model, which pointed to a spatial lag rather than a spatial error model. In turn, we decided to estimate the dynamic SDM specification using the bias-corrected ML estimator developed by Lee and Yu (2010). The estimation results are in the second column of Table 3. The results then serve to test H_0 : $\theta = \theta$ and $\eta = 0$ and H_0 : $\theta + \delta \beta = \theta$ and $\eta + \delta \tau = 0$. That is, we test whether the dynamic spatial Durbin might be simplified to a dynamic spatial lag model or dynamic spatial error model. Both tests follow a chi-squared distribution with K+1 degrees of freedom (number of spatially lagged explanatory variables and the spatially lagged dependent variable) and take the form of a Wald test, because the simplified models have not been estimated. The results reject both hypotheses, but again, a spatial econometric model extended to include a spatially lagged dependent variable is more likely than its counterpart with a spatial autocorrelated error term. Overall, the empirical results point to the utility function specified in Equation (18), which posits that the utility of individuals correlates negatively with the level of population (population size) and the population growth rate of their neighbors, and to the global spillover model, which posits that $\delta \neq 0$.

5.2 The determinants of Brazilian municipalities population dynamics

The results reported in second column of Table 3 show that six of the thirteen spatially lagged explanatory variables in the dynamic SDM specification appear statistically significant at the 5% level. The coefficients of the spatially lagged dependent variable at time t and t–1, WY_t and WY_{t-1} , are also significant. A necessary and sufficient condition for stationarity, $\tau + \delta + \eta = 0.0755 + 0.3439 + 0.0681 = 0.4875 < 1$, is satisfied. In Table 6, columns (I)-(III), we

⁶ This bias correction is needed because the dependent variables lagged in time and in both space and time on the right-hand side of Equation (20) are correlated with the spatial fixed effects μ , which is the spatial counterpart of the Nickell (1981) bias, as shown by Yu et al. (2008) and Lee and Yu (2010) for a dynamic spatial panel data model without and with time-period fixed effects, respectively.

report long-term estimates of the direct, spillover and total effects, derived from the parameter estimates using Equation (22).⁷ To draw inferences regarding the statistical significance of these effects, we used the variation of 100 simulated parameter combinations, drawn from the variance-covariance matrix implied by the ML estimates. The number of explanatory variables with significant (5%) spillover effects is three and with weakly significant (10%) spillover effects is two; this count is less than the number of significant spatial interaction effects because they depend on more than just one parameter—that is, five parameters in the long term (Equation 22).

<< Table 6 around here >>

First of all, the long-term, direct, spillover, and total effect estimates of the growth rate represent significant convergence and deconcentration effects. The direct effect amounts to -0.918, and the total effect is -0.781; they are both significant. That is, the greater the population growth in the MCA in the previous decade, the smaller it will be in the next decade, and vice versa. This finding points to convergence. The spillover effect of 0.137 is also significant, which indicates that population growth can be stimulated if population growth in neighboring MCAs has been greater in the previous decade. This movement or deconcentration of people to neighboring areas, perhaps to escape the bustle of the city, represents a convergence effect. However, as a feedback effect of this behavior, the city starts growing again, such that the total convergence effect diminishes. This rationale helps explain the reduction of the convergence effect from -0.918 to -0.781.

Regarding the influence of factors associated to local productivity, we first note that the literacy rate increases by one percentage point, the population growth rate in the area increases by 0.083 percentage points, and that the cumulative effect over all neighbors amounts to 0.143 percentage points. These effects are weakly significant (10%), and confirm the hypotheses that the population grows faster if it is relatively immobile; due to deconcentration, this growth partly spills over to neighboring areas. The positive relationship between educational attainment and population growth matches Glaeser and Saiz's (2003) and Da Mata et al.'s (2007) arguments that economies with better educated people generate positive amenities and are more adaptable to technological change. The positive relationship between educational attainment and population growth in the surroundings also aligns with the proposition introduced by Equation (2), namely, that knowledge accumulated in one economy depends on knowledge accumulated in others.

Just as the literacy rate, most variables associated with local productivity have the expected signs, although not all of them do produce significant spillover effects. As expected and in contrast to Chi and Voss (2011), the share of employment in agriculture has a negative effect of 0.247 percentage points on population growth in the long term, due to the reduction in economic opportunities, especially for women. A greater share of employment in manufacturing relative to services and GDP per capita instead have positive, significant

⁷ Since the analysis is based on data observed over 10-year time intervals, the short-term effects do not differ greatly from the long-term effects. For this reason they are not reported, but available upon request.

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effects. These two results are consistent with the idea that the growth of productivity is higher in municipalities with bigger markets and with stronger presence of manufacturing activities. Rural GDP per capita also has a positive and significant direct effect on population growth, such that municipalities that offer income opportunities remain attractive. However, they do not have positive effects on their environment, in that the spillover effects of these variables are insignificant. Da Mata et al. (2007) note that their rural variables perform poorly, due to limited variation and multicollinearity, but by decomposing the market potential variables, we avoid such problems.

In contrast to rural GDP per capita, the direct effect of the rural population is negative and significant. A one percentage point increase of the rural population has an adverse effect on population growth, equal to 0.043 percentage points. The spillover effect amounts to -0.021 and is statistically significant; rural municipalities surrounded by other rural municipalities tend to grow one and a half times slower than rural municipalities close to urban areas. These negative effects are probably explained by the strong correlation between this variable and the absence or insufficiency of local provision of basic household infrastructure in Brazilian municipalities with high rural population, making these localities less attractive.

The birth rate not only produces a significant direct effect, but also a significant spillover effect that, in terms of magnitude, is greater than the direct effect. If the birth rate increases by 1 child for every 1,000 inhabitants in a given area, the population growth rate in that area itself increases by 0.018 percentage points in the long term, and 0.027 percentage points in its surroundings. This latter figure represents the cumulative effect over all neighbors; considering our finding that the average number of neighbors is 4.98, the average spillover effect per neighbor is likely around 0.005. The impact of the mean age of the population is positive and significant. If this mean age increases by one year, the population growth rate increases by 0.01 percentage points. During our observation period, the mean age increased, from 23 in 1970 to 32 in 2010, and this finding corroborates the view that economic opportunities grow when the number of working-age adults increases, relative to the dependent population. Finally, consistent with Glaeser et al.'s (1995) idea that potential migrants do not move to areas with high unemployment rates, we obtain positive direct and total effects of the percentage of economically active population that is occupied on population growth of Brazilian municipalities.

As for the variables associated to local amenities, we note that all variables, when statically significant, have the expected signs, and some of them with important spillover effects. Specifically, the direct effect of population density is negative and significant, corroborating the hypothesis that densely populated cities deter prospective migrants with their poor living conditions. To some extent this negative effect may also be related with a kind of convergence in population size across cities. Interestingly, this adverse effect also spills over to neighboring MCAs. The spillover effect is negative and significant and, in terms of magnitude, almost as substantial as the direct effect. If population density in a city increases by one percentage point, the population growth rate falls by 0.145 percentage points in the long term in the city, and by 0.141 in its surroundings. We uncover even stronger

results related to homicide rates. The direct effect is insignificant, but the spillover effect is negative and weakly significant (10%), such that city surroundings pay the price for this disamenity. The negative relationship between the homicide rate and population growth in surroundings corroborates the proposition from Equation (7) that disamenities in one economy harm individuals and deter prospective migrants in neighboring economies.

The proportion of people with access to public water has a positive effect and significant effect on population growth, but the proportion of people with access to public sewer does not. This variable partly reflects the price of urban space: If the supply of housing with access to public sewer is relatively inelastic, the prices of this type of housing might increase so much that prospective migrants would be discouraged, and the population growth rate would decrease again. Research by FGV (2010) suggests that sanitation enables construction with higher added value and appreciation in the value of existing buildings.

The significant spillover effects obtained for some variables make it interesting to compare the long-term total effects reported in Table 4, derived from the dynamic SDM specification, against those from the non-spatial model reported in the first column of Table 1. The long-term total effect of the latter model can be obtained by calculating $\beta/(1-\tau)$, where β is the coefficient estimate of a particular explanatory variable and τ is the coefficient estimate of the dependent variable (population growth rate), lagged one decade. The results of these comparisons are presented in Table 6, column (IV). The long-term total effect of the rural population amounts to, according to the spatial model, -0.064 and, according to the non-spatial model, -0.0433/(1 - (-0.0271) = -0.0422. Therefore, the effect in the non-spatial model is underestimated by 34.1%. For the other variables that produce significant spatial spillover effects, we find 57.5% for population density, 61.9% for the birth rate, 41.4% for the literacy rate, and 58.3% for the homicide rate. The degree of underestimation averages 27% across all explanatory variables, thus a non-spatial modeling approach, as the previous ones applied to Brazilian cities' population dynamics, evidently does not reflect the full impact of policy measures that act on these variables.

5.3 Metropolitan and non-metropolitan heterogeneities

Table 6 also reports direct and spillover effects of the variables on population growth for non-metropolitan (columns V and VI) and metropolitan (columns VII and VIII) municipalities, to explore possible structural difference between them. In the empirical model, the differentiated effects by metropolitan and non-metropolitan conditions are obtained by interacting each variable with a dummy for the condition of belonging to a metropolitan area. The results appear to indicate important heterogeneities associated with belonging to a metropolitan region. Those that are statistically different from their counterparts in the homogenous model are marked grey.

Firstly, regarding the influence of variables associated with local productivity, we note that while the direct and spillover effects for non-metropolitan municipalities are basically similar to the ones in the homogenous model, those obtained for metropolitan localities present important particularities. Specifically, we do not find statistically significant direct or

spillover effects arising from the human capital variable (literacy rate), a result probably explained by the higher and more homogenous levels of schooling in these localities. Similarly, the direct effects arising from the variables rural GDP per capita and the manufacturing/services ratio are not statistically significant for MCAs belonging to metropolitan areas. These two results are consistent with a minor importance of rural markets and a more productive services sector in these areas. Related to this, the spillover effect of rural population is also no longer significant, both in metropolitan and non-metropolitan MCAs. This indicates that the significant spillover effect previously obtained for the entire set of MCAs is solely explained by the difference between non-metropolitan and metropolitan MCAs. Another spillover effect no longer significant for metropolitan MCAs is the birth rate. By contrast, only for metropolitan MCAs we do find a positive and significant spillover effect associated with the variable GDP per capita. Apparently, the belonging to a metropolitan area is important for a municipality to take advantage from markets in its surroundings. Note that this spillover effect is consistent with both the better road infrastructure and stronger returns to scale in the economic activities generally present in these MCAs.

The results for the influence of the amenity-related variables on population growth of Brazilian cities in the heterogeneous model are quite different from those in the homogeneous model. Differently from previous results, neither the direct nor the spillover effect of the homicide rate on population growth is significant, a result valid for both non-metropolitan and metropolitan MCAs. Just as for the rural population, this indicates that the weakly significant spillover effect previously obtained for the entire sample is solely explained by the difference between non-metropolitan and metropolitan MCAs. As for metropolitan MCAs, we also obtain different direct effects for the variables associated with the public availability of water and sewer: different from the previous results, the availability of public water now presents a negative direct effect on population growth and the availability of public sewer a positive one. The latter result is in line with the better conditions present in metropolitan regions, where the supply of housing with this service tends to be more elastic to market conditions. On the other hand, the negative direct effect of availability of water appears to reflect the difficulty in water provision or the exhaustion in the availability of this resource in metropolitan areas. This situation tends to be associated with higher prices of housing in these MCAs, making them less attractive.

We finally note that the direct and spillover effects of population density, both for non-metropolitan and metropolitan MCAs, are negative and significant; this once more corroborates the hypothesis that densely populated cities deter prospective migrants with their more expensive living conditions not only to their own MCAs but also to their surroundings.

6. Conclusion

The process of Brazilian urbanization represents a highly significant social phenomenon. Now the country presents around 85% of people living in urban centers and the movements of people among localities are mainly movements of people between cities. Given the few number of studies about the determinants of Brazilian cities' population growth,

we propose a spatial theoretical economic model for city population growth, derive an explicit econometric spatial model from it, and estimate the effects of variables associated with the population growth of Brazilian cities during the period 1970-2010.

Specifically, from a theoretical perspective, our contribution extends the population growth model developed by Glaeser et al. (1995) and Glaeser (2008) to include spatial interaction effects. Instead of treating cities as independent entities, we include interaction effects in the production function and assume that knowledge accumulated in one economy depends on knowledge accumulated in other economies; we also include interaction effects in the utility function by assuming that (dis)amenities in one economy may (dis)benefit people living in other economies. Depending on whether individual utility correlates negatively with population size and population growth of neighboring economies, we show that the population growth rate that can derived from our extended urban growth model eventually results in an econometric model known as either a dynamic spatial Durbin error model (SDEM) or a dynamic general spatial nesting (GNS) model.

We use a Bayesian approach to discriminate between these two models empirically, as well as different potential specifications of the neighborhood matrix. The strength of a comparison based on log-marginal values is that the outcome is independent of specific parameter values, as they have been integrated out in advance. The dynamic GNS model approach by the dynamic spatial Durbin model (SDM) in combination with the first-order binary contiguity matrix achieves the best performance of all 44 possibilities, and our empirical results thus point to global spillover effects and the utility function as a means to account for the population size and population growth of neighboring economies.

Consistent with the proposed spatial economic model of city growth, the parameter estimates of the variables associated with local productivity and city amenities generate a plausible model structure, i.e., they take the theoretically expected signs, with only one exception. In addition, population dynamics of Brazilian MCAs are substantively affected by their location, i.e., they are evidently associated to productivity and amenities of their neighbors.

More specifically, among the set of factors associated to local productivity, the results obtained indicate that the population growth of the Brazilian MCAs are positively affected by the level of human capital (literacy rate), the level of GDP per capita, and by the manufacturing/services employment shares ratio. Furthermore, in the case of human capital, there are spillovers arising from neighboring MCAs that also positively affect the population growth of the Brazilian MCAs. Regarding the set of variables associated with local amenities, the evidence indicates that population growth of Brazilian cities is positively affected by the level of public water provision and negatively by the share of employment in agriculture. There are also spillover effects related to some amenities: demographic density and homicide rates of neighboring MCAs negatively affect the population growth of Brazilian MCAs.

To investigate the extent to which our spatial extension of the population growth model makes a difference, we count the number of explanatory variables causing significant spatial interaction effects. Of the 12 determinants of population growth, 5 produce significant spillover effects in the long term: rural population size, population density, the birth

rate, the literacy rate, and the homicide rate. A change of one unit in one of these variables significantly affects population growth in other units, a phenomenon that has been ignored in most previous studies of population growth. By comparing the results with the evidence obtained from a non-spatial panel, we demonstrate that a non-spatial approach for Brazil substantively underestimates the long-term total effects of the explanatory variables: underestimation averages 27% across all explanatory variables. Regarding the last four determinants, we find that the magnitude of the cumulative effect across all neighbors is as great as the magnitude of the impact on the city itself.

In order to explore heterogeneities of the results associated with belonging to metropolitan areas, that includes the biggest cities of the country, additional results are generated for non-metropolitan and metropolitan MCAs. While for non-metropolitan MCAs these results are similar to the ones previously obtained, for the set of metropolitan MCAs we also find positive and significant spillover effects associated with the variable GDP per capita, but not for the human capital variable (literacy rate). These results are consistent, respectively, with both the better road infrastructure and stronger returns to scale in the economic activities in these MCAs and with the higher and more homogenous levels of schooling in these localities.

From the perspective of government policies directed to stimulate cities' population growth, the results not only suggest important determinants to focus on, but also the ones that tend to be more effective. Specifically, in addition to implement policies favoring economic activities and with special focus on manufacturing sector and policies to improve housing infrastructure, the government must mainly act on determinants that generate both direct and spatial spillover effects. Thus, localities that would hope to stimulate growth should better educate their population, offer good child-care facilities, reduce crime, and coordinate housing construction with neighboring localities, to spread the population over a larger area.

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Source: Geographical delineation by the authors.

Table 1. Relationship between econometric and theoretical model equations

Econometric model	Theoretical model
$ au oldsymbol{Y}_{t ext{-}l}$	$\psi au \log \left(rac{P_{it}}{P_{it-1}} ight)$
δWY_t	$\psi(-(\nu+\sigma))\left(\sum_{j\neq i}^{N}w_{ij}\log\frac{P_{jt+1}}{P_{jt}}\right)$
$oldsymbol{\eta WY_{t ext{-}l}}$	$\psi\sigma\!\!\left(\sum\limits_{j eq i}^{N}\! w_{ij}\log\!rac{P_{jt}}{P_{jt-1}} ight)$
$X_{i}\beta$	$\left(1 + \frac{(\delta - \alpha\delta + \alpha)}{\delta}\right) X'_{it}(\lambda_a + \lambda_\theta)$
$WX_i\theta$	$\left(\eta + \rho \frac{(\delta - \alpha \delta + \alpha)}{\delta}\right) \sum_{j \neq i}^{N} w_{ij} X'_{jt} (\lambda_a + \lambda_\theta)$
$oldsymbol{arepsilon}_t$	$\psi \left(\log \left(\frac{\overline{V}_{t+1}}{\overline{V}_{t}} \right) + \varepsilon_{it+1} + \xi_{it+1} \right)$
$\lambda W v_t$	$\psi\!\!\left(\eta\!\sum_{j\neq i}^{N}\!w_{ij}\varepsilon_{jt+1}\!+\!\rho\frac{(\delta\!-\!\alpha\delta\!+\!\alpha)}{\delta}\!\sum_{j\neq i}^{N}\!w_{ij}\xi_{jt+1}\right)$

Table 2. Description, type and data source of explanatory variables

Explanatory variables	Description	Data source
Dependent variable		
population growth rate	Population growth rate	IPEADATA
Productivity-related varie	ables (a), see equations (3) and (16a)	
literacy rate	Percentage of population (age>25) that is literate	Census/IBGE
In GDP per capita	Natural log of GDP per capita (prices of 2010)	IPEADATA
In rural GDP per capita	Natural log of rural GDP per capita (prices of 2010)	IPEADATA
In rural population	Natural log of share of population living in rural areas	IPEADATA
Agriculture	Percentage of people working in agriculture, livestock, hunting and related services (age>10)	Census/IBGE
manufacture/service	Relationship between the number of employees in manufacturing and the service sector	Census/IBGE
workforce occupied	Workforce occupied (employment rate)	IPEADATA
birth rate	(Mean of number of children born alive and still living)*(1000/Pop)	Census/IBGE
mean age	Mean age	Census/IBGE
Amenity-related variable.	$s(\theta)$, see equations (7) and (16b)	
In density	Natural log of people per squared kilometres	IPEADATA
homicide rate	(Number of homicides)*(100000/Pop)	IPEADATA
water company	Share of households supplied by water company	Census/IBGE
sewer company	Share of households supplied by sewer company	Census/IBGE

Table 3. Population Growth: Non-Spatial and Dynamic Spatial Models
OLS+

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
Explanatory VariablesEffectsEffects (bias correction)Dependent variable lagged in space and/or time $W*Y_t(\delta)$ 0.3439** $Y_{t-1}(\tau, \tau \text{ and } \eta)$ -0.0271**0.0755**0.0681**Productivity-related variables (a), see equations (3) and (16a)
Explanatory Variables Coeff t Coeff t Spatial t Dependent variable lagged in space and/or time $W^*Y_t(\delta)$ 0.3439 ** $Y_{t-1}(\tau, \tau \text{ and } \eta)$ -0.0271 ** 0.0755 ** 0.0681 ** Productivity-related variables (a), see equations (3) and (16a)
Dependent variable lagged in space and/or time $W^*Y_t(\delta)$ 0.3439 ** $Y_{t-1}(\tau, \tau \text{ and } \eta)$ -0.0271 ** 0.0755 ** 0.0681 ** Productivity-related variables (a), see equations (3) and (16a)
$W^*Y_t(\delta)$ 0.3439 ** $Y_{t-1}(\tau, \tau \text{ and } \eta)$ -0.0271 ** 0.0755 ** 0.0681 ** Productivity-related variables (a), see equations (3) and (16a)
$W^*Y_t(\delta)$ 0.3439 ** $Y_{t-1}(\tau, \tau \text{ and } \eta)$ -0.0271 ** 0.0755 ** 0.0681 ** Productivity-related variables (a), see equations (3) and (16a)
$Y_{t-1}(\tau, \tau \text{ and } \eta)$ -0.0271 ** 0.0755 ** 0.0681 ** Productivity-related variables (a), see equations (3) and (16a)
Productivity-related variables (a), see equations (3) and (16a)
0.12(1) ## 0.0001 ## 0.0205
literacy rate 0.1361 ** 0.0681 ** 0.0395
In GDP per capita 0.0513 ** 0.0527 ** -0.0248 **
In rural GDP per capita 0.0088 ** 0.0135 ** -0.0095 **
In rural population -0.0433 ** -0.0391 ** 0.0068
Agriculture -0.2612 ** -0.2315 ** 0.1063 **
manufacturing/service 0.0045 ** 0.0021 ** 0.0016
workforce occupied 0.4911 ** 0.3535 ** -0.0681
birth rate 0.0172 ** 0.0150 ** 0.0072 **
mean age 0.0135 ** 0.0089 ** -0.0020
Amenity-related variables (θ) , see equations (7) and (16b)
In density -0.1248 ** -0.1256 ** -0.0221 **
homicide rate -0.0030 ** 0.0006 -0.0042 *
water company 0.0081 0.0274 -0.0255
sewer company -0.0123 -0.0365 ** -0.0058
Regression diagnostics
No. Obs. 10977 10977
R-squared 0.711 0.743
Log Likelihood 11144 5580.37
Spatial lag, OLS model:
LM 909.32 ** Spatial lag, SDM model:
<i>LM(robust)</i> 114.89 ** <i>Wald</i> 54.39 **
Spatial error, OLS model:
LM 796.34 ** Spatial error, SDM model:
<i>LM(robust)</i> 1.91 <i>Wald</i> 134.23 **
Joint significance
<i>LR(spatial fe=0)</i> 8674.60 **
LR(time fe=0) 789.06 **

^{**} Significant at 1%. *Significant at 5%.

Table 4. Correlation Coefficients Across Explanatory Variables

In rural population	1.00												
In density	-0.25	1.00											
mean age	-0.32	0.13	1.00										
birth rate	-0.29	-0.16	-0.05	1.00									
literacy rate	-0.30	0.14	0.60	-0.15	1.00								
agriculture	0.23	-0.32	-0.31	0.21	-0.48	1.00							
manufacturing/service	-0.09	0.11	0.14	-0.07	0.20	-0.19	1.00						
workforce occupied	0.09	-0.04	-0.29	0.14	-0.20	0.48	-0.08	1.00					
In GDP per capita	-0.26	0.19	0.40	-0.13	0.76	-0.42	0.19	-0.07	1.00				
In rural GDP per capita	0.10	-0.44	0.02	0.19	0.05	0.46	-0.07	0.38	0.20	1.00			
homicide rate	-0.05	0.17	0.02	-0.13	0.14	-0.25	-0.03	-0.19	0.18	-0.19	1.00		
water company	-0.36	0.26	0.56	-0.18	0.65	-0.64	0.11	-0.40	0.55	-0.20	0.18	1.00	
sewer company	-0.30	0.26	0.51	-0.13	0.53	-0.45	0.11	-0.22	0.49	-0.10	0.07	0.64	1.00

Table 5. Comparison of Model Specifications and Neighborhood Matrices

W Matrix	Statistics	SAR	SDM	SEM	SDEM
Binary Contiguity	log marginal	3566.85	3616.03	3548.42	3611.80
Billary Configurty	model probabilities	0.0000	0.9855	0.0000	0.0145
First and Second Order	log marginal	3562.21	3574.79	3558.60	3579.41
That and Second Order	model probabilities	0.0000	0.0097	0.0000	0.9903
First, Second and Third Order	log marginal	3527.98	3528.75	3535.86	3536.28
riist, second and riint order	model probabilities	0.0001	0.0003	0.3974	0.6022
Inverse distance	log marginal	3368.78	3444.87	3363.32	3455.44
inverse distance	model probabilities	0.0000	0.0000	0.0000	1.0000
5 nearest neighbors	log marginal	3539.69	3601.04	3521.72	3597.88
5 hearest heighoofs	model probabilities	0.0000	0.9594	0.0000	0.0406
6 nearest neighbors	log marginal	3551.02	3613.06	3539.41	3613.60
o nearest neighbors	model probabilities	0.0000	0.3676	0.0000	0.6324
7 nearest neighbors	log marginal	3548.94	3606.39	3537.52	3606.54
, nearest neighbors	model probabilities	0.0000	0.4622	0.0000	0.5378
8 nearest neighbors	log marginal	3551.30	3607.94	3541.97	3610.07
o nearest neighbors	model probabilities	0.0000	0.1054	0.0000	0.8946
9 nearest neighbors	log marginal	3561.30	3610.94	3553.84	3613.93
y nearest neighbors	model probabilities	0.0000	0.0474	0.0000	0.9526
10 nearest neighbors	log marginal	3560.11	3607.68	3556.60	3609.52
10 hearest heighbors	model probabilities	0.0000	0.1373	0.0000	0.8627
20 nearest neighbors	log marginal	3526.87	3552.07	3534.30	3552.99
20 licarest heighbors	model probabilities	0.0000	0.2853	0.0000	0.7147

Source: Own calculations, based on LeSage (2013)

Table 6. Long-term Direct and Spillover Effects of Homogenous and Heterogeneous Dynamic Spatial Model

	•		Unde	restimation of	Long-Terr			-Term Effects	
				Γerm Effect in	Non-Metropol		Metropolitan Areas in Heterogenous		
Explanatory Variables			Non-	Spatial Model	Heterog				
Explanatory variables	Long-	Long-Term Effects		(%)		sion	Regression		
	Direct	Spillover	Total		Direct	Spillover	Direct	Spillover	
	(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)	
lagged population growth rate	-0.918	0.137	-0.781	=	-0.922	0.123	-0.922	0.123	
	(-113.15)	(7.03)	(40.17)		(-109.13)	(6.28)	(-109.13)	(6.28)	
literacy rate	0.083	0.143	0.226	41.4	0.076	0.127	-0.015	0.024	
	(2.38)	(1.73)	(2.48)		(2.68)	(1.65)	(-0.12)	(0.07)	
In of GDP per capita	0.057	-0.001	0.055	9.2	0.050	-0.008	0.079	0.072	
	(12.65)	(-0.11)	(3.96)		(11.57)	(-0.60)	(6.52)	(1.65)	
In of rural GDP per capita	0.014	-0.008	0.006	-42.8	0.019	-0.011	0.001	0.002	
	(5.61)	(-1.14)	(0.89)		(6.48)	(-1.36)	(0.29)	(0.17)	
In rural population	-0.043	-0.021	-0.064	34.1	-0.061	-0.011	-0.023	-0.015	
	(-13.99)	(-1.97)	(-5.55)		(-15.04)	(-0.80)	(-5.53)	(-1.06)	
agriculture	-0.247	0.004	-0.242	-5.1	-0.217	0.070	-0.414	-0.323	
	(-8.09)	(0.06)	(-2.97)		(-8.76)	(0.96)	(-3.78)	(-0.97)	
manufacturing/services	0.003	0.004	0.007	37.4	0.003	0.004	0.002	-0.005	
	(2.24)	(1.06)	(1.63)		(2.19)	(0.96)	(0.37)	(-0.42)	
workforce occupied	0.394	0.167	0.561	14.8	0.340	0.085	0.361	0.100	
	(10.35)	(1.51)	(4.47)		(8.56)	(0.75)	(3.29)	(0.42)	
birth rate	0.018	0.027	0.044	61.9	0.017	0.026	0.094	0.160	
	(9.52)	(3.37)	(5.48)		(9.29)	(4.30)	(2.17)	(1.18)	
mean age	0.010	0.004	0.013	-1.1	0.009	0.003	0.007	-0.000	
-	(7.14)	(1.16)	(4.53)		(6.29)	(0.81)	(1.66)	(-0.02)	
In density	-0.145	-0.141	-0.286	57.5	-0.135	-0.138	-0.175	-0.100	
	(-22.47)	(-6.72)	(-12.91)		(-20.28)	(-6.46)	(-14.04)	(-2.96)	
homicide rate	0.000	-0.007	-0.007	58.3	0.000	-0.000	-0.000	0.001	
	(0.36)	(-1.80)	(-1.57)		(0.91)	(-0.90)	(-0.72)	(0.73)	
water company	0.028	-0.028	0.001	0.0	0.041	-0.006	-0.226	0.170	
· ·	(1.93)	(-0.72)	(0.02)		(2.53)	(-0.14)	(-4.92)	(1.25)	
sewer company	-0.040	-0.043	-0.082	85.4	-0.043	-0.017	0.022	-0.114	
	(-2.71)	(-1.40)	(-2.56)		(-3.31)	(-0.52)	(0.64)	(-1.13)	

Note: t-values in parentheses. Significance levels of numbers marked grey in the heterogeneous model are different from their counterparts in the homogenous model.