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Does History Fully Determine the Spatial Distribution of Human Capital

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Does History Fully Determine the Spatial Distribution of Human Capital?

Henri Busson*

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Abstract

In the United States, cities with more human capital attract skilled workers, and as a result, convergence between states does not occur. Many among the most productive European workers try to migrate to the United States. Consequently, economic growth in Europe could be affected by such migrations. To comprehend these problems, we develop a theoretical Economic Geography model with heterogeneous skills for workers. The question is whether history completely determines the final equilibrium or whether is it possible to attract skilled workers to regions with less human capital. Our results suggest that history does not determine the final outcome conditional on economies of scales that are sufficiently high. This finding contradicts previous papers, which demonstrated that heterogeneity was ruling out indeterminacy.

JEL-Classification: J61, C62, R12

Keywords: economic geography; location choice; equilibrium paths; linear differential systems

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1 Introduction

Spatial inequalities can be very damaging by producing social tensions (Davezies [6]). As a consequence, inequality is a matter of concern for national politicians. National governments must address inequality because of its impact on the welfare of their citizens. In the past, local politicians have attempted to stem the decline of their city or state. For instance, in Spain, the regional Basque government has built a new museum in Bilbao (the Guggenheim museum) to make the city more dynamic (Plaza [27]). The most famous example of a declining city is Detroit, which was one of the most successful cities at the beginning of the twentieth century. The decline of Detroit can be attributed to the process of deindustrialization, the poor quality of its workforce and the city's racial tensions.

Spatial inequalities emerge because human capital is not equally distributed between cities, states or even countries. Glaeser and Berry [12] noted that in the US, cities with more human capital tend to attract more skilled workers than other cities. This unequal repartition of human capital produce a divergence in income between regions established by Sala i Martin and Barro [16]. Moreover, the same problem arises at an international level. Saint-Paul [30] demonstrated that many skilled European workers migrate to the United States. A similar phenomenon has occurred in Israel, many highly educated workers flew to the United States (Ben-David [3]). Europe growth prospects may be dramatically reduced if its best and brightest migrate to the United States.

However, the brain drain from India and China to the United States is being reversed. Many entrepreneurs are returning in their home countries (Wadhwa et al. [32]). The main explanations are family ties and economic opportunities. 72 percent of Indian returnees and 61 percent of Chinese returnees cited lower employees' wages as an important advantage of their home countries. Another famous example is the brain drain of the physician toward the United States that used to occur. Many medical activities are now outsourced in developed countries (Kshetri and Dholakia [19]) and many physicians are returning in their home countries.

This paper aims to answering the following questions: Are the lack of convergence between states that is occurring in the US or the brain drain from Europe to the United States completely determined by history? Or could it be reversed by making skilled people move toward cities or countries with insufficient human capital (in this case, could future policies be inspired by the example of the reverse brain drain)?

Our methodology is simple: we introduce heterogeneous qualifications for workers in a dy-

dynamic NEG (New Economic Geography) model inspired by Krugman [17]. The final equilibria and how they can be reached are considered. Our model addresses movement from a sector X with economies of scale to a sector C with constant wages. We construct the basic model by considering the difference in wages as the primary factor in location choice. We suppose that skilled workers have an impact on unskilled workers' productivity and vice versa. This assumption is supported by a recent paper made by Eeckhout et al. [9].

As demonstrated by Ottaviano et al. [24], a more general model including differences in workers' utilities could easily be developed. As a result, our model could also be used to study migration from a national or an international point of view. Specifically, our paper focuses on the dispersion of the skills across the landscape. Indeed, skilled workers could be attracted to one city (or to one sector, or to one country), while less skilled workers could be to other cities (or sectors). Therefore, the spatial dispersion of human capital, which does not exist in Krugman's model, is potentially a final equilibrium here. More precisely, this model enables us to understand whether history fully determines the final distribution of human capital among regions. It is a dynamic model. It helps to understand how the workers move from one city to another. Deardorff [8] examined why skilled labor industry may be outsourced. He explained it by fundamental differences between nations. But his model does not explain why some skilled workers decide to relocate and to return in their home countries and the dynamics of workers are not considered.

Many theories have attempted to explain spatial inequalities (for instance Blanchard and Katz [4], Puga [28] or the new economic geography initiated by Krugman [18]). However, previous research has not considered the dispersion of human capital in space.

Royer [29] argues that history plays a major role in the explanation of these inequalities. The initial conditions, the spatial specialization of economic activities, could explain a great deal of the variation and these conditions have not been studied sufficiently by economists. As Davis and Weinstein [7] noted, city size displays outstanding stability throughout the twentieth century, as demonstrated by the Japanese example. This stability persists despite external shocks. It demonstrates that history is decisive in the distribution of workers among cities.

If history matters, individual expectations could also be relevant. The role of expectations in migration was formalized by Krugman [17] and Matsuyama [21] by introducing levels of expected utility. Indeed, it is reasonable to believe that if someone expects future utility levels to be very different from the current levels, he will decide where to locate by taking this information into account. Indeed, even if region A is the wealthiest, if everybody believes that most people will be in region B in the future, people may move to region B. That is, people perform

an intertemporal optimization in these models; people select where the migration destination and the timing of the move. These models are dynamic. In previous models, workers are considered to be homogeneous. Several papers extended the original paper made by Krugman [17] (Acemoglu and Jackson [1], Oyama [25], Wirl and Feichtinger [33], Ottaviano [23], Baldwin [2]) but none of them introduced heterogeneity among workers.

Some papers have introduced heterogeneous workers within static models (Picard and Zeng [26], Zeng [34], Tabuchi and Thisse [31]). To our knowledge, there has been only one paper introducing heterogeneity in a dynamic model (Herrendorf et al. [15] which was based on the model made by Matsuyama [21]).

The following results were obtained:

- Two different kinds of workers may not move toward the same location. For instance, skilled workers may move toward city (or sector) X and unskilled workers may move toward city (or sector) C . As a result, several equilibria of spatial dispersion of skills (where the skilled workers are all located in one city and the unskilled workers are all in the other city) appear in the model. These equilibria were not present in the original model.
- Krugman established that if the interest rate and congestion costs are high and the economies of scale are low, expectations do not change the role of history. Contrary to Krugman, our model predicts that a change in expectations leading toward another equilibrium, even with a high interest rate and/or high moving costs, is possible. However such a result depends on the presence of less skilled workers in the areas where the skilled workers are originally leaving. Their impacts on the skilled workers' welfare need to be sufficiently high. Therefore, the brain drain from Europe to the United States or the lack of convergence observed between the states could be fought regardless of the interest rates and the congestion costs. The final equilibrium is not completely determined by history.
- Our result can also be viewed from a theoretical point of view. In several theoretical papers, e.g Morris and Shin [22], Herrendorf et al. [15], heterogeneity is predominantly viewed as a stabilizing force, providing one stable equilibrium. The opposite result is observed here; heterogeneity can increase the level of indeterminacy of the final equilibrium.

In the next section, we present the equations of our theoretical model. Several intermediate results are then elaborated, because these are necessary to the final results of this paper presented in the subsequent section. The last section concludes.

2 The model

2.1 Definitions and assumptions

There are two sectors in our economy, sector C and sector X and as in previous models. In contrast to Krugman, who assumes identical skills, this model assumes that two kinds of workers exist: skilled workers and unskilled workers. The model is described by four equations: two congestion cost equations and two shadow price equations (the opportunity cost of being employed in sector X instead of sector C) which depend on the definition of the wages. The shadow price equations indicate how the shadow price evolves. If that shadow price becomes positive, then the workers will shift toward sector X , and if the price becomes negative, workers will shift toward sector C . The congestion cost equations indicate how many workers will move from one sector to the other at a date, t . Workers cannot all move instantaneously because there are congestion costs (the more workers there are, the higher the costs). Workers move until the marginal gain of shifting sectors equals the marginal costs.

2.1.1 Wage definitions

$\Pi_s(L_{sx}, L_{ux})$ is the average productivity of the skilled workers and $\Pi_u(L_{sx}, L_{ux})$ is the average productivity of the unskilled workers. w_{UX} is the wage of the unskilled workers in the X sector and w_{SX} is the wage of skilled workers. L_{sx} is the number of skilled people in sector X and L_{ux} is the number of unskilled workers in that sector.

The production function is:

$$\Pi(L_{sx}, L_{ux}) = \Pi_s(L_{sx}, L_{ux})L_{sx} + \Pi_u(L_{sx}, L_{ux})L_{ux}$$

Since the economies of scale are external, each firm treats labor productivity as constant: $w_{SX} = \Pi_s(L_{sx}, L_{ux})$ and $w_{UX} = \Pi_u(L_{sx}, L_{ux})$.

Sector C exhibits a constant productivity level ($a < 1$). $\beta_1, \beta_2, \delta_1$ and δ_2 are positive constants, representing the economies of scale.

Considering skill levels, wages are

$$w_{SX} = 1 + \beta_1(L_{sx} - L_{sx1}^*) + \delta_1(L_{ux} - L_{ux1}^*) \quad (1)$$

$$w_{UX} = a + \beta_2(L_{sx} - L_{sx2}^*) + \delta_2(L_{ux} - L_{ux2}^*) \quad (2)$$

The existence of L_{sx1}^* , L_{sx2}^* , L_{ux1}^* and L_{ux2}^* is assumed. However when these equilibrium values exist, they are not necessarily unique. Skilled workers have an impact on unskilled workers' productivity and vice versa. For instance, an unskilled worker could have a positive impact on a skilled worker's productivity by performing simple tasks that allow the skilled worker to focus on complex tasks. A skilled worker could increase the unskilled worker's productivity through good management and improving the organization of work flow. We already pointed that these assumptions were confirmed in a paper made by Eeckhout et al. [9]. They found that in big cities there are bigger inequalities. This is due to the fact that highly skilled workers increase the productivity of low skilled workers and vice versa. Suppose also that the influence of one type of worker on the productivity of that type is stronger than the influence of the other type of worker. Then the following inequalities are true $\beta_1 > \delta_1$, $\beta_1 > \beta_2$, $\delta_2 > \beta_2$ and $\delta_2 > \delta_1$.

2.1.2 Shadow price equations

The shadow prices for skilled and unskilled workers are represented by the following equations:

$$q_s(t) = \int_t^{mt} (w_{SX} - 1)e^{-r(x-t)} dx \quad (3)$$

$$q_u(t) = \int_t^{mt} (w_{UX} - 1)e^{-r(x-t)} dx \quad (4)$$

Notice that the integrals range from t to mt because it has not yet been proven yet that the workers have all agglomerated in one region after a finite time, T . Therefore, mt belongs to the set $[t, \infty[\cup \infty$. This is exactly the same concept introduced in the previous section, we simply introduce two shadow prices, one per type of worker, instead of one price.

The world interest rate is equal to r and the rates of return for the two shadow prices must be equal to the following rates:

$$\dot{q}_s = r q_s - w_{SX} + 1 \quad (5)$$

$$\dot{q}_u = r q_u - w_{UX} + a \quad (6)$$

These two equations are obtained by deriving (3) and (4).

2.1.3 Congestion cost equations

The congestion cost equations are obtained by optimizing a production function (similar to Krugman's). If skilled and unskilled workers are both moving toward the same sector, the costs

of migration are as follows:

$$\dot{L}_{sx} + \dot{L}_{ux} = \gamma_1 q_s \quad (7)$$

$$\dot{L}_{ux} + \dot{L}_{sx} = \gamma_2 q_u \quad (8)$$

These equations state that the marginal cost of moving is equal to the marginal benefit. The benefit is the shadow price associated with the corresponding level of skill. The costs are congestion costs. We assumed that skilled workers are more mobile than unskilled workers ($\gamma_1 > \gamma_2$). The costs of migration are created by the migrants themselves.

If the skilled and unskilled workers are moving in opposite directions (for instance, a scenario in which the skilled workers are moving into sector C and the unskilled workers are moving into sector X , is quite different), then the effect depends on the type of externality under consideration. Three different cases arise when the parameter μ is introduced into the following equation: $\dot{L}_{sx} = \gamma_1 q_s - \mu \dot{L}_{ux}$

- When $\mu = 1$ the derivatives have opposite signs, and the movement of one type of workers reduces the congestion in the movement of the other type. For instance, consider the extension of the model in economic geography, which incorporates the choice between two cities with different utility function. If the number of houses in a city is fixed over the short term, an increase in the number of migrants produces congestion in the housing market. However, if one person leaves a city, a house or an apartment becomes available.
- When $\mu = 0$ The movement of one type of worker has absolutely no impact on the movement of the other type. Think of traffic congestion: the traffic on a highway from one place to another has no impact on the traffic in the other way.
- When $\mu = -1$ The externalities are cumulated: even if they are moving in opposite direction, the movement of one type of workers is congesting the movement of the other types. For instance, think of two computers in a network. The downloading from one computer to another could congest the downloading in the opposite direction. In his paper, Baldwin said that the arrival of new people were easier if they were not the first ones. So maybe we could adapt this idea and say that the presence of one type of worker is making easier the arrival of the other type and their departure is making it harder.

Baldwin [2] is the only author who has explained what type of externality could be produced. But none of his indications indicates us what could be the value of μ . The three cases could be studied. Nevertheless, we will not examine the three different cases, only the case when $\mu = 1$.

When $\mu = 0$ and $\mu = -1$, these cases are more complex, they will not be addressed in this paper.

Additional hypothesis We will demonstrate in the next section that only one type of worker is moving at the same time. For instance it is possible that only the skilled workers are moving at a date $t - dt$ and at a date $t + dt$, only the unskilled workers relocate. There is a shift from $\dot{L}_{sx} = \gamma_1 q_s, \dot{L}_{ux} = 0$ to $\dot{L}_{ux} = \gamma_2 q_u, \dot{L}_{sx} = 0$ at date t . As a result, a discontinuity exists when $q_s = \frac{\gamma_2}{\gamma_1} q_u$.

Nevertheless, the discontinuity corresponds to an extreme, unrealistic situation. These discontinuities appear when it is assumed that the process that leads to a full congestion by one type of worker is instantaneous (this full congestion will be demonstrated in the next section). It is more realistic to assume that this process lasts.

The equations are temporarily changed during a very small interval: if $\gamma_2 q_u - \epsilon_{c1} \leq \gamma_1 q_s \leq \gamma_2 q_u + \epsilon$, with ϵ, ϵ_{c1} very small

$$\dot{L}_{sx} = (1 - \lambda) \max(\gamma_1 q_s, \gamma_2 q_u) \quad (9)$$

$$\dot{L}_{ux} = (\lambda) \max(\gamma_1 q_s, \gamma_2 q_u) \quad (10)$$

with λ a continuous increasing function.

The intuitive idea behind these two equations is given: at the instant t , q_s and $\frac{\gamma_2}{\gamma_1} q_u$ have the same absolute value. At $t + dt$, q_u is greater than $\frac{\gamma_1}{\gamma_2} q_s$. The unskilled workers can begin to move but the skilled workers are still moving. In order to have the equation $\dot{L}_{sx} + \dot{L}_{ux} = \gamma_1 q_s$ verified, \dot{L}_{sx} has to decrease. But \dot{L}_{sx} will only decrease at $t + 2dt$. Because the skilled workers need a little amount of time to realize that more unskilled workers are moving. The process will continue until only the unskilled workers moves.

The system of differential equations is now continuous and K-lipschitz, and the Cauchy-Lipschitz theorem could be applied (proof given in Appendix A). That is, for any set of initial conditions, there exists a solution. As a result, if the terminal conditions of the movement are set to $T = 0$ (the movement occurs when time is negative), then a solution exists.

However, this solution is not unique because of the parameter ϵ : the solution is unique for a given value of ϵ . Indeed, ϵ is supposed to be taken small enough, but it could take different values, therefore changing the equations and the solutions. This parameter could vary continuously

inside a given interval. This change does not alter the total movement.

2.1.4 Extension to more general utility functions

Our model can be easily adapted to more general problems. Wages are replaced by utility functions and instead of a choice between sectors, workers are faced with a choice between cities, states or even countries.

U_{sa} is the utility function of the skilled workers and it depends on the fraction of the skilled L_{sa} and the unskilled L_{ua} present in city A . f and g are both real functions.

$$U_{sa} = f(L_{sa}) + g(L_{ua})$$

The demonstration made by Ottaviano, Tabuchi and Thisse could easily be adapted with the heterogeneity of workers. It is nevertheless needed to assume that f and g are both linear functions.

Application to Indian and Chinese entrepreneurs Our model could be applied to the problem that has been seen in the introduction. These entrepreneurs are better off in China and India partly because workers in their home countries are cheaper. This could be captured by the function g .

3 Simplification: only one type of worker is moving

In this section, we show that the problem could be simplified to a system of two differential equations. The shapes of the solutions are of the same form than in Krugman's model. The laws of motion are not the same throughout the entire process; migration depends on the relative signs of q_u and q_s . Two different cases of q_s and q_u will be considered: first they have the same sign; second they have opposite signs.

q_s and q_u have the same sign When q_s and q_u have the same sign and $q_s \neq \frac{\gamma_2}{\gamma_1} q_u$, then one type of worker will face stronger incentives to move than the other type of worker. If the two values are equal, then both types of workers will face the same incentive to move.

Lemma If $q_s \neq \frac{\gamma_2}{\gamma_1} q_u$, then only workers facing the highest shadow price will move.

Proof Suppose that $q_s > \frac{\gamma_2}{\gamma_1} q_u > 0$ and $\dot{L}_{ux} \neq 0$. Then, the skilled workers will move according to $\dot{L}_{sx} = \gamma_1 q_s - \dot{L}_{ux}$. However, the unskilled workers' costs are suddenly too high due

to the additional movement of skilled workers $\dot{L}_{sx} + \dot{L}_{ux} = \gamma_1 q_s > \gamma_2 q_u$. The cost of movement for an unskilled worker will be higher than the benefits; therefore, L_{ux} must decrease. The same demonstration could be repeated until $L_{ux} = 0$.

Intuitively, skilled workers face greater incentives to move. When the unskilled workers have no additional incentive to move, some skilled workers continue to migrate. This movement increases the marginal cost of moving beyond the marginal benefits for unskilled workers, and reduced the number of unskilled workers who are moving, thus decreasing the cost of moving for skilled workers. Consequently, skilled workers will continue to move and increase the cost of moving for unskilled workers who may be forced to abandon plans to migrate. This pattern will continue until no unskilled workers remain. The same demonstration can be easily adapted for cases where $\frac{\gamma_2}{\gamma_1} q_u > q_s$ or where q_s and q_u are both negative.

As a result, the aggregate movement can be divided into smaller migration flows in which only one type of worker moves (i.e., a submovement occurs when a particular type of worker moves; the movement of the other type marks the beginning of the next submovement).

Corollary: The equilibrium path for the type of worker who is moving exhibits the same shape as Krugman's model during a sub-movement. The workers' movements are described by four equations that form a linear differential system. Nevertheless during a submovement of the skilled workers, q_u has absolutely no influence on q_s and L_{sx} . q_u remains only relevant to determine the end of the submovement. Moreover L_{ux} is a constant and $\dot{L}_{ux} = 0$. Therefore only two equations are necessary to describe the path of the skilled workers' submovement. These two equations are almost the same than in Krugman's model. The matrix is the same; however, the constant is different. The constant does not change the eigenvalues, and we observe the same shape of the solutions (a spiral or an S curve). For instance, if the skilled are moving and $r^2 - 4\beta_1\gamma_1 > 0$ then the skilled workers path forms an S curve. On the contrary, if $r^2 - 4\beta_1\gamma_1 < 0$, a spiral emerges. However, as L_{sx} is moving, then q_u is also changing because the skilled workers exert an influence on unskilled wages.

q_s and $\frac{\gamma_2}{\gamma_1} q_u$ have opposite signs Remember that only the case $\mu = 1$ will be studied in this paper. The costs of moving are no longer a problem because the skilled workers and the unskilled are moving in opposite directions. Every time an unskilled worker shifts from one sector to another, it enables a skilled worker to shift without costs in the opposite direction. As a result, a problem arises. Suppose that at t_0 , $|q_s| > \frac{\gamma_2}{\gamma_1} |q_u| > 0$. At a certain date $t_1 > t_0$, assume that $q_u = 0$ with the derivative of q_u telling us that it will change its sign and q_s stays strictly positive. Therefore, the shadow prices of the two types will have a different sign and

the costs will be negligible. Fukao and Benabou [10] showed that, on an equilibrium path, if the shadow price is strictly positive and if the costs of moving become suddenly equals to zero, this cannot be an equilibrium path. Therefore, if $q_u = 0$ at t_1 then q_s must be equal to 0 at the same date t_1 .

The main conclusion of this section is that the differential system could be simplified locally, for a limited time, in a system of three differential equations when q_s and q_u have the same sign (four equations were originally in the model but as $\dot{L}_{ux} = 0$, only three remains relevant). Moreover, the two q_s and q_u starts with the same sign, they keep the same sign on the same equilibrium path at least for as long as one of the two types of workers will stop moving. Because if they move in the opposite direction, costs will become null and a worker could make a gain by deviating from the equilibrium path. As a result, the total movement could be divided in submovements where only one type of worker is moving. In a submovement, the shape of the solution is the same as in Krugman [17] for the type of worker who is moving.

4 The role of expectations

Expectations are taken into account where $q: \int_t^{mt} (w_{SX} - 1)e^{-\rho t} dt$ is the sum of the discounted difference in wages between the two sectors from time t until the end of the movement. A change in expectations is possible when, for a given spatial distribution of the economy, we observe different values of q that are compatible with the equations. If L_{sx} and L_{ux} are given, the actual difference in wages is also determined; therefore, different values of q represent a differences in future wages.

As a result, there exists a possibility of a change in anticipation of the skilled workers if and only if $\exists L_{sx}, L_{ux}$ and $q_s^1(L_{sx}, L_{ux}), q_s^2(L_{sx}, L_{ux})$ such that $q_s^1(L_{sx}, L_{ux}) \neq q_s^2(L_{sx}, L_{ux})$ and q_s^1 and q_s^2 are solutions to the differential equations of motion. However, this definition of change in expectations is incomplete. The definition does not tell us if the change will lead to a change in the final equilibrium. Krugman [17] solves the differential equations and then demonstrates that, when there are different deterministic paths, a change in expectations can produce a jump from one path to another. Each path leads to a different equilibrium. There are two different paths if and only if $r^2 - 4\beta\gamma < 0$. The variable q is a jumping variable, which can shift from an equilibrium path to another, depending on the expectations of the workers.

In our model, different results are found. Two different paths, leading toward two different equilibria, with each different shadow prices value for the same spatial configuration (L_{sx}, L_{ux}) , are found in the case $r^2 - 4\beta_1\gamma_1 > 0$ and $r^2 - 4\beta_2\gamma_2 > 0$. Therefore, a change in expectations

leading toward a different final equilibrium is possible in this case.

4.1 Solutions when $r^2 - 4\beta_1\gamma_1 > 0$

Before introducing the main result of this paper, we need to study the forms of the solution. In this paragraph, the solutions of the equations during a skilled workers' submovement when $r^2 - 4\beta_1\gamma_1 > 0$ (the condition that Krugman established to prevent expectations from playing a role in people's choices) will be examined. For these values, the solutions have the shape of an S curve during a submovement. We established in the previous section that two equations are necessary to describe the shape of the path taken by the skilled workers.

The system is easily solvable. The solution is described by the following equations:

$$q_s(t) = Aexp(\lambda_1 t) + Bexp(\lambda_2 t) \quad (11)$$

$$L_{sx}(t) = Cexp(\lambda_1 t) + Dexp(\lambda_2 t) \quad (12)$$

$$q_u(t) = Eexp(\lambda_1 t) + Fexp(\lambda_2 t) + Gexp(rt) \quad (13)$$

The equations (11) and (13) are used to describe the skilled workers' submovement. Equation (12) is used to follow the evolution of the shadow price of the unskilled workers during that submovement. The eigenvalues of the system are λ_1, λ_2, r . Notice that the constants were not present in the solutions. It could be demonstrated that the set the spatial configurations L_{sx}^*, L_{ux}^* that equalize the wages is a vector space of dimension one. By choosing the appropriate values for L_{sx}^*, L_{ux}^* and the appropriate origins for L_{sx} and L_{ux} , the constants could be taken equal to zero. Three conditions (final or initial) are required to produce a unique solution to the system. The initial conditions selected are the following: $L_{sx}(0) = K, q_s(0) = K_2, q_u(0) = K_2 - \epsilon$ where $K, K_2, K_2 - \epsilon$ are all positive constants.

The linear system given in equations (11)-(13) was solved utilizing Xcas software (freeware, developed by B Parisse, J Fourier University, Grenoble, France). The following results were obtained:

$$A = \frac{(-K)\beta_1 - K_2\lambda_2 + K_2r}{\lambda_1 - \lambda_2} \quad (14)$$

$$B = \frac{K\beta_1 + K_2\lambda_1 - K_2r}{\lambda_1 - \lambda_2} \quad (15)$$

$$C = \frac{(K\beta_1\lambda_1 - K\beta_1r + K_2\lambda_1\lambda_2 - K_2\lambda_1r - K_2\lambda_2r + K_2r^2)}{(\beta_1\lambda_1 - \beta_1\lambda_2)} \quad (16)$$

$$D = \frac{(-K\beta_1\lambda_2 + K\beta_1r - K_2\lambda_1\lambda_2 + K_2\lambda_1r + K_2\lambda_2r - K_2r^2)}{(\beta_1\lambda_1 - \beta_1\lambda_2)} \quad (17)$$

$$(18)$$

The results for the unskilled workers' shadow price are presented below:

$$q_u(t) = \frac{\beta_2}{\beta_1}q_s(t) + Ge^{rt} \quad (19)$$

$$G = \frac{(K_2\beta_1 - K_2\beta_2 - \beta_1\epsilon)}{\beta_1} \quad (20)$$

The shape of the solutions of the two equations of the skilled submovement are represented on graph 1 in the plane L, q .

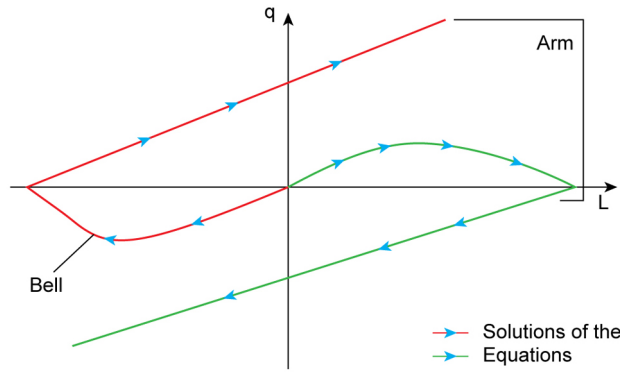


Figure 1: Form of the solution

The curves are separated into two areas in the rest of the paper.

- The arms begin after the paths have crossed the abscissa (by following the arrows in graph 1), and they diverge towards infinity.
- The bells: are located before the paths have crossed the abscissa.

It is important to note that this graph represents the global shape of the vector field of the solution. During a submovement, with the initial conditions set, only a part of these paths described on the figure are taken. Moreover, there is a difference between a solution to a set of

equations and an equilibrium path observed in our economic problems. It is necessary, but not sufficient, to be a solution to the equations because the terminal conditions need to be reached (Fukao and Benabou [10]). Graph 1 represents the mathematical solutions to the system of equations, but not all of these solutions necessarily represent an equilibrium path.

It is necessary to know whether the movement occurs on an arm or on the bell of the curve. In Krugman [17], the movement always occurs on the bell. He achieved this result by imposing a final condition at $\dot{q} = 0$, which is only possible on the bell. Fukao and Benabou [10] also impose a final condition at $q = 0$, which is, again, only possible outside the arms. However, here, the final condition for a submovement is the equality between q_s and $\frac{\gamma_2}{\gamma_1} q_u$; the final condition of the total movement remains $q_s = q_u = 0$. As a result, an equilibrium path could temporarily shift to an arm during a submovement under the condition that it shifts to the bell to reach the terminal condition.

If the movement is located on an arm, then the time when $q_s(t_0) = 0$ should be negative, and if not, the time should be positive because the movement produces divergence. By following the arrows on the graph, the path is located on the bell part of the curve before time $q_s = 0$ and after a movement on an arm.

Imposing $q_s(t_0) = 0$,

$$(\lambda_1 - \lambda_2)t_0 = \ln\left(-\frac{B}{A}\right)$$

Therefore, t_0 is negative if and only if $-\frac{B}{A} < 1$.

It could be easily demonstrated that the submovement is not on an arm if and only if $-K\beta_1 + K_2\lambda_1 < 0$ and $K\beta_1 - K_2\lambda_2 > 0$. It is sufficient to impose $K\beta_1 > K_2\lambda_1$ or $\beta_1 > \frac{K_2}{K}\lambda_1$. Consequently, the movement of the skilled workers is located on a bell if and only if the ratio $\frac{q_s}{L_{sx}}$ is low enough.

To sum up, we have established that the movement is located on a bell if and only if the ratio $\frac{q}{L}$ is sufficiently low. We also established that there exist a difference between a solution of the mathematical equation and an equilibrium path. But in our model, the total movement could be subdivided in small submovements where only one type of worker is moving. We will show in the next section that it is possible for a type of worker to take an arm during a submovement and to finish on a bell and reach the necessary conditions to be an equilibrium path.

4.2 Expectations when both shadow prices have the same sign

In this section, only the equations in which both the shadow prices have the same sign will be examined.

A change in expectations is possible even if $r^2 - 4\beta_1\gamma_1 > 0$ and $r^2 - 4\beta_2\gamma_2 > 0$ Suppose that L_{sx} is such that there are more workers in sector C and $r^2 - 4\beta\gamma > 0$. Under these conditions, Krugman [17] predicts that in the final equilibrium, all skilled workers are employed in sector C . However, with the heterogeneity of workers, an equilibrium path with these initial conditions can lead toward an equilibrium in which everyone is employed in sector X . A positive shadow price with a low value of L was not possible in Krugman's model because the terminal condition needs to be $q = 0$. With heterogeneity, the terminal condition of a submovement becomes $q_s = \frac{\gamma_2}{\gamma_1}q_u$. This different constraint enables a new equilibrium path to emerge. The final conditions of the movement are either $q_s = q_u = 0$ or $\dot{q}_s = \dot{q}_u = 0$. The rigorous demonstration is in the Appendix B. The intuition is given below.

Suppose that the state of the economy for the skilled workers is on an arm and the bell for the unskilled workers. Suppose also that the unskilled workers are moving. At this point, a shift could occur. It is possible that during the submovement of unskilled workers, the ratio $\frac{q_s}{L_{sx}}$ become low enough for skilled workers to be located on the bell given a sufficiently high coefficient δ_1 . On graph 2, the ratio is illustrated by the red line between the numbers 1 and 2. When unskilled workers are moving, the economy is on an arm for skilled workers at the beginning of the movement. The economy is on the bell for the skilled workers at the end of the submovement.

Therefore, when the skilled workers are moving again, the final conditions of the movement can be attained. Both of the shadow prices remain on the bell for the rest of the movement. Consequently, a new possible equilibrium path emerges.

There is an economic interpretation of this finding. The presence of unskilled workers in sector C is necessary and its influence on the productivity of skilled workers needs to be sufficiently high. The strength of the economies of scale from the other type of worker prevents the divergence of shadow prices. Therefore, a change in expectations is possible if another type of workers is already present in the city and creates sufficient agglomeration forces.

The paths are represented in the figure 3

At the point L_0 , two different equilibrium paths exist, in contrast to one in Krugman's model, leading toward two different equilibria.

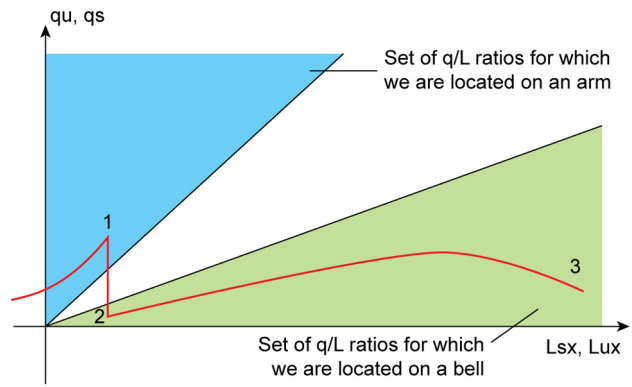


Figure 2: New equilibrium path

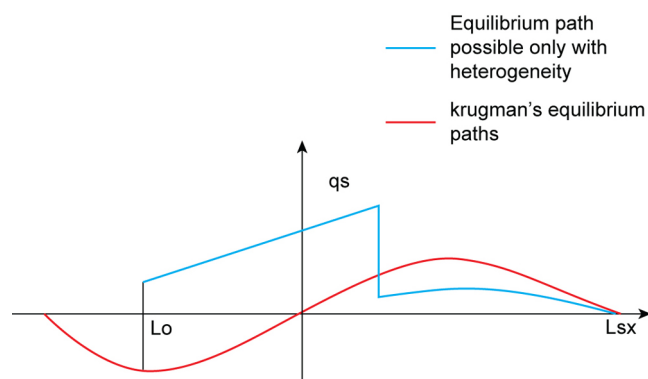


Figure 3: Possible Paths

Even with high interest rates, expectations could play a role. If workers are initially moving in opposite directions, the sudden shift by a worker of one type could create economies of scale. As a result, political interventions could fight the divergence in incomes observed in American cities by changing the expectations of workers.

L_{sx} and L_{ux} agglomerating toward the same sector If both types of workers shift into the same sector, then it is impossible for both types to move into the other sector because the two types of workers will therefore be located along the arms and none could reach the bell part of the curve. Examine the movement of one type of worker. During a submovement in which this type of worker moves, it is impossible to reach the bell part of the curve by definition. During a submovement where the other type is moving, the labor $L_{ix}, i = s, u$, remains constant and q_i is necessarily increasing. A quick look at the equations reveals that it is the linear combination of q_j and e^{rt} , which are both increasing. The ratio $\frac{q_i}{L_{ix}}$ is increasing; therefore, the movement of workers follows an arm.

5 Equilibria

In this section the final equilibria are given.

5.1 Stable equilibria

If dispersion is possible, there are at least four final equilibria that could be stable: everyone is employed in sector C , everyone is employed in sector X , the skilled are employed in sector C and the unskilled are employed in sector X , or the skilled are employed in sector X and the unskilled are employed in sector C .

Equilibrium of dispersion A dispersion (i.e., when all the skilled workers will agglomerate in one sector and all the unskilled workers are employed in the other sector) will be possible if and only if the wages for both type of workers are higher in the sector where they are employed than in the other sector.

5.1.1 Final conditions for the movement and stability of the equilibria

$r^2 - 4\beta_1\gamma_1 > 0$ and $r^2 - 4\beta_2\gamma_2 > 0$ The solutions have the shape of an S curve with arms at the end, as previously described. This is the condition that Krugman [17] established to prevent expectations to from playing a role in people's choices. The Poincare Bendixson theorem could not be applied here because the dimension of the differential system is four and not two. It is

demonstrated in the Appendix A that equilibrium is reached within a finite period of time.

If one shadow price changes its sign during a transition, the movement could continue and the equilibrium is not necessarily stable. Nevertheless, the equilibrium is stable if $\mu = 1$. As described earlier, in this case the two shadow prices become null at the same time. Suppose the movement is not finished when $q_s = q_u = 0$. Both shadow prices are located on an arm and they cannot reach equilibrium in that configuration. As a result, the movement ends when both shadow prices become null and they stay there forever.

6 Discussion

This paper aimed to establish whether spatial inequalities could be entirely explained by history or whether worker expectations could play a role. Introducing this heterogeneity in Krugman [17] increases the number of different equilibrium states. The model established that expectations can play a role, even with high interest rates, if increasing returns are strong enough. The explanation of this result lies in the difference in the terminal condition. In Krugman [17], that condition is $q = 0$, but with heterogeneity, the total movement can be subdivided into a number of small sub-movements where only one type of worker moves. Each of these sub-movements has a terminal condition different from $q = 0$ that enables a new equilibrium path. As a result, heterogeneity is a factor of instability and multiplicity.

The model could be extended to a new economic geography. It could for instance explain the return of Indian and Chinese entrepreneurs in their home countries. The actual wages are greater in the US but India and China seem to be more dynamic with more economic opportunities. Furthermore, the presence of a cheaper workforce clearly played a role in their decision to return. Our model could also be used to help future policy. The brain drain of the skilled workers from Europe to the United States and the actual divergence in income between states in the US are not a fatality. The highly skilled workers could be attracted to Europe or to US cities with too little human capital. But the influence of the low skilled on the high skilled workers productivity needs to be sufficiently high. In a recent paper made by Eeckhout et al. [9], they found an extreme-skill complementarity in big cities: highly skilled workers and low skilled workers mutually enhanced their productivity. This finding supports our main assumption.

Another assumption has been made that was not present in the original model. Discontinuous differential linear systems do not necessarily have a solution to their equations; therefore, the equations have been changed for a limited time through the parameter ϵ . This parameter is critical to the main proposition of this paper: changing the value of this parameter changes the

solution of the equations and enables many equilibrium paths to emerge.

Heterogeneity has been introduced by two discrete levels of skills. Other forms of heterogeneity (for instance, a continuum of skills) have been introduced in the papers previously reviewed. This difference in modeling heterogeneity may explain the differences observed in the papers.

From a political point of view, attracting skilled workers to a city with insufficient human capital has some advantages. Glaeser [11] argues in his book that poor areas in cities should be judged by their ability to make poor people poorer. Indeed, he argues that many poor people succeed in climbing the social ladder after arriving in a big city. However, this mobility is only observed if there are already rich people in the city. In a city like Detroit, people tend to remain poor, because there are too few skilled workers. The Michigan governor is trying to attract skilled immigrants in this city to stop its decline (Lichterman [20]). In another article, Glaeser and Mare [13] demonstrate that cities speed the accumulation of human capital. Glaeser and Saiz [14] argue that only high skill cities endure because they can adapt to economic shocks. These cities have grown more quickly than cities with less human capital because they are becoming more productive. Consequently, cities with insufficient human capital fall into a poverty trap, which makes regional specialization less desirable. However, cities that are too large produce negative externalities such as congestion and environmental problems (Brueckner [5]). In Europe, Davezies [6] wrote that with the decreases in public spending that will occur over the next few years, inequality will be harder to fight and redistribution programs may not provide a solution. He does not believe in making the poorer area more attractive; rather, governments should help dynamic cities in poorer areas.

A Appendix - The discontinuity problem

In the case where $q_s = \frac{\gamma_2}{\gamma_1} q_u$, the problem of discontinuity arises. Indeed, one type of worker suddenly stops moving and another begins to move with a derivative instantly equals to $\gamma_j q_k$. The continuity of the system is needed to meet the qualifications for the Cauchy-Lipschitz theorem. Without a loss of generality, consider the case where the skilled workers are moving, and then suddenly the unskilled workers are moving (the opposite case requires the same discussion). An additional assumption is made: when $\gamma_2 |q_u| - \epsilon_{c1} \leq \gamma_1 |q_s| \leq \gamma_2 |q_u| + \epsilon$ with ϵ, ϵ_{c1} very small, the laws of motion are slightly different.

The demonstration is made for q_s and q_u both positive but it could be easily adapted. Remember that $\beta_1 > \beta_2$ and $\delta_1 < \delta_2$.

We take a function λ which verify

$$\lambda(q_s, q_u, L_{sx}, L_{ux}) = \begin{cases} 0, & \text{as soon as both } \gamma_1 q_s < \gamma_2 q_u + \epsilon_{c1}, r(q_s - \frac{\gamma_2}{\gamma_1} q_u) < g(L_{sx}, L_{ux}) \text{ are verified} \\ \lambda_{TP}, & \text{if } \gamma_2 q_u = \gamma_1 q_s + \epsilon_1 \\ 1, & \text{if } \gamma_2 q_u = \gamma_1 q_s + \epsilon \end{cases}$$

$g(L_{sx}, L_{ux}) = L_{sx}(\beta_1 - \frac{\gamma_2}{\gamma_1} \beta_2) + L_{ux}(\delta_1 - \frac{\gamma_2}{\gamma_1} \delta_2)$. λ could take many forms but it is necessarily increasing: the transition begins at $\lambda = 0$ and it finishes at $\lambda = 1$. λ is taken continuous, differentiable, strictly increasing with bounded derivatives and $\lambda(0) = \lambda(1) = 0$. $\epsilon_1, \epsilon_{c1}$ is taken smaller than ϵ . $\epsilon, \epsilon_2, \epsilon_{c1}$ are all strictly positive.

The laws of the movement are now defined on the interval $\gamma_2 |q_u| - \epsilon_{c1} \leq \gamma_1 |q_s| \leq \gamma_2 |q_u| + \epsilon$ by

$$\dot{L}_{sx} = (1 - \lambda) \max(\gamma_1 q_s, \gamma_2 q_u) \quad (21)$$

$$\dot{L}_{ux} = (\lambda) \max(\gamma_1 q_s, \gamma_2 q_u) \quad (22)$$

We define λ_{TP} such as $f(\lambda_{TP}) = 0$ with $f(\lambda) = (1 - \lambda)(\beta_1 - \frac{\gamma_2}{\gamma_1} \beta_2) + (\lambda)(\delta_1 - \frac{\gamma_2}{\gamma_1} \delta_2)$. f is continuous, decreasing and $f(0) > 0$. If $f(1) > 0$ the demonstration is obvious. Suppose $f(1) < 0$. Therefore $0 < \lambda_{TP} < 1$.

We need to verify that our system is now continuous. It is not necessarily the case: now that the law of motions have been modified, it is necessary to verify that the unskilled workers are moving at the end of the transition. Notice that we have $\frac{\gamma_2}{\gamma_1} q_u(t_o) > q_s(t_o)$, t_o being the time at the beginning of the transition (i.e. when $\lambda = 0$). It is a direct consequence of the inequality $r(q_s - \frac{\gamma_2}{\gamma_1} q_u) < L_{sx}(\beta_1 - \frac{\gamma_2}{\gamma_1} \beta_2) + L_{ux}(\delta_1 - \frac{\gamma_2}{\gamma_1} \delta_2)$.

It is possible to show that during the transition defined above

$$\ddot{q}_s - \frac{\gamma_2}{\gamma_1} \ddot{q}_u = r(\dot{q}_s - \frac{\gamma_2}{\gamma_1} \dot{q}_u) - ((1 - \lambda)(\beta_1 - \frac{\gamma_2}{\gamma_1} \beta_2) - (\lambda)(\delta_1 - \frac{\gamma_2}{\gamma_1} \delta_2)) \max((\gamma_1 q_s, \gamma_2 q_u))$$

Therefore, at the beginning of the transition, $\lambda = 0$ and $\ddot{q}_s < \frac{\gamma_2}{\gamma_1} \ddot{q}_u$. λ increases but as long as $\lambda < \lambda_{TP}$, the inequality $\ddot{q}_s < \frac{\gamma_2}{\gamma_1} \ddot{q}_u$ is verified. Therefore $\dot{q}_u \frac{\gamma_2}{\gamma_1}$ stays higher than \dot{q}_s as long as $\lambda < \lambda_{TP}$. Therefore there exists a time t where $\frac{\gamma_2}{\gamma_1} q_u(t) = q_s(t)$. When $\lambda = \lambda_{TP}$, we have $\gamma_2 q_u = \gamma_1 q_s + \epsilon_1$, $\dot{q}_s < \frac{\gamma_2}{\gamma_1} \dot{q}_u$ and $\ddot{q}_s < \frac{\gamma_2}{\gamma_1} \ddot{q}_u$. Therefore, the shadow price of the unskilled is going to be greater than the shadow price of the skilled. The transition ends when $\gamma_2 q_u = \gamma_1 q_s + \epsilon$. This

states is always reached if ϵ_1 is taken close enough ϵ , and we have still $\frac{\gamma_2}{\gamma_1}q_u > q_s$. Therefore the unskilled workers are moving at the end of the transition, and the movement is now continuous.

It could be easily demonstrated that with bounded derivatives, the functions $\dot{q}_s, \dot{q}_u, \dot{L}_{sx}, \dot{L}_{ux}$ are locally K lipschitz. As a result the Cauchy-Lipschitz theorem can be applied.

B Appendix - A Change of expectations when $r^2 - 4\beta_1\gamma_1 > 0$ and $r^2 - 4\beta_2\gamma_2 > 0$

Initial conditions Suppose that at the end of a submovement, L_{sx}, L_{ux} are both positive. Suppose that we are on the bell part of the curve for the unskilled workers but are on an arm for the skilled workers. If the skilled workers are moving, is there a solution for which $q_s(t_1) = q_u(t_1)$? Suppose that $t = 0$ when the skilled workers starts to move; the same notation as subsection (4.1) is used and $K < 0, K_2 > 0$. As $t \rightarrow \infty$, q_u is greater than q_s . Because in $+\infty$, q_u is equivalent to $Gexp(rt)$, which is greater than the equivalent of q_s because $G > 0$. The equivalent of q_s equals $Aexp(\lambda_1 t)$ as $r > \lambda_1$. Therefore, t_1 exists and the unskilled workers beginning to move at $t > t_1$.

As previously demonstrated, to be located on the bell part of the curve, it is necessary and sufficient for $L_{ux}\delta_2 > q_u(0)\lambda_3$. To be located on an arm, we require $L_{sx}\beta_1 < q_s(0)\lambda_2$. We take L_{sx} very close to zero and $q_s(0)$ sufficiently high; we take $q_u(0)$ and $q_s(0)$ as close but with q_u greater than q_s .

Summary of the demonstration The demonstration proceeds in several steps. First, it is necessary to illustrate that the skilled workers could shift from an arm to a bell during a submovement of the unskilled workers. Then, this could lead to a final equilibrium where everyone is employed in sector X by respecting the equations and to reach the final global conditions $q_s = q_u = 0$ at the end of the total movement. Therefore, we describe an equilibrium path. Finally, an initial negative L_{sx} could lead to this path and allow a change in expectations.

Skilled workers shifts toward the bell due to the unskilled workers' economies of scale
The movement begins at 0 and ends at t_1 . Δ_k is a fixed exogenous constant. $q_s(0) = K_s, \Delta_k > 0, q_u(0) = K_s + \epsilon$. At the end of the submovement, it is necessary to have:

$$q_s(t_1) = K_s - \Delta_k \quad (23)$$

$$\frac{\gamma_2}{\gamma_1}q_u(t_1) = K_s - \Delta_k + \epsilon_{c1} \quad (24)$$

$$\dot{q}_s(t_1) > \dot{q}_u(t_1) \quad (25)$$

The equations from section (4.1) are adapted to the case where the unskilled workers are moving. For instance, by adapting the equation (19), we can write:

$$\frac{\delta_1 \gamma_1}{\gamma_2 \delta_2} (K_s - \Delta k - \epsilon_{c1}) + \left(1 - \frac{\delta_1}{\delta_2}\right) (K_s + \epsilon) - \epsilon e^{rt_1} = (K_s - \Delta k)$$

From this equation, we deduced that t_1 is positive if $\frac{1}{\delta_2 - \delta_1} h1(K_s, \frac{\gamma_2}{\gamma_1}, \delta_1, \delta_2, \epsilon_{c1}, \epsilon) > \Delta k$ with $h1$ a linear function.

From equation (24), we deduced that δ_2 is positive if and only if $\Delta k > h2(K_s)$ with h a linear function of K_s .

The equation (25) is verified if L_{ux} is high enough.

We assume from now on that these three conditions are verified. To sum up, the skilled workers shift from an arm toward a bell if δ_1 is sufficiently close to δ_2 .

Therefore q_s is allowed to go as low as $K_s - \Delta k$ before the end of the submovement. If Δk is high enough, then q_s is located on the bell part of the curve at the end of the submovement.

Both shadows prices stay on the bell for the rest of the movement Now, the two shadow prices are on the bell. It is necessary to find a condition under which both types of workers stay on the bell. Indeed, if skilled workers are located on the bell of the curve, they will remain on that part during a submovement of unskilled workers and vice versa. The workers could leave that path during a submovement of the other type only.

Firstly, we consider a finite number of submovements. We consider the case of the skilled workers but the case of the unskilled workers is similar.

q_s reach its maximum for $\dot{q}_s = r q_s^{max} - \beta_1 L_{sx} - \delta_1 L_{ux} = 0$. As a result, $q_s^{max} = \beta_1 L_{sx} + \delta_1 L_{ux}$. Therefore for a sufficiently high product $\delta_1 L_{ux}$, q_s would never become larger than a certain value and the ratio $\frac{q_s}{L_{sx}}$ would stay sufficiently low.

For a sufficiently large L , the derivative of q is negative. Consequently, the q 's should remain on the bell if sufficiently large L 's are reached in a finite number of submovement. This is the case: due to bounded derivative, a submovement has a minimal length (for a given value of the parameter ϵ). Therefore a minimal proportion of skilled workers are coming into sector X in a submovement. Therefore, for a sufficiently large, but finite, number of submovement, the necessary values of L_{sx} are reached.

Final conditions of the movement It is necessary to prove that the final conditions established by Fukao and Benabou [10] are met. Two cases arise: the boundaries are hit in a finite period of time or $q_s = q_u = 0$ before hitting a boundary. But as shown in section (5.1.1), none of the two types of workers have incentive to move when $q_u = q_s = 0$. Therefore, it could be an equilibrium even if the boundary is not hit. In the two cases, equilibrium is reached in a finite time. Because as previously demonstrated, there exist a minimal length for a submovement, with the consequence of a minimal increase in absolute value of L_{sx} or L_{ux} .

Suppose that the boundaries are not reached, we now demonstrate that q_s and q_u are converging toward 0. q_u and q_s are strictly decreasing, with a minimum absolute value. There exists a minimal length for a submovement for a given ϵ . As the time derivatives of the q 's are bounded, the submovement lasts at least for a time dt . Therefore the q 's should decrease for a minimal value noted q_{dc} . Suppose that they are always above a value q_{mn} that is strictly positive. Because it is decreasing and bounded, it is converging. Then, it is possible to take a q at the beginning of a submovement such as $q - q_{mn} = \frac{\delta_\epsilon}{2}$ with δ_ϵ as small as we want. As a result, if δ_ϵ is taken small enough, below q_{dc} , the demonstration is over.

Both shadows price arrives at 0.

Back to the initial conditions Remember that we imposed a very small but positive L_{sx} earlier. The skilled workers are on the arms, and the unskilled workers are on the bell part of the curve. The second step of the demonstration implies that an initially negative L_{sx} could lead to this equilibrium path. The unskilled workers were moving at time $t = 0$ under these conditions; therefore, there was a transition further back in time. The length of the transition is Δ_t . Therefore, we simply have to define an L_{sx} small enough in the final conditions of that submovement. Because there is a minimal non- null value for \dot{L}_{sx} (we are located on an arm), we noted $L_{sx}^{\dot{min}}$. Therefore, during the transition, L_{sx} is increasing for a minimal value of $\Delta_t * L_{sx}^{\dot{min}}$.

References

- [1] Acemoglu, D. and Jackson, M. (2011). History, expectations, and leadership in the evolution of social norms. *mimeo*.
- [2] Baldwin, R. (2001). Core-periphery model with forward looking expectations. *Regional Science and Urban Economics*, 31:21 – 49.
- [3] Ben-David, D. (2008). Brain drained: A tale of two countries.
- [4] Blanchard, O. and Katz, L. (1992). Regional evolutions. *Brookings Papers on Economic Activity*, 1.

- [5] Brueckner, J. K. (2000). Urban sprawl, diagnosis and remedies. *International Regional Science Review*, 23(2).
- [6] Davezies, L. (2012). *La Crise qui vient*. La Republique des Idees.
- [7] Davis, D. R. and Weinstein, D. E. (2002). Bones, bombs, and break points: The geography of economic activity. *American Economic Review*, 92(5):1269–1289.
- [8] Deardorff, A. V. (2005). A trade theorist's take on skilled labor outsourcing. *International Review of Economics and Finance*.
- [9] Eeckhout, J., Pinheiro, R., and Schimidheiny, K. (2014). Spatial sorting. *Journal of Political Economy*, 122(3):554–620.
- [10] Fukao, K. and Benabou, R. (1993). History versus expectations: a comment. *Quarterly Journal of Economics*, 108 (2):535–542.
- [11] Glaeser, E. (2012). *Triumph of the City*. Penguin Books.
- [12] Glaeser, E. and Berry, C. R. (2005). The divergence of human capital across cities. *Regional Science*, 84(3):407–444.
- [13] Glaeser, E. L. and Mare, D. C. (2001). Cities and skills. *Journal of Labor Economics*, 19(2).
- [14] Glaeser, E. L. and Saiz, A. (2001). The rise of the skilled city. *Journal of Labor Economics*, 19(2).
- [15] Herrendorf, B., Valentinyi, A., and Waldmann, R. (2000). Ruling out multiplicity and indeterminacy: The role of heterogeneity. *Review of Economic Studies*, 67:295–308.
- [16] i Martin, S. and Barro, R. (1992). Convergence. *Journal of Political Economy*, 100(2):223–251.
- [17] Krugman, P. (1991a). History versus expectations. *Quarterly Journal of Economics*, 106 (2):651–667.
- [18] Krugman, P. (1991b). Increasing returns and economic geography. *Journal of Political Economy*, 99 (3):483–499.
- [19] Kshetri, N. and Dholakia, N. (2011). Offshoring of healthcare services: the case of us-india trade in medical transcription services. *Journal of Health Organization and Management*, 25(1).

- [20] Lichterman, J. (2014). Michigan governor urges 50,000 visas for skilled detroit immigrants. *Reuters*.
- [21] Matsuyama, K. (1991). Increasing returns, industrialization and inderterminacy of equilibrium. *Quaterly Journal of Economics*, 106 (2):617–650.
- [22] Morris, S. and Shin, H. S. (2005). *The economy as an evolving complex system III*. Oxford University Press USA.
- [23] Ottaviano, G. (1999). Integration, geography and the burden of history. *Regional Science and Urban Economics*, 29(2):245–256.
- [24] Ottaviano, G., J.-F. Thisse, and Tabuchi, T. (2002). Agglomeration and trade revisited. *International Economic Review*, 43 (2):409–436.
- [25] Oyama, D. (2009). History versus expectations reconsidered. *Journal of Economic Dynamics and Control*, 33 (2):394–408.
- [26] Picard, P. M. and Zeng, D. Z. (2005). Agricultural sector and industrial agglomeration. *Journal of Development Economics*, 77(1):75–106.
- [27] Plaza, B. (2002). The guggenheim-bilbao museum effect: A reply to maria v. gomez " reflective images:the case of urban regeneration in glasgow and bilbao". *International Journal of Urban and Regional Research*, 23(3):589–592.
- [28] Puga, D. (1999). The rise and fall of regional inequalities. *European Economic Review*, 43:303–334.
- [29] Royer, J.-F. (2008). Disparites territoriales: effets et causes des comportements des agents. *Economie et Statistique*, 416-416.
- [30] Saint-Paul, G. (2008). The european brain drain: European workers living in the us. *Voxeu*.
- [31] Tabuchi, T. and Thisse, J.-F. (2002). Taste heterogeneity, labor mobility and economic geography. *Journal of Development Economics*, 69(1):155–177.
- [32] Wadhwa, V., Jain, S., Saxenian, A., Gereffi, G., and Wang, H. (2011). The grass is indeed greener in india and china for returnee entrepreneurs: America’s new immigrant entrepreneurs – part vi.
- [33] Wirl, F. and Feichtinger, G. (2006). History versus expectations: Increasing returns or social influence ? *The Journal of Socio-Economics*, 35:877–888.
- [34] Zeng, D. Z. (2008). New economic geography with heterogeneous preferences: An explanation of segregation. *Journal of Urban Economics*, 63(1):306–324.