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Toward a theory of monopolistic competition*

Mathieu Parenti[†] Philip Ushchev[‡] Jacques-François Thisse[§]

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Abstract

We propose a general model of monopolistic competition, which encompasses existing models while being flexible enough to take into account new demand and competition features. Using the concept of Frechet differentiability, we determine a general demand system. The basic tool we use to study the market outcome is the elasticity of substitution at a symmetric consumption pattern, which depends on both the per capita consumption and the total mass of varieties. We impose intuitive conditions on this function to guarantee the existence and uniqueness of a free-entry equilibrium. Our model is able to mimic oligopolistic behavior and to replicate partial equilibrium results within a general equilibrium framework. For example, an increase in per capita income or in population size shifts prices (outputs) downwards (upwards). When firms face the same productivity shock, they adopt an incomplete pass-through policy, except when preferences are homothetic. Finally, we show how our approach can be generalized to the case of a multisector economy and extended to cope with heterogeneous firms and consumers.

Keywords: monopolistic competition, general equilibrium, additive preferences, homothetic preferences

JEL classification: D43, L11, L13.

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1 Introduction

The theory of general equilibrium with imperfectly competitive markets is still in infancy. In his survey of the various attempts made in the 1970s and 1980s to integrate oligopolistic competition within the general equilibrium framework, Hart (1985) has convincingly argued that these contributions have failed to produce a consistent and workable model. Unintentionally, the absence of a general equilibrium model with oligopolistic competition has paved the way to the success of the CES model of monopolistic competition developed by Dixit and Stiglitz (1977), which has been applied to an amazingly large number of economic problems (Brakman and Heijdra, 2004). This state of affairs has led many scholars to believe that the CES model was *the* model of monopolistic competition. For example, Head and Mayer (2014) observe that this model is “nearly ubiquitous” in the trade literature. However, owing to its extreme simplicity, the CES model dismisses several important effects that contradict basic findings in economic theory as well as empirical evidence. To mention a few, unlike what the CES predicts, markups and firm sizes are affected by entry (Breshnahan and ...), market size (Handbury and Weinstein, 2013), consumers’ income (Simonovska, 2013), while markups are dependent of cost (De Loecker et al., 2012). In addition, “tweaking” the CES in the hope of obviating these difficulties, as done in many empirical papers, does not appear to be a satisfactory research strategy for at least two reasons. First, it does not permit a genuine comparison of results and, second, it hinders the development of new and more general models of monopolistic competition that could be brought to the data.

Different alternatives have been proposed to avoid the main pitfalls of the CES model. Behrens and Murata (2007) propose the CARA utility that captures both price and size effects, while Zhelobodko et al. (2012) use general additive preferences to work with a variable elasticity of substitution. Vives (1999), Ottaviano et al. (2002) and Melitz and Ottaviano (2008) show how the quadratic utility model obviates some of the difficulties associated with the CES model, while delivering a full analytical solution. More recently, Bertoletti and Etro (2013) consider an additive indirect utility function to study the impact of per capita income on the market outcome, but price and firm size are independent of population size in their setting. In sum, it seems fair to say that the state of the art looks like a scattered field of incomplete and insufficiently related contributions.

Our purpose is to build a general equilibrium model of monopolistic competition, which encompasses existing models and retains enough flexibility to take into account new demand and competition features. There are two main reasons explaining why working with monopolistic competition in general equilibrium looks more promising than oligopolistic competition. First, it is well known that the redistribution of firms’ profits is at the root of the non-existence of an equilibrium in general equilibrium with oligopolistic competition. Since entry drives profits down to zero in monopolistic competition, we get rid of this feedback effect and end up with a consistent and analytically tractable model.

Second, we capture Chamberlin's idea that the decision made by a firm has no impact on its competitors by assuming the existence of a continuum of firms.¹ However, firms are bound together through the marginal utility of income and substitution effects among varieties. This has an far-fetched implication: even though firms do not compete strategically, our model is able to mimic oligopolistic markets and to generate within a general equilibrium framework findings akin to those obtained in partial equilibrium analyses.

To prove the existence and uniqueness of an equilibrium and to study its properties, we need to impose some restrictions on the demand side of our model. Rather than making new assumptions on preferences and demands, we tackle the problem by building on the theory of product differentiation. Specifically, we exploit the symmetry of preferences over a continuum of goods to show that under the most general specification of preferences, at any symmetric outcome the elasticity of substitution between any two varieties is a function of two variables only: the common per capita consumption and the total mass of firms. Combining this with the absence of business-stealing effects reveals that, at the market equilibrium, *firms' markup must be equal to the inverse of the equilibrium value of the elasticity of substitution.*

This result agrees with one of the main messages of industrial organization: the higher is the elasticity of substitution, the less differentiated are varieties, and thus the lower are firms' markup. It should then be clear that the properties of the symmetric free-entry equilibrium depends on how the elasticity of substitution function behaves when the per capita consumption and the mass of firms, which are both endogenous, vary with the parameters of the economy. The above relationship, which links the supply and demand sides of the model in a very intuitive way, allows us to study the market outcome by means of simple analytical arguments. To be precise, by imposing plausible conditions to the elasticity of substitution function, we are able to disentangle the various determinants of firms' strategies. For example, we show how markups vary with market size and firms' productivity. This may help applied economists to improve their structural interpretation of the estimated elasticities.

Our main findings may be summarized as follows. First, using the concept of Frechet differentiability, we determine a general demand system, which includes a wide range of special cases such as the CES, quadratic, CARA, additive, indirectly additive, and homothetic preferences. At a symmetric market outcome, the individual demand for a variety depends only upon its consumption when preferences are additive. By contrast, when preferences are homothetic, the demand for a variety depends upon its relative consumption level and the mass of available varieties. Therefore, when preferences are neither additive nor homothetic, the demand for a variety must depend on both its consumption level and total mass of available varieties.

Second, to insulate the impact of various types of preferences on the market outcome, we focus on symmetric firms and, therefore, on symmetric free-entry equilibria. We provide weak sufficient

¹The idea of using a continuum of firms was already discussed by Dixit and Stiglitz in their 1974 working paper, which has been published in Brakman and Heijdra (2004).

conditions to be satisfied by the elasticity of substitution function for the existence and uniqueness of a symmetric free-entry equilibrium. Our setting is especially well adapted to conduct detailed comparative static analyses. The typical thought experiment is to study the impact of market size. What market size signifies is not always clear because it compounds two variables, i.e. the number of consumers and their willingness-to-pay for the product under consideration. Regarding the impact of population size and income level on prices, output and the number of firms, the nature of preferences does not matter for the impact of the former, but it does for the impact of the latter.

An increase in population or in income raises demand, thereby fostering entry and lower prices. But an income hike also raises consumers' willingness-to-pay, which tends to push prices upward. The final impact is thus ambiguous. However, the entry effect dominates the income effect if and only if the elasticity of substitution rises or does not fall too much with the number of competitors, that is, the entry of new firms does not entail a considerable augmentation in the degree of product differentiation. To sum up, like most oligopoly models, *our model exhibits the standard pro-competitive effects associated with entry*. However, it may also give rise to less standard findings, such as the anti-competitive effect that may be caused by an income hike. This echoes results obtained in industrial organization, but the channels are different (Amir and Lambson, 2000; Chen and Riordan, 2008). The CES is the only utility for which price and output are independent of both income and market size.

We also show that, when preferences are non-homothetic, a productivity hike affecting all firms leads to a lower market price, but a higher markup. In this event, there is incomplete pass-through, like in spatial pricing models. Interestingly, firm size need not increase under a positive productivity shock. Firm size increases when preferences are additive or homothetic, but it may decrease under indirectly additive or quadratic preferences.

Last, we discuss three major extensions of our baseline model. In the first one, we consider Melitz-like heterogeneous firms. In this case, the market outcome is not symmetric anymore. In addition, when preferences are non-additive, the profit-maximizing price of a firm depends directly on the prices set by other types' firms. This requires the use of Tarski's fixed point theorem to prove the existence of an equilibrium. The second extension focuses on a multisector economy. The main additional difficulty stems from the fact that the sector-specific expenditures depend on the upper-tier utility. Under fairly mild conditions on this function, we prove the existence of an equilibrium and show that many of our results hold true for the monopolistically competitive sector. This highlights the idea that our model can be used as a building block to embed monopolistic competition in full-fledged general equilibrium models coping various applications. Our last extension addresses the almost untouched issue of consumer heterogeneity in love-for-variety models of monopolistic competition. Consumers may be heterogeneous because of taste or income differences. Here, we will restrict ourselves to the discussions of some particular cases.

In the next section, we describe the demand and supply sides of our setting. The primitive of

the model being the elasticity of substitution function, we discuss in Section 3 how this function varies with per capita consumption and the mass of varieties. In Section 4, we prove the existence and uniqueness of a symmetric free-entry equilibrium and characterize its various properties. Three extensions are discussed in Section 5. First, we show how our model and findings can be extended to a multisector economy. We then address the case of heterogeneous firms and, last, discuss some possible lines of research to tackle the case of heterogeneous consumers. Section 6 concludes.

2 The model and preliminary results

Consider an economy with a mass L of identical consumers, one sector and one production factor – labor, which is used as the numéraire. Each consumer is endowed with y efficiency units of labor, so that the per capita income y is given and the same across consumers. This will allow us to discriminate between the effects generated by the consumer income, y , and the number of consumers, L . Firms produce a horizontally differentiated good under increasing returns. Each firm supplies a single variety and each variety is supplied by a single firm.

2.1 Consumers

Let \mathbb{N} , an arbitrarily large number, be the mass of “potential” varieties, e.g. the varieties for which a patent exists. Very much like in the Arrow-Debreu model where all commodities need not be produced and consumed, all potential varieties are not necessarily made available to consumers. We denote by $N \leq \mathbb{N}$ the endogenous mass of available varieties.

A *potential consumption profile* $\mathbf{x} \geq 0$ is a Lebesgue-measurable mapping from $[0, \mathbb{N}]$ to \mathbb{R}_+ . Since a market price profile $\mathbf{p} \geq \mathbf{0}$ must belong to the dual of the space of consumption profiles (Bewley, 1972), we assume that both \mathbf{x} and \mathbf{p} belong to $L_2([0, \mathbb{N}])$, which is its own dual. This implies that both \mathbf{x} and \mathbf{p} have a finite mean and variance. Furthermore, L_2 may be viewed as the most natural infinite-dimensional extension of \mathbb{R}^n . Indeed, as will be seen below, using L_2 allows us to write the consumer program in a simple way and to determine well-behaved demand functions by using the concept of Frechet-differentiability, which is especially tractable in L_2 (Dunford and Schwartz, 1988).

Preferences are described by a *utility functional* $\mathcal{U}(\mathbf{x})$ defined over $L_2([0, \mathbb{N}])$. In what follows, we make two assumptions about \mathcal{U} , which seem close to the “minimal” set of requirements for our model to be nonspecific while displaying the desirable features of existing models of monopolistic competition. First, for any N , the functional \mathcal{U} is *symmetric* in the sense that any Lebesgue measure-preserving mapping from $[0, N]$ into itself does not change the value of \mathcal{U} . Intuitively, this means that renumbering varieties has no impact on the utility level.

Second, the utility function exhibits *love for variety* if, for any $N \leq \mathbb{N}$, a consumer strictly prefers to consume the whole range of varieties $[0, N]$ than any subinterval $[0, k]$ of $[0, N]$, that is,

$$\mathcal{U}\left(\frac{X}{k}I_{[0,k]}\right) < \mathcal{U}\left(\frac{X}{N}I_{[0,N]}\right) \quad (1)$$

where $X > 0$ is the consumer's total consumption of the differentiated good and I_A is the indicator of $A \subseteq [0, N]$. Since (1) holds under any monotone transformation of \mathcal{U} , the nature of our definition of love for variety is ordinal. In particular, our definition does not appeal to any parametric measure such as the elasticity of substitution in CES-based models.

Proposition 1. *If $\mathcal{U}(\mathbf{x})$ is continuous and strictly quasi-concave, then consumers exhibit love for variety.*

The proof is given in Appendix 1. The convexity of preferences is often interpreted as a “taste for diversification” (Mas-Collel et al., 1995, p.44). Our definition of “love for variety” is weaker than that of convex preferences because the former, unlike the latter, involves symmetric consumption only. This explains why the reverse of Proposition 1 need not hold.

For any given N , the utility functional \mathcal{U} is said to be *Frechet-differentiable* in $\mathbf{x} \in L_2([0, N])$ when there exists a unique function $D(x_i, \mathbf{x})$ from $[0, N] \times L_2$ to \mathbb{R} such that, for all $\mathbf{h} \in L_2$, the equality

$$\mathcal{U}(\mathbf{x} + \mathbf{h}) = \mathcal{U}(\mathbf{x}) + \int_0^N D(x_i, \mathbf{x}) h_i di + o(\|\mathbf{h}\|_2) \quad (2)$$

holds, $\|\cdot\|_2$ being the L_2 -norm. In what follows, we restrict ourselves to utility functionals that are Frechet-differentiable for all $\mathbf{x} \geq 0$ such that $D(x_i, \mathbf{x})$ is decreasing and differentiable with respect to the consumption x_i of variety i . The function D is the *marginal utility of variety i* when there is a continuum of goods. That $D(x_i, \mathbf{x})$ does not depend directly on $i \in [0, N]$ follows from the symmetry of preferences. Moreover, $D(x_i, \mathbf{x})$ strictly decreases with x_i if \mathcal{U} is strictly concave. Thus, a strictly concave utility functional exhibits love for variety and generates a downward sloping demand function for every variety.

The reason for restricting ourselves to decreasing Frechet-derivatives is that this property allows us to work with well-behaved demand functions. Indeed, maximizing the functional $\mathcal{U}(\mathbf{x})$ subject to (i) the budget constraint

$$\int_0^N p_i x_i di = y \quad (3)$$

where y stands for the individual income, and (ii) the availability constraint

$$x_i \geq 0 \text{ for all } i \in [0, N] \quad \text{and} \quad x_i = 0 \text{ for all } i \in]N, N]$$

yields the following inverse demand function for variety i :

$$p_i = \frac{D(x_i, \mathbf{x})}{\lambda} \quad \text{for all } i \in [0, N] \quad (4)$$

where λ is the Lagrange multiplier of the consumer's optimization problem. Expressing λ as a function of y and \mathbf{x} , we obtain

$$\lambda(y, \mathbf{x}) = \frac{\int_0^N x_i D(x_i, \mathbf{x}) di}{y}, \quad (5)$$

which is the marginal utility of income at the consumption profile \mathbf{x} under income y .²

The marginal utility function D also allows determining the Marshallian demand. Indeed, because the consumer's budget set is convex and weakly compact in $L_2([0, \mathbb{N}])$, while \mathcal{U} is continuous and strictly quasi-concave, there exists a unique utility-maximizing consumption profile $\mathbf{x}^*(\mathbf{p}, y)$ (Dunford and Schwartz, 1988). Plugging $\mathbf{x}^*(\mathbf{p}, y)$ into (4) – (5) and solving (4) for x_i , we obtain the Marshallian demand for variety i :

$$x_i = \mathcal{D}(p_i, \mathbf{p}, y) \quad (6)$$

which is weakly decreasing in its own price.³ In other words, when there is a continuum of varieties, *decreasing marginal utilities are a necessary and sufficient condition for the Law of demand to hold.*

To illustrate how preferences shape the demand system, consider the following examples of utility functionals satisfying the condition (2).

1. Additive preferences.⁴ (i) Assume that preferences are additive over the set of available varieties (Spence, 1976; Dixit and Stiglitz, 1977):

$$\mathcal{U}(\mathbf{x}) \equiv \int_0^N u(x_i) di \quad (7)$$

where u is differentiable, strictly increasing, strictly concave and such that $u(0) = 0$. The CES and the CARA utility (Bertoletti, 2006; Behrens and Murata, 2007) are special cases of (7).

It is straightforward to show that (7) satisfies (2). The marginal utility of variety i depends only upon its own consumption:

$$D(x_i, \mathbf{x}) = u'(x_i).$$

Thus, the inverse demand functions satisfy the property of independence of irrelevant alternatives, whereas the demand functions

$$x_i = (u')^{-1}(\lambda p_i) \quad (8)$$

²If we apply to \mathcal{U} a monotonic transformation ψ , then $D(x_i, \mathbf{x})$ will change into $\psi'(\mathcal{U}(\mathbf{x})) D(x_i, \mathbf{x})$. However, as implied by (5), λ is also multiplied by $\psi'(\mathcal{U}(\mathbf{x}))$, which means that the inverse demand $D(x_i, \mathbf{x})/\lambda$ is invariant to a monotonic transformation of the utility functional.

³Since D is continuously decreasing in x_i , there exists at most one solution of (4) with respect to x_i . However, if a finite choke price exists, i.e. when $D(0, \mathbf{x}^*)/\lambda < \infty$, there may be no solution. To encompass this case, the Marshallian demand \mathcal{D} should be formally defined by $\mathcal{D}(p_i, \mathbf{p}, y) \equiv \inf\{x_i \geq 0 \mid D(x_i, \mathbf{x}^*)/\lambda(y, \mathbf{x}^*) \leq p_i\}$.

⁴The idea of additive utilities and additive indirect utilities goes back at least to Houthakker (1960).

do not because, as seen from (5), the multiplier λ captures information about the whole consumption profile.

(ii) Bertolotti and Etro (2013) have recently proposed a new approach to modeling monopolistic competition, in which preferences are expressed through the following indirect utility function:

$$\mathcal{V}(\mathbf{p}, y) \equiv \int_0^{\mathbb{N}} v(p_i/y) di \quad (9)$$

where v is differentiable, strictly decreasing and strictly convex. Using Roy's identity, the demand function for variety i is given by

$$x_i = \frac{v'(p_i/y)}{\int_0^{\mathbb{N}} (p_k/y)v'(p_k/y) dk} \quad (10)$$

where the denominator is an aggregate demand shifter that, by the envelope theorem, equals $-\lambda y$.

Clearly, the demand functions satisfy the property of independence of irrelevant alternatives, whereas the inverse demand functions

$$p_i = y(v')^{-1}(-\lambda y x_i)$$

do not. Thus, unlike the Marshallian demand (8) obtained under additive preferences, the Marshallian demand (10) now depends directly on y .

In brief, the link between a direct and an indirect additive utility goes through the demand functions under (9) and the inverse demand functions under (7), which both share the property of independence of irrelevant alternatives. Thus, we may already conclude that a direct and an indirect additive utility generate different market outcomes. This point is further developed in Sections 3 and 4.

2. Non-additive preferences. Consider first the quadratic utility proposed by Ottaviano et al. (2002):

$$\mathcal{U}(\mathbf{x}) \equiv \alpha \int_0^{\mathbb{N}} x_i di - \frac{\beta}{2} \int_0^{\mathbb{N}} x_i^2 di - \frac{\gamma}{2} \int_0^{\mathbb{N}} \left(\int_0^{\mathbb{N}} x_i di \right) x_j dj \quad (11)$$

where $\alpha, \beta,$ and γ are positive constants such that $\beta > \gamma$. In this case, the marginal utility of variety i is given by

$$D(x_i, \mathbf{x}) = \alpha - \beta x_i - \gamma \int_0^{\mathbb{N}} x_j dj$$

which is linear decreasing in x_i . In addition, D also decreases with the aggregate consumption across varieties:

$$X \equiv \int_0^{\mathbb{N}} x_j dj.$$

Another example of non-additive preferences, which also captures the idea of love for variety is given by the entropy utility proposed by Anderson et al. (1992):

$$\mathcal{U}(\mathbf{x}) \equiv U(X) + X \ln X - \int_0^{\mathbb{N}} x_i \ln x_i di$$

where U is increasing and strictly concave. The marginal utility of variety i is

$$D(x_i, \mathbf{x}) = U'(X) - \ln \left(\frac{x_i}{X} \right)$$

which decreases with x_i .

3. Homothetic preferences. A tractable example of non-CES homothetic preferences is the translog, as developed by Feenstra (2003). By appealing to the duality principle in consumption theory, these preferences are described by the following expenditure function:

$$\ln E(\mathbf{p}) = \ln \mathcal{U}_0 + \frac{1}{\mathbb{N}} \int_0^{\mathbb{N}} \ln p_i di - \frac{\beta}{2\mathbb{N}} \left[\int_0^{\mathbb{N}} (\ln p_i)^2 di - \frac{1}{\mathbb{N}} \left(\int_0^{\mathbb{N}} \ln p_i di \right)^2 \right].$$

A generalization of the translog is provided by the following expenditure function (Feenstra, 2014), which also portrays homothetic preferences:

$$E(\mathbf{p}) = \mathcal{U}_0 \cdot \left[\alpha \int_0^{\mathbb{N}} p_i^r di + \beta \left(\int_0^{\mathbb{N}} p_i^{r/2} di \right)^2 \right]^{1/r} \quad r \neq 0.$$

A large share of the literature focusing on additive or homothetic preferences, we find it important to provide a full characterization of the corresponding demands (the proof is given in Appendix 2).

Proposition 2. *The marginal utility $D(x_i, \mathbf{x})$ of variety i depends only upon (i) the consumption x_i if and only if preferences are additive and (ii) the consumption ratio \mathbf{x}/x_i if and only if preferences are homothetic.*

Proposition 2 can be illustrated by using the CES:

$$\mathcal{U}(\mathbf{x}) \equiv \left(\int_0^{\mathbb{N}} x_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the elasticity of substitution across varieties. The marginal utility $D(x_i, \mathbf{x})$ is given by

$$D(x_i, \mathbf{x}) = A(\mathbf{x}) x_i^{1/\sigma} = A(\mathbf{x}/x_i)$$

where $A(\mathbf{x})$ is the aggregate given by

$$A(\mathbf{x}) \equiv \left(\int_0^{\infty} x_j^{\frac{\sigma-1}{\sigma}} dj \right)^{-1/\sigma}.$$

Let $\bar{\eta}(x_i, \mathbf{x})$ be firm i 's inverse demand elasticity given by

$$\bar{\eta}(x_i, \mathbf{x}) \equiv -\frac{x_i}{D} \frac{\partial D}{\partial x_i} \quad (12)$$

which is conditional on the consumption profile \mathbf{x} .

Since each variety is negligible, it must be that

$$\frac{\partial D(x_i, \mathbf{x})}{\partial x_j} = \frac{\partial D(x_j, \mathbf{x})}{\partial x_i} = 0$$

for all $j \neq i$. Plugging this expression into the formulae provided by Nadiri (1982, p.442) for the elasticity of substitution between any two varieties i and j , setting $x_i = x_j = x$ and simplifying yield

$$\bar{\sigma}(x, \mathbf{x}) = -\frac{D(x, \mathbf{x})}{x} \frac{1}{\frac{\partial D(x, \mathbf{x})}{\partial x}} = \frac{1}{\bar{\eta}(x, \mathbf{x})}. \quad (13)$$

Because we focus on symmetric equilibria, we may restrict ourselves to symmetric consumption profiles:

$$\mathbf{x} = xI_{[0, N]}.$$

Therefore, in accordance with Proposition 2, we may redefine $\bar{\eta}(x_i, \mathbf{x})$ and $\bar{\sigma}(x, \mathbf{x})$ as follows:

$$\eta(x, N) \equiv \bar{\eta}(x, xI_{[0, N]}) \quad \sigma(x, N) \equiv \bar{\sigma}(x, xI_{[0, N]}).$$

Furthermore, (13) implies that

$$\sigma(x, N) = 1/\eta(x, N). \quad (14)$$

Therefore, along the diagonal, our original functional analysis problem boils down into a two-dimensional one.

The number of varieties as a consumption externality. In their 1974 working paper, Dixit and Stiglitz (1977) argued that the mass of varieties could enter the utility functional as a specific argument.⁵ In this case, the number of available varieties has the nature of a consumption externality, the reason being that the value of N stems from the entry decisions made by firms.

⁵Note that technically N could be written as a function of the consumption functional \mathbf{x} in the following way $N = \mu\{x_i > 0, \forall i \leq \mathbb{N}\}$ yet this would raise new issues regarding the Frechet differentiability of the utility function.

An example is given by the augmented-CES, which is defined as follows:

$$\mathcal{U}(\mathbf{x}, N) \equiv N^\nu \left(\int_0^N x_i^{\frac{\sigma-1}{\sigma}} di \right)^{\sigma/(\sigma-1)}. \quad (15)$$

In Benassy (1996), ν is a positive constant that captures the consumer benefit of a larger number of varieties. The idea is to separate the love-for-variety effect from the competition effect generated by the degree of product differentiation, which is inversely measured by σ . Blanchard and Giavazzi (2003) takes the opposite stance by assuming that $\nu = -1/\sigma(N)$ where $\sigma(N)$ increases with N . Under this specification, increasing the number of varieties does not raise consumer welfare but intensifies competition among firms.

Another example is the quadratic utility proposed by Shubik and Levitan (1971):

$$\mathcal{U}(\mathbf{x}, N) \equiv \alpha \int_0^N x_i di - \frac{\beta}{2} \int_0^N x_i^2 di - \frac{\gamma}{2N} \int_0^N \left(\int_0^N x_i di \right) x_j dj. \quad (16)$$

The difference between (11) and (16) is that the former may be rewritten as follows:

$$\alpha X - \frac{\beta}{2} \int_0^N x_i^2 di - \frac{\gamma}{2} X^2$$

which is independent of N , whereas the latter becomes

$$\alpha X - \frac{\beta}{2} \int_0^N x_i^2 di - \frac{\gamma}{2N} X^2$$

which ceteris paribus strictly increases with N .

One restrictive aspect of introducing N as an explicit argument in the utility functional $\mathcal{U}(\mathbf{x}, N)$ is that varying the number of varieties may change the indifference surfaces. In this case, preferences are no longer ordinal. Nevertheless, the analysis developed below remains valid in such cases. Indeed, the marginal utility function D already includes N as an argument because the support of \mathbf{x} varies with N .

2.2 Firms

Each firm supplies a single variety and each variety is produced by a single firm. Thus, a variety may be identified by its producer $i \in [0, N]$. Firms are homogeneous: to produce q units of its variety, a firm needs $F + cq$ efficiency units of labor, which means that F is the fixed production cost and c the marginal production cost. Being negligible to the market, each firm chooses its output (or price) while accurately treating some market aggregates as given. However, for the market to be in equilibrium, firms must accurately guess what these market aggregates will be.

In monopolistic competition, unlike oligopolistic competition, Cournot and Bertrand compe-

tion yield the same market outcome (Vives, 1999). However, unless explicitly mentioned, we assume that firms are quantity-setters. Thus, firm $i \in [0, N]$ maximize its profits

$$\Pi(q_i) = (p_i - c)q_i - F \quad (17)$$

with respect to its output q_i subject to the inverse market demand function $p_i = LD/\lambda$. Since consumers share the same preferences, the consumption of each variety is the same across consumers. Therefore, product market clearing implies $q_i = Lx_i$. Firm i accurately treats the market aggregates N and λ , which are endogenous, parametrically.

At any symmetric consumption outcome, the variety profile \mathbf{x} can be replaced with the common consumption level x and the mass N of varieties. Plugging D into (17), the program of firm i is given by

$$\max_{x_i} \pi(x_i, \mathbf{x}) \equiv \left[\frac{D(x_i, \mathbf{x})}{\lambda} - c \right] L x_i - F. \quad (18)$$

Because firm i accurately treats λ as a parameter, the first-order condition for profit-maximization

$$x_i \frac{\partial D(x_i, \mathbf{x})}{\partial x_i} + D(x_i, \mathbf{x}) = [1 - \bar{\eta}(x_i, \mathbf{x})] D(x_i, \mathbf{x}) = \lambda c \quad (19)$$

implies that the equilibrium output is such that a firm's markup is equal to $\bar{\eta} = 1/\bar{\sigma}$.

It is well known that (19) is both necessary and sufficient if the following condition holds:

(A.1) *the elasticity $\bar{\eta}(x_i, \mathbf{x})$ weakly increases with x_i* (Foellmi and Zweimuller, 2004).

This condition implies that the profit function π is strictly concave in x_i .

We propose below a different condition that builds on the concept of r -convexity, which generalizes the standard notion of convexity (Pearce et al., 1998). For (19) to have at least one solution regardless of $c > 0$, it is sufficient to assume that, for any \mathbf{x} , the following Inada conditions hold:

$$\lim_{x_i \rightarrow 0} D = \infty \quad \lim_{x_i \rightarrow \infty} D = 0. \quad (20)$$

Indeed, since $\bar{\eta}(0, \mathbf{x}) < 1$, (20) implies that $\lim_{x_i \rightarrow 0} (1 - \bar{\eta})D = \infty$. Similarly, since $0 < (1 - \bar{\eta})D < D$, it follows from (20) that $\lim_{x_i \rightarrow \infty} (1 - \bar{\eta})D = 0$. Because $(1 - \bar{\eta})D$ is continuous, it follows from the intermediate value theorem that (19) has at least one positive solution. The same holds if D displays a finite choke price exceeding the marginal cost.

For (19) to be a necessary and sufficient condition of profit maximization, it must be that the second derivative of profit π is negative:

$$x_i \frac{\partial^2 D}{\partial x_i^2} + 2 \frac{\partial D}{\partial x_i} < 0. \quad (21)$$

In this case, (19) has a single solution.

Solving (19) for x_i , plugging the result into (21) and multiplying both parts by $\partial D/\partial x_i$, we

obtain:

$$2 \left(\frac{\partial D}{\partial x_i} \right)^2 - (D - \lambda c) \frac{\partial^2 D}{\partial x_i^2} > 0 \quad (22)$$

which necessarily holds if $D - \lambda c$ is a strictly (-1) -convex function for all $x_i < D^{-1}(\lambda c)$.⁶ Indeed, if

$$\frac{\partial^2}{\partial x_i^2} \left(\frac{1}{D - \lambda c} \right) = \frac{2 \left(\frac{\partial D}{\partial x_i} \right)^2 - (D - \lambda c) \frac{\partial^2 D}{\partial x_i^2}}{(D - \lambda c)^3} > 0$$

then (22) also holds. Since

$$1 + \frac{\lambda c}{D - \lambda c} = \frac{p_i}{p_i - c}$$

the strict convexity of $1/(D - \lambda c)$ is equivalent to the strict convexity of $p_i/(p_i - c)$ in x_i . In other words, the profit function π is strictly quasi-concave in x_i if

(A.2) *the Lerner index $(p_i - c)/p_i$ is strictly (-1) -convex in x_i .*

Note that this condition holds if the marginal utility is concave or log-concave in x_i .

In what follows, we assume that at least one of the two conditions (A.1) and (A.2) holds.

3 How does the elasticity of substitution vary?

In what follows, we restrict ourselves to symmetric outcomes and, then, impose some intuitively plausible conditions on the elasticity of substitution σ at a symmetric outcome when each of the N available varieties is consumed in volume x .

Rewriting the equilibrium conditions (19) along the diagonal yields

$$\frac{p - c}{p} = \eta(x, N) = \frac{1}{\sigma(x, N)}. \quad (23)$$

We illustrate below what the function $\sigma(x, N)$ and (23) become for the different types of preferences discussed in the previous section.

(i) When the utility is additive, we have:

$$\frac{1}{\sigma(x, N)} = \eta(x, N) = r(x) \equiv -\frac{xu''(x)}{u'(x)} \quad (24)$$

which means that σ depends only upon the per capita consumption when preferences are additive.

⁶Following Pearce et al. (1998), given a real number $r \neq 0$, a function $f(x)$ is r -convex if and only if $[f(x)]^r$ is convex.

In this case, (23) yields

$$p = \frac{c}{1 - r(x)}.$$

Combining this expression with the budget constraint shows that, under additive preferences, the profit-maximizing price varies both with the per capita income y and the mass N of firms.

(ii) When the indirect utility is additive, it is shown in Appendix 3 that σ depends only upon the total consumption $X = Nx$. Since the budget constraint implies $X = y/p$, (23) may be rewritten as follows:

$$\frac{p - c}{p} = \theta(X) \equiv -\frac{v'(p/y) y}{v''(p/y) p}. \quad (25)$$

The per capita income y being given, the profit-maximizing price must solve the equation

$$p = \frac{c}{1 - \theta(y/p)} \quad (26)$$

which signifies that the profit-maximizing price varies only with the per capita income, but not with the mass of firms.

(iii) When preferences are homothetic, it follows from Proposition 2 and (12) that

$$\frac{1}{\sigma(x, N)} = \varphi(N) \equiv \eta(1, N) \quad (27)$$

and thus the profit-maximizing price

$$\bar{p} = \frac{c}{1 - \varphi(N)}$$

varies only with the mass of firms, but not with the per capita income.

For example, under translog preferences, we have

$$\mathcal{D}(p_i, \mathbf{p}, y) = \frac{y}{p_i} \left(\frac{1}{N} + \frac{\beta}{N} \int_0^N \ln p_j dj - \beta \ln p_i \right)$$

where $\varphi(N) = 1/(1 + \beta N)$.

(iv) In the CES case, the indirect utility is given by

$$\mathcal{V}(\mathbf{p}, y) = \int_0^N \left(\frac{p_i}{y} \right)^{-(\sigma-1)} di.$$

Since both the direct and indirect CES utilities are additive, the elasticity of substitution is constant. Furthermore, since the CES is also homothetic, it must be that

$$r(x) = \theta(X) = \varphi(N) = \frac{1}{\sigma}.$$

It is, therefore, no surprise that σ is the only demand side parameter that drives the market outcome under CES preferences.

(v) In the entropy utility case, it is readily verified that

$$\sigma(x, N) = U'(Nx) + \ln N$$

which decreases with x , whereas $\sigma(x, N)$ can be U-shaped in N according to the function form of U .

As illustrated in Figure 1, the CES is the *sole* function that belongs to the three classes of preferences. Furthermore, the expressions (24), (25) and (27) imply that the classes of additive, indirectly additive and homothetic preferences are disjoint, except for the CES that belongs to the three of them.

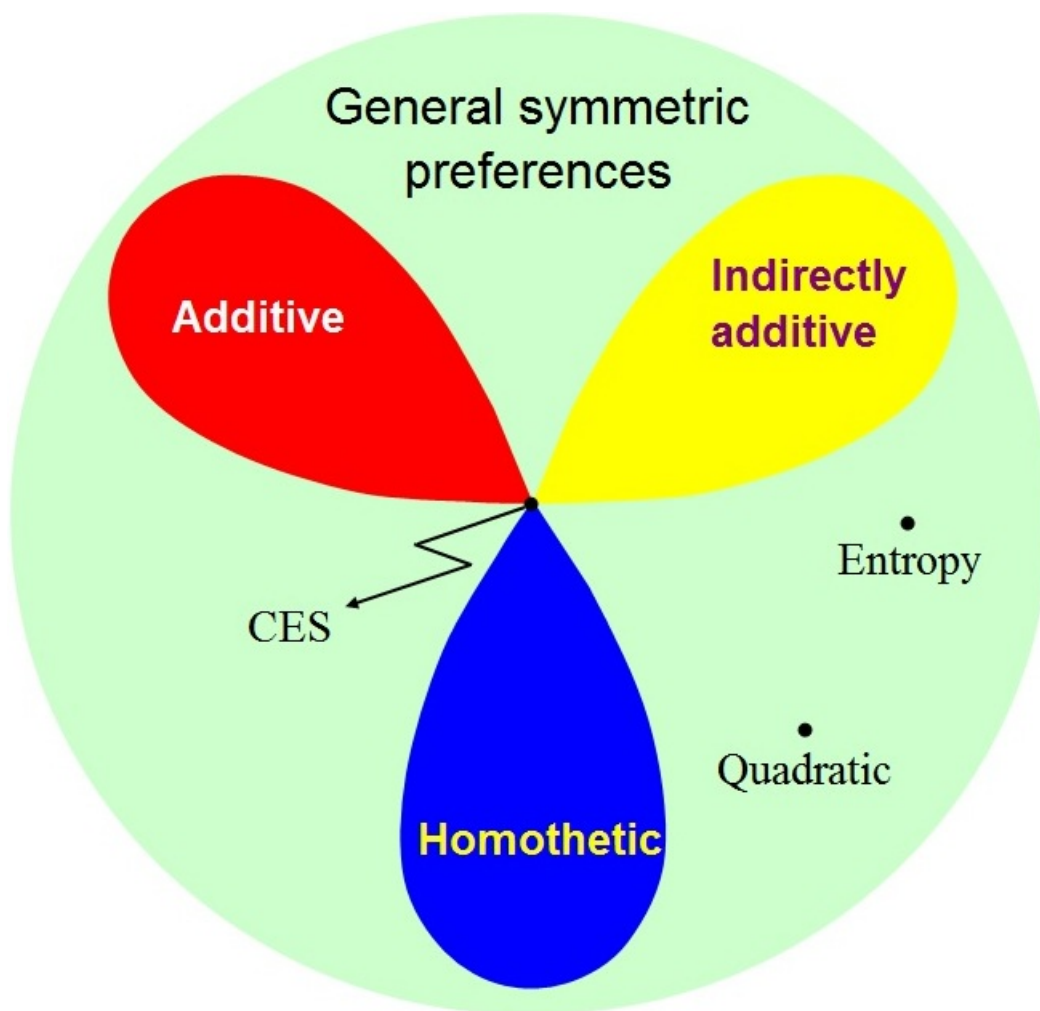


Fig. 1. The space of preferences

Properties of the function $\sigma(x, N)$. In what follows, we consider the function $\sigma(x, N)$ as the *primitive* of the model. This is because this function portrays preferences along the diagonal,

which is sufficient to characterize symmetric equilibria. Hence, we impose below some plausible restrictions on the behavior of σ with respect to x and N . For notational simplicity, we denote by $\mathcal{E}_z(f)$ the elasticity of $f(z)$ with respect to z .

First, when N is constant, assuming that $\sigma(x, N)$ increases (decreases) with x means that varieties become better (worse) substitutes. Although we acknowledge with Zhelobodko et al. (2012) that well-behaved additive preferences may give rise to price-increasing competition under additive preferences, we find it more in accordance with the folk wisdom of economic theory to focus on price-decreasing competition. As a consequence, we assume that $\sigma(x, N)$ weakly decreases with x :

$$\frac{\partial \sigma}{\partial x} \leq 0. \quad (28)$$

We now come to the impact of N on σ . The literature in industrial organization suggests that varieties become closer substitutes when N increases, the reason being that adding new varieties crowds out the product space (Salop, 1979; Tirole, 1988). However, in some standard models of monopolistic competition such as the quadratic utility, the elasticity of substitution decreases with N :

$$\sigma(x, N) = \frac{\alpha - \beta x}{\beta x} - \frac{\gamma}{\beta} N.$$

This should not come as a surprise. Indeed, although spatial models of product differentiation and symmetric models of monopolistic competition are not orthogonal to each other, they differ in several respects (Anderson et al., 1992). This implies that all their properties need not be the same. In particular, as consumers are endowed with a love for variety, they are likely to spread their consumption over a wider range of varieties at the expense of their consumption of each variety, whereas consumers buy one unit of a single variety in spatial models. It thus seems reasonable to assume that the consumption x falls when the number of varieties rises.

To capture the interaction between these two effects, it appears to be convenient to study the impact of N on σ when a consumer's total consumption Nx is constant. In this case, it is readily verified that the following two relationships must hold simultaneously:

$$\begin{aligned} \frac{dx}{x} &= -\frac{dN}{N} \\ \frac{d\sigma}{\sigma} &= \frac{\partial \sigma}{\partial N} \frac{N}{\sigma} \frac{dN}{N} + \frac{\partial \sigma}{\partial x} \frac{x}{\sigma} \frac{dx}{x}. \end{aligned}$$

Plugging the first expression into the second, we obtain

$$\left. \frac{d\sigma}{dN} \right|_{Nx=const} = \frac{\sigma}{N} (\mathcal{E}_N(\sigma) - \mathcal{E}_x(\sigma)).$$

Hence, when Nx is constant the elasticity of substitution weakly decreases with N if and only if

the condition

$$\mathcal{E}_x(\sigma) \leq \mathcal{E}_N(\sigma) \quad (29)$$

holds. Evidently, if $\partial\sigma/\partial N > 0$ is satisfied, (29) holds true, but the reverse is not true. In other words, (29) is a less stringent assumption than $\partial\sigma/\partial N > 0$, which allows the elasticity of substitution to decrease mildly with N . In other words, entry may increase or decrease the degree of product differentiation. Note that $\partial\sigma/\partial x = \partial\sigma/\partial N = 0$ in the CES case.

In what follows, unless explicitly mentioned, *the elasticity of substitution σ is assumed to satisfy the two conditions (28) and (29)*.

Properties of markup. Using the budget constraint, the equilibrium condition (23) may be restated as

$$p \left(1 - \frac{1}{\sigma(y/Np, N)} \right) = c. \quad (30)$$

It follows immediately from (28) that the left-hand side of (30) increases with p . Thus, (30) has a unique solution, which is the profit-maximizing price $\bar{p}(N)$ conditional on N . Note (29) amounts to assuming that the left-hand side of (30) increases with N . As a consequence, the equilibrium markup

$$m(N) \equiv \frac{\bar{p}(N) - c}{\bar{p}(N)} \quad (31)$$

decreases with N , which reflects the pro-competitive effects generated by the entry of new firms. This agrees with standard differentiated oligopoly models (Anderson et al., 1992; Vives, 1999). Note that the profit-maximizing price \bar{p} , whence the corresponding markup m , increases with y because a higher income raises consumers' willingness-to-pay, thus allowing the incumbent firms to sell at a higher price. Finally, as implied by (25), in the additive indirect utility case firms' markup is independent of the mass of firms: $m'(N) = 0$.

Given that

$$m(N) = \frac{1}{\sigma(y/N\bar{p}(N), N)} \quad (32)$$

always holds, a decreasing (increasing) markup means that the degree of product differentiation across varieties decreases (increases) when more firms are in the market. It should be stressed that this intuitive result is not trivial because the relationship (32) holds even when (29) is not satisfied, so that how $N\bar{p}(N)$ varies with N is a priori undetermined.

4 Symmetric free-entry equilibrium

In the case of a general strictly concave utility functional, a *symmetric free-entry equilibrium* (SFE) is described by the vector (p^*, x^*, q^*, N^*) , which solves the following four conditions:

(i) the profit-maximization condition:

$$\frac{p - c}{p} = \frac{1}{\sigma(x, N)} \quad (33)$$

(ii) the zero-profit condition:

$$(p - c)q = F \quad (34)$$

(iii) the product market clearing condition:

$$q = Lx$$

(iv) and budget constraint:

$$Npx = y.$$

The Walras Law implies that labor market balance $N(cLx + F) = yL$ holds. In what follows, we restrict ourselves to the domain of parameters for which $N^* < \mathbb{N}$.

We have seen above that conditions (i), (iii) and (iv) yield the equilibrium markup (31). Combining this expression with (ii), we obtain a single equilibrium condition given by

$$m(N) = \frac{NF}{Ly} \quad (35)$$

which means that, at the equilibrium mass of firms the equilibrium markup is equal to the share of the labor supply spent on overhead costs. Therefore, the larger F , the higher the markup m because there are fewer competitors.

4.1 Existence

Because the right-hand side of (35) increases from 0 with N whereas m is positive and decreasing with N , the two curves intersect each other, and only once. As a consequence, we get the following result.

Proposition 3. *Assume that (A.1) or (A.2) holds. If (28) and (29) hold, then there exists a unique symmetric free-entry equilibrium.*

Figure 2 illustrates this result.

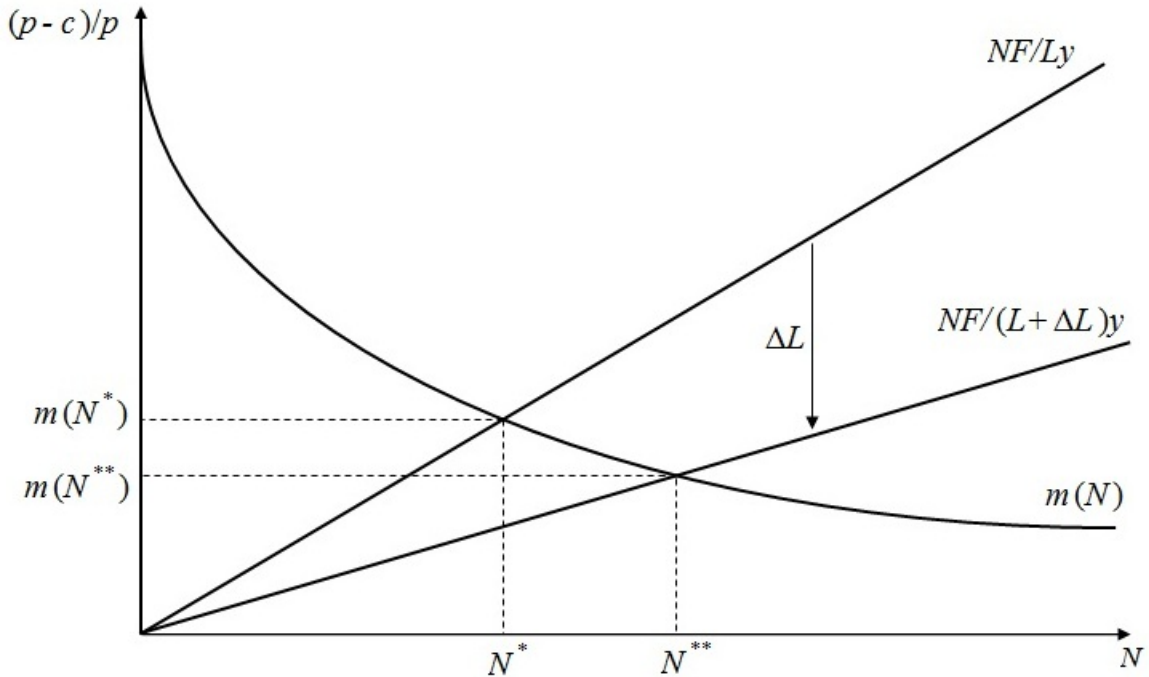


Fig. 2. Existence.

Note that Proposition 3 relies on a set of sufficient, but not necessary, conditions for the existence and uniqueness of an SFE. In particular, there exists a unique SFE when the markup function $m(N)$ increases with N provided that the slope of m is smaller than F/Ly (minus an arbitrarily small positive number). In other words, a market mimicking anti-competitive effects need not preclude the existence and uniqueness of a SFE (Chen and Riordan, 2008; Zhelobodko et al., 2012).

4.2 Comparative statics

In this subsection, we study the impact of a higher gross domestic product on the SFE. A higher total income may stem from a larger population L , a higher per capita income y , or both. Next, we will discuss the impact of firm's productivity.

4.2.1 Population size

A larger population leads to (i) a larger number of firms and (ii) a lower market price. To show it, we first note that the left-hand side of (35) is unaffected by a population hike because, as implied by (30) and (31), it does not involve L as a parameter. By contrast, the ray representing the right-hand side of (35) rotates clockwise (see Figure 2 for an illustration), so that the equilibrium mass of firms increases with L , while the equilibrium price falls with L . This is in accordance with Handbury and Weinstein (2013) who observe that price level for food products falls with city size.

Second, the zero-profit condition $(p - c)q = F$ implies that L always shifts p and q in opposite directions. Therefore, firm sizes are larger in larger markets. It remains to determine how the per variety consumption level x varies with an increase in population L and per capita income y .

When σ weakly decreases with N , the right-hand side of (33) increases with N . Since the left-hand side of this equation decreases with N , (28) implies that x^* decreases with L . This result thus holds for additive, indirectly additive and quadratic preferences. The situation is more ambiguous when σ increases with N .

To illustrate it, consider the case of homothetic preferences where σ depends upon N only. In this case, (30) boils down to

$$p(1 - \varphi(N)) = c. \quad (36)$$

Totally differentiating (36) with respect to L yields

$$\mathcal{E}_L(p^*) - \frac{N^* \varphi'(N^*)}{1 - \varphi(N^*)} \mathcal{E}_L(N^*) = 0.$$

In addition, the budget constraint $pNx = y$ implies

$$-\mathcal{E}_L(N^*) - \mathcal{E}_L(p^*) = \mathcal{E}_L(x^*).$$

Thus, adding the above two expressions, we get

$$-\left[1 + \frac{N^* \varphi'(N^*)}{1 - \varphi(N^*)}\right] \mathcal{E}_L(N^*) = \mathcal{E}_L(x^*)$$

which means that x^* decreases with L if and only if

$$1 + \frac{N^* \varphi'(N^*)}{1 - \varphi(N^*)} > 0.$$

Since $\varphi' < 0$, this inequality need not hold. However, in the case of the translog where $\varphi(N) = 1/(1 + \beta N)$, the desired inequality is satisfied, and thus x^* decreases with L .

The following proposition summarizes our results.

Proposition 4. *Assume that (28) and (29) hold. Then, a larger population leads to (i) more product diversity, (ii) a larger firm size and (iii) a lower markup. Furthermore, if preferences are additive, indirectly additive, quadratic, or translog, a larger population leads to a decrease in the per variety consumption.*

4.2.2 Income

We now come to the impact of the per capita income on the SFE. A positive shock on y triggers the entry of new firms because there is more labor available for production while consumers have a higher willingness-to-pay for the incumbent varieties. More precisely, we know that the curve $m(N)$

is shifted upward when y rises. Therefore, as shown by Figure 3, N^* increase with y . However, the impact of y on market price p^* and firms' size q^* is less straightforward.

We show in Appendix 4 that the equilibrium price decreases (increases) with y when $\sigma(x, N)$ increases (decreases) with N . In other words, a higher per capita income may give rise to both price-decreasing and price-increasing competition. This extends Zhelobodko et al. (2012) who show that a larger market size generates price-decreasing (price-increasing) competition under additive preferences when $r(x)$ increases (decreases) with x . Under indirectly additive preferences, (28) and (29) implies $\theta'(X) > 0$. As a consequence, a higher income always triggers a lower market price, a finding obtained by Bertolotti and Etro (2013) through a different approach.

Finally, note that the product market balance $q = Lx$ implies that firm's size and the per variety consumption always vary in the same direction under a per capita income shock.

Proposition 5. *Assume that (28) and (29) holds. Then, a higher per capita income leads to more product diversity. Furthermore, a higher per capita income results in (i) a lower markup, (ii) a larger firm size, and (iii) a larger per variety consumption if and only if $\sigma(x, N)$ increases with N .*

Propositions 4 and 5 imply that, under (28) and (29) an increase in y and L leads to similar pro-competitive effects if $\sigma(x, N)$ increases with N . Otherwise, a positive shock on y generates an anti-competitive effect, i.e. an increase in price and a decrease in output, whereas a larger population gives rise to pro-competitive effects. Therefore, an increase in L (y) cannot be used as a substitute for an increase in y (L) in comparative statics (except in the case of homothetic preferences as shown below). This should not come as a surprise because an increase in y affects the shape of individual demands when preferences are non-homothetic, whereas an increase in L shifts upward the market demand without changing its shape.

Observe that using (indirectly) additive utilities allows capturing the effects generated by shocks on population size (income), but disregard the impact of the other magnitude. Propositions 4 and 5 thus extend results obtained by Zhelobodko et al. (2012) and Bertolotti and Etro (2013).

Finally, if preferences are homothetic, it is well known that the effects of L and y on the market variables p , q and N are exactly the same. To check it within our framework, it suffices to notice that (30), whence m , does not involve y as a parameter because σ depends solely on N . Therefore, it follows from (35) that the equilibrium price, firm size, and number of firms depend only upon total income yL .

4.2.3 Productivity

A firm's productivity is typically measured by its marginal cost level. Thus, to uncover the impact on the market outcome of a productivity shock common to all firms, we conduct a comparative static analysis of the SFE with respect to c .

Assume, first, that preferences are not homothetic, which means that σ strictly decreases with

x . It then follows from (30) that a drop in c implies that $\bar{p}(N)$ gets lower for all values of N . Therefore, since (32) implies that the curve $m(N)$ is shifted upward, both the markup $m(N^*)$ and the mass of firms N^* increase from (35).

Intuitively, we expect p^* to decrease with a drop in c . Figure 4 depicts (30) and (35) in the plane (N, p) .

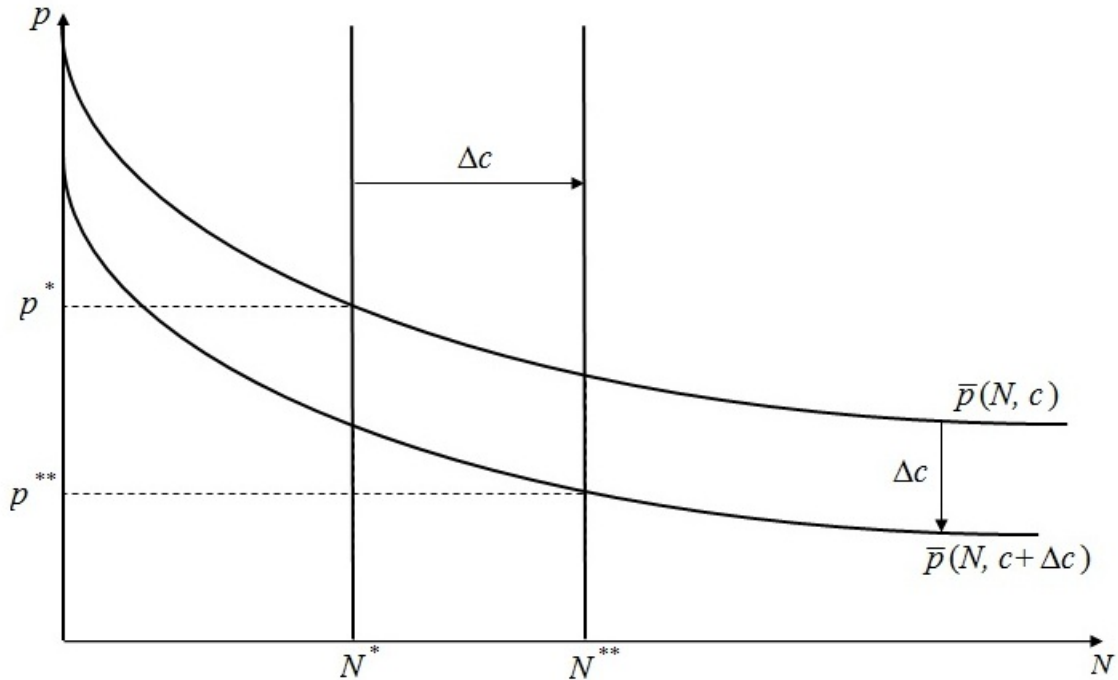


Fig. 4. The impact of a productivity shock.

It is straightforward to check that a drop in c moves the vertical line rightward while the p^* -locus is shifted downward. Thus, p^* decreases together with c . Because the markup increases, firms follow a pricing policy that involves *incomplete pass-through*. Thus, firms partially offset their reduction in marginal costs by raising markups, very much like in oligopoly theory.

Consider now the case of homothetic preferences. It follows from (36) that

$$\bar{p}(N) = \frac{c}{1 - \varphi(N)} \implies m(N) = \varphi(N).$$

As a consequence, (35) does not involve c as a parameter. This implies that a technological shock does affect the number of firms. In other words, the markup remains the same regardless of the productivity shocks, thereby implying that under homothetic preferences firms follow a complete pass-through policy. This explains why trade models that combine CES preferences and iceberg trade costs are unable to exhibit freight absorption or phantom freight.

The impact of technological shocks on firms' size is more involved. Assume, first, that $\sigma(x, N)$ is a weakly increasing function of N . Using (33) and (34), we obtain

$$\frac{x}{\sigma(x, N) - 1} = \frac{F}{cL}.$$

Combining this equation and (28) shows that x^* increases, and thus firms' size q^* decreases. By contrast, as illustrated by the following examples, when $\sigma(x, N)$ decreases with N , the impact of technological shocks on q^* is ambiguous.

Consider the case of indirectly additive preferences (9). The profit-maximizing condition boils down to $p[1 - \theta(y/p)] = c$, thus implying that the equilibrium price always decreases when c falls. Furthermore, the zero-profit condition amounts to

$$q = \frac{F}{p - c}$$

and thus firms' size increases in response to a positive technological shock if and only if the unit margin $p - c = p\theta(y/p)$ decreases with c . For example, if $v(p/y) \equiv (a - p/y)^{1+b}$, with $a, b > 0$, we have $q^* = F(1 + b)/(ay - c)$, which decreases in response to a positive technological shock.

We now come to the case of quadratic preferences in which the parameter α is normalized to 1 by choosing the unit of the differentiated product. Using (30) and (34), it is readily verified that the following two relationships hold:

$$N = \frac{1}{\gamma} \left(\frac{L}{q} - 2\beta - \frac{\beta c}{F} q \right) \quad (37)$$

$$q = \frac{LN}{\beta N + \gamma N^2 + \beta y(L/F)}. \quad (38)$$

Note that (38) is independent of c and is described by a bell-shaped curve. Therefore, the impact of a technological shock on q^* has the same sign as the impact on N^* if and only if N^* belongs to the interval $[0, N_0]$, where N_0 maximizes the right-hand side of (38):

$$N_0 \equiv \sqrt{\frac{\beta y L}{\gamma F}}$$

that is, when the (38)-locus is upward-sloping.

Plugging (38) into (37) and rearranging terms shows that N^* must solve the following equation:

$$\gamma N^3 + \left(\beta + c \frac{L}{F} - \gamma \frac{yL}{F} \right) N^2 - \beta \left(\frac{yL}{F} \right)^2 = 0. \quad (39)$$

Since the left-hand side of (39) is either increasing or U-shaped, the solution N^* is unique. Therefore, $N^* \leq N_0$ if and only if the left-hand side of (39) is non-negative at $N = N_0$, or,

equivalently, if and only if the following inequality holds:

$$c \geq \max\{\hat{c}, 0\} \quad \text{where} \quad \hat{c} \equiv 2\gamma y - \sqrt{\beta\gamma y(F/L)} - \beta(F/L).$$

To sum up, firms' size increases with productivity when $\hat{c} \leq 0$, or first increases and, then, decreases when $\hat{c} > 0$.

The following proposition comprises a summary.

Proposition 6. *Assume that (28) and (29) hold. Then, when marginal costs decrease, (i) prices decrease whereas markups weakly increase; (ii) there is more diversity when preferences are non-homothetic; (iii) under additive or homothetic preferences, firms' size increases, but firms' size may decrease under indirectly additive or quadratic preferences.*

4.2.4 Monopolistic or oligopolistic competition

It should be clear that Propositions 4-6 have the same nature as results obtained in similar comparative analyses conducted in oligopoly theory (Vives, 1999). They may also replicate the less standard anti-competitive effects that a larger market size may trigger under some specific conditions. Therefore, we find it fair to say that *our model of monopolistic competition mimics oligopolistic competition*.

Observe that the markup (33) stems directly from preferences through the sole elasticity of substitution because we focus on monopolistic competition. However, in symmetric oligopoly models the markup emerges as the outcome of the interplay between preferences *and* strategic interactions. To illustrate, consider the case of quantity-setting firms and additive preferences over a finite-dimensional consumption set:

$$\mathcal{U}(x_1, \dots, x_n) = \sum_{i=1}^n u(x_i)$$

where n is an integer. The inverse demands are given by

$$p_i = \frac{u'(x_i)}{\lambda} \quad \lambda = \frac{1}{y} \sum_{i=1}^n x_i u'(x_i).$$

Unlike monopolistic competition, each firm can manipulate λ . This is captured by the first-order conditions for profit maximization:

$$\frac{p_i - c}{p_i} = r(x_i) + \mathcal{E}_{x_i}(\lambda).$$

At the symmetric outcome, the expression boils down to

$$\frac{p - c}{p} = r(x) + \frac{1}{n}(1 - r(x)) \quad (40)$$

while, under monopolistic competition with additive preferences, we have

$$\frac{p - c}{p} = r(x). \quad (41)$$

Comparing (40) and (41) shows that, when preferences are additive, the markup decreases directly with n under Cournot competition, exactly like it does under monopolistic competition under non-additive preferences.

We would be the last to say that monopolistic competition models are able to replicate the whole range of results obtained in industrial organization. For example, these models are useless for studying the incumbents' strategies vis-a-vis potential entrants when there is a handful of firms.

4.3 When is the SFE socially optimal?

The social planner faces the following optimization problem:

$$\begin{aligned} & \max \mathcal{U}(\mathbf{x}) \\ \text{s.t. } & Ly = cL \int_0^N x_i di + NF. \end{aligned}$$

The first-order condition with respect to x_i implies that the problem may be treated using symmetry, so that the above problem may be reformulated as maximizing

$$\phi(x, N) \equiv \mathcal{U}(xI_{[0, N]}) \quad (42)$$

subject to $Ly = N(cLx + F)$.

The ratio of the first-order conditions with respect to x and N leads to

$$\frac{\phi_x}{\phi_N} = \frac{NcL}{cLx + F}. \quad (43)$$

It is well known that the comparison of the social optimum and market outcome leads to ambiguous conclusions for the reasons highlighted by Spence (1976). We illustrate here this difficulty in the special case of homothetic preferences. Without loss of generality, we can write $\phi(N, x)$ as follows:

$$\phi(N, x) = N\psi(N)x$$

where $\psi(N)$ is an increasing function of N . In this event, we get $\phi_{xx}/\phi = 1$ and $\phi_N N/\phi =$

$1 + N\psi'/\psi$, which is equal to the constant ν in Benassy (1996). Therefore, (43) becomes

$$\mathcal{E}_N(\psi) = \frac{F}{cLx}$$

while the market equilibrium condition (35) is given by

$$\frac{\varphi(N)}{1 - \varphi(N)} = \frac{F}{cLx}.$$

The social optimum and the market equilibrium are identical if and only if

$$\mathcal{E}_N(\psi) = \frac{\varphi(N)}{1 - \varphi(N)}. \quad (44)$$

It should be clear that this condition is unlikely to be satisfied unless strong restrictions are imposed on the utility. To be concrete, denote by $A(N)$ the solution to

$$\mathcal{E}_N(A) + \mathcal{E}_N(\psi) = \frac{\varphi(N)}{1 - \varphi(N)}$$

which is unique up to a positive coefficient. It is then readily verified that (44) holds for all N if and only if $\phi(x, N)$ is replaced with $A(N)\phi(x, N)$. Thus, contrary to the folk wisdom, the equilibrium and the optimum may be the same for utility functions that differ from the CES (Dhingra and Morrow, 2013). This finding has an unexpected implication: *when preferences are homothetic, whatever the market outcome there exists a consumption externality such that the equilibrium is optimal regardless of the values taken by the parameters of the economy.* Hence, the choice of a particular consumption externality has subtle welfare implications, which are often disregarded in the literature.

Last, in the case of additive preferences, (44) amounts to the condition:

$$r(x^*) = 1 - \frac{x^*u'(x^*)}{u(x^*)}.$$

In other words, the market outcome is optimal if and only if the equilibrium markup is equal to what Kuhn and Vives (1999) call the preference for variety.

5 Extensions

In this section, we extend our baseline model to cope with a multisector economy as well as with heterogeneous firms and consumers.

5.1 Multisector economy

Following Dixit and Stiglitz (1977), we consider a two-sector economy involving a differentiated good supplied under increasing returns and monopolistic competition, and a homogeneous good supplied under constant returns and perfect competition. Both goods are normal. Labor is the only production factor and is perfectly mobile between sectors. Consumers share the same preferences given by $U(\mathcal{U}(\mathbf{x}), x_0)$ where the functional $\mathcal{U}(\mathbf{x})$ satisfies the properties stated in Section 2, while x_0 is the consumption of the homogeneous good. The upper-tier utility U is strictly quasi-concave, once continuously differentiable, strictly increasing in each argument, and such that the demand for the differentiated product is always positive.⁷

Choosing the unit of the homogeneous good for the marginal productivity of labor to be equal to 1, the equilibrium price of the homogeneous good is equal to 1. Since profits are zero, the budget constraint is given by

$$\int_0^N p_i x_i di + x_0 = E + x_0 = y$$

where the expenditure E is endogenous because competition across firms is affected by the relative preference between the differentiated and homogeneous goods.

Applying the first-order condition for utility maximization yields

$$p_i = \frac{U'_1(\mathcal{U}(\mathbf{x}), x_0)}{U'_2(\mathcal{U}(\mathbf{x}), x_0)} D(x_i, \mathbf{x}) \quad (45)$$

which generalizes (4) to the case of an economy with two sectors.

When varieties of the differentiated product are equally priced, (45) becomes

$$p = \frac{U'_1(\phi(x, N), x_0)}{U'_2(\phi(x, N), x_0)} D(x, xI_{[0, N]}). \quad (46)$$

where $\phi(x, N)$ is defined by (42). Since U is strictly quasi-concave and both goods are normal, the marginal rate of substitution decreases with $\phi(x, N)$ and increases with x_0 . Therefore, for any given (p, x, N) , (46) has a unique solution in $x_0^*(p, x, N)$, which increases in p and x . Plugging this solution into the symmetrized budget constraint, we get

$$pNx + x_0^*(p, x, N) = y. \quad (47)$$

The left-hand side of this expression increases with p and x . Therefore, this equation has a single solution $x^*(p, N)$, which decreases with p .

Since the marginal rate of substitution in (45) is independent of x_i , the elasticity of the inverse demand for variety i is the same as in the one-sector economy studied in the previous sections. Therefore, we may still consider the elasticity of substitution $\sigma(x, N)$ as a primitive. As in 4.1,

⁷In fact, our results hold true if the choke price is finite and sufficiently high.

there exists a unique equilibrium price $\hat{p}(N)$.

Plugging $x^*(p, N)$ and $\hat{p}(N)$ into the zero-profit condition that prevails in the differentiated sector, we obtain

$$\Pi(N) \equiv (\hat{p}(N) - c)x^*(\hat{p}(N), N) = F/L. \quad (48)$$

Since $\Pi(N)$ is continuous, for this equation to have a solution, it is sufficient that $\Pi(N)$ gets arbitrarily large (small) when N is arbitrarily low (high). The argument goes as follows. (i) Since $x^*(\hat{p}(N), N) < x^*(c, N)$ while $x^*(c, N)$ tends to 0 when N is arbitrarily large by the budget constraint, it must be that $x^*(\hat{p}(N), N) \rightarrow 0$ when $N \rightarrow \infty$. (ii) If $N \rightarrow 0$, then $x^*(\hat{p}(N), N) \rightarrow \infty$. Indeed, note first that $x^*(\hat{p}(N), N) > x^*(\sup \hat{p}(N), N)$. Therefore, we get the desired result since, for $p = \sup \hat{p}(N)$, $x^*(p, N) \rightarrow \infty$ when $N \rightarrow 0$, for otherwise the demand for the differentiated product would converge to 0.

The above boundary conditions imply that at least one solution of (48) is stable, that is, $\Pi'(N^*) < 0$. Moreover, from the continuity of $\Pi(N)$ it must be that the set of SFEs has a supremum and an infimum that are also stable SFEs.

Finally, it is straightforward from (48) that, for any stable SFE, a population hike triggers more entry.⁸ Note also that the market price (the firm size) decreases (shrinks) with L as long as $\hat{p}(N)$ decreases with N . For example, under homothetic preferences, we have

$$\hat{p}(N) = \frac{c}{1 - \varphi(N)}$$

so that $\hat{p}(N)$ decreases as implied by (29). Under additive preferences, (46) becomes

$$p = \frac{U'_1(\phi(x, N), x_0)}{U'_2(\phi(x, N), x_0)} u'(x)$$

thereby implying that $x_0^*(p, x, N)$ increases with N . It then follows from (47) that $x^*(p, N)$ decreases with N . Therefore, using (41), $\hat{p}(N)$ must solve

$$\frac{p - c}{p} = r(x^*(p, N)).$$

Since r increases with x , raising N lowers the right-hand side of this expression. Thus, $\hat{p}(N)$ decreases with N .

Our results are summarized in the next proposition.⁹

Proposition 7. *Assume an upper-tier utility such that the demand for the differentiated product is always positive. Then, (i) the set of stable SFEs is non-empty and has a supremum and an infimum, which are both stable; (ii) in any stable SFE, the number of firms rises in response to an*

⁸When the SFE is unstable, an increase in L leads to less product diversity.

⁹Note that the proof developed above differs from and is more general than that proposed in Zhelobodko et al. (2012).

increase in market size; and (iii) the market price (firm size) decreases (increases) with population size under additive or homothetic preferences.

5.2 Heterogeneous firms

One may wonder how the approach developed in this paper can cope with Melitz-like heterogeneous firms. In this case, the equilibrium ceases to be symmetric, for firms are now parametrized by their marginal cost c_i that differs across firms. The equilibrium distribution of prices, firm sizes and the number of firms must be determined as the fixed point of a mapping describing the equilibrium conditions in a functional space. For conciseness, we use the one-period timing proposed by Melitz and Ottaviano (2008). The mass of potential firms is \mathbb{N} . Prior to entry, risk-neutral firms face uncertainty about their marginal cost and entry requires a sunk cost F_e . Once the entry cost is paid, firms observe their marginal cost drawn randomly from the probability distribution $G(c)$ defined over $[0, \infty)$. After observing its type c , each entrant decides to produce or not, given that an active firm incurs a fixed production cost F . Even though varieties are differentiated from the consumer's point of view, firms sharing the same marginal cost c behave in the same way. As a consequence, we may refer to any available variety by its c -type only.

Studying monopolistic competition as a price game appears to be more convenient when firms are heterogeneous, the reason being that we can use the concept of complementarity. To describe the price game, we have to define the demand mapping away from the diagonal. Note that the consumer demand $\mathcal{D}(p_i, \mathbf{p}, y)$ is well defined when the utility functional \mathcal{U} is strictly quasi-concave. Indeed, in this case there exists a unique solution to the consumer's problem.

The new equilibrium conditions are as follows:

(i) the profit-maximization condition for firms of c -type:

$$\pi^*(c, \mathbf{p}, y) \equiv \max_{p \geq 0} \{L(p - c)\mathcal{D}(p, \mathbf{p}, y) - F\}$$

(ii) the zero-profit condition for the cut-off firm \bar{c} :

$$(p_{\bar{c}} - \bar{c})q_{\bar{c}} = F$$

(iii) the product market clearing condition:

$$q_c = Lx_c \quad c \in [0, \bar{c}]$$

(iv) the budget constraint:

$$N \int_0^{\bar{c}} p_c x_c dG(c) = y$$

(v) firms enter the market until their expected profits net of entry costs F_e are zero:

$$\int_0^{\bar{c}} \Pi^*(c) dG(c) = F_e \quad (49)$$

where $\Pi^*(c)$ is the equilibrium value of a type c -firm profit.

In what follows, we prove the existence of a Nash price equilibrium when $\lambda \geq 0$ and $\bar{c} > 0$ are given but arbitrary. To this end, we assume that $\mathcal{D}(p_c, \mathbf{p}, y)$ is differentiable with respect to p_c while its elasticity

$$\bar{\varepsilon}(p_c, \mathbf{p}) \equiv -\frac{\partial \mathcal{D}}{\partial p_c} \frac{p_c}{\mathcal{D}} > 1$$

increases in p_c and decreases in \mathbf{p} , where the inequality is required for profit maximization conditions to have a solution. Given these conditions, the following proposition holds true.

Proposition 8. *Assume that (i) firms are heterogenous while λ and \bar{c} are given and (ii) the direct demand elasticity $\bar{\varepsilon}(p_c, \mathbf{p})$ increases in p_c and decreases in \mathbf{p} . Then, a Nash equilibrium in the price game exists.*

The sketch of the proof is as follows. The first-order condition for a c -type firm, conditional on the vector of prices \mathbf{p} charged by the other firms, is given by

$$\frac{p_c - c}{p_c} = \frac{1}{\bar{\varepsilon}(p_c, \mathbf{p})}. \quad (50)$$

Since the left- (right-)hand side of (50) continuously increases (decreases) with p_c for any given \mathbf{p} , there exists a unique $\hat{p}_c(\mathbf{p})$ that solves (50) for any given $\mathbf{p} \in L_2([0, \bar{c}])$ and $c \in [0, \bar{c}]$. Because $\bar{\varepsilon}$ decreases with \mathbf{p} , $\hat{p}_c(\mathbf{p})$ increases both in \mathbf{p} . Finally, since the left-hand side of (50) decreases in c , $\hat{p}_c(\mathbf{p})$ also increases in c .

Let us define the best-reply mapping \mathcal{P} from $L_2([0, \bar{c}])$ into itself:

$$\mathcal{P}(\mathbf{p}; c) \equiv \hat{p}_c(\mathbf{p}).$$

The proposition holds true if \mathcal{P} has a fixed point. This is done in Appendix 5 by using Tarski's theorem (Vives, 1999, ch.2).

Remark 1. Under additive preferences, that $\bar{\varepsilon}(p_c, \mathbf{p})$ increases in p_c and decreases in \mathbf{p} is equivalent to the assumptions that (i) $r(x_c)$ is an increasing function of x_c and (ii) $r(x_c) < 1$ for all $x \geq 0$. Indeed, $(u')^{-1}(\lambda p_c)$ is the demand for variety c , while its elasticity with respect to p_c is given by

$$\bar{\varepsilon}(p_c, \mathbf{p}) = \frac{1}{r[(u')^{-1}(\lambda(\mathbf{p})p_c)]} > 1 \quad (51)$$

where $\lambda(\mathbf{p})$ is the implicit solution to the budget constraint:

$$\int_0^{\bar{c}} (u')^{-1}(\lambda p_c) p_c dG(c) = y. \quad (52)$$

Therefore, the elasticity of a c -type firm's revenue with respect to p_c is negative, thereby implying that a firm's revenue $(u')^{-1}(\lambda p_c) p_c$ decreases with p_c . It then follows from (52) that $\lambda(\mathbf{p})$ also decreases with \mathbf{p} . Combining this with $r'(x) > 0$, we obtain from (51) that $\bar{\varepsilon}(p_c, \mathbf{p})$ increases in p_c and decreases in \mathbf{p} .

Remark 2. For indirectly additive preferences, the elasticity $\bar{\varepsilon}(p_c, \mathbf{p})$ is independent of \mathbf{p} . Existence thus follows immediately when $\bar{\varepsilon}$ increases with p_c .

Remark 3. In the case of translog preferences, we have:

$$\mathcal{D}(p_i, \mathbf{p}, y) = \frac{y}{p_i} \left(\frac{1}{N} + \frac{\beta}{N} \int_0^N \ln p_j dj - \beta \ln p_i \right)$$

whereas the elasticity is given by

$$\bar{\varepsilon}(p_i, \mathbf{p}) = 1 + \beta \frac{p_i}{\frac{1}{N} + \frac{\beta}{N} \int_0^N \ln p_j dj - \beta \ln p_i}$$

which is increasing in p_i and decreasing in \mathbf{p} when keeping N constant.

We now come to the cut-off cost. Consider two firms with marginal costs c and c' such that $c > c'$. Evidently, we have

$$(p_c - c)\mathcal{D}(p_c, \mathbf{p}, y) < (p_c - c')\mathcal{D}(p_c, \mathbf{p}, y) \quad \forall (p_c, \mathbf{p}) \in \mathbb{R}_+ \times L_2([0, N])$$

which implies the perfect sorting of firms along their cost type. As a consequence, if there exists a solution $\bar{c}(\mathbf{p})$ to the equation

$$\pi^*(c, \mathbf{p}, y) = F/L$$

this solution is unique. Furthermore, when the Inada conditions (20) hold, the above equation has a solution.

Without imposing more structure on preferences and the cost distribution, the cutoff cost need not be monotone in the price vector \mathbf{p} and, therefore, competitive shocks generate convoluted effects in the selection of firms.

To gain insights about the equilibrium, we investigate whether the main comparative static results of Section 4 still hold when firms are heterogenous. Since obtaining results for the case of unspecified preferences seems to be hopeless, we focus on particular preferences that lead to unambiguous predictions.

1. Additive preferences. When preferences are additive, Zhelobodko et al. (2012) show that the cutoff cost is independent of the population size if and only if preferences are CES. In addition, the per capita income has no impact on the cutoff cost and the equilibrium price and

firm size distributions. By contrast, when preferences are indirectly additive, Bertolotti and Etro (2013) show that the cutoff cost is independent of income if and only if preferences are CES, while population size has no impact on the cutoff cost and the equilibrium price and firm size distributions. Interestingly enough, these results hold regardless of the cost distribution.

2. Homothetic preferences. As shown by (51), when preferences are additive \mathcal{D} depends on the price profile \mathbf{p} solely through the scalar aggregate λ . This ceases to hold for non-CES homothetic preferences. However, we can show that our result on complete pass-through at the industry level still holds under firm heterogeneity. To this end, consider a proportionate reduction in marginal costs by a factor $\mu > 1$. In other words, the distribution of marginal costs is now given by $G(\mu c)$.

Let us first investigate the impact of μ on firms' operating profits when the cutoff \bar{c} is unchanged. The cutoff firms now have a marginal cost equal to \bar{c}/μ . Furthermore, under homothetic preferences, $\bar{\varepsilon}(p_i, \mathbf{p})$ does not depend on the income y and is positive homogenous of degree 0. Therefore, (50) is invariant to the same proportionate reduction in c , p_c and \mathbf{p} . As a consequence, the new price equilibrium profile over $[0, \bar{c}]$ is obtained by dividing all prices by μ . To put it differently, regardless of the cost distribution, *under homothetic preferences the equilibrium price distribution changes in proportion with the cost distribution*, thereby leaving unchanged the distribution of equilibrium markups.

We now show that the profits of the \bar{c} -type firms do not change in response to the cost drop. Indeed, both marginal costs and prices are divided by μ , while homothetic preferences imply that demands are shifted upwards by the same factor μ . Therefore, the operating profit of the \bar{c} -type firms is unchanged because

$$\left(\frac{p_{\bar{c}}}{\mu} - \frac{\bar{c}}{\mu}\right) L\mu x_{\bar{c}} = (p_{\bar{c}} - \bar{c}) Lx_{\bar{c}} = F$$

which also shows that the new cutoff is given by \bar{c}/μ .

Last, like in the symmetric firm case, any shock on L and y that keeps Ly unchanged does not affect the equilibrium outcome. The reason is that all the equilibrium conditions (i)-(v) depends only upon the aggregate income Ly .

5.3 Heterogeneous consumers

Accounting for consumer heterogeneity in models of monopolist competition is not easy but doable. Let $\mathcal{D}(p_i, \mathbf{p}; y, \theta)$ be the Marshallian demand for variety i of a (y, θ) -type consumer where θ is the taste parameter. The aggregate demand faced by firm i is then given by

$$\Delta(p_i, \mathbf{p}) = L \int_{\mathbb{R}_+ \times \Theta} \mathcal{D}(p_i, \mathbf{p}; y, \theta) dG(y, \theta) \quad (53)$$

where G is a joint probability distribution of income y and taste θ . As in the foregoing, $\Delta(p_i, \mathbf{p})$ is decreasing in p_i . A comparison with Hildenbrand (1983) and Grandmont (1987), who derived fairly sophisticated conditions for the Law of demand to hold when the number of goods is finite, shows how working with a continuum of goods, which need not be the varieties of a differentiated product, vastly simplifies the analysis.

The properties of \mathcal{D} crucially depend on the relationship between income and taste. Indeed, since firm i 's profit is given by $\pi(p_i, \mathbf{p}) = (p_i - c)\Delta(p_i, \mathbf{p}) - F$, the first-order condition (30) for a symmetric equilibrium becomes

$$p \left[1 - \frac{1}{\varepsilon(p, N)} \right] = c \quad (54)$$

where $\varepsilon(p, N)$ is the elasticity of $\Delta(p, \mathbf{p})$ evaluated at the symmetric outcome. If $\varepsilon(p, N)$ is an increasing function of p and N , most of the results derived above hold true. Indeed, integrating consumers' budget constraints across $\mathbb{R}_+ \times \Theta$ and applying the zero-profit condition yields the markup.

$$m(N) = \frac{NF}{LY} \quad \text{where} \quad Y \equiv \int_{\mathbb{R}_+ \times \Theta} y dG(y, \theta). \quad (55)$$

Note that (55) differs from (35) only in one respect: the individual income y is replaced with the mean income Y , which is independent of L . Consequently, if $\varepsilon(p, N)$ decreases both with p and N , a population hike or a productivity shock affects the SFE as in the baseline model (see Propositions 4 and 6). By contrast, the impact of an increase in Y is ambiguous because it depends on how θ and y are related.

Ever since the Sonnenschein-Mantel-Debreu theorem (Mas-Colell et al., 1995, ch.17), it is well known that the aggregate demand (53) need not inherit the properties of the individual demand functions. In particular, there is no reason to expect the aggregate demand to exhibit an increasing price elasticity even if the individual demands satisfy this property.

To highlight the nature of this difficulty, we show in Appendix 6 that

$$\begin{aligned} \frac{\partial \varepsilon(p, N)}{\partial p} &= \int_{\mathbb{R}_+ \times \Theta} \frac{\partial \varepsilon(p, N; y, \theta)}{\partial p} s(p, N; y, \theta) dG(y, \theta) - \\ &\quad - \frac{1}{p} \int_{\mathbb{R}_+ \times \Theta} [\varepsilon(p, N; y, \theta) - \varepsilon(p, N)]^2 s(p, N; y, \theta) dG(y, \theta) \end{aligned} \quad (56)$$

where $\varepsilon(p, N; y, \theta)$ is the elasticity of the individual demand $\mathcal{D}(p_i, \mathbf{p}; y, \theta)$ evaluated at a symmetric outcome ($p_i = p_j = p$), while $s(p, N; y, \theta)$ stands for the share of demand of (y, θ) -type consumers in the aggregate demand, evaluated at a symmetric outcome:

$$s(p, N; y, \theta) \equiv \frac{\mathcal{D}(p, \mathbf{p}; y, \theta)}{\Delta(p, \mathbf{p})} \Bigg|_{\mathbf{p}=pI_{[0, N]}}. \quad (57)$$

Because the second term of (56) is negative, the market demand may exhibit decreasing price elasticity even when individual demands display increasing price elasticities. This term being the variance of individual demand elasticities across consumers (up to the coefficient $-1/p$), loosely speaking, we may say that *the less heterogeneous consumers are, the more likely the aggregate demand elasticity increases with p .*

In the special case where consumers are endowed with different incomes and CES preferences such that the elasticity of substitution $\sigma(\theta)$ depends on θ , Osharin et al. (2014) show that the market price and the mass of varieties are given, respectively, by $p^* = c\bar{\sigma}/(\bar{\sigma} - 1)$ and $N^* = Y/\bar{\sigma}F$ where

$$\bar{\sigma} \equiv \frac{\int_{\Omega} \sigma(\omega)y(\omega)dG(\theta, y)}{\int_{\Omega} y(\omega)dG(\theta, y)}.$$

Thus, the market outcome depends on the income and taste distribution through the sole value of $\bar{\sigma}$.

6 Concluding remarks

We want to stress that our equilibrium conditions are all the same as those obtained in a standard CES model in which the constant elasticity of substitution is equal to the value taken by the elasticity of substitution function at the equilibrium per capita consumption and mass of firms. Because the values of these variables depend on the population size, income level, and production costs, it is, therefore, not surprising that empirical studies find in the data that these parameters have an impact on the market outcome. Our model displays enough versatility to obviate several of the analytical difficulties encountered in oligopoly theory, as well as the main pitfalls of the CES model of monopolistic competition. Yet, it must be acknowledged that working with heterogeneous firms and/or consumers remains a hard task. Furthermore, in order to bring our model to the data, it seems promising to develop new parametrization methods along the line of what Mrazova and Neary (2013) do. Clearly, more work is called for.

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Appendix

Appendix 1. Proof of Proposition 1.

(i) We first show (1) for the case where N/k is a positive integer. Note that

$$1_{[0,N]} = \sum_{i=1}^{N/k} 1_{[(i-1)k, ik]} \quad (\text{A.1})$$

while symmetry implies

$$\mathcal{U} \left(\frac{X}{k} 1_{[(i-1)k, ik]} \right) = \mathcal{U} \left(\frac{X}{k} 1_{[0,k]} \right) \quad \text{for all } i \in \{2, \dots, N/k\}. \quad (\text{A.2})$$

Together with quasi-concavity, (A.1) – (A.2) imply

$$\mathcal{U} \left(\frac{X}{N} 1_{[0,N]} \right) = \mathcal{U} \left(\frac{k}{N} \sum_{i=1}^{N/k} \frac{X}{k} 1_{[(i-1)k, ik]} \right) > \min_i \mathcal{U} \left(\frac{X}{k} 1_{[(i-1)k, ik]} \right) = \mathcal{U} \left(\frac{X}{k} 1_{[0,k]} \right).$$

Thus, (1) holds when N/k is a positive integer.

(ii) We now extend this argument to the case where N/k is a rational number. Let r/s , where both r and s are positive integers and $r \geq s$, be the irredundant representation of N/k . It is then readily verified that

$$s 1_{[0,N]} = \sum_{i=1}^r 1_{[N\{(i-1)k/N\}, N\{ik/N\}]} \quad (\text{A.3})$$

and

$$\mathcal{U} \left(\frac{X}{k} 1_{[N\{(i-1)k/N\}, N\{ik/N\}]} \right) = \mathcal{U} \left(\frac{X}{k} 1_{[0,k]} \right) \quad \text{for all } i \in \{2, \dots, r\} \quad (\text{A.4})$$

where the fractional part of the real number a is denoted by $\{a\}$.

Using (A.3) – (A.4) instead of (A.1) – (A.4) in the above argument, we obtain

$$\mathcal{U}\left(\frac{X}{N}1_{[0,N]}\right) = \mathcal{U}\left(\frac{1}{r}\sum_{i=1}^r\frac{X}{k}1_{[N\{(i-1)k/N\}, N\{ik/N\}]}\right) > \mathcal{U}\left(\frac{X}{k}1_{[0,k]}\right).$$

Thus, (1) holds when N/k is rational.

(iii) Finally, since \mathcal{U} is continuous while the rational numbers are dense in \mathbb{R}_+ , (1) holds for any real number $N/k > 1$. Q.E.D.

Appendix 2. Proof of Proposition 2.

It is readily verified that the inverse demands generated by preferences (7) are given by $D(x_i, \mathbf{x}) = u'(x_i)$. The uniqueness of the Frechet derivative implies that preferences are additive. This proves part (i).

Assume now that \mathcal{U} is homothetic. Since a utility is defined up to a monotonic transformation, we may assume without loss of generality that \mathcal{U} is homogenous of degree 1. This, in turn, signifies that $D(x_i, \mathbf{x})$ is homogenous of degree 0 with respect to (x_i, \mathbf{x}) . Indeed, because $t\mathcal{U}(\mathbf{x}/t) = \mathcal{U}(\mathbf{x})$ holds for all $t > 0$, (2) can be rewritten as follows:

$$\mathcal{U}(\mathbf{x} + \mathbf{h}) = \mathcal{U}(\mathbf{x}) + \int_0^N D\left(\frac{x_i}{t}, \frac{\mathbf{x}}{t}\right) h_i di + o(\|\mathbf{h}\|_2). \quad (\text{A.5})$$

Uniqueness of the Frechet derivative together with (A.5) implies that

$$D\left(\frac{x_i}{t}, \frac{\mathbf{x}}{t}\right) = D(x_i, \mathbf{x}) \text{ for all } t > 0$$

which shows that D is homogenous of degree 0. As a result, there exists a functional Φ belonging to $L_2([0, N])$ such that $D(x_i, \mathbf{x}) = \Phi(\mathbf{x}/x_i)$. Q.E.D.

Appendix 3. Let

$$\bar{\varepsilon}(p_i, \mathbf{p}, y) \equiv -\frac{\partial \mathcal{D}(p_i, \mathbf{p}, y)}{\partial p_i} \frac{p_i}{\mathcal{D}(p_i, \mathbf{p}, y)}$$

be the elasticity of the Marshallian demand (6). At any symmetric outcome, we have

$$\varepsilon(p, N) \equiv \bar{\varepsilon}(p, pI_{[0,N]}).$$

Using the budget constraint $p = y/Nx$ and (14) yields

$$\varepsilon(y/Nx, N) = \eta(x, N) = \frac{1}{\sigma(x, N)}. \quad (58)$$

When preferences are indirectly additive, it follows from (10) that $\varepsilon(y/Nx, N) = 1 - \theta(y/p)$ where θ is given by (25). Combining this with (A.6), we get $\sigma(x, N) = 1/\theta(Nx)$.

Appendix 4. Setting

$$\varsigma \left(\frac{y}{p}, N \right) \equiv \frac{1}{\sigma \left(\frac{y}{Np}, N \right)} \quad (59)$$

and plugging ς into (30), we obtain

$$p \left[1 - \varsigma \left(\frac{y}{p}, N \right) \right] = c. \quad (60)$$

Furthermore, combining (33) and (34) yields

$$\frac{p - c}{p} = \frac{NF}{Ly}.$$

Differentiating (A.7) and (A.8) with respect to y and rearranging terms yields the following two conditions:

$$-(1 - \varsigma)\mathcal{E}_y(p^*) + \varsigma\mathcal{E}_y(N^*) = \varsigma$$

$$(1 - \varsigma + \varsigma\mathcal{E}_{y/p}(\varsigma)) \mathcal{E}_y(p^*) - \varsigma\mathcal{E}_\varsigma(N)\mathcal{E}_y(N^*) = \varsigma\mathcal{E}_{y/p}(\varsigma).$$

Solving these equations for $\mathcal{E}_{y/p}(p^*)$ and $\mathcal{E}_y(N^*)$ yields

$$\mathcal{E}_y(p^*) = \frac{\varsigma}{1 - \varsigma} \frac{\mathcal{E}_{y/p}(\varsigma) + \mathcal{E}_N(\varsigma)}{1 + \varsigma\mathcal{E}_{y/p}(\varsigma) - \mathcal{E}_N(\varsigma)} \quad (A.9)$$

$$\mathcal{E}_y(N^*) = \frac{1 - \varsigma + \mathcal{E}_{y/p}(\varsigma)}{\varsigma\mathcal{E}_{y/p}(\varsigma) + (1 - \varsigma)(1 - \mathcal{E}_N(\varsigma))}. \quad (A.10)$$

It is readily verified that

$$\mathcal{E}_{y/p}(\varsigma) = -\mathcal{E}_x(\sigma)|_{x=y/Np} \quad \mathcal{E}_N(\varsigma) = \mathcal{E}_x(\sigma) - \mathcal{E}_N(\sigma)|_{x=y/Np} < 0 \quad (A.11)$$

Combining (A.11) with (28)-(29) implies $\mathcal{E}_N(\varsigma) < 0 < \mathcal{E}_{y/p}(\varsigma)$. As a consequence, it follows from (A.9) that $\mathcal{E}_y(p^*) < 0$ if and only if $\mathcal{E}_{y/p}(\varsigma) + \mathcal{E}_N(\varsigma) < 0$, that is,

$$\mathcal{E}_N(\sigma) > 0.$$

Finally, (A.10) implies that N^* always increases with income y . Q.E.D.

Appendix 5. \mathcal{P} has a fixed point.

Since $\hat{p}_c(\mathbf{p})$ increases in \mathbf{p} , \mathcal{P} is an increasing operator. To check the assumption of Tarski's fixed-point theorem, it suffices to construct a set $S \subset L_2([0, \bar{c}])$ such that (i) $\mathcal{P}S \subseteq S$, i.e. \mathcal{P} maps the lattice S into itself and (ii) S is a complete lattice.

Denote by \bar{p} the unique symmetric equilibrium price when all firms share the marginal cost \bar{c} (which exists as shown in Section 4) and observe that $\bar{p} = \hat{p}_{\bar{c}}(\bar{\mathbf{p}})$, where $\bar{\mathbf{p}} \equiv \bar{p}1_{[0, \bar{c}]}$. Since $\hat{p}_c(\mathbf{p})$ increases in c , we have $\bar{p} \geq \hat{p}_c(\bar{\mathbf{p}})$ for all $c \in [0, \bar{c}]$ or, equivalently, $\bar{\mathbf{p}} \geq \mathcal{P}\bar{\mathbf{p}}$. Furthermore, because \mathcal{P} is an increasing operator, $\mathbf{p} \leq \bar{\mathbf{p}}$ implies $\mathcal{P}\mathbf{p} \leq \mathcal{P}\bar{\mathbf{p}} \leq \bar{\mathbf{p}}$. In addition, $\mathcal{P}\mathbf{p}$ is an increasing function of c because $\hat{p}_c(\mathbf{p})$ increases in c . In other words, \mathcal{P} maps the set S of all non-negative weakly increasing functions bounded above by \bar{p} into itself. It remains to show that S is a complete lattice, i.e. any non-empty subset of S has a supremum and an infimum that belong to S . This is so because pointwise supremum and pointwise infimum of a family of increasing functions are also increasing.

To sum-up, since S is a complete lattice and $\mathcal{P}S \subseteq S$, Tarski's theorem implies that \mathcal{P} has a fixed point. Q.E.D.

Appendix 6. At a symmetric outcome the aggregate demand elasticity is given by

$$\varepsilon(p, N) = \int_{\mathbb{R}_+ \times \Theta} \varepsilon(p, N; y, \theta) s(p, N; y, \theta) dG(y, \theta) \quad (\text{A.12})$$

where $s(p, N; y, \theta)$ is the share of the (y, θ) -type consumer's individual demand in the aggregate demand.

Differentiating (A.12) with respect to p yields

$$\frac{\partial \varepsilon(p, N)}{\partial p} = \int_{\mathbb{R}_+ \times \Theta} \left(\frac{\partial \varepsilon(p, N; y, \theta)}{\partial p} s + \varepsilon(p, N; y, \theta) \frac{\partial s}{\partial p} \right) dG(y, \theta). \quad (\text{A.13})$$

Using (57), we obtain

$$\mathcal{E}_p(s) = \varepsilon(p, N) - \varepsilon(p, N; y, \theta).$$

Hence,

$$\frac{\partial s}{\partial p} = \frac{s}{p} [\varepsilon(p, N) - \varepsilon(p, N; y, \theta)].$$

Finally, note that

$$\int_{\mathbb{R}_+ \times \Theta} s(p, N; y, \theta) dG(y, \theta) = 1 \Rightarrow \int_{\mathbb{R}_+ \times \Theta} \frac{\partial s}{\partial p} dG(y, \theta) = 0. \quad (\text{A.14})$$

Plugging (A.14) into (A.13) and subtracting $(\varepsilon(p, N)/p) \int_{\mathbb{R}_+ \times \Theta} (\partial s / \partial p) dG(y, \theta) = 0$ from both parts of (A.13), we obtain (56). Q.E.D.