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# Birth of Cities: Division of Labor and Transportation Costs

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# Birth of Cities: Division of Labor and Transportation Costs\*

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## Abstract

In this paper, we construct a model with two-region, two sector and two types of workers. One type of workers has a comparative advantage in agricultural production and mobile between two regions, while the other type of workers has a comparative advantage in producing manufactured goods and immobile between regions. We assume that intra-regional (inter-regional) trade incurs transaction (transportation) costs. This paper shows that when the productivity of workers is low relative to the intra-regional transaction costs, division of labor is not performed and the market does not exist. In this economy, both workers are self-sufficient. When the productivity of workers becomes medium level, division of labor is progressed in a region where both types of workers exist, and the market is formed. The number of people living in this region increases and this region becomes the city. When the productivity of workers becomes high, inter-regional trade starts and the division of labor is also progressed in the other region.

JEL Classification: R10

Key words: city formation; division of labor; transportation costs; comparative advantage;

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# 1 Introduction

The purpose of this paper is to investigate how cities appear in the world and what is the elements to form cities. Bairoch (1988) points out properties of urbanization: The existence of full-time craftsmen, furnishing evidence of a division of labor which enables a population of sufficient size and, above all, density in the city. Weber (1958) thinks that in the "city", the local inhabitants satisfy an economically substantial part of their daily wants in the local market, and to an essential extent by products which the local population and that of the immediate hinterland produced for sale in the market or acquired in other ways.

Smith (1776) in book III, also points out that division of labor is progressed within cities and market are formed city creation. He explains the origins of cities as follows: "artificers, too, stand occasionally in need of the assistance of one another; and as their residence is not, like that of the farmer, necessarily tied down to a precise spot, they naturally settle in the neighbourhood of one another, and thus form a small town or village. The butcher, the brewer, and the baker, soon join them, together with many other artificers and retailers, necessary or useful for supplying their occasional wants, and who contribute still further to augment the town. The inhabitants of the town, and those of the country, are mutually the servants of one another. The town is a continual fair or market, to which the inhabitants of the country resort, in order to exchange their rude for manufactured produce. It is this commerce which supplies the inhabitants of the town, both with the materials of their work, and the means of their subsistence." Smith (1776) also notes that through international or long-distant trade, people can obtain the necessities cheaper than they produce and trade encourages division of labor. To consider international or long-distant trade, this paper incorporates the transportation costs of goods. We show that progress of division of labor, formation of markets, and international or long-distant trade are main elements in account of the origins of cities.

Followed above statements, we focus on the two properties of urbanization. In the city, there have local markets to exchange goods and people specialize to produce their comparative advantage goods. This paper constructs a two-region model to investigate the formation of cities. In this paper, a city in a region has a larger population than the other region. In the city, workers specialize to produce their comparative advantage goods. We consider two sectors; agricultural sector and manufacturing sector. In our model, there are two types of workers, farmers and craftsmen. Farmers have a comparative advantage to produce agricultural goods and craftsmen have a comparative advantage to produce manufactured goods. Both workers have fixed time allocation and they allocate their time to produce goods. Farmers can be mobile between regions and all of craftsmen live in region 1 for simplicity. We assume that when workers sell agricultural goods or manufactured goods to a region they live or the other region, workers have to incur both intra-regional transaction costs and inter-regional transportation costs to show the roles of transaction and transportation costs. The inter-regional transportation costs is higher than the intra-regional trans-

action costs. In our model, there are fixed amount of lands in each region, and agents consume lands to live in a region. Then, when number of workers in a region increases, the price of land is raised, which lowers the number of workers in a region.<sup>1</sup>

The ratio of the productivity to the transaction costs or the transportation costs determines which goods each worker produces. When the productivity is low relative to both intra-regional transaction costs and the inter-regional transportation costs, not only inter-regional trade but also intra-regional trade are costly. Then, they do not exchange the goods they produce and the economy is in self-sufficient. In this regime, farmers disperse between two regions. When the productivity becomes medium level, workers in a region where both types of workers live specialize into their comparative advantage sector, and intra-regional trade starts. With this process, some farmers immigrate to this region. Thus, a city is formed in the region where both types of workers live. The division of labor and market formation is observed, which create a city. However, in this regime, the other region where only farmers live is in the self-sufficient economy, since inter-regional transportation costs is high. When the productivity becomes sufficiently high relative to both the intra-regional transaction costs and the inter-regional transportation costs, the inter-regional trade starts, which promote the division of labor and market formation in the other region where only farmers live.

This paper can show the birth of city: the population in a region where both types of workers live increases and a city is formed, there are markets in the city, and the workers in the city specialize to produce their comparative advantage goods which Smith (1776) points. Suppose that the number of farmers relative to that of craftsmen is sufficiently large. When the productivity of craftsmen is sufficiently low relative to the intra-regional transaction costs, there is no trade and both farmers and craftsmen are self-sufficient. In this economy, the number of population in the city does not depend on the intra-regional transaction costs and inter-regional transportation costs. When the productivity of craftsmen is medium level, there exists intra-regional trade in the city where craftsmen live but there is no inter-regional trade. When the productivity of craftsmen is low relative to the intra-regional transaction costs, the farmers produce both goods and craftsmen specialize to produce the manufactured goods. In this economy, benefits of trade for farmers are small but benefits of trade for craftsmen are large. Then, the number of farmers living in the city decreases. When the productivity becomes high relative to the intra-regional transaction costs, both farmers and craftsmen specialize to their comparative advantage sectors. In this economy, intra-regional trade improves the welfare of both workers and the number of population in the city increases monotonically. When the productivity is sufficiently high relative to the inter-regional transportation costs, there exist both intra-regional and inter-regional trade. Farmers in both regions and craftsmen specialized to their comparative advantage sectors. Because there

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<sup>1</sup>Helpman (1998) showed that fixed supply of lands in a region works as dispersion force of population in a two region model.

exist intra-regional and inter regional trade and the prices of manufactured goods in two regions becomes the same, the rate of the number of population in the city to productivity of workers becomes constant. Therefore, both farmers and craftsmen specialize to produce their comparative advantage goods and the number of population in the city increases. In addition, in the city, there are markets.

Some researchers investigate the basis of formation of cities. Kanemoto (1980) shows the three elements that cities will arise: 1. Concentration of immobile factors, 2. Increasing returns to scale or indivisibility, 3. Externalities or public goods. O'sullivan (2009) points out that comparative advantage among regions makes trade between regions and inter-regional trade causes the development of market cities. Our model focuses on the role of comparative advantage of individuals, not on the country or region. We points out that comparative advantage among individuals is the basis of division of labor, which is progressed by the decreases in intra-regional transaction costs. Our paper is also related with Starrett (1978). Starrett (1978) presented "Spatial Impossibility Theorem". Fujita and Thisse (2013) shows

"Assume a two-region economy with a finite number of consumers and firms. If space is homogenous, transport is costly, and preferences are locally non-satiated, there is no competitive equilibrium involving transportation."

In our model, there is comparative advantage of individual level, which departs from the conditions supporting Spatial Impossibility Theorem. Then, in our model, we can observe the equilibrium under which agents agglomerate in a region and there are inter-regional trade.

Behrens, Duranton, and Robert-Nicoud (2010), Behrens and Robert-Nicoud (2011), Davis and Dingel (2012) construct the model to show how cities emerge with skill heterogeneity. In these models, before workers draw their productivity, both cities and workers are symmetric. In Behrens, Duranton, and Robert-Nicoud (2010) and Behrens and Robert-Nicoud (2011), highly productive workers choose to become entrepreneur to hire the unskilled labor and to live in large city because of agglomeration economies. Then, in the city where highly productive workers live, the number of population becomes large. In Davis and Dingel (2012), agents agglomerate at the large city to exchange productive ideas. When a highly productive worker live in a city, in this city there have large externality of idea exchange and this large externality attracts highly productive workers to the city.

## 2 Model

There are two regions named, 1 and 2. In each regions, there is an one-dimensional land space. The land spaces are assumed to be owned by an absentee land owner. In this model, there are two goods, agricultural goods and manufactured goods. Both agricultural goods market and manufactured goods market is perfectly competitive. Intra-regional and inter-regional trade of both goods incur 'iceberg'-type transaction costs and transportation costs, respec-

tively. We assume that there are two types of worker, farmer and craftsman and both types of worker have one unit of labor. The number of farmers is  $L$  and the number of craftsmen is 1. Both farmers and craftsmen can produce both goods. Farmers have a comparative advantage in producing agricultural goods and craftsmen have a comparative advantage in producing manufactured goods. Farmers can be mobile between regions and we assume that all of craftsmen live in region 1.<sup>2</sup> Farmers can produce  $a > 1$  unit of agricultural goods and 1 unit of manufactured goods to use one unit of labor. Craftsmen can produce 1 unit of agricultural goods and  $b > 1$  unit of manufactured goods to use one unit of labor. If a worker in a region sells agricultural goods (manufactured goods) to the same region, it must dispatch  $\tau_A > 1$  ( $\tau_M > 1$ ) units of goods. Then,  $\tau_A - 1$  ( $\tau_M - 1$ ) represents the transaction costs of agricultural goods (manufactured goods). If a worker in a region sells agricultural goods (manufactured goods) to the other country, it also must dispatch  $\tau_A^* > 1$  ( $\tau_M^* > 1$ ) units of agricultural goods (manufactured goods). We assume that  $\tau_A^* > \tau_A > 1$  and  $\tau_M^* > \tau_M > 1$ . We take the agricultural goods in region 1 as the numeraire.  $p_{A2}$  denotes the price of the agricultural goods in region 2 and  $p_{Mi}$  denotes the price of the manufactured goods in region  $i$ .

Firstly, we focus on the intra-regional trade in region 1. When  $p_{M1} < a/\tau_A$ , farmers in region 1 specialize to produce agricultural goods and sell it to craftsmen in region 1. When  $a/\tau_A < p_{M1} < a\tau_M$ , farmers in region 1 produce both goods and do not trade with craftsmen. When  $p_{M1} > a\tau_M$ , farmers in region 1 specialize to produce manufactured goods and sell it to craftsmen. On the other hand, when  $p_{M1} < 1/b\tau_A$ , craftsmen specialize to produce agricultural goods and to sell it to farmers in region 1. When  $1/b\tau_A < p_{M1} < \tau_M/b$ , craftsmen produce both goods and do not trade with farmers in region 1. When  $p_{M1} > \tau_M/b$ , craftsmen specialize to produce manufactured goods and sell it to farmers in region 1. When  $ab < \tau_A\tau_M$ , that is  $a/\tau_A < \tau_M/b$ , there is no incentives to trade between farmers and craftsmen in region 1. Whereas, when  $ab > \tau_A\tau_M$ , that is  $\tau_M/b < a/\tau_A$ , and  $\tau_M/b \leq p_{M1} \leq a/\tau_A$ , there is an incentive to trade between farmers and craftsmen in region 1.

Next, we investigate conditions that craftsmen trade with farmers in region 2. When  $p_{M2}/p_{A2} < a/\tau_A^*$ , farmers in region 2 specialize to produce agricultural goods and sell it to craftsmen. When  $a/\tau_A^* < p_{M2}/p_{A2} < a\tau_M^*$ , farmers in region 2 produce both goods and do not trade with craftsmen. When  $p_{M2}/p_{A2} > a\tau_M^*$ , farmers in region 2 specialize to produce manufactured goods and sell it to craftsmen. On the other hand, when  $p_{M2}/p_{A2} < 1/b\tau_A^*$ , craftsmen specialize to produce agricultural goods and sell it to farmers in region 2. When  $1/b\tau_A^* < p_{M2}/p_{A2} < \tau_M^*/b$ , craftsmen produce both goods and do not trade with farmers in region 2. When  $p_{M2}/p_{A2} > \tau_M^*/b$ , craftsmen specialize to produce manufactured goods and sell it to farmers in region 2. When  $ab < \tau_A^*\tau_M^*$ , that is  $a/\tau_A^* < \tau_M^*/b$ , there is no incentives to trade between farm-

<sup>2</sup>If we assume that craftsmen are mobile between region, our model can derive qualitatively similar results. However, the analysis becomes very complex. In addition, if we assume that farmers are immobile and craftsmen are mobile between regions, our results are not changed. Then, we assume that craftsmen are immobile between regions.

ers in region 2 and craftsmen . Whereas, when  $ab > \tau_A^* \tau_M^*$ , that is  $a/\tau_A^* > \tau_M^*/b$  and  $\tau_M^*/b \leq p_{M2}/p_{A2} \leq a/\tau_A^*$ , there is an incentive to trade between farmers in region 2 and craftsmen. Summarize above statement, the following proposition can be obtained:

**Proposition 1** *When  $ab < \tau_A \tau_M$ , there is no trade between farmers in both region and craftsmen. When  $\tau_A \tau_M < ab < \tau_A^* \tau_M^*$  and  $\tau_M/b \leq p_{M1} \leq a/\tau_A$ , there is an incentive to trade between farmers in region 1 and craftsmen but there is no inter-regional trade. When  $ab > \tau_A^* \tau_M^*$  and  $\tau_M^*/b \leq p_{M2}/p_{A2} \leq a/\tau_A^*$ , there is an incentive to trade between farmers in region 2 and craftsmen.*

The utility functions of farmers in region  $i$  and craftsmen are given by

$$U_i^F = (c_{Li}^F)^\alpha (c_{Mi}^F)^\beta (c_{Ai}^F)^{1-\alpha-\beta}, \quad (1)$$

$$U_1^C = (c_{L1}^C)^\alpha (c_{M1}^C)^\beta (c_{A1}^C)^{1-\alpha-\beta}, \quad (2)$$

where  $c_{Li}^j$  denotes the consumption level of land space by type  $j$  in region  $i$ ,  $c_{Mi}^j$  denotes the consumption level of manufactured goods by type  $j$  in region  $i$ , and  $c_{Ai}^j$  denotes the consumption level of land space by type  $j$  in region  $i$ . From Proposition 1, to solve the utility maximization problems, we separate three cases. The first case is that the productivity of farmers or productivity of craftsmen is sufficiently low, that is  $ab < \tau_A \tau_M$ . We name this economy as self-sufficient economy. The second case is that the productivity of farmers or productivity of craftsmen is medium, that is  $\tau_A \tau_M < ab < \tau_A^* \tau_M^*$ . We name this economy as intra-regional trade economy. The third case is that the productivity of farmers or productivity of craftsmen is sufficiently high, that is  $ab > \tau_A^* \tau_M^*$ . We name this economy as inter-trade economy.

## 2.1 Self-sufficient economy

From Proposition 1, when  $ab < \tau_A \tau_M$ , farmers in region 1 do not exchange their production to craftsmen. Therefore, in both regions, both farmers and craftsmen consume the goods that they produce. The budget constraints of farmers in region  $i$  are given by  $c_{Mi}^F = 1 - l_i^F$  and  $p_{Li} c_{Li}^F + c_{Ai}^F = a l_i^F$  where  $l_i^F$  is the time allocation of labor to produce agricultural goods by farmers in region  $i$  and  $p_{Li}$  is the price level of land space in region  $i$ . In this model, we assume that both workers in both regions pay agricultural goods for land rent. Then, from the utility maximization problems of farmers in region  $i$ , the following demand functions can be obtained:

$$p_{Li} c_{Li}^F = \alpha a, \quad (3)$$

$$c_{Mi}^F = \beta, \quad (4)$$

$$c_{Ai}^F = a(1 - \alpha - \beta). \quad (5)$$

The budget constraints of craftsmen are given by  $c_{Mi}^C = b(1 - l^C)$  and  $p_{L1}c_{L1}^C + c_{A1}^C = l^C$  where  $l^C$  is the time allocation of labor to produce agricultural goods by craftsmen. To solve the utility maximization problems of craftsmen, the demand functions are given by

$$p_{L1}c_{L1}^C = \alpha, \quad (6)$$

$$c_{M1}^C = \beta b, \quad (7)$$

$$c_{A1}^C = 1 - \alpha - \beta. \quad (8)$$

In the self-sufficient economy, the land market equilibrium conditions in both regions are given by

$$c_{L1}^C + L_1c_{L1}^F = 1, \quad (9)$$

$$L_2c_{L2}^F = 1, \quad (10)$$

where  $L_i$  is the number of farmers living in region  $i$  and  $L_1 + L_2 = L$  holds. From (9) and (10), the price levels of land space in both regions become

$$p_{L1} = \alpha(1 + aL_1), \quad (11)$$

$$p_{L2} = \alpha aL_2. \quad (12)$$

Then, using price levels of land space in both regions, the utility levels of farmers in both regions are given by

$$U_1^F = \left(\frac{\alpha a}{p_{L1}}\right)^\alpha (\beta)^\beta (a(1 - \alpha - \beta))^{1-\alpha-\beta},$$

$$U_2^F = \left(\frac{\alpha a}{p_{L2}}\right)^\alpha (\beta)^\beta (a(1 - \alpha - \beta))^{1-\alpha-\beta}.$$

In this model, we assume that farmers can be mobile between regions freely. Therefore, in the equilibrium, the utility levels of farmers in both regions must be equal. Therefore,  $p_{L1} = p_{L2}$  holds in the equilibrium. From (9) and (10), the numbers of farmers in both regions can be obtained as follows:

$$L_1^S = \frac{aL - 1}{2a}, \quad (13)$$

$$L_2^S = \frac{aL + 1}{2a}. \quad (14)$$

When  $L < 1/a \equiv \underline{L}^S$  holds, all of farmers live in region 2. From (13) and (14), the number of farmers in region 1 is smaller than that in region 2. The number of population in region 1 is larger than that in region 2.<sup>3</sup>

<sup>3</sup>Subtracting from the number of farmer in region 1 plus the number of craftsmen, to the number of farmer in region 2,  $1 + L_1^S - L_2^S = (a - 1)/a > 0$ , because  $a > 1$ . Then, population in region 1 is larger than the population in region 2.



## 2.2 Intra-regional trade economy

In this economy, from Proposition 1, there exists trade between farmers in region 1 and craftsmen and there does not exist trade between farmers in region 2 and craftsmen. In this economy, we firstly focus on the equilibrium price of manufactured goods in region 1 that is in  $\tau_M/b < p_{M1} < a/\tau_A$  and then we investigate the equilibrium price of manufactured goods that is  $p_{M1} = \tau_M/b$  or  $p_{M1} = a/\tau_A$ .

### 2.2.1 The case of $\tau_M/b < p_{M1} < a/\tau_A$

When equilibrium price of manufactured goods is in  $\tau_M/b < p_{M1} < a/\tau_A$ , farmers in region 1 specialize to produce the agricultural goods and craftsmen specialize to produce the manufactured goods. Then, farmers in region 1 trade the agricultural goods for the manufactured goods with craftsmen. In this economy, the budget constraint of farmers in region 1 is  $p_{M1}c_{M1}^F = (a - p_{L1}c_{L1}^F - c_{A1}^F)/\tau_A$ . From the utility maximization problem of farmers in region 1, the demand functions of each goods are given by

$$p_{L1}c_{L1}^F = \alpha a, \quad (15)$$

$$p_{M1}c_{M1}^F = \frac{\beta a}{\tau_A}, \quad (16)$$

$$c_{A1}^F = (1 - \alpha - \beta)a. \quad (17)$$

Next, we solve the utility maximization problem of craftsmen. The budgets constraints of craftsmen is  $p_{L1}c_{L1}^C + c_{A1}^C = p_{M1}(b - c_{M1}^C)/\tau_M$ . Then, the demand functions of craftsmen can be obtained as follows:

$$p_{L1}c_{L1}^C = \frac{\alpha p_{M1}b}{\tau_M}, \quad (18)$$

$$c_{M1}^C = \beta b, \quad (19)$$

$$c_{A1}^C = (1 - \alpha - \beta)\frac{p_{M1}b}{\tau_M}. \quad (20)$$

The demand functions of farmers in region 2 are the same to the demand functions of farmers in Self-sufficient economy. In this economy, the equilibrium condition of land space in region 1 is given by

$$c_{L1}^C + L_1c_{L1}^F = 1. \quad (21)$$

From (15), (18), and (21), the price of land space in region 1 is given by

$$p_{L1} = \alpha \left[ \frac{p_{M1}b}{\tau_M} + aL_1 \right]. \quad (22)$$

The equilibrium condition of manufactured goods in region 1 is

$$\frac{b - c_{M1}^C}{\tau_M} = L_1c_{M1}^F. \quad (23)$$

Substituting (16) and (19) into (23), the price level of manufactured goods in region 1 can be obtained as follows:

$$p_{M1} = \frac{\tau_M}{\tau_A} \frac{\beta}{1-\beta} \frac{a}{b} L_1. \quad (24)$$

Substituting (24) into (22), the land price can be rewritten as follows:

$$p_{L1} = \alpha a L_1 \left[ 1 + \frac{\beta}{(1-\beta)\tau_A} \right].$$

From (15), (16), and (17), the utility levels of farmers in two regions are given by

$$U_1^F = \left( \frac{\alpha a}{p_{1L}} \right)^\alpha \left( \frac{\beta a}{\tau_A p_{M1}} \right)^\beta ((1-\alpha-\beta)a)^{1-\alpha-\beta},$$

$$U_2^F = \left( \frac{\alpha a}{p_{L2}} \right)^\alpha (\beta)^\beta (a(1-\alpha-\beta))^{1-\alpha-\beta},$$

where the utility level of farmers in region 2 is the same to that in self-sufficient economy. Because farmers can be mobile between two regions freely, the utility levels in both regions are equalized. Then, from  $U_1^F = U_2^F$ , we can obtain the following equation:

$$L_2 = \left( 1 + \frac{\beta}{(1-\beta)\tau_A} \right) \left( \frac{1-\beta}{\beta} \frac{b}{\tau_M} \right)^{-\frac{\beta}{\alpha}} L_1^{\frac{\alpha+\beta}{\alpha}} \equiv \phi(L_1).$$

$\phi(0) = 0$ ,  $\phi(L_1)$  is increasing in  $L_1$ , and convex function.<sup>4</sup> From  $L_1 + L_2 = 1$  and  $L_2 = \phi(L_1)$ , the equilibrium number of population of farmers in each regions  $L_1^{NT}$  and  $L_2^{NT}$  are determined uniquely in Figure 1. In the equilibrium that both farmers and craftsmen specialize to produce one type of goods, the price of manufactured goods is in  $\tau_M/b < p_{M1} < a/\tau_A$ . Then, from (24), the condition that both farmers and craftsmen specialize to produce one type of goods is given by

$$\frac{1-\beta}{\beta} \frac{\tau_A}{a} < L_1^{NT} < \frac{1-\beta}{\beta} \frac{b}{\tau_M}.$$

To rewrite this condition using  $L_2^{NT} = \phi(L_1^{NT})$  and  $\phi(L_1)$  is increasing in  $L_1$ ,  $\phi(\frac{1-\beta}{\beta} \frac{\tau_A}{a}) < L_2^{NT} < \phi(\frac{1-\beta}{\beta} \frac{b}{\tau_M})$  holds. Then, from this inequality, we can obtain

<sup>4</sup>We can obtain first and second derivative equations of  $\phi(L_1)$  as follows:

$$\phi'(L_1) = \frac{\alpha+\beta}{\alpha} \left( 1 + \frac{\beta}{(1-\beta)\tau_A} \right) \left( \frac{1-\beta}{\beta} \frac{b}{\tau_M} \right)^{-\frac{\beta}{\alpha}} L_1^{\frac{\beta}{\alpha}} > 0,$$

$$\phi''(L_1) = \frac{\alpha+\beta}{\alpha} \frac{\beta}{\alpha} \left( 1 + \frac{\beta}{(1-\beta)\tau_A} \right) \left( \frac{1-\beta}{\beta} \frac{b}{\tau_M} \right)^{-\frac{\beta}{\alpha}} L_1^{-\frac{\beta-\alpha}{\alpha}} > 0.$$

Then,  $\phi(L_1)$  is increasing in  $L_1$  and convex function.

the following equation:<sup>5</sup>

$$\hat{L}^{NT} < L < \tilde{L}^{NT},$$

where

$$\hat{L}^{NT} \equiv \frac{1-\beta}{\beta} \frac{\tau_A}{a} \left[ 1 + \left( 1 + \frac{\beta}{(1-\beta)\tau_A} \right) \left( \frac{\tau_A \tau_M}{ab} \right)^{\frac{\beta}{\alpha}} \right], \quad (25)$$

$$\tilde{L}^{NT} \equiv \frac{1-\beta}{\beta} \frac{b}{\tau_M} \left[ 2 + \frac{\beta}{(1-\beta)\tau_A} \right]. \quad (26)$$

When  $ab = \tau_A \tau_M$ ,  $\hat{L}^{NT} = \tilde{L}^{NT}$  holds. From (25),  $\partial \hat{L}^{NT} / \partial a < 0$ ,  $\partial \hat{L}^{NT} / \partial b < 0$ ,  $\partial \hat{L}^{NT} / \partial \tau_A > 0$ , and  $\partial \hat{L}^{NT} / \partial \tau_M > 0$ . From (26),  $\partial \tilde{L}^{NT} / \partial a = 0$ ,  $\partial \tilde{L}^{NT} / \partial b > 0$ ,  $\partial \tilde{L}^{NT} / \partial \tau_A < 0$ , and  $\partial \tilde{L}^{NT} / \partial \tau_M < 0$ . Because in intra-regional trade economy,  $ab > \tau_A \tau_M$  holds,  $\hat{L}^{NT} < \tilde{L}^{NT}$ . When  $\underline{L}^{NT} < L < \hat{L}^{NT}$ ,<sup>6</sup>  $p_{M1} = \tau_M/b$  holds and craftsmen are indifferent between obtaining the agricultural goods from the market and producing it themselves. When  $L > \tilde{L}^{NT}$ ,  $p_{M1} = a/\tau_A$  holds and farmers in region 1 are indifferent between buying manufactured goods from the market and producing it themselves.<sup>7</sup>

### 2.2.2 The cases of $p_{M1} = \tau_M/b$ or $p_{M1} = a/\tau_A$

When  $p_{M1} = \tau_M/b$ , that is  $\underline{L}^{NT} < L < \hat{L}^{NT}$ , craftsmen are indifferent between buying agricultural goods from the market and producing it themselves. Farmers in region 1 specialize to produce agricultural goods. Because the behavior of farmers in region 1 is same as the equilibrium when  $\tau_M/b < p_{M1} < a/\tau_A$ , we focus on the behavior of craftsmen. The budget constraint of craftsmen is given by  $p_{L1} c_{L1}^C + c_{A1}^C = l^C + p_{M1} [b(1-l^C) - c_{M1}^C] = 1 - c_{M1}^C/b$ . Then, from the utility maximization problems, the demand functions of craftsmen become

$$p_{L1} c_{L1}^C = \alpha,$$

<sup>5</sup>We can rewrite the left inequality of  $\phi(\frac{1-\beta}{\beta} \frac{\tau_A}{a}) < L_2^{NT}$  to use  $L_1^{NT} > \frac{1-\beta}{\beta} \frac{\tau_A}{a}$  and  $L_1^{NT} + L_2^{NT} = L$  as follows:

$$\begin{aligned} L - L_1^{NT} &> \phi\left(\frac{1-\beta}{\beta} \frac{\tau_A}{a}\right) \\ L &> \frac{1-\beta}{\beta} \frac{\tau_A}{a} + \phi\left(\frac{1-\beta}{\beta} \frac{\tau_A}{a}\right) \hat{L}^{NT} \equiv \hat{L}^{NT}. \end{aligned}$$

We can also rewrite the right inequality of  $L_2^{NT} < \phi(\frac{1-\beta}{\beta} \frac{b}{\tau_M})$  to use  $L_1^{NT} < \frac{1-\beta}{\beta} \frac{b}{\tau_M}$  and  $L_1^{NT} + L_2^{NT} = L$  as follows:

$$\begin{aligned} L - L_1^{NT} &< \phi\left(\frac{1-\beta}{\beta} \frac{b}{\tau_M}\right) \\ L &< \frac{1-\beta}{\beta} \frac{b}{\tau_M} + \phi\left(\frac{1-\beta}{\beta} \frac{b}{\tau_M}\right) \tilde{L}^{NT}. \end{aligned}$$

<sup>6</sup>We show the definition of  $\underline{L}^{NT}$  later. Intuitively, when the number of farmers is sufficiently small, all of farmers prefer to live in region 2 and there is no farmers in region 1.

<sup>7</sup>When  $L < \underline{L}^{NT}$ , the farmers prefer to live in region 2. Then, there is no farmer in region 1 and there is no trade between farmers and craftsmen in region 1.

$$c_{M1}^C = \beta b,$$

$$c_{A1}^C = 1 - \alpha - \beta.$$

The land market equilibrium condition in region 1 is given by

$$c_{L1}^C + L_1 c_{L1}^F = 1.$$

To rewrite above equation, the land price in region 1 becomes

$$p_{L1} = \alpha(1 + aL_1).$$

Then, the utility levels of farmers in both regions are

$$U_1^F = \left( \frac{\alpha a}{p_{L1}} \right)^\alpha \left( \frac{\beta a b}{\tau_A \tau_M} \right)^\beta ((1 - \alpha - \beta)a)^{1 - \alpha - \beta},$$

$$U_2^F = \left( \frac{\alpha a}{p_{L2}} \right)^\alpha (\beta)^\beta (a(1 - \alpha - \beta))^{1 - \alpha - \beta}.$$

Because the farmers can be mobile between regions freely, the utility levels become same. From  $U_1^F = U_2^F$  and  $L_1 + L_2 = 1$ , the numbers of farmers in both regions are given by

$$L_1 = \frac{L - \frac{1}{a} \left( \frac{\tau_A \tau_M}{ab} \right)^\frac{\beta}{\alpha}}{1 + \left( \frac{\tau_A \tau_M}{ab} \right)^\frac{\beta}{\alpha}}, \quad (27)$$

$$L_2 = \frac{L \left( \frac{\tau_A \tau_M}{ab} \right)^\frac{\beta}{\alpha} + \frac{1}{a} \left( \frac{\tau_A \tau_M}{ab} \right)^\frac{\beta}{\alpha}}{1 + \left( \frac{\tau_A \tau_M}{ab} \right)^\frac{\beta}{\alpha}}. \quad (28)$$

When  $L < \underline{L}^{NT} \equiv \frac{1}{a} \left( \frac{\tau_A \tau_M}{ab} \right)^\frac{\beta}{\alpha}$ , farmers prefer to live in region 2 and all of farmers inhabit region 2. When  $L < \underline{L}^{NT}$ , because the number of farmers is scarce, they can consume larger land space to live in region 2 where it is cheaper than to make manufactured goods by themselves to buy it from craftsmen. Therefore, all of farmers live in region 2.

Next, we investigate the behavior of the farmers when  $L > \tilde{L}^{NT}$  holds, that is  $p_{M1} = a/\tau_A$ . When  $L > \tilde{L}^{NT}$ , farmers in region 1 are indifferent between buying manufactured goods from the market and producing it themselves. Because the behavior of craftsmen are same as the equilibrium when  $\tau_M/b < p_{M1} < a/\tau_A$ , we focus on the behavior of farmers in region 1. The budget constraint of farmers in region 1 is given by  $\tau_A p_{M1} [c_{M1}^F - (1 - l_1^F)] = a l_1^F - p_{L1} c_{L1}^F - c_{A1}^F$ , that is,  $a c_{M1}^F = a - p_{L1} c_{L1}^F - c_{A1}^F$ . Then, from the utility maximization problems, the demand functions of farmers in region 1 become

$$p_{L1} c_{L1}^F = \alpha a,$$

$$c_{M1}^F = \beta,$$

$$c_{A1}^F = a(1 - \alpha - \beta).$$

The land market equilibrium condition in region 1 is given by

$$p_{L1} = \alpha a \left( L_1 + \frac{b}{\tau_A \tau_M} \right).$$

Then, the utility levels of farmers in both regions are

$$U_1^F = \left( \frac{\alpha a}{p_{L1}} \right)^\alpha (\beta)^\beta ((1 - \alpha - \beta)a)^{1 - \alpha - \beta},$$

$$U_2^F = \left( \frac{\alpha a}{p_{L2}} \right)^\alpha (\beta)^\beta (a(1 - \alpha - \beta))^{1 - \alpha - \beta}.$$

From  $U_1^F = U_2^F$ , then  $p_{L1} = p_{L2}$  holds. The numbers of farmers in both regions are

$$L_1 = \frac{1}{2} \left( L - \frac{b}{\tau_A \tau_M} \right), \quad (29)$$

$$L_2 = \frac{1}{2} \left( L + \frac{b}{\tau_A \tau_M} \right). \quad (30)$$

Summarizing Intra-regional trade economy, we can obtain the following proposition.

**Proposition 2** *Suppose that  $\tau_A \tau_M < ab < \tau_A^* \tau_M^*$ .*

1. *When  $L < \underline{L}^{NT}$ , all of farmers live in region 2 and both farmers and craftsmen produce both goods.*
2. *When  $\underline{L}^{NT} < L < \hat{L}^{NT}$ , farmers in region 1 specialize to produce agricultural goods and craftsmen produce both goods.*
3. *When  $\hat{L}^{NT} < L < \check{L}^{NT}$ , price of manufactured goods in region 1 is in  $\tau_M/b < p_{M1} < a/\tau_A$ , farmers in region 1 specialize to produce agricultural goods and craftsmen specialize to produce manufactured goods.*
4. *When  $L > \check{L}^{NT}$ , farmers in region 1 produce both goods and craftsmen specialize to produce manufactured goods.*

### 2.3 Inter-regional trade economy

When  $ab > \tau_A^* \tau_M^*$  holds, craftsmen may exchange the manufactured goods for agricultural goods produced by farmers in region 2. When craftsmen supply the manufactured goods to both regions, the relative price of manufactured goods in region 2 is given by

$$\frac{p_{M1}}{\tau_M} = \frac{p_{M2}}{\tau_M^* p_{A2}}. \quad (31)$$

To use (31), we can rewrite the condition that craftsmen supply their manufactured goods to region 2 as follows:

$$\frac{\tau_M}{b} \leq p_{M1} \leq \frac{\tau_M a}{\tau_A^* \tau_M^*}.$$

From the results of Intra-regional trade economy, when  $ab > \tau_A^* \tau_M^*$  holds, we can obtain the following proposition:

**Proposition 3** Suppose that  $ab > \tau_A^* \tau_M^*$ .

1. When  $p_{M1} = \frac{\tau_M}{b}$ , craftsmen produce both goods and farmers in both regions specialize to produce agricultural goods.

2. When  $\frac{\tau_M}{b} < p_{M1} < \frac{\tau_M a}{\tau_A^* \tau_M^*}$ , craftsmen specialize to produce manufactured goods and farmers in both regions specialize to produce agricultural goods.

3. When  $\frac{\tau_M a}{\tau_A^* \tau_M^*} \leq p_{M1} < \frac{a}{\tau_A}$ , there is no trade between region 1 and 2 and craftsmen specialize to produce manufactured goods. Farmers in region 1 specialize to produce agricultural goods and farmers in region 2 produce both goods.

4. When  $p_{M1} = \frac{a}{\tau_A}$ , craftsmen specialize to produce manufactured goods and farmers in both regions produce both goods.

From Proposition 3, we separate four cases. The cases of 3 and 4 are the same to Intra-regional trade economy. Then, we focus on the cases of 1 and 2.

### 2.3.1 The case of $\frac{\tau_M}{b} < p_{M1} \leq \frac{\tau_M a}{\tau_A^* \tau_M^*}$

When  $\frac{\tau_M}{b} < p_{M1} \leq \frac{\tau_M a}{\tau_A^* \tau_M^*}$  holds, from Proposition 3, craftsmen specialize to produce manufactured goods and sell their goods for farmers living in both regions. Farmers in both region 1 and 2 also specialize to produce agricultural goods and exchange the agricultural goods for manufactured goods. The budget constraint of craftsmen is given by  $p_{L1} c_{L1}^C + c_{A1}^C = p_{M1}(b - c_{M1}^C)/\tau_M$ . From the utility maximization problem, the demand functions can be obtained as follows:

$$p_{L1} c_{L1}^C = \frac{\alpha p_{M1} b}{\tau_M}, \quad (32)$$

$$c_{M1}^C = \beta b, \quad (33)$$

$$c_{A1}^C = (1 - \alpha - \beta) \frac{p_{M1} b}{\tau_M}. \quad (34)$$

The utility maximization problems of farmers in region 1 are the same to Intra-regional trade economy. Then, the demand functions of farmers in region 1 are (15), (16), and (17). The budget constraint of farmers in region 2 is given by  $c_{M2}^F = \frac{\tau_M}{\tau_A^* \tau_M^* p_{M1}} (a - p_{L2} c_{L2}^F - c_{A2}^F)$ . Then, the demand functions of farmers in region 2 are given by

$$p_{L2} c_{L2}^F = \alpha a, \quad (35)$$

$$c_{M2}^F = \frac{\tau_M \beta a}{\tau_A^* \tau_M^* p_{M1}}, \quad (36)$$

$$c_{A1}^F = (1 - \alpha - \beta) a. \quad (37)$$

The market equilibrium condition of manufactured goods becomes as follows:

$$b - c_{M1}^C = \tau_M L_1 c_{M1}^F + \tau_M^* L_2 c_{M2}^F,$$

where left-hand side represents the supply of manufactured goods and the right-hand side represents the demand of manufactured goods. Then, rewriting above

equation, we can obtain the price level of manufactured goods in region 1 as follows:

$$p_{M1} = \frac{\tau_M \beta a}{(1-\beta)b} \left( \frac{L_1}{\tau_A} + \frac{L_2}{\tau_A^*} \right). \quad (38)$$

The market equilibrium conditions of land space in both regions are given by

$$c_{L1}^C + L_1 c_{L1}^F = 1, \quad (39)$$

$$L_2 c_{L2}^F = 1. \quad (40)$$

Substituting (15), (32), and (38) into (39), we can obtain the price of land space in region 1 as follows:

$$p_{L1} = \alpha a \left[ L_1 + \frac{\beta}{1-\beta} \left( \frac{L_1}{\tau_A} + \frac{L_2}{\tau_A^*} \right) \right]. \quad (41)$$

Substituting (35) into (40), the price of land space in region 2 is given by

$$p_{L2} = \alpha a L_2. \quad (42)$$

Then, the utility levels of farmers in both regions become as follows:

$$U_1^F = \left( \frac{\alpha a}{p_{L1}} \right)^\alpha \left( \frac{\beta a}{\tau_A p_{M1}} \right)^\beta ((1-\alpha-\beta)a)^{1-\alpha-\beta},$$

$$U_2^F = \left( \frac{\alpha a}{p_{L2}} \right)^\alpha \left( \frac{\tau_M}{\tau_A^* \tau_M^*} \frac{\beta a}{p_{M1}} \right)^\beta (a(1-\alpha-\beta))^{1-\alpha-\beta}.$$

Because the farmers can be mobile between regions, the utility levels in both regions must be equalized. Then, from  $U_1^F = U_2^F$ , following equation can be obtained:

$$L_2 = \frac{1 + \frac{\beta}{(1-\beta)\tau_A}}{\gamma - \frac{\beta}{(1-\beta)\tau_A^*}} L_1 \equiv \psi(L_1), \quad (43)$$

where  $\gamma \equiv \left( \frac{\tau_A^* \tau_M^*}{\tau_A \tau_M} \right)^\frac{\beta}{\alpha} > 1$ . We assume  $\gamma > \frac{\beta}{(1-\beta)\tau_A^*}$ .<sup>8</sup> Because total number of farmers is unity, the equilibrium numbers of farmers in both regions are determined uniquely in Figure 2. The equilibrium numbers of farmers in both regions of  $L_1^T$  and  $L_2^T$  are given by

$$L_1^T = \frac{\gamma - \frac{\beta}{(1-\beta)\tau_A}}{1 + \gamma + \frac{\beta(\tau_A^* - \tau_A)}{(1-\beta)\tau_A \tau_A^*}} L, \quad (44)$$

$$L_2^T = \frac{1 + \frac{\beta}{(1-\beta)\tau_A}}{1 + \gamma + \frac{\beta(\tau_A^* - \tau_A)}{(1-\beta)\tau_A \tau_A^*}} L. \quad (45)$$

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<sup>8</sup>If  $\gamma \leq \frac{\beta}{(1-\beta)\tau_A^*}$ , all of farmers prefer to locate in region 2.

Substituting (44) and (45) into (38), we can obtain the equilibrium price level of manufactured goods in region 1 as follows:

$$p_{M1} = \frac{\beta}{1-\beta} \frac{\tau_M a}{b} \frac{\frac{\gamma}{\tau_A} + \frac{1}{\tau_A^*}}{1 + \gamma + \frac{\beta(\tau_A^* - \tau_A)}{(1-\beta)\tau_A\tau_A^*}} L. \quad (46)$$

From Proposition 3, the equilibrium that craftsmen and farmers in both regions specialize to produce their comparative advantage goods is  $\frac{\tau_M}{b} < p_{M1} \leq \frac{\tau_M a}{\tau_A^* \tau_M^*}$ . To use the equilibrium price level of manufactured goods in region 1, we can rewrite the condition as follows:

$$\hat{L}^T < L < \tilde{L}^T,$$

where

$$\hat{L}^T \equiv \frac{1-\beta}{\beta a} \frac{(1+\gamma)\tau_A\tau_A^* + \frac{\beta}{1-\beta}(\tau_A^* - \tau_A)}{\gamma\tau_A^* + \tau_A}, \quad (47)$$

$$\tilde{L}^T \equiv \frac{1-\beta}{\beta} \frac{b}{\tau_A\tau_A^*} \frac{(1+\gamma)\tau_A\tau_A^* + \frac{\beta}{1-\beta}(\tau_A^* - \tau_A)}{\gamma\tau_A^* + \tau_A}. \quad (48)$$

When  $ab = \tau_A^* \tau_M^*$  holds,  $\hat{L}^T = \tilde{L}^T$ . From (47),  $\partial \hat{L}^T / \partial a < 0$ , and  $\partial \hat{L}^T / \partial b = 0$ . From (48),  $\partial \tilde{L}^T / \partial a = 0$ , and  $\partial \tilde{L}^T / \partial b > 0$ . When  $L > \hat{L}^T$  holds, craftsmen specialize to produce manufactured goods. When  $L \leq \hat{L}^T$  holds, craftsmen produce both goods. When  $L < \tilde{L}^T$  holds, farmers in region 2 specialize to produce agricultural goods and exchange agricultural goods for manufactured goods produced by craftsmen. When  $L \geq \tilde{L}^T$  holds, farmers in region 2 do not trade with craftsmen in region 1 and region 2 is self-sufficient economy. Therefore, when  $L \geq \tilde{L}^T$  holds, the economy becomes intra-regional trade economy in which farmers and craftsmen in region 1 trade with each other while region 2 is self-sufficient economy.

### 2.3.2 The case of $p_{M1} = \tau_M/b$

When  $p_{M1} = \tau_M/b$  that is  $L \leq \hat{L}^T$ , craftsmen produce both goods and farmers in both regions specialize to produce agricultural goods. The budget constraint of craftsmen is  $p_{L1}c_{L1}^C + c_{A1}^C = l^C + \frac{p_{M1}}{\tau_M}[b(1-l^C) - c_{M1}^C] = 1 - \frac{c_{M1}^C}{b}$ . Then, from the utility maximization problem, the demand functions of craftsmen are given by

$$p_{L1}c_{L1}^C = \alpha, \quad (49)$$

$$c_{M1}^C = \beta b, \quad (50)$$

$$c_{A1}^C = 1 - \alpha - \beta. \quad (51)$$

To use (15) and (49), the land space equilibrium condition in region 1 becomes as follows:

$$p_{L1} = \alpha(1 + aL_1). \quad (52)$$



Then, the utility levels of farmers in both regions become as follows:

$$U_1^F = \left( \frac{\alpha a}{p_{1L}} \right)^\alpha \left( \frac{\beta a}{\tau_{APM1}} \right)^\beta ((1 - \alpha - \beta)a)^{1-\alpha-\beta},$$

$$U_2^F = \left( \frac{\alpha a}{p_{L2}} \right)^\alpha \left( \frac{\tau_M}{\tau_A^* \tau_M^*} \frac{\beta a}{p_{M1}} \right)^\beta (a(1 - \alpha - \beta))^{1-\alpha-\beta}.$$

Because the farmers can be mobile between regions, the utility levels in both regions should be equalized. Then, substituting (42) and (52) into  $U_1^F = U_2^F$ , following equation can be obtained:

$$L_2 = \frac{1 + aL_1}{\alpha\gamma}. \quad (53)$$

Because  $L_1 + L_2 = 1$ , the numbers of farmers living in region 1 and 2 are given by

$$L_1 = \frac{1}{a} \frac{a\gamma L - 1}{1 + \gamma}, \quad (54)$$

$$L_2 = \frac{1}{a} \frac{aL + 1}{1 + \gamma}. \quad (55)$$

When  $L < \underline{L}^T \equiv \frac{1}{a\gamma}$ , all of farmers prefer to live in region 2. Summarizing Inter-regional trade economy, we can obtain the following proposition.

**Proposition 4** *Suppose that  $ab > \tau_A^* \tau_M^*$ . When  $L < \tilde{L}^T$ , there exist both intra-regional and inter-regional trade. When  $L > \tilde{L}^T$ , the economy becomes intra-regional trade economy in which farmers and craftsmen in region 1 trade with each other while region 2 is self-sufficient economy.*

1. *When  $L < \underline{L}^T$ , all of farmers live in region 2 and specialize to produce agricultural goods, and craftsmen produce both goods.*

2. *When  $\underline{L}^T < L < \hat{L}^T$ , farmers in both regions specialize to produce agricultural goods and craftsmen produce both goods.*

3. *When  $\hat{L}^T < L < \tilde{L}^T$ , price of manufactured goods in region 1 is in  $\tau_M/b < p_{M1} \leq \tau_M a / \tau_A^* \tau_M^*$ , farmers in both regions specialize to produce agricultural goods and craftsmen specialize to produce manufactured goods.*

### 3 Development of the technology of agricultural goods

In this section, we investigate the development of the technology of agricultural goods given the technology of manufactured goods, transaction costs, and transportation costs. Then, we can depict Figures 3, 4, and 5 to use (25), (26), (47), and (48).<sup>9</sup> In Figures 3, 4, and 5, there are eight areas. When  $a < \tau_A \tau_M / b$ ,

<sup>9</sup>In the Appendix, we explain the relationship between  $\hat{L}^T, \tilde{L}^T, \hat{L}^{NT}, \tilde{L}^{NT}, \underline{L}^{NT}$ , and  $\underline{L}^T$  in the space of  $a$  and  $L$ .

the economy prevails in self-sufficient economy. When  $\tau_A \tau_M / b < a < \tau_A^* \tau_M^* / b$ , the economy prevails in intra-trade economy. When  $a > \tau_A^* \tau_M^* / b$ , the economy prevails in inter-regional trade economy. The area of A1 represents the economy that there is no trade and in both regions, both farmers and craftsmen consume the goods that they produce. In the area of A2, the number of farmers is larger than  $\tilde{L}^{NT}$ . Then, in the area of A2, farmers in both regions produce both goods and craftsmen specialize to produce manufactured goods. In the area of A3, there exists intra-regional trade but there is no inter-regional trade. In the area of A3, the number of farmers is between  $\tilde{L}^{NT}$  and  $\tilde{L}^{NT}$ . In this area, both farmers in region 1 and craftsmen specialize to produce their comparative advantage goods and farmers in region 2 produce both goods. In the area of A4, the number of farmers is less than  $\tilde{L}^{NT}$  and the economy is intra-regional trade economy. In this area, farmers in region 1 specialize to produce agricultural goods and craftsmen produce both goods. Farmers in region 2 produce both goods and they are self-sufficient. In area A5, the number of farmers is between  $\tilde{L}^T$  and  $\tilde{L}^T$ . Then, farmers in both regions and craftsmen specialize to produce their comparative advantage goods. In this area, there exist both intra-regional trade and inter-regional trade. In the area of A6, the number of farmers is less than  $\tilde{L}^T$ . In this area, farmers in both regions specialize to produce agricultural goods and craftsmen produce both goods. In this area, there also exist both intra-regional trade and inter-regional trade. In area of A7, the economy is self-sufficient economy and the number of farmers is less than  $\underline{L}^S$ . Therefore, in area of A7, there is no farmers in region 1. In areas of A8 and A9, because farmers prefer to live in region 2, there are not any farmers in region 1. Therefore, there is no intra-regional trade. However, in area of A9, there exists inter-regional trade between craftsmen and farmers in region 2.

When the productivity of farmers is low ( $a$  is low), the economy is the self-sufficient economy (A1 or A7). In A7, there are not any farmers in region 1. If there exists sufficiently large amount of farmers, the economy can switch from self-sufficient economy to intra-regional trade economy (A2 or A3 or A4) with the improvement of the technology of farmers for agriculture. In the intra-regional trade economy, farmers live both in region 1 and 2. With the development of agricultural technology, some farmers migrate from region 2 to 1 and start to trade with craftsmen in region 1. In this area, technological improvement of farmers for agriculture induces the inter-regional migration of farmers from region 2 to 1. The farmers in region 1 specialize to agriculture and trade with craftsmen who produce manufactured goods. In self-sufficient economy (A1 or A7), farmers and craftsmen do not live in the same region, since the land price in the region is raised with increase in population in the region. In the intra-regional trade economy (A2 or A3 or A4), some farmers start to live with craftsmen in the same region and trade with each other. Technological improvement of farmers in agriculture induces farmers to specialize in agriculture and start trade with craftsmen. Since inter-regional trade incurs more transportation costs, farmers start to live in the same region to craftsmen to start inter-regional trade. In this process, we can see the city formation. Improvement agricultural technology enables farmers to overcome the increase

in land price of population concentration in a region.

When the level of agricultural technology is raised further, the economy come to be in area  $A5$ ,  $A6$ , when the population of farmers is in intermediate range. In area  $A5$ ,  $A6$ , the economy is in the inter-regional trade economy. In these areas, farmers in both regions specialize to agriculture and trade with craftsmen. The improvement of agricultural technology enables farmers who live in region 2 to trade with craftsmen in region 1. The technological progress of agriculture makes the inter-regional trade possible.

## 4 Development of the technology of manufactured goods

In this section, we investigate the development of the of manufacturing technology of craftsmen given the agricultural technology, transaction costs, and transportation costs. Then, we can depict Figures 6, 7, and 8 to use (25), (26), (47), and (48).<sup>10</sup> In Figures 6, 7, and 8, there are eight areas. When  $b < \tau_A \tau_M / a$ , the economy prevails in the self-sufficient economy. When  $\tau_A \tau_M / a < b < \tau_A^* \tau_M^* / a$ , the economy prevails in the intra-trade economy. When  $b > \tau_A^* \tau_M^* / a$ , the economy prevails in the inter-trade economy. The area of  $B1$  represents the economy that there is no trade and in both regions, both farmers and craftsmen consume the goods that they produce. The area of  $B2$  is that the number of farmers is larger than  $\tilde{L}^{NT}$ . Then, In the area of  $B2$ , farmers in both regions produce both goods and craftsmen specialize to produce manufactured goods. In the area of  $B2$ , there exists intra-regional trade but there is no inter-regional trade. The area of  $B3$  is that the number of farmers is between  $\hat{L}^{NT}$  and  $\tilde{L}^{NT}$ . In this area, both farmers in region 1 and craftsmen specialize to produce their comparative advantage goods and farmers in region 2 produce both goods. The area of  $B4$  represents that the number of farmers is less than  $\hat{L}^{NT}$  and the economy is intra-regional trade economy. In this area, farmers in region 1 specialize to produce agricultural goods and craftsmen produce both goods. farmers in region 2 produce both goods and they are self-sufficient. In area  $B5$ , the number of farmers is between  $\hat{L}^T$  and  $\tilde{L}^T$ . Then, farmers in both regions and craftsmen specialize to produce their comparative advantage goods. In this area, there exist both intra-regional trade and inter-regional trade. The area of  $B6$ , the number of farmers is less than  $\hat{L}^T$ . In this area, farmers in both regions specialize to produce agricultural goods and craftsmen produce both goods. In this area, there also have both intra-regional trade and inter-regional trade. In area of  $B8$ , the economy is self-sufficient economy and the number of farmers is less than  $\underline{L}^S$ . Therefore, in area of  $B8$ , there is no farmers in region 1. In areas of  $B8$  and  $B9$ , because farmers prefer to live in region 2, there is no farmers in

<sup>10</sup>In the Appendix, we explain the relationship between  $\hat{L}^T, \tilde{L}^T, \hat{L}^{NT}, \tilde{L}^{NT}, \underline{L}^{NT}$ , and  $\underline{L}^T$  in the space of  $b$  and  $L$ .

region 1. Therefore, there is no intra-regional trade. However, in area of B9, there exists inter-regional trade between craftsmen and farmers in region 2.

From Figures 6, 7, and 8, we investigate the relationship between the number of farmers in region 1 and the technology of manufactured goods. We focus on the case in Figure 6 when  $\beta < \beta_3$  holds. Suppose that  $L > \frac{(1-\beta)\tau_A}{\beta a}(2 + \frac{\beta}{(1-\beta)\tau_A}) \equiv L^A$  holds. When  $b < \tau_A\tau_M/a$ , the area is B1. Then, in this area, the economy is in self-sufficient economy and the number of farmers in region 1 is (13). The number of farmers is constant. When  $\tau_A\tau_M/a < b < \hat{b}^{NT}$  where  $\hat{b}^{NT}$  satisfies  $L = \hat{L}^{NT}|_{b=\hat{b}^{NT}}$ , the area is B2. In this area, the number of farmers in region 1 is (27). The number of farmers in region 1 is decreasing in the technology of manufactured goods. When  $\hat{b}^{NT} < b < \tilde{b}^T$  where  $\tilde{b}^T$  satisfies  $L = \tilde{L}^T|_{b=\tilde{b}^T}$ , the area is B3. In this area, the number of farmers in region 1 is  $L^{NT}$  and increasing in  $b$ . When  $b > \tilde{b}^T$ , the area is B5. In this area, the number of farmers in region 1 is (44) and is constant. To summarize above statement, we can depict the relationship between the number of farmers in region 1 and the technology of manufactured goods in Figure 9.

Suppose that  $L^B \equiv \frac{(1-\beta)\tau_A}{\beta a}(1 + \frac{(1-\beta)\tau_A + \beta}{(1-\beta)\tau_A\gamma}) < L < L^A$ . When  $b < \tau_A\tau_M/a$ , the area is B1. Then, in this area, the economy is in self-sufficient economy and the number of farmers in region 1 is (13). The number of farmers is constant. When  $\tau_A\tau_M/a < b < \hat{b}^{NT}$  where  $\hat{b}^{NT}$  satisfies  $L = \hat{L}^{NT}|_{b=\hat{b}^{NT}}$ , the area is B4. In this area, the number of farmers in region 1 is (29). The number of farmers in region 1 is increasing in the technology of manufactured goods. When  $\hat{b}^{NT} < b < \tilde{b}^T$ , the area is B3. In this area, the number of farmers in region 1 is  $L^{NT}$  and increasing in  $b$ . When  $b > \tilde{b}^T$ , the area is B5. In this area, the number of farmers in region 1 is (44) and is constant. To summarize above statement, we can depict the relationship between the number of farmers in region 1 and the technology of manufactured goods in Figure 10. Summarizing Figures 9 and 10, we can obtain the following proposition:

**Proposition 5** *Suppose that  $\beta < \beta_3$ . When  $L > L^A$ , the relationship between the number of farmers in region 1 and the technology level of manufacturing is U-shaped. When  $L^B < L < L^A$ , an increase in the technology level of manufactured centralizes farmers to region 1 monotonically.*

We explain Proposition 5 with Figures 9 and 10 intuitively. In Figures 9 and 10, there are two features. One is that when  $\tau_A\tau_M/a < b < \tilde{b}^T$ , the relationship between the productivity of craftsmen and the number of farmers in region 1 is different in Figure 9 and Figure 10. The other feature is that when  $b = \tilde{b}^T$ , the number of farmers in region 1 decreases suddenly. We explain these two features intuitively. When  $b < \tau_A\tau_M/a$ , because both farmers and craftsmen are self-sufficient, the number of farmers is constant. When  $\tau_A\tau_M/a < b < \tilde{b}^T$ , the relationship between productivity of craftsmen and the number of farmers is U-shaped in Figure 9 and is monotonically increasing in Figure 10. When  $\tau_A\tau_M/a < b < \tilde{b}^T$  holds, there exists intra-regional trade but there does not

exist inter-regional trade. There are three effects when  $\tau_{ATM}/a < b < \tilde{b}^T$ : *Trade effect*, *Price effect*, and *Land price effect*. *Trade effect* is a positive effect on the number of farmers in region 1. *Trade effect* is that an increase in the productivity of craftsmen enlarges the volume of trade between craftsmen and farmers. This raises the utility of farmers that trading with craftsmen and this increases the number of farmers in region 1. *Price effect* is also positive effect on the number of farmers in region 1. An increase in the productivity of craftsmen increases the supply of manufactured goods. Then, price of manufactured goods in region 1 decreases. Therefore, this improves the utility of farmers in region 1 and the number of farmers in region 1 is raised. *Land price effect* is the negative effect on the number of farmers in region 1. An increase in the productivity of craftsmen increases the income level of craftsmen and this increases the demand of land space. Therefore, since the land price in region 1 becomes high, the utility of farmers in region 1 decreases and the number of farmers in region 1 decreases. In case of Figure 9  $L > L^A$  is satisfied, that is the relative number of farmers is sufficiently large. When  $\tau_{ATM}/a < b < \hat{b}^{NT}$ , because the number of craftsmen is scarce relative to farmers, the supply of manufactured goods produced by craftsmen is small. Then, farmers in region 1 produce both goods and the price of manufactured goods is high and constant. Therefore, *Price effect* is zero. Because the relative number of craftsmen is small, *trade effect* becomes small. Because the price of manufactured goods is high, the income level of craftsmen is large relative to farmers. Then, *Land price effect* is large. Therefore, when  $\tau_{ATM}/a < b < \hat{b}^{NT}$ , *Land price effect* is larger than *trade effect* and an increase in the productivity of craftsmen decreases the number of farmers in region 1. When  $\hat{b}^{NT} < b < \tilde{b}^T$ , both craftsmen and farmers in region 1 specialize to produce their comparative advantage goods and the price of manufactured goods decreases in the productivity of craftsmen.<sup>11</sup> Therefore, when  $\hat{b}^{NT} < b < \tilde{b}^T$ , *Price effect* is not zero and two positive effects are larger than one negative effect. Then, an increase in the productivity of craftsmen increases the number of farmers in region 1. Suppose that  $L^B < L < L^A$  in case of Figure 10. When  $\tau_{ATM}/a < b < \hat{b}^{NT}$ , because the number of craftsmen is large relative to farmers, the supply of manufactured goods produced by craftsmen is large. Then, farmers in region 1 specialize to produce agricultural goods and craftsmen produce both goods. Then, the price of manufactured goods is low and constant. Therefore, *Price effect* is also zero. Because the relative number of craftsmen is large, *trade effect* becomes large for farmers in region 1. Because the price of manufactured goods is low, the income level of craftsmen is small relative to farmers. Then, *Land price effect* is small. Therefore, when  $\tau_{ATM}/a < b < \hat{b}^{NT}$ , *Land price effect* is smaller than *trade effect* and an increase in the productivity of craftsmen increases the number of farmers in region 1. When  $\hat{b}^{NT} < b < \tilde{b}^T$ , *Price effect* is not zero and an increase in the productivity of craftsmen increases the number of farmers in region 1.

When  $b = \tilde{b}^T$ , the number of farmers decreases suddenly. When  $b = \tilde{b}^T$ , the inter-regional trade begins. Then, farmers in region 2 also can buy the

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<sup>11</sup>See Appendix for proof.

cheaper manufactured goods from craftsmen and the benefits of living in region 1 decreases. Therefore, the number of farmers living in region 1 decreases at  $b = \tilde{b}^T$ . When  $b > \tilde{b}^T$ , both farmers and craftsmen specialize to produce their comparative advantage goods and there exist intra-regional trade and inter-regional trade. Then, the relative price of manufactured goods in both regions are constant to the technology of manufactured goods and the number of farmers in region 1 is also constant.

## 5 Conclusion

This paper constructs a two-region model to investigate how cities are born. In this paper, we define that cities have a large number of population, markets, and the workers in the city specialize to produce their comparative advantage goods. This paper focuses on the three elements of city formation: international or long-distant trade, division of labor, and productivity of workers. In this paper, there are two goods, agricultural goods and manufactured goods and two types of workers: farmers and craftsmen. Both workers can produce both goods. Farmers have a comparative advantage to produce agricultural goods and craftsmen have a comparative advantage to produce manufactured goods. Farmers can be mobile between regions and all of craftsmen live in region 1. To transport both goods both intra-regionally and inter-regionally, workers have to incur the intra-regional transaction costs and inter-regional transportation costs, respectively. We assume that intra-regional transaction costs is smaller than inter-regional transportation costs. When the technology of workers is sufficiently small relative to the intra-regional transaction costs, there is no market to exchange both goods and both workers are self-sufficient. When the technology of workers is medium in intra-regional transaction costs and inter-regional transportation costs, there exists markets intra-regionally but there is no inter-regional market. When the technology of workers is large relative to the inter-regional transportation costs, there exists markets both intra-regionally and inter-regionally.

The results of this paper are following. Suppose that the number of farmers is sufficiently large. When the technology of craftsmen is small relative to the intra-regional transaction costs, both farmers and craftsmen are self-sufficient. When the technology of craftsmen is medium in intra-regional transaction costs and inter-regional transportation costs, there exists a intra-regional markets. Craftsmen specialize to produce their comparative advantage goods and farmers in both region produce both goods. Income level of craftsmen increases and the demand for land space increases. Then, the relative land price in region 1 increases and farmers prefer to live in region 2. Therefore, the number of population in region 1 decreases. When the technology of craftsmen is similar to the inter-regional transportation costs, both craftsmen and farmers in region 1 specialize to produce their comparative advantage goods and there exists a intra-regional markets. In this economy, the number of population in region 1

increases. Therefore, this represents the born of city. When the productivity of craftsmen is larger relative to the inter-regional transportation costs, both craftsmen and farmers in both region specialize to produce their comparative advantage goods and there exist intra-regional and inter-regional markets.

This paper is very simple model to focus on the investigating the birth of cities. There are many extension ways to study the city formation. In this model, we assume that labor is the only input to produce agricultural goods. To relax this assumption, both labor and land space are the inputs to produce agricultural goods. Second, because the city formation is dynamic problem, we construct the dynamic model to incorporate stock variables of technologies.

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## 6 Appendix

### 6.1 Relationship between $\hat{L}^{NT}$ , $\tilde{L}^{NT}$ , $\hat{L}^T$ , $\tilde{L}^T$ , $\underline{L}^{NT}$ , and $\underline{L}^T$ in space of $a$ and $L$

To draw Figures 3, 4, and 5, we separate two steps. The first step is to investigate the relationship between  $\hat{L}^{NT}$ , and  $\hat{L}^T$  when  $a = \tau_A^* \tau_M^* / b$ . In the second step, we show the value of  $\underline{L}^{NT}$  when  $a = \tau_A \tau_M / b$ . In the first step, subtracting from  $\hat{L}^{NT}$  to  $\hat{L}^T$  when  $a = \tau_A^* \tau_M^* / b$ , the following equation can be obtained

$$\begin{aligned} \hat{L}^{NT} \Big|_{a=\frac{\tau_A^* \tau_M^*}{b}} - \hat{L}^T \Big|_{a=\frac{\tau_A^* \tau_M^*}{b}} &= \frac{(1-\beta)}{\beta \tau_A^* \tau_M^*} \left[ \frac{(1+\gamma)\tau_A}{\gamma} + \frac{\beta}{(1-\beta)\gamma} - \frac{(1+\gamma)\tau_A \tau_A^* + \frac{\beta}{1-\beta}(\tau_A^* - \tau_A)}{\gamma \tau_A^* + \tau_A} \right] \\ &= \frac{(1-\beta)}{\beta \tau_A^* \tau_M^*} \frac{\tau_A(1+\gamma)(\tau_A + \frac{\beta}{1-\beta})}{\gamma(1-\beta)(\gamma \tau_A^* + \tau_A)} > 0. \end{aligned}$$

Then, we can show that  $\hat{L}^{NT}$  is larger than  $\hat{L}^T$  when  $a = \tau_A^* \tau_M^* / b$ . In the second step, we compare the value of  $\hat{L}^T$  when  $a = \tau_A^* \tau_M^* / b$  and  $\underline{L}^T$  when  $a = \tau_A \tau_M / b$ .

$$\hat{L}^T \Big|_{a=\frac{\tau_A^* \tau_M^*}{b}} - \underline{L}^T \Big|_{a=\frac{\tau_A \tau_M}{b}} = \frac{b}{\beta \tau_A^* \tau_M^*} \frac{(1+\gamma)\tau_A \tau_A^* - \beta \left( (1+\gamma)\tau_A \tau_A^* + (\gamma^{\frac{\alpha+\beta}{\beta}} - 1)\tau_A^* + (1+\gamma^{\frac{\alpha}{\beta}})\tau_A \right)}{\gamma \tau_A^* + \tau_A}.$$

Therefore, when  $\beta < (>) \beta_1 \equiv \frac{(1+\gamma)\tau_A \tau_A^*}{(1+\gamma)\tau_A \tau_A^* + (\gamma^{\frac{\alpha+\beta}{\beta}} - 1)\tau_A^* + (1+\gamma^{\frac{\alpha}{\beta}})\tau_A}$ ,  $\hat{L}^T \Big|_{a=\frac{\tau_A^* \tau_M^*}{b}}$  is larger (smaller) than  $\underline{L}^T \Big|_{a=\frac{\tau_A \tau_M}{b}}$ . Next, we compare the value of  $\hat{L}^{NT}$  when  $a = \tau_A^* \tau_M^* / b$  and  $\underline{L}^T$  when  $a = \tau_A \tau_M / b$ .

$$\hat{L}^{NT} \Big|_{a=\frac{\tau_A^* \tau_M^*}{b}} - \underline{L}^T \Big|_{a=\frac{\tau_A \tau_M}{b}} = \frac{b}{\beta \tau_A^* \tau_M^* \gamma} \left[ (1+\gamma)\tau_A - \beta \left( (1+\gamma)\tau_A + \gamma^{\frac{\alpha+\beta}{\beta}} - 1 \right) \right].$$

Therefore, when  $\beta < (>) \beta_2 \equiv \frac{(1+\gamma)\tau_A}{(1+\gamma)\tau_A + \gamma^{\frac{\alpha+\beta}{\beta}} - 1}$ ,  $\hat{L}^{NT} \Big|_{a=\frac{\tau_A^* \tau_M^*}{b}}$  is larger (smaller) than  $\underline{L}^T \Big|_{a=\frac{\tau_A \tau_M}{b}}$ . We can show that  $\beta_2$  is larger than  $\beta_1$  as follows:

$$\beta_2 - \beta_1 = \frac{(1+\gamma)^{\frac{\alpha+\beta}{\beta}} \tau_A^2}{\left[ (1+\gamma)\tau_A \tau_A^* + (\gamma^{\frac{\alpha+\beta}{\beta}} - 1)\tau_A^* + (1+\gamma^{\frac{\alpha}{\beta}})\tau_A \right] \left[ \gamma^{\frac{\alpha+\beta}{\beta}} - 1 + (1+\gamma)\tau_A \right]} > 0.$$

Therefore, when  $0 < \beta < \beta_1$ ,  $\underline{L}^T \Big|_{a=\frac{\tau_A \tau_M}{b}}$  is smaller than  $\hat{L}^T \Big|_{a=\frac{\tau_A^* \tau_M^*}{b}}$ . When  $\beta_1 < \beta < \beta_2$ ,  $\underline{L}^T \Big|_{a=\frac{\tau_A \tau_M}{b}}$  is between  $\hat{L}^T \Big|_{a=\frac{\tau_A^* \tau_M^*}{b}}$  and  $\hat{L}^{NT} \Big|_{a=\frac{\tau_A^* \tau_M^*}{b}}$ . When  $\beta_2 < \beta < 1$ ,  $\underline{L}^T \Big|_{a=\frac{\tau_A \tau_M}{b}}$  is larger than  $\hat{L}^{NT} \Big|_{a=\frac{\tau_A^* \tau_M^*}{b}}$ . From these two steps, we can obtain Figures 3, 4, and 5.



## 6.2 Relationship between $\hat{L}^{NT}$ , $\tilde{L}^{NT}$ , $\hat{L}^T$ , $\tilde{L}^T$ , $\underline{L}^{NT}$ , and $\underline{L}^T$ in space of $b$ and $L$

To draw Figures 6, 7, and 8, we separate two steps. The first step is to investigate the relationship between  $\hat{L}^{NT}$ , and  $\hat{L}^T$  when  $b = \tau_A^* \tau_M^* / a$ . This first step is same as the above Appendix. Then, we can show that  $\hat{L}^{NT}$  is larger than  $\hat{L}^T$  when  $b = \tau_A^* \tau_M^* / a$ . In the second step, we show the value of  $\underline{L}^{NT}$  when  $b = \tau_A \tau_M / a$ . We compare the value of  $\hat{L}^T$  and  $\underline{L}^{NT}$  when  $b = \tau_A \tau_M / a$ .

$$\hat{L}^T - \underline{L}^{NT} \Big|_{b=\frac{\tau_A \tau_M}{a}} = \frac{1}{\beta a} \frac{(1+\gamma)\tau_A \tau_A^* - \beta[(\gamma-1)\tau_A^* + 2\tau_A + (1+\gamma)\tau_A \tau_A^*]}{\gamma \tau_A^* + \tau_A}.$$

Therefore, when  $\beta < (>) \beta_3 \equiv \frac{(1+\gamma)\tau_A \tau_A^*}{(1+\gamma)\tau_A \tau_A^* + (\gamma-1)\tau_A^* + 2\tau_A}$ ,  $\hat{L}^T$  is larger (smaller) than  $\underline{L}^T \Big|_{b=\frac{\tau_A \tau_M}{a}}$ . Next, we compare the value of  $\hat{L}^{NT}$  when  $b = \tau_A^* \tau_M^* / a$  and  $\underline{L}^{NT}$  when  $a = \tau_A \tau_M / b$ .

$$\hat{L}^{NT} \Big|_{b=\frac{\tau_A^* \tau_M^*}{a}} - \underline{L}^{NT} \Big|_{b=\frac{\tau_A \tau_M}{a}} = \frac{(1+\gamma)\tau_A - \beta[(1+\gamma)\tau_A + \gamma - 1]}{\beta \gamma a}.$$

Therefore, when  $\beta < (>) \beta_4 \equiv \frac{(1+\gamma)\tau_A}{(1+\gamma)\tau_A + \gamma - 1}$ ,  $\hat{L}^{NT} \Big|_{b=\frac{\tau_A^* \tau_M^*}{a}}$  is larger (smaller) than  $\underline{L}^T \Big|_{b=\frac{\tau_A \tau_M}{a}}$ . We can show that  $\beta_4$  is larger than  $\beta_3$  as follows:

$$\beta_4 - \beta_3 = \frac{2(1+\gamma)\tau_A^2}{[(1+\gamma)\tau_A \tau_A^* + (\gamma-1)\tau_A^* + 2\tau_A][(1+\gamma)\tau_A + \gamma - 1]} > 0.$$

Therefore, when  $0 < \beta < \beta_3$ ,  $\underline{L}^T \Big|_{b=\frac{\tau_A \tau_M}{a}}$  is smaller than  $\hat{L}^T$ . When  $\beta_3 < \beta < \beta_4$ ,  $\underline{L}^T \Big|_{b=\frac{\tau_A \tau_M}{a}}$  is between  $\hat{L}^T \Big|_{b=\frac{\tau_A^* \tau_M^*}{a}}$  and  $\hat{L}^{NT} \Big|_{b=\frac{\tau_A^* \tau_M^*}{a}}$ . When  $\beta_4 < \beta < 1$ ,  $\underline{L}^T \Big|_{b=\frac{\tau_A \tau_M}{a}}$  is larger than  $\hat{L}^{NT} \Big|_{b=\frac{\tau_A^* \tau_M^*}{a}}$ . From these two steps, we can obtain Figures 6, 7, and 8.

## 6.3 Relationship between $b$ and $p_{M1}$ in Intra-regional trade economy

Differentiating (24) with respect to  $b$ , we can obtain the following equation:

$$\frac{\partial p_{M1}}{\partial b} = \frac{\tau_M}{\tau_A} \frac{a\beta}{1-\beta} \left( -\frac{L_1^{NT}}{b^2} + \frac{1}{b} \frac{\partial L_1^{NT}}{\partial b} \right).$$

To obtain  $\partial L_1^{NT} / \partial b$ , we totally differentiate  $\phi(L_1^{NT}) = L - L_1^{NT}$  as follows:

$$\frac{\partial L_1^{NT}}{\partial b} = \frac{\frac{\beta}{\alpha} \left( 1 + \frac{\beta}{(1-\beta)\tau_A} \right) \left( \frac{1-\beta}{\beta} \frac{b}{\tau_M} \right)^{-\frac{\beta}{\alpha}} L_1^{NT} \frac{\alpha+\beta}{b}}{1 + \left( 1 + \frac{\beta}{(1-\beta)\tau_A} \right) \left( \frac{1-\beta}{\beta} \frac{b}{\tau_M} \right)^{-\frac{\beta}{\alpha}} L_1^{NT} \frac{\beta}{\alpha}}.$$

Then, substituting  $\partial L_1^{NT}/\partial b$  into  $\partial p_{M1}/\partial b$ , following equation can be obtained:

$$\frac{\partial p_{M1}}{\partial b} = -\frac{\tau_M}{\tau_A} \frac{a\beta}{1-\beta} \frac{L_1^{NT}}{b^2} \frac{F}{1 + \left(1 + \frac{\beta}{(1-\beta)\tau_A}\right) \left(\frac{1-\beta}{\beta} \frac{b}{\tau_M}\right)^{-\frac{\beta}{\alpha}} L_1^{NT\frac{\beta}{\alpha}}},$$

$$F \equiv 1 - \frac{\beta - \alpha}{\alpha} \left(1 + \frac{\beta}{(1-\beta)\tau_A}\right) \left(\frac{1-\beta}{\beta} \frac{b}{\tau_M}\right)^{-\frac{\beta}{\alpha}} L_1^{NT\frac{\beta}{\alpha}}.$$

The sign of  $\partial p_{M1}/\partial b$  depends on the sign of  $F$ . We assume that  $\alpha > \frac{\beta^2}{2(1-\beta)\tau_A + \beta}$ . Because  $L_1^{NT} < \frac{1-\beta}{\beta} \frac{b}{\tau_M}$ , the value of  $F$  becomes

$$\begin{aligned} F &= 1 - \frac{\beta - \alpha}{\alpha} \left(1 + \frac{\beta}{(1-\beta)\tau_A}\right) \left(\frac{1-\beta}{\beta} \frac{b}{\tau_M}\right)^{-\frac{\beta}{\alpha}} L_1^{NT\frac{\beta}{\alpha}} \\ &> 1 - \frac{\beta - \alpha}{\alpha} \left(1 + \frac{\beta}{(1-\beta)\tau_A}\right) \left(\frac{1-\beta}{\beta} \frac{b}{\tau_M}\right)^{-\frac{\beta}{\alpha}} \left(\frac{1-\beta}{\beta} \frac{b}{\tau_M}\right)^{\frac{\beta}{\alpha}} \\ &= 1 - \frac{\beta - \alpha}{\alpha} \left(1 + \frac{\beta}{(1-\beta)\tau_A}\right) > 0, \end{aligned}$$

because  $\alpha > \frac{\beta^2}{2(1-\beta)\tau_A + \beta}$ .