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A Dynamic Rural-Urban-Natural Environment Interactive Spatial Model of Palangkaraya City in Indonesia

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1. Introduction

Permana and Miyata (2012) showed a partial equilibrium urban economic model to explain the existence of illegal settlements in flood prone areas in Palangkaraya City in Central Kalimantan Province, introducing the expected damage rate on household asset. Applying this new idea, one can derive the conclusion where the bid rents by low income households get higher than those by high income households in flood prone areas. This is the contrary conclusion being highlighted as compared with that in the traditional urban economics.

Following this paper, Permana and Miyata (2012) extended the partial equilibrium model into a general equilibrium model. And then Permana and Miyata (2009) developed a two dimensional city model applying Miyata's achievement (2011). However the study region, Palangkaraya City and its surrounding area, shows a complicated interaction between natural environment and human activities. Therefore this article aims at developing a rural and urban economic model with natural environment considerably extending our previous literature.

2. Assumptions in the Model

(1)The study region consists of Palangkaraya City and its surrounding rural area. The city shape is assumed to be a disk where there are three flood prone areas. The land other than the flood prone areas is called normal land. The normal land is assumed to have no flood risk, while the flood prone areas are facing the flood risk with occurrence probabilities. The rural area is specified as dimensionless i.e. there is no spatial structure.

(2)There are two types of households which are high income households (H.I.H.) and low income ones (L.I.H.) in the city. High income households are assumed to reside on the normal land, while low income households are supposed to live in the flood prone areas. The city is assumed to be closed for high income households, and open for low income households. This reflects the fact that the utility of a high income household is much higher than that in rural areas, thus has no incentive to live outside the city. The low income households in rural areas expect the higher utilities in the city thus want to live in the city, but most of them results in residing in the flood prone areas due to the income gap. In the rural area there live only low income households. The number of the high income households is denoted by N_1 , that of the low income households in the city by N_2 , that of the low income households in the rural area by N_3 .

(3)We consider different types firms in the study region. In the city all firms are assumed to be homogeneous and produce single type of goods, i.e. urban goods. In the rural area there are three types of industries, namely, agriculture, forestry and rural general goods. The number of firms in the city is M , while firms in the rural area are aggregated into the three types.

(4)Land in the city is owned by absentee landowners who reside outside the city. There is a unique local government in the city, and it rents all land in the city from the absentee landowners. The local government rents the land to households and firms at market rent, and then redistributes the rent revenues to the two types of households. In the rural area, land types are differentiated, namely, agricultural land, forest land, general firms' land and residential land. Supply of each type of land is exogenously given.

(5)Capital stock in the city is assumed to be freely mobile across firms. Thus the capital return rate is uniquely determined being irrespective firms' location. The capital service is assumed to be numerare.

(5)The parameters in the locational potential function for each firm are sufficiently large. In this case a simple von Thünen ring becomes an equilibrium urban configuration (Miyata (2011)).

3. Firms Behavior in the City

The production function of a firm at location $\mathbf{x} = (x_1, x_2)$ in the city is specified as a Cobb-Douglas-CES type of homogeneous degree of unity, and agglomeration economy is taken into account. The agglomeration economy is represented by locational potential function $\Omega(\mathbf{x})$, which was introduced by Fujita and Ogawa (1982). It is defined as follows:

$$\Omega(\mathbf{x}) \equiv \iint_A \mu b(\mathbf{y}) \exp(-\omega \|\mathbf{x} - \mathbf{y}\|) dy_1 dy_2 \quad (1)$$

where

A : city area

$b(\mathbf{y})$: density of firms at location \mathbf{y}

μ : monetary conversion parameter in locational potential

ω : parameter expressing the effects of distance between different points

$\|\mathbf{x} - \mathbf{y}\|$: distance between location \mathbf{x} and \mathbf{y}

Then the production by a firm at location \mathbf{x} may be written as follows:

$$q_1(\mathbf{x}) = \Omega(\mathbf{x}) q_{A1} \prod_{i=1}^4 q_{i1}(\mathbf{x})^{\alpha_{i1}} \left[\zeta^{\frac{1}{\sigma_1}} ld_1(\mathbf{x})^{\frac{\sigma_1-1}{\sigma_1}} + (1-\zeta)^{\frac{1}{\sigma_1}} ld_2(\mathbf{x})^{\frac{\sigma_1-1}{\sigma_1}} \right]^{\frac{\sigma_1}{\sigma_1-1} \alpha_{l1}} \cdot \quad (2)$$

$$kd_1(\mathbf{x})^{\alpha_{k1}} m_{B1}(\mathbf{x})^{\alpha_{m1}} n_{U0}^{\alpha_{U01}} n_{R3}^{\alpha_{R31}}$$

where

$q_1(\mathbf{x})$: output of a firm at location \mathbf{x}

q_{A1} : efficient parameter

$q_{11}(\mathbf{x})$: intermediate input of urban goods at location \mathbf{x}

$q_{21}(\mathbf{x})$: intermediate input of agricultural products at location \mathbf{x}

$q_{31}(\mathbf{x})$: intermediate input of forestry products at location \mathbf{x}

$q_{41}(\mathbf{x})$: intermediate input of rural goods at location \mathbf{x}

$ld_1(\mathbf{x})$: input of labor of high income type at location \mathbf{x}

$ld_2(\mathbf{x})$: input of labor of low income type at location \mathbf{x}

$kd_1(\mathbf{x})$: capital input at location \mathbf{x}

$m_{B1}(\mathbf{x})$: land input at location \mathbf{x}

n_{U0} : natural environmental level in the city (e.g. air and water)

n_{R3} : forest volume in the rural area

ζ : share parameter

σ_1 : elasticity of substitution ($0 < \sigma_1 < 1$)

$\alpha_{i1}, \alpha_{l1}, \alpha_{k1}, \alpha_{m1}, \alpha_{U01}, \alpha_{R31}$: elasticity parameters ($\alpha_{i1} + \alpha_{l1} + \alpha_{k1} + \alpha_{m1} + \alpha_{U01} + \alpha_{R31} = 1$)

We assume that each firm is a price taker for commodities and production factors, and the impact of production activity of each firm on the natural environment and forest is negligible. The firms' locational equilibrium condition is that the profit in each firm is equalized at every point. Due to the linear homogeneity of degree one in each firm's technology, the equilibrium profit in each firm becomes zero. Then the bid rent function, conditional de-

mands for intermediate goods, labors of the two types, capital stock and bid max lot size are obtained.

$$g_{B1}(\mathbf{x}) \equiv \max \left[\frac{p_1(\mathbf{x})q_1(\mathbf{x}) - \sum_{i=1}^4 p_i(\mathbf{x})q_{i1}(\mathbf{x}) - w_1(\mathbf{x})ld_1(\mathbf{x}) - w_2(\mathbf{x})ld_2(\mathbf{x}) - r_1(\mathbf{x})kd_1(\mathbf{x})}{m_{B1}(\mathbf{x})} \right] \quad (3)$$

with respect to $q_{i1}(\mathbf{x})$, $ld_1(\mathbf{x})$, $ld_2(\mathbf{x})$, $kd_1(\mathbf{x})$ and $m_{B1}(\mathbf{x})$

subject to

$$q_1(\mathbf{x}) = \Omega(\mathbf{x})q_{A1} \prod_{i=1}^4 q_{i1}(\mathbf{x})^{\alpha_{i1}} \left[\zeta^{\frac{1}{\sigma_1}} ld_1(\mathbf{x})^{\frac{\sigma_1-1}{\sigma_1}} + (1-\zeta)^{\frac{1}{\sigma_1}} ld_2(\mathbf{x})^{\frac{\sigma_1-1}{\sigma_1}} \right]^{\frac{\sigma_1-1}{\sigma_1} \alpha_{11}} \quad (4)$$

$$kd_1(\mathbf{x})^{\alpha_{k1}} m_{B1}(\mathbf{x})^{\alpha_{m1}} n_{U0}^{\alpha_{U01}} n_{R3}^{\alpha_{R31}}$$

$$\pi_{B1}(\mathbf{x}) = 0 \quad (5)$$

where

$p_1(\mathbf{x})$: price of urban good at location \mathbf{x}

$p_2(\mathbf{x})$: price of agricultural product at location \mathbf{x}

$p_3(\mathbf{x})$: price of forestry product at location \mathbf{x}

$p_4(\mathbf{x})$: price of rural good at location \mathbf{x}

$w_1(\mathbf{x})$: wage rate of a high income household at location \mathbf{x}

$w_2(\mathbf{x})$: wage rate of a low income household at location \mathbf{x}

$r_1(\mathbf{x})$: capital return rate at location \mathbf{x}

$g_{B1}(\mathbf{x})$: bid rent by a firm at location \mathbf{x}

$\pi_{B1}(\mathbf{x})$: profit in a firm at location \mathbf{x}

To obtain the bid rent function and the bid max lot size in a firm, we consider the cost function.

$$C(q_1(\mathbf{x})) \equiv \min \sum_{i=1}^4 p_i(\mathbf{x})q_{i1}(\mathbf{x}) + w_1(\mathbf{x})ld_1(\mathbf{x}) + w_2(\mathbf{x})ld_2(\mathbf{x}) + r_1(\mathbf{x})kd_1(\mathbf{x}) + g_{B1}(\mathbf{x})m_{B1}(\mathbf{x}) \quad (6)$$

with respect to $q_{i1}(\mathbf{x})$, $ld_1(\mathbf{x})$, $ld_2(\mathbf{x})$, $kd_{d1}(\mathbf{x})$ and $m_{B1}(\mathbf{x})$

subject to

$$q_1(\mathbf{x}) = \Omega(\mathbf{x})q_{A1} \prod_{i=1}^4 q_{i1}(\mathbf{x})^{\alpha_{i1}} \left[\zeta^{\frac{1}{\sigma_1}} ld_1(\mathbf{x})^{\frac{\sigma_1-1}{\sigma_1}} + (1-\zeta)^{\frac{1}{\sigma_1}} ld_2(\mathbf{x})^{\frac{\sigma_1-1}{\sigma_1}} \right]^{\frac{\sigma_1-1}{\sigma_1} \alpha_{11}} \quad (7)$$

$$kd_1(\mathbf{x})^{\alpha_{k1}} m_{B1}(\mathbf{x})^{\alpha_{m1}} n_{U0}^{\alpha_{U01}} n_{R3}^{\alpha_{R31}}$$

Then the cost function is solved as follows:

$$C(q_1(\mathbf{x})) = \frac{q_1(\mathbf{x})}{\Omega(\mathbf{x})q_{A1}n_{U0}^{\alpha_{U01}}n_{R3}^{\alpha_{R31}}} \prod_{i=1}^4 \left[\frac{p_i(\mathbf{x})}{\alpha_{i1}} \right]^{\alpha_{i1}} \cdot \left[\frac{[\zeta w_1(\mathbf{x})^{1-\sigma_1} + (1-\zeta)w_2(\mathbf{x})^{1-\sigma_1}]^{\frac{1}{1-\sigma_1}}}{\alpha_{11}} \right]^{\alpha_{11}} \left[\frac{r_1(\mathbf{x})}{\alpha_{k1}} \right]^{\alpha_{k1}} \left[\frac{g_{B1}(\mathbf{x})}{\alpha_{m1}} \right]^{\alpha_{m1}} \quad (8)$$

Since the equilibrium profit is zero, the following equation holds.

$$p_1(\mathbf{x})q_1(\mathbf{x}) = C(q_1(\mathbf{x})) \quad (9)$$

Then the bid rent function in the firm is solved as follows:

$$g_{B1}(\mathbf{x}) = \alpha_{m1} (p_1(\mathbf{x})\Omega(\mathbf{x})q_A n_{U0}^{\alpha_{U01}} n_{R3}^{\alpha_{R31}})^{\frac{1}{\alpha_{m1}}} \cdot \prod_{i=1}^4 \left[\frac{\alpha_{i1}}{p_i(\mathbf{x})} \right]^{\frac{\alpha_{i1}}{\alpha_{m1}}} \left[\frac{\alpha_{l1}}{[\zeta w_1(\mathbf{x})^{1-\sigma_1} + (1-\zeta)w_2(\mathbf{x})^{1-\sigma_1}]^{\frac{1}{1-\sigma_1}}} \right]^{\frac{\alpha_{l1}}{\alpha_{m1}}} \left[\frac{\alpha_{k1}}{r_1(\mathbf{x})} \right]^{\frac{\alpha_{k1}}{\alpha_{m1}}} \quad (10)$$

$$q_{i1}(\mathbf{x}) = \alpha_{i1} p_1(\mathbf{x}) q_1(\mathbf{x}) / p_i(\mathbf{x}) \quad (i = 1, 2, 3, 4) \quad (11)$$

$$ld_1(\mathbf{x}) = \frac{\alpha_{l1} \zeta p_1(\mathbf{x}) q_1(\mathbf{x})}{w_1(\mathbf{x})^{\sigma_1} [\zeta w_1(\mathbf{x})^{1-\sigma_1} + (1-\zeta)w_2(\mathbf{x})^{1-\sigma_1}]^{\frac{\sigma_1}{\sigma_1-1}}} \quad (12)$$

$$ld_2(\mathbf{x}) = \frac{\alpha_{l1} (1-\zeta) p_1(\mathbf{x}) q_1(\mathbf{x})}{w_2(\mathbf{x})^{\sigma_1} [\zeta w_1(\mathbf{x})^{1-\sigma_1} + (1-\zeta)w_2(\mathbf{x})^{1-\sigma_1}]^{\frac{\sigma_1}{\sigma_1-1}}} \quad (13)$$

$$kd_1(\mathbf{x}) = \alpha_{k1} p_1(\mathbf{x}) q_1(\mathbf{x}) / r_1(\mathbf{x}) \quad (14)$$

$$m_{B1}(\mathbf{x}) = (\Omega(\mathbf{x})q_A n_{U0}^{\alpha_{U01}} n_{R3}^{\alpha_{R31}})^{\frac{1}{\alpha_{m1}}} p_1(\mathbf{x})^{1-1/\alpha_{m1}} \cdot \prod_{i=1}^4 \left[\frac{\alpha_{i1}}{p_i(\mathbf{x})} \right]^{\frac{\alpha_{i1}}{\alpha_{m1}}} \left[\frac{\alpha_{l1}}{[\zeta w_1(\mathbf{x})^{1-\sigma_1} + (1-\zeta)w_2(\mathbf{x})^{1-\sigma_1}]^{\frac{1}{1-\sigma_1}}} \right]^{\frac{\alpha_{l1}}{\alpha_{m1}}} \left[\frac{\alpha_{k1}}{r_1(\mathbf{x})} \right]^{\frac{\alpha_{k1}}{\alpha_{m1}}} q_1(\mathbf{x}) \quad (15)$$

4. Households Behavior in the City

The household utility functions in the both types of households at location \mathbf{x} is assumed to be expressed as follows:

$$u_i(c_{1i}(\mathbf{x}), c_{2i}(\mathbf{x}), c_{3i}(\mathbf{x}), c_{4i}(\mathbf{x}), c_{Fi}(\mathbf{x}), m_{Hi}(\mathbf{x}); n_{U0}, n_{R3}) \equiv \frac{\prod_{k=1}^4 c_{ki}(\mathbf{x})^{\beta_{k1}} c_{Fi}(\mathbf{x})^{\beta_{F1}} m_{Hi}(\mathbf{x})^{\beta_{m1}} n_{U0}^{\beta_{U01}} n_{R3}^{\beta_{R31}}}{1 + cn_{R3}^{-\omega} Y_i(\mathbf{x})^\varepsilon} \quad (i = 1, 2) \quad (16)$$

$$Y_i(\mathbf{x}) \equiv w_i(\mathbf{x}) + r_1(\mathbf{x})k s_i(\mathbf{x}) + \pi_{Hi}(\mathbf{x}) \quad (17)$$

where

i : $i = 1$ for a high income household and $i = 2$ for a low income household

u_i : household utility function at location \mathbf{x}

$c_{1i}(\mathbf{x})$: household consumption of urban goods at location \mathbf{x}

$c_{2i}(\mathbf{x})$: household consumption of agricultural products at location \mathbf{x}

$c_{3i}(\mathbf{x})$: household consumption of forestry products at location \mathbf{x}

$c_{4i}(\mathbf{x})$: household consumption of rural goods at location \mathbf{x}

$c_{Fi}(\mathbf{x})$: household future consumption at location \mathbf{x}

$m_{Hi}(\mathbf{x})$: household land input at location \mathbf{x}

n_{U0} : natural environmental level (e.g. air and water)

n_{R3} : forest volume in the rural area

$\beta_{k1}, \beta_{F1}, \beta_{m1}, \beta_{U01}$ and β_{R31} : elasticity parameters ($\beta_{11} + \beta_{21} + \beta_{31} + \beta_{41} + \beta_{F1} + \beta_{m1} + \beta_{U01} + \beta_{R31} = 1$)

c : expected damage rate on household asset ($c = 0$ in the normal land, $0 < c < 1$ in the flood prone areas)

$Y_i(\mathbf{x})$: household income at location \mathbf{x}

$w_i(\mathbf{x})$: wage rate at location \mathbf{x}

$r_1(\mathbf{x})$: capital return rate at location \mathbf{x}

$ks_i(\mathbf{x})$: capital stock endowment of a household of type i at location \mathbf{x}

$\pi_{Hi}(\mathbf{x})$: redistributed income from the local government to the household at location \mathbf{x}

Each household endows available working time l_{si} , and is assumed to perfectly inelastically supply it to firms obtaining income of $w_i(\mathbf{x})l_{si}$ plus redistributed income from the local government $\pi_{Hi}(\mathbf{x})$. In household locational equilibrium, the utility levels in the both types of households take the respective same values u_i^* , being irrespective of household residential places. u_1^* is endogenously determined while u_2^* is endogenously determined in the rural area. Therefore the household current and future consumptions, the bid max lot size and the bid rent function in the two types of households are derived.

Moreover the future consumption becomes saving and then it is invested to the capital stock of each household. To explain the household behavior, first, derivation of the price of future good is described here. The future good implies the future consumption which derived from household saving, however, the saving formulates capital investment. Therefore capital good can be regarded as saving good. Investment is made by using only urban good q_1 . Then the price of investment good is identified as $p_1(\mathbf{x})$. This can be regarded as the price of saving good $p_s(\mathbf{x})$.

Since the capital returns by a unit of capital injection is equal to $r_1(\mathbf{x})$, the expected return rate of the price of saving good $p_s(\mathbf{x})$, that is, the expected net return rate of household saving $r_s(\mathbf{x})$ is written as $r_s(\mathbf{x}) = r_1(\mathbf{x}) / p_s(\mathbf{x})$. It is assumed that the expected returns of saving finance the future consumption. Regarding the price of future good as the price of the current consumption good under the myopic expectation, and denoting the household real saving by $s_i(\mathbf{x})$, the following equation holds.

$$p_{Gi}(\mathbf{x}) \cdot c_{Fi}(\mathbf{x}) = r_1(\mathbf{x}) \cdot s_i(\mathbf{x}) \quad (18)$$

$$p_{Gi}(\mathbf{x}) \equiv \prod_{k=1}^4 \left[\frac{p_k(\mathbf{x})}{\beta_{k1}} \right]^{\beta_{k1} / \beta_{T1}} \left[\frac{p_F(\mathbf{x})}{\beta_{F1}} \right]^{\beta_{F1} / \beta_{T1}} \left[\frac{g(\mathbf{x})}{\beta_{m1}} \right]^{\beta_{m1} / \beta_{T1}} \quad (19)$$

where $\beta_{T1} \equiv \sum_{k=1}^4 \beta_{k1} + \beta_{F1} + \beta_{m1}$ and $g(\mathbf{x})$ is the market land rent at location \mathbf{x} .

This yields $[p_s(\mathbf{x})p_{Gi}(\mathbf{x}) / r_1(\mathbf{x})]c_{Fi}(\mathbf{x}) = p_s(\mathbf{x})s_i(\mathbf{x})$, and we set the price of future good $p_{Fi}(\mathbf{x})$ associated with the real saving $s_i(\mathbf{x})$ as;

$$p_{Fi}(\mathbf{x}) = p_s(\mathbf{x})p_{Gi}(\mathbf{x}) / r_1(\mathbf{x}) \quad (20)$$

Then $p_s(\mathbf{x})s_i(\mathbf{x}) = p_{Fi}(\mathbf{x})c_{Fi}(\mathbf{x})$ is realized.

Now the household bid rent function for land is specified as follows:

$$g_{Hi}(\mathbf{x}) \equiv \max \frac{w_i(\mathbf{x}) + r_1(\mathbf{x})k s_i + \pi_{Hi}(\mathbf{x}) - \sum_{k=1}^4 p_k(\mathbf{x})c_{ki}(\mathbf{x}) - p_{Fi}(\mathbf{x})c_{Fi}(\mathbf{x})}{m_{Hi}(\mathbf{x})} \quad (21)$$

with respect to $c_{ki}(\mathbf{x})$, $c_{Fi}(\mathbf{x})$ and $m_{Hi}(\mathbf{x})$

$$\text{subject to } u_i(\mathbf{x}) = u_i^* \quad (22)$$

where $g_{Hi}(\mathbf{x})$ is household bid rent function of type i at location \mathbf{x} , and π_H is defined as follows:

$$\pi_{Hi}(\mathbf{x}) \equiv \frac{\theta_i}{N} \iint_A [g_B(\mathbf{x})b(\mathbf{x})m_B(\mathbf{x}) + g_{H1}(\mathbf{x})h_1(\mathbf{x})m_{H1}(\mathbf{x}) + g_{H2}(\mathbf{x})h_2(\mathbf{x})m_{H2}(\mathbf{x})] dx_1 dx_2 \quad (23)$$

where $h_i(\mathbf{x})$: density of households of type i at location \mathbf{x} and $\theta_1 + \theta_2 = 1$

To solve the maximization problem (21), (22) and (23), we consider the expenditure function.

$$E_i(\mathbf{x}) \equiv \min \sum_{k=1}^4 p_k(\mathbf{x})c_{ki}(\mathbf{x}) + p_{Fi}(\mathbf{x})c_{Fi}(\mathbf{x}) + g_{Hi}(\mathbf{x})m_{Hi}(\mathbf{x}) \quad (i=1, 2) \quad (24)$$

with respect to $c_{ki}(\mathbf{x})$, $c_{Fi}(\mathbf{x})$ and $m_{Hi}(\mathbf{x})$

$$\text{subject to } u_i^* = \frac{\prod_{k=1}^4 c_{ki}(\mathbf{x})^{\beta_{k1}} c_{Fi}(\mathbf{x})^{\beta_{F1}} m_{Hi}(\mathbf{x})^{\beta_{m1}} n_{U0}^{\beta_{U01}} n_{R3}^{\beta_{R31}}}{1 + cn_{R3}^{-\omega} Y_i(\mathbf{x})^\varepsilon} \quad (25)$$

The expenditure function is solved as follows:

$$E_i(\mathbf{x}) \equiv \beta_{T1} \left[\frac{(1 + cn_{R3}^{-\omega} Y_i(\mathbf{x})^\varepsilon) u_i^*}{n_{U0}^{\beta_{U01}} n_{R3}^{\beta_{R31}}} \right]^{1/\beta_{T1}} \prod_{k=1}^4 \left[\frac{p_k(\mathbf{x})}{\beta_{k1}} \right]^{\beta_{k1}/\beta_{T1}} \left[\frac{p_{Fi}(\mathbf{x})}{\beta_{F1}} \right]^{\beta_{F1}/\beta_{T1}} \left[\frac{g_{mi}(\mathbf{x})}{\beta_{m1}} \right]^{\beta_{m1}/\beta_{T1}} \quad (26)$$

The expenditure function must be equal to the household income yielding the bid rent function.

$$g_{Hi}(\mathbf{x}) \equiv \frac{\beta_{m1}}{\beta_{T1}^{\beta_{T1}/\beta_{m1}}} \left[\frac{n_{U0}^{\beta_{U01}} n_{R3}^{\beta_{R31}}}{(1 + cn_{R3}^{-\omega} Y_i(\mathbf{x})^\varepsilon) u_i^*} \right]^{1/\beta_{m1}} \prod_{k=1}^4 \left[\frac{\beta_{k1}}{p_k(\mathbf{x})} \right]^{\beta_{k1}/\beta_{m1}} \left[\frac{\beta_{F1}}{p_F(\mathbf{x})} \right]^{\beta_{F1}/\beta_{m1}} Y_i(\mathbf{x})^{\beta_{T1}/\beta_{m1}} \quad (27)$$

Then demands for commodities, future good and the bid max lot size are solved as follows:

$$c_{ki}(\mathbf{x}) = \frac{\beta_{k1}}{\beta_{T1} p_k(\mathbf{x})} Y_i(\mathbf{x}) \quad (k = 1, 2, 3, 4) \quad (28)$$

$$c_{Fi}(\mathbf{x}) = \frac{\beta_{F1}}{\beta_{T1} p_F(\mathbf{x})} Y_i(\mathbf{x}) \quad (29)$$

$$s_i(\mathbf{x}) = p_{Fi}(\mathbf{x})c_{Fi}(\mathbf{x}) / p_s(\mathbf{x}) \quad (30)$$

$$I_i(\mathbf{x}) = s_i(\mathbf{x}) \quad (31)$$

$$m_{Hi}(\mathbf{x}) = \left[\frac{(1 + cn_{R3}^{-\omega} Y_i(\mathbf{x})^\varepsilon) u_i^*}{n_{U0}^{\beta_{U01}} n_{R3}^{\beta_{R31}}} \right]^{\beta_{T1}} \prod_{k=1}^4 \left[\frac{\beta_{T1} p_k(\mathbf{x})}{\beta_{k1} Y_i(\mathbf{x})} \right]^{\beta_{k1}/\beta_{m1}} \left[\frac{\beta_{T1} p_F(\mathbf{x})}{\beta_{F1} Y_i(\mathbf{x})} \right]^{\beta_{F1}/\beta_{m1}} \quad (32)$$

Thus the following dynamic equation holds.

$$k \dot{s}_i(\mathbf{x}) = I_i(\mathbf{x}) - \delta \cdot k s_i(\mathbf{x}) - (\dot{N}_i / N_i) k s_i(\mathbf{x}) \quad (33)$$

$$KS(t) \equiv \iint_A (ks_1(\mathbf{x})h_1(\mathbf{x}) + ks_2(\mathbf{x})h_2(\mathbf{x}))dx_1dx_2 \quad (34)$$

where

$ks(\mathbf{x})$: capital stock endowed by a household of type i at time t and location \mathbf{x}

$KS(t)$: aggregate capital stock at time t

$I_i(\mathbf{x})$: investment by a household of type i at location \mathbf{x}

δ : capital depreciation rate

Moreover the natural environment in the city is specified as follows:

$$z_1 = \iint_A \eta_1 q_1(\mathbf{x})b(\mathbf{x})dx_1dx_2 + \iint_A \sum_{k=1}^4 \sum_{i=1}^2 \mu_{ki} c_{ki}(\mathbf{x})h_i(\mathbf{x})dx_1dx_2 \quad (35)$$

$$n_{U0} = \bar{n}_{U0} - \frac{z_1}{q_{A5} m_{B2}^{\gamma_{B5}} n_{R3}^{\gamma_{R5}}} \quad (36)$$

where

z_1 : pollution

η_1, μ_{ki} : emission factors

\bar{n}_{U0} : natural environment before pollution

m_{A3} : agricultural land size in the rural area

n_{R3} : forest volume in the rural area

5. Firms in the Rural Area

We consider an aggregate production function in respective sectors in the rural area. The production functions are specified as follows:

$$q_2 = q_{A2} q_{12}^{\alpha_{12}} q_{22}^{\alpha_{22}} q_{32}^{\alpha_{32}} q_{42}^{\alpha_{42}} l d_2^{\alpha_{l2}} k d_2^{\alpha_{k2}} m_{B2}^{\alpha_{m2}} n_{R0}^{\alpha_{R02}} n_{R3}^{\alpha_{R32}} \quad (37)$$

$$q_3 = \min (q_{A3} q_{13}^{\alpha_{13}} q_{23}^{\alpha_{23}} q_{33}^{\alpha_{33}} q_{43}^{\alpha_{43}} l d_3^{\alpha_{l3}} k d_3^{\alpha_{k3}}, y_3 / \alpha_{y3}) \quad (38)$$

$$q_4 = q_{A4} q_{14}^{\alpha_{14}} q_{24}^{\alpha_{24}} q_{34}^{\alpha_{34}} q_{44}^{\alpha_{44}} l d_4^{\alpha_{l4}} k d_4^{\alpha_{k4}} m_{B4}^{\alpha_{m4}} n_{R0}^{\alpha_{R04}} n_{R3}^{\alpha_{R34}} \quad (39)$$

where

$$\alpha_{12} + \alpha_{22} + \alpha_{32} + \alpha_{42} + \alpha_{l2} + \alpha_{k2} + \alpha_{m2} = 1$$

$$\alpha_{13} + \alpha_{23} + \alpha_{33} + \alpha_{43} + \alpha_{l3} + \alpha_{k3} = 1$$

$$\alpha_{14} + \alpha_{24} + \alpha_{34} + \alpha_{44} + \alpha_{l4} + \alpha_{k4} + \alpha_{m4} = 1$$

q_2 : agricultural output

q_3 : forestry output

q_4 : output of rural general goods

q_{ki} : intermediate input

$l d_{ki}$: labor input

$k d_i$: capital input

m_{A2} : agricultural land

m_{B4} : land input in rural general firm

y : cut volume of forest

n_{R0} : natural environmental level in the rural area

n_{R3} : forest volume in the rural area

q_{Ai} : efficient parameter

$\alpha_{ki}, \alpha_{ki}, \alpha_{ij}, \alpha_{ki}, \alpha_{mi}$ and α_{ni} : elasticity parameters

α_{y3} : Leontief parameter

The natural environment and the forest volume in the production functions are treated as externalities. So the production functions are homogeneous of degree unity with respect to factor inputs. Then we consider the cost minimization in firms' behavior due to linear homogeneity of degree one leading to the conditional demands for intermediate goods, labor, capital, land and cut volume of forest.

$$q_{ij} = \left[\frac{q_j \alpha_{ij}}{q_{Aj} n_{R0}^{\alpha_{R0j}} n_{R3}^{\alpha_{R3j}} p_i} \right] \prod_{i=1}^4 \left[\frac{p_i}{\alpha_{ij}} \right]^{\alpha_{ij}} \left[\frac{w_3}{\alpha_{lj}} \right]^{\alpha_{lj}} \left[\frac{r_3}{\alpha_{kj}} \right]^{\alpha_{kj}} \left[\frac{g_{Bj}}{\alpha_{mj}} \right]^{\alpha_{mj}} \quad (j = 2, 4) \quad (40)$$

$$q_{i3} = \left[\frac{q_3 \alpha_{i3}}{q_{A3} p_i} \right] \prod_{i=1}^4 \left[\frac{p_i}{\alpha_{i3}} \right]^{\alpha_{i3}} \left[\frac{w_3}{\alpha_{l3}} \right]^{\alpha_{l3}} \left[\frac{r_3}{\alpha_{k3}} \right]^{\alpha_{k3}} \left[\frac{g_{B3}}{\alpha_{m3}} \right]^{\alpha_{m3}} \quad (41)$$

$$ld_j = \left[\frac{q_j \alpha_{lj}}{q_{Aj} n_{R0}^{\alpha_{n0}} n_{R3}^{\alpha_{n3}} w_3} \right] \prod_{i=1}^4 \left[\frac{p_i}{\alpha_{ij}} \right]^{\alpha_{ij}} \left[\frac{w_3}{\alpha_{lj}} \right]^{\alpha_{lj}} \left[\frac{r_3}{\alpha_{kj}} \right]^{\alpha_{kj}} \left[\frac{g_{Bj}}{\alpha_{mj}} \right]^{\alpha_{mj}} \quad (j = 2, 4) \quad (42)$$

$$ld_3 = \left[\frac{q_3 \alpha_{l3}}{q_{A3} w_3} \right] \prod_{i=1}^4 \left[\frac{p_i}{\alpha_{i3}} \right]^{\alpha_{i3}} \left[\frac{w_3}{\alpha_{l3}} \right]^{\alpha_{l3}} \left[\frac{r_3}{\alpha_{k3}} \right]^{\alpha_{k3}} \left[\frac{g_{B3}}{\alpha_{m3}} \right]^{\alpha_{m3}} \quad (43)$$

$$kd_j = \left[\frac{q_j \alpha_{kj}}{q_{Aj} n_{R0}^{\alpha_{n0}} n_{R3}^{\alpha_{n3}} r_3} \right] \prod_{i=1}^4 \left[\frac{p_i}{\alpha_{ij}} \right]^{\alpha_{ij}} \left[\frac{w_3}{\alpha_{lj}} \right]^{\alpha_{lj}} \left[\frac{r_3}{\alpha_{kj}} \right]^{\alpha_{kj}} \left[\frac{g_{Bj}}{\alpha_{mj}} \right]^{\alpha_{mj}} \quad (j = 2, 4) \quad (44)$$

$$kd_3 = \left[\frac{q_3 \alpha_{k3}}{q_{A3} r_3} \right] \prod_{i=1}^4 \left[\frac{p_i}{\alpha_{i3}} \right]^{\alpha_{i3}} \left[\frac{w_3}{\alpha_{l3}} \right]^{\alpha_{l3}} \left[\frac{r_3}{\alpha_{k3}} \right]^{\alpha_{k3}} \left[\frac{g_{B3}}{\alpha_{m3}} \right]^{\alpha_{m3}} \quad (45)$$

$$m_{Bj} = \left[\frac{q_j \alpha_{mj}}{q_{Aj} n_{R0}^{\alpha_{n0}} n_{R3}^{\alpha_{n3}} g_{Bj}} \right] \prod_{i=1}^4 \left[\frac{p_i}{\alpha_{ij}} \right]^{\alpha_{ij}} \left[\frac{w_3}{\alpha_{lj}} \right]^{\alpha_{lj}} \left[\frac{r_3}{\alpha_{kj}} \right]^{\alpha_{kj}} \left[\frac{g_{Bj}}{\alpha_{mj}} \right]^{\alpha_{mj}} \quad (j = 2, 4) \quad (46)$$

$$y_3 = \alpha_{y3} q_3 \quad (47)$$

6. Households in Rural Area

In the rural area we do not taken into account the area size and location. Thus we can specify the household behavior as intertemporal utility maximization. The representative household behavior may be illustrated as follows:

$$\max \int_0^{\infty} \frac{\sigma_3}{\sigma_3 - 1} \left[\prod_{k=1}^4 c_{k3}^{\beta_{k3}} m_{H3}^{\beta_{m3}} n_{R0}^{\beta_{R03}} n_{R3}^{\beta_{R33}} \right]^{(\sigma_3 - 1) / \sigma_3} e^{-\xi t} dt \quad (48)$$

subject to

$$\begin{aligned} \dot{k} s_3 &= [w_3 + (r_3 - p_1(\mathbf{x}_B) \delta) k s_3 + g_{B2} m s_{B2} + g_{B4} m s_{B4} + g_{H3} m s_{H3} \\ &\quad - \sum_{i=1}^4 p_i c_{i3} - g_{H3} m_{H3}] / p_1(\mathbf{x}_B) - (\dot{N}_3 / N_3) k s_3 \end{aligned} \quad (49)$$

$$I_3 \equiv [w_3 + r_3 k s_3 + g_{B2} m s_{B2} + g_{B4} m s_{B4} + g_{H3} m s_{H3} - \sum_{k=1}^4 p_k c_{k3} - g_{H3} m_{H3}] / p_1(\mathbf{x}_B) \quad (50)$$

where

- σ_3 : intertemporal elasticity of substitution
- c_{13} : household consumption of urban goods
- c_{23} : household consumption of agricultural products
- c_{33} : household consumption of forestry products
- c_{43} : household consumption of rural goods
- m_{H3} : residential land
- ξ : subjective discount rate
- ks_3 : capital stock endowed by a household
- w_3 : wage rate prevailing in the rural area
- r_3 : capital return rate prevailing in the rural area
- δ : capital depreciation rate
- p_i : price of good i
- g_{H3} : residential land rent
- \dot{N}_3 / N_3 : population growth rate in the rural area

Moreover we should take into account the natural environment and forest. Those are specified as follows:

$$z_3 = \sum_{k=2}^4 \eta_k q_k + \sum_{k=1}^4 \mu_k N_3 c_{k3} \quad (51)$$

$$n_{R0} = \bar{n}_{R0} - \frac{z_3}{q_{B6} m_{B2}^{\gamma_{B6}} n_{R3}^{\gamma_{R6}}} \quad (52)$$

$$\dot{n}_{R3} = (\varepsilon_3 - \theta_3 n_{R3}) n_{R3} - y_3 \quad (53)$$

where

- z_3 : pollution in the rural area
- η_k and μ_k : emission factors
- \bar{n}_{R0} : natural environmental level before pollution
- ε_3 : carrying capacity
- θ_3 : ecological parameter

To solve the rural household optimization behavior, we introduce the current value Hamiltonian which is as follows:

$$H_3 \equiv \frac{\sigma_3}{\sigma_3 - 1} \left[\prod_{k=1}^4 c_{k3}^{\beta_{k3}} m_{H3}^{\beta_{m3}} n_{R0}^{\beta_{R03}} n_{R3}^{\beta_{R33}} \right]^{(\sigma_3 - 1) / \sigma_3} \\ + \lambda_3 \{ [w_3 + (r_3 - p_1(\mathbf{x}_B) \delta) ks_3 + g_{B2} ms_{B2} + g_{B4} ms_{B4} + g_{H3} ms_{H3} - p_1(\mathbf{x}_B) c_{13} - \sum_{k=2}^4 p_k c_{k3} \\ - g_{H3} m_{H3}] / p_1(\mathbf{x}_B) - (\dot{N}_3 / N_3) ks_3 \} \quad (54)$$

Applying the optimal control theory, we obtain the current consumption of goods, residential land, the dynamic equation for the costate variable.

The necessary and sufficient conditions for optimality are as follows:

$$\dot{\lambda}_3 = -\partial H_3 / \partial ks_3 + \xi \lambda_3 \quad (55)$$

$c_{12}, c_{22}, c_{32}, c_{42}$ and m_{H2} maximize Hamiltonian in each time

$$\text{transversality condition: } \lim_{t \rightarrow \infty} \lambda_3 \cdot k s_3 \cdot e^{-\xi t} = 0 \quad (56)$$

Those conditions are expressed as follows:

$$\dot{k} s_3 = [w_3 + (r_3 - p_1(\mathbf{x}_B)\delta)k s_3 + g_{B2} m s_{B2} + g_{B4} m s_{B4} + g_{H3} m s_{H3} - p_1(\mathbf{x}_B)c_{13} - \sum_{k=2}^4 p_k c_{k3} - g_{H3} m_{H3}] / p_1(\mathbf{x}_B) - (\dot{N}_3 / N_3) k s_3 \quad (57)$$

$$\dot{\lambda}_3 = \lambda_3 [\xi - r_3 / p_1(\mathbf{x}_B) + \delta + \dot{N}_3 / N_3] \quad (58)$$

$$c_{i3} = \left[\frac{p_1(\mathbf{x}_B)}{\lambda_3} \right]^{\frac{\sigma_3}{\sigma_3 + (1-\sigma_3)\beta_{T3}}} \left[\frac{\beta_{i3}}{p_i(\mathbf{x}_B)} \right]^{\frac{(\sigma_3-1)\beta_{k3}}{\sigma_3 + (1-\sigma_3)\beta_{T3}}} \prod_{k=1}^4 \left[\frac{\beta_{k3}}{p_k} \right]^{\frac{(\sigma_3-1)\beta_{m3}}{\sigma_3 + (1-\sigma_3)\beta_{T3}}} \left[\frac{\beta_{m3}}{g_{H3}} \right]^{\frac{(\sigma_3-1)\beta_{m3}}{\sigma_3 + (1-\sigma_3)\beta_{T3}}} \cdot \frac{\sigma_3 \beta_{R03}}{n_{R0}^{\sigma_3 + (1-\sigma_3)\beta_{T3}}} \frac{\sigma_3 \beta_{R33}}{n_{R3}^{\sigma_3 + (1-\sigma_3)\beta_{T3}}} \quad (59)$$

$$m_{H3} = \left[\frac{p_1(\mathbf{x}_B)}{\lambda_3} \right]^{\frac{\sigma_3}{\sigma_3 + (1-\sigma_3)\beta_{T3}}} \left[\frac{\beta_{m3}}{g_{H3}} \right]^{\frac{(\sigma_3-1)\beta_{k3}}{\sigma_3 + (1-\sigma_3)\beta_{T3}}} \prod_{k=1}^4 \left[\frac{\beta_{k3}}{p_k} \right]^{\frac{(\sigma_3-1)\beta_{m3}}{\sigma_3 + (1-\sigma_3)\beta_{T3}}} \left[\frac{\beta_{m3}}{g_{H3}} \right]^{\frac{(\sigma_3-1)\beta_{m3}}{\sigma_3 + (1-\sigma_3)\beta_{T3}}} \cdot \frac{\sigma_3 \beta_{R03}}{n_{R0}^{\sigma_3 + (1-\sigma_3)\beta_{T3}}} \frac{\sigma_3 \beta_{R33}}{n_{R3}^{\sigma_3 + (1-\sigma_3)\beta_{T3}}} \quad (60)$$

where $\beta_{T3} \equiv \beta_{13} + \beta_{23} + \beta_{33} + \beta_{43} + \beta_{m3}$

7. Property of the Hamiltonian

Let us further examine the implication of Hamiltonian. Along the optimal trajectory, all the variables in the model can be represented by functions of the stock variables and their associated costate variables. Thus time derivative in the Hamiltonian on the optimal path H^* can be calculated by applying the canonical system of equations of the stock variables as;

$$\frac{dH_3^*}{dt} = \frac{\partial H_3^*}{\partial k_3} \frac{dk_3}{dt} + \frac{\partial H_3^*}{\partial \lambda_3} \frac{d\lambda_3}{dt} = (\xi \lambda_3 - \frac{d\lambda_3}{dt}) \frac{dk_3}{dt} + \frac{dk_3}{dt} \frac{d\lambda_3}{dt} = \xi \lambda_3 \frac{dk_3}{dt} = \xi (H_3^* - u_3^*) \quad (61)$$

where u_3^* stands for the value of utility function on the optimal trajectory. Solving this differential equation, one can obtain another expression of the *Hamiltonian*.

$$H_3^*(t) = \xi \int_t^\infty u_3^*(\tau) e^{-\xi(\tau-t)} d\tau \quad (62)$$

That is, the value of *Hamiltonian* at time t , $H_3^*(t)$, is calculated as the integration of the present value of the maximized utility being multiplied by the subjective discount rate ξ . Further calculation on $H_3^*(t)$ yields;

$$\int_t^\infty H_3^*(t) e^{-\xi(\tau-t)} d\tau = \int_t^\infty u_3^*(\tau) e^{-\xi(\tau-t)} d\tau \quad (63)$$

Equation (63) in turn indicates that the integration of discounted constant income stream $H_3^*(t)$ equals the integration of the present value of the household utility. Therefore $H_3^*(t)$ can be regarded as a social welfare index giving the stationary equivalent to the future utility.

8. Land Market Equilibrium Conditions

Denoting the agricultural land rent by g_{B2} , which is determined in the rural area, the market rent function $g(\mathbf{x})$

over the city in equilibrium is described as follows:

$$g(\mathbf{x}) \equiv \max \{g_B(\mathbf{x}), g_{H1}(\mathbf{x}), g_{H2}(\mathbf{x}), g_{B2}\} \quad (\text{on the normal land}) \quad (64)$$

$$g(\mathbf{x}) \equiv \max \{g_B(\mathbf{x}), g_{H1}(\mathbf{x}), g_{H2}(\mathbf{x})\} \quad (\text{in the flood prone area}) \quad (65)$$

The reason of formula (65) is that the flood prone area can not be used for agriculture because of the periodical inundation. The flood prone area is assumed to be located within the residential area. When we assume that the business area is located around the city center, and the residential area is located surrounding the business area, the land equilibrium conditions are expressed as follows (discussions about this point is shown in Miyata (2011)):

$$g(\mathbf{x}) = g_{B1}(\mathbf{x}) \geq g_{H1}(\mathbf{x}) \text{ and } g_{H2}(\mathbf{x}) \text{ for } \mathbf{x} \in \text{business area} \quad (66)$$

$$g(\mathbf{x}) = g_{B1}(\mathbf{x}) = g_{H1}(\mathbf{x}) \quad \text{for } \mathbf{x} \in \text{the boundary between the business and the residential areas} \quad (67)$$

$$g(\mathbf{x}) = g_{H1}(\mathbf{x}) \geq g_{B1}(\mathbf{x}) \text{ and } g_{H2}(\mathbf{x}) \text{ for } \mathbf{x} \in \text{residential area on the normal land} \quad (68)$$

$$g(\mathbf{x}) = g_{H2}(\mathbf{x}) \geq g_{B1}(\mathbf{x}) \text{ and } g_{H1}(\mathbf{x}) \text{ for } \mathbf{x} \in \text{flood prone area} \quad (69)$$

$$g(\mathbf{x}) = g_{H1}(\mathbf{x}) = g_{B2} \text{ for } \mathbf{x} \in \text{city boundary} \quad (70)$$

$$N_3 m_{S_{B2}} = m_{B2} \quad \text{agricultural land equilibrium condition in the rural area} \quad (71)$$

$$N_3 m_{S_{B4}} = m_{B4} \quad \text{firms land equilibrium condition in the rural area} \quad (72)$$

$$N_3 m_{S_{H3}} = N_3 m_{H3} \quad \text{residential land equilibrium condition in the rural area} \quad (73)$$

9. Local Balance Equations for Commodity and Labor

Let us assume the transport technology for commodities as von Thünen technology with a cost ratio a_i . That is, the transport cost is incurred in the transported commodities being expressed as $a_i q_i$ in carrying q_i unit of commodities in the unit distance. Let $\varphi_i(\mathbf{x})$ be a two dimensional vector of commodities of type i , that is urban good, agricultural good, forestry good and rural good, transported to location \mathbf{x} in unit time and in unit area size. Then the following local balance equations for commodities hold (Beckmann and Puu (1985)).

$$\text{div } \varphi_1(\mathbf{x}) = q_1(\mathbf{x})b(\mathbf{x}) - I_1(\mathbf{x})b(\mathbf{x}) - c_{11}(\mathbf{x})h_1(\mathbf{x}) - c_{21}(\mathbf{x})h_2(\mathbf{x}) - a_1 \|\varphi_1(\mathbf{x})\| \quad (74)$$

$$\text{div } \varphi_2(\mathbf{x}) = -b(\mathbf{x})q_{21}(\mathbf{x}) - c_{21}(\mathbf{x})h_1(\mathbf{x}) - c_{22}(\mathbf{x})h_2(\mathbf{x}) - a_2 \|\varphi_2(\mathbf{x})\| \quad (75)$$

$$\text{div } \varphi_3(\mathbf{x}) = -b(\mathbf{x})q_{31}(\mathbf{x}) - c_{31}(\mathbf{x})h_1(\mathbf{x}) - c_{32}(\mathbf{x})h_2(\mathbf{x}) - a_3 \|\varphi_3(\mathbf{x})\| \quad (76)$$

$$\text{div } \varphi_4(\mathbf{x}) = -b(\mathbf{x})q_{41}(\mathbf{x}) - c_{41}(\mathbf{x})h_1(\mathbf{x}) - c_{42}(\mathbf{x})h_2(\mathbf{x}) - a_4 \|\varphi_4(\mathbf{x})\| \quad (77)$$

where

$b(\mathbf{x})$: density of firms at \mathbf{x} in the city area

$h_1(\mathbf{x})$: density of high income households at \mathbf{x} in the city

$h_2(\mathbf{x})$: density of low income households at \mathbf{x} in the city

a_i : von Thünen coefficient

$\|\cdot\|$: norm of a two dimensional vector

Similarly, let b_i denote the transport cost in transporting unit labor of high (low) income households in unit distance. This cost is incurred in labor itself as well. Let $\psi_i(\mathbf{x})$ express a two dimensional vector of labor in unit time

and in unit area size transported to location \mathbf{x} . Then the local balance equation for labor held at location \mathbf{x} is expressed as follows (Beckmann and Puu (1985)):

$$\text{div } \psi_i(\mathbf{x}) = ls_i h_i(\mathbf{x}) - ld_i(\mathbf{x})b(\mathbf{x}) - b_i \|\psi_i(\mathbf{x})\| \quad (i = 1, 2) \quad (78)$$

10. The Local Government and Absentee Landowners

The city area is assumed to be occupied by absentee landowners. Absentee landowners reside outside of the city. The local government rents all land in the city from absentee landowners. It rents the land to firms and households in the city at market rents. The revenue of the local government from the land rent is redistributed to households. The redistributed household income $\pi_{Hh}(\mathbf{x})$ is indicated as formula (23) mentioned earlier. $\theta_1(\mathbf{x})$ and $\theta_2(\mathbf{x})$ in formula (23) are policy parameters which aim to reduce the illegal settlements in the flood prone area.

11. Global Equilibrium Condition in Commodity and Labor Markets

Integrating the commodity local balance equation (74), and applying Gauss divergence theorem, one can obtain the global equilibrium condition on commodities as presented in equation (79).

$$\begin{aligned} \iint_A \text{div } \varphi_1(\mathbf{x}) dx_1 dx_2 &= \iint_A [q_1(\mathbf{x})b_1(\mathbf{x}) - I_1(\mathbf{x})b_1(\mathbf{x}) - c_{11}(\mathbf{x})h_1(\mathbf{x}) - c_{12}(\mathbf{x})h_2(\mathbf{x}) \\ &\quad - a_1 \|\varphi_1(\mathbf{x})\|] dx_1 dx_2 \\ &= \int_{\partial A} q_{n1}(\mathbf{x}(s)) ds = q_{12} + q_{13} + q_{14} + c_{13} + I_3 \end{aligned} \quad (79)$$

Next integrating the labor local balance equation (78) with Gauss divergence theorem, one can obtain the global equilibrium equation for labor. Since we assume that there is no in- and out-migration at the city boundary in equilibrium, the line integral of labor migration along the city boundary becomes zero as well.

$$\begin{aligned} \iint_A \text{div } \psi_i(\mathbf{x}) dx_1 dx_2 &= \iint_A [ls_i h_i(\mathbf{x}) - ld_i(\mathbf{x})b_1(\mathbf{x}) - b_i \|\psi_i(\mathbf{x})\|] dx_1 dx_2 \\ &= \int_{\partial A} l_{ni}(\mathbf{x}(s)) ds = 0 \end{aligned} \quad (80)$$

12. Commodity Transport Agent (C.T.A.) and Labor Transport Agents (L.T.A₁ and L.T.A₂)

Transport of commodities is assumed to be done by the commodity transport agent (C.T.A.) (Beckmann and Puu (1985)). The C.T.A. buys $p_1(\mathbf{x})q_1(\mathbf{x})b_1(\mathbf{x})$ of commodities at point \mathbf{x} , and sells $p_1(\mathbf{x})c_1(\mathbf{x})h_1(\mathbf{x}) + p_1(\mathbf{x})c_2(\mathbf{x})h_2(\mathbf{x})$ of commodities to households. Thus the profit of C.T.A. at \mathbf{x} is expressed as follows:

$$\begin{aligned} \pi_{Tq_1}(\mathbf{x}) &= p_1(\mathbf{x})c_{11}(\mathbf{x})h_1(\mathbf{x}) + p_1(\mathbf{x})c_{21}(\mathbf{x})h_2(\mathbf{x}) + p_1(\mathbf{x})I_1(\mathbf{x})b(\mathbf{x}) - p_1(\mathbf{x})q_1(\mathbf{x})b(\mathbf{x}) \\ &= -p_1(\mathbf{x})\text{div } \varphi_1(\mathbf{x}) - p_1(\mathbf{x})a_1 \|\varphi_1(\mathbf{x})\| \end{aligned} \quad (81)$$

$$\begin{aligned} \pi_{Tq_1}(\mathbf{x}_B) &= p_1(\mathbf{x}_B)c_{11}(\mathbf{x}_B)h_1(\mathbf{x}_B) + p_1(\mathbf{x}_B)c_{21}(\mathbf{x}_B)h_2(\mathbf{x}_B) + p_1(\mathbf{x}_B)I_1(\mathbf{x}_B)b(\mathbf{x}_B) \\ &\quad + p_1(\mathbf{x}_B)q_{12}(\mathbf{x}_B) + p_1(\mathbf{x}_B)q_{13}(\mathbf{x}_B) + p_1(\mathbf{x}_B)q_{14}(\mathbf{x}_B) + p_1(\mathbf{x}_B)c_{13}(\mathbf{x}_B) + p_1(\mathbf{x}_B)I_3(\mathbf{x}_B) \\ &\quad - p_1(\mathbf{x}_B)q_1(\mathbf{x}_B)b(\mathbf{x}_B) = -p_1(\mathbf{x}_B)\text{div } \varphi_1(\mathbf{x}_B) - p_1(\mathbf{x}_B)a_1 \|\varphi_1(\mathbf{x}_B)\| \end{aligned} \quad (82)$$

The C.T.A. aims to find the optimal route which maximizes the profit earned over the entire city area. The profit of the C.T.A. over the entire city area is written as follows:

$$\begin{aligned}
& \iint_A \pi_{Tq_1}(\mathbf{x}) dx_1 dx_2 = \iint_A [p_1(\mathbf{x})c_{11}(\mathbf{x})h_1(\mathbf{x}) + p_1(\mathbf{x})c_{21}(\mathbf{x})h_2(\mathbf{x}) \\
& \quad + p_1(\mathbf{x})I_1(\mathbf{x})b(\mathbf{x}) - p_1(\mathbf{x})q_1(\mathbf{x})b(\mathbf{x})] dx_1 dx_2 \\
& \quad + \int_{\partial A} [p_1(\mathbf{x}_B(s))q_{12}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{14}(\mathbf{x}_B(s)) \\
& \quad + p_1(\mathbf{x}_B(s))c_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))I_3(\mathbf{x}_B(s)) - p_1(\mathbf{x}_B(s))q_1(\mathbf{x}_B(s))b(\mathbf{x}_B(s))] ds \\
& = - \iint_A [p(\mathbf{x})\text{div } \varphi(\mathbf{x}) + p(\mathbf{x})a \|\varphi(\mathbf{x})\|] dx_1 dx_2 \\
& \quad + \int_{\partial A} [p_1(\mathbf{x}_B(s))q_{12}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{14}(\mathbf{x}_B(s)) \\
& \quad + p_1(\mathbf{x}_B(s))c_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))I_3(\mathbf{x}_B(s)) - p_1(\mathbf{x}_B(s))q_1(\mathbf{x}_B(s))b(\mathbf{x}_B(s))] ds
\end{aligned} \tag{83}$$

The necessary and sufficient condition for the profit maximization in the *C.T.A.* is derived from the calculus variation (Gelfand and Fomin [11] (1963)). The Euler-Lagrange equation in the calculus variation is as follows:

$$\frac{d}{dx_1} \frac{\partial \pi_{Tq_1}}{\partial(\partial \varphi_{1j} / \partial x_1)} + \frac{d}{dx_2} \frac{\partial \pi_{Tq_1}}{\partial(\partial \varphi_{1j} / \partial x_2)} - \frac{\partial \pi_{Tq_1}}{\partial \varphi_{1j}} = 0 \tag{84}$$

where $\mathbf{x} = (x_1, x_2)$ and $\varphi_1(\mathbf{x}) = (\varphi_{11}(x_1, x_2), \varphi_{12}(x_1, x_2))$.

Transforming the profit in the *C.T.A.* in the x_1 - x_2 coordinate, we have;

$$\pi_{Tq_1}(\mathbf{x}) = -p_1(\mathbf{x})(\text{div } \varphi_1(\mathbf{x}) - p_1(\mathbf{x})a_1 \|\varphi_1(\mathbf{x})\|) = -p_1(x_1, x_2) \left\{ \frac{\partial \varphi_{11}}{\partial x_1} + \frac{\partial \varphi_{12}}{\partial x_2} + a(\varphi_{11}^2 + \varphi_{12}^2)^{1/2} \right\} \tag{85}$$

Therefore the Euler-Lagrange equation is concretely expressed as follows:

$$\frac{\partial \pi_{Tq_1}}{\partial(\partial \varphi_{1i} / \partial x_i)} = -p_1(\mathbf{x}) \tag{86}$$

$$\frac{d}{dx_i} \frac{\partial \pi_{Tq_1}}{\partial(\partial \varphi_{1i} / \partial x_i)} = -\frac{\partial p_1(\mathbf{x})}{\partial x_i} \tag{87}$$

$$\frac{\partial \pi_{Tq_1}}{\partial \varphi_{1i}} = -p_1(\mathbf{x})a_1 \frac{\varphi_{1i}(\mathbf{x})}{\|\varphi_1(\mathbf{x})\|} \tag{88}$$

$$\therefore p_1(\mathbf{x})a_1 \frac{\varphi_1(\mathbf{x})}{\|\varphi_1(\mathbf{x})\|} = \text{grad } p_1(\mathbf{x}) \tag{89}$$

In equation (89), $\varphi_1(\mathbf{x}) / \|\varphi_1(\mathbf{x})\|$ stands for the direction along which commodities are transported, and that direction coincides the gradient of commodity price. This condition is the optimal one for commodity transport. In other words, the direction along which commodities are carried is the direction where commodity price gets highest.

$p_1(\mathbf{x})a_1\varphi_1(\mathbf{x}) / \|\varphi_1(\mathbf{x})\|$ depicts the cost of carrying a unit commodity in a unit distance. Let us calculate the transport cost in carrying a unit commodity from the point \mathbf{x}_A to \mathbf{x}_B on the optimal route. We denote the route by $D(s) = (x_1(s), x_2(s))$ ($0 \leq s \leq 1$, $\mathbf{x}_A = (x_1(0), x_2(0))$, $\mathbf{x}_B = (x_1(1), x_2(1))$). Thus the transport cost is expressed as follows:

$$\int_0^1 p_1(\mathbf{x}(s))a_1 \frac{\varphi_1(\mathbf{x}(s))}{\|\varphi_1(\mathbf{x}(s))\|} \frac{d\mathbf{x}(s)}{ds} ds = \int_0^1 \text{grad } p_1(\mathbf{x}(s)) \frac{d\mathbf{x}(s)}{ds} ds = p_1(\mathbf{x}_B) - p_1(\mathbf{x}_A) \tag{90}$$

This equation implies that the transport cost in transporting the unit commodity on the optimal route is the dif-

ference between commodity prices at different points. This also asserts that the transport cost is incurred in commodity price. Then let us calculate the profit in the *C.T.A.* earned in the entire city area. Multiplying the both hand sides in equation (89) by $\varphi_1(\mathbf{x})$ in the sense of scalar product, we have;

$$p_1(\mathbf{x})a_1 \parallel \varphi_1(\mathbf{x}) \parallel = \varphi_1(\mathbf{x})grad p_1(\mathbf{x}) \quad (91)$$

$$\begin{aligned} \therefore \int \int_A \pi_{Tq_1}(\mathbf{x})dx_1dx_2 &= -\int \int_A [p_1(\mathbf{x})div \varphi_1(\mathbf{x}) + p_1(\mathbf{x})a_1 \parallel \varphi_1(\mathbf{x}) \parallel]dx_1dx_2 \\ &+ \int_{\partial A} [p_1(\mathbf{x}_B(s))q_{12}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{14}(\mathbf{x}_B(s)) \\ &+ p_1(\mathbf{x}_B(s))c_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))I_3(\mathbf{x}_B(s)) - p_1(\mathbf{x}_B(s))q_1(\mathbf{x}_B(s))b(\mathbf{x}_B(s))]ds \\ &= -\int \int_A [p_1(\mathbf{x})div \varphi_1(\mathbf{x}) + \varphi_1(\mathbf{x})grad p_1(\mathbf{x})]dx_1dx_2 \\ &+ \int_{\partial A} [p_1(\mathbf{x}_B(s))q_{12}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{14}(\mathbf{x}_B(s)) \\ &+ p_1(\mathbf{x}_B(s))c_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))I_3(\mathbf{x}_B(s)) - p_1(\mathbf{x}_B(s))q_1(\mathbf{x}_B(s))b(\mathbf{x}_B(s))]ds \end{aligned} \quad (92)$$

By the way, $div p_1(\mathbf{x})\varphi_1(\mathbf{x}) = p_1(\mathbf{x})div \varphi_1(\mathbf{x}) + \varphi_1(\mathbf{x})grad p_1(\mathbf{x})$ holds. Thus equation (92) can further be transformed as;

$$\begin{aligned} \int \int_A \pi_{Tq_1}(\mathbf{x})dx_1dx_2 &= -\int \int_A div p_1(\mathbf{x})\varphi_1(\mathbf{x})dx_1dx_2 \\ &+ \int_{\partial A} [p_1(\mathbf{x}_B(s))q_{12}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{14}(\mathbf{x}_B(s)) \\ &+ p_1(\mathbf{x}_B(s))c_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))I_3(\mathbf{x}_B(s)) - p_1(\mathbf{x}_B(s))q_1(\mathbf{x}_B(s))b(\mathbf{x}_B(s))]ds \\ &= -\int_{\partial A} [p_1(\mathbf{x}_B(s))q_{12}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{14}(\mathbf{x}_B(s)) \\ &+ p_1(\mathbf{x}_B(s))c_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))I_3(\mathbf{x}_B(s)) - p_1(\mathbf{x}_B(s))q_1(\mathbf{x}_B(s))b(\mathbf{x}_B(s))]ds \\ &+ \int_{\partial A} [p_1(\mathbf{x}_B(s))q_{12}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))q_{14}(\mathbf{x}_B(s)) \\ &+ p_1(\mathbf{x}_B(s))c_{13}(\mathbf{x}_B(s)) + p_1(\mathbf{x}_B(s))I_3(\mathbf{x}_B(s)) - p_1(\mathbf{x}_B(s))q_1(\mathbf{x}_B(s))b(\mathbf{x}_B(s))]ds = 0 \end{aligned} \quad (93)$$

Equation (93) asserts that the total profit in the *C.T.A.* in the city area becomes zero.

In turn, we formulate the behavior of the labor transport agents (*L.T.A.*₁ and *L.T.A.*₂) (Beckmann and Puu (1985)). The *L.T.A.*_{*i*} receives wages of $w_i(\mathbf{x})l_{di}(\mathbf{x})b(\mathbf{x})$ from firms at location \mathbf{x} , and pays wages of $w_i(\mathbf{x})l_{si}h_i(\mathbf{x})$ to households of type *i*. As mentioned earlier, it is assumed that there is no migration from and to the outside of the city in equilibrium. Therefore the profit in the *L.T.A.*_{*i*} at \mathbf{x} is denoted as follows:

$$\pi_{Ti}(\mathbf{x}) = w_i(\mathbf{x})l_{di}(\mathbf{x})b(\mathbf{x}) - w_i(\mathbf{x})l_{si}h_i(\mathbf{x}) = -w_i(\mathbf{x})div \psi_i(\mathbf{x}) - w_i(\mathbf{x})b_i \parallel \psi_i(\mathbf{x}) \parallel \quad (94)$$

The *L.T.A.*_{*i*} determines labor transport routes so as to maximize the profit which will be gained over the entire city area. The optimal condition can be obtained by applying the calculus of variation as well.

$$w_i(\mathbf{x})b_i \frac{\psi_i(\mathbf{x})}{\parallel \psi_i(\mathbf{x}) \parallel} = grad w_i(\mathbf{x}) \quad (95)$$

13. Commodity Transport Agent (*C.T.A.*)

Transport of commodities is assumed to be done by the commodity transport agent (*C.T.A.*) (Beckmann and Puu (1985)). The *C.T.A.* buys $p_2(\mathbf{x}_B)q_2(\mathbf{x}_B)$ of agricultural products at a point on the city boundary \mathbf{x}_B , and sells

$p_2(\mathbf{x})q_{21}(\mathbf{x})b(\mathbf{x})+p_2(\mathbf{x})c_{21}(\mathbf{x})h_1(\mathbf{x}) + p_2(\mathbf{x})c_{22}(\mathbf{x})h_2(\mathbf{x})$ of commodities to firms and households. Thus the profit of *C.T.A.* at \mathbf{x} is expressed as follows:

$$\begin{aligned}\pi_{Tq_2}(\mathbf{x}) &= p_2(\mathbf{x})q_{21}(\mathbf{x})b(\mathbf{x}) + p_2(\mathbf{x})c_{21}(\mathbf{x})h_1(\mathbf{x}) + p_2(\mathbf{x})c_{22}(\mathbf{x})h_2(\mathbf{x}) \\ &= -p_2(\mathbf{x})\text{div } \varphi_2(\mathbf{x}) - p_2(\mathbf{x})a_2 \parallel \varphi_2(\mathbf{x}) \parallel\end{aligned}\quad (96)$$

$$\begin{aligned}\pi_{Tq_2}(\mathbf{x}_B) &= p_2(\mathbf{x}_B)q_{21}(\mathbf{x}_B)b(\mathbf{x}_B) + p_2(\mathbf{x}_B)c_{21}(\mathbf{x}_B)h_1(\mathbf{x}_B) + p_2(\mathbf{x}_B)c_{22}(\mathbf{x}_B)h_2(\mathbf{x}_B) \\ &\quad - p_2(\mathbf{x}_B)q_{n_2}(\mathbf{x}_B) \\ &= -p_2(\mathbf{x}_B)\text{div } \varphi_2(\mathbf{x}_B) - p_2(\mathbf{x}_B)a_2 \parallel \varphi_2(\mathbf{x}_B) \parallel\end{aligned}\quad (97)$$

$$\begin{aligned}\therefore \iint_A \pi_{Tq_2}(\mathbf{x})dx_1dx_2 &= -\iint_A [p_2(\mathbf{x})\text{div } \varphi_2(\mathbf{x}) + p_2(\mathbf{x})a_2 \parallel \varphi_2(\mathbf{x}) \parallel]dx_1dx_2 - \int_{\partial A} [p_2(\mathbf{x}_B(s))q_{n_2}(\mathbf{x}_B(s))]ds \\ &= -\iint_A \text{div } p_2(\mathbf{x})\varphi_2(\mathbf{x})dx_1dx_2 - \int_{\partial A} [p_2(\mathbf{x}_B(s))q_{n_2}(\mathbf{x}_B(s))]ds \\ &= \int_{\partial A} p_2(\mathbf{x}_B(s))q_{n_2}(\mathbf{x}_B(s))ds - \int_{\partial A} p_2(\mathbf{x}_B(s))q_{n_2}(\mathbf{x}_B(s))ds = 0\end{aligned}\quad (98)$$

Similarly we have the zero profit equations.

$$\iint_A \pi_{Tq_k}(\mathbf{x})dx_1dx_2 = 0 \quad (k=3, 4) \quad (99)$$

14. Commodity Price and Wage Rates

Equation (89) can be transformed into the following nonlinear first order partial differential equation.

$$\left(\frac{\partial \ln p_1(\mathbf{x})}{\partial x_1}\right)^2 + \left(\frac{\partial \ln p_1(\mathbf{x})}{\partial x_2}\right)^2 = a_1^2 \quad (100)$$

This equation can mathematically be solved. Here we consider a special case where the initial manifold is degenerated to one point. Thus the initial condition of $x_1(s, t)$, $x_2(s, t)$, and $\ln p_1(x_1, x_2)$ is specified as $x_1(0, t) = 0$, $x_2(0, t) = 0$, and $\ln p_1(0, t) = p_{10}$, respectively. Then the solution surface for the commodity price is written as equation (101). This equation stands for a cone with the vertex of $(0, 0, p_{10})$ and is said to be integral conoid (Courant and Hilbert (1962)).

$$p_1(x_1, x_2) = p_{10} \exp a_1 \sqrt{x_1^2 + x_2^2} \quad (101)$$

Similarly solution surfaces of other prices are as follows:

$$p_k(x_1, x_2) = p_{k0} \exp(a_k \sqrt{x_1^2 + x_2^2}) \quad (k=2, 3, 4) \quad (102)$$

As for the wage rates, the same discussion is possible. The solution surface for the wage rate i is expressed by;

$$w_k(x_1, x_2) = w_{k0} \exp(b_k \sqrt{x_1^2 + x_2^2}) \quad (k=1, 2) \quad (103)$$

When we assume that the business district encircles the origin, and the residential area is located outside the business area, that is, a simple von Thünen ring, the wage rate prevailing at the city center w_{k0} is given by the wage rate on the city boundary which is determined to satisfy the equilibrium conditions mentioned below. Contrary to the urban commodity price surface, the wage profile shows a decrease with exponential order from the city center to the city boundary.

15. Equilibrium Conditions

Skipping other details because of page limitation, we can finally derive the following equilibrium conditions.

Urban good market

$$\begin{aligned} \|\varphi_1(\mathbf{x}(S))\| = & \int_0^S [q_1(\mathbf{x}(\tau))b(\mathbf{x}(\tau)) - c_{11}(\mathbf{x}(\tau))h_1(\mathbf{x}(\tau)) - c_{12}(\mathbf{x}(\tau))h_2(\mathbf{x}(\tau)) \\ & - I_1(\mathbf{x}(\tau))b_2(\mathbf{x}(\tau))] \cdot (\tau/S) \exp a_1(\tau - S) d\tau = \sum_{j=2}^4 q_{n1j} + N_3 c_{n13} + I_{n3} \end{aligned} \quad (104)$$

Agricultural, forestry and rural goods market

$$\begin{aligned} \|\varphi_k(\mathbf{x}(0))\| = & \lim_{s \rightarrow S} \int_0^s [-q_{k1}(\mathbf{x}(S - \tau))b(\mathbf{x}(S - \tau)) \\ & - c_{k1}(\mathbf{x}(S - \tau))h_1(\mathbf{x}(S - \tau)) - c_{k2}(\mathbf{x}(S - \tau))h_2(\mathbf{x}(S - \tau))] (S - \tau / S - s) \quad (k=2, 3, 4) \quad (105) \\ & \exp a_k(\tau - S) d\tau + q_{nk} = 0 \end{aligned}$$

$$q_k = \int_{cA} q_{nk}(S) dS + \sum_{j=2}^4 q_{kj} + c_{k3} \quad (k=2, 3, 4) \quad (106)$$

Labor market for high income households

$$\begin{aligned} \|\psi_1(\mathbf{x}(0))\| \\ = \lim_{s \rightarrow S} \int_0^s [l_{s1}h_1(\mathbf{x}(S - \tau)) - l_{d1}(\mathbf{x}(S - \tau))b(\mathbf{x}(S - \tau))] (S - \tau) / (S - s) \exp b_1(\tau - S) d\tau = 0 \end{aligned} \quad (107)$$

Labor market for low income households

$$\begin{aligned} \|\psi_2(\mathbf{x}(0))\| \\ = \lim_{s \rightarrow S} \int_0^s [l_{s2}h_2(\mathbf{x}(S - \tau)) - l_{d2}(\mathbf{x}(S - \tau))b(\mathbf{x}(S - \tau))] (S - \tau) / (S - s) \exp b_2(\tau - S) d\tau = 0 \end{aligned} \quad (108)$$

Labor market in the rural area

$$N_3 = ld_2 + ld_3 + ld_4 \quad (109)$$

Capital market in the urban area

$$KS_1 = \iint_A kd_1(\mathbf{x})b(\mathbf{x})dx_1dx_2 \quad (110)$$

Capital market in the rural area

$$KS_3 = kd_2 + kd_3 + kd_4 \quad (111)$$

Land market

Formulae (66) to (73)

Location equilibrium conditions

$$\pi_{B1}(\mathbf{x}) = 0 \quad (\mathbf{x} \in \text{business area}) \quad (112)$$

$$u_1(\mathbf{x}) = u_1^* \quad (\mathbf{x} \in \text{normalland}) \quad (113)$$

$$u_2(\mathbf{x}) = H_3^* \quad (\mathbf{x} \in \text{flood prone area. } H_3^* \text{ is the rural utility level endogenous ly determined.}) \quad (114)$$

Constraints on the numbers of firms and households

$$M = \iint_{A_B} \frac{1}{m_B(\mathbf{x})} dx_1 dx_2 \quad (115)$$

$$N_1 = \iint_{A_{H1}} \frac{1}{m_{H1}(\mathbf{x})} dx_1 dx_2 \quad (116)$$

$$N_2 = \iint_{A_{H2}} \frac{1}{m_{H2}(\mathbf{x})} dx_1 dx_2 \quad (117)$$

$$N_3 : \text{exogenously given} \quad (118)$$

In the equations mentioned above, S , A_B , A_{H1} and A_{H2} depict the parameter value expressing the city boundary, business area, normal land, flood prone area, respectively. The model mentioned above describes the rural-urban-natural environment-flood interaction observed in Palangkaraya city in Indonesia.

16. Comparative Static Analysis

Here we consider the stationary state, and derive some propositions from the comparative static analysis.

Proposition 1.

The flood prone areas are occupied by low income households, while the normal land is occupied by high income households. The reason is that $\partial g_{Hi} / \partial Y_i < 0$ holds from equation (27). Therefore if the per capita income increases, then the bid rent function decreases. So the flood prone areas are occupied by the low income households.

Proposition 2.

A slight increase in income of L.I.H. decreases the bid rent and increase the bid max lot size. So parameter θ_i in equation (23) can decrease the number of L.I.H. in the flood prone areas.

Proposition 3.

An increase in the residential area in the rural area increases the current value Hamiltonian, so the supreme utility of L.I.H. in the flood prone areas is increased. Thus the number of L.I.H. is decreased in the flood prone areas.

Proposition 4.

If the carrying capacity of forest in the rural area is increased, the bid rent by L.I.H. is increased and the bid max lot size is decreased, resulting in an increase in the population in the flood prone areas although the flood risk is decreased.

Proposition 5.

If the von Thünen parameters a_k ($k = 1, 2, 3, 4$), b_i ($i = 1, 2$) are decreased by a transportation project, the business district and residential area (normal land) increases and utility level of H.I.H. increases. In the flood prone areas, the bid max lot size decreases leading to an increase in the number of L.I.H.

17. Concluding Remarks

Up to now the authors have developed partial and general equilibrium urban economic models for Palangkaraya City. In these models we have obtained a conclusion where the bid rent by L.I.H. is higher than that by

H.I.H. in flood prone area. This is due to the fact of introduction of the expected damage on household asset. This result is perfectly contrary to the result of the traditional urban economics.

Flood is a great concern in Palangkaraya City, and one of its causes is harvest of forest in rural area surrounding Palangkaraya City. Hence one must take into account the socio-economic activities and forest in the rural area. One of motivations of this study is this point. Moreover it is important to consider the plane city rather than a linear city for reality.

In this study Palangkaraya City is regarded as a plane city, and the external area of the city is assumed to be a point area (i.e. dimensionless). In the city and the external area, the natural environmental level and the externality of forest are taken into account. The role of forest is a reduction in flood damage in Palangkaraya City. Moreover the natural environmental levels in the city and the external area are improved by forest.

The important policy target of the Palangkaraya City government is to reduce the illegal settlements in the flood prone areas. For this policy target, an increase in the supreme utility level in the external area, a redistribution income policy and fostering the forest in the external area are studied by the comparative static analysis. And then this study suggests a possibility of reduction in the illegal settlements in the flood prone area.

By the way this analysis heavily depends on specific parameter values, hence it is necessary to estimate parameters by employing empirical data and to present more realistic policies. These are left for a future study.

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