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Garage and Curbside Parking Competition with Search Congestion*

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Abstract

In many downtown areas, privately operated parking garages compete with each other and with publicly operated curbside parking. Garages exercise market power by charging fees that vary with parking duration. Curbside space is scarce, and drivers have to search for it. This creates a congestion externality and enhances garages' market power. We show that with inelastic parking demand setting differentiated hourly curbside parking fees can support the social optimum without regulating garage fees. Second-best uniform curbside fees can also perform well. In general, first-best and second-best parking fees are sensitive to parking supply and demand conditions, and therefore should be tailored to local circumstances.

Keywords: endogenous outside option; parking; price discrimination; search costs; spatial competition

JEL Codes: D62; L13; R41; R48

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1 Introduction

Downtown parking markets can be rather complex. Parking is often available both on and off the street. Parking garages provide bulk capacity at discrete locations and can extend over multiple storeys above or below ground. The friction of space gives them market power. Curbside parking, on the other hand, is located more widely but it is frequently in short supply and difficult to find. According to some estimates, cruising for parking accounts for roughly thirty percent of traffic at certain times of day (Shoup, 2005, 2006). Time spent searching for parking increases the full price or generalized cost of curbside parking, and limits the degree to which it constrains garages' market power.

Garage and curbside parking differ in how they are priced. Garages generally cater to drivers who park for different lengths of time, and they usually charge hourly parking fees that vary with parking duration. Curbside parking is typically priced at a uniform hourly rate in North American cities where conventional parking meters are used. However, non-linear pricing is sometimes practiced in cities where labor is relatively cheap. For example, in Istanbul employees collect parking fees using hand terminal technology. Finally, administration arrangements vary. Curbside parking is publicly operated in most cities whereas garage parking can be publicly or privately operated and/or regulated. For example, some Dutch cities such as Maastricht and Almere regulate garage parking fees whereas garages in London and Boston are free to choose their prices.

In this paper we study downtown parking markets in which spatial competition between garage and curbside parking, nonlinear pricing, and curbside parking search congestion are simultaneously at play. To facilitate analysis, the model is kept simple by treating total parking demand as fixed, ignoring through traffic congestion, and considering only two types of individuals that differ in the amount of time they wish to park. Nevertheless, curbside parking search congestion creates an interdependence between parking submarkets and non-convexities in garages' profits, and the derivation of market equilibria in this setting is new to the spatial competition literature. In such an environment, we attempt to answer some questions about downtown parking markets: How does competition between parking garages play out when curbside parking is available as a substitute? How can garage parking fee schedules be explained? How should curbside parking fees be set to control cruising congestion and parking garage market power? Is a uniform hourly fee optimal, or should hourly fees be varied with parking duration? Is regulation of garage parking necessary to achieve a social optimum, or can curbside parking fees do the job?

Several strands of literature cover part of the ground required to address these questions

(see Arnott, 2011 for a literature review). Some studies consider curbside parking in isolation (Arnott and Inci, 2006, 2010). Others incorporate garage parking but omit heterogeneity with respect to parking durations (Arnott and Rowse, 2009; Arnott, Inci and Rowse, 2013). Parking duration is considered by Calthrop and Proost (2006) and Glazer and Niskanen (1992), and Arnott and Rowse (2013) analyze parking time limits. Spatial competition between parking garages has been studied (Anderson and de Palma, 2004, 2007; Arnott and Rowse, 1999, 2009; Calthrop and Proost, 2006) but without including price discrimination. Yet other studies analyze parking and pricing of parking when traffic congestion occurs at a bottleneck (Arnott, de Palma and Lindsey, 1991; Zhang *et al.*, 2005; Fosgerau and de Palma, 2013). Shoup (2005, 2006) and Arnott and Inci (2006, 2010) emphasize the importance of curbside parking search congestion externalities in downtown districts, and van Ommeren, Wentink, and Dekkers (2011) and van Ommeren, Wentink, and Rietveld (2012) estimate the externalities empirically. Perhaps the most thorough analysis of parking market competition to date is by Arnott (2006) who analyzes spatial competition between parking garages in his core model, which he later enriches to include curbside parking. However, he assumes that drivers are identical and he does not consider price discrimination.

Spatial competition and price discrimination have been extensively studied in the industrial organization literature (see Gabszewicz and Thisse (1986), Varian (1989), and Stole (2007) for literature reviews). Spatial competition models such as Salop's (1979) allow for the possibility that some potential customers choose not to buy a product or service from any firm, but select an outside option instead. These models can be adapted to the downtown parking market by treating parking garages as firms offering services that differ by location, and curbside parking as an outside option that is ubiquitous. The models typically assume that utility from the outside option is exogenous. However, in the parking market expected utility from curbside parking decreases with the number of individuals who use it because of search congestion. Our setting is unique in incorporating such an endogenous outside option into a Salop-type model.

A few empirical studies of competition in parking markets have recently appeared. Kobus, Gutierrez-i-Puigarnau, Rietveld and van Ommeren (2012) examine the effects of parking fees on drivers' choice between curbside and garage parking. Froeb, Tschantz, and Crooke (2003), De Nijs (2012), and Choné and Linnemer (2012) focus on the effects of mergers in the parking industry. Lin and Wang (2012) examine the relationship between competition and price discrimination. Several general lessons emerge from these studies which inspired the general structure of our model. First, hourly garage parking fees generally decline steeply with parking duration. Put another way, total payment or outlay is an increasing but steeply

curved concave function of parking duration. Second, the degree of curvature in the outlay curve declines with increased competition. Third, the marginal supply cost of parking is close to zero for garages. Fourth, drivers are reluctant to walk more than a few blocks from a parking garage to their destination.

The paper is organized as follows. Section 2 describes the model. Section 3 characterizes various possible equilibrium allocations of driver types between garage and curbside parking. Section 4 derives the socially optimal allocation of driver types between garage and curbside parking space, and shows how the allocation can be decentralized using differentiated hourly curbside parking fees. Section 5 uses a numerical example to illustrate how the welfare gains from implementing optimal differentiated fees depend on such parameters as the distance between parking garages, parking search costs, and walking time costs. Section 5 also assesses the relative efficiency of setting optimal uniform curbside fees. Section 6 summarizes and identifies directions for further research.

2 The Model

Consider a fixed set of individuals who travel to a downtown area by car and will be called drivers. Drivers differ in their destinations and lengths of stay. There are two types: High (H) and Low (L). A high-type driver is a *long-term parker* who requires parking for l_H hours, and a low-type driver is a *short-term parker* who requires parking for l_L hours, where $l_H > l_L$.¹ A type i driver, $i = H, L$, receives a benefit of B_i from a trip. The B_i s are large enough that all potential trips are made and total parking demand is therefore price inelastic.

Each driver has a given trip destination. Destinations are uniformly distributed around a circle with densities d_H for long-term parkers and d_L for short-term parkers. Parking is available at parking garages and on the curb.² Curbside parking is operated publicly and distributed continuously around the circle. Parking garages are privately operated and have fixed locations a distance D apart. Garage parking space is lumpy because of scale economies in garage capacity (Arnott, 2006).

¹One interpretation is that long-term parkers are commuters and short-term parkers are making business trips. Another is that long-term parkers are multipurpose or comparison shoppers who shop for an extended period of time, whereas short-term parkers are one-stop shoppers who need to park for a shorter time.

²Parking is sometimes also available at surface lots. Surface lots are typically built as transitory uses of land after buildings are torn down and therefore offer only temporary additional space to park. Surface lots are similar to garages in that they do not contribute appreciably to search congestion. In our model setting, lots can be treated as equivalent to garages.

Curbside parking in many cities is priced at a constant fee per hour. However, to allow for price discrimination and the use of curbside parking fees to enhance market efficiency, it is assumed that curbside parking fees can differ, with type i drivers paying an hourly fee of f_i , $i = H, L$. Short-term parkers therefore pay $f_L l_L$ to park for l_L hours, and long-term parkers pay $f_H l_H$ to park for l_H hours. Depending on how parking fees are levied and enforced, incentive compatibility constraints may apply. If $f_L < f_H$, a long-term parker might be able to save money by interrupting his visit, returning to his car, and either moving it to another parking spot or feeding the meter. But doing so would be inconvenient, and we rule it out. If, alternatively, $f_L > f_H$, it is possible that $f_L l_L > f_H l_H$. A short-term parker could then stay at the destination an extra $l_H - l_L$ hours and save $f_L l_L - f_H l_H$ on the parking bill. The driver might also be able to pay the long-term charge and then leave after l_L hours. To admit this possibility we will entertain the incentive compatibility constraint

$$f_L l_L \leq f_H l_H. \tag{1}$$

This constraint is not imposed in the analysis of Sections 3 and 4, but it is addressed in the numerical analysis of Section 5. To concentrate on the behavior of garage operators, curbside parking fees f_H and f_L are treated as exogenous until Section 4.

The time required to find a curbside parking space is assumed to be proportional to the total number of hours of curbside parking occupied, T , which is determined endogenously. A type i driver incurs a search cost of $k_i T$, where $k_i > 0$ is the type-specific unit search cost.³ The generalized cost of curbside parking for a type i driver is therefore $f_i l_i + k_i T$, and the net benefit from a trip is

$$B_i - f_i l_i - k_i T, \quad i = H, L. \tag{2}$$

Parking garages charge an hourly fee of s_H for parking l_H hours, and an hourly fee of s_L for parking l_L hours. Because parking durations are fixed, garage operators cannot affect the proportions of short-term and long-term parkers. The time required to locate a garage and park a vehicle there is assumed to be negligible. However, garage customers have to walk from the garage to their destination and back. A type i driver incurs a round-trip walking

³One interpretation is that drivers require time to find a vacant spot, but can secure one close enough to their destination that walking time between the parking space and the destination is negligible. An alternative interpretation is that on-street parking is available not only on the circle, but also on intersecting roads and/or on roads that run parallel to the circle one or more blocks away. As nearby spots fill up, drivers have to travel further from the circle in order to find parking. Time is spent driving to a vacant spot, and walking to the circle and back. Since occupancy of space is proportional to length of stay, in both interpretations the time cost is proportional to the total number of hours that curbside parking is used rather than the number of vehicles that use it.

time cost of $w_i x$, where x is the distance between the parking garage and the destination, and w_i is the type-specific walking time cost per unit distance. Similar to the case with curbside parking, an incentive compatibility constraint may apply:

$$s_L l_L \leq s_H l_H. \quad (3)$$

As discussed later, it is also possible for the short-term garage parking fee to exceed the long-term fee (*i.e.*, $s_L > s_H$).

A type i driver who parks at a garage a distance x from his destination incurs a generalized cost of $s_i l_i + w_i x$ and gains a net benefit of

$$B_i - s_i l_i - w_i x, \quad i = H, L. \quad (4)$$

This specification embodies the assumption that a driver has a fixed *parking* duration which is equal to the sum of walking time to/from the destination and the *visit* duration. Visit duration therefore decreases with distance from a parking garage. This assumption simplifies the analysis. It also precludes the possibility for garage operators to price discriminate between drivers on the basis of their walking distance.

A parking garage incurs a cost of c for each hour that a car is parked.⁴ Thus, it earns a profit of $(s_i - c)l_i$ from a type i driver. To assure that parking garages can earn positive profits in equilibrium, it is assumed that the generalized cost of curbside parking when all drivers of both types park on the curb exceeds the supply cost of garage parking during their visit:

$$f_i l_i + k_i (d_H l_H + d_L l_L) D > c l_i, \quad i = H, L. \quad (5)$$

Following common practice in the literature on spatial competition, attention is focused on symmetric equilibria in which all garages employ the same parking fee schedules. Consider one garage called the “home garage.” Given the simple, deterministic nature of demand, competition on the circle is localized and the home garage only competes directly either with curbside parking or with the nearest garages on either side.⁵ In any candidate symmetric

⁴This cost includes any costs related to transactions, security, vehicle damage, and wear and tear on the garage. It could also reflect the shadow cost of parking space due to capacity constraints although capacity is not featured anywhere in the model.

⁵If the home garage sets parking fees much lower than its neighbors’ fees, it would gain the business not only of all customers located between it and its nearest neighbors but also some customers located on the far side of the neighbors. The literature has sometimes adopted a ‘no-mill-price-undercutting’ assumption to preclude hyper-competitive pricing and customer ‘leapfrogging’ of this sort. The potential profitability of undercutting is muted in the model here because utility from the outside alternative — curbside parking —

equilibrium, the nearest neighbors adopt the same parking fee schedules as the home garage. The home garage therefore attracts type i drivers from a distance x_i on either side. It earns a profit of $2(s_i - c)d_i l_i x_i$ from type i drivers, and a total profit of

$$\pi(s_H, s_L) = 2(s_H - c)d_H l_H x_H + 2(s_L - c)d_L l_L x_L. \quad (6)$$

The home garage chooses s_H and s_L to maximize (6).

3 Market Equilibrium

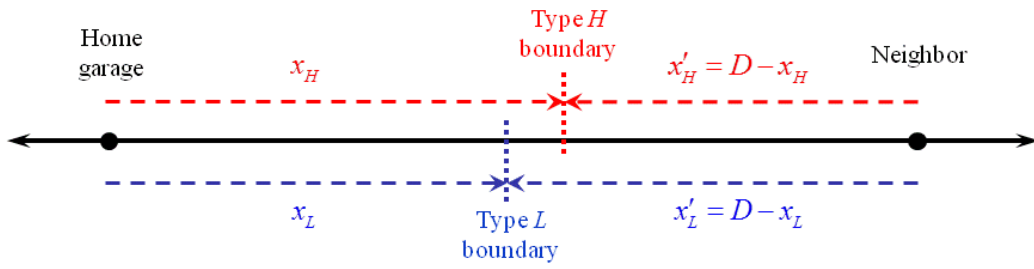
In any symmetric equilibrium, the home garage competes for each type of driver either with curbside parking or with its nearest garage neighbors. There are nine possible market configurations in all. Two are illustrated in Figure 1 by showing a section of the circle between the home garage and one of its neighbors. In panel (a), garage parking fees are so low that no one parks on the curb. The market boundary for type i drivers between the home garage and its neighbor is located at a distance x_i from the home garage and a distance $x'_i = D - x_i$ from the neighbor. A type i driver with a destination on the market boundary is indifferent between parking at the home garage and parking at the neighbor, while preferring both to parking on the curb. We call this driver the “marginal type i driver”. The boundaries can differ for long-term and short-term parkers although in a symmetric equilibrium both boundaries are located mid-way between the garages (*i.e.*, $x_H = x_L = D/2$). Because no drivers park on the curb, $T = 0$ in this configuration. We call this market configuration regime $Hg + Lg$, meaning that both driver types use only parking garages.

Another configuration in which some drivers park on the curb is shown in panel (b) of Figure 1. Type i drivers located within a distance x_i of the home garage park there, those located within a distance x'_i of the neighbor park there, and those located in the central region with a span of $D - x_i - x'_i$ park on the curb. Thus, the marginal type i driver who is located a distance x_i from the home garage is indifferent between parking at the home garage and on the curb. Similarly, the marginal type i driver located distance x'_i from the neighbor is indifferent between parking at the neighbor and curbside parking. Hence, the total number of hours spent parking on the curb is

$$T = d_H l_H (D - x_H - x'_H) + d_L l_L (D - x_L - x'_L). \quad (7)$$

decreases as the price of garage parking drops. Undercutting is not profitable in the symmetric equilibria we derive.

(a) No curbside parking; garage market areas abut (Regime $Hg+Lg$)



(b) Garage parking and curbside parking (Regime Int)

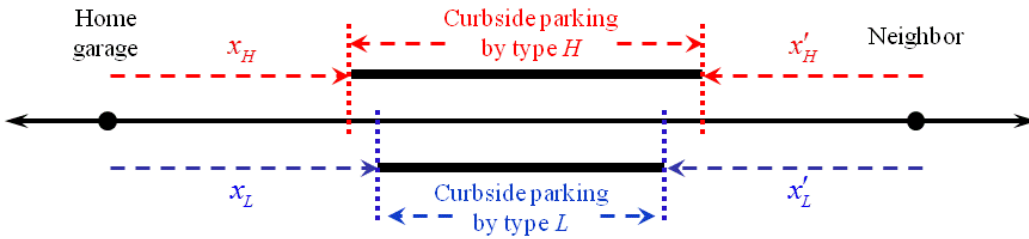


Figure 1: Alternative market configurations

We call this market configuration regime *Int*, meaning that each driver type uses both garage and curbside parking, and the market boundaries are in the interior of the market segment (i.e., $0 < x_H < D/2$ and $0 < x_L < D/2$).

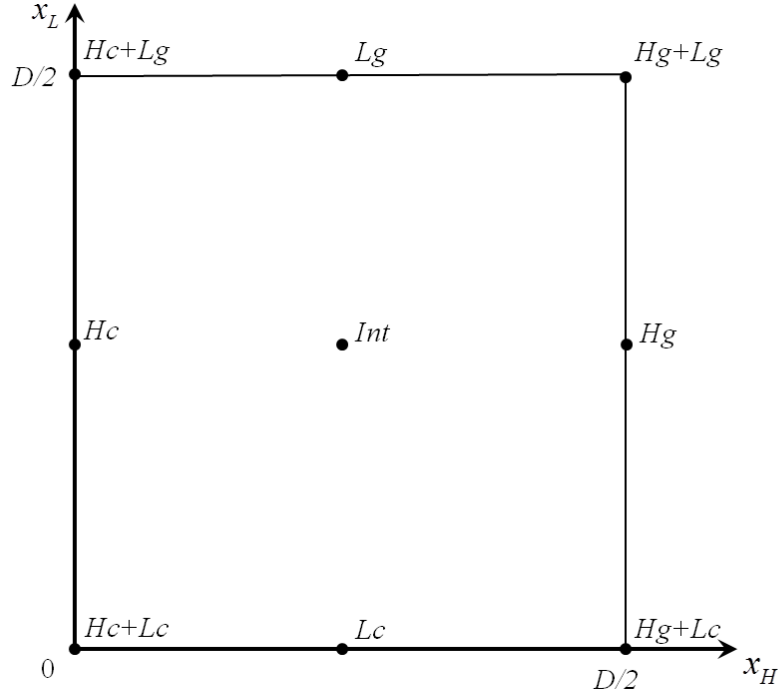


Figure 2: Candidate equilibrium regimes

The full set of nine market configurations is depicted in Figure 2. The location of the home garage's market boundary for long-term parkers is plotted on the horizontal axis, and the location of the boundary for short-term parkers is plotted on the vertical axis. In regime $Hg + Lg$, shown in panel (a) of Figure 1, both driver types park exclusively at garages. In regime *Int*, shown in panel (b) of Figure 1, both types split between garages and the curb. In regime *Hg*, long-term parkers use only garages while short-term parkers split between garages and curbside parking (i.e., $x_H = D/2$ and $0 < x_L < D/2$). Regime *Lg* is defined analogously. In regime *Hc*, long-term parkers use only curbside parking while short-term parkers split between garages and the curb (i.e., $x_H = 0$ and $0 < x_L < D/2$). Regime *Lc* is defined analogously. In regime $Hc + Lc$, both types of drivers park only on the curb (i.e., $x_H = x_L = 0$). In regime $Hg + Lc$, long-term parkers use only garages while short-term parkers use only the curb (i.e., $x_H = D/2$ and $x_L = 0$). Regime $Hc + Lg$ is defined analogously. Equation (7) is modified accordingly for each regime. Which one of the regimes prevails in equilibrium depends on parameter values including the curbside parking fees, f_H and f_L . Since f_H and f_L are treated as given in this section, the equilibrium will be called

the “current equilibrium”. Regimes *Int* and *Hg* are examined in the next two subsections in more detail in order to provide a better sense of the model’s properties and the nature of equilibrium. All other regimes are examined in the appendix.

3.1 Equilibrium for regime *Int*

In regime *Int*, shown in panel (b) of Figure 1, some drivers of each type park at garages and the rest park on the curb. The home garage chooses s_H and s_L to maximize its profit given in (6). Several conditions must hold for this regime to be an equilibrium. First, the marginal driver of each type must be indifferent between parking at the home garage and parking on the curb:

$$B_i - s_i l_i - w_i x_i = B_i - f_i l_i - k_i T, \quad i = H, L. \quad (8)$$

Second, the net benefit to a marginal type i driver of parking at the home garage must be strictly greater than the net benefit of parking at the neighbor:

$$B_i - s_i l_i - w_i x_i > B_i - s'_i l_i - w_i (D - x_i), \quad i = H, L. \quad (9)$$

This guarantees that some drivers of each type park on the curb. Third, if practically relevant the incentive compatibility constraint in (3) must hold.

The market boundaries, x_H , x_L , x'_H , x'_L , and total time spent parking on the curb, T , can be solved using the two conditions in (8), the two analogous conditions for the neighboring garage, and equation (7). There are five linear equations in five unknowns. The solution for x_H is

$$x_H = \frac{(d_H l_H + d_L l_L) k_H w_H w_L D + d_H k_H w_L l_H^2 (s'_H - s_H)}{w_H (w_H w_L + 2d_H l_H k_H w_L + 2d_L l_L k_L w_H)} + \frac{(w_L + 2d_L l_L k_L) l_H (f_H - s_H) - d_L k_H l_L^2 (2f_L - s_L - s'_L)}{w_H w_L + 2d_H l_H k_H w_L + 2d_L l_L k_L w_H}. \quad (10)$$

The formula for x_L is obtained from (10) by interchanging the H and L subscripts. As shown in Appendix A.1, the equilibrium is stable in the sense that a perturbation in the number of drivers of either type who park on the curb induces adjustments that return the system to equilibrium asymptotically.

A notable feature of (10) is that the home garage’s market boundary depends on the parking fees set by the neighbor, s'_H and s'_L , even though the garages do not compete directly

with each other. The garage markets are interdependent because the fees charged by one garage affect the number of drivers who choose to park on the curb, the intensity of curbside parking search congestion, and hence the demand for parking at the neighboring garages.

The first-order conditions for an interior profit maximum for the home garage are

$$\frac{\partial \pi}{\partial s_H} = 2d_H l_H \left(x_H + (s_H - c) \frac{\partial x_H}{\partial s_H} \right) + 2d_L l_L (s_L - c) \frac{\partial x_L}{\partial s_H} = 0, \quad (11)$$

$$\frac{\partial \pi}{\partial s_L} = 2d_L l_L \left(x_L + (s_L - c) \frac{\partial x_L}{\partial s_L} \right) + 2d_H l_H (s_H - c) \frac{\partial x_H}{\partial s_L} = 0. \quad (12)$$

These equations yield closed-form but very long expressions for s_H and s_L , as well as long expressions for x_H and x_L when the formulas for s_H and s_L are substituted into (10) and its counterpart for x_L . Clear analytical results can be obtained only in special cases. One such case occurs when curbside parking search costs are proportional to parking durations:

$$\frac{k_H}{k_L} = \frac{l_H}{l_L}. \quad (13)$$

The difference in garage parking fees is then given by the simple formula:

$$s_H - s_L = \frac{f_H - f_L}{2}. \quad (14)$$

Given condition (13), both the monetary costs and the search costs of curbside parking are proportional to parking duration. Curbside parking then imposes the same effective constraint on garage pricing in each type's parking markets. Accordingly, garages charge fees for the two driver types that differ by half the difference in drivers' reservation prices, $f_H - f_L$, in the same way that a monopolist facing a linear demand curve sets a monopoly price. If, in addition to condition (13), curbside parking fees are equal (*i.e.*, $f_H = f_L$) then $s_H = s_L$. Hourly fees for the two driver types are then the same for both curbside and garage parking.⁶ Within a neighborhood of the point $f_H = f_L$ and $k_H/k_L = l_H/l_L$ in parameter space, there are parameter combinations for which $s_H > s_L$ so that the hourly fee for garage parking is actually higher for long-term parkers (as shown in (14), this is the case if $f_H > f_L$). In practice, most parking garages offer quantity discounts in their parking prices, but some do feature escalating marginal price increments. In any case, the practical relevance of condition (13) is debatable since it is not obvious why search costs would be proportional to parking duration.

⁶If, in addition, unit walking time costs are proportional to parking durations (*i.e.*, $w_H/w_L = l_H/l_L$), then the parking boundaries for the two types are also equal (*i.e.*, $x_H = x_L$).

A second, degenerate case in which (11) and (12) yield simple solutions occurs if search costs for curbside parking are zero (*i.e.*, $k_H = k_L = 0$). In this case, garage parking fees and market boundaries work out to

$$s_i = \frac{c + f_i}{2}, \quad i = H, L \quad (15)$$

$$x_i = \frac{(f_i - c) l_i}{2w_i}, \quad i = H, L. \quad (16)$$

If $k_H = k_L = 0$, curbside parking serves as an outside good with *exogenous* utility as in Salop's (1979) model. Garages can then attract a type i driver without losing money only if $f_i > c$. They set their fees for each driver type half way between the cost of providing garage parking, c , and the fee for curbside parking, f_i , as per equation (15) and also similar to (14). The garage market for each type in (16) varies proportionally with the type's parking duration, l_i , and inversely with its walking cost to and from garages, w_i . The simplicity of formulas (15) and (16) shows that search costs are essential to make the model interesting and useful for policy analysis.

	l_H	l_L	w_H	w_L	f	$d_H, d_L,$ k_H, k_L
s_H		– if $k_H = k_L$	+ if $k_H = k_L$	+ if $l_H = l_L$ and $k_H = k_L$	+ if $k_H = k_L$?
x_H	Same sign as $f - c$	Opposite sign to $f - c$	– if $l_H = l_L$ and $k_H = k_L$	+ if $l_H = l_L$ and $k_H = k_L$	+ if $l_H = l_L$ and $k_H = k_L$?

Table 1: Comparative statics properties of interior equilibrium

In the model with positive search costs, the comparative statics properties of equilibrium garage parking fees and market boundaries are mostly ambiguous. Table 1 lists some properties for s_H and x_H for the case where $f_H = f_L = f$.⁷ The garage market boundary for long-term parkers, x_H , expands with parking duration, l_H , if the curbside parking fee is higher than the cost of garage parking, and decreases if the curbside parking fee is lower. This is because the relative monetary costs of garage and curbside parking become more important compared to curbside parking access costs as parking duration increases. By contrast, an increase in parking duration of short-term parkers, l_L , causes the garage market for long-term parkers to contract if $f > c$ because it induces short-term parkers to use less curbside parking. This reduces search costs for curbside parking, and attracts some long-term parkers away from garages. The market boundary x_H tends to contract if walking cost,

⁷The comparative statics properties for s_L and x_L are mirror images of those for s_H and x_H except for the condition $l_H \geq l_L$ which is unchanged.

w_H , rises. But garages tend to raise the parking fee, s_H , because garage parking demand becomes less price elastic. An increase in w_L has the opposite effect on x_H , but it has a similar effect on s_H because curbside parking becomes less elastic. Finally, as expected, both the garage market and the garage parking fee tend to increase if curbside parking becomes more expensive.

As noted above for the case in which condition (13) holds, it is possible in regime *Int* for the hourly garage parking fee to be higher for long-term parkers than short-term parkers. Conversely, it is also possible for the long-term fee to be so much lower that incentive compatibility constraint (3) is violated. For example, this occurs if l_H and l_L are similar, and if short-term parkers have higher walking and search costs than long-term parkers. A numerical example in which this happens is provided in Section 5.

The comparative statics properties of the model are simpler in the degenerate case where drivers are identical so that there is effectively only one type. Garages then set only one parking fee, s , and there is only one market boundary, x , between garage parking and curbside parking. The home garage's problem then is to choose a single parking fee, s , to maximize profit, $2(s-c)dx$, subject to the constraints $sl+wx = fl+kT$, $s'l+wx' = fl+kT$, and $T = dl(D-x-x')$. (There is no incentive compatibility constraint with just one type.) The symmetric equilibrium solution is given by

$$s = \frac{fw + wdkD + c(w + dkl)}{2w + dkl}, \quad (17)$$

$$x = \frac{dkl(w + dk)D + (f - c)l(w + dk)}{(2w + dlk)(w + 2dlk)}. \quad (18)$$

	l	w	f	d	k
s	-	+	+	+	+
x	+/-	-	+	+/-	+/-
	Same sign as $f - c$			+ if $f \geq c$	+ if $f \leq c$

Table 2: Comparative statics properties of interior equilibrium with homogeneous drivers

The comparative statics properties of this simplified model, listed in Table 2, follow by differentiating (17) and (18). The effects of market density (d) and search time costs (k) are now definitive. If the density of parkers increases, and the curbside parking fee exceeds the supply cost of garage parking, the garage parking fee increases and the garage market expands. This happens because the garage market operates with constant returns to scale whereas, due to search costs, curbside parking suffers from diminishing returns. An increase in the size of the market therefore favors garage parking. If search costs for curbside parking

increase, and the curbside parking fee is less than the supply cost of garage parking, the garage parking fee increases and the garage market expands. However, if curbside parking is very costly, garages have substantial market power and they can increase their fees so much that the garage market area contracts.

3.2 Equilibrium for regime *Hg*

In regime *Hg*, all long-term parkers park at garages while short-term parkers split between garages and the curb. The home garage maximizes its profit, given by (6), subject to several constraints. First, the marginal long-term parker must be indifferent between parking at the home garage and parking at the neighbor:

$$B_H - s_H l_H - w_H x_H = B_H - s'_H l_H - w_H (D - x_H). \quad (19)$$

Second, the marginal short-term parker must be indifferent between parking at the home garage and curbside parking:

$$B_L - s_L l_L - w_L x_L = B_L - f_L l_L - k_L T. \quad (20)$$

This condition is the same as condition (8) for regime *Int*, except now $T = d_L l_L (D - x_L - x'_L)$ because only short-term parkers use curbside parking. Third, the marginal long-term parker must weakly prefer parking at the home garage to curbside parking:

$$B_H - s_H l_H - w_H x_H \geq B_H - f_H l_H - k_H T. \quad (21)$$

Fourth, the marginal short-term parker must strictly prefer parking at the home garage to parking at the neighbor:

$$B_L - s_L l_L - w_L x_L > B_L - s'_L l_L - w_L (D - x_L). \quad (22)$$

This condition assures that some short-term parkers prefer curbside parking to garage parking. Finally, if practically relevant, the incentive compatibility constraint given in (3) must hold. The equilibrium solution depends on whether condition (21) binds or not. The two cases are examined in turn.

3.2.1 Long-term parkers strictly prefer garage parking to curbside parking

If long-term parkers strictly prefer garage parking to curbside parking, the symmetric-equilibrium parking fees are as follows (see Appendix A.2.1 for details).

$$s_H = c + \frac{w_H D}{l_H}, \quad (23)$$

$$s_L = \frac{(w_L + k_L d_L l_L) c + k_L d_L w_L D + w_L f_L}{2w_L + k_L d_L l_L}. \quad (24)$$

Market boundaries are

$$x_H = \frac{D}{2}, \quad (25)$$

$$x_L = \frac{w_L + k_L d_L l_L}{(w_L + 2k_L d_L l_L)(2w_L + k_L d_L l_L)} (k_L d_L l_L D + l_L (f_L - c)). \quad (26)$$

Unlike regime *Int*, in regime *Hg* the formulas for garage parking fees and market boundaries for one driver type do not depend on parameters specific to the other type because the two types no longer interact. Equation (23) can be rewritten as $(s_H - c)l_H = w_H D$ which shows that the profit earned from a long-term parker is independent of parking duration, l_H .⁸ The profit increases with the market power of a garage, which varies proportionally with the distance between garages, D , and the walking time cost for long-term parkers, w_H . The equilibrium garage parking fee for short-term parkers in (24) is a more complicated function. It increases, but less than proportionally, with the cost of providing service, c , and the curbside parking fee, f_L .⁹ The fee s_L also increases with the distance between garages because more drivers then park on the curb, which increases search costs and makes curbside parking less attractive as a substitute for garage parking. Finally, s_L decreases with parking duration, l_L , but unlike for long-term parkers, the profit earned from short-term parkers increases with parking duration.

Similar to regime *Int*, it is possible for the garage parking fee to be higher for long-term parkers than short-term parkers (*i.e.*, $s_H > s_L$), and conversely also possible for the total outlay on garage parking to be lower for long-term parkers than short-term parkers (*i.e.*, $s_H l_H < s_L l_L$). That is, the incentive compatibility constraint can be active. An example with $s_H > s_L$ is provided in Section 5.

⁸This result is attributable to the linearity of the model.

⁹If c and f_L both increase by some amount Δ , then s_L also increases by Δ because total parking demand is price inelastic.

Two further derivatives of interest are

$$\frac{\partial s_L}{\partial w_L} \stackrel{s}{=} f_L + k_L d_L D - c, \quad (27)$$

$$\frac{\partial s_L}{\partial k_L} \stackrel{s}{=} c + \frac{2w_L D}{l_L} - f_L > 0, \quad (28)$$

where $\stackrel{s}{=}$ means “identical in sign.” Assumption (5) does not assure that the right-hand side of (27) is positive, but it is likely to be satisfied unless the supply cost of garage parking is high. If the right-hand side is positive, the garage parking fee for short-term parkers rises as their walking time cost increases. Using (26) and the condition $x_L < D/2$, it is easy to show that the right-hand side of (28) is positive. Therefore, s_L rises with the search cost for curbside parking, as expected.

3.2.2 Long-term parkers indifferent between garage and curbside parking

If long-term parkers are indifferent between garage and curbside parking, constraint (21) binds and multiple symmetric equilibria for s_H and s_L can exist. This complication arises because the garage’s profit function is kinked. To see this, consider the candidate symmetric equilibrium shown in Figure 3. The full cost of parking at the home garage, line AB , and the full cost of parking at the neighbor, CD , intersect at point E where $x_H = x'_H = D/2$. The cost of curbside parking, FG , intersects the other two curves at point E as well. If the home garage lowers s_H , line AB shifts down and intersects line CD at point E_1 . All long-term parkers now strictly prefer parking at one of the garages to parking on the curb. If the home garage instead raises s_H , line AB shifts up. The market boundary with the neighbor is broken, and some long-term parkers now park on the curb. This raises the search cost of curbside parking, and line FG shifts upwards. A new market boundary between the home garage and curbside parking forms at point E_2 , and the equilibrium changes from regime Hg to regime Int .

As shown in Appendix A.2.2, the home garage’s market shifts more quickly when s_H increases than when it decreases, which suggests that the profit function has a “downward” kink at the candidate equilibrium. However, because the cost of curbside parking rises if s_H is increased, some short-term parkers shift from curbside parking to the home garage, which increases the home garage’s profits from short-term parkers. The direction of the kink in the profit function is thus ambiguous.

The profit function is also kinked for changes in s_L . If s_L rises, more short-term parkers park on the curb, and curbside parking becomes less attractive for long-term parkers. Line

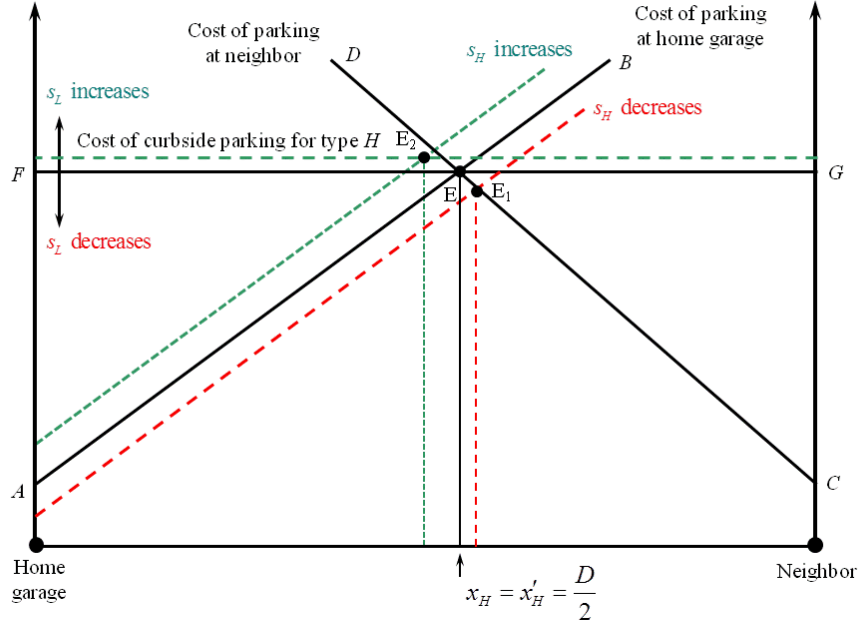


Figure 3: Costs of parking in regime Hg

FG rises and the equilibrium for long-term parkers is unaffected. If s_L drops instead, fewer short-term parkers park on the curb, and curbside parking becomes more attractive for long-term parkers. Line FG drops, and the home garage loses some long-term parkers. The home garage's profit function therefore has a downward kink with respect to s_L at the candidate equilibrium. The kink occurs for similar reasons as in Salop's (1979) model although utility from the outside good is exogenous in his model whereas utility from curbside parking is endogenous here.

The kinks explained above in the home garage's profit function complicate the derivation of equilibria for regime Hg . Details are provided in Appendix A.2.3.

Derivations of equilibria for the remaining regimes follow similar lines to regimes Int and Hg , and descriptions are relegated to Appendices A.3-A.9.

4 Social Optimum

Since total parking demand is fixed, the social optimum corresponds to a total cost minimum. Total costs are determined by how many drivers of each type park in garages, and how many park on the curb. All drivers who park at a garage should patronize the nearest one, and

with garages spaced at equal intervals the social optimum can be derived by minimizing total costs within a distance $D/2$ on either side of the home garage.

Total costs (TC) are the sum of garage supply costs (GC), walking time costs (WC), and curbside parking search costs (SC). The component costs in a symmetric equilibrium are tallied as follows. The number of type i drivers on either side of the home garage who park there is $2d_i x_i$. Type i drivers park for l_i hours, and it costs c dollars to provide an hour of parking. Therefore, garage operating costs are

$$GC = 2c(d_H x_H l_H + d_L x_L l_L). \quad (29)$$

A type i driver bound for a destination x units away from a garage incurs a walking cost for the round-trip of $w_i x$. Total walking time for type i drivers is $2d_i \int_0^{x_i} x dx$. Total walking time costs for all drivers are therefore

$$WC = 2d_H w_H \int_0^{x_H} x dx + 2d_L w_L \int_0^{x_L} x dx = d_H w_H x_H^2 + d_L w_L x_L^2. \quad (30)$$

There are $d_i(D - 2x_i)$ type i drivers who each park for l_i hours on the curb and incur a search cost of $k_i T$, with T given by (7). Total curbside parking search costs are therefore

$$SC = [k_H d_H (D - 2x_H) + k_L d_L (D - 2x_L)] [d_H l_H (D - 2x_H) + d_L l_L (D - 2x_L)]. \quad (31)$$

The social optimum is derived by choosing x_H and x_L to minimize total costs, $TC = GC + WC + SC$. In principle, the optimum can fall into any one of the nine regimes described in Section 3 for the current equilibrium. However, assumption (5) rules out regime $Hc + Lc$ in which garage parking is not used. The model assumptions also rule out regime $Hg + Lg$ in which curbside parking is not used. The reasoning is as follows. First, the supply of curbside parking is fixed so that providing curbside parking has no opportunity cost. Second, administration costs of curbside parking and fee collection are assumed to be zero. Third, drivers do not contribute to through traffic congestion while they are searching for parking. Fourth, the marginal external cost of search approaches zero as hours of curbside parking used, T , approach zero. Finally, garage parking involves a supply cost per vehicle-hour of $c > 0$, and drivers incur walking time costs when they use garages. Therefore, the marginal social cost of using a small amount of curbside parking is lower than the marginal social cost of using a small amount of garage parking so that some curbside parking should

always be used.¹⁰

It is straightforward to derive the social optimum for each of the remaining seven regimes. However, formulas for the optimal market boundaries are complicated and opaque (formulas for regime *Int* are given in Appendix A.10). In the balance of this section we offer some general insights about the social optimum and how it can be decentralized. A numerical example is developed in the next section.

The social optimum can be achieved in several ways. The most direct approach is for the city to regulate garage parking fees or take over control of garages altogether. The city can implement first-best pricing of both garage and curbside parking. Denote this approach with a superscript o . Garage parking is priced at marginal supply cost so that $s_H^o = s_L^o = c$. Curbside parking is priced to internalize the marginal external cost of search. Because this externality is proportional to parking duration, hourly curbside parking fees will be equal, (i.e., $f_H^o = f_L^o$). We will refer to the common value, f^o , as the “first-best uniform curbside parking fee”. Given total curbside parking search costs in (31), the marginal externality cost imposed by a type i driver is $[k_H d_H(D - 2x_H) + k_L d_L(D - 2x_L)]l_i$, and the first-best uniform curbside fee is therefore $f^o = k_H d_H(D - 2x_H) + k_L d_L(D - 2x_L)$.

Since total parking demand for each driver type is fixed, first-best pricing is not the only way to support the social optimum. Indeed, any set of garage and curbside parking fees that maintains the appropriate differential between garage and curbside parking fees for each driver type does the job. For example, in the symmetric equilibrium for regime *Int* the market boundary for long-term parkers given in (10) is a function of $f_H - s_H$ and $f_L - s_L$, but it does not depend on the levels of the fees. If the city can regulate garage fees, it can fix curbside fees, f_H and f_L , at any level (subject to nonnegativity constraints) and then set s_H and s_L , correspondingly. If parking garage operators can set their rates freely, the city can still attain the optimum by setting curbside fees appropriately. The hourly curbside parking fees will typically differ because garage parking demand elasticities typically differ for the two driver types, and garages therefore impose different markups on their rates. We call the curbside parking fees in this case “first-best differentiated curbside parking fees” and denote them with a caret (i.e., \hat{f}_H and \hat{f}_L).

Differentiating curbside fees is difficult using conventional parking meter technology, and it may be opposed by interest groups or ruled out on public policy grounds. If so, hourly curbside fees are constrained to be uniform and the social optimum cannot, in general, be

¹⁰This differs from some other parking models such as that in Arnott, Inci and Rowse (2013) where allocating space to curbside parking reduces road capacity, and cruising for parking contributes to congestion for through traffic.

reached if garage parking is privately controlled. We denote this scheme by a superscript u , and refer to the common fee, f^u , as the “second-best uniform curbside parking fee”.

Regardless of which equilibrium regime prevails at the optimum, the first-best and second-best curbside parking fees defined above are positive for both driver types. This follows from two observations. First, curbside parking creates a negative search congestion externality. The first-best (Pigouvian) curbside parking fees are therefore positive. Second, when garage parking is privately controlled it is overpriced because the operators exercise market power. Because garage operators do not suffer from congestion themselves, they do not internalize congestion costs in the way that private toll road operators do (Verhoef, Nijkamp and Rietveld, 1996). Indeed, garage operators benefit from the congestion induced by curbside parking search. Since curbside parking is a substitute for garage parking, second-best curbside parking fees exceed first-best fees.

5 A Numerical Example

The numerical example is designed to establish a sense of which of the various candidate equilibrium regimes prevails under alternative parameter assumptions. It also compares the first-best and second-best pricing schemes described in Section 4. Appendix A.11 summarizes the numerical solution procedure that was used to derive equilibria.

5.1 Parameterization

Base-case parameter values for the example are listed in Table 3. Some of the values are taken from Arnott (2006), Arnott and Inci (2006, 2010), and Arnott and Rowse (2009). Suitable values for the d_i , l_i , w_i , and k_i parameters depend on such factors as trip purpose and income, and possibly also trip frequency and familiarity with parking availability. There is a large empirical literature on values of driving time and walking time, and how they vary with individual socioeconomic characteristics. However, we are not aware of specific estimates of the value of time spent cruising for parking. Another complication is that suitable parameter values depend on how long-term and short-term parkers are interpreted. To be agnostic, we have chosen for the base case identical parameter values for the two driver types except for parking duration.

The curbside parking fee for both types is \$1.00 per hour; the distance between two consecutive parking garage is 0.125 miles; the cost of providing a parking garage space is

$f_H = f_L$	D	c	l_H	l_L	$w_H = w_L$	$k_H = k_L$	$d_H = d_L$
\$1.00/hr	0.125 miles	\$2.50/hr	2 hr	1 hr	\$16/mile	\$0.16/vehicle-hr	100/mile

Table 3: Base-case parameter values

\$2.50 for each hour that a car is parked; parking duration is 2 hours for long-term parkers and 1 hour for short-term parkers; walking time cost for both types is \$16 per mile; unit parking search cost for both types is \$0.16 per vehicle-hour; and the density of trip destinations for each type is 100 per mile. With these parameter values, the cost of walking for the round-trip between a garage and the mid-point between parking garages is \$2.00 for each type. For short-term parkers, the generalized cost of curbside parking ranges from $f_L l_L = \$1.00$ if no one uses curbside parking to $f_L l_L + k_L T = f_L l_L + k_L (d_H l_H + d_L l_L) D = \7.00 if all drivers of both types park on the curb. The corresponding minimum and maximum values for long-term parkers are \$2.00 and \$8.00, respectively.

5.2 Results

Results for various parameter combinations are reported in Tables 4-6. For the base case (case 1), the current equilibrium, denoted by a *, falls in regime *Int*. Hourly garage parking fees are $s_H^* = \$2.81$ and $s_L^* = \$3.74$. Only 9 percent of long-term parkers use garages, but 97 percent of short-term parkers do so. Total costs are about \$85 per garage. This value should be interpreted as costs incurred every 2 hours: the parking duration of long-term parkers.

The social optimum falls into regime *Lg* (Table 5). All short-term parkers use garage parking, and a larger fraction (33 percent) of long-term parkers uses garages than in the current equilibrium. This pattern reflects the underusage of garages in the current equilibrium, described earlier, that results from a combination of underpricing of curbside parking and exercise of garage market power. Total costs in the social optimum are about \$81 which is roughly 5 percent lower than in the current equilibrium.

The social optimum can be attained by pricing garage parking at marginal social cost (*i.e.*, $s_H^o = s_L^o = \$2.50$), and charging curbside users the marginal search congestion externality cost. For the base-case parameter values the optimal curbside fee is $f^o = \$1.33/hr$ which is not much higher than the \$1.00/hr assumed for the current equilibrium.

As explained in Section 4, even if garages are operated privately and free of regulation, the social optimum can be decentralized by setting first-best differentiated curbside parking fees. For Case 1, the fees are $\hat{f}_H = \$1.61$, and \hat{f}_L equal to \$2.83 or higher to deter short-term

parkers from parking on the curb. Starting from the current equilibrium, garages raise the rate for long-term parkers from $s_H^* = \$3.74$ to $\widehat{s}_H = \$4.50$. However, they lower the rate for short-term parkers slightly from $s_L^* = \$2.81$ to $\widehat{s}_L = \$2.78$ because short-term parkers no longer park on the curb which makes curbside parking more attractive to long-term parkers. This illustrates how curbside parking congestion creates an interdependence between the parking markets for long-term and short-term parkers.

The second-best uniform curbside parking fee is $f^u = \$1.85$ (see Table 6) which is intermediate between the first-best differentiated fees. The uniform fee fails to support the social optimum. Its *relative efficiency* can be measured by the ratio

$$eff \equiv \frac{TC^* - TC^u}{TC^* - TC^o}, \quad (32)$$

where TC^* denotes total costs in the current equilibrium, TC^o denotes total costs in the first-best optimum, and TC^u denotes total costs with the uniform curbside parking fee. For the base-case parameter values, $eff = 0.77$ so that the second-best uniform fee achieves more than three-quarters of the efficiency gains from first-best differentiated fees.

Results for eleven alternative parameter combinations are reported in Tables 4-6, with parameter variations identified in bold type in Table 4.

Case 2: The hourly curbside parking fee for each driver type is raised from \$1.00 to \$4.00. The current equilibrium shifts to regime *Hg*. Because long-term parkers park for twice as long as short-term parkers, they are hit twice as hard by the higher curbside parking fees and all of them end up parking at garages. By contrast, short-term parkers shift toward curbside parking because of reduced search congestion. Similar to the base case, congestion in curbside parking creates an interdependence between the parking markets. The social optimum is unchanged, of course, by the change in current curbside parking fees. However, the welfare gain from optimal curbside parking fees more than quadruples compared to the base case, and the relative efficiency of the second-best uniform curbside parking fee rises to 94 percent.

Case 3: Curbside parking is made free. The current equilibrium shifts to regime *Hc*, and both driver types use less garage parking than in the base case. The welfare gain from optimal curbside parking fees is similar to Case 2, and the relative efficiency of the second-best uniform curbside parking fee rises to 93 percent.

Case 4: Long-term parkers are assumed to be commuters who park for 9 hours. The current equilibrium shifts to regime *Lg* with all short-term parkers using garages. Long-term

parkers patronize mainly curbside parking although the proportion is less than in the base case equilibrium. Total parking costs increase to nearly three times their base-case level. The increase is due to the higher resource costs of accommodating long-term parkers at garages for 9 hours as well as the greater contribution of long-term parkers to curbside congestion due to their longer stays. The social optimum is similar to the current equilibrium, and the efficiency gains from optimal curbside parking are modest. Moreover, the optimum can be supported by charging any non-negative hourly fee for short-term curbside parking so that the fee can be set at the optimal level for long-term parking of \$1.35 and the second-best uniform curbside parking fee achieves 100 percent efficiency.

Case 5: Distance between garages is doubled. The current equilibrium remains in regime *Int*, and the social optimum remains in regime *Lg*. But total costs increase greatly, and optimal curbside parking fees are appreciably higher than in previous cases. Garage profits are also much higher. The second-best uniform curbside parking fee of \$3.34 achieves about 90 percent of the welfare gains from the first-best differentiated curbside fees. A problematic feature of the differentiated fees is that short-term parkers pay a total of $f_L l_L = \$5.39$ for parking which slightly exceeds the payment by long-term parkers of $f_H l_H = \$5.14$. Incentive compatibility constraint (1) is therefore violated. If the incentive compatibility constraint is practically relevant, it can be accommodated by imposing it on the optimization problem while still treating f_H and f_L as independent control variables. Doing so would affect the solution very little in this instance because the constraint is only violated by a small margin.

Case 6: Curbside parking search costs are doubled. The current equilibrium shifts to regime *Lg*. It does not differ markedly from the social optimum so that the benefits from adjusting curbside parking fees are modest.

Case 7: Curbside parking search costs are reduced by half. Both the current equilibrium and the social optimum shift to regime *Hc*. The social optimum is supported by rather modest first-best differentiated curbside fees although incentive compatibility constraint (1) is again violated.

Case 8: Walking time costs are doubled. The market power of garages increases and they increase their hourly fees slightly. The combined effect of more onerous walking and higher garage parking fees induces short-term parkers to reduce their use of garage parking.

Case 9: Walking time costs are halved. All short-term parkers now use garage parking in both the current equilibrium and the social optimum, and the optimum can be supported by charging for short-term parking any fee over \$1.41/hr. Since this minimum is below the optimal fee of \$1.45/hr for long-term parking, the second-best uniform curbside parking fee

is \$1.45/hr and supports the social optimum.

Case 10: Curbside parking search and walking time costs for long-term parkers are set at twice the values for short-term parkers while holding average costs at their base-case levels. Condition (13) is then satisfied and, as explained in Section 3, with $f_H = f_L$ as well, garages set the same fee (\$3.00) for short-term and long-term parking. The social optimum is supported by charging a uniform hourly curbside parking fee of \$2.20/hr.

Case 11: In a mirror image of case 10, curbside parking search and walking time costs for short-term parkers are set at twice their values for long-term parkers. The current equilibrium shifts to regime $Hc + Lg$ so that long-term and short-term parkers are fully segregated. The social optimum is nearly identical to the current equilibrium and can be supported by charging a uniform hourly fee of \$1.29/hr.

Case 12: Several parameter changes are made for this final case. Curbside parking is assumed to be free, and the supply cost of garage parking is set to zero. Parking duration for long-term parkers is reduced from 2 hr to 1.75 hr, and the costs of curbside parking search and walking are increased. In the current equilibrium, long-term parkers end up paying less in total for garage parking than short-term parkers ($s_H^* l_H = \$2.30$ vs. $s_L^* l_L = \$2.81$). The social optimum calls for all long-term parkers to park in garages, and this allocation can be realized by imposing a uniform curbside parking fee of \$4.00/hr.

Overall, the numerical example illustrates a range of ways in which drivers are allocated to parking space in the various pricing schemes. Consistent with usual practice, in most cases the hourly fee for long-term garage parking is lower than the fee for short-term parking but the total outlay is higher for long-term parking. (Cases 10 and 12 are the exceptions.) In general, the current equilibrium features too little garage parking because curbside pricing is underpriced. (Case 2 is an exception.) The welfare gain from efficient curbside parking varies strongly with parameter values. It is largest in Case 5, where the distance between garages is doubled, because costs are tallied over a large market area and because the greater friction of space endows garages with more market power.

A second-best uniform hourly curbside parking fee supports the social optimum in some cases where the allocation of driver types is partly segregated between garages and the curb. In other cases, it falls short of the social optimum but still performs relatively well. Finally, the first-best uniform curbside fees are always lower than the first-best differentiated and second-best uniform curbside parking fees. This is because garage pricing in the first-best case is priced at marginal social cost rather than priced at a markup by private operators. The first-best uniform curbside fees vary relatively little over the 12 cases compared to

the first-best differentiated and second-best uniform curbside parking fees because curbside congestion varies less from case to case than does garage market power.

Case	Parameter value										Current equilibrium				
	$f_H = f_L$ (\$/hr)	D (\$/hr)	l_H (hr)	l_L (hr)	w_H (\$/hr)	w_L (\$/hr)	k_H (\$/hr)	k_L (\$/hr)	Regime	s_H^* (\$/hr)	s_L^* (\$/hr)	x_H^* (frac. of $D/2$)	x_L^* (frac. of $D/2$)	TC^* (\$)	π^* (\$)
1	1.00	1/8	2	1	16	16	0.16	0.16	<i>Int</i>	2.81	3.74	0.09	0.97	85.4	15.7
2	4.00	1/8	2	1	16	16	0.16	0.16	<i>Hg</i>	3.50	3.67	1.00	0.78	98.1	36.3
3	0.00	1/8	2	1	16	16	0.16	0.16	<i>Hc</i>	<u>2.50</u>	3.67	0.00	0.78	96.0	11.3
4	1.00	1/8	9	1	16	16	0.16	0.16	<i>Lg</i>	2.55	4.50	0.22	1.00	236.9	26.1
5	1.00	1/4	2	1	16	16	0.16	0.16	<i>Int</i>	3.47	4.88	0.35	0.88	229.8	69.7
6	1.00	1/8	2	1	16	16	0.32	0.32	<i>Lg</i>	2.92	4.30	0.46	1.00	96.6	27.3
7	1.00	1/8	2	1	16	16	0.08	0.08	<i>Hc</i>	2.50	3.10	0.00	0.45	64.7	3.4
8	1.00	1/8	2	1	32	32	0.16	0.16	<i>Int</i>	2.95	4.00	0.15	0.60	98.5	12.9
9	1.00	1/8	2	1	8	8	0.16	0.16	<i>Lg</i>	2.58	3.50	0.19	1.00	79.3	12.9
10	1.00	1/8	2	1	21.3	10.7	0.213	0.107	<i>Int</i>	3.00	3.00	0.43	0.43	91.5	8.0
11	1.00	1/8	2	1	10.7	21.3	0.107	0.213	<i>Hc + Lg</i>	2.50	5.00	0.00	1.00	72.9	31.3
12	0.00	1/8	1.75	1	26.7	40	0.267	0.40	<i>Int</i>	1.32	2.81	0.55	0.81	42.9	44.5

Notes:

- * denotes current equilibrium.
- Parameter values held constant are $c = 2.5$, $d_H = 100$, and $d_L = 100$. For case 12, $c = 0$.
- Parameter variations are identified in bold type.
- Minimum values are underlined.

Table 4: Results for numerical example: current equilibrium

Case	Regime	Social optimum				MSC pricing of curbside and garage parking			Differentiated curbside fees and private garage operation			
		x_H^o (frac. of $\frac{D}{2}$)	x_L^o ($\frac{D}{2}$)	TC^o (\$)	$Gain$ (\$)	f^o (\$/hr)	s_H^o (\$/hr)	s_L^o (\$/hr)	\hat{f}_H (\$/hr)	\hat{f}_L (\$/hr)	\hat{s}_H (\$/hr)	\hat{s}_L (\$/hr)
1	<i>Lg</i>	0.33	1.00	81.3	4.1	1.33	2.5	2.5	1.61	<u>2.83</u>	2.78	4.50
2	<i>Lg</i>	0.33	1.00	81.3	16.8	1.33	2.5	2.5	1.61	<u>2.83</u>	2.78	4.50
3	<i>Lg</i>	0.33	1.00	81.3	14.7	1.33	2.5	2.5	1.61	<u>2.83</u>	2.78	4.50
4	<i>Lg</i>	0.36	1.00	231.7	5.2	1.27	2.5	2.5	1.35	<u>0.00</u>	2.58	4.50
5	<i>Lg</i>	0.61	1.00	203.5	26.3	1.56	2.5	2.5	2.57	5.39	3.52	6.50
6	<i>Lg</i>	0.65	1.00	93.0	3.6	1.41	2.5	2.5	1.99	<u>2.68</u>	3.08	4.50
7	<i>Hc</i>	0.00	0.83	62.0	2.8	1.17	2.5	2.5	1.00	2.28	2.50	3.61
8	<i>Int</i>	0.38	0.88	87.9	10.6	1.50	2.5	2.5	2.38	3.75	3.38	4.75
9	<i>Lg</i>	0.35	1.00	77.8	1.5	1.29	2.5	2.5	1.45	<u>1.41</u>	2.66	3.50
10	<i>Int</i>	0.63	0.63	84.6	6.9	1.46	2.5	2.5	2.20	2.20	3.24	3.24
11	<i>Lg</i>	0.06	1.00	72.8	0.1	1.26	2.5	2.5	1.29	<u>0.36</u>	2.53	4.07
12	<i>Hg</i>	1.00	0.80	22.9	20.0	1.00	0	0	<u>2.48</u>	4.00	<u>1.90</u>	3.00

Notes:

1. o denotes first-best optimum with uniform curbside parking fees and marginal social cost pricing at garages.
2. $\hat{\cdot}$ denotes first-best optimum with differentiated curbside parking fees.
3. *MSC* denotes marginal social cost.
4. Parameter values held constant are $c = 2.5$, $d_H = 100$, and $d_L = 100$. For case 12, $c = 0$.
5. Minimum values are underlined. Maximum values are overlined.

Table 5: Results for numerical example: first best

Uniform curbside fee							
Case	Regime	f^u (\$/hr)	s_H^u (\$/hr)	s_L^u (\$/hr)	x_H^u (frac. of $D/2$)	x_L^u (frac. of $D/2$)	eff
1	<i>Int</i>	1.85	3.01	3.59	0.39	0.87	0.77
2	<i>Int</i>	1.85	3.01	3.59	0.39	0.87	0.94
3	<i>Int</i>	1.85	3.01	3.59	0.39	0.87	0.93
4	<i>Lg</i>	1.35	2.58	4.50	0.36	1.00	1.00
5	<i>Int</i>	3.34	4.02	4.74	0.77	0.74	0.90
6	<i>Int</i>	2.70	3.37	3.90	0.78	0.92	0.64
7	<i>Int</i>	1.51	2.65	3.23	0.04	0.60	0.65
8	<i>Int</i>	2.71	3.50	4.16	0.54	0.60	0.90
9	<i>Lg</i>	1.45	2.66	3.50	0.35	1.00	1.00
10	<i>Int</i>	2.20	3.24	3.24	0.63	0.63	1.00
11	<i>Lgint</i>	1.29	2.53	4.99	0.06	1.00	1.00
12	<i>Hg</i>	4.00	1.90	3.00	1.00	0.80	1.00

Notes:

1. u denotes second-best optimum with uniform curbside fees.
2. Parameter values held constant are $c = 2.5$, $d_H = 100$, and $d_L = 100$. For case 12, $c = 0$.

Table 6: Results for numerical example: second best

6 Conclusions

In this paper, we have developed a simple spatial parking model. Individuals drive downtown to visit destinations that are uniformly distributed around a circle. There are two types of drivers who differ in how long they need to park. Privately operated parking garages compete for business with each other and with publicly operated curbside parking space. Garage parking is free of traffic congestion, but curbside parking is scarce and drivers incur search costs that increase with the total number of vehicle-hours of curbside parking that is used.

The model has several notable properties. First, unlike in standard models of spatial competition, the utility derived from curbside parking (the outside good) is endogenous because it decreases with the number of drivers who use it. This creates an interdependence between parking garages even when they do not compete for customers with each other directly. If one garage changes its parking fees, it affects the number of drivers who park on the curb, and the change in parking search congestion affects demand at neighboring garages. An increase in fees benefits rivals (and a reduction hurts them) in the same qualitative way as when garages compete directly across a common border. Second, curbside parking congestion creates an interdependence between the demand for garage parking by the two driver types. For example, if a garage raises its fee for short-term parking, some short-term parkers switch to curbside parking. This increases search congestion for long-term parkers and induces some of them to switch from curbside parking to the garage. Demand for garage parking from the two market segments is therefore interdependent, and partly substitutable, even though garages are not constrained by capacity in the number of vehicles they can accommodate.

The parking market in the model is distorted in two ways. First, search for curbside parking creates a negative congestion externality among drivers looking for curbside parking. Second, parking garages have market power due to their discrete locations and the friction of space, and they exploit it by setting parking fees above marginal costs. They also price discriminate by setting different hourly fees for short- and long-term parking. Under plausible assumptions, the parking outlay is increasing and concave with parking duration, but exceptions are possible.

The degree of market failure varies with the distance between parking garages, parking search costs, and (crucially) the “current” fees charged for curbside parking. In the base model, the public authority lacks regulatory power over garage prices, but it can set curbside parking fees to alleviate search congestion. Moreover, since total parking demand is inelastic it can achieve the social optimum by charging differentiated curbside parking fees. In some

cases where the social optimum entails partial segregation of driver types between garage and curbside parking, a uniform hourly curbside parking fee can also achieve full efficiency. If garage parking rates can be regulated, the social optimum can also be reached regardless of curbside fees by adjusting garage fees to maintain the appropriate differential between garage and curbside rates.

Overall, the model shows that optimal parking policy is sensitive to parameter values which vary with local parking conditions.¹¹ Thus, each city (and municipality) should set its own parking fees and parking policies.

The model and analysis in the paper could be modified or extended in various directions. One obvious priority is to extend it from two driver types to multiple types or a continuum of types. Doing so would not only be more realistic, but also permit a more precise analysis of price discrimination according to parking duration as well as a comparison with garage pricing in practice. A typical garage in Lin and Wang's (2012) dataset posts five fees, and most garages set between three and eight fees. Working with a continuum of types would be easier than with discrete types insofar as there would be fewer equilibrium regimes to deal with. However, it would be necessary to adopt a joint frequency distribution of types in three dimensions (*i.e.*, parking duration, walking time cost, and search time cost) for which empirical data are lacking. It would still be necessary to contend with possible non-concave garage profit functions. Furthermore, basic insights would probably be more difficult to obtain from a continuum model.

A second extension would be to allow parking duration to depend on the cost of parking rather than being fixed. Third, alternative functional forms for curbside parking search costs could be entertained. The linear function used here is analytically tractable, and it is a natural choice if curbside parking is located perpendicular to the circle where trip destinations are located. If, instead, the supply of curbside parking is fixed, it would be more realistic to adopt a convex function as in Anderson and de Palma (2004).

Fourth, garage capacity constraints could be introduced. Parking capacity is an effective policy tool that can be used by public authorities. Many cities and towns regulate parking capacity by imposing minimum and maximum parking requirements. Unfortunately, adding capacity constraints to a model can introduce kinks in profit functions and create problems with the existence of pure-strategy equilibria. Froeb, Tschantz and Crooke (2003) and Arnott (2006) discuss these problems in the context of parking markets. These difficulties can be finessed by assuming Cournot competition, but this approach is unsatisfactory for studying

¹¹Local differences are evident from the *SFpark* experiment where it has been found that even adjacent blocks can differ widely in occupancy rate (Pierce and Shoup, 2013).

driver heterogeneity and price discrimination. Finally, alternative market structures could be examined. Monopoly control of garage parking or free entry into the garage parking market are two possibilities. Another is an oligopoly in which each firm controls more than one parking garage.

A Appendix

A.1 Stability of equilibrium in regime *Int*

Following standard procedures (see, for example, Silberberg, 1978, pp. 516-521), the stability of an interior equilibrium for given garage and curbside parking fees can be established by constructing, for each driver type, excess demand functions for parking at the home garage and parking at the neighbor. To economize on writing, define $\tilde{d}_H \equiv d_H l_H$ and $\tilde{d}_L \equiv d_L l_L$. Using (7), equations (8)-(9) can be written as excess demand functions:

$$E_H = f_H l_H + k_H \left(\tilde{d}_H (D - x_H - x'_H) + \tilde{d}_L (D - x_L - x'_L) \right) - s_H l_H - w_H x_H \quad (\text{A.1})$$

$$E_L = f_L l_L + k_L \left(\tilde{d}_H (D - x_H - x'_H) + \tilde{d}_L (D - x_L - x'_L) \right) - s_L l_L - w_L x_L \quad (\text{A.2})$$

$$E'_H = f_H l_H + k_H \left(\tilde{d}_H (D - x_H - x'_H) + \tilde{d}_L (D - x_L - x'_L) \right) - s'_H l_H - w_H x'_H \quad (\text{A.3})$$

$$E'_L = f_L l_L + k_L \left(\tilde{d}_H (D - x_H - x'_H) + \tilde{d}_L (D - x_L - x'_L) \right) - s'_L l_L - w_L x'_L. \quad (\text{A.4})$$

Drivers are assumed to adjust their parking location choices when out of equilibrium following the equations of motion

$$\frac{dx_H}{dt} = \lambda_H E_H \quad (\text{A.5})$$

$$\frac{dx_L}{dt} = \lambda_L E_L \quad (\text{A.6})$$

$$\frac{dx'_H}{dt} = \lambda_H E'_H \quad (\text{A.7})$$

$$\frac{dx'_L}{dt} = \lambda_L E'_L, \quad (\text{A.8})$$

where the speeds of adjustment for long and short-term parkers, λ_H and λ_L respectively, are both positive and can differ. Let x_H^* , x_L^* , x'_H^* , and x'_L^* denote the equilibrium values of x_H , x_L , x'_H , and x'_L , and define new variables $y_H \equiv x_H - x_H^*$, $y_L \equiv x_L - x_L^*$, $y'_H \equiv x'_H - x'_H^*$, and

$y'_L \equiv x'_L - x'_L^*$. Given equations (A.1)-(A.8), the y variables follow the equations of motion

$$\begin{bmatrix} \frac{dy_H}{dt} \\ \frac{dy_L}{dt} \\ \frac{dy'_H}{dt} \\ \frac{dy'_L}{dt} \end{bmatrix} = \begin{bmatrix} -\lambda_H (k_H \tilde{d}_H + w_H) & -\lambda_H k_H \tilde{d}_L & -\lambda_H k_H \tilde{d}_H & -\lambda_H k_H \tilde{d}_L \\ -\lambda_L k_L \tilde{d}_H & -\lambda_L (k_L \tilde{d}_L + w_L) & -\lambda_L k_L \tilde{d}_H & -\lambda_L k_L \tilde{d}_L \\ -\lambda_H k_H \tilde{d}_H & -\lambda_H k_H \tilde{d}_L & -\lambda_H (k_H \tilde{d}_H + w_H) & -\lambda_H k_H \tilde{d}_L \\ -\lambda_L k_L \tilde{d}_H & -\lambda_L k_L \tilde{d}_L & -\lambda_L k_L \tilde{d}_H & -\lambda_L (k_L \tilde{d}_L + w_L) \end{bmatrix} \begin{bmatrix} y_H \\ y_L \\ y'_H \\ y'_L \end{bmatrix} \quad (\text{A.9})$$

Finally, assume that the y variables evolve out of equilibrium according to the equations $y_H = Z_H e^{\mu t}$, $y_L = Z_L e^{\mu t}$, $y'_H = Z'_H e^{\mu t}$ and $y'_L = Z'_L e^{\mu t}$, where the Z variables are constants. The characteristic equation for the system (A.9) is

$$\begin{vmatrix} -\lambda_H (k_H \tilde{d}_H + w_H) & -\lambda_H k_H \tilde{d}_L & -\lambda_H k_H \tilde{d}_H & -\lambda_H k_H \tilde{d}_L \\ -\mu & & & \\ -\lambda_L k_L \tilde{d}_H & -\lambda_L (k_L \tilde{d}_L + w_L) & -\lambda_L k_L \tilde{d}_H & -\lambda_L k_L \tilde{d}_L \\ & -\mu & & \\ -\lambda_H k_H \tilde{d}_H & -\lambda_H k_H \tilde{d}_L & -\lambda_H (k_H \tilde{d}_H + w_H) & -\lambda_H k_H \tilde{d}_L \\ & & -\mu & \\ -\lambda_L k_L \tilde{d}_H & -\lambda_L k_L \tilde{d}_L & -\lambda_L k_L \tilde{d}_H & -\lambda_L (k_L \tilde{d}_L + w_L) \\ & & & -\mu \end{vmatrix} = 0, \quad (\text{A.10})$$

which reduces to

$$(\lambda_H w_H + \mu)(\lambda_L w_L + \mu)(\mu^2 + B\mu + C) = 0, \quad (\text{A.11})$$

where $B \equiv \lambda_L w_L + \lambda_H w_H + 2\lambda_H k_H \tilde{d}_H + 2\lambda_L k_L \tilde{d}_L$, and $C \equiv \lambda_H \lambda_L w_H w_L + 2\lambda_H \lambda_L (w_L k_H \tilde{d}_H + w_H k_L \tilde{d}_L)$. The first two terms of the product in (A.11) yield roots $\mu_1 = -\lambda_H w_H$ and $\mu_2 = -\lambda_L w_L$ which are both negative. It is straightforward to show that the third, quadratic, term in (A.11) has a positive discriminant. The two roots are therefore both real and take the form

$$\mu_3 = \frac{-B - \sqrt{B^2 - 4C}}{2}, \quad \mu_4 = \frac{-B + \sqrt{B^2 - 4C}}{2}. \quad (\text{A.12})$$

Since B and C are both positive, μ_3 and μ_4 are both negative. All four roots of the characteristic equation are therefore negative, which proves that the system is stable.

A.2 Equilibrium for regime *Hg*

A.2.1 Long-term parkers strictly prefer garage parking to curbside parking

In this case, constraint (21) does not bind. The market boundary for long-term parkers is obtained from (19):

$$x_H = \frac{D}{2} + \frac{s'_H - s_H}{2w_H} l_H. \quad (\text{A.13})$$

The market boundary for short-term parkers is determined by solving (20) and its counterpart for the neighboring garage. The solution for x_L is

$$x_L = \frac{k_L d_L l_L D + l_L f_L}{w_L + 2k_L d_L l_L} + \frac{k_L d_L l_L s'_L - (w_L + k_L d_L l_L) s_L}{w_L (w_L + 2k_L d_L l_L)} l_L. \quad (\text{A.14})$$

Substituting (A.13) and (A.14) into (6), and differentiating with respect to s_H and s_L , yields the home garage's best response functions for garage parking fees:

$$s_H = \frac{c}{2} + \frac{w_H D}{2l_H} + \frac{s'_H}{2} \quad (\text{A.15})$$

$$s_L = \frac{c}{2} + \frac{k_L d_L w_L D + w_L f_L + k_L d_L s'_L l_L}{2(w_L + k_L d_L l_L)}. \quad (\text{A.16})$$

In a symmetric equilibrium, $s'_H = s_H$ and $s'_L = s_L$. Equation (A.15) yields (23) in the text, and (A.16) yields (24). Equation (25) follows from (A.13) in the symmetric equilibrium. Finally, (26) is obtained by substituting (24) into (A.14).

A.2.2 Kink in demand curve

Suppose that the marginal long-term parker is indifferent between the home garage, the neighboring garage, and curbside parking. If the home garage reduces s_H , the marginal long-term parker now strictly prefers garage parking. Regime *Hg* applies and the market boundary for long-term parkers is defined by (A.13): $x_H = (D/2) + ((s'_H - s_H)l_H)/(2w_H)$. The slope of the home garage's market demand is

$$\left. \frac{\partial x_H}{\partial s_H} \right|_{Hg} = -\frac{l_H}{2w_H}.$$

If the home garage instead increases s_H , regime *Int* prevails and the market boundary

for long-term parkers is given by (10). The slope of the home garage's market demand is

$$\begin{aligned} \left. \frac{\partial x_H}{\partial s_H} \right|_{Int} &= \frac{-d_H l_H^2 k_H w_L + w_H (w_L + 2d_L l_L k_L) l_H}{w_H (w_H w_L + 2d_H l_H k_H w_L + 2d_L l_L k_L w_H)} \\ &= -\frac{l_H}{2w_H} \frac{2d_H l_H k_H w_L + 2w_H (w_L + 2d_L l_L k_L)}{2d_H l_H k_H w_L + w_H (w_L + 2d_L l_L k_L)} < -\frac{l_H}{2w_H}. \end{aligned} \quad (\text{A.17})$$

Since $\left\| \partial x_H / \partial s_H \big|_{Hg} \right\| < \left\| \partial x_H / \partial s_H \big|_{Int} \right\|$, the market demand curve for long-term parkers is kinked.

A.2.3 Long-term parkers indifferent between garage parking and curbside parking

Let (s_H^*, s_L^*) be candidate symmetric equilibrium fees, π^{Int} the home garage's profit function in regime *Int*, and π^{Hg} the home garage's profit function in regime *Hg*. If condition (21) binds, several local conditions must be satisfied for a candidate symmetric equilibrium in regime *Hg* to prevail. First, raising s_H slightly should not increase profit. Raising s_H slightly shifts the equilibrium from regime *Hg* to regime *Int*. An increase is unprofitable if

$$\lim_{s_H \rightarrow s_H^* +} \frac{\partial \pi^{Int}}{\partial s_H} \leq 0.$$

This condition can be written as a lower bound on s_H^* :

$$s_H^* \geq s_H^{Min}, \quad (\text{A.18})$$

where s_H^{Min} is a complicated expression.

Second, lowering s_H slightly should not increase profit. A slight reduction leaves equilibrium in regime *Hg*. The requisite condition is therefore

$$\lim_{s_H \rightarrow s_H^* -} \frac{\partial \pi^{Hg}}{\partial s_H} \geq 0.$$

This condition can be written as an upper bound on s_H^* :

$$s_H^* \leq s_H^{Max} = c + \frac{w_H D}{l_H}. \quad (\text{A.19})$$

Third, raising s_L should not increase profits. Since a slight increase in s_L leaves equilib-

rium in regime Hg , the condition for deviation to be unprofitable is

$$\lim_{s_L \rightarrow s_L^*} \frac{\partial \pi^{Hg}}{\partial s_L} \leq 0,$$

which can be written as a lower bound on s_L^* :

$$s_L^* \geq s_L^{Min} = \frac{(w_L + k_L d_L l_L) c + k_L d_L w_L D + w_L f_L}{2w_L + k_L d_L l_L}. \quad (\text{A.20})$$

Fourth, lowering s_L should be unprofitable. Since lowering s_L shifts equilibrium to regime Int , the relevant condition is

$$\lim_{s_L \rightarrow s_L^e} \frac{\partial \pi^{Int}}{\partial s_L} \geq 0.$$

This can be written as an upper bound on s_L^* :

$$s_L^* \leq s_L^{Max}, \quad (\text{A.21})$$

where s_L^{Max} is a complicated expression.

A final local condition for a symmetric equilibrium is that condition (21) in the text holds as an equality with $x_H = D/2$; this yields

$$s_L = \frac{2l_H (w_L + 2d_L l_L k_L) s_H - 2l_H (w_L + 2k_L d_L l_L) f_H + 4k_H d_L l_L^2 f_L}{4d_L l_L^2 k_H} + \frac{(w_H (w_L + 2d_L l_L k_L) - 2d_L l_L w_L k_H) D}{4d_L l_L^2 k_H}. \quad (\text{A.22})$$

Figure 4 illustrates conditions (A.18)-(A.22) for a numerical example with parameter values $f_H = f_L = 2.5$, $D = 0.125$, $c = 2.5$, $l_H = 2$, $l_L = 1$, $w_H = 8$, $w_L = 8$, $k_H = 0.2$, $k_L = 0.08$, $d_H = 100$, and $d_L = 100$. Inequality conditions (A.18)-(A.21) are all satisfied within the polygonal region of (s_H, s_L) with vertices at points A , B , C and D . All points within this region are candidate equilibria. Condition (A.22) lies within the region between points E and F . All points on the line segment EF are therefore candidate symmetric equilibria. We arbitrarily choose point E with the lowest garage parking fees as the equilibrium for regime Hg .

The necessary conditions for a profit maximum listed above are all local conditions. To confirm that regime Hg is a global equilibrium it is also necessary to check that it yields a higher profit to the home garage than any other regime. Doing so entails consideration

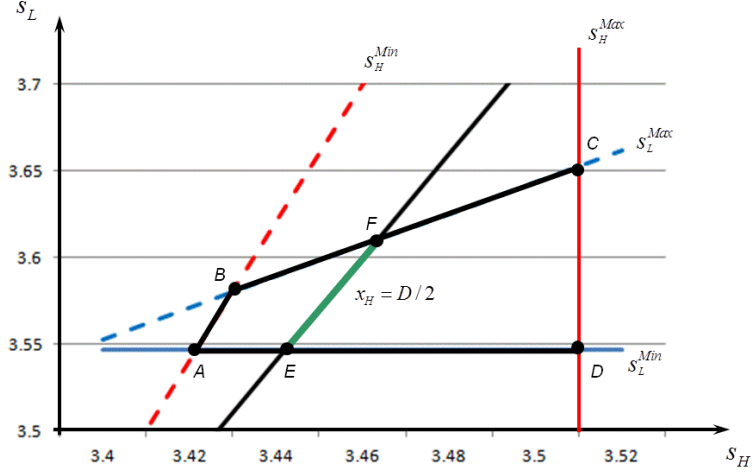


Figure 4: Example with multiple equilibria in Regime Hg

of non-local deviations by the home garage in garage parking fees that shift equilibrium to other regimes. This process is described in Appendix A.11 below. A systematic check is not practical analytically, but it is straightforward to do numerically as is done in Section 5.

A.3 Equilibrium for regime Lg

Regime Lg is a mirror image of regime Hg . The solution can therefore be obtained from the solution to regime Hg described in the text by interchanging H and L subscripts.

A.4 Equilibrium for regime Hc

In regime Hc , all long-term parkers park on the curb. Short-term parkers split between garages and the curb, and the marginal short-term parker is indifferent between parking at a garage and parking on the curb. Thus, the market boundary for short-term parkers is solved by using (8) with $T = d_H l_H D + d_L l_L (D - x_L - x'_L)$ and the counterpart of (8) for the neighbor. In a symmetric equilibrium with $s'_L = s_L$, the solution is

$$x_L = \frac{k_L (d_L l_L + d_H l_H) D + l_L (f_L - s_L)}{w_L + 2k_L d_L l_L}. \quad (\text{A.23})$$

The home garage's profit is given by (6) with $x_H = 0$: $\pi = 2(s_L - c)d_L l_L x_L$. The local profit-maximizing parking fee for short-term parkers works out to

$$s_L^* = \frac{(w_L + k_L d_L l_L) l_L c + w_L l_L f_L + w_L k_L (d_L l_L + d_H l_H) D}{(2w_L + k_L d_L l_L) l_L}. \quad (\text{A.24})$$

Several conditions must be satisfied for this solution to be consistent with regime *Hc* as well as with a global profit maximum. First, s_H must be high enough that no long-term parkers are drawn to park at a garage. The lower bound on s_H is an increasing function of s_L because a higher value of s_L induces more short-term parkers to use curbside parking, which makes curbside parking less attractive for long-term parkers, and increases their demand for garage parking. The constraint thus takes the form

$$s_H^* \geq s_H^{Min}(s_L), \quad (\text{A.25})$$

where $s_H^{Min}(s_L)$ is a complicated, increasing, linear function of s_L .

A second condition is that the home garage does not find it profitable to attract long-term parkers by setting s_H low enough. If it does attract any long-term parkers, the market moves into regime *Int*. Thus, the profit in regime *Int* should be non-decreasing at $s_H^{Min}(s_L)$:

$$\left. \frac{\partial \pi^{Int}}{\partial s_H} \right|_{s_H^{Min}(s_L)} \geq 0.$$

This condition can be written as an upper bound on s_H^* :

$$s_H^* \leq s_H^{Max}(s_L), \quad (\text{A.26})$$

where $s_H^{Max}(s_L)$ is another complicated, increasing, linear function of s_L .

Further conditions must be imposed because the profit function is not globally concave. If s_L is increased sufficiently above the level given in (A.24), curbside parking may become so crowded that long-term parkers start to use garage parking. Let $s_L|_{x_H=0}$ denote the threshold value of s_L if it exists. At this point, the profit function is kinked upward with respect to s_L . A necessary condition for (A.24) to be a global profit maximum is therefore that the local maximum of regime *Hc* be lower than the threshold parking fee at which the regime switches from regime *Hc* to regime *Int*.

$$s_L^* < s_L|_{x_H=0}. \quad (\text{A.27})$$

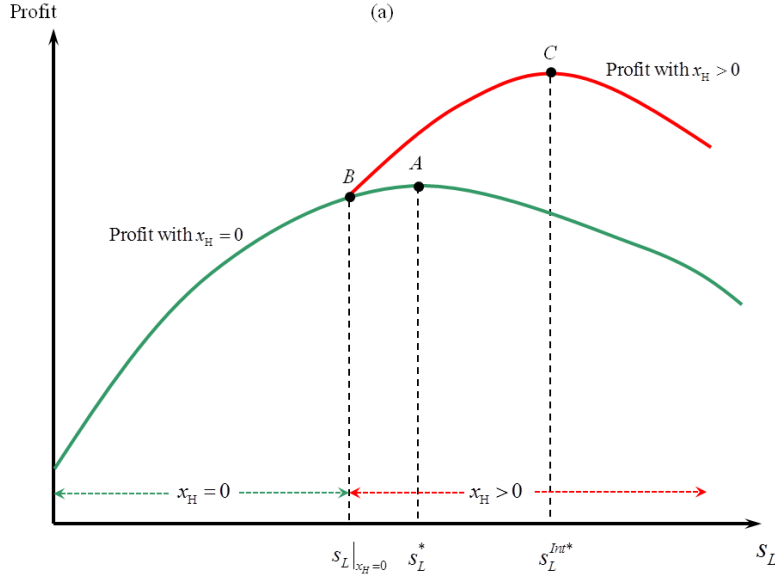


Figure 5: Profit function in regime Hc , case (a)

Figure 5 depicts a case where inequality (A.27) is violated. In this example, the parking market switches from regime Hc to regime Int at point B . Thus, by default, the local maximum in regime Hc , represented by point A , must be below the local maximum in regime Int , represented by point C . This means that point A cannot be the global maximum under these conditions. If inequality (A.27) is satisfied, profit may still increase enough within the interior region that the global profit maximum lies within regime Int rather than regime Hc . A fourth condition is therefore

$$\pi^{Hc*} > \pi^{Int*}, \quad (\text{A.28})$$

where $*$ denotes the maximum of the respective regime.

In Figure 6, regime switching occurs (at point B) after the local maximum of regime Hc (point A); yet the local maximum of regime Int , point C , is higher. Thus, in this figure, condition (A.27) is satisfied but condition (A.28) is violated. Figures 7 and 8 show two further cases for which both conditions are satisfied.

Finally, multiple equilibria are a possibility. Figure 9 depicts an example with the same parameter values as for the base case of the numerical example in Section 5 except for the curbside parking fees: $f_H = f_L = 0.7$, $D = 0.125$, $c = 2.5$, $l_H = 2$, $l_L = 1$, $w_H = 16$, $w_L = 16$, $k_H = 0.16$, $k_L = 0.16$, $d_H = 100$, and $d_L = 100$. The locally optimal parking fee for short-term parkers in regime Hc is $s_L^* = 3.9$. Inequality conditions (A.25) and (A.26) are satisfied on line segment AB . Conditions (A.27) and (A.28) are also satisfied since profits within regime Int are decreasing to the right of the line $\partial\pi^{Int}/\partial s_L = 0$. The choice of s_H on

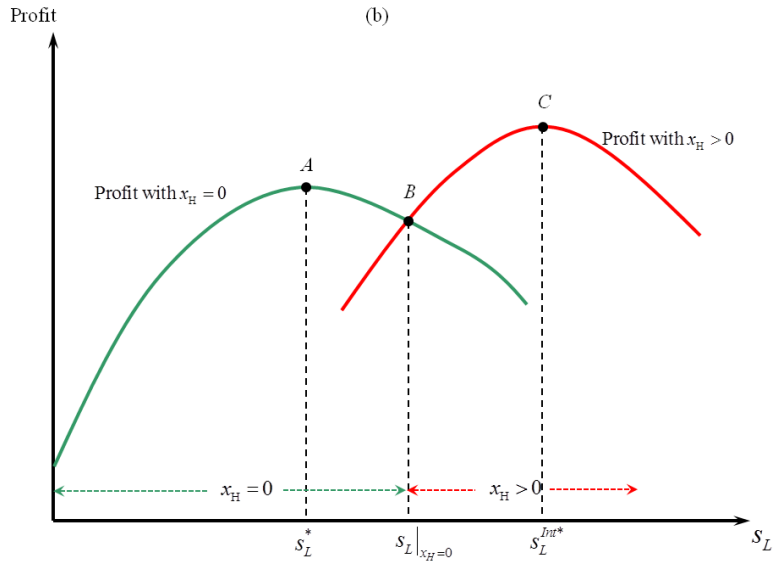


Figure 6: Profit function in regime Hc , case (b)

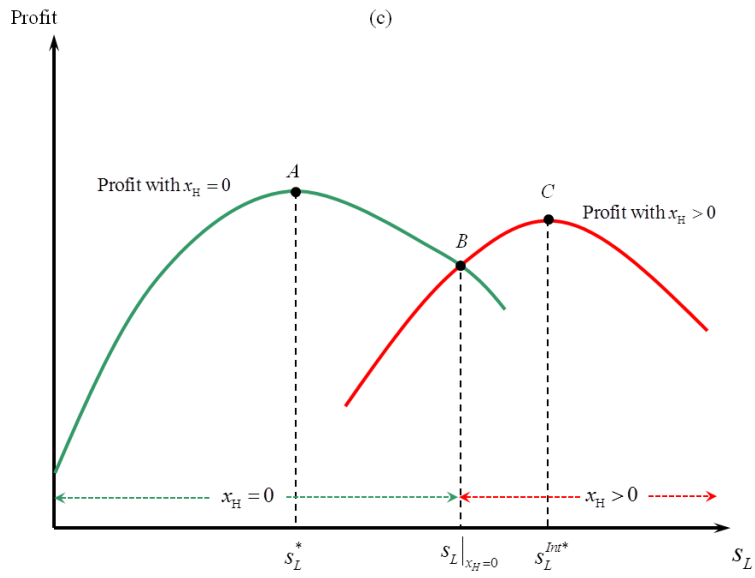


Figure 7: Profit function in regime Hc , case (c)

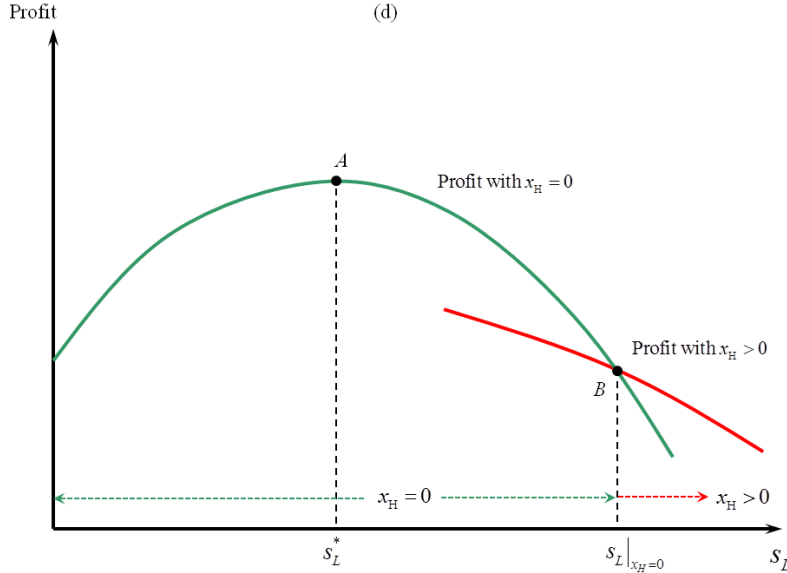


Figure 8: Profit function in regime Hc , case (d)

segment AB is inconsequential because it does not affect parking choices for either type of driver.

The equilibrium conditions listed above are necessary conditions for an equilibrium in regime Hc to prevail. To confirm that regime Hc is a global equilibrium, it is also necessary to check that it yields a higher profit to the home garage than any other regime.

A.5 Equilibrium for regime Lc

Regime Lc is a mirror image of regime Hc , and the solution can be obtained from the solution to regime Hc by interchanging H and L subscripts.

A.6 Equilibrium for regime $Hc+Lg$

In this regime, all long-term parkers park on the curb while all short-term parkers park at a garage. Thus, $x_H = 0$ and $x_L = D/2$ in a symmetric equilibrium. Total parking time on the curb is $T = d_H l_H D$. Several conditions must be satisfied for this regime to be an equilibrium. First, s_H must be high enough that no long-term parkers are drawn to park at

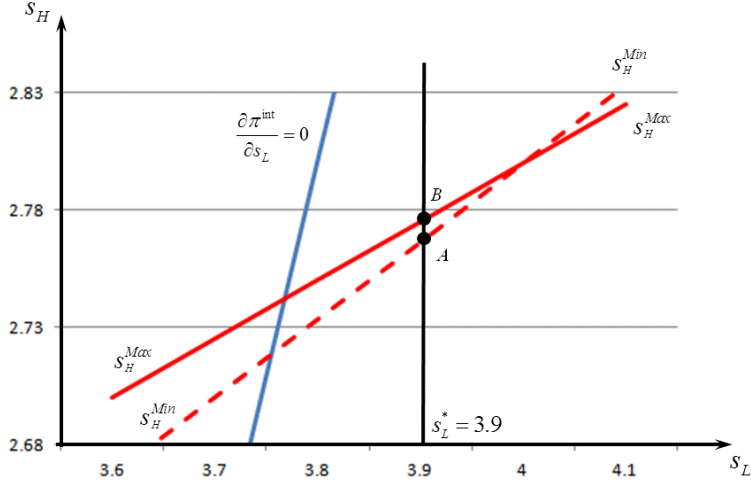


Figure 9: Example with multiple equilibria in regime Hc

a garage, a condition that works out to

$$s_H \geq \frac{f_H l_H + k_H d_H l_H D}{l_H} \equiv s_H^{Min}. \quad (\text{A.29})$$

Second, the curbside parking fee must be high enough that no short-term parkers are drawn to curbside parking, which defines an upper bound on the garage parking fee that can be charged to short-term parkers:

$$s_L \leq \frac{f_L l_L + k_L d_H l_H D - \frac{w_L D}{2}}{l_L} \equiv s_L^{Max}. \quad (\text{A.30})$$

Third, it cannot be profitable to decrease s_H and attract long-term parkers, which would cause a shift to regime Lg :

$$\left. \frac{\partial \pi}{\partial s_H} \right|_{s_H^{Min}} \geq 0. \quad (\text{A.31})$$

Fourth, it cannot be profitable to increase s_L and induce some short-term parkers to shift to the curb which would cause a shift to regime Hc :

$$\left. \frac{\partial \pi}{\partial s_L} \right|_{s_L^{Max}} \leq 0. \quad (\text{A.32})$$

As is the case with other regimes, these four conditions are necessary conditions for a local maximum. To confirm that regime $Hc + Lg$ is a global equilibrium, it is also necessary to check that it yields a higher profit to the home garage than any other regime.

A.7 Equilibrium for regime $Hg + Lc$

This regime is a mirror image of regime $Hc + Lg$. The solution can therefore be obtained from the solution to regime $Hc + Lg$ by interchanging H and L subscripts.

A.8 Equilibrium for regime $Hg + Lg$

All drivers of both types in this regime use garage parking. Thus, $x_H = D/2$ and $x_L = D/2$ in a symmetric equilibrium. For this regime to be an equilibrium, the following conditions must be satisfied. First, s_H must be low enough that no long-term parkers are drawn to park on the curb:

$$s_H \leq \frac{2f_H l_H - w_H D}{2l_H} \equiv s_H^{Max}. \quad (\text{A.33})$$

Second, a similar condition applies for short-term parkers:

$$s_L \leq \frac{2f_L l_L - w_L D}{2l_L} \equiv s_L^{Max}. \quad (\text{A.34})$$

Third, it cannot be profitable to increase s_H above s_H^{Max} which would cause a shift to regime Lg :

$$\left. \frac{\partial \pi^{Lg}}{\partial s_H} \right|_{s_H^{Max}} \leq 0. \quad (\text{A.35})$$

Fourth, it cannot be profitable to increase s_L above s_L^{Max} which would cause a shift to regime Hg :

$$\left. \frac{\partial \pi^{Hg}}{\partial s_L} \right|_{s_L^{Max}} \leq 0. \quad (\text{A.36})$$

These four conditions are necessary conditions for a local maximum. To confirm that regime $Hg + Lg$ is an equilibrium regime it is also necessary to check numerically that it yields a higher profit to the home garage than any other regime.

A.9 Equilibrium for regime $Hc+Lc$

In this regime, all drivers of both types park on the curb. Thus, $x_H = 0$, $x_L = 0$, and $T = (d_H l_H + d_L l_L)D$. Several conditions must be satisfied for this regime to be an equilibrium. First and second, garage parking fees must be high enough that all drivers prefer to park on the curb. The requisite conditions are

$$s_H \geq \frac{f_H l_H + k_H(d_H l_H + d_L l_L)D}{l_H} \equiv s_H^{Min} \quad (\text{A.37})$$

$$s_L \geq \frac{f_L l_L + k_L(d_H l_H + d_L l_L)D}{l_L} \equiv s_L^{Min}. \quad (\text{A.38})$$

Third, it cannot be profitable to decrease s_H to attract long-term parkers and induce a shift to regime Lc :

$$\left. \frac{\partial \pi^{Lc}}{\partial s_H} \right|_{s_H^{Min}} \geq 0. \quad (\text{A.39})$$

Fourth, it cannot be profitable to decrease s_L to attract short-term parkers and induce a shift to regime Hc :

$$\left. \frac{\partial \pi^{Hc}}{\partial s_L} \right|_{s_L^{Min}} \geq 0. \quad (\text{A.40})$$

Again, these four conditions are necessary conditions for a local maximum. To confirm that regime $Hc + Lc$ is an equilibrium regime, it is also necessary to check numerically that the home garage cannot obtain positive profits in another regime.

A.10 The social optimum: regime Int

Because it is an interior solution, the social optimum in regime Int is characterized by first-order conditions $\partial TC / \partial x_H = 0$ and $\partial TC / \partial x_L = 0$ where TC is given by the the sum of (29), (30), and (31). The solution is

$$x_H^{SO} \Big|_{Int} = \frac{\left(\begin{array}{l} 2d_L l_L [d_H(k_H l_L - k_L l_H)]^2 D - c(k_H + k_L)l_L + ck_L l_H \\ -2d_H l_H k_H D w_L - d_L l_L (k_H + k_L) D w_L + cl_H w_L \end{array} \right)}{4d_H d_L (k_H l_H - k_L l_L)^2 - 4d_L l_L k_L w_H - 4d_H l_H k_H w_L - w_H w_L} \quad (\text{A.41})$$

$$x_L^{SO} \Big|_{Int} = \frac{\left(\begin{array}{l} 2d_H l_H [d_L(k_H l_L - k_L l_H)]^2 D - c(k_H + k_L)l_H + ck_H l_L \\ -2d_L l_L k_L D w_H - d_H l_H (k_H + k_L) D w_H + cl_L w_H \end{array} \right)}{4d_H d_L (k_H l_H - k_L l_L)^2 - 4d_L l_L k_L w_H - 4d_H l_H k_H w_L - w_H w_L}. \quad (\text{A.42})$$

The second-order condition for a total cost minimum is $(4k_H d_H l_H + w_H)(4k_L d_L l_L + w_L) - 4d_H d_L (k_H l_L + k_L l_H)^2 > 0$.

A.11 Testing for a global equilibrium

As noted above, to establish that a candidate symmetric equilibrium for a given regime is a global equilibrium, it is necessary to check that the home garage cannot earn higher profits by deviating from the candidate and bringing about a change of regime. The following procedure was used for the numerical examples.

Let (s_H^e, s_L^e) denote candidate symmetric equilibrium parking fees and set the neighboring garage's fees at these values. Then, allow the home garage to experiment with different combinations of (s_H, s_L) that shift equilibrium into other regimes. For example, suppose that the candidate equilibrium regime is *Int*. Set $(s'_H, s'_L) = (s_H^{Int}, s_L^{Int})$. Then, consider deviations from (s_H^{Int}, s_L^{Int}) by the home garage. One possibility is that the home garage will reduce s_H to the point where all long-term parkers park in garages. If short-term parkers continue to park both in garages and on the curb, the new regime becomes *Hg*. Assume that regime *Hg* does prevail and derive profit-maximizing values of (s_H, s_L) for the home garage while holding (s'_H, s'_L) fixed at (s_H^{Int}, s_L^{Int}) . Several consistency conditions have to be satisfied (the x_i must be non-negative, etc.). If the new regime passes this initial test, check whether the home garage's profits are higher than in the candidate equilibrium. If they are higher, the candidate equilibrium fails. Otherwise, the candidate equilibrium passes this test and the next alternative regime is examined.

Because deviation by the home garage breaks symmetry, it is possible that the home garage and the neighbor will serve different markets. Consequently, several asymmetric regimes must be tested in addition to the symmetric regimes described in the text. The additional regimes are as follows:

- $Hc(-)$: The home garage does not serve any long-term parkers, but the neighbor does serve some of them. Both home garage and neighbor serve some short-term parkers while the remaining short-term parkers park on the curb.
- $-(Hc)$: Same as $Hc(-)$ with the roles of home garage and neighbor reversed.
- $Lc(-)$: The home garage does not serve any short-term parkers, but the neighbor does serve some of them. Both home garage and neighbor serve some long-term parkers while the remaining long-term parkers park on the curb.

- $-(Lc)$: Same as $Lc(-)$ with the roles of home garage and neighbor reversed.
- $Hc(Hc)$: Neither the home garage nor the neighbor serves long-term parkers. This regime is qualitatively the same as regime Hc although it is not symmetric.
- $Lc(Lc)$: Neither the home garage nor the neighbor serves short-term parkers. This regime is qualitatively the same as regime Lc although it is not symmetric.
- $Hg + Lc(-)$: The home garage and neighbor together serve all long-term parkers. The home garage does not serve any short-term parkers, but the neighbor does serve some of them.
- $Hc(-) + Lg$: The home garage does not serve any long-term parkers, but the neighbor does serve some. The home garage and neighbor together serve all short-term parkers.
- $Hc(Hc) + Lg$: Neither the home garage nor the neighbor serves long-term parkers. Together they serve all short-term parkers. This regime is qualitatively the same as regime $Hc + Lg$ although it is not symmetric.
- $Hg + Lc(Lc)$: The home garage and neighbor together serve all long-term parkers. Neither serves any short-term parkers. This regime is qualitatively the same as regime $Hg + Lc$ although it is not symmetric.

Depending on which candidate symmetric equilibrium is being tested, some of these additional regimes cannot be reached by profitable deviations by the home garage. But it is simpler to run through all the possibilities numerically than to determine analytically which are potentially profitable.

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