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MEASUREMENTS OF VALUE OF TIME AND TRANSPORTATION BENEFITS USING OBSERVABLE DEMAND

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ABSTRACT

The subjective value of time is the marginal rate of substitution between travel time and travel cost under constant utility or profit level. It is commonly referred to as the monetary appraisal of value of time or the willingness to pay for savings in travel time. This study proposes methods to measure the true value of time from *observable* demand functions applicable to non-business transport services modelled both for persons and freight. The methods proposed are different from all previous studies in the valuation of value of time which assumed either a specified utility function then applying true VOT under constant utility or a specified demand function applied using VOT under constant demand. This study showed the capability of the proposed methods to measure the components of the value of time, i.e., value of time as a resource and value of time as a commodity calculated independently or combined using observable demand for non-business person trips. The key factor in deriving the VOT in terms of observable demand is the application of the time-extended Roy's identity. The VOT as a resource defined by the marginal substitution ratio of transport cost and travel time is expressed as the observable transport demand and its derivatives. In deriving the VOT as a commodity, tx is a measure of disutility of traveling. VOT as a commodity varies depending on the mode, length of trip, destination, timing, among other, contrary to the VOT as a resource, which is fix regardless of those mentioned transport characteristics. The total VOT is the combination of the VOT as a resource and VOT as a commodity. Two methods are shown on how to measure the time savings benefits per VOT component in terms of observable Marshallian demand and VOT. The first method is by expressing the Marshallian demand with adjustment of the marginal utility ratio of income, $e_y x$. The second method is by expressing the compensated demand h in terms of observable Marshallian demand. As it is the Hicksian compensated demand and VOT that need to be measured, this study succeeded in deriving the time extended Slutsky equations that relates the compensated demand and VOT to the observable Marshallian demand and VOT. The result of non-business freight trips is exactly identical to the case of VOT as a commodity for non-business person trips. The discussion of VOT for business person and freight trips will not included due to the space limit.

Keywords: value of time as a resource, value of time as a commodity, value of time for freight trips, time saving benefit

1. INTRODUCTION

Value of time is one of the most important variables for both trip demand forecasting and transportation project appraisal because its value is a key factor in the generalized travel cost. The value of time for household is defined as the change in travel fee corresponding to the change in travel time with the utility level kept constant, i.e., the willingness to pay for marginal change in travel time. Value of time for firms is defined as the same with the profit kept constant. Mathematically, it is defined as

$$VOT \equiv -\frac{dp}{dt} \Big|_{u \text{ or } \pi = \text{const}} \quad (1.1)$$

where p is transport cost, t is necessary transport time, u is utility and π is profit. The subjective value of time is the marginal rate of substitution between travel time and travel cost under constant utility or profit level. It is commonly referred to as the monetary appraisal of value of time or the willingness to pay for savings in travel time. In practice, value of time is obtained by making use of the formulation in (1.2), as it is difficult to grasp the function form of the utility or profit given by (1.1).

$$VOT \equiv -\frac{dp}{dt} \Big|_{X \text{ or } c = \text{const}} \quad (1.2)$$

where p is transport cost, t is necessary transport time, X is demand quantity and c is transportation generalized cost. The value of time obtained from (1.2) is under the assumption of constant demand or generalized cost.

The value for the marginal substitution of traffic cost p relative to travel time t differs depending on the method applied to derive the value of time. Using (1.1) is not equal to using (1.2), as in the latter assumes the demand quantity or the generalized cost as fixed. Needless to say, it is dubious to obtain an accurate value of time from (1.2). Thus, the formulation in (1.1) is used in exact measurement of value of time. However, there has been no previous study, to my knowledge, that specifically dealt with this fundamental question discussing if the exact value of time, that is under constant utility, is equal to the practical value of time derived under constant demand in practice. Also, there has been no study discussing the implications of using the formulation of (1.2) instead of (1.1).

The current practice derives the value of time by two ways, either by the wage rate approach or by mode choice analysis using Stated Preference (SP) or Revealed Preference (RP) data. The wage rate approach is appropriate for business trips with VOT as a resource as it is the firm that dictates behavior but does not hold true for private or non-business trips. However, both methods do not consider changes in value of time relative to changes in economic factors like prices, travel time, among others. Also, previous papers on the valuation of passenger travel time using SP or RP data have assumed a specified utility and demand functions. The limitation of this method is that the results are purely indicative, coming from specified functional form. In addition, there is difficulty in data acquisition. RP method requires mode choice data or route choice data, which in some kind of trips are not available

as there are no choices between modes or routes. The main objective of this chapter is to measure the value of time and its accrued time savings benefits for non-business person trips using observable demand. The components of value of time that will be analyzed are value of time as a resource, value of time as a commodity and the case combining both.

To express the value of time in terms of observable demand for non-business person trip, the indirect utility function and expenditure function are derived based on the given demand. This is different from the more common approach of starting from a specified utility function like trans-log then estimating the unknown parameters from the derived market functions. However, the analysis on how to choose the appropriate demand function for the given demand will not be discussed here. In other words, it is assumed that market demand is observable, and that the preference is not observed. Instead, the focus is on the next step, on the indirect utility function and expenditure function, on how to express the value of time in terms of observable demand.

The process applied here is an extension of the Larson and Shaikh's (2001) approach, which they introduced to measure recreation demand. With the extension, it is then possible to estimate the general case of an endogenous value of time jointly with observable behavior in a utility-consistent framework, which utilizes full price – full income specification subject to two binding constraints (i.e., income and time constraints) on demand parameters. This approach, with the indirect utility function and expenditure function, lead to exact welfare measurement derived consistently from observable demand for person trips.

This paper will be organized as follows. Section 3 is the case of value of time as a resource, then in Section 4 is the value of time as a commodity and in Section 5 is analysis of the combination of both. Each will have sub-sections dealing with the description of individual behavior followed by the derivation of the value of time in full form in terms of observable demand under constant utility then the measurement of time savings benefits by deriving the compensated value of time from the expenditure function in terms of observable Marshallian demand. The extended Slutsky equations will result from this sub-section. In The discussion of VOT for business person and freight trips will not included due to the space limit.

2. PREVIOUS STUDIES

Since the early attempts to incorporate time into consumer theory, such as those by Becker (1965), previous approaches primarily relied on the wage rate to provide information about a person's willingness to trade time with money. In applied economic research, there have been criticisms on the use of the wage rate as the relevant opportunity cost of time. Recent approaches, for example Bockstael et al (1987) and Feather and Shaw (2000) have explicitly recognized that people might not be able to freely choose their work hours, a such have allowed the shadow value of time to deviate from the wage rate. Even with the recent developments that clarified more explicitly the nature of the value of time, there is still no consensus yet on a workable method for estimating the exact value of time. Other than the current discrete choice models approach, the concept of measuring the value of time in terms of observable demand from market data is yet under explored. Although this concept is already popular in valuing non-market goods in environmental economics, its application in the context of value of time analysis is not yet tapped. There are series of papers by Larson and Shaikh (2001, 2003, 2004) that mentioned the observability of demand in the context of

valuing leisure time. However, their analysis only considered VOT as a resource and made no mention of the case of VOT as a commodity. Larson and Shaikh (2001) modified Becker's full price approach to introduce the two-constraints model for recreation choice wherein time is a binding constraint. With this approach, it is possible to extend it to evaluate the general case of person trips and carry out exact welfare measurement that is consistent to the utility theoretic model. They first mentioned the observability of VOT as a resource and applied it to derive recreation demand. However, there is no approach yet wherein the components of VOT can be specifically derived based on actual observable demand. Moreover, measurement of transport benefits in terms of observable demand is a novel approach that has not been introduced to the value of time research yet. Further thorough analysis of this approach will be of use to establish the theoretical basis for future empirical applications.

Based on the recent reviews and theoretical development on the study of the value of time derived from the studies of De Serpa (1971), De Donnea (1972), Evans (1972), Small (1982), Gronau (1986), Bates (2001) and Jara-Diaz (2003), theoretical analysis concludes that value of time consists of three components: 1) value of time as a resource (VTR), which represents value of re-assignment of time to other activities, 2) value of time as a commodity (VTC), which represents direct variation in the satisfaction of traveling, and 3) value of changes in the consumption pattern (VCC), which is caused by the arrangement of time. There have been numerous studies measuring the value of travel time savings like those of Ben-Akiva et al (1994), Bates et al (2001), Gunn (2001,) Wardman (2001), and Hensher (2001). However, few studies paid attention in analyzing the variation mechanisms of the value of time and its components. Notable studies in the literature are those of De Serpa (1971), De Donnea (1972), Kono and Morisugi (2000) and Jiang (2003). Most studies measured only the part where value of time is considered as a resource partly because the components of the value of time are considered to be largely theoretical constructs. Now the question is, is there any other way to incorporate the effects of the other components of the value of time?

This study attempts to fill the research gaps mentioned in the background and motivation of the study vis-à-vis in the definition and welfare measurement of the value of time and transport benefits.

3. VALUE OF TIME AS A RESOURCE: NON BUSINESS PERSON TRIPS

3.1 Modeling individual behavior

First, to formulate the VOT, think of an individual on a non-business trip, which is, either commuting or shopping. Let $u(z, x, l)$ be the individual's preference function. The utility of an individual is assumed to be a function of the demand for commodity goods z whose price is normalized to 1, transport service demand x and leisure time l . This formulation assumes neutrality in time t where disutility induced by the trip duration such as fatigue, inconvenience and discomfort are not included in the analysis. These kinds of effects are discussed in the next chapter.

Also, assume that working time is fixed then referring to (3.1) and (3.2), the individual maximizes utility subject to income y and total available time T . The indirect utility function, $V(p, t, T, y)$, is given by

$$V(p, t, T, y) = \max_{z, x, l} u(z, x, l) \quad (3.1)$$

$$\text{s.t. } z + px = y, \quad \text{and} \quad tx + l = T \quad (3.2)$$

The utility function is assumed to be differentiable and quasi-concave in x , and the constraints are differentiable and linear in x and in both money and time prices and money and time budgets.

3.2 Envelope theorem

The Lagrangian function of (3.1) and (3.2) is represented by

$$V(p, t, T, y) = \max_{z, x, l} u(z, x, l) + \lambda(z - y) + \mu(T - tx - l) \quad (3.3)$$

The Lagrange multipliers λ and μ represent the shadow values of money and time, respectively. Intuitively, λ is the marginal utility of income and μ is the marginal utility of time. By applying envelope theorem to (3.3), the following are derived:

$$V_y = \lambda(p, t, T, y) \quad (3.4a)$$

$$V_T = \mu(p, t, T, y) \quad (3.4b)$$

$$V_p = -\lambda(p, t, T, y) x(p, t, T, y) \quad (3.4c)$$

$$V_t = -\mu(p, t, T, y) x(p, t, T, y) \quad (3.4d)$$

where V_y, V_T, V_p, V_t are partial derivatives wrt the subscript

Using the envelope results, the Marshallian demand $x(p, t, T, y)$ can be recovered through two separate versions of Roy's identity,

$$V_p = -V_y x \quad (3.5a)$$

$$V_t = -V_T x \quad (3.5b)$$

3.3 VOT as a resource expressed by observable Marshallian demand

Value of time is defined as the marginal substitution rate for utility between price p and duration time t . Recalling (1.1) and applying Roy's identities above, the value of time can be expressed as

$$VOT \equiv -\frac{dp}{dt} \Big|_{v=const} = \frac{V_t}{V_p} = \frac{-V_T x}{-V_y x} = \frac{V_T}{V_y} = \frac{\mu}{\lambda} \quad (3.6)$$

where notation with subscripts indicate partial derivatives wrt the subscript

To express (3.6) in terms of observable demand, the approach is to take the derivative of (3.5a) and (3.5b) with respect to t and p to express the VOT in terms of x ,

$$V_{pt} = \left(\frac{V_y x}{p} \right)' = -V_{ty} x - V_y x_t = - \left(\frac{V_T x}{p} \right)' x - V_y x_t = V_{Ty} x^2 + V_T x_y x - V_y x_t \quad (3.7)$$

$$V_{tp} = \left(\frac{\partial V_T x}{\partial p} \right) = -V_{pTx} - V_T x_p = - \left(\frac{\partial V_y x}{\partial p} \right) x - V_T x_p = V_y T x^2 + V_y x_T x - V_T x_p \quad (3.8)$$

Note that (3.7) and (3.8) are equal, so equating (3.7) and (3.8),

$$V_T \left(\frac{\partial p}{\partial x} + x x_y \right) = V_y \left(\frac{\partial t}{\partial x} + x T x \right) \quad (3.9)$$

Therefore, it follows from (3.9) that VOT as a resource can be expressed in terms of observable demand x as

$$VOT|_{v=const} = \frac{V_T}{V_y} = \frac{x_t + x x_T}{x_p + x x_y} \quad (3.10)$$

Analyzing (3.10), value of time under the assumption of constant utility can be measured from observable changes in the Marshallian demand x where x_t is the change in demand with respect to change in time, x_p is the change in demand with respect to change in transport cost, x_T is the change in demand with respect to change in total available time and x_y is the change in demand with respect to change in income. The denominator in (3.10) is the price substitution effect while the numerator is the time substitution effect. Looking at it, it shows that VOT is observable just by examining only the price and time substitution effects of transport demand. Also, it shows at the same time that VOT is a function of p, t, T, y and other individual characteristics, but it is identical for any mode, OD, route transportation of a specific individual.

3.4 Application of how VOT can be derived by observable demand

To illustrate how (3.10) could be applied in practice, i.e., how VOT can be derived by observable demand, take these demand functions as examples.

a) Linear demand function

Two examples of linear demand function are shown. Let

$$x_1 = a + bp + ct + dy + fT \quad (3.11a)$$

Then applying (3.10), VOT can be derived as

$$VOT_1 = \frac{x_t + x x_T}{x_p + x x_y} = \frac{c + f x_1}{b + d x_1} \quad (3.11b)$$

Note that the true value of VOT in (3.11b) is not simply equal to $\frac{c}{b}$ or $\frac{f}{d}$.

Now, let

$$x_2 = a + bp + ct + fby + fcT \quad (3.12a)$$

Accordingly, VOT is derived as

$$VOT_2 = \frac{x_t + x x_T}{x_p + x x_y} = \frac{c(1 + f x_2)}{b(1 + f x_2)} = \frac{c}{b} = \frac{x_t}{x_p} = VOT|_{x=const.} \quad (3.12b)$$

Note that in (3.11a) and (3.12a), $d \neq fb$ and $f \neq fc$ so the resulting (3.11b) and (3.12b) are not equal.

b) *Log-linear demand function*

Let the demand function be

$$x_3 = ay^b T^c p^d t^f \quad (3.13a)$$

Substituting the demand function x and its derivatives to derive the value of time is shown as,

$$VOT_3 = \frac{x_t + xx_T}{x_p + xx_y} = \frac{\frac{fx_3}{t} + x_3 \frac{cx_3}{T}}{\frac{dx_3}{p} + x_3 \frac{bx_3}{y}} = \frac{f/t + c/T x_3}{d/p + b/y x_3} \quad (3.13b)$$

c) *Semi-log linear demand function*

Now consider x as a semi-log linear function,

$$x_4 = e^{a+by+cT+dp+ft} \quad (3.14a)$$

And the resulting VOT is shown as,

$$VOT_4 = \frac{x_t + xx_T}{x_p + xx_y} = \frac{fx_4 + x_4 cx_4}{dx_4 + x_4 bx_4} = \frac{f + cx_4}{d + bx_4} \quad (3.14b)$$

From the different demand functions shown, as long as the formulation is expressed in full form where time is a binding constraint, then it is possible to simply substitute the demand function to the formulation of VOT in terms of x and its derivatives.

3.5 Compensated demand function and expenditure function

The dual approach to the problem is to consider the associated minimization problem, which defines the expenditure function

$$e(p, t, T, \bar{u}) \equiv \min_{h_0, h} h_0 + ph \quad (3.15)$$

$$\text{s.t. } th + h_2 = T, \quad u(h_0, h, h_2) = \bar{u} \quad (3.16)$$

where $h_i \in [0, 2]$ represents the demand for commodity goods z and leisure l respectively, h is the compensated transport demand, t is transport time, T is total available time and \bar{u} is given level of utility, and $e(p, t, T, \bar{u})$ is expenditure function.

The expenditure function may be expressed as the minimization of the Lagrange function,

$$e = L = h_0 + ph + \xi(-th - h_2) + \varphi(-u(h_0, h, h_2) - \bar{u}) \quad (3.17)$$

Then the following are derived from the envelope theorem applied to (3.17):

$$e_p = h \quad (3.18a)$$

$$e_t = -\xi h = -e_T h \quad (3.18b)$$

$$e_T = \xi \quad (3.18c)$$

3.6 Compensated VOT and Time-extended Slutsky Equations

The compensated VOT is defined as the marginal substitution rate for expenditure function between price p and time t , which is equal to the marginal value of available time T . The compensated VOT in terms of the expenditure function is given as,

$$\text{compensated VOT} = -\frac{dp}{dt} \Big|_{\substack{e=\text{const} \\ \bar{u}=\text{const}}} = \frac{e_t}{e_p} = \frac{-e_T h}{h} = -e_T \equiv \rho \quad (3.19)$$

where $\left(e_T \right)$ can be interpreted as the compensated value of time

Now, to express the compensated VOT in terms of the compensated demand h , take the derivative of the $e_p = h$ with respect to t and $e_t = -\xi h = -e_T h$ with respect to p , which results to

$$e_{pt} = h_t \quad (3.20a)$$

$$e_{tp} = \left(e_T h \right)_p = -e_{pT} h - e_T h_p = -h_T h - e_T h_p \quad (3.20b)$$

Then equating (3.20a) and (3.20b), the compensated VOT is equal to

$$\rho = \frac{\left(h_t + h h_T \right)}{h_p} \quad (3.21)$$

Note that the compensated VOT, i.e. (3.21), is not equal to the Marshallian VOT given by (3.10), which is $VOT|_{v=\text{const}} = \frac{V_T}{V_y} = \frac{x_t + x x_T}{x_p + x x_y}$.

This compensated VOT can be expressed by the compensated demand function. The compensating demand function is always equal to the usual demand function at income equal to $e(p, t, T, \bar{u})$, which can be expressed as,

$$h(p, t, T, \bar{u}) \equiv x(p, t, T, e(p, t, T, \bar{u})) \quad (3.22)$$

Taking the derivative of (3.22) with respect to p , t and T , and recall that when u is at $V(p, t, T, y)$ then it follows that $e(p, t, T, V(p, t, T, y)) \equiv y$ and accordingly, it can be derived that $x_e = x_y$, $h = x$ and $VOT = \rho = -e_T$, then it can be shown that

$$h_p = x_p + x x_y \quad (3.23a)$$

$$h_t = x_t + \left(VOT \right) x x_y \quad (3.23b)$$

$$h_T = x_T - \left(VOT \right) x y \quad (3.23c)$$

From (3.23b) and (3.23c), this relationship can be derived showing how h_t can be expressed in terms of h_T ,

$$h_t + x h_T = x_t + x x_T \quad (3.23d)$$

Notice that (3.23a)-(3.23d) show the relationships between the observable uncompensated Marshallian demand and the unobservable Hicksian demand, that is, the changes in compensated demand h with respect to travel cost $\left(p \right)$, travel time $\left(t \right)$ and total available time $\left(T \right)$ can be expressed in terms of the changes in observable Marshallian demand x . These expressions lead to evaluating the compensated demand in terms of observable demand. These relationships of the uncompensated demand to observable demand have not

been expressed this way before. These formulations will be more beneficial in the succeeding texts where transport benefits will be derived based on observable demand.

Following the same logic, the compensating VOT is always equal to the usual Marshallian VOT at income equal to $e(p, t, T, u)$, which can be expressed as, $\rho(p, t, T, u) \equiv VOT(p, t, T, e(p, t, T, u))$. Accordingly, the compensated VOT can be expressed in terms of observable VOT, which take the form similar to a Slutsky equation as follows:

$$\rho_p = VOT_p + x VOT_y \quad (3.24a)$$

$$\rho_t = VOT_t + VOT_x VOT_y \quad (3.24b)$$

$$\rho_T = VOT_T + VOT_x VOT_y \quad (3.24c)$$

It is also necessary to calculate the combined compensated demand and VOT. With $h(p, t, T, u) \equiv x VOT(p, t, T, e(p, t, T, u))$, the derivatives of h_p with respect to p and t can be derived as,

$$h_{p_p} = x VOT_{p_p} + x x VOT_{p_y} \quad (3.25a)$$

$$h_{p_t} = x VOT_{t_t} + x VOT_{t_x} VOT_y \quad (3.25b)$$

$$h_{p_T} = VOT_T + VOT_x VOT_y \quad (3.25c)$$

These extended Slutsky formulations of the compensated VOT in terms of observable VOT will come in handy in the measurement of welfare changes as it is only the Marshallian VOT that is observable and yet the compensated VOT is the vital component that needs to be measured.

Finally, at $A = (p^A, t^A, T^A, y^A)$, comparing the Marshallian VOT to the Hicksian VOT, recall (3.10), $VOT|_{v=const} = \frac{V_T}{V_y} = \frac{x_t + x x_T}{x_p + x x_y}$, and (3.21), $\rho = \frac{q_t + h h_T}{h_p}$. Referring to (3.23a),

where $h_p = x_p + x x_y$, then we could say that the denominators of the Marshallian VOT and Hicksian VOT are equal.

Looking at the numerator of $\rho = \frac{q_t + h h_T}{h_p}$ and referring to (3.23b) and (3.23c), which are

$h_t = x_t + VOT_x x_y$ and $h_T = x_T - VOT_x x_y$ respectively, then from (3.23b) and (3.23c), this equation can be derived relating $h_t = x_t + x x_T - x h_T$. Substituting that formulation of h_t to the

numerator of $\rho = \frac{q_t + h h_T}{h_p}$, given that $x = h$, then it becomes equal to the numerator of

$VOT|_{v=const} = \frac{V_T}{V_y} = \frac{x_t + x x_T}{x_p + x x_y}$. Therefore, with the derived Slutsky equations, it is possible

to relate the observable Marshallian VOT to the compensated Hicksian VOT.

3.7 Measurement of time saving benefits using observable demand

The indirect utility function and the expenditure function provide the theoretical structure for welfare estimation. The theoretical method is to use information available about the expenditure function or its accompanying Hicksian demand curves for market goods to obtain compensating or equivalent variation measures of a change in a nonmarket good.

Applying the concept of equivalent variation to the value of time problem, it is not only the price change that needs to be evaluated. As the definition of value of time implies, it is the willingness to pay for savings in travel time so the changes that we will consider to measure the exact welfare changes between the before (A) and after (B) scenarios are time change $(A \rightarrow t_B)$, price change $(p_A \rightarrow p_B)$, income change $(y_A \rightarrow y_B)$ and change in utility $(V^A = v(p^A, t^A, T, y^A) \rightarrow v^B = v(p^B, t^B, T, y^B))$. It is possible that these changes may occur individually or simultaneously depending on the given circumstances or the need of evaluation but to measure the exact welfare changes, we will consider a whollistic approach including all the changes.

Two expressions of equivalent variation, EV , will be considered. It will be evaluated with respect to the Marshallian demand with adjustment of the marginal ratio of income, $e_{y,x}$, and with respect to the compensated demand, h .

3.7.1 Exact welfare measurement formula wrt $e_{y,x}$

First, looking at the reference point A, EV can be expressed as

$$EV = e(A, V^B) - y^A = e(A, V^B) - e(A, V^A) = e(p^A, t^A, T, V^B) - e(p^A, t^A, T, V^A) \quad (3.26a)$$

Another way of expressing EV is by

$$\begin{aligned} EV &= e(A, V^B) - y^A = e(A, V^B) - y^B + y^B - y^A \\ &= e(A, V^B) - e(B, V^B) + y^B - y^A \\ &= e(p^A, t^A, T, V^B) - e(p^B, t^B, T, V^B) + y^B - y^A \end{aligned} \quad (3.26b)$$

Now, to evaluate EV in terms of Marshallian demand and applying Roy's identity where $V_p = -V_y x$, $V_t = -V_T x$, $VOT = V_T/V_y$,

$$\begin{aligned} EV &\equiv e(A, V^B) - e(A, V^A) = \int_{V^A}^{V^B} e_v dV \\ &= \int_{A \rightarrow B} (e_v V_p dp + e_v V_t dt + e_v V_y dy) \\ &= \int_{A \rightarrow B} (e_v V_y (-x) dp + e_v V_y (T/V_y) (-x) dt + e_v V_y dy) \\ &= \int_{A \rightarrow B} (e_y (A, V) (-x dp - x VOT dt + dy)) \end{aligned} \quad (3.26c)$$

Looking at (3.26c), it is necessary to express the term $e_y(A, V)$ in terms of demand x .

Evaluating $e_y(A, V)$, it can be shown that,

$$e_y(A, V) = e_y(A, V) \frac{V_y(p, t, T, y)}{V_y(p^A, t^A, T, y)} \quad (3.26d)$$

So e_y may be called as the marginal utility ratio of income. Note that $e(A, V, y) \equiv y$, so $e_y(A, V, y) = y$ at any y , therefore $e_{yy} = 0$.

3.7.2 Exact welfare measurement formula wrt h

Another way of expressing EV is in terms of compensated demand h by applying Sheppard's lemma $e_p = h$, $e_t = h\rho$ such as,

$$\begin{aligned} EV &= e(p^A, t^A, T, V^B) - e(p^B, t^B, T, V^B) + (y^B - y^A) \\ &= \int_{B \rightarrow A} e_p dp + e_t dt + (y^B - y^A) \\ &= \int_{B \rightarrow A} h(p, t, T, V^B) dp + h(p, t, T, V^B) \rho(p, t, T, V^B) dt + (y^B - y^A) \end{aligned} \quad (3.27)$$

In order to calculate and express (3.26a) and (3.27) in terms of consumer's surplus with respect to the Marshallian and compensated demand functions x, h and to multiply by marginal utility ratio of income $e_{y,x}$ the first case. *Linear approximation for the marginal utility ratio of income, $e_{y,x}$*

By linear approximation of the demand function, that is, second-order approximation of EV , (3.26a) can be re-written as

$$\begin{aligned} EV &= e(A, V^B) - e(A, V^A) \\ &= \int_{A \rightarrow B} e_y(A, V) [x dp - x VOT dt + dy] \\ &= \frac{1}{2} (e_y^A x^A + e_y^B x^B) (p^A - p^B) + \frac{1}{2} (e_y^A x^A VOT^A + e_y^B x^B VOT^B) (t^A - t^B) \\ &\quad + \frac{1}{2} (e_y^A + e_y^B) (y^B - y^A) \end{aligned} \quad (3.28)$$

since $e_y^A = 1$, it needs to express only $e_y^B = e_y(A, V^B)$ in terms of observable demand. So e_y^B is linearly approximated and to be expressed in terms of observable demand,

$$e_y^B = e_y^A + e_{yp}^A (p_B - p_A) + e_{yt}^A (t_B - t_A) + e_{yy}^A (y_B - y_A) \quad (3.29)$$

where $e_{yy}^A = 0$,

$$\begin{aligned} e_{yp}^A &= e_{py}^A = \left[\frac{\partial e_y(A, V)}{\partial p} \right]_y = \left[e_y(A, V) \right]_x = -x_y^A \\ e_{yt}^A &= e_{ty}^A = \left[\frac{\partial e_y(A, V)}{\partial t} \right]_y = \left[e_y V_T x \right]_x = -[VOT]_y^A \end{aligned}$$

Simplifying, it becomes,

$$e_y^B = 1 - x_y^A (p_B - p_A) - [VOT]_y^A (t_B - t_A) \quad (3.30)$$

From (3.30), it shows that e_y^B can be approximately expressed in terms of observable demand as VOT is expressed in terms of observable demand in (3.10).

The above formula, (3.28), is the well-known trapezoidal formula for area with modification of x^B by $e_y^B x^B$ where e_y^B is an addition by income effect. This is not similar to Willig's consumer's surplus approach wherein fixed income is a valid assumption. However, if the income effect e_y^B is negligible, the modified demand function could be equal to the Marshallian demand function.

3.7.4 Linear approximation for compensated demand, h

From (3.27), it can be shown that by linear approximation for h ,

$$\begin{aligned}
 EV &= e \int_{p^A}^{p^B} x^A dt + \int_{t^A}^{t^B} x^B dt - e \int_{p^B}^{p^A} x^B dt + y^B - y^A \\
 &= \int_{B \rightarrow A} h dp + h \rho dt + y^B - y^A \\
 &= -\frac{1}{2} h(A, V^B) + h^B (p^B - p^A) - \frac{1}{2} h(A, V^B) \rho(A, V^B) + h^B \rho^B (t^B - t^A) \\
 &\quad + y^B - y^A
 \end{aligned} \tag{3.31}$$

where $h^B = h(p^B, V^B)$

$$\rho^B = \rho(p^B, V^B)$$

Referring to the equation above, terms $h(A, V^B)$ and $h(A, V^B) \rho(A, V^B)$ can be linearly approximated around h^B and $h \rho^B$, respectively.

$$h(A, V^B) \approx h^B + h_p^B (p^A - p^B) + h_t^B (t^A - t^B) \tag{3.32a}$$

$$h(A, V^B) \rho(A, V^B) \approx h^B \rho^B + \rho_p^B (p^A - p^B) + \rho_t^B (t^A - t^B) \tag{3.32b}$$

Substituting Slutsky equations for h_p , h_t , ρ_p and ρ_t ,

$$h(A, V^B) \approx x^B + \left(\frac{B}{p} + x^B x_y^B \right) (p^A - p^B) + \left(\frac{B}{t} + \text{VOT}^B x^B x_y^B \right) (t^A - t^B) \tag{3.33a}$$

$$\begin{aligned}
 h \rho(A, V^B) \approx & x^B \text{VOT}^B + \left[\text{VOT}_p^B + x^B \left(\text{VOT}_y^B \right) \left(\frac{A}{p} - p^B \right) \right. \\
 & \left. + \left[\text{VOT}_t^B + \left(\frac{B}{t} \text{VOT}^B \right) \left(\text{VOT}_y \right) \right] (t^A - t^B) \right]
 \end{aligned} \tag{3.33b}$$

Thus $h(A, V^B)$ and $h \rho(A, V^B)$ can be expressed by observable demand and observable Marshallian VOT as defined in (3.10). Note also the application of the extended Slutsky equations in the derivations of (3.33a) and (3.33b).

4. VALUE OF TIME AS A COMMODITY:NON-BUSINESS FREIGHT TRIPS

4.1 Modeling individual behavior

In the case of valuing time as a commodity, travel time is affected by factors relating to satisfaction or dissatisfaction of traveling such as comfort, fatigue, convenience, and appreciation of scenery among others, which is denoted in the formulation of the preference function as tx , which is also interpreted as the service level for freight trips. Also, an interpretation of this formulation of the preference function for person trips is the situation where time is not binding therefore leisure time no longer a variable and the time constraint is deleted. So the resulting utility function is now given as

$$\max u(z, x, tx) \quad (4.1)$$

$$\text{s.t.} \quad z + px = y \quad (4.2)$$

The notation are as follows: z is the demand for commodity goods whose price is normalized to 1, x is transport service demand, p is transport cost, tx is the disutility of traveling and y is income.

The Lagrange function of (4.1) and (4.2) is

$$V(p, t, y) \equiv L \equiv u(z, x, tx) - \lambda (y - z - px) \quad (4.3)$$

where:

$$\begin{aligned} z &= z(p, t, y) \\ x &= x(p, t, y) \\ \lambda &= \lambda(p, t, y) \end{aligned} \quad (4.4)$$

Applying envelope theorem to (4.3), the following could be derived:

$$V_p = -\lambda x \quad (4.5)$$

$$V_y = \lambda \quad (4.6)$$

$$V_t = u_{tx} x \quad (4.7)$$

4.2 VOT as commodity expressed by observable demand

From (4.6) substituted to (4.5), $V_p = -V_y x$. Then to derive the VOT, substituting (4.7) yields,

$$VOT \equiv -\frac{dp}{dt} \Big|_{v=const} = \frac{V_t}{V_p} = \frac{u_{tx} x}{-V_y x} = -\frac{u_{tx}}{V_y} \quad (4.8)$$

Now to express (4.8) in terms of observable demand x , take the derivative of V_p in (4.5) with respect to t and V_t in (4.7) with respect to p .

$$V_{pt} = -V_{yt} x - V_y x_t = -u_{tx} x_t x - V_y x_t = -u_{ytx} x^2 - u_{tx} x x_y - V_y x_t \quad (4.9)$$

$$V_p = u_{tx} x \quad V_{pt} = u_{ptx} x + u_{tx} x_p = -V_{yt} x + u_{tx} x_p = -V_{ytx} x^2 - V_y x_{tx} x + u_{tx} x_p \quad (4.10)$$

Equating (4.9) and (4.10), that is, $V_{pt} = V_{tp}$,

$$\begin{aligned} -u_x x y - V_y x &= -V_y x x + u_x x \\ -u_x (x_y + x_p) &= V_y (-x x) \end{aligned} \quad (4.11)$$

And accordingly, VOT as commodity can be expressed in terms of x as,

$$VOT = -\frac{u_x}{V_y} = -\frac{x_t - x x}{x_p + x x_y} \quad (4.12)$$

The numerator can be further simplified as $\left(\frac{\partial x}{\partial t}\right) = 1/\frac{\partial t}{\partial x} = 1/t$. So substituting it to

(4.11), VOT as a commodity becomes,

$$VOT = \frac{u_x}{V_y} = -\frac{x_t - x/t}{x_p + x x_y} \quad (4.13)$$

The VOT as a commodity is also expressed in terms of transport demand as (4.13). The denominator in (4.13) is the price substitution effect which is same as VOT as a resource. The numerator is the marginal disutility of total transportation time. VOT as a commodity varies depending on the mode, length of trip, destination, timing, among others, contrary to the VOT as a resource that is fixed regardless of those mentioned transport characteristics.

4.3 Compensated demand function and expenditure function

To measure the time savings benefits in terms of observable demand, the compensated Hicksian demand needs to be evaluated and be expressed in terms of observable Marshallian demand. Dual to the utility maximization in (4.1) and (4.2) is a typical expenditure minimization given as

$$e_1, p, t, \bar{u} \equiv \min h_0 + ph, \quad s.t. \quad u_{h_0, h, th} = \bar{u} \quad (4.14)$$

where h_0 is the compensated demand for commodity goods h is the compensated transport demand, t is transport time and \bar{u} is the given level of utility. It is well known that the following could be derived:

$$e_p = h \quad (4.15)$$

$$e_t = e_{th} h \quad (4.16)$$

4.4 Compensated VOT and time-extended Slutsky equations

The compensated VOT as a commodity can then be derived as,

$$\text{compensated VOT} \equiv -\frac{dp}{dt} \Big|_{\substack{e=\text{const.} \\ \bar{u}=\text{const.}}} = \frac{e_t}{e_p} = \frac{e_{th} h}{h} = e_{th} \equiv \rho \quad (4.17)$$

Now, to express the compensated VOT in (4.17) in terms of compensated demand h , take the derivative of e_p with respect to t and e_t with respect to p .

$$e_{pt} = h_t \quad (4.18)$$

$$e_{tp} = e_{th} h_p = e_{th} h + e_{th} h_p = h_{th} + e_{th} h_p \quad (4.19)$$

Then equating (4.18) and (4.19) to get the compensated VOT, note that $e_p = h$ and

$\left(\frac{\partial h}{\partial th}\right) = 1/\frac{\partial th}{\partial h} = 1/t$ so $h_t = h/t + \rho h_p$. Rearranging to derive the compensated VOT,

$$\rho = \frac{h_t - h/t}{h_p} \quad (4.20)$$

As shown in (4.20), the compensated VOT can be expressed by the compensated demand and its derivatives. But since the compensated demand cannot be observed, it is imperative to express it in terms of observable Marshallian demand.

The compensating demand is always equal to the usual demand function at income equal to $e(p, t, T, u)$, which can be expressed as,

$$h(p, t, T, u) \equiv x(p, t, T, e(p, t, T, u)) \quad (4.21)$$

Taking the derivative of (4.21) with respect to p and t , and recall that when u is at $V(p, t, T, y)$ then it follows that $e(p, t, T, V(p, t, T, y)) \equiv y$ and accordingly, it can be derived that $x_e = x_y$, $h = x$ and $VOT = \rho = e_{th}$, then it can be shown that

$$h_p = x_p + x x_y \quad (4.22a)$$

$$h_t = x_t + VOT x x_y \quad (4.22b)$$

Similarly, the compensating VOT is always equal to the usual Marshallian VOT at income equal to $e(p, t, T, u)$, which can be expressed as, $\rho(p, t, T, u) \equiv VOT(p, t, T, e(p, t, T, u))$. Accordingly, the compensated VOT can be expressed in terms of observable VOT, which take the form similar to a Slutsky equation as follows:

$$\rho_p = VOT_p + x VOT_y \quad (4.23a)$$

$$\rho_t = VOT_t + VOT_x VOT_y \quad (4.23b)$$

It is also necessary to calculate the combined compensated demand and VOT. With $h_p = x_p + x x_y$, the derivatives of h_p with respect to p and t can be derived as,

$$h_{pp} = x_{pp} + x x_{py} \quad (4.24a)$$

$$h_{pt} = x_{pt} + x_{py} VOT_y \quad (4.24b)$$

4.5 Estimation of time savings benefits using observable demand

After the compensated demand and VOT are expressed in terms of Marshallian demand and VOT, we can now proceed with the benefit measurement for p, t, y changes. Two cases will be analyzed. The first case is considering the change in utility and the second is change in compensated demand. The succeeding process of calculation is exactly identical to

the case of time as a resource above 3.7.

5. VALUE OF TIME AS A RESOURCE AND A COMMODITY

5.1 Modeling individual behavior

Section 3 discussed the value of time as a resource while in Section 4 is the analysis of value of time as a commodity. In this section the combination of both will be analyzed simultaneously making the complete approach. The utility of an individual is assumed to be a function of the demand for commodity goods z whose price is normalized to 1, transport service demand x , service level relating to the satisfaction of traveling $s(t)x$ and leisure time l . In the preference function, the variable $s(x)$ denotes the time as a commodity, which is equal to (x) in Section 3.3. The change in notation is only to preclude any confusion in the succeeding derivations. Individual behavior and its indirect utility function is given as,

$$V(p, t, T, y, s) = \max_{x, s} u(x, s, l) \quad (5.1)$$

$$\text{s.t.} \quad z + px = y \quad tx + l = T \quad (5.2)$$

The Lagrangian function is shown as

$$u(p, t, T, y, s) = \max_{x, s} u(x, s, l) - \lambda(y - z - px) - \mu(-tx - l) \quad (5.3)$$

By applying the envelope theorem to (5.3), the following are derived:

$$V_y = \lambda \quad (5.4)$$

$$V_T = \mu \quad (5.5)$$

$$V_p = -V_y x \quad (5.6)$$

$$V_t = u_{xs} s_t - \mu x = (u_{xs} s_t - \mu x) = (u_{xs} s_t - V_T x) \quad (5.7)$$

5.2 VOT expressed by observable demand

Substituting the values derived from the envelope results, the value of time can be expressed as,

$$VOT \equiv - \left. \frac{dp}{dt} \right|_{V=const.} = \frac{V_t}{V_p} = - \frac{u_{xs} s_t - V_T}{V_y} = \frac{V_T - u_{xs} s_t}{V_y} \quad (5.8)$$

The first term is VOT as a resource while the second term is VOT as a commodity. In order to express (5.8) in terms of observable demand, note that $x \equiv x(p, t, T, y, s)$ and that $\frac{dx}{dt} = x_t + x_s s_t$. Then taking the derivatives of (5.6) and (5.7) with respect to t and p respectively, the following can be derived,

$$\begin{aligned} V_{pt} &= (V_{yx} x_t - V_{ty} x - V_y (x_t + x_s s_t)) - (u_{xs} s_t - V_T x)_y x - V_y (x_t + x_s s_t) \\ &= - (u_{xs} s_t - V_T x)_y x^2 - (u_{xs} s_t - V_T x)_y x - V_y (x_t + x_s s_t) \end{aligned} \quad (5.9)$$

$$\begin{aligned}
V_{tp} &= u_{sx} s_t - V_T x_p = u_{sx} s_t - V_T x_p + u_{sx} s_t - V_T x_p \\
&= u_{sx} s_{tp} - V_{pT} x + u_{sx} s_t - V_T x_p \\
&= u_{sx} s_{tp} x - V_y x_T x + u_{sx} s_t - V_T x_p \\
&= u_{sx} s_{tp} x + V_{yT} x^2 + V_y x x_T + u_{sx} s_t - V_T x_p
\end{aligned} \tag{5.10}$$

To further simplify (5.10), note that $u_p = V_p$ and looking at the first term on the right-hand side,

$$u_p x = u_p x = (V_y x) x = -V_y x^2 - V_y x x_T \tag{5.11}$$

Substituting (5.11) back to (5.10),

$$\begin{aligned}
V_{tp} &= -u_{sx} s_t x^2 - V_y x x_T x + V_{yT} x^2 + V_y x x_T + u_{sx} s_t - V_T x_p \\
&= -u_{sx} s_t x^2 + V_{yT} x^2 + V_y x x_T - x x_{sx} + u_{sx} s_t - V_T x_p
\end{aligned} \tag{5.12}$$

Now equating $V_{pt} = V_{tp}$,

$$\begin{aligned}
-(u_{sx} s_t - V_T) x^2 - V_y x x_T &= V_y (x x_T - x x_{sx}) + (u_{sx} s_t - V_T) x_p \\
(u_{sx} s_t - V_T) x_p + x x_{sx} &= V_y (x_T + x_s s_t + x x_T - x x_{sx})
\end{aligned} \tag{5.13}$$

Finally, VOT can be expressed in terms of observable demand as,

$$VOT = \frac{V_T - u_{sx} s_t}{V_y} = \frac{x_t + x_s s_t + x (x_T - x_{sx})}{x_p + x x_y} \tag{5.14}$$

Further simplifying of (5.14) given that $\frac{\partial x}{\partial (x)} = \frac{1}{\frac{\partial (x)}{\partial x}} = \frac{1}{s}$ results to,

$$VOT = \frac{(x_t + x x_T) + (x_s s_t - x/s)}{x_p + x x_y} \tag{5.15}$$

Analyzing (5.15), note that it is a combination of the value of time as a resource (3.10) and value of time as a commodity (4.12).

$$\begin{aligned}
VOT_{resource} &= \frac{V_T}{V_y} = \frac{x_t + x x_T}{x_p + x x_y} \\
VOT_{commodity} &= \frac{u_{sx} s_t}{V_y} = \frac{x_s s_t - x/s}{x_p + x x_y}
\end{aligned}$$

As previously indicated, $s(t)$ denoting service level of satisfaction of traveling is a function of the duration of the total travel time, which could be interpreted simply as equal to time, so it can be observed and measured along with the demand and its derivatives.

5.3 Estimation of time savings benefits using observable demand

The resulting main equations for the benefit estimation are the same as in the case of VOT as a resource. However, in the expansion of the terms $h(V^B)$ and $h(V^B) \frac{\partial (V^B)}{\partial (V^B)}$ in the

case of the estimation based on the compensated demand, the effect of variable s_t is included.

6. CONCLUDING REMARKS

This paper focused on how value of time and its accrued benefits for non-business person and freight trips can be expressed and measured in terms of observable demand. Three cases signifying the variation of the value of time components, that is, as a resource, a commodity and a combination of both, are considered.

This study develops an approach for welfare measurement of the value of time based on observable demand. While the current practice derives the value of time assuming specified utility and demand functions, analysis through observable demand is possible and is consistent with utility-theoretic model. It is an important finding as this approach expresses the VOT in terms of Marshallian observable demand and even capable to analyze the components of the value of time.

It has been known that the use of compensated demand curves lead to appropriate welfare measures. However, as Hausman (1981) commented, 'While this result has been known for a long time by economic theorists, applied economists have only a limited awareness of its application'. True enough, its application to the benefit measurement of the value of time has not been shown, how VOT can be derived and its benefits be measured from observable demand. This study showed that the indirect utility function and expenditure function provide the appropriate compensated demand curve and thus the appropriate welfare function.

VOT as a resource

VOT as a resource assumes utility neutrality in travel time. For the exact welfare measurement of VOT, two approaches by equivalent variation considering time change, price change, income change and utility change are demonstrated. These two methods resulted from the transformation of the compensated demand and VOT in terms of Marshallian demand and VOT by applying the extended Slutsky equations. The first method is by expressing the marginal utility ratio of income in terms of Marshallian demand and VOT while the second method is by expressing the compensated demand and VOT in terms of Marshallian demand and VOT.

VOT is expressed as $\frac{x_t + xx_T}{x_p + xx_y}$, which is not equal to $\frac{x_t}{x_p}$. Also, the fundamental question

of the relationship between the value of time derived under constant utility and under constant demand is discussed. Generally, the resulting VOT under constant utility is not equal to the VOT derived under constant demand. The two-constraints model adopted from Larson and Shaikh (2001), lead to a function of full price – full income form wherein when x is

a function of full price, $VOT|_{v=const} = \frac{V_T}{V_y} = \frac{x_t + xx_T}{x_p + xx_y}$ reduces to $VOT|_{x=const} = \frac{x_t}{x_p}$.

However, contrary to their claim, it does not follow that it could lead to solution where

$$VOT = \frac{x_t}{x_p} = \frac{x_T}{x_y}.$$

VOT as a commodity

In the definition of VOT as a commodity for non-business person trip, tx is a measure of disutility of traveling. VOT as a commodity varies depending on the mode, length of trip, destination, timing, among others, contrary to the VOT as a resource that is fixed regardless of those mentioned transport characteristics. The formulations for the measurement of benefits are the same as in the case of VOT as a resource.

Total VOT

The total VOT defined as the marginal substitution ratio of transport costs and travel time taking into consideration the disutility of travel can be expressed as the observable transport demand and its derivatives.

$$VOT = \frac{C_t + x x_T}{x_p + x x_y} \left(\frac{C_s}{C} - x/s \right)$$

Looking at the equation above, it is the combination of VOT as a resource and VOT as a commodity. Total VOT also varies depending upon different transport characteristics since the satisfaction of traveling represented by VOT as a commodity is considered in the analysis.

Furthermore, the total VOT under constant utility is not identical to the total VOT under constant demand.

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