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# THE CONCEPT OF FUZZY CENTRAL PLACE AS THE APPROACH TO COMPARE DISTRIBUTION OF CENTRAL FUNCTIONS WITHIN URBAN AGGLOMERATIONS

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**Keywords:** *fuzzy central place, central functions, quasi relief, index of the key centers equilibrium, urban agglomeration, Korea, fractal.*

**Introduction.** The central place (CP) theory developed by W. Christaller was a smart attempt to find regularities in the urban population distribution (Christaller, 1966). The Christaller's ideas were further developed by A. Lösch (1967), J.B. Parr (1969), M. Dacey (1965), B. Berry (1958, 1960), R. White (1974, 1977), V.A. Shuper (1995) and others. According to that theory, CP is a city performing central functions (CF). Their occurrence depended on the ability of the people living outside the given territory to satisfy their demands for central products (CP). In the Christaller's hexagon the city area was designated as a point in a homogeneous space. The presentation of CP as a point is not very convenient at present, as individual elements of the urban settlement systems are large and heterogeneous, that their reduction to a point level makes the theory meaningless. CF value may vary over a broad range within the boundaries of a single city. The CF localization is not confined to urban settlements. In the countries with a high urban census there are occasional rural settlements with population of several tens of thousands. They have a variety of CF aimed at servicing of small rural settlements nearby. Though CF are limited both in assortment and in amount, they should not be ignored. Besides, settlements are distributed in space irregularly; that renders impossible to use the Christaller's hexagon based on assumption of a uniform distribution of cities. In the fuzzy central place (FCP) conception we attempt to consider CF distribution within the limits of a set of points.

## **Axiomatic apparatus of the conception of FCP.**

An *element* of the FCP system is identified as a bounded region of a set of points. An element with CF value equal to  $n$  is taken as *FCP*, while an element devoid of CF forms a *service area*. By the CF value is meant a quantitative characteristic within the limits of FCP. To make easier finding of representative data, we suggest using outlines of the lower elements of legal division as the boundaries of the FCP system elements.

FCP is a complex of socio-economic subsystems. The key subsystem is the population – the main producer and consumer of CP. Its needs exert primary control over the mechanism of the FCP system formation. Usually the location of service provider indicates the presence of user contingent in its vicinity. With increasing density of enterprises rendering CF, the assortment of services they supply is expanding. Accordingly, the possibilities to satisfy the needs for CP are also growing, both in amount and in quality. The correlation analysis has been performed between the density of service enterprises and population density. In the Republic of Korea (RK) boundaries of FCP system elements correspond to those of counties and cities in provinces, and to municipal areas in special cities. Seoul became a *Special City* in 1946. After that the status of *Special Autonomous City* was granted to several cities: Busan in 1963, Daegu and Incheon in 1981, Gwangju in 1986, Daejeon in 1989 and Ulsan in 1997. In the Democratic People's Republic of Korea (DPRK), Pyongyang became the first *municipality* in 1946. Rason acquired the status in 1993 and is the only city except Pyongyang still keeping it. The coefficient of correlation equal to 0,72 calculated for 2000–2009 period proves a close relationship between the above considered values. In view of those data, it is assumed that *CF value in a FCP is in direct proportion to the average population density*. For convenience, it is measured in conventional units.

Distribution of population and elements of economy, and consequently of CF, was never uniform. Taking this into consideration, we assume the *CF distribution to be originally not uniform*. Such a proposition makes us to revise other postulates of classical CP theory.

The notion of FCP system elements as a set of points leads us to reject the Christaller's CP hierarchy, as well as the multiformity of CP systems. Changes of postulates of the CP theory in the context of the FCP conception exerts no effect on the rational behavior of consumers whose requirements are mostly satisfied within the limits of their residence area. If the needed product or service cannot be found in the residence area, the choice of a suitable element of the system for the purchase depends on several factors, the most important being the type of CP and the distance to be travelled. The latter factor requires spending of additional time and resources. The greater is the FCP attractiveness, the less important for the customer are additional expenditures of time and money for travelling to the FCP.

According to the logic of the classical CP theory, CP of the lowest level included only basic institutions, such as elementary schools. The next level institutions were middle schools, and so on, up to universities providing the highest level services. With a rise of the hierarchic level, the number of centers decreases, and their service area increases. In the FCP system elements with low CF value not only high schools but even universities may occur. FCP including a university campus located far from densely populated regions would have a low

CF value, though it provides services of the highest level. Therefore, the above stated principle of the classical CP theory appears to be wrong.

To distribute FCP system elements by levels of hierarchy we attempted to find a relationship between medium CF values corresponding to different levels of hierarchy. With this object in view we have studied the distribution of educational, health care, financial institutions, postal offices, as well as commercial enterprises, catering facilities and law enforcement agencies in the RK and in combinations of its provinces and special cities from 1990 to 2009. In every case a specific principle was applied to the distribution of the studied social infrastructure objects based on their qualitative and quantitative characteristics. By applying different principles to distribution of elements of FCP systems by levels of hierarchy in other spheres of social infrastructure, there have been found the ratio between the average CF value in elements attributed to the highest hierarchic level and analogous values in other level elements. The ratio (from IV to I level) equals to 1 – 0.5 – 0.25 – 0.1, respectively. Further it is necessary to set numeric limits for every hierarchic level in accordance with above presented ratio. For example, if the maximum recorded CF value in a hypothetical FCP system with four levels is 15 000 units, the elements of the first level will be placed from 0 to 999, those of the second level – from 1000 to 2999, the third level – from 3000 to 6999, and the fourth level – from 7000 to 15 000. If the number of hierarchic levels in the FCP system is more (less) than in the example discussed, the auxiliary segment length increases (decreases) according to the rule established.

The key characteristic in the FCP conception is the *index of key centers equilibrium*. The equation for its calculation is as follows:

$$\frac{S_c}{S_h} = m - 1 - c \quad (1),$$

where  $S_c$  is average distance between the administrative centers of FCP of two neighbor levels of hierarchy,  $S_h$  – average distance between their centers of gravity,  $m$  – total number of levels,  $c$  – number of missing levels. The distances are calculated from the lower hierarchic levels towards the elements of higher level located at minimum distance.

The  $\frac{S_c}{S_h}$  index permits to estimate the equilibrium between positions of expected and real focus of the CF localization. Ideally, points of administrative centers would coincide with FCP centers of gravity. The  $S_c$  index possesses greater stability as it depends only on the location of administrative centers which are “shifted” by a small distance.  $S_c$  is often tolerant even for changes in the FCP system boundaries, while  $S_h$  responds not only to changes in boundaries of FCP system elements, but also to their CF value. The CF centers of gravity and geometric centers in FCP were found using Arc GIS 9 software.

When constructing a graphic model of the FCP conception we proceeded from assumption that the CF value in an element of the system corresponds to its value in the centers of gravity. Using them as elevations and applying *IDW interpolation* we drew the *CF quasi-relief (QR)*. It is a graphic model of a non-uniform dispersal of CF in the space. The idea of the CF QR was offered by Prof. V.A. Shuper in a personal interview with the author. The CF QR modeling was performed using Arc GIS 9 and Global Mapper software.

As relative elevations of the CF QR, elements of an arithmetic progression were used, with initial term being zero and the common difference of successive members equal to 50. After every relative elevation had been established, the key index was tested for its consistency with the theoretical value. Such a “play” with elevations continued until the index of key centers equilibrium fitted the theoretical value best of all; thus found modification of the FCP system was considered to be optimal. The CF QR is related to the index of key centers equilibrium and to the process of establishing the optimum relative elevation. The FCP system elements with the basic CF value lesser than the established relative elevation would perform a service area and would be “flooded” in the CF QR.

### **The evolution of FCP systems on the Korean Peninsula.**

The FCP system of the RK was essentially transformed between 1952 and 2011. Due to administrative reforms the system elements increased from 145 in 1952 to 230 in 2011. Because of greater CF concentration the CF value in the FCP of the first rank increased in this time interval by the factor of 2.5: from 10532 to 26679 units. Elements of FCP system of the RK were distributed over four hierarchic levels (  $\frac{S_c}{S_h} \rightarrow 3$  ).

In 1952, at the basic modification of the CF QR,  $\frac{S_c}{S_h} = 3.13$  (Table 1). The best correspondence with the theoretical value of the key index in the FCP system of the RK was in 1952 at the relative elevation of 100 units (Table 1). At the given configuration, the fourth level included Mokpo and Seoul, with 7.9 % of the RK population (Table 1). Elements of the third and second levels formed moderately elevated “mounts” on the CF QR (Fig. 1). 115 elements with minimum CF value formed the first level with 77.4% of the total population (Table 1). In the CF QR (Fig. 2) they form “plains”. 16 county elements in Gangwon-do and Jeju-do provinces with 6.3% of the population turned out to be “flooded” (Fig. 1, Table 1).

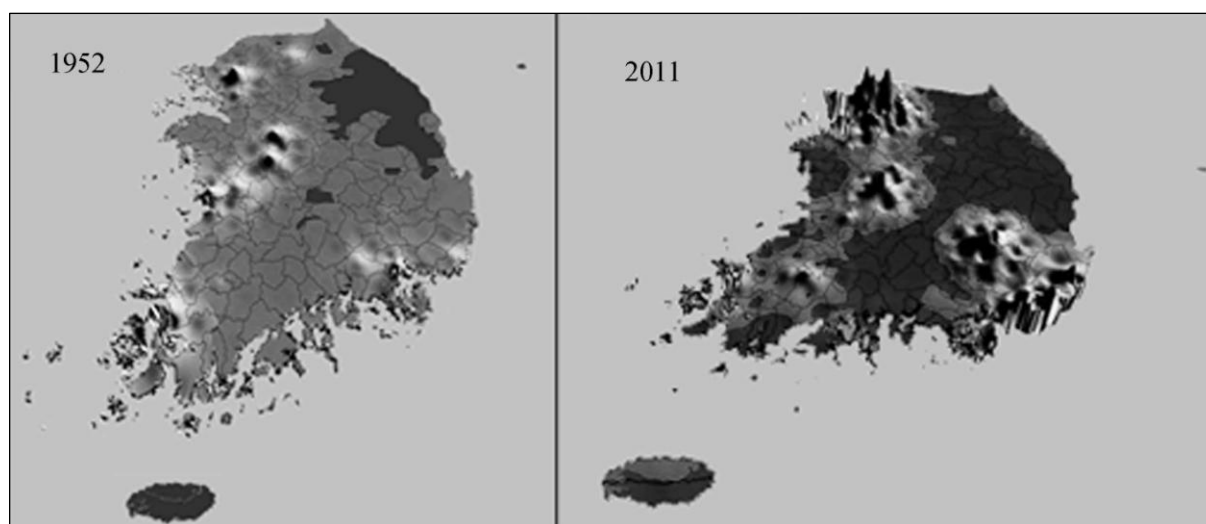
In 2011 the best correspondence of the key index with theoretical value (3.19) was attained at the relative elevation of 300 units (Table 1). The highest level consisted of 28 FCP concentrating 24.2% of the population. It was only in Gyongi-do province that provincial elements of the highest level occurred. The largest portion of FCP with 25.7% of population

formed the first level. The service area was composed of 109 elements with 15.8% of population in them (Table 1).

**Table 1.** The main indexes of the FCP system of the RK from 1952 to 2011

Year	Value of relative elevation	Number of FCP on different levels: 1–2–3–4	Number of elements without CF	Population on different levels, %				Population of the service area, %	$\frac{S_c}{S_h}$
				1	2	3	4		
1952	0 и 50	130-8-5-2	0	82.1	7.9	2.1	7.9	0	3.13
	<b>100</b>	<b>115-7-5-2</b>	<b>16</b>	<b>77.4</b>	<b>6.3</b>	<b>2.1</b>	<b>7.9</b>	<b>6.3</b>	<b>3.07</b>
	150	94-7-5-2	37	66.1	6.3	2.1	7.9	17.6	3.15
2011	250	48-21-26-29	106	27.2	16.6	16.9	24.9	14.4	3.28
	<b>300</b>	<b>43-21-29-28</b>	<b>109</b>	<b>25.7</b>	<b>16.6</b>	<b>17.7</b>	<b>24.2</b>	<b>15.8</b>	<b>3.19</b>
	350	45-20-27-28	110	26	16.3	17.7	24.2	15.8	3.25

While in 1952 the highest elements of the CF QR were in the Northwest and Southwest of the RK, by 2011 the main centers of CF were localized in the Capital and Southeastern agglomerations (Fig. 1). Therefore, if in 1952 the main positive landforms of CF QR were dispersed, by 2011 they were positioned around agglomerations or within them (Fig. 1). In 2011 the concentration of the higher elements towards the seaside caused formation of “cliffs” in the CF QR (Fig. 1). The conspicuous rising of the FCP elevations was related to the economic breakout of the RK. In 1963 the GDP per capita in the RK was comparable with the poorest countries of Africa and Asia, and in 2010 it was commensurable to those in France and the UK.

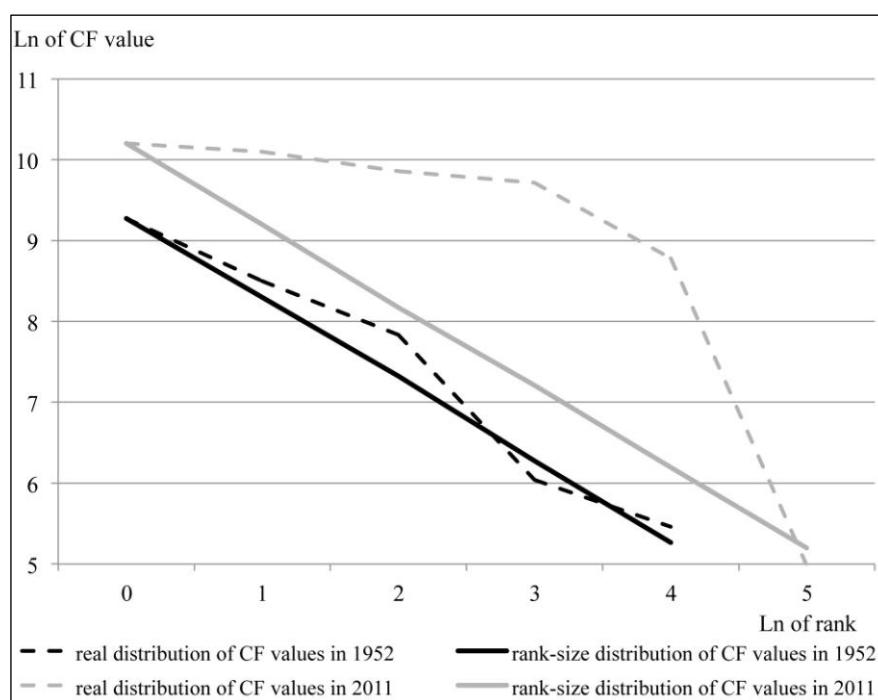


**Figure 1.** The CF QR of the RK in 1952 and in 2011

The dynamics of the CF QR of the RK from 1952 to 2011 makes it clear that in the second half of the 20<sup>th</sup> century urban agglomerations became centers of attraction for the

population having absorbed a sizeable part of the CF value of the FCP system in the RK: the most part of the country territory appeared “flooded” relative to the agglomerations (Fig. 1). Due to increasing relative elevation, the most part of FCP system elements that formed the lowest level in 1952 became a part of the service area by 2011.

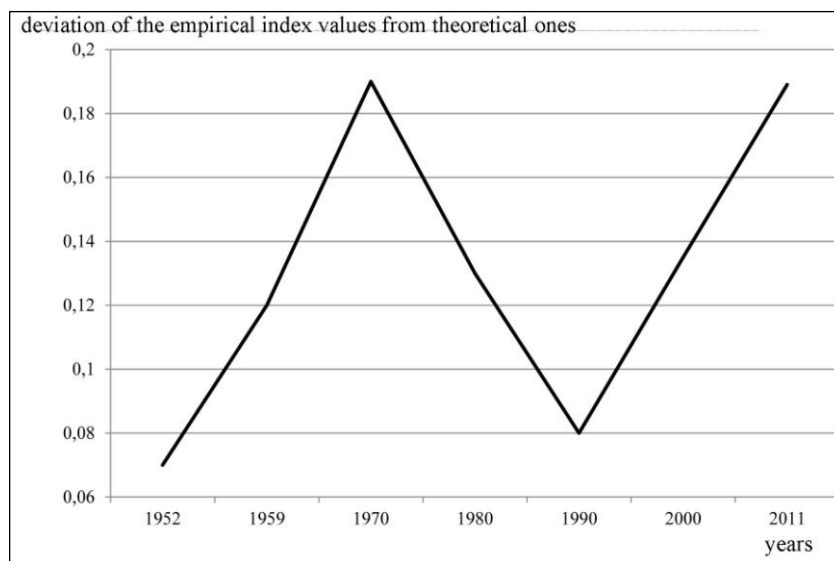
In an earlier work (2013) we considered a possibility of the rank-size rule application to the analysis of the CF value distribution in the FCP system. The figure formed by trends of real and theoretical distributions of the CF values in the RK FCP system elements in 2011 is much larger in area than that formed by analogous trends in 1952 (Fig. 2). Such a circumstance suggests a noticeably worse agreement between the trend line of the real distribution and the curve constructed according to the rank-size rule. It should be noted that along with the enlargement of the figures delineated by curves of real and theoretical distributions (calculated following the rank-size rule), the service «flooded» area also grew in size between 1952 and 2011 (Fig. 1, 2). It seems probable that there is an interrelation between the CF QR and the trends calculated using the rank-size rule. To gain better insight into this interrelation, further researches are needed.



**Figure 2.** The dynamics of FCP systems elements distribution in the RK in accordance with the “rank-size” rule from 1952 to 2011

The more is the  $\frac{S_c}{S_h}$  deviation from 3 at four levels of hierarchy in the FCP system of the RK, the less stable is the system, and vice versa. To estimate deviation of the empirical index values from theoretical ones, we calculated difference between them. The changes in the calculated value reflect the wave-like dynamics of the FCP system stability in the RK

between 1952 and 2011 (Fig. 3). A similarity between empirical ratio  $\frac{S_c}{S_h}$  and theoretical value in 1970–1990 increased in parallel with high rates of economic growth in the RK. In 1952–1970 the  $\frac{S_c}{S_h}$  lessened along with the development of the RK economics, and a similar tendency in 1990–2011 period may be attributed to the influence of the Asian financial crisis of 1997–1998 and to the World economic crisis of 2008. Reasoning from the above data, we suggest that the balance of the FCP system in the RK is related to the level of the country economic development.



**Figure 3.** The dynamics of the empirical index of key centers equilibrium deviation from the theoretical value in the RK from 1952 to 2011

The application of the FCP system study methods to DPRK is complicated by the meagerness of the statistic base. In 2008 the FCP system of DPRK was noted for extremely high hypertrophy in the two elements of the Capital (Ujon-gu and Gandon-gu). A difference in the CF value between the FCP of the first and the rank on one hand and the rest elements of the system on the other resulted in elimination of the third and the second hierarchic levels elements. Therefore, the key index in the FCP system in the DPRK in 2008 tends to one because the distances between the first and the fourth levels elements are taken as key ones.

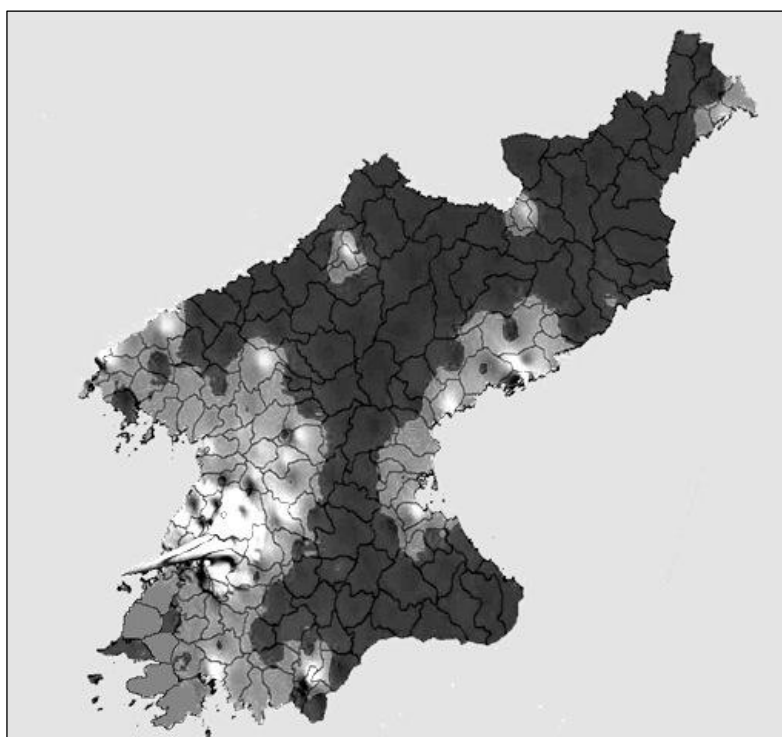
The optimum meaning  $\frac{S_c}{S_h} = 1.21$  in the FCP system of DPRK was recorded in 2008 at a relative elevation of 100 units (Table 2). 59 elements of the system with 12.4% of the country population formed a service area. 10.5% of the DPRK population lived at the fourth hierarchic level, and 77.1% – at the first one. In 2008 about 85% of FCPs in DPRK had the basic CF value less than 500 units! A fact of that kind is a testimony to the underdevelopment of the territory and the lack of the free competition on the market.



**Table 2.** The main indexes of the FCP system of DPRK in 2008

Value of relative elevation	Number of FCP on different levels: 1–2–3–4	Number of elements without CF	Population on different levels, %				Population of the service area, %	$\frac{S_c}{S_h}$
			1	2	3	4		
50	144-0-0-2	24	85.9	0	0	10.5	3.6	1.35
<b>100</b>	<b>109-0-0-2</b>	<b>59</b>	<b>77.1</b>	<b>0</b>	<b>0</b>	<b>10.5</b>	<b>12.4</b>	<b>1.21</b>
150	89-0-0-2	79	76.1	0	0	10.5	13.4	1.47

The greater part of the CF QR is “flooded” (Fig. 4). According to the constructed CF QR, the principal place where requirements for CP may be satisfied is the «peak» formed by CF concentrations in two FCP of the Capital (Fig. 4). South and North of the Capital «peak» the “isthmus” extends composed of the first level elements (Fig. 4). Aside from the principal «peak» there are also some other landforms of the CF QR in the Northern and Northeastern parts of the country and near the Eastern coast. Their CF value exceeds that of the rest elements of the first hierarchic level, though it is not sufficient for identifying them as belonging to a higher level. An assortment of services provided in such cities practically does not differ from that in rural settlements. Noteworthy are “uplands” of Sinuiju City (at the DPRK-China border) and Kaesong City (at the border between DPRK and the RK). Most part of the Goods from China enters DPRK through Sinuiju City, and a special administrative region was formed in Kaesong City in order to attract Capital from the RK. Another high “hill” in the CF QR corresponds to a large city of Hamhin (Fig. 4).



**Figure 4.** The central functions QR of DPRK in 2008

In 2008 more than 70% of the DPRK population lived in system elements of the first hierarchic level (Table 2), and therefore – in practically equal living conditions. The *Juche* ideology of total equality of population assuming the CP being equally available to everybody is Utopian. The Pyongyang residents have access to a greater variety of products than their rural fellow citizens. The CF QR of the DPRK on Fig. 4 is realistic. So, the method suggested may be also applied to studies of economically underdeveloped countries.

Some rural settlements, paradoxically, are actually elements of urban systems, as they perform CF at lower price and are providing a wide choice and better quality. As follows from the analysis performed, the number of rural FCP in the RK decreased by factor of 22.4, from 112 to 5, their population – more than tenfold, from 96% to 7.9% (Table 3)! The main cause of those changes was a dramatic increase in the urbanization rate from 21.4% in 1950 to 81.9% in 2010, along with increasing number of cities from 19 in 1950 to 84 in 2010. Over the years most of rural FCP elements in the RK increased the CF value. Such changes necessitated administrative reforms that favored changing their status from rural to urban. Hence the FCP of rural status may be considered as potential cities.

**Table 3.** *The rural FCP in Korea*

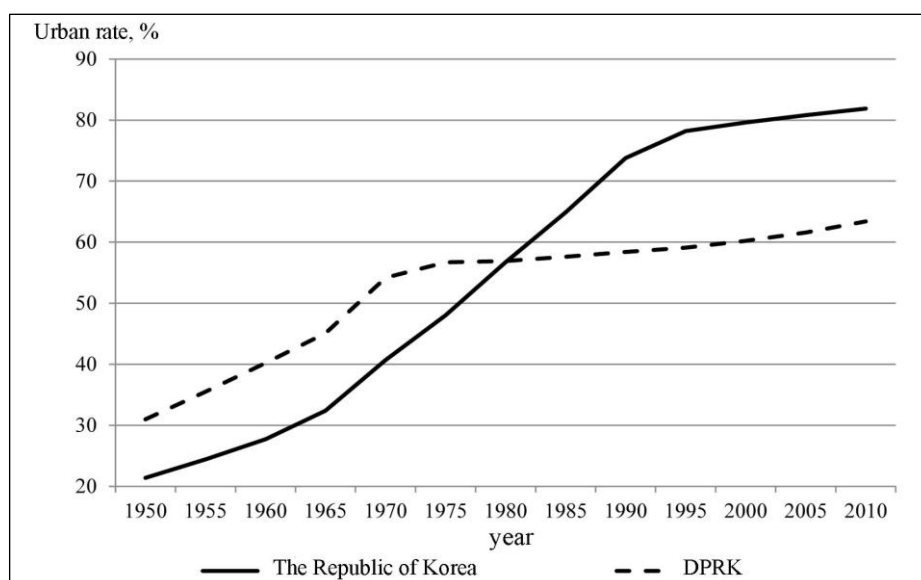
Country	Year	Quantity of rural FCP on the first level of hierarchy	Inhabitants of rural FCP among the first level population, %	CF value inside rural FCP in elements of the first level, %	Number of cities without CF
The Republic of Korea	1952	112	96	97.4	0
	1970	106	78,8	95.6	0
	1990	47	39,3	20.5	0
	2011	5	7,9	14.1	25
DPRK	2008	119	57,3	82.3	0

According to estimated data on 2008, the countryside inhabitants in DPRK constituted more than a half of population (57.3%) in elements of the first level (Table 3). The FCP system elements of rural status concentrated 82.3% of CF value at the first hierarchic level (Table 3). So, the case of DPRK evidently confirms the CF presence in the countryside.

The appearance of rural FCP, as well as urban elements within the service area, is conditioned by objective factors. Having enlarged their functionality and increasing population, rural elements became CF providers for the nearby elements of the service area. Most of them became cities as the time went on. On the other hand, early in the 20<sup>th</sup> century some cities without CF appeared in FCP system of the RK. It could be a consequence of their implementation of bedroom district functions.

### **Specific features of urbanization. Urban agglomerations in Korea as the FCP systems.**

Within the interval of 1950–2010 the population increased by factor of 2.6 (from 18.9 million to 48.7 million) in the Southern part of Korean Peninsula and by the factor of 2.5 (from 9.7 million to 24 million) in the Northern part. The second half of the 20<sup>th</sup> century was a time of explosive urbanization in Korea. In the period 1950–2010 the urbanization rate increased from 21.4% to 81.9% in the RK and from 31% to 63.4% in DPRK (Fig. 5). Between 1975 and 1980 the RK overcame the urban transition of 50% (Fig. 5). A higher initial urbanization level in DPRK made it possible to pass the 50% level earlier than in the RK – between 1965 and 1970 (Fig. 5). Thus it can be seen that urbanization in the RK proceeded from a very low initial level at a very high increasing rate. In DPRK a high rate of urbanization growth was recorded only until 1970, after that the rate decreased.



**Figure 5.** *The dynamics of urban rate in the RK and DPRK from 1950 to 2010*

The total amount of cities in the RK increased from 1950 to 2010 by factor of 4.4 – from 19 to 84. In 1950 only the Capital was a city with a million-plus population; in 2010 eight cities had such a status and concentrated almost 50.4% of the population. In DPRK cities grew in number from 19 to 27 in the period of 1963–2008. During that interval the population of Pyongyang grew from 1 to 3.26 million. The Capital still keeps its status of the only city with a million-plus population; in 2008 31.4% of urban population lived there.

In the period from 1985 to 1990 there began an active development of areas surrounding the main cities. The Seoul and Busan agglomerations began to develop immediately after the end of the Korean War. An economic development of the RK promoted stronger relations between the largest cities and settlements in their vicinities encouraged

commuting and development of suburbs and satellite towns. The cited facts indicate a rapid development of urban agglomerations resulting in “dispersed” borders of individual cities.

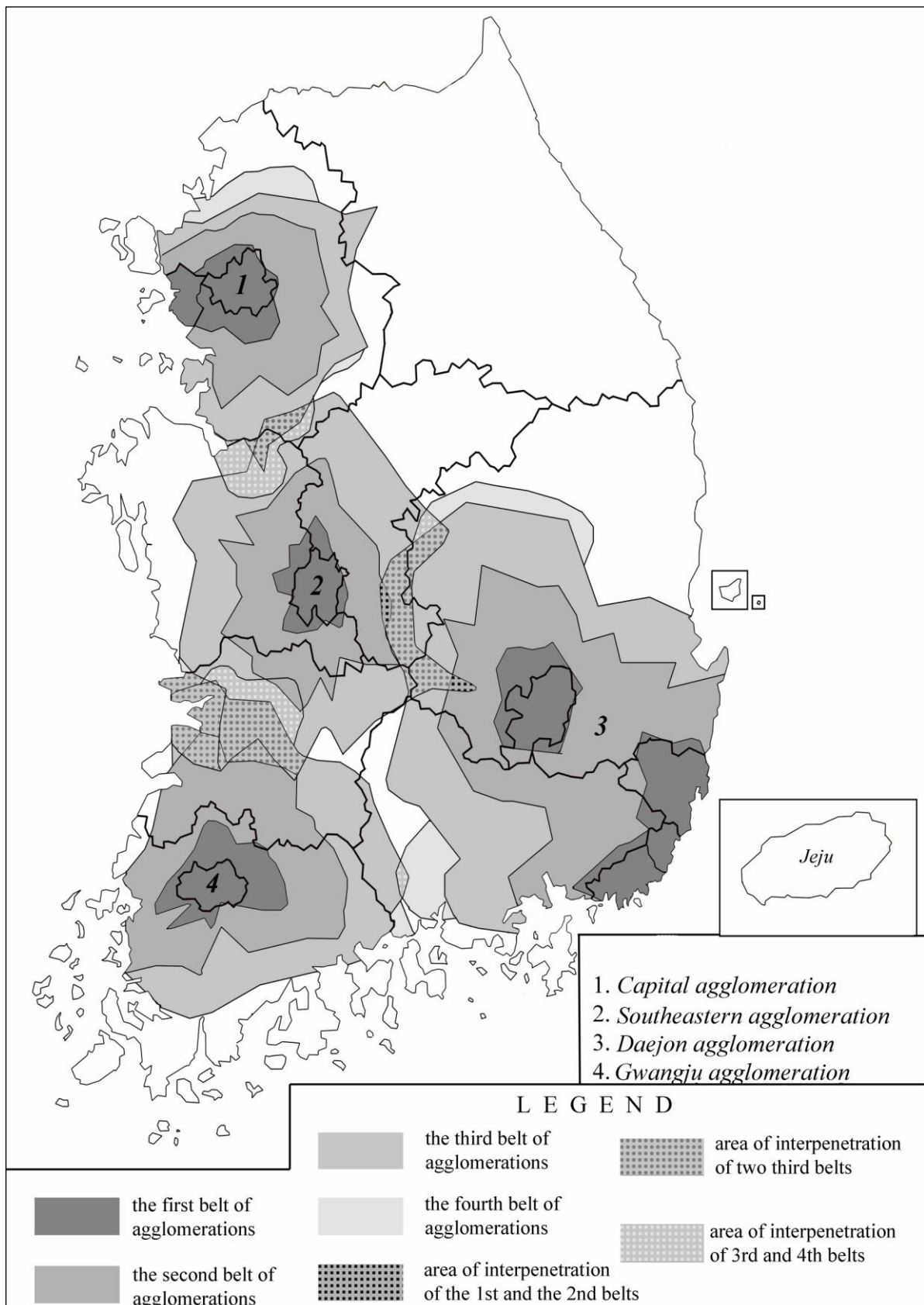
To define limits of agglomerations having grown around special cities we used the method suggested by P.M. Polyan (1988). Its main criterion consisted in estimating transport accessibility of points of space from the core city. 0.5-, 1- and 1.5-hour isochrones have been drawn corresponding to belts of remoteness from the core city. Satellite towns with population exceeding 50 000 inhabitants located outside the 1.5-hour isochrones are included into the agglomeration if the area of their own 0.5 isochrone overlaps the 1.5-hour isochrones of the core city. If a satellite is located within the third belt, the portion of its area lying within 0.5-hour isochrones is included into the agglomeration. That area composes the fourth belt. Borders of agglomerations are drawn around the cities of special status following the outlines of their combined third and fourth belts.

To determine the distance of a specific point from the agglomeration cores in the RK in case of traveling by car we used the *Daum Chido* system. The virtual route was laid following highways when possible and supposing to be passed at night so as to eliminate the effect of traffic jams. In case of travelling by public transport the time needed was estimated using the *Google Map*. The usage of public transport is only of correcting importance when drawing final isochrones because the RK is a country with a high mean number of cars per capita. The index of agglomeration development is calculated by the equation:

$$K_d = P * (M * m + N * n) \quad (2),$$

where  $P$  – is urban population of the agglomeration,  $M$  is a number of cities in the agglomeration,  $N$  – a number of rural settlements,  $m$  and  $n$  – proportions of urban and rural population in the agglomeration (Polyan, 1988).  $K_d$  for an agglomeration should exceed 1.

The main constituent of the settlement system in the RK was always the *Seoul City*. A continuous growth of its core area was conditioned by its Capital status. *Incheon City* developed independently in its vicinity. The two were drawn together and in the late 1980s formed a *dual-core Capital agglomeration*. It enlarged gradually and occupied practically the entire Gyonggi-do province and small areas in adjacent provinces (Fig. 6). In 2005 Suwon City with a million-plus population came into being between Seoul and Daejeon. It forms the third core of the Capital agglomeration, the main elements of which are connected together by well-developed lines of metro. People migrate from the noisy Capital to cheaper and ecologically clean regions in the Seoul vicinities.



**Figure 6.** The Belts of urban agglomeration in the RK in 2010

In 2010 within the limits of the Capital agglomeration with total population of 23.8 million inhabitants there were 73 settlements including 45 rural elements. 49.6% of the population lived outside the core city, though only 0.6% of the population was really leading

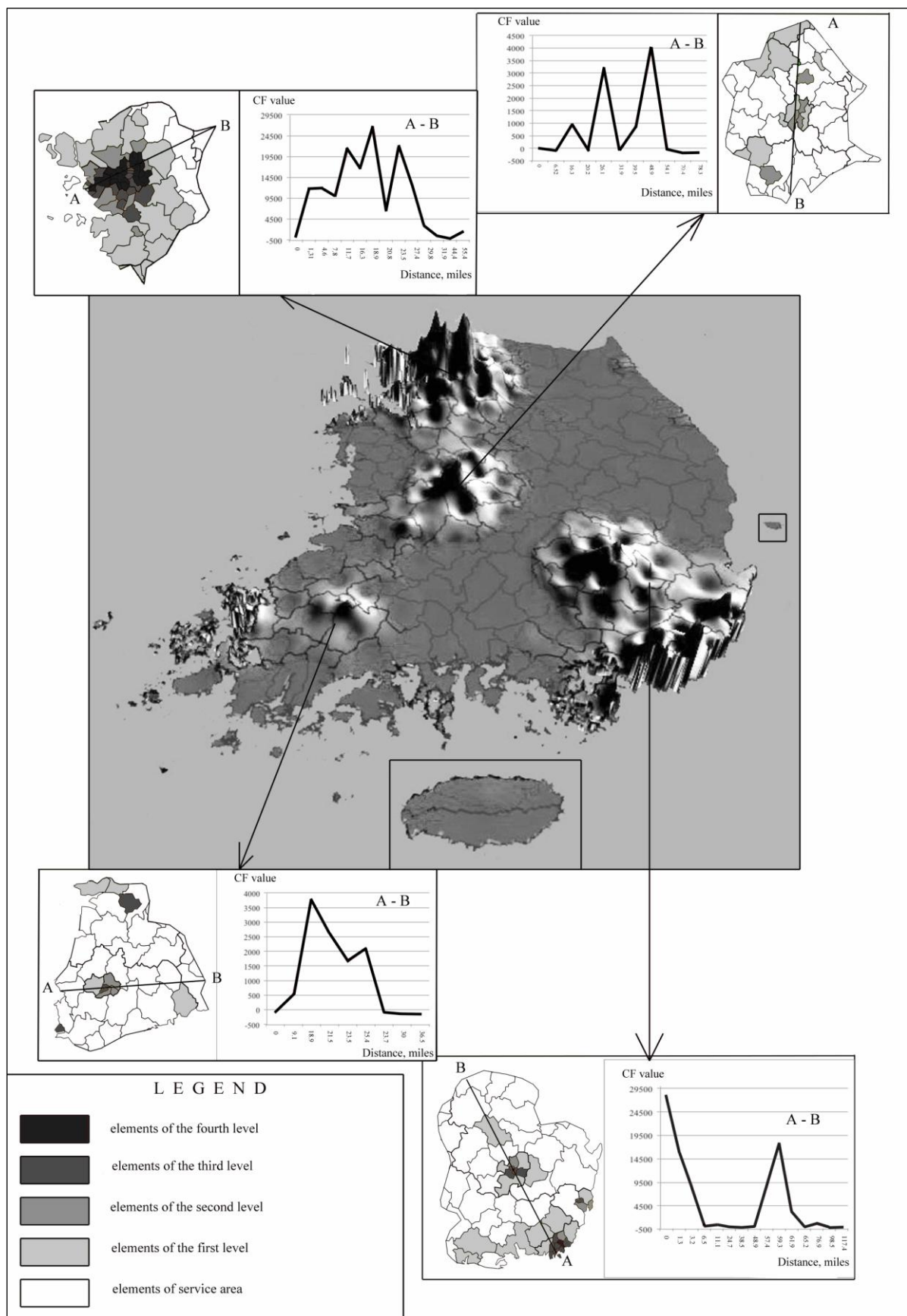
rural life. In 2010 the Capital agglomeration had  $K_d = 32.7$  and was attributable to a class of well-developed agglomerations.

Let us consider differentiation in CF distribution within the limits of the Capital agglomeration. The latter had both FCPs of a great CF value, such as Yancheon-gu municipal area with maximum CF value in the RK ( $> 26\,700$  units), and elements devoid of CF completely. In 2011 the agglomeration included 24 elements of the highest hierarchic level, 13 elements of the third level, 8 of the second and 16 of the first level. The dispersion of the CF values in the FCP system elements in the Capital agglomeration may be illustrated in the CF QR profile. The profile goes from the Jun-gu municipal area within the Incheon City (A) to Chungcheon City (B) (Fig. 7). The profiles thus obtained reflected a considerable dispersion of the CF values in the Capital agglomeration elements (Fig. 7).

In the Southeast of the RK there are three cities with a million-plus population not more than 100 km apart. One of them, Busan, often called the second Capital of the country or its sea gate. Daegu was progressing rapidly due to its convenient location between two capitals. The advancement of Ulsan benefited greatly from the copious inflow of Capital investments after head offices of large companies moved into it. The listed cities formed the *Southeastern polycentric agglomeration* (Fig. 6). In 2010 its population amounted to 9.7 million inhabitants; there were 144 settlements including 19 cities. The *Southeastern agglomeration* occupies the most parts of Gyongsan-Namdo and Gyongsan-Bukdo provinces, and also parts of the adjacent provinces. The share of the suburban zone in the total population of the agglomeration is 7.3%, only 0.05% of them living in rural settlements. The *Southeastern agglomeration* with  $K_d = 16.8$  is also well developed.

In 2011 the *Southeastern agglomeration* included 64 elements of the RK FCP system. 4 elements were at the fourth hierarchic level, 14 – at the third, 4 – at the second and 13 – at the first level. 29 elements of the system composed the service area. The profile line across the CF QR of the Southeastern agglomeration was drawn from the Jun-gu municipal area in Busan (A) to the Mungeon district (B) (Fig. 7). The profile has reflected a noticeable differentiation of CF value between its elements. They abruptly go down towards the seaside where the highest elements are concentrated.





**Figure 7.** The profiles of the CF QR of the RK agglomerations in 2011

Two homocentric agglomerations with population of 3.9 and 5.3 million respectively were formed around Gwangju City and Daejeon City (Fig. 6). The first one is the main focal

point of the Southwest economics, while the second occupies position of advantage between the two capitals. In 2011 the Daejon agglomeration included 42 elements of the RK FCP system. The agglomeration is noted for a lack of the fourth and the third hierarchic levels elements. 5 FCP are at the second hierarchic level and 9 at the first one. The rest 28 elements of the system formed service area. The CF QR profile of the Daejon agglomeration was constructed from Anson City (A) to Mujun-gun district (B) (Fig. 7). The profile reveals noticeable variations in the CF QR elements elevations.

In 2011 the Gwangju agglomeration included 3 FCP of the third and the second levels, and 4 FCP of the first hierarchic level (Fig. 7). 27 of the agglomeration elements were “flooded” area. It is at that elevation that the index of key centers equilibrium is the best fitted. No elements of the fourth hierarchic level are presented in the Gwangju agglomeration. The CF QR profile was constructed from the Hamphyon-gun district (A) to Kanseo-gun district (B) (Fig. 7). The trend thus revealed confirms the conjecture as to the considerable differentiation of elevations in the FCP system elements (Fig. 7). Homocentric agglomerations in the RK feature typically the least  $K_d$  values: 12.2 in Gwangju and 17.4 in Daejon agglomeration. The exceedingly forced urbanization in the RK was not favorable for appearance of an agglomeration with  $K_d \geq 50$  within its limits. In 2011 the maximum  $K_d = 32.7$  was recorded in the Capital agglomeration.

When moving from the central belts of the RK agglomerations towards peripheral ones, there has not been found a distinct tendency for the average CF value decreasing. This value first decreases when moving from the first belt to the periphery but then it rises again. The exception is the Southeastern polycentric agglomeration with CF value regularly decreasing from the inner belt to the outer one.

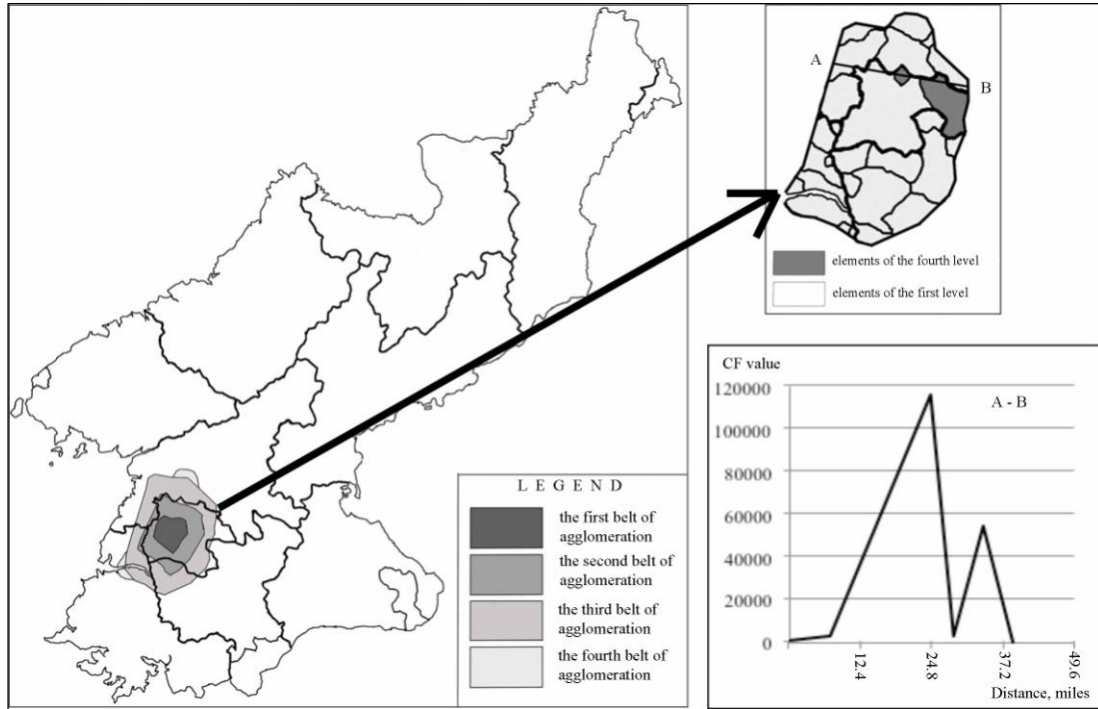
The agglomerations in the RK have vast areas of interpenetration. There are 15 cities and counties located within two agglomerations. The most pronounced superposition is found in case of the Capital and Daejon agglomerations (6 elements). So, the greater part of the country falls into a vast *urbanized zone* (Fig. 7). The CF value within the zones of interpenetration of the urban agglomerations is greater than in preceding belts (Fig. 7).

In 2008 there were two municipalities in DPRK, though a suburban zone formed only around the Capital. The great majority of the DPRK citizens have no privately owned cars, so the accessibility belts were distinguished using data on the public transport. The Capital of DPRK covered almost completely the Pyongyang municipality and some regions in adjoining provinces. Its population of 4.8 million settles in 18 settlements including 4 cities (Fig. 8). According to estimated data, 32.7% of population in the Pyongyang agglomeration lives



outside its core, and 13.1% are employed in agriculture. The Pyongyang agglomeration has  $K_d = 4.13$  and is the least developed agglomeration.

The Pyongyang agglomeration includes 26 elements of the FCP system of DPRK. There have been identified 2 system elements of the fourth hierarchic level and 24 elements of the first level. The profile of CF QR of the Pyongyang agglomeration was made from Phyongwon City (A) to Hijan City (B) (Fig. 8). The FCPs of the fourth level form “peaks” prominent against the mostly plain CF QR in DPRK. In the Pyongyang agglomeration the medium CF value decreases regularly from the first to the fourth belt.



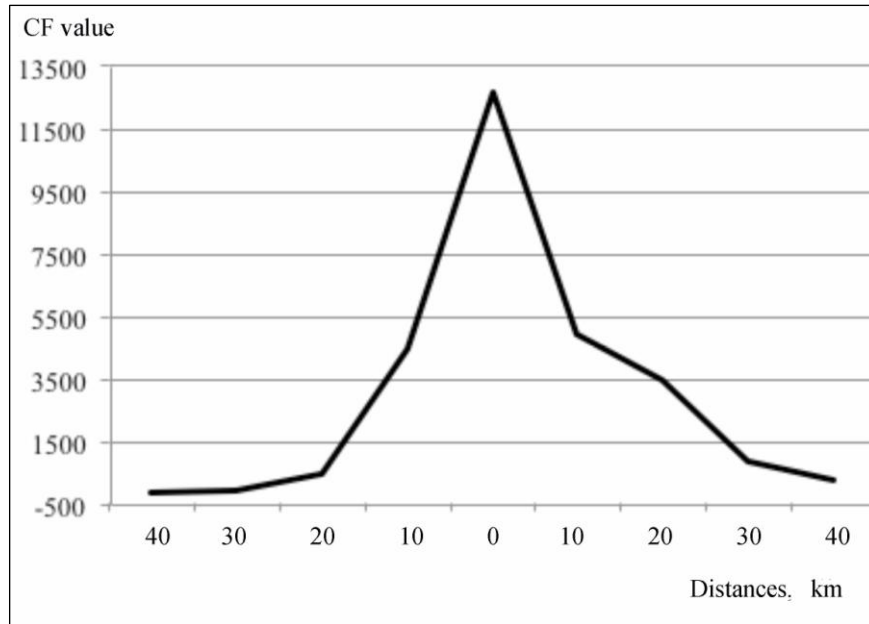
**Figure 8.** The time accessibility belts of the Pyongyang agglomeration in 2008

A profile has been constructed based on averaged data for the studied agglomerations. To construct the profile, points with maximum CF QR elevation have been chosen in every agglomeration. Then the points were plotted on the profile at distances of 10, 20, 30 and 40 km, where the CF QR elevation had been measured. The height of a point in the profile was calculated as an average of heights in the considered agglomerations. An analysis of the trend thus obtained revealed its general similarity to the trend established using the *Clark's model* (Fig. 10). The latter starts from the assumption that the population density in a city decreases as an exponential function of distance:

$$y = Ae^{-bx} \quad (3),$$

where  $y$  is population density at a distance  $x$  from the center,  $b$  is a slope of the density curve, and  $A$  is the density in the central zone extrapolated from the curve slope in outer zones (Clark, 1951). The right slope of the resulting averaged profile is gentler than the left one due

to specific features of the CF value distribution in the plane of the FCP system to the construction features.



**Figure 9.** The average profile of the Korean agglomerations from 1980 to 2011

### Fractals – a new approach to the spatial analysis.

A great majority of geographical studies assume a postulate that the space is absolutely uniform. Most of geographers could not dismiss the homogeneity as they could only think within the frame of Euclidean geometry. Against that background, the fractal theory presented by B. Mandelbrot in 1977 appeared as a jolt of fresh air.

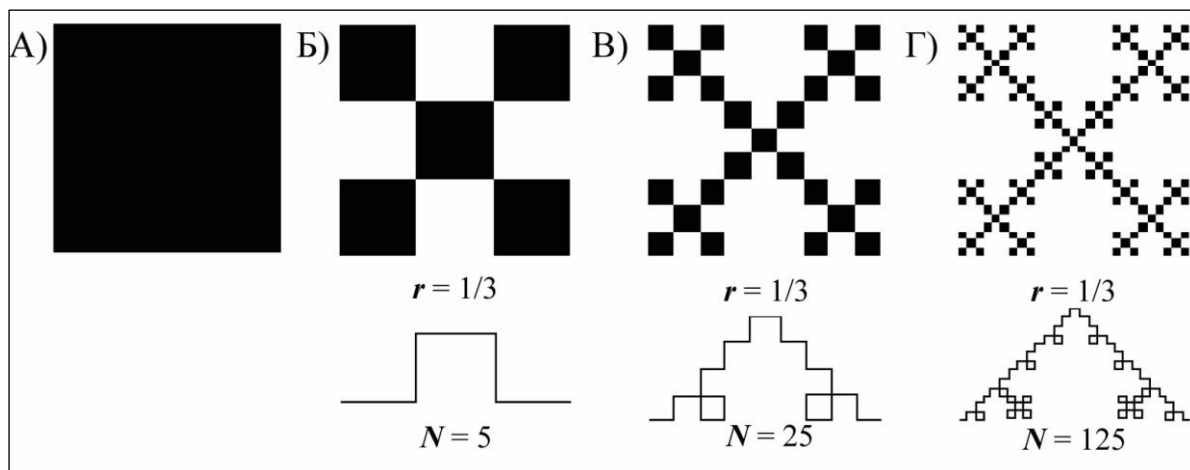
*Fractal* is a geometrical figure composed of  $n$  parts, each of them being similar to the entire figure. Every fractal has an initial figure as a basis. Let us consider forming of the *Sierpinski carpet* with a square in its basis and a side of the square equal to  $L$ . The generator will be given as formed by 5 squares ( $N = 5$ ), with factor  $r = 1/3$ . It appears as a part of a chessboard (Fig. 10-B). The Fig. 10-C is no longer similar to the chessboard due to empty zones referred to as lacunas. It is the presence of the latter that accounts for a spatial hierarchy of elements within the system. A hierarchy inside the Sierpinsky carpet suggests a heterogeneous distribution of its elements.

Multiplication of the initial figure may be repeated infinitely ( $n$  times). Here we give equations for calculating its key characteristics. The number of elements in the fractal under consideration is:

$$\square_n = N^n \quad (4).$$

The length of the each element perimeter is calculated by the following equation:

$$l_n = r^n * L \quad (5).$$



**Figure 10.** The construction of the Sierpinski carpet

Therefore, the resulting fractal area may be calculated as follows:

$$S_n = l_n^2 \quad (6).$$

According to the Euclidean geometry, a line is one-dimensional. However, with increasing  $n$ ,  $S_n$  will tend to zero, while *perimeter* – to an infinitely large value. If the perimeter length tends to infinity, then the dimension of the obtained figure exceeds 1. So, taking into consideration values of area and perimeter, the fractal dimension ( $D$ ) would not agree with the notion of the traditional geometry. As stated by Prof. Pierre Frankhauser «*fractal geometry contains conventional geometry as a limiting case*» (1988, p. 210).  $D$  is determined as follows:

$$D = \frac{\log N}{\log(1/r)} \quad (7).$$

The  $D$  values are governed by strict inequalities  $1 < D < 2$ .  $D = 2$  when the mass under study is distributed homogeneously over the studied surface. Such a case is not realistic. The more  $D$  departs from 2, the greater is number of lacunas in the studied structures.

The fractals are self-similar geometrical figures with hierarchic structure of their components. This fact accounts for their internal *heterogeneity*. Therefore, fractals may be instrumental in studying heterogeneity in FCP with CF also distributed nonuniformly.

We start from the assumption that an overwhelming part of CF is rendered to consumer in certain buildings. Therefore, the higher is built-up density, the greater CF volume may be available to the consumers. The fractal dimension calculated from the build-up outlines would provide an insight into morphological characteristics of the CF distribution. In this research was used *Fractalyse 2.4.1* software. It makes possible the fractal analysis based on two-color map. To prepare some auxiliary cartographic material, satellite images were obtained from the *Google Map*. Further on, a new layer was made using

*Adobe Photoshop CS5*, and the buildings seen in the image were plotted on it as points of different size depending on the building size in the image. As a result, there were obtained adequate cartograms of various buildings in the studied FCP elements. Such an approach gives an insight into degree of the building heterogeneity, and therefore – in a degree of uniformity of the CF spatial distribution.

There are several alternative ways of finding the fractal dimension of the area under study. Here we dwell on the method of *dilatation analysis* which our colleagues abroad and the author consider to be optimal (Tannier, Pumain, 2005; Thomas, Frankhauser, De Keersmaecker, 2007). In the process of the first step of dilatation every pixel in the auxiliary map is enclosed in a square frame of a thickness of 1 pixel. Further on, the frame thickens in accordance with the size of the dilatation step. As the size of squares keeps increasing, any detail of a size less than that of the basic element is generalized. In this way we approach gradually the initial form, and then calculate the number of squares necessary to a complete coverage of the studied image.

No less than 10 empirical simulations at different dilatation steps have been performed for every municipal district that is a constituent of any FCP in Capital agglomerations of the RK. The Fractalyse software calculates not only  $D$ , but a series of additional coefficients as well. Of the series of empirical data on the structural elements of the administrative division within FCP those  $D$  values were taken as optimal where the additional coefficients of correlation  $K_a$  exceeded 0.999. Using the dilatation analysis, however, we succeeded in finding such a fractal dimension for all the considered elements of the system, that  $K_a = 1$ . Empirical testing of fractal applicability to studies of FCP systems in Capital agglomeration of the RK was demonstrated with the elements that had been used as a basis for the CF QR profiles.

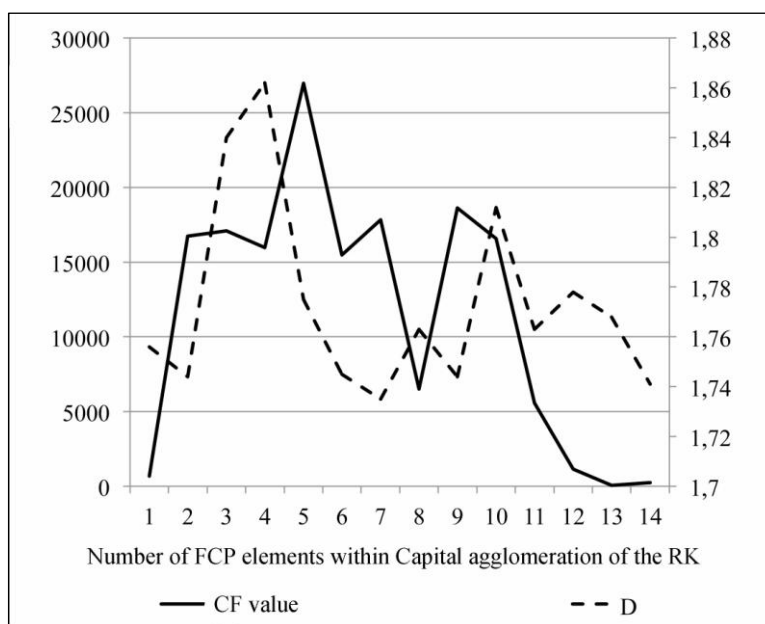
The considered FCP have been ranked according to their fractal dimension. The first group with low  $D$  value includes Nam-gu, Mapo-gu, Seodaemun-gu, Seonbuk-gu, and Chungcheon, their  $D$  being less than 1.75 (Table 4). That may be attributed to the prevalence of traditional one- and two-story residences in those elements of FCP system. Most of buildings are dated to the first decade after the Korean War. In some FCP of that group lacunas decrease the regularity in CF distribution over the territory, which is initially low even without that. The second group with medium  $D$  value includes Jun-gu, Yangcheon-gu, Jeonno-gu, Guri, Namyangju, Gapheon-gu. The fractal dimension values in this group fit inequation  $1.75 < D < 1.8$  (Table 4). Those elements are dominated by traditional type of built-up, though vast blocks of modern houses are also present. The third group marked by a highly regular CF distribution includes Bupheon-gu, Bucheon, and Nowon-gu, with  $D > 1.8$

(Table 4). Dominant in the structure of Bupheon-gu and Nowon-gu within the Capital agglomeration are communities built up regularly with high-rise buildings. So there is a distinct tendency that may be stated as follows: the more recent is the studied FCP built-up, the more regularly is the CF value distributed in it.

**Table 4.** The results of the empirical validation of the dilatation analysis for determination of the fractal dimension value of the elements of the FCP systems within Korean Capital agglomerations

№	FCP within Korean Capital agglomerations	Province or Special City of location	$D$	$K_a$
1	Jun-gu	Incheon City	1.756	1
2	Nam-gu	Incheon City	1.744	1
3	Bupheon-gu	Incheon City	1.84	1
4	Bucheon	Gyeongido province	1.862	1
5	Yangcheon-gu	Seoul City	1.775	1
6	Mapo-gu	Seoul City	1.745	1
7	Seodaemun-gu	Seoul City	1.735	1
8	Jeonno-gu	Seoul City	1.763	1
9	Seonbuk-gu	Seoul City	1.744	1
10	Nowon-gu	Seoul City	1.812	1
11	Guri City	Gyeongido province	1.763	1
12	Namyangju	Gyeongido province	1.778	1
13	Gapheon-gu	Gyeongido province	1.768	1
14	Chungcheon City	Gyeongido province	1.741	1

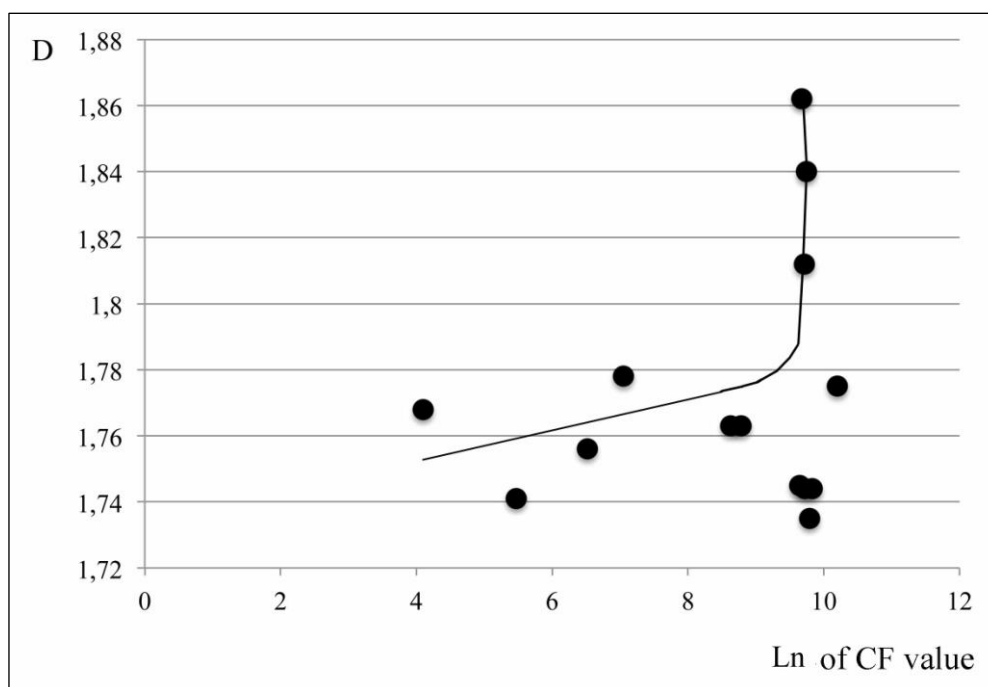
To estimate changes in regularity of CF distribution over the Capital agglomeration of the RK, we traced the trend in the  $D$  index changes when moving from point A to point B in the CF QR profile (Fig. 11). The trends thus calculated revealed an interrelation between the CF value and the regularity of CF distribution within FCP.



**Figure 11.** The dynamics of the CF value and fractal dimension value in the CF QR of the Capital agglomeration of the RK

The dynamics of changes in the direction of trends calculated from empirical data on CF value and  $D$  when moving from element 1 to element 14 in the CF QR profile. According to calculations, a change in the CF value trend within one interval leads to analogous change of the  $D$  value trend in the next interval in 75% of cases. An increase or decrease in CF value in one FCP would most likely result in an increase of  $D$  value in the adjacent element of the system.

We tried to reveal an interrelation between the CF value and the  $D$  (Fig. 12). For that purpose a graph was constructed with CF values –  $Ln$  – plotted against  $D$ . The trend thus obtained bears a general resemblance to a hyperbolic curve, as well as an exponential curve. Interrelationships between the discussed indexes call for further investigations. It should be noted that there may exist an exponential relation between CF value and  $D$ .



**Figure 12.** The relationships between the CF value and the fractal dimension value in the different elements of the FCP system of Korean Capital agglomerations

## Conclusion.

The following conclusions were made in the course of the performed research.

1. The usage of the suggested FCP conception apparatus made it possible to understand the spatial structure and dynamics of development in the RK urban systems with agglomeration effects, as well as in the relatively economically undeveloped DPRK. The deviations of the equilibrium index of key centers from theoretical values changed cyclically with time.

2. There have been established relationships between average CF values at different

hierarchic levels (from IV to I):  $1 - 0.5 - 0.25 - 0.1$ . With a rising to a higher hierarchic level, not only a number of institutions and facilities increases, but also the provided services are improved in quality.

3. A considerable differentiation in CF value has been found within the limits of urban agglomerations in the RK and DPRK. The urban agglomerations are the greatest CF focus. A vast majority of elements belonging to the highest levels of hierarchy in the urban agglomerations of the Korean Peninsula are located within the limits of the first and the second belts. It has been found that average CF values in the RK agglomerations do not necessarily tend to decrease from the center to periphery. The CF value is greater in the zones of contact of several urban agglomerations than in the preceding belts due to pooling the potency of several agglomerations. In the Pyongyang agglomeration the average CF value successively decreases from the first to the fourth belt. The constructed profiles of CF QR show conspicuous variations in CF values between the system elements in urban agglomerations. An analysis of the CF QR profile averaged over the agglomerations under study reveals a similarity between the CF distribution curve from the QR peak towards periphery and the curve in the Clark's model.

4. In the RK trends in actual and theoretical distribution of CF values in the FCP system elements were calculated using the rank-size rule for different years; the agreement between actual and theoretical distribution appeared to be much worse in 2011 than in 1952. That is indicated by a noticeably larger area of the figure formed by the above trends in the graph (Fig. 3). Accordingly, the «flooded» service area in the FCP system also increased from 1952 to 2011 (Fig. 2). Seemingly, there is a relationship between the CF QR and the rank-size rule to be investigated later.

5. In the RK the rural areas subjected to a transformation; the result was that along with their basic function – providing people with food products – they offer a package of other services. Many FCP of rural status increased their CF value and population and so became cities. There have been also found some urban system elements with a large area and disproportionately small population. They «lost» CF, which were attracted by some adjacent “stronger” elements of the system. So some rural FCP may be considered as potential cities.

6. The fractal theory seems very fruitful in the analysis of FCP system. It may be an effective tool in studying morphology of the CF distribution within the system elements. In doing so, the key index  $D$  provides a means for finding the degree of regularity in the CF distribution within FCP. An analysis of FCP classification in Capital agglomeration of the RK has shown that the regularity in CF distribution is much higher within the areas recently developed as compared with those of traditional built-up. The CF value and  $D$  are in direct

relation in the FCP system elements: changes in the direction of the former index lead to analogous change of the latter in the adjacent element. It has been established that the more is CF value in FCP, the higher is degree of heterogeneity in the CF distribution in the adjacent element of the system. In search of interrelation between CF value and the  $D$ , there have been advanced a hypothesis about an exponential relation existing between the considered indexes.

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