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**Conference Paper**

## Spatial Competition and Flexible Manufacturing with Spatially Discriminatory Pricing

54th Congress of the European Regional Science Association: "Regional development & globalisation: Best practices", 26-29 August 2014, St. Petersburg, Russia

**Provided in Cooperation with:**

European Regional Science Association (ERSA)

*Suggested Citation:* Liang, Wen-Jung; Wang, Kuang-Cheng; Din, Hong-Ren (2014) : Spatial Competition and Flexible Manufacturing with Spatially Discriminatory Pricing, 54th Congress of the European Regional Science Association: "Regional development & globalisation: Best practices", 26-29 August 2014, St. Petersburg, Russia, European Regional Science Association (ERSA), Louvain-la-Neuve

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**Spatial Competition and Flexible Manufacturing with Spatially Discriminatory  
Pricing**

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Current Version: February 06, 2014

JEL Classification: R32, L22

Keywords: Spatial Agglomeration; Flexible Manufacturing; Spatially Discriminatory  
Pricing

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## **Abstract**

This paper develops a two-dimensional spatial framework, in which firms have the technique of flexible manufacturing and engage in spatially discriminatory pricing, to explore the firms' optimal locations and optimal attributes of basic products under linear transportation costs. The paper shows that the two firms will agglomerate at the center of the location line and the optimal attributes of the two basic products will be located at the first and third quartiles of the attribute line, respectively, when the ratio of the marginal modification rate to the transport rate is high. It also shows that the two firms will locate separately on the location line and that the optimal attributes of the two basic products will remain at the first and third quartiles, when this ratio is moderate. Moreover, this paper proves that the two firms will locate at the first and third quartiles of the location line, respectively, and that the attribute addresses of the basic products will agglomerate at the center of the attribute line, when this ratio is low.

## 1. Introduction

This paper aims to provide a new explanation to the previous literature, in which firms have the technique of flexible manufacturing and engage in spatial price discrimination, to justify the existence of locational agglomeration among firms. We show that both spatial agglomeration and spatial dispersion can emerge by developing a two-dimensional spatial framework under linear transportation costs.

As indicated by Eaton and Schmitt (1994), the key feature of flexible manufacturing is *economies of scope*, which can be represented by the production of an array of differentiated products extended by a *basic product* using the same manufacturing process.<sup>1</sup> The production of the basic product incurs a sunk cost of product development, whose feature can be described by a point on Hotelling's attribute line. This basic product can be modified to produce extended variant products by incurring additional costs. The additional cost of producing an extended variant product is denoted by a per-unit modification cost that is proportional to the distance of the attribute line between the attribute addresses of the basic product and the extended variant product.

We can observe in the real world that the technique of flexible manufacturing has been widely adopted by most major manufacturing industries, such as those for aircraft, automobiles, the IT industry, steel, machinery, ready-mixed concrete, and rail transportation and long-distance coach services, etc.<sup>2</sup> In particular, a concrete producer constructs the factory with a sunk cost, and then produces various ingredients of the ready-mixed concrete with small modification costs. A rail company expends a large amount of sunk cost to build tracks among cities, and then offers various rail

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<sup>1</sup> Please refer to Eaton and Schmitt (1994, p. 875-876).

<sup>2</sup> See Eaton and Schmitt (1994), Gil-Moltó and Poyago-Theotoky (2008), Vogel (2011), and Matsumura and Shimizu (2012).

transportation services for customers through the use of various types of locomotives and carriages, which involve considerable amounts of modification costs. It is worth noting that firms in these two industries charge spatially discriminatory prices to customers. Next, it is worth indicating that firms not only produce various arrays of differentiated products, but they also make a variety of location decisions. We can observe that train and bus stations are usually located closely in cities, while we rarely see ready-mixed concrete producers agglomerating at the same site. Based on the above observations, it is interesting to take into account the characteristic that firms own the technique of flexible manufacturing in exploring their optimal location choices.

It is noteworthy that firms have two decisions to make in a location model involving flexible manufacturing. In one of them they choose an optimal location to settle their facilities, and in the other they select an optimal attribute of the basic product to develop an array of differentiated products. In order to manipulate the decisions on the equilibrium location and the optimal attribute of the basic product, we develop a two-dimensional spatial model, in which the horizontal axis is denoted as the location line and the vertical axis is represented as the attribute line.

In a seminal paper, Hotelling (1929) first proposed what has subsequently been termed the *Principle of Minimum Differentiation*, in which two firms producing a homogeneous product agglomerate at the center of the market under a linear transportation cost. This principle was challenged by D'Aspremont *et al.* (1979), who pointed out that when both firms locate together, price competition will drive the price and profit down to zero. Many economists have tried to come up with the conditions under which the Principle of Minimum Differentiation is restored. These researchers

include: Stahl (1982), De Palma *et al.* (1985), Hurter and Lederer (1985), Lederer and Hurter (1986), Rhee *et al.* (1992), De Fraja and Norman (1993), Neven and Thisse (1990), Anderson and Neven (1991), Jehiel (1992), Friedman and Thisse (1993), Tabuchi (1994), Veendorp and Majeed (1995), Zhang (1995), Irmén and Thisse (1998), Mai and Peng (1999), Liang and Mai (2006), Matsushima and Matsumura (2006), Tseng *et al.* (2010), and Liang *et al.* (2012). Most of the above literature focuses on the use of mill pricing by competing firms, except Hurter and Lederer (1985), Lederer and Hurter (1986), De Fraja and Norman (1993) and Liang *et al.* (2012), who analyze the optimal location by using spatially discriminatory pricing.

By employing spatially discriminatory pricing, firms are capable of charging different prices to consumers residing at different locations. The earlier literature, which introduces spatial price discrimination to study spatial competition, includes the following: Hurter and Lederer (1985) assumed that a single monopolist controlled both firms by using a two-dimensional model. The monopolist locates the firms to minimize the total production and transportation cost in their model, which is exactly identical to the location choice by a welfare-maximizing social planner. The equilibrium location combination is that firms agglomerate at the horizontal axis, but locate separately at the first and third quartiles of the vertical axis, respectively.<sup>3</sup> Lederer and Hurter (1986) considered a market consisting of circular submarkets and showed that firms will never agglomerate if they have identical production costs and transport rates. Fraja and Norman (1993) showed that central agglomeration is the *unique* equilibrium under spatially discriminatory pricing, when products are horizontally differentiated and the demand at each point of the Hotelling line is elastic. Liang *et al.* (2012) extended Fraja and

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<sup>3</sup> This is equivalent to  $(0, 1/2)$  and  $(0, -1/2)$  in Hurter and Lederer's (1985) model.

Norman's (1993) model by using a more reasonable market-serving condition. They found that for a low transport rate or a high degree of product differentiation, central agglomeration is the only location configuration; but for a high transport rate or a low degree of product differentiation, dispersion becomes the only location configuration. The present paper differs from the earlier literature by taking into account the technique of flexible manufacturing owned by firms in many industries in the analysis of firms' optimal location choices.

The main findings of the paper are as follows. First of all, we show that the two firms will agglomerate at the center of the location line and the optimal attributes of the two basic products will be located at the first and third quartiles of the attribute line, respectively, when the ratio of the marginal modification rate to the transport rate is high, say,  $(r/t) \geq (2/3)$ . Secondly, the two firms will locate separately on the location line and the optimal attributes of the two basic products will remain at the first and third quartiles when this ratio is moderate, say,  $(1/2) \leq (r/t) < (2/3)$ . Moreover, the two firms will locate at the first and third quartiles of the location line, respectively, and the optimal attributes of the basic products will agglomerate at the center of the attribute line when this ratio is low, say,  $(r/t) < (1/2)$ .

The remainder of the paper is organized as follows. Section 2 develops a two-dimensional spatial model, in which the location and product attributes are taken into account. Section 3 explores the optimal attributes of the basic products and the equilibrium locations in the case where the marginal consumer lies in between the attributes of the two basic products for every point on the location line. Section 4 extends the model to the case where the ratio of the marginal modification rate to the transport

rate is so low that the marginal consumer may lie beyond the attributes of the two basic products. Section 5 concludes the paper.

## 2. The Two-dimensional Model

Consider a two-dimensional spatial framework where the horizontal axis measures the location of the firms denoted by the location line, and the vertical axis measures the ideal product characteristics of consumers represented by the attribute line, as shown in Figure 1. Two firms, firms 1 and 2, are located at  $x_1$  and  $x_2$ , respectively, with  $x_1 \leq x_2$  along a line segment of length  $L_H = 1$  on the location line. A continuum of consumers, whose ideal product attributes are located along a line segment of length  $L_V = 1$  on the attribute line at any site  $x$  along the location line, are uniformly distributed over the rectangle with area 1. Each consumer purchases one unit of product. The transport cost function is assumed to be linear in distance, taking the following form:  $T_i(x) = t|x - x_i|$ , where  $T$  is the transportation cost,  $t$  is the transport rate per unit of output per unit of distance,  $x_i$  denotes firm  $i$ 's location, and  $x$  represents the site where consumer  $x$  resides. Following Eaton and Schmitt (1994), we assume that the incremental cost of modification from producing a further variant product extended by the basic product is linear and takes the following form:  $r|y_{ki} - y_{bi}|$ , where  $r$  is the marginal modification rate of producing a unit of a further variant product,  $y_{bi}$  denotes the attribute address of firm  $i$ 's basic product with  $y_{b1} \leq y_{b2}$  on the attribute line,  $y_{ki}$  represents the set of the attribute addresses of firm  $i$ 's variant products, and the distance from variant product  $y_{ki}$  to the attribute address of firm  $i$ 's basic product is  $|y_{ki} - y_{bi}|$ . Furthermore, the consumer residing at  $x$  with an ideal product



characteristic  $y$  suffers a disutility from buying product  $y_{ki}$ , whose product characteristic does not fit the consumer's ideal characteristic. This disutility can be expressed as  $s|y - y_{ki}|$ , where  $s$  represents the marginal disutility rate.

(Insert Figure 1 here)

The game in question is a three-stage game. Firms simultaneously select their equilibrium locations in the first stage. Then, they simultaneously choose the optimal attributes of the basic products in the second stage. Finally, firms engage in spatially discriminatory pricing in the third stage. The subgame-perfect Nash equilibrium can be solved by backward induction, beginning with the final stage.

In stage 3, the indirect utility of a consumer residing at  $x$  with an ideal product characteristic  $y$  and purchasing one unit of variant product  $y_{ki}$  from firm  $i$  is defined as:

$$U(x, y) = v - p_i(x, y_{ki}) - s|y_{ki} - y|, i = 1, 2, k \in [0, 1], \quad (1)$$

where  $v$  is the reservation utility of consuming one unit of product; and  $p_i(x, y_{ki})$  is the delivered price of variant product  $y_{ki}$  charged by firm  $i$  at site  $x$ .

Eaton and Schmitt (1994) indicated that when the marginal modification rate  $r$  is big relative to the marginal disutility rate  $s$ , only basic products are produced, and the model will degenerate into one in which the technology is completely inflexible. In order to highlight the importance of the technique of flexible manufacturing, we shall ignore this case and follow Eaton and Schmitt (1994) by focusing on the case where the marginal disutility  $s$  exceeds the marginal modification rate  $r$ . When  $s > r$  and firms charge a spatially discriminatory price on product  $y$  purchased by the consumer residing at  $x$ , i.e., the combination  $(x, y)$ , firms will produce the product fitting this consumer's ideal attribute. This result can be explained by the following instance. Suppose that there

is a consumer residing at  $(x, 0.8)$ , and that firms provide this consumer either with a product  $y_{ki} = 0.8$  or a product  $y_{ki} = \delta$ ,  $y_{bi} \leq \delta < 0.8$ . By charging a spatially discriminatory price with zero marginal production cost, the difference in profit between providing a product  $y_{ki} = 0.8$  and  $y_{ki} = \delta$  equals  $p_i(x, 0.8) - p_i(x, y_{ki}) = s(0.8 - \delta) > r(0.8 - \delta)$  if  $s > r$ . Thus, the firms will provide this consumer with a product that fits its ideal attribute for earning a higher profit. The same result can be derived if firms provide this consumer either with a product  $y_{ki} = 0.8$  or a product  $y_{ki} = \delta$ ,  $\delta > 0.8$ .

We then obtain the following result:<sup>4</sup>

**Lemma 1.** *When the marginal disutility exceeds the marginal modification rate and firms engage in spatially discriminatory pricing, firms will produce products fitting each consumer's ideal attribute and meanwhile the consumers will buy their ideal products.*

It is worth emphasizing that, by Lemma 1, the disutility of consuming a product that deviates from the ideal product in (1) vanishes due to every consumer purchasing its ideal product, i.e.,  $s|y_{ki} - y| = 0$ . Thus, the equilibrium delivered price  $p(x, y)$  in stage 3 equals the marginal cost (the transportation cost plus the marginal modification cost) of the second most efficient firm, and this equilibrium delivered price can also be referred to as the limit price.<sup>5</sup> Accordingly,  $p(x, y)$  can be expressed as the maximum marginal cost of the two firms at  $(x, y)$  as follows:

$$p(x, y) = \max \left\{ t|x - x_i| + r|y - y_{bi}|, t|x - x_j| + r|y - y_{bj}| \right\}, i, j = 1, 2 \quad i \neq j. \quad (2)$$

<sup>4</sup> Please see Eaton and Schmitt (1994, p. 878-879) for the details.

<sup>5</sup> The same price schedule for spatially discriminatory pricing can be found in Hurter and Lederer (1985), Lederer and Hurter (1986), Hamilton *et al.* (1989) and Vogel (2011).

Define  $z(x)$  as the marginal consumer at site  $x$  who is indifferent between buying from firm 1 and firm 2. Recall that  $x_1 \leq x_2$ ,  $y_{b1} \leq y_{b2}$  and  $s|y_{ki} - y| = 0$  in (1). By considering (1) and (2), we can derive  $z(x)$  when  $z(x)$  lies in between the attributes of the two basic products for every  $x$ , i.e.,  $y_{b1} \leq z(x) \leq y_{b2}$  for  $x \in [0, 1]$ , as follows:

$$z(x) = \begin{cases} [t(x_2 - x_1) + r(y_{b1} + y_{b2})]/2r, & \text{if } x \leq x_1, \\ [t(x_2 + x_1 - 2x) + r(y_{b1} + y_{b2})]/2r, & \text{if } x_1 \leq x \leq x_2, \\ [-t(x_2 - x_1) + r(y_{b1} + y_{b2})]/2r, & \text{if } x_2 \leq x. \end{cases} \quad (3)$$

(Insert Figure 2 here)

We can use Figure 2 to illustrate the result derived in (3). In Figure 2, when  $y_{b1} \leq z(x) \leq y_{b2}$  for  $x \in [0, 1]$ , the locus of  $z(x)$  remains unchanged in regions  $[0, x_1]$  and  $[x_2, 1]$ , while it is negatively-sloping in  $[x_1, x_2]$ . Since we assume  $x_1 \leq x_2$  and  $y_{b1} \leq y_{b2}$ , it follows that firm 1 will serve the consumers residing beneath the locus of the marginal consumer  $z(x)$ , while firm 2 will serve those who reside above  $z(x)$  due to having a cost advantage.

Define the condition for  $y_{b1} \leq z(x) \leq y_{b2}$ , which requires that at least part of the consumers residing at every site  $x$  in the interval  $[0, 1]$  be served by both firms. We find from (3) that this condition is fulfilled when the following inequality holds:<sup>6</sup>

$$t(x_2 - x_1) \leq r(y_{b2} - y_{b1}). \quad (4)$$

The intuition behind the result in (4) is as follows. It should be noticed that the transport cost function is linear in distance on the location line and the modification cost function is also linear in distance on the attribute line. When the modification cost

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<sup>6</sup> For  $x \in [0, x_1]$ ,  $z(x) = [t(x_2 - x_1) + r(y_{b1} + y_{b2})]/2r \leq y_{b2}$ . Similarly, for  $x \in [x_2, 1]$ ,  $z(x) = [-t(x_2 - x_1) + r(y_{b1} + y_{b2})]/2r > y_{b1}$ .

between the two basic products is bigger than the transportation cost between the two firms, firm 2 is capable of selling products to part of the consumers residing in firm 1's hinterland,  $x \in [0, x_1]$ , due to having a cost advantage in terms of the transportation cost. Likewise, firm 1 is capable of selling its products to part of the consumers residing in firm 2's hinterland  $x \in [x_2, 0]$ . As a result, both firms can serve part of the consumers residing at every site  $x$  in the interval  $[0, 1]$ .

### 3. Equilibrium Locations and Attributes

In this section, we examine firms' equilibrium locations and the attributes of the basic products when the marginal consumer lies in between the attributes of the two basic products for every  $x$  on the location line. In stage 2, firms simultaneously choose the optimal attributes of the basic products to maximize their aggregate profit. Firm  $i$ 's operating profit function at site  $x$ ,  $\pi_i(x)$ , which equals the integration of the delivered price minus the marginal cost (modification cost plus transportation cost) for each variety of product with respect to the varieties produced by firm  $i$  at site  $x$ , can be expressed as:

$$\begin{aligned} \pi_1(x) &= \int_0^{y_{b1}} [p_1(x, y) - (r(y_{b1} - y) + t|x - x_1|)] dy + \int_{y_{b1}}^{z(x)} [p_1(x, y) - (r(y - y_{b1}) + t|x - x_1|)] dy, \end{aligned} \quad (5.1)$$

$$\begin{aligned} \pi_2(x) &= \int_{z(x)}^{y_{b2}} [p_2(x, y) - (r(y_{b2} - y) + t|x - x_2|)] dy + \int_{y_{b2}}^1 [p_2(x, y) - (r(y - y_{b2}) + t|x - x_2|)] dy. \end{aligned} \quad (5.2)$$

Firm  $i$ 's aggregate profit function  $\Pi_i$  is denoted as its aggregate operating profit minus fixed cost  $k$ , which is expressed as:

$$\Pi_i = \int_0^1 \pi_i(x) dx - k = \int_0^{x_i} \pi_i(x) dx + \int_{x_1}^{x_2} \pi_i(x) dx + \int_{x_2}^1 \pi_i(x) dx - k, i=1,2, \quad (6)$$

where  $k$  denotes the sunk cost of the development for a basic product.

Substituting (2), (3) and (5) into (6), and differentiating the resulting equation with respect to  $y_{bi}$ , we obtain:<sup>7</sup>

$$\partial \Pi_1 / \partial y_{b1} = -r(y_{b1} + y_{b2})/2 + r(y_{b2} - y_{b1}) + t(x_2 - x_1)(x_2 + x_1 - 1)/2 = 0, \quad (7.1)$$

$$\partial \Pi_2 / \partial y_{b2} = r(2 - y_{b1} - y_{b2})/2 - r(y_{b2} - y_{b1}) + t(x_2 - x_1)(x_2 + x_1 - 1)/2 = 0. \quad (7.2)$$

Recall that  $0 \leq x_1 \leq x_2 \leq 1$  and  $0 \leq y_{b1} \leq y_{b2} \leq 1$ . It follows that the first term on the right-hand side of (7.1) is negative while that of (7.2) is positive, the second term of (7.1) is non-negative while that of (7.2) is non-positive, and the third term in both equations is non-positive (non-negative) when  $(x_1 + x_2 - 1) < (>) 0$ . These three terms are referred to as the competition effect, the hinterland effect and the transport-cost-advantage effect, in that order. The optimal attributes of the two firms' basic products  $y_{b1}$  and  $y_{b2}$  are jointly determined by these three effects. Intuitively, the competition effect shows that when the attributes of the two firms' basic products move apart, the differentiation between the two firms increase so that the competition between firms is mitigated. As a result, firms can charge higher prices and then earn larger profits. Consequently, the competition effect is a centrifugal force, attracting firm 1 (2) to move its attribute of the basic product downward (upward) on the attribute line. Next, the hinterland effect can increase the firm's hinterland and the market share by moving closer to the attribute of the rival's basic product. Thus, the hinterland effect is a centripetal force. Lastly, the

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<sup>7</sup> The second-order and the stability conditions are as follows:

$$\partial^2 \Pi_1 / \partial y_{b1}^2 = \partial^2 \Pi_2 / \partial y_{b2}^2 = -3r/2 < 0, \text{ and}$$

$$(\partial^2 \Pi_1 / \partial y_{b1}^2)(\partial^2 \Pi_2 / \partial y_{b2}^2) - (\partial^2 \Pi_1 / \partial y_{b1} \partial y_{b2})(\partial^2 \Pi_2 / \partial y_{b1} \partial y_{b2}) = 2r^2 > 0.$$

transport-cost-advantage effect attracts the attributes of both firms' basic products to move downward (upward) on the attribute line when  $(x_2+x_1) < (>) 1$ , while this effect vanishes when  $(x_2+x_1) = 1$ . When  $x_1 > 1-x_2$ , firm 1 (2) is closer to (farther away from) the center of the location line, the selling of the products from firm 1 (2) to the whole market gives rise to a transportation cost advantage (disadvantage). As a result, the competition competence of firm 1 (firm 2) is enhanced (weakened), such that firm 1's attribute of the basic product moves upward on the attribute to capture more product varieties, while firm 2's attribute of the basic product also moves upward on the attribute in order to mitigate the competition between the firms. On the contrary, firm 1 (2) has the transportation cost disadvantage (advantage) so that both firms' attributes of the basic products move downward on the attribute line, when  $x_2 < 1-x_1$ . In addition, if the two firms are symmetric, implying  $(x_2+x_1) = 1$ , the transport-cost-advantage effect vanishes. In sum, the optimal attributes of the basic products hinge upon the relative strength of these three effects.

Solving (7.1) and (7.2), we derive the two firms' optimal attributes of the basic products as follows:

$$y_{b1} = 1/4 + 2t(x_2 - x_1)(x_2 + x_1 - 1)/(4r), \quad (8.1)$$

$$y_{b2} = 3/4 + 2t(x_2 - x_1)(x_2 + x_1 - 1)/(4r). \quad (8.2)$$

Equations (8) show that if  $(x_2+x_1) = 1$ , implying that the transport-cost-advantage effect vanishes, the two firms' optimal attributes of the basic products are solved by (8) as  $1/4$  and  $3/4$ , respectively. This result occurs when the competition effect and the hinterland effect are balanced. Moreover, if firm 1 has a transportation cost disadvantage (advantage), i.e.,  $(x_2+x_1) < (>) 1$ , then both firms move downward (upward) on the

attribute line simultaneously.

Based on the above analysis, we establish:

*Proposition 1. Assume that firms having the technique of flexible manufacturing engage in spatially discriminatory pricing under linear transportation costs. When  $(x_2+x_1) = 1$ , the optimal attributes of the two basic products are located at the first and third quartiles of the attribute line, respectively. Moreover, the optimal attributes move downward (upward) on the attribute line simultaneously when  $(x_2+x_1) < (>) 1$ .*

We proceed to explore the impact of each firm's location on two firms' optimal attributes of the basic products. By differentiating (8.1) and (8.2) with respect to  $x_1$  and  $x_2$ , we obtain:

$$\partial y_{b1} / \partial x_1 = \partial y_{b2} / \partial x_1 = (t / 2r)(1 - 2x_1) \geq 0 \text{ if } x_1 \leq (1/2), \quad (9.1)$$

$$\partial y_{b2} / \partial x_2 = \partial y_{b1} / \partial x_2 = (t / 2r)(2x_2 - 1) \geq 0 \text{ if } x_2 \geq (1/2). \quad (9.2)$$

Equation (9.1) shows that a rise in firm 1's location attracts the two firms to move their attributes of the basic products upward on the attribute line while the distance between these two attributes remains unchanged, when firm 1 locates on the left-hand side of the center of the location line, i.e.,  $x_1 < (1/2)$ . The result occurs because firm 1 is closer to the center of the location line so that the selling of the products from firm 1 to the whole market has a transportation cost advantage, when  $x_1 < (1/2)$ . It follows that when  $x_1 < (1/2)$ , an increase in  $x_1$  strengthens the transport-cost-advantage effect, attracting firm 1 to move its attribute of the basic product closer to firm 2's attribute

while forcing firm 2 to move farther away from firm 1's.<sup>8</sup> It is worth noting that the distance between the attributes of the two basic products remains unchanged as a result of increasing  $x_1$ . Moreover, a larger transport rate enhances the transport-cost-advantage effect through decreasing total transportation costs, whereas a larger marginal modification rate has an opposite impact through increasing the modification cost of the increase in product varieties. Similarly, a rise in firm 2's location attracts the two firms to move their attributes of the basic products upward on the attribute line while the distance between these two attributes remains unchanged, when  $x_2 > (1/2)$ . To sum up, equations (9.1) and (9.2) show that when  $x_1 < (1/2)$  and  $x_2 > (1/2)$ , a move by each firm to the center of the location line attracts a firm to move its attribute of the basic product toward that of the rival and meanwhile the rival will move its attribute of the basic product farther away from that of the other firm. Accordingly, we have:

*Proposition 2. The distance between the attributes of the two basic products is always 1/2. When  $x_1 < (1/2)$  and  $x_2 > (1/2)$ , a move by each firm to the center of the location line attracts a firm to move its attribute of the basic product toward that of the rival and meanwhile the rival will move its attribute of the basic product farther away from that of the other firm. Moreover, the lower that the ratio of the marginal modification rate to the transport rate is, the bigger will be the magnitude of the move of these attributes.*

We now turn to the first stage to determine the firms' equilibrium locations.

Substituting (2), (3), (5) and (8) into (6), we obtain the reduced aggregate profit function

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<sup>8</sup> The transport-cost-advantage effect is denoted by the term,  $t(x_2 - x_1)(x_2 + x_1 - 1)/r$ , in (7.1) and (7.2). When  $x_1 < (1/2)$ , a rise in  $x_1$  increases the magnitude of this term regardless of the sign of this term being positive or negative. As a result, an increase in  $x_1$  strengthens the transport-cost-advantage effect.



and then by differentiating it with respect to  $x_i$  we derive the following profit-maximizing conditions:

$$\begin{aligned} \partial \Pi_1 / \partial x_1 = & t\{-t(x_2 - x_1)(1 - x_2 + x_1) / (2r) + 3[(1/2) - x_1] / 2 \\ & + t(x_2 - x_1)(x_2 + x_1 - 1)(1 - 2x_1) / r\} = 0, \end{aligned} \quad (10.1)$$

$$\begin{aligned} \partial \Pi_2 / \partial x_2 = & t\{t(x_2 - x_1)(1 - x_2 + x_1) / (2r) + 3[(1/2) - x_2] / 2 \\ & - t(x_2 - x_1)(x_2 + x_1 - 1)(1 - 2x_2) / r\} = 0. \end{aligned} \quad (10.2)$$

Recall that  $0 \leq x_1 \leq x_2 \leq 1$  and  $x_2 \geq x_1$ . The first term in the braces on the right-hand side of (10.1) is negative, while that in (10.2) is positive. This term is referred to as the competition effect, which serves as a centrifugal force in separating the two firms on the location line. The intuition can be stated as follows. The competition effect consists of a direct and a strategic effect, in which the direct effect emerges from the differentiation caused by the location distance between the two firms on the location line, while the strategic effect arises from the differentiation caused by the attribute distance between the attributes of the two basic products on the attribute line. The direct effect indicates that when the locations of the two firms move apart, the differentiation between the two firms increases. This will reduce the competition between firms, implying that firms can charge higher prices and then earn larger profits. The lower that the ratio of the marginal modification rate to the transport rate is, the stronger will be the direct effect. On the other hand, a rise in firm 1's location creates a strategic effect through moving the attributes of the two basic products upward on the attribute line in stage 2. Analytically, firm 1's mark-up generated by the modification costs at site  $x$  can be measured by  $p(x) - MC(x) = r(y_{b1} - y_{b2})$  for  $y \in [0, y_{b1}]$  and  $p(x) - MC(x) = r(y_{b1} + y_{b2} - 2y)$  for  $y \in [y_{b1}, z(x)]$ .<sup>9</sup>

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<sup>9</sup> Note that the equilibrium price equals the transportation cost plus the marginal modification cost of the second efficient firm.

Recall Proposition 2 which states that a rise in firm 1's location attracts the two firms to move their attributes of the basic products upward on the attribute line while the distance between these two attributes remains unchanged. It follows that a rise in firm 1's location increases this mark-up for the region  $y \in [y_{b1}, z(x)]$ , but has no impact for the region  $y \in [0, y_{b1}]$ . This strategic effect attracts firm 1 to move closer to its rival and weakens the competition effect in the location decision. We can also find from Proposition 2 that a lower ratio of the marginal modification to the transport rate will enhance this strategic effect. It should be noted that the direct effect outweighs the strategic effect so that the net competition effect is a centrifugal force, and is decreasing in the ratio of the marginal modification rate to the transport rate.

Next, the second term in (10.1) is positive (negative) when firm 1 is located on the left-hand (right-hand) side of the center of the location line (i.e.,  $x_1 < (>) 1/2$ ), while that in (10.2) is negative (positive) if  $x_2 > (<) 1/2$ . This term is referred to as the transport-cost-saving effect, which acts as a centripetal force in attracting the two firms to move toward the center of the location line. If the two firms serve part of the consumers residing at every site on the location line, then the minimum transportation cost can be reached when firms locate at the center of the location line. Thus, this effect can decrease the transportation costs by moving towards the center of the location line.

Finally, the third term is called the transport-cost-advantage effect, which is similar to that in (7.1) and (7.2). When  $x_1 > 1 - x_2$  and  $x_1 < (1/2)$ , firm 1 is closer to the center of the location line, and the selling of the products by firm 1 to the whole of the market gives rise to a transportation cost advantage. A rise in firm 1's location will increase its market share and profit by capturing larger demands, attracting firm 1 to move toward its

rival. Similarly, when  $x_2 < 1 - x_1$  and  $x_2 > (1/2)$ , firm 2 is closer to the center of the location line so that firm 2 will move toward its rival in order to capture a larger market share and profit. In addition, when the two firms are symmetric, implying  $(x_2 + x_1) = 1$ , the transport-cost-advantage effect vanishes due to the two firms having equal market shares. Consequently, these three effects jointly determine the firms' equilibrium locations.

Following Sydsaeter and Hammond (1995), the location equilibria are subject to second-order and stability conditions as follows:

$$\partial^2 \Pi_1 / \partial x_1^2 = t[3(t-r) - 4t(x_2^2 - 3x_1^2) + 2t(x_2 - 5x_1)] / (2r) \leq 0, \quad (11.1)$$

$$\partial^2 \Pi_2 / \partial x_2^2 = t[3(t-r) + 4t(3x_2^2 - x_1^2) - 2t(7x_2 - 3x_1)] / (2r) \leq 0, \quad (11.2)$$

$$J = t^2 / (4r^2) \{ 12tr[(x_1 + 3x_2) - 2(x_1^2 + x_2^2)] + 9r(r - 2t) - 48t^2(x_1^4 + x_2^4) + 4t^2[4(7x_1^3 + 5x_2^3) + 4x_1x_2(6x_1x_2 + 5 - 3x_2 - 9x_1) - 10(x_1^2 + x_2^2)] \} \geq 0. \quad (11.3)$$

Solving (10.1) and (10.2), we obtain the location equilibria as follows:

$$x_1 = x_2 = 1/2, \quad (12.1)$$

$$x_1 = 3r/4t, x_2 = 1 - 3r/4t. \quad (12.2)$$

There are two possible location equilibria, namely, central agglomeration and spatial dispersion. Substituting the agglomeration equilibrium of (12.1) into (11.1) - (11.3) and (4), we figure out that the second-order condition is fulfilled when  $(r/t) \geq (1/3)$ , the stability condition is satisfied when  $(r/t) \geq (2/3)$ , and the constraint for  $z(x)$  lying within the two basic products is definitely valid. Taking into account these three constraints, we conclude that the two firms will agglomerate at the center of the location line when the ratio of the marginal modification rate and the transport rate is high, say,  $(r/t) \geq (2/3)$ . Similarly, substituting the dispersion equilibrium of (12.2) into (11.1) - (11.3) and (4), we obtain that spatial dispersion occurs when this ratio is moderate, say,  $(1/2) \leq (r/t) < (2/3)$ .

It is worth pointing out that since firms engage in spatially discriminatory pricing in this paper, the equilibrium price can be referred to as the limit price equaling the rivals' marginal cost at each location-attribute combination  $(x, y)$ . The limit price is essentially itself an undercutting price. Thus, the location equilibrium derived in this paper has already taken into account the undercutting problem indicated by D'Aspremont *et al.* (1979).

Based on the above analysis, we establish the following theorem:

*Theorem 1. Assume that firms having the technique of flexible manufacturing engage in spatially discriminatory pricing under linear transportation costs. We propose that:*

*(i) the two firms agglomerate at the center of the location line and the optimal attributes of the two basic products are located at the first and third quartiles of the attribute line, respectively, when the ratio of the marginal modification rate to the transport rate is high, say,  $(r/t) \geq (2/3)$ .*

*(ii) the two firms locate separately at  $x_1 = 3r/4t$  and  $x_2 = 1 - 3r/4t$  along the location line and the optimal attributes of the two basic products are located at the first and third quartiles of the attribute line, respectively, when this ratio is moderate, say,  $(1/2) \leq (r/t) < (2/3)$ .*

The present paper is closely related to Hurter and Lederer (1985), in the sense that they also study firms' optimal locations through the use of a two-dimensional framework and firms implementing spatially discriminatory pricing. Theorem 1 is significantly different from the result derived in Hurter and Lederer (1985), in which the equilibrium

location combination is that where firms agglomerate on the horizontal axis, but locate separately at the first and third quartiles along the vertical axis, respectively. The difference occurs because the firms in the present paper have the technique of flexible manufacturing causing them to be able to produce different arrays of differentiated products, but they produce a single product in Hurter and Lederer (1985).

Theorem 1 can explain some real world phenomena. For example, we observe that train and bus stations are usually located closely in cities, while we rarely see ready-mixed concrete producers agglomerating coincidentally. This former observation arises because the ratio of the marginal modification rate to the transport rate is high, while the latter emerges because this ratio is low.

#### 4. An Extension

In this section, we extend our analysis to the case where the modification cost between the two basic products is smaller than the transportation cost between the two firms, such that the marginal consumer  $z(x)$  may lie beyond the attributes of the two basic products. This is the case where the inequality  $t(x_2 - x_1) > r(y_{b2} - y_{b1})$  holds by referring to (4). It follows from (2) that all consumers located in the region  $x \leq x_1$  ( $x \geq x_2$ ) will buy products from firm 1 (firm 2).<sup>10</sup> This means that the marginal consumers can only exist in the region  $x_1 \leq x \leq x_2$ . Thus, by assuming that the inequality of  $t(x_2 - x_1) > r(y_{b2} - y_{b1})$  holds, we can solve for the threshold  $x_a$  from (3) that  $z(x_a) =$

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<sup>10</sup> By using the inequality  $t(x_2 - x_1) > r(y_{b2} - y_{b1})$ , we can obtain from (2) that firm 1's marginal cost is smaller (larger) than that of firm 2, i.e.,  $[t|x - x_1| + r|y - y_{b1}| - t|x - x_2| + r|y - y_{b2}|] \leq (\geq) 0$ , when  $x \leq x_1$  ( $x \geq x_2$ ). Thus, all consumers located in  $x \leq x_1$  ( $x \geq x_2$ ) will buy products from firm 1 (firm 2).

$[t(x_2 + x_1 - 2x_a) + r(y_{b1} + y_{b2})]/2r = y_{b2}$  and the threshold  $x_b$  that  $z(x_b) = [t(x_2 + x_1 - 2x_b) + r(y_{b1} + y_{b2})]/2r = y_{b1}$  for  $x_1 \leq x \leq x_2$  as follows:

$$x_a = [t(x_1 + x_2) - r(y_{b2} - y_{b1})]/(2t), \quad (13.1)$$

$$x_b = [t(x_1 + x_2) + r(y_{b2} - y_{b1})]/(2t). \quad (13.2)$$

Since both the modification cost and the transport cost functions are linear in distance, it is obvious that the consumers residing above  $y_{b2}$  at the site  $x_a$  along the attribute line are also marginal consumers. Likewise, the consumers residing beneath  $y_{b1}$  at the site  $x_b$  along the attribute line are also marginal consumers. We can derive from (2) and (3) that firm 1 becomes a local monopolist in the region  $x \in [0, x_a]$ , while firm 2 is a local monopolist in the region  $x \in [x_b, 1]$  due to having a cost advantage.

For the region  $x \in [x_a, x_b]$ , both firms can serve part of the consumers at each site  $x$  and the marginal consumer can be expressed as:

$$z(x) = t(x_2 + x_1 - 2x) + r(y_{b1} + y_{b2})/2r, \quad \text{if } x_a \leq x \leq x_b \quad (13.3)$$

We use Figure 3 to illustrate the case where the modification cost between the two basic products is smaller than the transportation cost between the two firms. In Figure 3,  $z(x)$  is negatively-sloping for  $x \in [x_a, x_b]$ , while it is a vertical line segment for  $x \in [0, x_a]$  and  $x \in [x_b, 1]$ . Since we assume that  $x_1 \leq x_2$  and  $y_{b1} \leq y_{b2}$ , firm 1 will serve the consumers residing beneath the locus of the marginal consumer  $z(x)$ , while firm 2 will serve those who reside above  $z(x)$  for  $x \in [x_a, x_b]$ . Moreover, firm 1 (2) is a local monopolist in the regions  $x \in [0, x_a]$  ( $x \in [x_b, 1]$ ).

(Insert Figure 3 here)

The equilibrium prices solved in stage 3 are identical to those in (2). In stage 2, since

firm 1 (2) becomes a local monopolist in the rectangle  $[0, x_a] \times [0, 1]$  ( $[x_b, 1] \times [0, 1]$ ), firm  $i$ 's aggregate profit function  $\Pi_i$  can be expressed as follows:

$$\Pi_1 = \int_0^{x_b} \pi_1(x) dx - k = \int_0^{x_1} \pi_1(x) dx + \int_{x_1}^{x_a} \pi_1(x) dx + \int_{x_a}^{x_b} \pi_1(x) dx - k, \quad (14.1)$$

$$\Pi_2 = \int_{x_a}^1 \pi_2(x) dx - k = \int_{x_a}^{x_b} \pi_1(x) dx + \int_{x_b}^{x_2} \pi_1(x) dx + \int_{x_2}^1 \pi_1(x) dx - k. \quad (14.2)$$

Substituting (13.1) – (13.3) into (14) and differentiating them with respect to  $y_{bi}$ , respectively, we derive the following profit-maximizing conditions:

$$\begin{aligned} \partial \Pi_1 / \partial y_{b1} = & r[2t(x_1 + x_2)(1/2 - y_{b1}) + r(y_{b2} - y_{b1})(3y_{b1} + y_{b2} - 1)] / (2t) \\ & + 2r^2(y_{b2} - y_{b1})^2 / (3t) - 2r^2(2y_{b1} + y_{b2})(y_{b2} - y_{b1}) / (3t) = 0, \end{aligned} \quad (15.1)$$

$$\begin{aligned} \partial \Pi_2 / \partial y_{b2} = & r[-2t(2 - x_1 - x_2)(y_{b2} - 1/2) + r(y_{b2} - y_{b1})(-3 + y_{b1} + 3y_{b2})] / (2t) \\ & - 2r^2(y_{b2} - y_{b1})^2 / (3t) + 2r^2(3 - y_{b1} - 2y_{b2})(y_{b2} - y_{b1}) / (3t) = 0. \end{aligned} \quad (15.2)$$

The first term on the right side of (15.1) represents the monopoly effect. Because firm 1 becomes the local monopoly between  $[0, x_a]$ , competition between the firms is reduced. Therefore, firm 1 has the incentive to push the basic product closer to the center of the attribute line in order to capture larger product varieties. Thus, the monopoly effect works as a centripetal force. The second and the third effects are referred to as the hinterland and the competition effects are just as those described in Section 3. In sum, the optimal attributes of the basic products hinge upon the relative strength of these three effects.

Following Sydsaeter and Hammond (1995), the optimal attributes of the two firms' basic products are subject to the second-order and stability conditions as follows:

$$\partial^2 \Pi_1 / \partial y_{b1}^2 = r[r(1 + 2y_{b1} - 2y_{b2}) - 2t(x_1 + x_2)] / 2t \leq 0, \quad (16.1)$$

$$\partial^2 \Pi_2 / \partial y_{b2}^2 = r[r(1+2y_{b1}-2y_{b2})-2t(2-x_1-x_2)]/2t \leq 0, \quad (16.2)$$

$$\begin{aligned} & \left( \partial^2 \Pi_1 / \partial y_{b1}^2 \right) \left( \partial^2 \Pi_2 / \partial y_{b2}^2 \right) - \left( \partial^2 \Pi_1 / \partial y_{b1} \partial y_{b2} \right) \left( \partial^2 \Pi_2 / \partial y_{b1} \partial y_{b2} \right) \\ & = r^2 [t(x_1+x_2)(2-x_1-x_2) - r(1+2y_{b1}-2y_{b2})] / t \geq 0. \end{aligned} \quad (16.3)$$

Solving (15.1) and (15.2), we derive the optimal attributes of the two firms' basic products as follows:

$$y_{b1} = y_{b2} = 1/2, \quad (17.1)$$

$$\begin{aligned} y_{b1} &= \{(x_1+x_2)[t(x_1+x_2)^2 - 4t(x_1+x_2) + 4t+r] - r\} / (2r), \text{ and} \\ y_{b2} &= \{(x_1+x_2)[t(x_1+x_2)^2 - 2t(x_1+x_2) + r] + r\} / (2r). \end{aligned} \quad (17.2)$$

There are two possible equilibrium attributes of the basic products, namely, central agglomeration and spatial dispersion.

In stage 1, we analyze two possible equilibrium locations based on the two possible equilibrium attributes of the basic products in (17). Firstly, for the case of central agglomeration on the attribute line, by substituting (17.1) into (13.1) and (13.2) we obtain that  $x_a = x_b = (x_1+x_2)/2$ . Since the thresholds  $x_a$  and  $x_b$  are located at the same site, it follows that the rival region  $[x_a, x_b]$  vanishes. Firm 1 captures the whole market in  $[0, x_a]$ , while firm 2 grasps the entire market in  $[x_b, 1]$ . It is worth pointing out that the third term on the right-hand side of (14.1) and the first term on the right-hand side of (14.2) vanish in this case. Substituting (17.1) and (13) into (14) gives the reduced aggregate profit functions for firm 1 and 2. Differentiating them with respect to  $x_i$ , respectively, we derive the following profit-maximizing conditions:

$$\partial \Pi_1 / \partial x_1 = 2t[(x_1+x_2)/4 - x_1] = 0, \quad (18.1)$$

$$\partial \Pi_2 / \partial x_2 = 2t[(x_1+x_2+2)/4 - x_2] = 0. \quad (18.2)$$

Recall that there is no rival region between the two firms and both firms become local monopolists in this case. When contrasted with the case where the modification cost



between the two basic products is bigger than the transportation cost between the two firms in Section 3, the competition and the transport-cost-advantage effects in (10) vanish. The only effect left is the transport-cost-saving effect in attracting the two firms to move toward the center of the market segment on the location line in order to minimize the transportation costs.<sup>11</sup>

Solving (18.1) and (18.2), we obtain the equilibrium location as follows:<sup>12</sup>

$$x_1 = 1/4, x_2 = 3/4. \quad (19)$$

Equation (19) shows that firms 1 and 2 locate at the first and the third quartiles, respectively, which are the centers of their own respective market areas. Each firm captures half of the market share and becomes a local monopolist.

Substituting (19) into (17), we figure out that the second-order condition is fulfilled when  $(r/t) < (1/2)$ , the stability condition is satisfied when  $(r/t) < 1$  and eq. (4) is violated when  $(r/t) < (1/2)$ . By taking into account these three constraints, we conclude that the two firms are located at the first and third quartiles of the location line, when the ratio of the marginal modification rate to the transport rate is low, say,  $(r/t) < (1/2)$ .

Next, in the case of the dispersion equilibrium for the attributes, by substituting (17.2), (2) and (13) into (14), we derive the equilibrium locations as follows:

$$x_1 = -1/4 + (r^2 + t^2)/4rt, x_2 = 5/4 - (r^2 + t^2)/4rt. \quad (20)$$

Substituting (20) into (17.2), we obtain the optimal attribute address of the basic product as follows:

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<sup>11</sup> Recall that  $x_a = x_b = (x_1 + x_2)/2$ . It follows that firm 1's market segment on the location line lies in  $[0, x_a = (x_1 + x_2)/2]$  and firm 2's in  $[x_b = (x_1 + x_2)/2, 1]$ . Thus, the market center is  $x_a/2 = (x_1 + x_2)/4$  for firm 1 and  $(x_b + 1)/2 = (2 + x_1 + x_2)/4$  for firm 2, respectively.

<sup>12</sup> The second-order and the stability conditions are as follows:

$\partial^2 \Pi_1 / \partial x_1^2 = \partial^2 \Pi_2 / \partial x_2^2 = -3/(2t) < 0$ , and  
 $(\partial^2 \Pi_1 / \partial x_1^2)(\partial^2 \Pi_2 / \partial x_2^2) - (\partial^2 \Pi_1 / \partial x_1 \partial x_2)(\partial^2 \Pi_2 / \partial x_1 \partial x_2) = 2t^2 > 0$ .

$$y_1 = t/2r, y_2 = 1 - t/2r. \quad (21)$$

However, the stability condition in stage 2 contradicts the condition of the violation of eq. (4). Thus, the results derived in (20) and (21) can never be equilibria.

Accordingly, we obtain the following theorem:

*Theorem 2. Assume that firms having the technique of manufacturing engage in spatially discriminatory pricing under linear transportation costs. The two firms are located at the first and third quartiles of the location line and the attributes of the basic products agglomerate at the center of the attribute line, when the ratio of the marginal modification rate to the transportation rate is low, say,  $(r/t) < (1/2)$ .*

(Insert Figures 4 and 5 here)

We use Figures 4 and 5 to illustrate the results derived in Theorems 1 and 2. Figure 4 depicts the relationship between the firms' equilibrium locations and the ratio of the marginal modification rate to the transport rate, and Figure 5 depicts the relationship between the firms' equilibrium attributes of the basic products and this ratio. Figures 4 and 5 demonstrate that when the ratio of the marginal modification rate to the transport rate is low, say,  $(r/t) < (1/2)$ , the two firms move further apart and locate at the first and third quartiles of the location line, respectively, while the equilibrium attributes of the basic products agglomerate at the center of the attribute line. Since the two firms produce an identical array of differentiated products in this case, the firms' attributes of products are identical. As a result, the two firms move further apart on the location line to increase the differentiation for minimizing the transportation costs. This implies that the technique

of flexible manufacturing does not provide any incentive for firms to locate closer to each other on the location line when the ratio is small. Next, when this ratio is moderate, say,  $(1/2) \leq (r/t) < (2/3)$ , the two firms still locate some distance apart but move closer to each other on the location line while the equilibrium attributes of the basic products are separated and located at the first and third quartiles of the attribute line, respectively. Finally, when this ratio is high, say,  $(r/t) \geq (2/3)$ , the two firms agglomerate at the center of the location line while the equilibrium attributes of the basic products are still separated and located at the first and third quartiles of the attribute line, respectively. The last two cases show that the technique of flexible manufacturing does matter in the firms' location decision, when this ratio is not low. Thus, we have the following corollary:

*Corollary 1. The technique of flexible manufacturing does not matter in firms' location decisions under linear transportation costs when the ratio of the marginal modification rate to the transport rate is low, whereas it does matter otherwise.*

## **5. Concluding Remarks**

This paper has developed a two-dimensional spatial framework, in which firms have the technique of flexible manufacturing and engage in spatially discriminatory pricing, to explore the optimal attributes of the basic products and firms' equilibrium locations under linear transportation costs. The focus of this paper is on the impact of the technique of flexible manufacturing on firms' location decisions. We have shown that the technique of flexible manufacturing with spatially discriminatory pricing will cause the firms to choose different attributes for the basic products on the attribute line, so that the two

firms are differentiated by producing different arrays of differentiated products, when the ratio of the marginal modification rate to the transport rate is not low. This will mitigate the competition between firms, resulting in the emergence of firms that will agglomerate coincidentally on the location line. Thus, the technique of flexible manufacturing does matter in firms' location decisions, when this ratio is not low.

This paper can be extended to the consideration of mill pricing taken by firms. We derive the results as follows. First, the firms will agglomerate on the location line but the optimal attributes of the basic products will be located separately at the first and third quartiles of the attribute line, respectively, regardless of the ratio of the marginal modification rate to the transport rate, when the transportation cost is linear in distance. Second, the firms will separate to locate at the endpoints of the location line, respectively, but the optimal attributes of the basic products will be agglomerated at the same site of the attribute line, regardless of the ratio of the marginal modification rate to the transport rate, when the transportation cost is quadratic.

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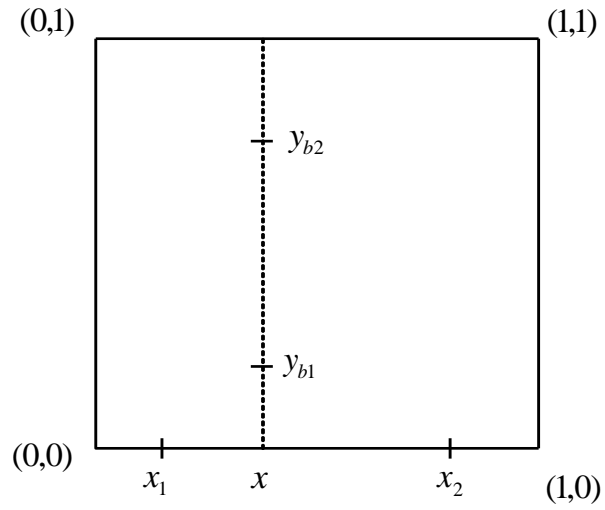


Figure 1. The two-dimensional spatial framework

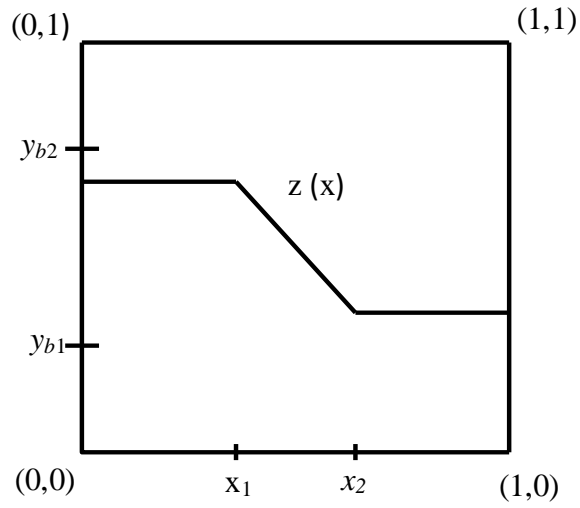


Figure 2. The locus of the marginal consumer when the marginal consumer lies in between the attributes of the two basic products.



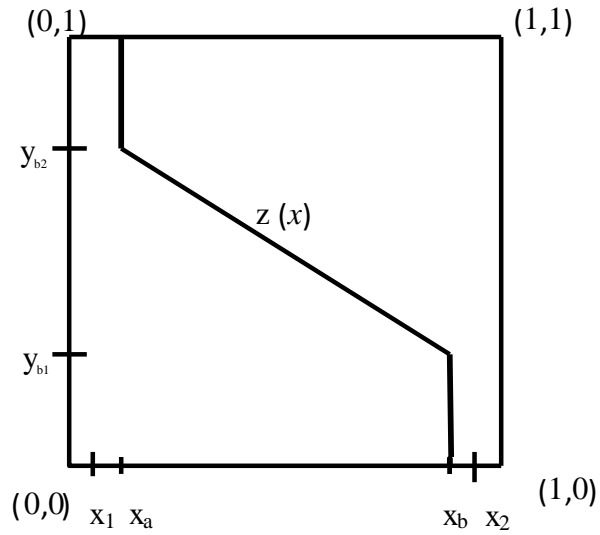


Figure 3. The locus of the marginal consumer when the marginal consumer may lie beyond the attributes of the two basic products.

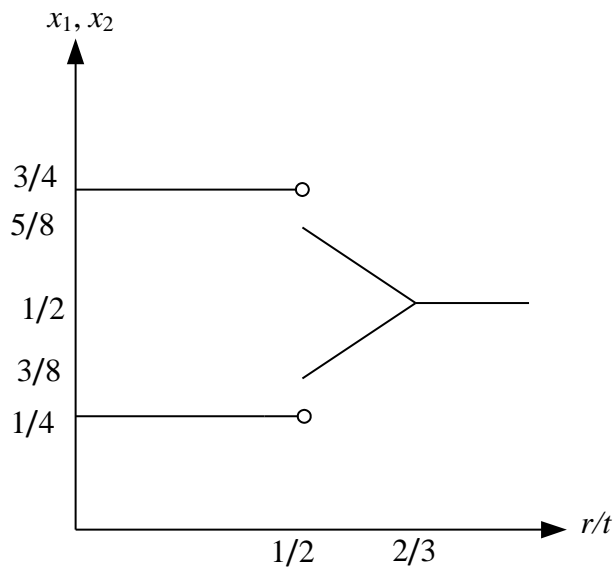


Figure 4. The equilibrium locations.

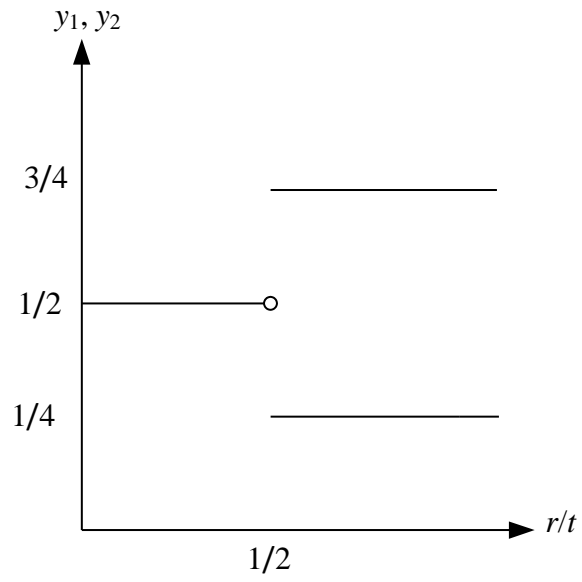


Figure 5. The equilibrium attributes of the basic products.