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Technological Progress and Economic Geography*

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Abstract

New economic geography focuses on the impact of falling transport costs on the spatial distribution of activities. However, it disregards the role of technological innovations, which are central to modern economic growth, as well as the role of migration costs, which are a strong impediment to moving. We show that this neglect is unwarranted. Regardless of the level of transport costs, rising labor productivity fosters the agglomeration of activities, whereas falling transport costs do not affect the location of activities. When labor is heterogeneous, the number of workers residing in the more productive region increases by decreasing order of productive efficiency when labor productivity rises.

Keywords: new economic geography, technological progress, labor productivity, migration costs, labor heterogeneity

JEL Classification: J61, R12

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1 Introduction

The Industrial Revolution has exacerbated regional disparities by an order of magnitude that was unknown before. For example, the English historian Pollard (1981), who paid special attention to the geographical characteristics of the Industrial Revolution, claimed that “the industrial regions colonize their agricultural neighbours [and take] from them some of their most active and adaptable labour, and they encourage them to specialize in the supply of agricultural produces, sometimes at the expense of some preexisting industry, running the risk thereby that this specialization would permanently divert the colonized areas from becoming industrial themselves.” (Pollard, 1981, 11).

In a path-breaking paper, Krugman (1991) proposed to explain this rapid and abrupt redistribution of economic activities, and the concomitant urbanization, by the integration of markets. Specifically, Krugman argued that manufacturing activities are dispersed across regions and countries when transport costs are high because local producers are protected against imported goods. As transport costs steadily decline, firms and consumers tend to agglomerate in a handful of places where firms are able to better exploit increasing returns by supplying larger markets and exporting their output at low cost. In the benchmark case of two identical regions studied in the literature, the symmetric distribution of manufacturing firms breaks down when transport costs decrease sufficiently to reach a minimum threshold. Once transport costs fall below this threshold, the manufacturing sector gets agglomerated in what becomes the core region, while the now-peripheral region is specialized in farming. This explanation has been embraced by a great number of authors under the heading of “new economic geography” (NEG). Their contributions are discussed and surveyed in great detail by Fujita et al. (1999b) and, more recently, by Combes et al. (2008b).

The empirical evidence collected by economic historians seems to give credence to this explanation. For example, Bairoch (1997) observed that “between 1800 and 1910, it can be estimated that the lowering of the real average prices of transportation was of the order of 10 to 1.” In the same vein, O’Rourke and Williamson (1999) attributed to the transportation revolution the near disappearance of commodity price gaps within the United States and in the Atlantic economy that took place between 1820 and 1914. Transportation costs continued to fall after World War I. For example, in the United States, Glaeser and Kohlhase (2004) noted that over the twentieth century, the costs of moving manufactured goods have declined by over 90 per cent in real terms. According to NEG, such numbers explain the growing concentration of manufacturing activities that started with the Industrial Revolution in the 19th and 20th centuries in many developed countries (see, e.g. Kim, 1995; Rosés et al., 2010; Combes et al., 2011).

Yet the most distinctive feature of the Industrial Revolution was probably a sharp rise in output per worker in the manufacturing sector. Ever since Schumpeter and Kuznets, the development of new technologies has long been recognized as the main engine of modern economic growth. According to Bairoch (1997) again, “the global productivity of production factors was multiplied on average in Western developed countries by 40 to 45 between 1700 and 1990.” However crude and controversial this estimation (see, e.g. Maddison, 2001), it seems unquestionable that, ever since the beginning of the Industrial Revolution, a sustained flow of technological innovations has dramatically increased labor productivity (Crafts, 2004). This makes it hard to believe that the collapse in transport costs was the only reason for the rising unevenness of economic development. Therefore, in
this paper, we have chosen to focus on falling production costs, rather than falling transport costs. We are agnostic about the concrete form taken by the various innovations developed before, during and after the Industrial Revolution. Indeed, our model is consistent with different narrative approaches to modern economic growth.

To achieve our goal, we develop a parsimonious model with one sector featuring increasing returns and monopolistic competition, thus remaining in the wake of NEG. Allen (2009) has convincingly argued that the relative scarcity of labor in Britain, where wages were remarkably high, had fostered the development of new labor-saving technologies that permit the substitution of capital and energy for labor. For this reason, we find it reasonable as a first-order approximation to focus on labor as the main production factor. In our model, labor productivity is expressed through the marginal and fixed labor requirements needed to produce one good in the manufacturing sector. In this context, technological progress takes the form of steadily decreasing marginal or fixed requirements of labor. Even though the price structure is likely to have fueled a biased technological progress, we will not try to trace back the reasons for the development of specific innovations. Like Krugman who does not explain why transport costs fall, we will consider an exogenous technological progress that permits an increase in the output per worker.

Although we recognize that consumers are mobile, it is unquestionable that they bear positive costs when they change location. These costs are often considered a one-time expenditure but this view strikes us as being too extreme. Indeed, migration generates substantial non-pecuniary costs created by differences in languages, cultures, or religions, which have a lasting influence on individual well-being. In addition, migrants often get a lower pay than local consumers who have a better tacit knowledge of social rules that make them more productive. Summarizing the state of the art, Collier (2013) asserts that “migrants tend to be less happy than the indigenous host population.” In this context, migration costs are to be interpreted as the difference in the degree of well-being enjoyed by the two groups of workers.

Temporary and return migration is evidence that migrants bear permanent social dislocation costs when they live away from their country or region of origin (Dustmann and Kirchkamp, 2002; Dustmann and Mestres, 2010). In this paper, migration costs act as the dispersion force that explains why not all consumers become concentrated in a single large region. They are born in different places and do not necessarily want to move away. As a result, a relatively large number of consumers choose to stay put even when they may be guaranteed a higher living standard in other places. Our setting differs from Krugman’s but they both share several common features, which should ease comparison between results. Despite numerous differences, our approach remains in the tradition of NEG because we study how an exogenous force—technological progress—affects urbanization.

Our main two findings may be summarized as follows. First, when labor productivity starts rising, the set of stable equilibria shrinks. In the limit, when one region is initially bigger than another—even by a trifle—all firms and consumers get agglomerated in the larger region. To put it differently, we will show that, even in the absence of falling transport costs, a sufficiently high labor productivity is sufficient to explain why

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1 Even today, these effects remain important. For example, Belot and Ederveen (2012) study the 22 OECD countries over the period 1990-2003. The authors find that international migration flows between countries with closely related languages are much larger than between countries with unrelated languages. They also show that religious and cultural proximity facilitates migrations.
the manufacturing sector is agglomerated. In our model, the distribution of activities is determined by the interplay between labor productivity and migration costs. We thus provide a new and historically relevant explanation for the geographical concentration of economic activities that started with the Industrial Revolution.

How does this compare to NEG? When labor productivity is low, many different distributions may be sustained as stable spatial equilibria. In particular, both the symmetric and the agglomerated patterns are always stable equilibria. In addition, these various configurations remain stable even when transport costs take on very low values. In other words, when production costs are high, firms and consumers that are a priori dispersed will remain so even when markets are very integrated. These two results clash with what NEG tells us. The reason for such a major difference in results is found in the migration costs. Regardless of the level of transport costs, positive migration costs always prevent a marginal change in locations from destabilizing an equilibrium distribution. Does this mean that migration costs must be absent when explaining the agglomeration of economic activities in a few regions? Happily enough, we show that the answer is no.

Second, very much like in NEG, the initial distribution of activities displays some sluggishness during the first phases of technological progress but then abruptly takes the form of a large economic agglomeration of firms and consumers, such as the Manufacturing Belt in the U.S. However, this sudden change in locations relies on the extreme assumption of homogeneous firms and consumers. If firms are heterogeneous à la Melitz (2003), the agglomeration process is gradual. More precisely, the most productive firms located in the smaller region are the first ones to move toward the larger region because they are the ones enjoying the greatest hike in profits (Okubo et al., 2010). Rather than pursuing this line of research, we assume that workers are heterogeneous: they are endowed with different amounts of efficiency units of labor. The reason for this choice lies in the empirical evidence showing that skilled workers tend to cluster in a small number of highly productive places (Glaeser and Maré, 2001; Combes et al., 2008a; Combes et al., 2012a; Moretti, 2012).

Assuming heterogeneous labor, we show that workers living in the less productive region move toward the more productive region by decreasing order of productive efficiency. In other words, migration goes hand in hand with labor productivity sorting, an empirically well-documented fact (Docquier and Rapoport, 2012). This is not a new phenomenon. Pollard (1981) argued that, during the Industrial Revolution, the core regions attracted from the peripheral regions “some of their most active and adaptable labour.” Focussing on the contemporary period, Moretti (2012) asserts that “geographically, American workers are increasingly sorting along educational lines.” As a consequence, the larger region is also the more productive one, so that income and welfare differences reflect differences in the spatial distribution of skills and know-how.

The paper is organized as follows. In the next section, we present our model and derive some preliminary properties of the market outcome. In Section 3, we characterize the spatial equilibria and study their stability. In Section 4, we show how technological progress may lead to the emergence of a core-periphery structure. In Section 5, we relax the assumption of homogeneous labor and recognize that workers have different skills. Section 6 concludes.
2 The Model and Preliminary Results

The economy is endowed with two regions, denoted \( r, s = 1, 2 \), a manufacturing (or tradable service) sector producing a horizontally differentiated good, one production factor (labor), and a population of consumers of mass \( L \).\(^2\) Therefore, unlike Krugman who considers a two-sector setting (manufacturing and agriculture) with two types of labor (workers and farmers), the dispersion force can no longer stem from the immobility of one type of workers, i.e. farmers.

The differentiated good is made available under the form of a continuum \( n \) of varieties. Consumers are endowed with one efficiency unit of labor and share the same preferences. The preferences of a consumer located in region \( r = 1, 2 \) are given by the CES utility:

\[
U_r = \left( \sum_s \int_0^{n_s} q_{sr}(i)^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}}
\]  

(1)

where \( n_s \) is the number of varieties produced in region \( s = 1, 2 \), \( q_{sr}(i) \) the consumption of variety \( i \) produced in region \( s \) and consumed in region \( r \), and \( \sigma > 1 \) the elasticity of substitution between any two varieties.

The budget constraint of a consumer located in region \( r \) is given by

\[
\sum_s \int_0^{n_s} p_{sr}(i)q_{sr}(i) \, di = w_r
\]

where \( p_{sr}(i) \) is the price of variety \( i \) produced in region \( s \) and consumed in \( r \), while \( w_r \) is the wage rate in region \( r \).

Labor markets are competitive and local, thus implying that wages need not be equal between the two regions. The equilibrium wage in region \( r \) is determined by a bidding process in which the region \( r \)-firms compete for workers by offering them higher wages until no firm earns strictly positive profits. Thus, a firm’s operating profits are equal to its wage bill.

The individual demand in region \( r \) for variety \( i \) produced in region \( s \) is then as follows:

\[
q_{sr}(i) = \frac{p_{sr}(i)^{-\sigma}}{P_r^{1-\sigma}}w_r
\]

(2)

where the price index \( P_r \) that prevails in region \( r \) is given by

\[
P_r \equiv \left( \sum_s \int_0^{n_s} p_{sr}^{1-\sigma}(i) \, di \right)^{\frac{1}{1-\sigma}}.
\]

(3)

Each firm therefore produces a single variety and each variety is produced by a single firm, so that \( n_s \) is also the number of firms set up in region \( s \). The technology is identical in all locations - regions have no specific comparative advantage - and for all the varieties - firms are homogeneous. There are increasing returns at the firm level, but no scope economies that would induce a firm to produce several varieties. Because firms are

\(^2\)Helpman (1998) also works with a single sector, while the dispersion force lies in the crowding out of the housing stock. Helpman ends up with conclusions that are at odd with Krugman’s.
homogeneous, we may drop the variety-index $i$ hereafter. The production of a variety needs a fixed requirement of $f > 0$ units of labor and a marginal requirement of $c > 0$ units of labor.\footnote{Note that $c$ is often used as a proxy for the total factor productivity.}

Following the new trade literature, we assume iceberg transport costs: $\tau_{rs} = \tau > 1$ units of a variety have to be shipped from region $r$ for one unit of that variety to be available in region $s \neq r$, while transport costs are zero when a variety is sold in the region where it is produced ($\tau_{rr} = \tau_{ss} = 1$). Therefore, we have $p_{rr} = p_r$ and $p_{sr} = \tau p_s$. If $\lambda_s$ denotes the share of consumers living in region $s$ (with $\lambda_1 + \lambda_2 = 1$), for the demand $\lambda_s L_q q_{rs}$ in region $s$ to be satisfied, each firm in region $r$ must produce $\tau \lambda_s L q_{rs}$ units. The profits earned by a firm located in region $r$ are thus given by

$$\pi_r = p_r L \left( \sum_s \lambda_s \tau_{rs} q_{rs} \right) - w_r \left( f + c L \sum_s \lambda_s \tau_{rs} q_{rs} \right).$$

(4)

When $f$ is replaced with $f/L$ in (4), $L$ is a simple scaling factor. Therefore, without loss of generality we may assume that $L = 1$, so that $f$ is to be interpreted as the fixed cost per capita. The manufacturing sector is thus modeled as in Krugman (1991).

Given the individual demand (2), the profit-maximizing price is

$$p_r = \frac{\sigma c}{\sigma - 1} w_r.$$  

(5)

Assuming free entry and exit in the manufacturing sector, profits (4) are zero in equilibrium:

$$(p_r - cw_r) \sum_s \lambda_s \tau_{rs} q_{rs} = w_r f.$$  

(6)

Plugging (2) into (6) and solving for the total output $q_r = \lambda_r q_{rr} + \tau \lambda_s q_{rs}$ yields

$$q^*_r = \frac{(\sigma - 1)f}{c}.$$  

(7)

Last, labor market balance in region $r$ implies

$$n_r \left( f + c \sum_s \lambda_s \tau_{rs} q_{rs} \right) = \lambda_r.$$  

(8)

Using (5), (6) and (8), we obtain:

$$n_r = \frac{\lambda_r}{\sigma f}.$$  

(9)

Plugging (5) and (9) into (4), we obtain the wage equation in region $r$:

$$\sum_s \frac{\phi_{rs} \lambda_s w_s}{\sum_t \phi_{rt} \lambda_t w_t^{1-\sigma}} = w^*_r$$  

(10)

where $\phi_{rs} \equiv \tau_{rs}^{1-\sigma} \in [0, 1)$. Choosing labor in region 2 as the numéraire, we have $w_1 = w$ and $w_2 = 1$. Setting $\lambda_1 \equiv \lambda$ and $\lambda_2 \equiv 1 - \lambda$, the wage equation (10) for $r = 1$ may then be rewritten as follows:

$$\lambda = \frac{w^* - \phi}{w^* - (w + 1) \phi + w^{1-\sigma}}$$  

(11)
where \( \phi \equiv \tau^{1-\sigma} \in [0, 1] \). The Walras law implies that the labor balance condition in region 2 is satisfied.

Observe, first, that \( w^* = 1 \) when \( \lambda = 1/2 \) while \( w^* = \phi^{-1/\sigma} > 1 \) when \( \lambda = 1 \). Furthermore, we show in the Appendix 1 that the right-hand side of (11) increases over the interval \([1/2, 1]\). Therefore, for any given \( \lambda \geq 1/2 \) there exists a unique equilibrium wage \( w^*(\lambda) \geq 1 \). Although the labor and product markets are more competitive in region 1 than in region 2, the nominal wage is higher in the larger market. In addition, the nominal wage prevailing in region 1 rises with the relative size of the corresponding market. As a result, the interregional wage gap widens when the two regions become more asymmetric.

Note, however, that the wage gap shrinks when \( \phi \) rises, that is, when the two regions get more integrated. This is because the interregional difference in prices get smaller when \( \phi \) increases, which fosters the interregional convergence of wages. In the limit, when the two markets are fully integrated (\( \phi = 1 \)), the size difference becomes immaterial and there is wage equalization (\( w^* = 1 \)).

Furthermore, using (3), (5) and (11) as well as the inequality \( w^* > 1 \), we get

\[
P_1^{1-\sigma} - P_2^{1-\sigma} = K \frac{(w^\sigma - 1)w^{1-\sigma}}{w^\sigma - (w + 1)\phi + w^{1-\sigma}} > 0
\]

where \( K \) is a positive constant. It then follows from this expression that \( P_1(\lambda) < P_2(\lambda) \). Thus, even though wages are higher in region 1 than in region 2, the price index in the larger region is lower than that in the smaller one. Hence, consumers residing in the larger region enjoy both higher wages and lower prices than those located in the smaller region.

Since the indirect utility of an individual living in region \( r \), which is equal to her real wage, is given by

\[
V_r(\lambda) = \frac{w_r(\lambda)}{P_r(\lambda)},
\]

\( V_1(\lambda) \) exceeds \( V_2(\lambda) \) if and only if \( \lambda > 1/2 \). Let \( \Delta V(\lambda) \equiv V_1(\lambda) - V_2(\lambda) \) be the interregional utility differential. Then, using Appendix 1, we obtain

\[
\frac{d\Delta V(\lambda)}{d\lambda} = \frac{\partial \Delta V(\lambda)}{\partial \lambda} + \frac{\partial \Delta V(\lambda)}{\partial w^*} \frac{dw^*}{d\lambda} > 0,
\]

which means that the utility differential increases with the size of the larger region. In other words, the incentive to move from region 2 to region 1 gets stronger as the larger region grows in size. It is worth stressing, however, that this incentive weakens as the two regional markets get more integrated, the reason being that the economic differences between regions fade away.

Thus, we have the following proposition.

**Proposition 1** Assume any given distribution of firms and consumers such that \( \lambda > 1/2 \). Then, the real wage in the larger region exceeds that in the smaller region. Furthermore, the interregional gap widens when regions become more asymmetric.

Because the local demand is higher in the larger region, firms located there can pay a higher wage to their workers, a result supported by robust empirical evidence (Head

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\(^4\)We show in the Appendix 1 that the denominator of (11) is positive.
and Mayer, 2011; Redding, 2011). Furthermore, since more varieties are produced in the larger region, the corresponding price index is lower, which also agrees with the empirical evidence provided by Handbury and Weinstein (2013) who observe that price level for food products falls with city size. Therefore, migration flows (if any) are unidirectional: consumers move from the smaller to the larger region but never from the larger to the smaller region.

As long as firms and consumers do not change location, technological progress makes all consumers equally better off because prices decrease at the same rate in both regions, while wages remain constant. In contrast, when technological progress leads to the relocation of some firms and consumers in the larger region, consumers residing there enjoyed a wage hike as well as a drop in the price index. Simultaneously, the price index in region 2 rises because fewer varieties

### 3 Spatial Equilibrium

The decision made by a consumer to migrate relies on the utility differential $\Delta V(\lambda)$ and the interregional migration cost $m > 0$. Because the equilibrium wage $w$ is uniquely determined by the wage equation (11), the interregional utility differential can be expressed as a function of $\lambda$:

$$\Delta V(\lambda) = \frac{\sigma - 1}{c_f \frac{1}{\sigma - 1} \frac{1}{\sigma - 1}} \left[ w \left( \phi - \lambda \phi + \lambda w^{1-\sigma} \right)^{\frac{1}{\sigma - 1}} - (1 - \lambda + \lambda w^{1-\sigma})^{\frac{1}{\sigma - 1}} \right]. \quad (14)$$

As argued in the Introduction, moving from one region to the other involves various psychological adjustments that adversely affect a migrant. Migration cost is not a one-time expense: if consumers migrate, they incur a permanent cost $m > 0$ to adjust to their new place. Therefore, measuring $m$ in utility terms, consumers choose to stay put if

$$|\Delta V(\lambda)| \leq m. \quad (15)$$

Otherwise, consumers migrate to the larger region where the utility level is higher than in the smaller region. Note that our approach can easily be extended to cope with consumers bearing different migration costs. In this case, consumers move by increasing order of migration costs instead of moving anonymously.

A spatial equilibrium is a consumer distribution $\lambda^* \in [0, 1]$ such that no consumer has an incentive to migrate away from the region where she is located.

Since $\Delta V(\lambda)$ increases with $\lambda$ and $\Delta V(1/2) = 0$, the equation $\Delta V(\lambda) = m$ as at most one solution $\bar{\lambda} > 1/2$. The function $\Delta V(\lambda)$ being point symmetric, $\Delta V(1/2 + x) = -\Delta V(1/2 - x)$, $1 - \bar{\lambda}$ is the solution to $\Delta V(\lambda) = -m$. If $|\Delta V(\lambda)| = m$ has no solution in $(1/2, 1)$, then migration costs are so high that no distribution exists that yields a positive utility gain net of migration costs. In other words, migration costs are large enough for any distribution to be a spatial equilibrium. From now on, we rule out this case by assuming that

$$\frac{\sigma - 1}{c_f \frac{1}{\sigma - 1} \frac{1}{\sigma - 1}} > m$$

for the equation $\Delta V(\lambda) = m$ to have a solution in $(1/2, 1)$.

As in Krugman (1991), two types of equilibria may arise. In the first one, $\lambda \in [1 - \bar{\lambda}, \bar{\lambda}]$ so that firms and consumers are partially dispersed. In this case, no consumers
migrate because the utility gains do not exceed their mobility cost, thus implying that any \( \lambda \in [1 - \bar{\lambda}, \bar{\lambda}] \) is a spatial equilibrium. The second type of equilibrium involves the agglomeration of activities in a single region: \( \lambda^* = 0, 1 \). When \( \lambda^* = 0 \), we get \( w^* = \phi^{1/\sigma} \), and thus \( \Delta V < 0 \); when \( \lambda^* = 1 \), we get \( w^* = \phi^{-1/\sigma} \), and thus \( \Delta V > 0 \). In either case, regardless of the values of the parameters of the economy no migration occurs. The reason for this result is the absence of immobile farmers who lead firms and consumers to leave the cluster when transport costs are high.

### 3.1 The set of stable spatial equilibria

When several equilibria exist, it is commonplace to use stability to discriminate between the different equilibria. This requires the specification of an adjustment process. To this end, we use the myopic evolutionary dynamics of NEG:\(^5\)

\[
\dot{\lambda} = \begin{cases} 
-\lambda(1 - \lambda)(\Delta V(\lambda) - m) & \text{for } 0 \leq \lambda < 1 - \bar{\lambda} \\
0 & \text{for } 1 - \bar{\lambda} \leq \lambda \leq \bar{\lambda} \\
\lambda(1 - \lambda)(\Delta V(\lambda) - m) & \text{for } \bar{\lambda} < \lambda \leq 1.
\end{cases}
\]  

Clearly, a spatial equilibrium is a steady-state of (16), while the utility differential \( \Delta V(\lambda) \) must exceed \( m \) for \( \lambda > 1/2 \) to increase.

A spatial equilibrium \( \lambda^* \) is said to be asymptotically stable when, for every marginal deviation from \( \lambda^* \), the adjustment process (16) leads the off-equilibrium consumers back to \( \lambda^* \). This stability concept is not appropriate here. Indeed, the change in the utility level triggered by the deviation is very small relative to the cost \( m \) the off-equilibrium consumers must bear to move to \( \lambda^* \). Therefore, these consumers choose to stay put, which implies that \( \lambda^* \) is unstable. As a consequence, we need a weaker concept of stability.

In what follows, we say that the equilibrium \( \lambda^* \) is **Lyapunov stable** for (16) if a distribution that starts out near the equilibrium \( \lambda^* \) stays near \( \lambda^* \) forever. The existence of positive migration costs implies that any distribution belonging to the interval \( (1 - \bar{\lambda}, \bar{\lambda}) \) are spatial equilibria and Lyapunov stable because this interval is an open set. In other words, **migration costs stabilizes a wide range of distributions of activities.** When the equality holds in (15), there exist two other equilibria given by \( \lambda^* = 1 - \bar{\lambda}, \bar{\lambda} \). However, both are Lyapunov unstable as shown by computing the derivative of the utility differential (13). These two equilibria bear some resemblance with the two asymmetric equilibria identified by Krugman (1991), which are unstable. Last, because the inequality \( \Delta V < 0 \) (\( \Delta V > 0 \)) when \( \lambda^* = 0 \) (\( \lambda^* = 1 \)) is strict, these two configurations are also Lyapunov stable equilibria.

To sum up, we present the next proposition.

**Proposition 2** *In the presence of migration costs, there exists a continuum of Lyapunov stable equilibria given by \((1 - \bar{\lambda}, \bar{\lambda})\) and \( \lambda^* = 0, 1 \).*

Note here the difference with Krugman’s model where the number of equilibria is always finite while the only stable equilibria involve full dispersion or full agglomeration.

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\(^5\)A myopic evolutionary dynamics is a good approximation when the discount rate is high, migration costs are large, or both. Things become very different, however, when workers care about their future and/or are very mobile. See Oyama (2009) for a stability analysis involving a forward-looking dynamics.
(Krugman, 1991; Robert-Nicoud, 2005). This is because migration costs act as a stabilizing force whose intensity is unaffected by the spatial distribution of activities. For example, the symmetric and agglomerated configurations on which NEG typically focuses are always stable equilibria, thus destroying the main prediction of NEG saying that dispersion (agglomeration) prevails when transport costs are high (low).\footnote{Introducing migration costs into Krugman’s setting generates a set of equilibria similar to the one described in Proposition 2.} The difference with Krugman’s results is thus striking and may suggest that no economic force is able to trigger the relocation of economic activities in the presence of migration costs. In Section 4, we show how technological progress in manufacturing can destabilize dispersed configurations, thus leading firms and consumers to gather in a single region that becomes the core of the economy.

### 3.2 Do transport costs matter?

In the presence of multiple stable equilibria, it is hard to predict which equilibrium emerges. A standard way out is to start from an arbitrary initial equilibrium $\lambda_0 \in (1 - \bar{\lambda}, \bar{\lambda})$ and to study its evolutionary path by changing steadily a key-parameter of the model. The standard thought experiment of NEG focuses on the impact of falling transport costs on the distribution of the manufacturing sector. In what follows, we thus assume that the economy starts with sufficiently high values of $\tau$ and study how the initial distribution $\lambda_0 \geq 1/2$ reacts to steady decreases in $\tau$.

When $\tau = 1$, $\Delta V(\lambda) = 0$ regardless of the value of $\lambda$. Therefore, by continuity, $\tau > 1$ exists such that $\Delta V < m$ for any $\lambda$ and all $1 < \tau < \bar{\tau}$. In other words, when transport costs are very low, the set of stable spatial equilibria encompasses the unit interval. But what happens when $\tau$ exceeds $\bar{\tau}$? To answer this question, we have to find how $\bar{\lambda}$ varies with $\tau$.

Figure 1 depicts the relationship between the equilibrium distributions and the level of transport costs. The interior of the shaded domain describes the continuum of dispersed equilibria satisfying (15), while the two bold horizontal lines describe the two agglomerated equilibria. As shown in Appendix 2, $\bar{\lambda}(\tau)$ increases when $\tau$ decreases when $\sigma \geq \bar{\sigma} \equiv 1 + 1/\sqrt{2} \approx 1.71$.\footnote{If $\sigma < \bar{\sigma}$, $\bar{\lambda}$ first decreases and then increases with falling the transport costs. In this case, $\lambda_0$ ceases to be a stable equilibrium at the first value of $\tau$ such that $\lambda = \bar{\lambda}(\tau)$. The equilibrium is given by $\lambda = 1$ for lower transport costs.} Since the empirical estimates of $\sigma$ are all much larger than $\bar{\sigma}$ (Head and Mayer, 2004), we may assume without much of generality that this condition holds. In this case, the interval $(1 - \bar{\lambda}, \bar{\lambda})$ expands as $\tau$ decreases. As a consequence, when the initial distribution $\lambda_0$ belongs to $(1 - \bar{\lambda}, \bar{\lambda})$ for some $\bar{\tau}$, this distribution remains a stable equilibrium for all $\tau < \bar{\tau}$. This is very different from the main finding of NEG where a steady decrease in $\tau$ always moves the economy from dispersion to agglomeration (Krugman, 1991; Fujita et al., 1999b).

Insert Figure 1 about here

The reason for this change in results may be explained as follows. Because differences between the interregional price and the wage gaps shrink when transport costs fall, the larger region becomes relatively less attractive. As a consequence, if the initial utility differential is not large enough to trigger consumers’ migration, this holds true even more
so when transport costs are lower because the cost-of-living difference has decreased. In addition, as illustrated by the shaded area of Figure 1, smaller transport costs allow sustaining a larger domain of spatial equilibria. In the limit, as said above, when \( \tau \) gets close to 1, the domain of spatial equilibria often encompasses the unit interval, which implies that any initial distribution of activities is a spatial equilibrium. To put it differently, when transport costs are positive but low enough, location no longere matters provided than the initial distribution is not too unbalanced. In sum, since Proposition 1 implies that no migration from the larger to the smaller region occurs, there is no force incentivizing consumers to migrate. Thus, contrary to the main prediction of NEG, we may conclude that the integration of regional markets does not necessarily trigger the agglomeration of the manufacturing sector.

By contrast, if \( \lambda_0 \) belongs to the upper (lower) non-shaded domain of Figure 1 while transport costs are high, the initial distribution is not a spatial equilibrium. If \( \lambda_0 > 1/2 \), Proposition 1 implies that the spatial equilibrium is unique and given by \( \lambda^* = 1 \). Indeed, owing to its size advantage, region 1 produces a much wider range of varieties than region 2, while high transport costs make these varieties much more expensive in region 2 than in region 1. As a consequence, the cost-of-living difference is large enough to trigger the relocation of consumers from region 2 to region 1. In this event, the larger region can be viewed as a “black hole” that accommodates the entire manufacturing sector (Fujita et al., 1999b). Note that a strong initial concentration of firms may stem from the uneven distribution of natural resources (e.g. coal or iron ore) needed to produce the manufactured good.

The foregoing results clash with what NEG tells us. Yet they are both intuitive and plausible. First, when one region is much bigger than the other, firms located in the latter are poorly protected against the import of a wide range of varieties produced in the former. Second, the price index in the smaller region is much higher than in the other, thus lowering the real income of the local consumers. Under such circumstances, being agglomerated allows firms to better exploit the internal scale economies that characterize their production, while the benefits accruing to the consumers compensate them for the migration costs they have to bear.

In contrast, when regions are not too different (\( \lambda_0 \) is not too high), in each region consumers have access to a fairly large number of locally produced varieties. In this event, the local market is sufficiently big to reduce the wage gap, while the additional benefit generated by better access to the entire range of varieties no longer compensates consumers in the smaller region for the migration costs they would have to bear to live in the larger region.

4 Does Technological Progress Foster the Agglomeration of Activities?

In this section, we turn our attention to the effect of a rising labor productivity and show that, regardless of its concrete form, a steadily rising labor productivity always brings about the agglomeration of the manufacturing sector. To avoid undue complexity, we assume that productivity gains stem from exogenous technological progress. Although both \( f \) and \( c \) are likely to be affected by technological progress, falling marginal and fixed requirements of labor do not have exactly the same impact on the economy. We thus
study each one separately. Since the literature often uses the marginal production cost as a proxy for firms’ productivity, we first study the impact of regular decreases in the marginal labor requirement $c$ on the distribution of firms and consumers.

**Falling marginal requirement of labor.** We consider a new thought experiment and show that a steady decrease in the marginal labor requirement $c$ has an impact that greatly differs from that generated by falling transport costs, which is described in Proposition 2. Figure 2, very much like Figure 1, shows how the spatial distribution of economic activities and the marginal labor requirement are related.

Let $\lambda_0$ be the initial distribution of economic activities. Since $\Delta V(\lambda)$ decreases with $c$, the equation $\Delta V(\lambda_0) = -\Delta V(1 - \lambda_0) = m$ has a unique solution $c_0$. The shaded domain describes the continuum of dispersed equilibria associated with any $c$ exceeding $c_0$. Note that the vertical distance between these two curves now increases with the marginal requirement $c$. Indeed, since $\partial \Delta V/\partial c > 0$, $\lambda$ decreases when $c$ falls. Since the spatial equilibria arising under $\lambda_0 \in [0, 1/2)$ are the mirror images of those arising under $\lambda_0 \in (1/2, 1]$, we assume without loss of generality that region 1 accommodates a priori a larger number of firms and consumers than region 2: $\lambda_0 \in (1/2, 1]$.

**Proposition 3** Assume that the marginal labor requirement falls steadily. Then, for any initial distribution of activities $\lambda_0 \in (1/2, \bar{\lambda})$, there exists a threshold $c_0$ such that (i) $\lambda_0$ is a Lyapunov stable spatial equilibrium for all $c > c_0$; and (ii) $\lambda^* = 1$ is a Lyapunov stable spatial equilibrium for all $c \leq c_0$.

This is reminiscent of Krugman’s (1991) core-periphery structure: the evolutionary process involves, first, the dispersion of economic activities and, then, their sudden agglomeration. However, there is a significant difference: our thought experiment is about a falling marginal labor requirement $c$ instead of a falling transport cost $\tau$.

The reason for Proposition 3 is easy to grasp. When $c$ falls, the following three effects are at work. First, because $\lambda_0$ exceeds $1/2$, it follows from Proposition 1 that the nominal wage is higher in region 1 than in region 2. As long as $\lambda = \lambda_0$, (11) implies that a decreasing marginal labor requirement does not affect the equilibrium wage $w^*$. By contrast, when
\[ \lambda \text{ starts rising above } \lambda_0, \]  
(11) shows that the nominal wage in region 1 also rises. Second, 
when \( \lambda = \lambda_0, \) (5) shows that a fall in \( c \) translates into a lower equilibrium price for the 
existing varieties, regardless of where they are produced. However, when \( \lambda \) starts rising 
above \( \lambda_0, \) the wage paid in region 1 also increases, which may result in a price hike in 
region 1. Nevertheless, (5) implies that the equilibrium wage \( w^* \) rises faster than the 
equilibrium price \( p_1^*. \)

Third, and last, the productivity hike implies that fewer workers are needed to produce 
the existing varieties. Although the equilibrium output \( q_r^* \) increases with falling \( c \) from 
(7), every firm hires the same number of workers to produce its larger output because 
\( cq_r^* + f \) is independent of \( c. \) This implies that the total number of varieties does not 
change, as also shown by (9). As a consequence, when \( c \) falls, \( 1/P_1 - 1/P_2 \) rises, and 
thus the indirect utility differential \( \Delta V(\lambda_0) \) increases. As long as \( \Delta V(\lambda_0) \) remains smaller 
than the migration cost \( m, \) no region 2’s consumer moves \( (\lambda = \lambda_0), \) but all consumers are 
better off because of the price drop and the production hike. Note that the number of 
varieties remains the same.

Once \( c \) falls below the threshold \( c_0, \) \( \Delta V(\lambda_0) \) exceeds the migration cost \( m \) and a 
few consumers living in the smaller region move to the larger one. As a consequence, 
more (fewer) varieties are produced in region 1 (2). Since \( w^* \) rises faster than \( p_1^* \), \( w_r^*/P_1^* \) 
increases at a higher rate than \( 1/P_2^*. \) Therefore, the difference \( \Delta V(\lambda) - \Delta V(\lambda_0) \) gets bigger 
when \( c \) falls. Thus, as in Myrdal (1957) and Krugman (1991), the interplay between these 
various effects generates the cumulative causality that feeds the migration process until all 
consumers are agglomerated in region 1, and so even when \( c < c_0 \) has stopped decreasing.

Observe that \( c_0 \) decreases with migration costs but increases with transport costs. 
Therefore, lowering mobility costs of goods and people gives rise to opposite effects on 
the location of economic activity.

Falling fixed requirement of labor. Consider now a fall in the fixed requirement 
of labor. As shown by (5), the price of existing varieties is unaffected. Even though a 
firm’s output \( q_r^* \) increases with falling \( f, \) the number of firms and varieties in each region 
increases from (9). In other words, the productivity hike implies that some workers are 
freed from producing the existing varieties. Since their number is greater in region 1 than 
in region 2, a larger number of new varieties are launched in region 1 than in region 2, 
which implies that \( 1/P_1 - 1/P_2 \) increases with falling \( f. \) In this case, the total number 
of varieties produced in the economy increases, but it does so more in region 1 than in 
region 2.

Because \( \Delta V(\lambda_0) \) is decreasing in \( f, \) the equation \( \Delta V(\lambda_0) = m \) has a single solution, 
which is denoted \( f_0. \) Applying the argument used to prove Proposition 3, we obtain the 
following result.

**Proposition 4** Assume that the fixed labor requirement falls steadily. Then, for any 
initial distribution of activities \( \lambda_0 \in (1/2, \lambda), \) there exists a threshold \( f_0 \) such that (i) \( \lambda_0 \) is 
a Lyapunov stable spatial equilibrium for all \( f > f_0; \) and (ii) \( \lambda^* = 1 \) is a Lyapunov stable 
spatial equilibrium for all \( f \leq f_0. \)

Although falling marginal and fixed labor requirements are not totally congruent in 
terms of their effects on the economy, the above two propositions have a clear implication: a steady flow of labor-saving innovations brings about a transition from a (partially)
dispersed configuration of activities to an agglomerated one. Hence, when we disregard
the problematic existence of immobile farmers whose role is to hold back industrial work-
ers living in the less prosperous region, the effects of a rising labor productivity are in
sharp contrast to those generated by falling transport costs. More precisely, a growing
labor productivity widens the interregional utility differential, which eventually outweig
migration costs and generates interregional migration. In contrast, steady drops in trans-
port costs reduce the interregional utility differential and keep the distribution of activities
unaltered.

5 Heterogeneous Labor

The assumption of identical workers is a very strong one. In this section, we assume that
an $e$-type worker is endowed with $e > 0$ efficiency units of labor, while individual types
are distributed according to the continuous density function $g_r(e) > 0$ defined over $(0, \infty)$
with a unit mass. Observe that density functions are not necessarily the same between
regions ($g_1 \neq g_2$). When labor is heterogeneous, the distributions of $e$-type workers now
matters to define the productive size of a region. In particular, we say that region 1 is
said to be more productive than region 2 if the total number of efficiency units of labor
available in the former exceeds that in the latter, that is, $E_1 > E_2$. This is not equivalent
to assuming that region 1 is larger ($\lambda_0 > 1/2$) because a much higher number of inefficient
workers may be located in region 2 than in region 1. The initial regional labor supply
functions are given by

$$E_1 = \lambda_0 \int_0^\infty e g_1(e) \, de \quad E_2 = (1 - \lambda_0) \int_0^\infty e g_2(e) \, de.$$  \hspace{1cm} (17)

Since $f$ is expressed in efficiency units of labor, labor market clearing implies $E_r = f n_r$
for $r = 1, 2$.

For any given initial distribution $E_1 > E_2$, let $w_r$ be the price of one efficiency unit
of labor in region $r$, so that the income of an $e$-type individual is equal to $ew_r$. While $e$
varies across individual types according to $g_r(e)$, the variables $w_r$ and $P_r$ are common to
all individuals residing in region $r$. Therefore, the indirect utility of an $e$-type worker is
given by

$$V_r = \frac{ew_r}{P_r},$$  \hspace{1cm} (18)

which increases linearly with $e$. Since a region endowed with $E$ efficiency units of labor
is equivalent to a region endowed with $E$ workers having the same productivity, we can
call on Proposition 1 to assert that $w_1 > w_2$ and $P_1 < P_2$. Thus, the interregional utility
differential

$$\Delta V(e) = V_1 - V_2 = e \left( \frac{w_1}{P_1} - \frac{w_2}{P_2} \right).$$  \hspace{1cm} (19)

is always positive and increasing in $e$.

Since $\Delta V(e)$ becomes arbitrarily large with $e$, the utility differential of the workers
endowed with an arbitrarily large number of efficiency units of labor always exceeds their
migration cost. As a consequence, region 2’s most productive workers choose to migrate
to region 1. But how many workers in region 2 want to migrate?
Let $e^* \in (0, \infty)$ be the marginal worker who is indifferent between moving to the more productive region or staying put in the less productive one. Since the number of migrants is equal to
\[ (1 - \lambda_0) \int_{e^*}^{\infty} g_2(e) \, de, \]
the equilibrium number of workers residing in region 1 is given by
\[ \lambda^* = \lambda_0 + (1 - \lambda_0) \int_{e^*}^{\infty} g_2(e) \, de > \lambda_0. \quad (20) \]

In this event, the equilibrium regional labor supply functions are given by
\[ E_1(e^*) = \lambda_0 \int_0^{\infty} eg_1(e) \, de + (1 - \lambda_0) \int_{e^*}^{\infty} eg_2(e) \, de \quad E_2(e^*) = (1 - \lambda_0) \int_0^{e^*} eg_2(e) \, de \]
while the wage equation (11) becomes
\[ \frac{E_1(e^*)}{E_2(e^*)} = \frac{w^{\sigma-1} (w^\sigma - \phi)}{1 - \phi w^\sigma}. \quad (21) \]

Clearly, the left-hand side of this expression decreases with $e^*$, whereas the right-hand side increases with $w$. The implicit function theorem thus implies that (21) has a unique solution $w = w(e^*)$ while $w'(e^*) < 0$ for all $e^* \in (0, \infty)$. In other words, when the number of workers in region 1 increases, the price of one efficiency unit of labor increases.

The expression (20) implies that there is a one-to-one correspondence between $e^*$ and $\lambda^*$. As a consequence, the utility differential may be written as a function of $e^*$ only. An interior equilibrium $e^*$ is then determined by the solution to the spatial equilibrium condition:
\[ \Delta V(e^*, w(e^*)) = e^* \left( \frac{w_1^*(e^*)}{P_1^*(e^*)} - \frac{w_2^*(e^*)}{P_2^*(e^*)} \right) = m. \quad (22) \]

Unlike (19), both the wages and price indices in (22) now depend on $e^*$. Set
\[ h(e) \equiv \Delta V(e, w(e)) - m. \quad (23) \]
We have $h(0) = -m < 0$ and $h(\infty) = \infty > 0$. Hence, there exists an equilibrium $e = e^*$ where $h'(e^*) > 0$. This inequality implies that $e^*$ is stable because $e^*$ decreases with $\lambda^*$. Since $h(e) = 0$ has a finite number of solutions, labor heterogeneity plays the role of an equilibrium refinement. Indeed, we know from Proposition 2 that there is a continuum of stable equilibria when labor in homogeneous, whereas we have a finite number of equilibria under heterogeneous labor.

We can repeat the analysis of Section 4 and show that the equilibrium price $w^*$ of one efficiency unit of labor rises when $c$ decreases. Similarly, the price index difference $1/P_1(e) - 1/P_2(e)$ increases when $c$ falls. As a consequence, $\Delta V(e, w(e))$ increases when $c$ decreases. In other words, $h(e)$ is shifted upward, which implies that $e^*$ decreases when $c$ falls. Note that the decrease in $e^*$ is not necessarily continuous. Indeed, if there are multiple stable equilibria, some of them may disappear as $c$ falls. In this case, the economy jumps to another stable equilibrium having a larger number of workers in region 1 because this region is more attractive. However, when there is a unique stable equilibrium, $e^*$ gradually decreases when $c$ steadily decreases.

Falling fixed requirements $f$ yield the same qualitative result. Thus, we have the following result.
Proposition 5 Assume that $E_1 > E_2$. If the marginal or fixed labor requirement steadily decreases, the number of individuals residing in region 1 monotonically increases by attracting workers whose productive efficiency decreases.

Hence, the skilled workers living in less efficient areas move toward more efficient areas. Through the migration of skilled workers, the economy may end up with one large and prosperous region, while the other gets smaller and relatively poorer, confirming the observations made by both Pollard (1981) and Moretti (2012) who focus on different periods and different countries. We may thus conclude that Proposition 5 highlights a fundamental trend of the evolution of the space-economy. Moreover, the interregional income gap is strengthened when more productive individuals exhibit lower migration costs.

What is more, a growing number of empirical contributions show that the concentration of firms and workers increases their productivity through various channels gathered under the term “agglomeration economies” (Rosenthal and Strange, 2004; Ellison et al., 2010; Jofre-Monseny et al., 2011; Combes et al., 2012b). In this event, we normally expect $c$ and/or $f$ to decrease faster in region 1 than region 2. This raises the relative attractiveness of the core, thus generating new migrant flows and exacerbating the process of divergence.

As discussed in the Introduction, the spatial concentration of skilled workers seems to be a major trend in many industrialized countries where the large extent of regional disparities reflects the unequal distribution of skills across space. Proposition 5 provides a rationale for this fact. However, empirical evidence also suggests that large and prosperous cities are also characterized by a growing skill and income polarization of their population (Berry and Glaeser, 2005). In our setting, this can be explained by the fact that the core region hosts both skilled and unskilled workers, i.e. the skilled from regions 1 and 2 as well as the unskilled from region 1, whereas the peripheral region accommodates only unskilled workers.

Last, Proposition 5 holds true when (i) consumers can be ranked according to a one-dimensional type $t$ that embodies different forms of heterogeneity across individuals (e.g. productivity and mobility) and (ii) the indirect utility of a type $t$-consumer residing in region $r$ is given by $V_r(t) = A(t)w_r/P_r$.

6 Conclusion

In this paper, we have proposed a new explanation for the emergence of a core-periphery structure, which is based only upon technological progress in the manufacturing sector. Given the dramatic labor productivity growth observed since the beginning of the Industrial Revolution, we find this explanation both plausible and relevant. Therefore, the prime mover responsible for the emergence of a core-periphery structure would be technological innovation in the manufacturing sector rather than in the transportation sector. In other words, falling production costs take the place of falling transport costs as the main explanation for the persistence of an uneven distribution of activities across space.

We would be the last, however, to claim that market integration does not play any role. Quite the opposite, we believe that market integration has been, and still is, one of the main drivers shaping the space-economy. For example, it is well documented that the commercial revolution in the 17th century, which has been facilitated by a large
number of improvements in transportation techniques, went with the relocation of textile production. Likewise, larger and integrated markets make R&D more profitable and lead to more invention. To a large extent, explaining the spatial pattern of production in various countries requires combining technological progress and market integration.

In contrast, we do not believe that the existence of immobile farmers explains the existence of dispersed patterns of activities. It is common in NEG to work with a setting in which farmers’ wages are equalized across space; this is guaranteed by the assumption of zero transport cost for the agricultural good. As argued by Davis (1998), it is hard to see why trading the agricultural good is costless in a model seeking to ascertain the overall impact of transport costs on the location of economic activity. Migration is often governed by push and pull effects that greatly restrict individual choices. Therefore, the existence of significant and continuing migration costs strikes us as a more sensible dispersion force to take into account in the system of forces that determines the economic landscape.

We have shown that, once labor productivity has increased sufficiently, the interplay between the agglomeration and dispersion forces triggers the (partial) concentration of activities. However, there is no reason to expect the resulting pattern of activities to prevail forever. Indeed, we have assumed in the foregoing sections that technological progress affected all regions equally. It is reasonable, however, to believe that labor requirement declines at different rates in various regions. In this case, even when region 1 is the core of the economy, a reversal of fortune becomes possible if region 2 experiences a stronger wave of innovations. In this event, the peripheral region or country is able to throw off its history (Landes, 1998). Such a redrawing of the map of economic activities is difficult to obtain in standard NEG models.

Note that our results may be reinterpreted in terms of population growth rather than technological progress. Since \( f \) is the per capita fixed cost, an increase in total population amounts to a decrease in \( f \). Therefore, Proposition 4 holds true. In other words, a growing population widens the interregional utility differential and, eventually, triggers consumers’ migration toward the larger region. Population growth is thus a new agglomeration force.

Our model, owing to its extreme flexibility, can be extended in several directions. First, once it is recognized that innovation follows a large variety of trajectories across industries, our approach should allow explaining why different industries display contrasted location patterns (Duranton and Overman, 2005). Second, while Krugman’s core-periphery model can hardly cope with an arbitrary number of regions, we may expect our simpler setting to permit such a generalization. To rule out trivial equilibria, we have to assume that regions are endowed with specific comparative advantage while consumers are heterogeneous (Tabuchi and Thisse, 2002). Third, Krugman’s core-periphery model is hard to generalize to non-CES preferences that capture the pro-competitive price effect generated by a larger market. By contrast, our results are likely to hold true in alternative settings, such as those involving quadratic preferences or additive utilities (Ottaviano et al., 2002; Zhelobodko et al., 2012). For our main results to hold, we need only the following two intuitive conditions: \( dV/d\lambda > 0 \) and \( \partial V/\partial c < 0 \).

Last, the model could also be extended to account for the internal functioning of regions, which do not often grow at the same pace. This could be done by introducing different microeconomic mechanisms that generate agglomeration (dis)economies, such as

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Footnote 8: Fujita, et al. (1999a) also study the impact of population growth in an NEG model. However, owing to the existence of farmers, population growth acts as a dispersion force in their model, whereas it acts as an agglomeration force in ours.
those analyzed by Duranton and Puga (2004). In such a context, it would be natural to focus on endogenous technological progress, which is often place-specific, and to add a housing sector to the model. Hopefully, such a microscopic extension of our macroscopic model would find out why some regions fare better than others. This will also pave the way for a deeper study of how innovation and urbanization interact.

References


Appendix 1

1. The denominator of (11) is positive because

\[ w^\sigma - \phi (w + 1) + w^{1-\sigma} > w^\sigma - (w + 1) + w^{1-\sigma} (w^\sigma - 1) (w^{\sigma-1} - 1) \geq 0 \]

for all \( \phi \in [0, 1) \).

2. Differentiating (11) with respect to \( w \), we get

\[
\frac{d\lambda}{dw} = \frac{H(w^\sigma)}{w^\sigma [w^\sigma - \phi (w + 1) + w^{1-\sigma}]^2}
\]

where

\[ H(w^\sigma) \equiv -(\sigma - 1) \phi w^{2\sigma} + (2\sigma - 1 - \phi^2) w^\sigma - (\sigma - 1) \phi. \]

Computing \( H \) at \( w^\sigma = 1 \) and \( w^\sigma = 1/\phi \) yields

\[ H(1) = (2\sigma - 1 + \phi)(1 - \phi) > 0 \quad \text{and} \quad H(1/\phi) = \sigma \frac{1 - \phi^2}{\phi} > 0. \]

Since \( H(w^\sigma) \) is concave, it must be that \( H(w^\sigma) > 0 \) over the interval \([1, 1/\phi]\). Therefore, \( d\lambda/dw > 0 \) for all \( w^\sigma \in [1, 1/\phi] \).

Appendix 2

We show that \( d\tilde{\lambda}(\tau)/d\tau < 0 \) or, equivalently, \( d\tilde{\lambda}(\phi)/d\phi > 0 \) over \((0, 1)\) for all \( \sigma \geq \bar{\sigma} \). The variable \( \lambda(\tau) \in (1/2, 1) \) must satisfy the following two equilibrium conditions:

\[
\begin{align*}
F_1(\lambda, w) &\equiv w^\sigma - \phi - [w^\sigma - (w + 1) \phi + w^{1-\sigma}] \lambda = 0 \quad (25) \\
F_2(\lambda, w) &\equiv \Delta V(\lambda) - m = 0.
\end{align*}
\]

It is readily verified from comparative statics that

\[
\frac{d\tilde{\lambda}}{d\phi} = \frac{\partial F_1(\lambda, w)}{\partial \phi} \frac{\partial F_2(\lambda, w)}{\partial \phi} - \frac{\partial F_2(\lambda, w)}{\partial \lambda} \frac{\partial F_1(\lambda, w)}{\partial \lambda}
\]

where \( \lambda \) and \( w \) solve (25) and (26).

The denominator of (27) is negative from (13). Plugging (11) into the numerator of (27), we get

\[ G(W) \equiv G_1(W) [G_2(W) - G_3(W)] \]

where \( W \equiv w^\sigma \in (1, 1/\phi) \) while

\[
\begin{align*}
G_1(W) &\equiv (1 - \phi^2) \frac{2}{\sigma - 1} W^{\frac{2}{\sigma - 1}} \frac{\sigma + 1}{\sigma - 1} \geq 0 \\
G_2(W) &\equiv W^{\frac{2}{\sigma - 1}} (1 - \phi W) [2\sigma - 1 - \phi^2 - 2(\sigma - 1) \phi W] > 0 \\
G_3(W) &\equiv (W - \phi) [(2\sigma - 1 - \phi^2) W - 2(\sigma - 1) \phi] > 0.
\end{align*}
\]
Thus, $G(W)$ is positive if and only if $G_2(W) - G_3(W) > 0$. Since $G_2$ and $G_3$ are positive, the sign of $G_2(W) - G_3(W)$ is the same as the sign of

$$G_4(W) \equiv \log G_2(W) - \log G_3(W).$$

Differentiating this expression yields

$$G'_4(W) = G_5(W) G_6(W)$$

where

$$G_5(W) \equiv \frac{2 \sigma - 1}{(\sigma - 1) W^{\frac{\sigma^2 - 3 \sigma + 3}{\sigma - 1}} G_2(W) G_3(W)}$$

is positive, while

$$G_6(W) \equiv 2(\sigma - 1) \phi^2 (2\sigma - 1 - \phi^2) W^4 + \phi (4\sigma - 3 - \phi^2) [\sigma^2 - 3\sigma + 1 - (\sigma^2 + \sigma - 3) \phi^2] W^3$$

$$- [4\sigma^3 - 10\sigma^2 + 6\sigma - 1 - (2\sigma - 1) (10\sigma - 11) \phi^2 - (4\sigma^3 - 6\sigma^2 - 10\sigma + 11) \phi^4 - \phi^6] W^2 + \phi (4\sigma - 3 - \phi^2) [\sigma^2 - 3\sigma + 1 - (\sigma^2 + \sigma - 3) \phi^2] W + 2\phi^2 (\sigma - 1) (2\sigma - 1 - \phi^2)$$

is negative as shown by studying the derivatives of this function.

(i) Since $G_6'''(W) \geq 0$, $G_6''(W)$ is increasing over $(1, \phi^-)$.

(ii) We have

$$G_6''(\phi^-) = 6\phi (1 - \phi^2) [4\sigma^3 + \sigma^2 - 11\sigma + 5 - (\sigma^2 + \sigma - 3) \phi^2]$$

$$\geq 6\phi (1 - \phi^2) [4\sigma^3 + \sigma^2 - 11\sigma + 5 - (\sigma^2 + \sigma - 3)]$$

$$= 24\phi (1 - \phi^2) (\sigma - 1)^2 (\sigma + 2)$$

$$\geq 0$$

where the first inequality holds because $\sigma^2 + \sigma - 3 > 0$ for all $\sigma \geq \bar{\sigma}$.

(iii) We have

$$G_6'' (\phi^-) = 2 (1 - \phi^2) [8\sigma^3 - 11\sigma^2 - 3\sigma + 4 - (4\sigma^3 - 3\sigma^2 - 7\sigma + 3) \phi^2 - \phi^4]$$

$$\geq 2 (1 - \phi^2) [8\sigma^3 - 11\sigma^2 - 3\sigma + 4 - (4\sigma^3 - 3\sigma^2 - 7\sigma + 3) - 1]$$

$$= 8\phi \sigma (1 - \phi^2) (\sigma - 1)^2$$

$$\geq 0$$

where the first inequality follows from $4\sigma^3 - 3\sigma^2 - 7\sigma + 3 > 0$ for all $\sigma \geq \bar{\sigma}$.

(iv) The signs of $G_6'''$ (1) and $G_6''(1)$ are indeterminate. However, if $G_6''(1) \geq 0$, then $G_6'''(1) \geq 0$ for all $\sigma \geq \bar{\sigma}$. Two subcases may arise.

(iv-a) If $G_6''(1) \geq 0$, then $G_6''(1) \geq 0$. Since $G_6''(W)$ is increasing, $G_6''(W) \geq 0$ always holds. Since $G_6''(1) \geq 0$, $G_6'(W) \geq 0$ always holds too, i.e., $G_6'(W)$ is increasing.

(iv-b) If $G_6''(1) < 0$, then $G_6''(1)$ is indeterminate. However, since $G_6''(W)$ is increasing and $G_6''(\phi^-) \geq 0$, it must be that $G_6''(W) < 0$ for small $W$, and then $G_6''(W) \geq 0$ for large $W$, i.e., $G_6'(W)$ is U-shaped.

(v) We have

$$G_6'(1) = -2(1 - \phi^3) (2\sigma - 1 + \phi) \left[2 \left(\sigma - \bar{\sigma} + \sqrt{2}\right) (\sigma - \bar{\sigma}) + 2 (\sigma^2 - 1) \phi + \phi^2\right]$$

$$\leq -2 (1 - \phi^3) (2\sigma - 1 + \phi) 2 \left(\sigma - \bar{\sigma} + \sqrt{2}\right) (\sigma - \bar{\sigma})$$

$$\leq 0$$
where the second inequality holds if and only if $\sigma \geq \sigma$. Since $G'_6(W)$ is either increasing or U-shaped from (iv-a) and (iv-b), it must be that $G_6(W)$ is either decreasing or U-shaped.

We have

$$G_6(1) = \frac{1}{2}G'_6(1) \leq 0$$
$$G_6(\phi^{-1}) = -\frac{(1 - \phi^2)^3}{\phi^2} (\sigma - 1) < 0.$$ 

Thus, $G_6(W) < 0$ for all $W \in (1, \phi^{-1}]$.

(vi) Since $\text{sgn}G_6(W) = \text{sgn}G'_4(W)$ and $G_4(1) = 0$, we get $G_4(W) < 0$ for all $W \in (1, \phi^{-1}]$, which implies $d\lambda(\phi)/d\phi > 0$. 


Figure 1: Stable equilibria for $\tau$ with $\sigma=3$, $c=1$, $m=1$, and $f=1/100$

Figure 2: Stable equilibria for $c$ with $\sigma=3$, $\phi=1/2$, $m=1$, and $f=1/50$