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The City as a Small Open Economy

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Abstract

We set out a city as a price-taking exporter and importer with its own local structure (housing (land per household), a local non-traded good and a local pure public good). We improve labor efficiency in the export sector, observe a jump in the local wage, and trace the impacts. For the "traditional" case, the population, output of the public good, and residential density expand (the law of urban growth) and in another, population, output of the public good and density contract.

- key words: small open city; law of urban growth

1 Introduction

We treat a city as a trading entity, a small, open economy living off exports so to speak. Early urban economists spoke of a city's basic activity (exporting key commodities) and its non-basic activity (local retail and production for the local market). Our model of a city as trader, below, will comprise the city's export activity, its residential activity, its government activity and its production of a local or non-traded good. Of central interest is how our city responds to a productivity improvement for labor in its export sector. We also take up the response of our city to an exogenous improvement in its climate amenity. The labor shift experiment for our model yields results that capture "the law of urban growth": the city becomes larger in area and population; residential density and the wage rise. There is a curious exception we observe, an exception turning on how significant the government good is to a representative household. Our amenity shift experiment provides support for the view that amenity improvement is capitalized primarily in local "house value" (site rent).

2 Commodity Trade of Large Cities in the United States

First we look at some recent information on US cities as trading entities, information compiled by the Brookings Institution in Washington, D.C. (Tomer, Puentes and Kane (2013)). The new data come in twelve categories (five commodities associated with "Advanced Industries" and seven associated with non-advanced industries). The typical observation is aggregate flow in dollars of a commodity INTO AND OUT OF a city. The Brookings-JP Morgan "Global Cities Initiative" establishes "that the 100 largest American metropolitan areas trade the greatest volume of and the most valuable goods. Products leaving these areas are worth more than the products entering them... every metro area has trade surpluses in at least one commodity and almost all in more than one." (Tomer, Puentes and Kane (2013; p 1.)). The 100 most active trading cities trade commodities with an average value of \$1563 per ton, about 40% above the average value of all traded goods in the US, while non-metropolitan regions trade commodities worth \$680 per ton (p. 7). The 100 large-trade cities dominate American international trade as well (some 63 percent of total foreign trade). Goods traded internationally tend to be in the high value per ton category (precision instruments, electronics, machinery/tools, and transportation equipment). New York, Houston and San Jose are in the top five of cities doing international trade, by weight. Jackson, MS, Honolulu and Tulsa are not huge trading cities *per se* but their trade is oriented mostly to international markets (p. 12).

We turn to the top twenty five cities ranked by the value of their domestic and international trade. This set is not quite the same as the top twenty five cities by value of production (local GDP (gross domestic product) but there is much overlap in these two lists (the top 25 commodity traders have local GDP's that sum to 49% of the national GDP for 2010). Drawing on tables in the Brookings report, we linked large trade flows by commodity type to these top trading cities in Table A. Table A has the twenty-five largest trading cities by value of total commodity flows (in plus out flows) listed in descending order, on the left side. Across the top are first 5 "advanced" commodities from left

to right followed by 7 "non-advanced" commodities. Entries in the table are total trade flow values per commodity and city.¹ We observe a pattern. The largest traders are fairly diversified across commodity types in their trade but tend to be large traders in advanced industrial activities (the five noted above). The medium sized large trading cities are also quite diversified but tend not to exhibit concentrations in particular commodities. The smaller large traders tend to be specialized in certain non-advanced commodities. See Table 1 for details. The Brookings people have not developed trade flows for services such as government, advertising, legal work and medicine. Almost every city has a market area for such services and certain key large cities have large market areas.² Thus the portrait of cities as traders and specialists that we have in Table A is somewhat partial but it still tells an important story. Cities tend to be very large traders when measured against their aggregate annual product and the largest traders are the largest cities (only six had a commodity trade value lower than the value of their GDPs for 2010). The largest traders tend to dominate the trade in advanced commodities.

Since larger cities are overall larger traders, we expect that the larger cities will have more entries for large trades in various commodities. This is what we observe. Larger traders have more entries for more commodity groups, with a tilt in favor of large trades in the five "advanced" commodities. Secondly cities down the large trade list will by definition be doing less trade and hence we expect

¹The flow values entered are taken from two tables in Tomer, Puentes and Kane (2013) based on large flows BY COMMODITY, with the relevant city noted. One table was for the 5 advanced commodities and the other table was for the 7 non-advanced commodities.

²"How do metropolitan areas offset trade deficits in particular commodities? Some metro areas use their surpluses in other traded goods to pay for these products, but they also use tradable services. These include a range of occupations, from financiers and product designers to hotel managers and professional athletes. When service firms and workers sell their products to outside markets, they bring new income to a metropolitan area. The United States already runs an international surplus in tradable services – primarily driven by services exported from the 100 largest metropolitan areas – and there is every reason to believe the same is occurring on the domestic side. These services help fund inflows, and in some places may be enough to offset local goods deficits. Unfortunately there are no metro measures for services trade." (Tomer, Puentes and Kane (2013; p. 17)).

"Washington's service-based economy depends on global goods trade to operate, even though it primarily functions as a consumer rather than a producer. In stark contrast to industrial hubs like Houston and Portland, Washington does not have many surplus goods to distribute beyond its borders, except waste. However, that deficit should not diminish the metro area's importance in global value chains, as it adds value in ways beyond physical production through its many service activities." (Tomer, Puentes and Kane (2013; p. 19))

them to register smaller entries. Thirdly, the largest overall traders are generally the largest cities by population and GDP and we hypothesize they will be most diversified in industrial structure. This we observe. A city higher up on the list is typically registering large trades in more than one commodity. And Miami, San Francisco, Riverside and Washington are sufficiently diversified that they exhibit no concentration in trade in a particular commodity. These are middle-weight trading cities. We want to qualify this here. Three of these cities are relatively large providers of services to a region and their exports of these services (legal services, advertising services, government services, etc.) are not counted in the commodity categories *a priori* (we noted this above). Nevertheless for Miami, San Francisco and Riverside we can infer that they are quite diversified in their trading. Fourthly, when we get to the smaller "large traders" down our list, we observe a quite regular pattern of "large trade" in a commodity, generally one from the non-advanced list (the remaining seven of the twelve categories). We infer that these cities are less diversified in their trade and less diversified in their industrial structures and are also likely not large providers of services such as advertising, finance, government, etc. Smaller "large traders" are also, we hypothesize, less diversified in their industrial structures. Observe that the large "large traders" and the small "large traders" tend to both have a concentration in "mixed freight", essentially warehousing, whole-saling, logistics and trans-shipment activity. These cities are commodity flow nodes. Roughly speaking, Table A has most entries along the left to right diagonal. This patterns would be more stark if we excised a few outlying data points (dropped outliers) from the top right and lower left parts of Table A.

Table A: Large-trade US Cities

and Large Flows of Selected Commodities Traded (billion 2010 dollars)

	chm*	tran	elec	mach	pr-in	enr	tex	mxF	met	agr	furn	wd
NYC	28.0				6.4		13.5					
LA	3.9	22.6	18.4	15.6	10.6		13.5	13.4			2.8	
Chi	31.4	5.7	4.2	6.7				5.4	12.1			
Hou	33.8	4.4		5.4								
Dal	6.8			6.9				7.1			2.1	
Phi	6.0			4.5				3.7				
Atl									2.8			
Det		30.2										
SF												
Bos			4.5		7.1							
Min				8.6	13.4							
SJ			11.8		7.8							
Phx	6.0		4.7					2.4				
StL		4.8				2.7						
Sea		9.5										
Mem	32.2			3.3	6.4			3.9		2.0		
Col	7.5	4.5										
Cin		4.2		10.2				5.7				
Mia												
Was												
Rvs												
Bal								3.0				
KC							3.3	3.6				
Ind								3.8				2.2
Clv				7.8					2.7			

*chm (chemicals/plastics); tran (transportation equipment); elec (electronics); mach (machinery/tools); pr-in (precision instruments); enr (energy products); tex (textiles); mxF (mixed freight); met (metals); agr (agricultural products); furn (furniture); wd (wood products).

Column-wise, only Memphis and Cleveland are "large traders" with a specialization in their own particular commodity (agricultural products for Memphis and wood products for Cleveland). An anomaly is that Houston has no entry for energy. New Orleans and Tulsa are major energy product traders but are not in the list of twenty-five "large traders".

3 A Trading City

We turn to a model of a small, open city exporting q_x at world prices and importing q_c in return, and we report on the city's response to an improvement in labor efficiency in the export sector. The higher wage associated with the efficiency improvement goes along typically with a city that is larger and more densely settled (the law of urban growth; more detail below). We observe this for a city with a sector producing housing, an additional non-traded good (haircuts), as well as a local public good (a government sector that we require to be small as defined below³). Equilibrium size for our city turns on within-city mobility costs incurred by a household, costs that rise with city size for each household. A household moving into our city faces a locally fixed level of utility (the so-called open city assumption). Distinctly urban about our small open economy is labor supply turning on the local utility level facing a household rather than the local wage and the within-city mobility costs a representative household cannot avoid.

Empirical work has established that larger cities in many places including the United States are observed to have a higher productivity for labor, controlling for a worker's education and experience, as well as higher wages and average housing prices (Ciccone and Hall (1996), Glaeser and Mare (2001) and Van Nieuwerburgh and Weill (2010)). Settlement density is higher in the larger cities as well. Ciccone and Hall estimated labor productivity in various counties in the United States as a function of the density of local residents. They controlled for worker education and local infrastructure abundance. They observed strong gains in worker productivity with increased local density. Glaeser and Mare present the standard evidence for this "law of urban growth": a positive coefficient on city size in estimations of Mincer equations. The basic Mincer equation "explains" a worker's wage as a weighted combination of her educa-

³With a "large" government sector producing a local public good, we observe a quite general violation of the law of urban growth (we see a smaller, less dense city; one with the size of the public sector shrinking with the wage increase). The scenario with the public sector shrinking as the city-wide wage increases echoes somewhat Baumol's (1967) argument that technical progress in the private sector of a city raises the local wage and makes local government higher cost and in turn makes budget balance for the local government difficult to achieve. In our framework with local government always in budget balance, the higher wage induces a contraction in the quantity of the local government good produced and a compensating rise in the quantity of housing consumed by each household.

tion and experience. Higher wages correlate positively with city size, even when worker education and experience are controlled for.⁴ Van Nieuwerburgh and Weill (2010) estimate a model of a many-city labor market that includes the local cost of housing. They have worker heterogeneity, housing heterogeneity within a city and city-specific heterogeneity of the industrial structure. They conclude: "House price differentials between metropolitan areas compensate for the income differential of the marginal, lowest ability household in the location, making that household indifferent between staying and moving to the next best metropolitan area. Households also live in smaller and more expensive quarters if they choose to work in higher income metropolitan areas. Lastly, higher income metropolitan areas have on average a larger housing stock and a larger workforce." (p. 1568). Part of the positive correlation of city wage and size has obviously to do with (a) larger cities exhibiting higher proportions of better educated residents and (b) the high average wage in a large city discouraging firms in certain "low-wage" industries from operating in a particular city.⁵ There are then these tricky labor force and industry composition effects at work in the larger city, higher wage relationship. Our model of a city abstracts from the issue of composition effects. Our worker-households are homogeneous. We adhere to the idea that the extra productivity a worker exhibits in a larger city is due to some complicated urban-ness factor, such as easier and more productive net-working with fellow workers, when a worker is employed in a higher density arrangement. We remain open-minded on the source of the extra productivity a worker gains from being in a larger city. We simply take the extra productivity as real and explore its implications in a model of a small open city.

Our point of departure for this work was the characterization of a city as a small, open trading entity. We took for granted that our framework would have a local public good endogenous. We were surprised to find that the Law of urban growth turned out to be sensitive to the way we introduced a local public

⁴Glaeser and Gyourko (2005) note that there is a lag in the productivity improvement exhibited by an in-migrant to a large city.

⁵Duranton and Puga (2001) provide evidence for firms doing innovative activity tending to be located in larger centers and firms doing more routine activity to be located in smaller, lower-cost places. Moretti (2012; Map 2, p. 91) emphasizes that a worker in a city with more residents with college degrees will earn more than her counterpart in a city with residents who lack as many college degrees.

good into our model. We are not claiming that our avenue to the violation of the law of urban growth is unique. There are many parameters in our model to experiment with. However the particular violation which we came upon and which we report on below is we believe worthy of much reflection.

Our model also allows us to work through an exogenous boost in the climate amenity enjoyed by citizens of our city. For the case of our public sector small or non-existent we observe the "free utility" from the boost to climate amenity showing up by making our city larger and more densely settled. For the case of our public sector large, the boost to the amenity makes our city smaller and less densely settled. We also note that the introduction of a local, non-traded goods sector (eg. haircuts) does not change our results. We turn now to our model of a small, open city and its response to parameter changes.

4 The Basic Model

Our city imports consumption goods, q_C at "world" price p_C and exports a good q_X at "world" price p_X . Capital K in the city is available in unlimited quantities at rent r and labor enters or exists the city when a worker's utility is lower or higher than \bar{u} . The export good is produced with labor and capital in fixed coefficients

$$q_X * an = N, \tag{1}$$

$$q_X * ak = K. \tag{2}$$

We assume that the production of this good requires no land. The value of imports equals the value of exports

$$q_C = q_X * [p_X / q_C].$$

The local wage will be $w = p_X * (1/an)$. A resident-worker in our city will have her utility $u(\frac{q_C}{N}, \frac{L}{N})$ equal to \bar{u} in⁶

$$u(\frac{q_C}{N}, \frac{L}{N}) = \bar{u}, \quad (3)$$

with $\frac{q_C}{N}$ per capita consumption good and $\frac{L}{N}$ per capita housing (a house is simply a land area⁷). A resident's income-expenditure equality is

$$w + \frac{rK}{N} + \frac{L * p_L}{N} - t(L) = \frac{q_C p_C}{N} + \frac{L * p_A}{N}$$

for p_L the local land rent and p_A the land rent at the edge of the occupied area (land rent for agricultural activity). Total land rent is returned to households as a per capita, lump-sum grant and a household pays out $\frac{L * p_A}{N}$ to land owners "abroad". A resident is assumed to own K/N and get rental income from this holding.⁸ $t(L)$ is a geographic friction cost incurred by a household, largely a commuting cost for getting to one's place of work. This per capita cost increases with city area.⁹ Each household is desiring more space for a residence and low-cost access to, on average, all other places in the city. This spatial friction cost prevents our city from having an indeterminant number of residents. If each resident lived above her workplace and did no commuting or shopping or visiting then the city would have an indeterminant size.¹⁰

Since $wN + rK = p_X q_X = p_C q_C$, our income-expenditure relationship reduces to

$$\frac{L * p_L}{N} - t(L) = \frac{L * p_A}{N}. \quad (4)$$

"High" land rent reflects "large" costs for a resident to move about the city. A household is assumed to take p_L as the cost of a marginal unit of residential land

⁶This approach is referred to as "the open city assumption". A better approach would be to have migration determined by a response to lifetime welfare attainable in various places (see for example Eaton and Eckstein (1997) and Lucas (2004)). The better approach would complicate our analysis greatly. In a somewhat stationary world, current utility for a person is a reasonable proxy for lifetime utility.

⁷We take up the possibility of a house comprising some capital as well as land below.

⁸Capital income is mutualized. An entrant to the city is assume to acquire an ownership share in total K .

⁹The consumer price of a home comprises land-input cost plus a city-wide accessibility "charge". An alternative to the accessibility charge based on city area, one might consider a charge based on city density. We do not distinguish diverse accessibility costs based on different locations on say a disc-shaped city. See for example Lucas and Rossi-Hansberg (2003). The internal movement of people from home to workplace typically generates a particular pattern of demand for land in a city, a so-called rent function.

¹⁰Lucas and Rossi-Hansberg (2003) allow for the possibility of a household having no motion in the city. They make firms gain an efficiency advantage by locating near other firms.

(a household does not connect its per capita land rent "dividend" as related to its "purchase" of land (housing)). Hence

$$\frac{u_{q_C/N}}{u_{L/N}} = \frac{p_C}{p_L}. \quad (5)$$

We can set $p_C = 1$ and eliminate q_C with $q_C = q_X * [p_X/q_C]$. K can be solved for after q_X has been obtained. Hence we have a 4 equation system to solve for q_X , N , L and p_L . Matlab software was used for solving this system and our others below.

We select the utility function to be Cobb-Douglas in $[q_C/N]^a [L/N]^{(1-a)}$ and can solve our system. For $a = 0.75$, $an = 0.1$, $p_X = 1.6$, $p_A = 0.01$, $\bar{u} = 11$ (and $w = (1/an) * p_X$), we solve to get results in line 1 of Table 1.

Table 1

	q_X	N	L	p_L	w	N/L
$an = 0.1$	6.4391	0.6439	2.3016	1.4921	16.00	0.2798
$an = 0.09$	10.3572	0.9321	2.4290	2.2741	17.78	0.3837

We improve the productivity of labor (an to 0.09 from 0.10) and resolve our system, getting results in line 2 in Table 1. The changes characterize "the law of urban growth": output, q_X and the wage each rise. Area occupied, population and the density of settlement in the city rises.

5 The Addition of a Local Public Sector

We take the local government as Samuelsonian in the sense that it supplies a local public good in a non-varying quantity to each household and charges in accord with marginal benefits, here simply p_G/N . We proceed then with our above model but with q_G produced with its distinctive Cobb-Douglas production function,

$$q_G = [K - K_X]^{ag} [N - N_X]^{(1-ag)}.$$

Efficient use of inputs involves

$$\frac{ag * (1/[K - K_X])}{(1 - ag) * (1/[N - N_X])} = \frac{r}{w},$$

and zero profits requires

$$p_G q_G = r * [K - K_X] + w * [N - N_X].$$

These are three new equations for our economy with a local public good. In addition, utility per household is now

$$u\left(\frac{q_C}{N}, \frac{L}{N}, q_G\right) = \bar{u},$$

and we require the additional marginal condition on consuming the public good to be satisfied

$$\frac{u_{q_C/N}}{u_{q_G}} = \frac{p_C}{p_G/N}.$$

Our budget constraint is unchanged since income from the production of the public good accrues to households and this income equals the expenditure by a household. Hence our new system is 7 equations in q_X, N, K, L, p_L, q_G and p_G . We make use of $K_X = ak * q_X$ and $N_X = an * q_X$ in solving the 7 equation system.

The utility function is now $[q_C/N]^a [L/N]^b [q_G]^{1-a-b}$, with $a = 0.4, b = 0.2$. For the production function for the government good, we take $ag = 0.1$. For the production of q_X we have $K_X = ak * q_X$ and $N_X = an * q_X$ for $ak = 0.3$ and $an = 0.1$. In addition we have $p_C=1$; $p_X = 1.6$; $\bar{u} = 11$; $r_A = 0.01$; and $r = 3.2$. We obtain results from solving in Table 2.

Table 2

	q_X	N	K	L	p_L	q_G	p_G	p_G/N	w	N/L
$an = 0.1$	16859.	3203.	5901.	4.	11384.	1431.	19.	0.0059	16.00	800.75
$an = 0.09$	14094.	2410.	4933.	4.	09029.	1088.	21.	0.0087	17.78	602.50

With an improvement in labor productivity in the export sector (an to 0.09 from 0.1) we get the results in line 2. Output, population, density and government output decline. We refer to such scenarios as violations of the law of urban growth. The cost of the public good has risen because the wage in the economy has risen. Each household sees its charge per unit of q_G (namely p_G/N) rise considerably and there is a general contraction in q_G being supplied by the government. To maintain a "reasonable level" of q_G , resources get drawn out of the export sector, including labor and essentially the city contracts, including its area occupied. Recall that Baumol (1967) argued that improvements in labor productivity in the non-government sector would lead to wage increases in a city and to a rise in costs for operating a city's public sector. We are seeing a variant of this Baumol phenomenon ("unbalanced growth") here. Our public sector is always in budget balance.

It is not surprising and perhaps reassuring to see that when the government good figures "small" in the utility function of the household that the law of urban growth re-appears. For example when we switch a from 0.4 to 0.79 in the utility function (b remains at 0.2 and the exponent on q_G becomes 0.05) and resolve the model, we observe that an improvement in labor productivity in the export sector leads to an expansion in output, q_X , population and density of settlement. Table 3 reports on solving the model for this experiment with a "small" coefficient on q_G . Line 1 in Table 3 is the base case and line 2 has the outputs for the case of labor productivity improved in the export sector.

Table 3

	q_X	N	K	L	p_L	q_G	p_G	p_G/N	w	N/L
$an = 0.1$	5.2790	0.5339	1.5870	2.0413	1.0999	0.0057	18.8541	35.31	16.00	.2615
$an = 0.09$	9.6332	0.8769	2.8961	2.1559	1.9004	0.0094	20.7294	23.64	17.78	.4067

Observe that q_G has increased as charge per unit p_G/N has declined. Also though the rent per unit of land has almost doubled, consumption of land per household has increased, albeit by a small amount. This scenario with productivity improvement and "small government" yields a reverse-Baumol phenomenon. The wage increase goes along with an increase in q_G and a decline in charge per unit, namely p_G/N . We observe a tension between (a) scenarios that conform to the law of urban growth and (b) scenarios that exhibit the Baumol phenomenon of "high cost" local government. Our take on this issue is that scenarios that exhibit "the law of urban growth" are the empirically plausible cases. The scenarios that "violate" the law of urban growth are doing so because our public sector has been set up as Samuelsonian and the Samuelson public goods approach is not the one that we can link to Baumol's argument. Baumol was working with stylized facts taken from real world local public sectors and real world public sectors appear to be quite different from what the Samuelson formulation, which we are relying on, dictates. In short we are getting good simulations of actual cities when our public sector "counts small" in the utility function of a household and we are getting theoretical *curiosa* when our public sector "counts large" in the utility function of a household. Our model is doing well at capturing "the law of urban growth", given the government good "counting small" in the representative household's utility function, but poorly

at capturing Baumol's "cost disease" at the municipal government level.

6 A Local Non-traded Good (haircuts)

We restructure our basic model without a government sector by introducing a locally produced, non-traded consumption good, q_H .¹¹ We do this to make our basic model more complete and thus to have a more detailed base-case scenario into which we introduce a local government sector. (We add the government sector in the following section.) One can interpret what we are doing here as changing the government good in the previous section into a local, non-traded private good. Relative to our basic model at the outset (4 equations in 4 unknowns) we add three equations related to the production of the non-traded good:

$$q_H = [K - K_X]^{ah} [N - N_X]^{(1-ah)}.$$

Efficient use of inputs involves

$$\frac{ah * (1/[K - K_X])}{(1 - ah) * (1/[N - N_X])} = \frac{r}{w},$$

and zero profits requires

$$p_N q_N = r * [K - K_X] + w * [N - N_X].$$

These are three new equations for our economy with a local non-traded good.

In addition, utility per household is now¹²

$$u\left(\frac{q_C}{N}, \frac{L}{N}, \frac{q_H}{N}\right) = \bar{u},$$

and we require the additional marginal condition on consuming the non-traded good to be satisfied

$$\frac{u_{q_C/N}}{u_{q_H/N}} = \frac{p_C}{p_H}.$$

¹¹ "the vast majority of jobs in a modern society are in local services... This sector exists only to serve the needs of a region's residents and is largely insulated from national and international competition." (Moretti (2012; p. 56). "most jobs in innovative industries belong to the traded sector... productivity growth is different in the two sectors... in many parts of the non-traded sector, labor productivity does not grow very much." (p. 57)

¹² "The average American spends only 14 percent of her income on food and beverages and 17 percent on transportation... By far the largest item in the budget is housing, which accounts for 40 percent of spending." (Moretti 2012; p. 169)

Our budget constraint is unchanged since income from the production of the non-traded good accrues to households and this income equals the expenditure by a household. Hence our new system is 7 equations in q_X, N, K, L, p_L, q_H and p_H . We make use of $K_X = ak * q_X$ and $N_X = an * q_X$ in solving the 7 equation system.

The utility function is now $[q_C/N]^a [L/N]^b [q_H/N]^{1-a-b}$, with $a = 0.3, b = 0.4$. For the production function for the non-traded good, we take $ah = 0.2$. For the production of q_X we have $K_X = ak * q_X$ and $N_X = an * q_X$ for $ak = 0.3$ and $an = 0.1$. In addition we have $p_C=1$; $p_X =1.6$; $\bar{u} =11$; $r_A =0.01$; and $r =3.2$. We solve with Matlab and obtain results in Table 4.

Table 4

	q_X	N	K	L	p_L	q_H	p_H	w	N/L
$an = 0.1$	0.1298	0.0234	0.0519	3.2353	0.0856	0.0109	19.127	16.00	0.00723
$an = 0.09$	0.1691	0.0274	0.0676	3.4509	0.1045	0.0130	20.8091	17.78	0.00794

We observe in Table 4 that the improvement in labor efficiency in the export sector leads to increases in $q_X, N, L, q_H, N/L$ and the wage. The law of urban growth is being displayed in this "experiment". The price of the non-traded good has increased somewhat while the aggregate quantity consumed has increased marginally. Also the rent per unit of land has increased somewhat while the aggregate quantity consumed has also increased somewhat. Moretti [2012; p. 90] argues that the wage increase that "starts" in the export sector spills over to wage increases for workers in the non-traded goods sector and adds general dynamism to the whole local economy. "The presence of many college-educated residents [in the local leading sector] changes the local economy in profound ways, affecting both the kinds of jobs available to residents and the productivity of all workers. In the end, this results in high wages not just for the skilled workers but also for workers with limited skills." And there would be a feedback effect: increased local buying power of those with higher wages would result in an expansion of sectors selling goods being consumed locally. Moretti's point appears to turn crucially on there being at least two types of workers, skilled and unskilled, but his wage spillover argument continues to operate in our model with only one type of worker, albeit in a slightly different way.

7 The Basic Model with a Non-traded and a Government Good

We add a Samuelsonian government sector to the model in the previous section. Each household consumes q_G , the current production of the government good and pays p_G/N per unit, where p_G is the current cost of producing q_G . Hence to the model above we add three equations pertaining to the production of q_G :

We add the Cobb-Douglas production function,

$$q_G = [K_G]^{ag}[N_G]^{(1-ag)}.$$

Efficient use of inputs in producing q_G involves

$$\frac{ag * (1/[K_G])}{(1-ag) * (1/[N_G])} = \frac{r}{w},$$

and zero profits requires

$$p_G q_G = r * [K_G] + w * [N_G].$$

These are three new equations for our economy with a local public good. In addition, utility per household is now

$$u\left(\frac{q_C}{N}, \frac{L}{N}, \frac{q_H}{N}, q_G\right) = \bar{u},$$

and we require the additional marginal condition on consuming the public good to be satisfied

$$\frac{u_{q_C/N}}{u_{q_G}} = \frac{p_C}{p_G/N}.$$

Our reduced budget constraint is unchanged since income from the production of the public good accrues to households and this income equals the expenditure by a household. Hence our new system is 11 equations in q_X , N , K , L , p_L , q_H , p_H , q_G , p_G , K_G and N_G . We make use of $K_X = ak * q_X$ and $N_X = an * q_X$ in solving the 11 equation system. The utility function is now $[q_C/N]^a [L/N]^b [q_H/N]^c [q_G]^{1-a-b-c}$, with $a = 0.3, b = 0.4, c = 0.25$. For the production function for the government good, we take $ag = 0.3$. For the production of q_N , we have $ah = 0.2$. For the production of q_X we have $K_X = ak * q_X$ and $N_X = an * q_X$ for $ak = 0.3$ and $an = 0.1$. In addition we have $p_C=1$; $p_X=1.6$; $\bar{u}=11$; $r_A=0.01$; and $r=3.2$. Note that the exponent on q_G in the utility

function is small for this solving, namely at 0.05. We obtain results from solving in Table 5.

Table 5

	q_X	N	K	L	p_L	q_H	p_H	p_G
$an = 0.1$	0.0454	0.0082	0.0181	2.8845	0.0336	0.0032	19.1270	18.7050
$an = 0.09$	0.0627	0.0102	0.0251	3.1697	0.0422	0.0040	20.8091	20.1367

Table 5 continued

	p_G/N	q_G	K_G	N_G	w	N/L
$an = 0.1$	2281.1	0.0006	0.0008	0.0006	16.00	.002843
$an = 0.09$	1974.2	0.0008	0.0010	0.0008	17.78	.003218

When we introduce an exogenous improvement in labor productivity in the export sector, we observe results in line 2 of Table 5. The law of urban growth is being displayed in the data. Observe that the charge per unit of q_G , namely p_G/N , has declined while the quantity consumed per household has risen.

We turn to solving with the exponent on q_G in the utility function being relatively large. In the utility function we now have $a = 0.1$, $b = 0.2$ and $c = 0.05$. Other parameters are unchanged.¹³ Results from solving are in Table

6

Table 6

	q_X	N	K	L	p_L	q_H	p_H	p_G
$an = 0.1$	314.4363	207.5279	314.4363	2.2019	456.9665	13.1515	19.1270	18.7050
$an = 0.09$	317.7668	188.7535	317.7668	2.3210	438.1085	12.2164	20.8091	20.1367

Table 6 continued

	p_G/N	q_G	K_G	N_G	w	N/L
$an = 0.1$	0.090	174.8272	204.3836	163.5069	16.00	94.25
$an = 0.09$	0.107	164.1174	206.5484	148.7148	17.78	81.32

q_X and L rise with the improvement in labor productivity in the export sector. q_H and q_G decline. The charge per unit of q_G , namely p_G/N , has increased while the quantity of q_G consumed per household has declined. The decline in N and in density conforms to our earlier notion of a violation of the law of urban growth.

¹³Note that we have made the exponent in the utility function for the non-traded good small. We did this to achieve "solvability" of our system with Matlab's simultaneous equation solver. Matlab requires an initial guess at the solution values. Solving successfully was difficult and to achieve a solving we ended up making the non-traded good count small in the solution. Hence this current specification of the problem numerically resembles our simple model with a government sector fairly closely.

We can summarize our model in a national account framework in Table 7. The top row and left side have labels. Net "national" product is the sum of the entries in the right hand column and "national" income is the sum of entries in the final row of the Table. Entries inside the Table for a column (row) sum to the entry on the bottom (right side).

Table 7

	$K - inpt$	$N - inpt$	$L - inpt$	NNP
$q_X - otpt$	rK_X	wN_X	0	$= q_X p_X = p_C q_C$
$q_H - otpt$	rK_H	wN_H	0	$= p_H q_H$
$q_G - otpt$	rK_G	wN_G	0	$= p_G q_G = \left[\frac{p_G}{N}\right] [Nq_G]$
$L - otpt$	0	0	$p_A L$	$= p_L L - [t(L) * N]$
NNI	$= rK$	$= wN$	$= p_A L$	

We could have housing produced with both land and capital. We introduce a constant returns to scale production function with inputs K_R and L for the production of q_R . We then have a zero profit condition $\overline{p_R} * q_R = rK_R + p_A L$, and an efficiency condition for the use of inputs (three new equations in K_R, q_R and $\overline{p_R}$). Since land rent is at the agricultural value, p_A we expect the capital to land ratio to not be large. In place of our earlier budget constraint we now have

$$\frac{p_R q_R}{N} - t(L) = \frac{\overline{p_R} * q_R}{N}$$

with new variable p_R (replaces variable p_L from the formulation of a plot of land as a house). The value of capital here will cancel from each side leaving the consumer rent for land above the production rent by the cost per household for mobility, $t(N)$. Alternatively, the $L - otpt$ row in Table 7 gets replaced by $rK_R + p_A L = \overline{p_R} * q_R = p_R q_R - [t(L) * N]$. Again if each household were to live on marginally less land, each household could access the rest of city at lower cost ($t(L)$ would decline for each household).

8 Sunnier Cities

We turn to consider the incidence of a climate premium for a city, say more temperate and sunny days per year for one city relative to another. Such amenities are generally priced into a household's equilibrium bundle and possibly wage indirectly. No agency charges at the entrance for a household re-locating to a city

with a better climate. Formally then a household will have a different amenity value A in the utility function when living in different cities; that is, we have $\bar{u} = u(\frac{q_C}{N}, \frac{q_H}{N}, \frac{L}{N}, q_G, A)$.¹⁴ Our interest is in the incidence of say an increase in the value of A , or in the indirect pricing of "more A ". We take up this issue by solving our complete model city in the previous section with different values of A (the wage remains unchanged¹⁵). Formally, we choose a simple channel for a local amenity premium to be registered, namely $\bar{u} = A * u(\frac{q_C}{N}, \frac{q_H}{N}, \frac{L}{N}, q_G)$, for $u(.)$ our base Cobb-Douglas utility function. More local amenities correspond to a larger value for A .

We take our base case from the first line in Table 6 (the case of q_G registering large in the utility function of a household). We have $a = 0.1$, $b = 0.2$ and $c = 0.05$. Other parameters are unchanged. Our equilibrium city is defined by results in the first line in Table 8. Parameter $A = 1$. We change the value of A to 1.05 and re-solve. The results are in line 2 of Table 8.

Table 8

	q_X	N	K	L	p_L	q_H	p_H	p_G
$A = 1.00$	314.4363	207.5279	314.4363	2.2019	456.9665	13.1515	19.1270	18.7050
$A = 1.05$	282.1276	186.2042	282.1276	2.2019	410.0132	11.8002	19.1270	18.7050

Table 8 continued

	p_G/N	q_G	K_G	N_G	w	N/L
$A = 1.00$	0.090	174.8272	204.3836	163.5069	16.00	94.25
$A = 1.05$	0.1005	156.8635	183.3829	146.7063	17.78	84.57

The city experiences a large contraction in population and production activity. The area occupied is unchanged and hence density declines. Our "sunnier city" is less populous and more spread out. Each agent is consuming less government services, q_G at a higher charge per unit. To a first approximation, we say that the price of more sunshine to an agent is having to live with relatively less government good. Since the city actually contracts in population and density, a second order effect would be a decline in worker productivity and a shift down in the average wage in the city. We have abstracted from this second order

¹⁴Haurin (1980) allows for firms to respond directly to a climate amenity. We only have worker-households responding directly. Firms are assumed to not be climate sensitive.

¹⁵Rosen's pioneering 1979 study of the pricing of amenities and disamenities in various cities (a sample of 19) assumed that the local wage reflected the indirect pricing of the amenities. Roback (1982) allowed for local "cost of living" differences (largely land prices) to register part of the indirect pricing of amenity premia. Here we leave local wages unchanged and let other variables register the indirect pricing of say a better local climate.

effect here. Recall that the basic city in this analysis is one with q_G "counting" relatively large in a household's utility function. We turn to the case of q_G "counting" relatively small.

The first line of Table 5 is our base case now. The results are reproduced as line 1 in Table 9 below. The utility function now has $a = 0.3, b = 0.4, c = 0.25$. The exponent on q_G in the utility function is 0.05. We obtain results from solving in Table 9.

Table 9

	q_X	N	K	L	p_L	q_H	p_H	p_G
$A = 1.00$	0.0454	0.0082	0.0181	2.8845	0.0336	0.0032	19.1270	18.7050
$A = 1.05$	0.0538	0.0097	0.0215	2.9651	0.0387	0.0038	19.1270	18.7050

Table 9 continued

	p_G/N	q_G	K_G	N_G	w	N/L
$A = 1.00$	2281.1	0.0006	0.0008	0.0006	16.00	.002843
$A = 1.05$	1928.4	0.0008	0.0009	0.0007	17.78	.003271

The city with the amenity increase is reported in the second line of Table 9. Population, city area and density have increased. The outputs of q_X, q_H and q_G have each increased. The rent on land has increased while the charge per unit of q_G has decreased. Roughly speaking, we would say that the amenity premium is being charged for to a household by an increase in "rationing" of residential land per household. Each household ends up with a smaller lot size or more generally with less "housing" per family. The price that a household is paying for enjoying more sunshine is living with less housing in the sunnier city. Since the city has a larger, more densely arranged population, labor productivity and the wage should rise somewhat. This would be a second-order force causing the city to expand. We abstract from this force here.

We see two very different scenarios of the capitalization of amenity improvement for a city, one with the public sector "counting large" in utility and the other with the public sector "counting small". In the second case the city is larger, more populous and denser. This case conforms to my prior on the effect of a climate premium for city i. This case is in a sense the companion scenario to "the law of urban growth" above, a scenario with solid empirical support. Hence we infer that the presence of a relatively large local government sector is causing the capitalization of a local amenity premium for city i "to go the wrong way",

as we argued above for the case of an exogenous boost in labor productivity in the export sector. More data analysis of city size and structure might clarify whether the size of local government affects overall city structure in unexpected ways. Certainly only a handful of cities are major seats of government.

9 Concluding Remarks

We set out an urban version of a small open economy involved in trade with a hinterland system of related small open cities and observed that a boost in labor productivity in the export sector of our city resulted in a larger, denser and higher wage city. This held for a very stripped down small open city as well as for one with a local non-traded good and a locally produced public good, a public good "counting" small in the utility function of our representative household. There is much good quality empirical evidence for larger cities to be higher wage and denser cities. Thus our model provides a simple framework for illustrating "the law of urban growth". Our model allowed for a labor productivity improvement in the export sector to yield smaller and less dense cities for the special case of the local public good "counting" large in the utility function of the representative agent. We dismiss these cases as theoretical curiosa that are turning on the way that public goods work their way through our model. Public good theory appears to be failing to provide a good way to model the local public sector, certainly for our model of a small open city. This is an issue that demands the attention of government sector modelers, but is tangential to our focus here. Probing the law of urban growth with our model with a different sort of local government seems like a good course of action. Our model suffers from the familiar problem of failing to deal with a household living in one place and earning income from say capital in a different location. We have mutualized the local ownership of capital and most of land. This has of course made our model easy to solve but is obviously at odds with the way the world works. We can be happy that our model behaves so well, given this way of treating the incomes of households but when we think about carrying out data analysis of cities we need a more detailed treatment of household income. Even household expenditures may be directed outside of our city and this also needs to be taken

care of in high quality econometric work.

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