Vasilev, Aleksandar

Research Report

The welfare effect of at income tax reform: the case of Bulgaria. Technical Appendix

The welfare effect of flat income tax reform: the case of Bulgaria. Technical Appendix

Aleksandar Vasilev

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1 Marginal and average tax rate under progressive taxation

Denote $y_t = r_t k_t + w_t h_t$ to be pre-tax income, and y_a to be the after-tax income.

Model progressive taxation $\tau_t = \tau(y_t) = \eta(\frac{y_t}{y})^{\phi}$ (taxation depends on the level of income). Let $T(y_t)$ denote total tax revenue. Then

$$T(y_t) = \tau(y_t)y_t = y_t\eta\left(\frac{y_t}{y}\right)^{\phi} = \eta\frac{y_t^{1+\phi}}{y^{\phi}} \tag{1}$$

$$T(y_t)/y_t = \eta\left(\frac{y_t}{y}\right)^{\phi} = \tau_t \tag{2}$$

$$T'(y) = \frac{dT(y_t)}{dy_t} = \eta(1+\phi)\left(\frac{y_t}{y}\right)^{\phi} = (1+\phi)\tau_t \tag{3}$$

Since $T(y_t)/y_t = \tau_t$ is the average tax rate, $T'(y)$ is the marginal tax rate, and $\phi > 0$ it is obvious that

$$T'(y_t) > T(y_t)/y_t, \tag{4}$$

or that the marginal tax rate is higher than the average tax rate under progressive taxation.

2 Firm Problem

The firm maximizes a sequence of static profits:

$$\pi_t = Ak_t^{\theta} e_t^{1-\theta} (g_t^c)^{\epsilon} - r_t k_t - w_t e_t. \tag{5}$$

To obtain optimal capital input rented, set $\pi_k = 0$ to obtain

$$\theta A k_t^{\theta-1} e_t^{1-\theta} (g_t^c)^{\epsilon} - r_t = 0 \tag{6}$$

or

$$\theta A k_t^{\theta-1} e_t^{1-\theta} (g_t^c)^{\epsilon} = r_t \tag{7}$$

After some rearrangement,

$$\theta \frac{y_t}{k_t} = r_t \tag{8}$$

To obtain the optimal of efficiency units hired, set $\pi_e = 0$ to obtain

$$(1-\theta) A k_t^{\theta} e_t^{-\theta} (g_t^c)^{\epsilon} - w_t = 0 \tag{9}$$

or

$$(1-\theta) A k_t^{\theta} e_t^{-\theta} (g_t^c)^{\epsilon} = w_t \tag{10}$$

After some rearrangement,

$$(1-\theta) \frac{y_t}{s_t h_t} = w_t \tag{11}$$

3 Household Problem

Set up the Lagrangian of the household (in equilibrium $\pi_t = 0$, $\forall t$):

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \ln(c_t) - \lambda_t \left[c_t + k_{t+1} - (1-\delta^k) k_t + s_{t+1} - (1-\delta^s) s_t - (1-\tau_t) \left(r_t k_t + w_t s_t h_t \right) \right] \right\} \tag{12}$$

To find the optimal consumption path, set $\mathcal{L}_c = 0$ to obtain

$$c_t^{-1} - \lambda_t = 0, \tag{13}$$

or

$$c_t^{-1} = \lambda_t. \tag{14}$$

To find the optimal physical capital stock, set $\mathcal{L}_k = 0$ to obtain

$$-\lambda_{t-1} + \beta\lambda_t\left(1 - \delta^k + (1-\tau_t)r_t - \frac{\partial \tau_t}{\partial k_t}y_t\right) = 0, \tag{15}$$

where

$$\frac{\partial \tau_t}{\partial k_t}\eta\left(\frac{1}{y^\phi}\right)\phi y_t^{\phi-1}\frac{\partial y_t}{\partial k_t} = \eta(\frac{1}{y^\phi})\phi y_t^{\phi-1}r_t = \frac{\tau_t\phi r_t}{y_t} \tag{16}$$

$$\tag{17}$$

or

$$\lambda_{t-1} = \beta\lambda_t\left(1 - \delta^k + (1-\tau_t)r_t - \frac{\tau_t\phi r_t}{y_t}y_t\right) \tag{18}$$

$$\lambda_{t-1} = \beta\lambda_t\left(1 - \delta^k + (1-\tau_t)r_t - \tau_t\phi r_t\right) \tag{19}$$

$$\lambda_{t-1} = \beta\lambda_t\left[1 - \delta^k + \left(1 - (1+\phi)\tau_t\right)r_t\right] \tag{20}$$

Shifting the equation one period forward to obtain

$$\lambda_t = \beta\lambda_{t+1}\left[1 - \delta^k + \left(1 - (1+\phi)\tau_{t+1}\right)r_{t+1}\right]. \tag{21}$$

Use the fact that the real interest rate is the marginal product of capital to obtain

$$\lambda_t = \beta\lambda_{t+1}\left[1 - \delta^k + \left(1 - (1+\phi)\tau_{t+1}\right)\theta\frac{y_{t+1}}{k_{t+1}}\right] \tag{22}$$

Finally, to find the optimal skill level, set $\mathcal{L}_s = 0$ to obtain

$$-\lambda_{t-1} + \beta\lambda_t\left[1 - \delta^s + (1-\tau_t)w_t h_t - \frac{\partial \tau_t}{\partial s_t}y_t\right] = 0, \tag{23}$$

where

$$\frac{\partial \tau_t}{\partial s_t}\eta\left(\frac{1}{y^\phi}\right)\phi y_t^{\phi-1}\frac{\partial y_t}{\partial s_t} = \eta\left(\frac{1}{y^\phi}\right)\phi y_t^{\phi-1}w_t = \frac{\tau_t\phi w_t h_t}{y_t} \tag{24}$$

$$\tag{25}$$

or

$$\lambda_{t-1} = \beta\lambda_t\left[1 - \delta^s + (1-\tau_t)w_t - \frac{\tau_t\phi w_t h_t}{y_t}y_t\right] \tag{26}$$

$$\lambda_{t-1} = \beta\lambda_t\left[1 - \delta^s + (1-\tau_t)w_t - \tau_t\phi w_t h_t\right] \tag{27}$$

$$\lambda_{t-1} = \beta\lambda_t\left[1 - \delta^s + \left(1 - (1+\phi)\tau_t\right)w_t h_t\right] \tag{28}$$

Shifting the equation one period forward to obtain

$$\lambda_t = \beta\lambda_{t+1}\left[1 - \delta^s + \left(1 - (1+\phi)\tau_{t+1}\right)w_{t+1}h_{t+1}\right]. \tag{29}$$

Use the fact that the real interest rate is the marginal product of capital to obtain

$$\lambda_t = \beta\lambda_{t+1}\left[1 - \delta^k + \left(1 - (1+\phi)\tau_{t+1}\right)\theta\frac{y_{t+1}}{k_{t+1}}\right] \tag{30}$$

To show how the transversality conditions are derived, we will first take the finite horizon version of the problem, and then let the terminal time period (N) diverge to infinity.[1]

$$\mathcal{L} = \sum_{t=0}^{N}\beta^t\left\{\ln(c_t) - \lambda_t\left[c_t + k_{t+1} - (1-\delta^k)k_t + s_{t+1} - (1-\delta^s)s_t - (1-\tau_t)\left(r_t k_t + w_t s_t h_t\right)\right]\right\} \tag{31}$$

Then, using the complementary-slackness condition, the optimal choice for terminal capital stock, k_{N+1} (which is the "last" first-order condition, or the boundary condition), is $-\beta^N\lambda_N k_{N+1} = 0$, or $\beta^N\lambda_N k_{N+1} = 0$. That is, from the Kuhn-Tucker conditions it follows that either $\lambda_N \geq 0$, or $k_{N+1} \geq 0$. Similarly, for human capital, $\beta^N\lambda_N s_{N+1} = 0$, hence either $\lambda_N \geq 0$, or $s_{N+1} \geq 0$. Furthermore, since the constraint set is a sequence of linear constraints, Slater's sufficiency condition for regularity is satisfied (Simon and Blume 1994, p. 477). In plain words, since $\beta^N > 0$, this means either $\lambda_N > 0$ and $k_{N+1} = s_{N+1} = 0$, or $\lambda_N = 0$ and $k_{N+1} > 0$, $s_{N+1} > 0$. In economic terms, the two results mean that one period after the end of the optimization horizon, either both capital stocks are zero, while the terminal-period price is positive, or we have positive quantities of both physical and human capital, but price in period N is nil. In both cases, the present value of both capital stocks is zero.

Now letting $N \to \infty$, in the limit

$$\lim_{t\to\infty}\beta^t\lambda_t k_{t+1} = \lim_{t\to\infty}\beta^t\frac{1}{c_t}k_{t+1} = 0 \tag{32}$$

Also

$$\lim_{t\to\infty} \beta^t \lambda_t s_{t+1} = \lim_{t\to\infty} \beta^t \frac{1}{c_t} s_{t+1} = 0 \tag{33}$$

4 Decentralized Equilibrium System

$$\begin{aligned}
c_t^{-1} &= \lambda_t && (34)\\
\lambda_t &= \beta\lambda_{t+1}\left[(1-\delta^k) + \Big(1-(1+\phi)\tau_{t+1}\Big) r_{t+1}\right] && (35)\\
\lambda_t &= \beta\lambda_{t+1}\left[(1-\delta^s) + \Big(1-(1+\phi)\tau_{t+1}\Big) w_{t+1}h_{t+1}\right] && (36)\\
r_t &= \theta\frac{y_t}{k_t} && (37)\\
w_t &= (1-\theta)\frac{y_t}{s_t h_t} && (38)\\
g_t^c &= \tau[r_t k_t + w_t s_t h_t] && (39)\\
Ak_t^{\theta}(s_t h_t)^{1-\theta}(g_t^c)^{\epsilon} &= c_t + k_{t+1} - (1-\delta^k)k_t + s_{t+1} - (1-\delta^s)s_t + g_t^c && (40)
\end{aligned}$$

5 Proof that the TVCs are respected

It needs to be demonstrated that

$$\lim_{t\to 0} \beta^t \frac{1}{c_t} k_{t+1} = 0 \tag{41}$$

and that

$$\lim_{t\to 0} \beta^t \frac{1}{c_t} s_{t+1} = 0 \tag{42}$$

Under the BGP,

$$k_t = k_0(1+\gamma)^t \tag{43}$$

$$c_t = c_0(1+\gamma)^t. \tag{44}$$

From the physical capital accumulation equation in period 1, it follows that

$$k_1 = i_0^k + (1-\delta^k)k_0 \tag{45}$$

$$k_0(1+\gamma) = i_0^k + (1-\delta^k)k_0 \tag{46}$$

$$\gamma k_0 = i_0^k - \delta^k k_0 \tag{47}$$

$$i_0^k = (\gamma+\delta_k)k_0 > 0 \tag{48}$$

Also, from the human capital accumulation equation in period 1, it follows that

$$s_1 = i_0^s + (1 - \delta^s)s_0 \tag{49}$$

$$s_0(1 + \gamma) = i_0^s + (1 - \delta^s)s_0 \tag{50}$$

$$\gamma s_0 = i_0^s - \delta^s s_0 \tag{51}$$

$$i_0^s = (\gamma + \delta_s)s_0 > 0 \tag{52}$$

From the government budget constraint in period 0, it follows that

$$\tau_0 y_0 = g_0^c > 0 \tag{53}$$

Then

$$c_0 = y_0 - i_0^k - i_0^k - g_0^c = \tag{54}$$

$$= y_0 - (\gamma + \delta_k)k_0 - (\gamma + \delta_s)s_0 - \tau_0 y_0 = \tag{55}$$

$$= (1 - \tau_0)y_0 - (\gamma + \delta_k)k_0 - (\gamma + \delta_s)s_0, \tag{56}$$

which under some restrictions on the initial values for the physical and human capital stock would be positive.

Returning to the TVC for physical capital, one can obtain

$$\lim_{t\to 0} \beta^t \frac{1}{c_t} k_{t+1} = \lim_{t\to 0} \beta^t \frac{1}{c_0(1+\gamma)^t} k_0(1+\gamma)^{t+1} = \tag{57}$$

$$= \lim_{t\to 0} \beta^t \frac{k_0(1+\gamma)}{c_0} = \tag{58}$$

$$= \frac{k_0(1+\gamma)}{c_0} \lim_{t\to 0} \beta^t = 0 \tag{59}$$

Analogously, for human capital

$$\lim_{t\to 0} \beta^t \frac{1}{c_t} s_{t+1} = \lim_{t\to 0} \beta^t \frac{1}{c_0(1+\gamma)^t} s_0(1+\gamma)^{t+1} = \tag{60}$$

$$= \lim_{t\to 0} \beta^t \frac{s_0(1+\gamma)}{c_0} = \tag{61}$$

$$= \frac{s_0(1+\gamma)}{c_0} \lim_{t\to 0} \beta^t = 0 \tag{62}$$

6 Construction of the Average Effective Income Tax Rate

Computing the average effective tax rate requires data on the revenues collected from the tax and the tax base. The data on income tax base are from the national income accounts, and the data on the tax revenues collected are from the World Development Indicators (WDI).

Effective Income tax rate
$R_{inc,t}$ = revenue from taxes on income, profits, and capital gains of individuals
GNI = gross national income
SSE_t = employers' contribution to social security
δK_t^h = household consumption of fixed capital

$$\tau_t = \mu \frac{R_{inc,t}}{GNI - SSE_t - \delta K_t^h}. \tag{63}$$

The progressivity of the income tax system implies that marginal tax rates tend to be larger than the average tax rates we are computing. The term μ is an adjustment factor that transforms average tax rates to marginal tax rates. Following Prescott (2002), to take into account the progressivity of the tax system, we multiply by the fraction by a scaling factor $\mu > 1$. For Bulgaria we set $\mu = 1.8$.[2]

7 Deriving the Balanced Growth Path (BGP) rate

show all variables grow at the same rate, starting from physical capital.

Starting with capital. Let physical capital grow at the rate g, i.e.

$$\frac{k_{t+1}}{k_t} = 1 + \gamma \tag{64}$$

Then it is easy to show that investment in physical capital grows at the same rate

$$k_{t+1} = i_t^k + (1 - \delta^k)k_t, \tag{65}$$

or

$$\frac{k_{t+1}}{k_t} = 1 + \gamma = \frac{i_t^k}{k_t} + 1 - \delta^k \tag{66}$$

Prices are constant, skills act like a labor-augmenting endogenous technological progress, so on the BGP

$$\frac{s_{t+1}}{s_t} = 1 + \gamma \tag{67}$$

Then it is easy to show that investment in physical capital grows at the same rate

$$s_{t+1} = i_t^s + (1 - \delta^s)s_t, \tag{68}$$

or

$$\frac{s_{t+1}}{s_t} = 1 + \gamma = \frac{i_t^s}{s_t} + 1 - \delta^s. \tag{69}$$

Output grows at the same rate since

$$y_t = A(k_t)^\theta (s_t h_t)^{1-\theta}, \tag{70}$$

and

$$y_t = A(k_0)^\theta (s_0 h_t)^{1-\theta}(1+\gamma)^{\theta t + (1-\theta)t} = A(k_0)^\theta (s_0 h_t)^{1-\theta}(1+\gamma). \tag{71}$$

Using the market clearing condition, it is trivial to show that consumption will also grow at the same rate as output.

Derivation of the compensatory variation

Since labor supply is constant, we can ignore the term containing utility of leisure, as it is an arbitrary constant. Next, we can ignoring the effect on initial consumption ($c_0 = c_{2008}$). Total welfare is then summarized by the formula

$$\sum_{t=0}^{T} \beta^t \ln\left[(1+\lambda)c_t^{PROG}\right] = \sum_{t=0}^{T} \beta^t \ln\left[c_t^{FLAT}\right] \tag{72}$$

Using that $c_t = c_0(1+\gamma)^t$, we can obtain that

$$\sum_{t=0}^{T} \beta^t \ln\left[(1+\lambda)c_0(1+\gamma^{PROG})^t\right] = \sum_{t=0}^{T} \beta^t \ln\left[c_0(1+\gamma^{FLAT})^t\right]. \tag{73}$$

Expand the expressions on both sides to obtain

$$\sum_{t=0}^{T} \beta^t \left[\ln(1+\lambda) + \ln c_0 + t\ln(1+\gamma^{PROG})\right] = \sum_{t=0}^{T} \beta^t \left[\ln c_0 + t\ln(1+\gamma^{FLAT})\right]. \tag{74}$$

Cancel out common terms on both sides to obtain

$$\sum_{t=0}^{T} \beta^t \left[\ln(1+\lambda) + t\ln(1+\gamma^{PROG}) \right] = \sum_{t=0}^{T} \beta^t t \ln(1+\gamma^{FLAT}). \tag{75}$$

Open brackets to obtain

$$\frac{1-\beta^T}{1-\beta} \ln(1+\lambda) + \ln(1+\gamma^{PROG}) \sum_{t=0}^{T} \beta^t t = \ln(1+\gamma^{FLAT}) \sum_{t=0}^{T} \beta^t t \tag{76}$$

Rearrange terms to obtain

$$\frac{1-\beta^T}{1-\beta} \ln(1+\lambda) = \ln \left[\frac{1+\gamma^{FLAT}}{1+\gamma^{PROG}} \right] \sum_{t=0}^{T} \beta^t t \tag{77}$$

Further simplifications lead to

$$\ln(1+\lambda) = \ln \left[\frac{1+\gamma^{FLAT}}{1+\gamma^{PROG}} \right] \frac{1-\beta}{1-\beta^T} \sum_{t=0}^{T} \beta^t t. \tag{78}$$

Therefore,

$$\lambda = \exp \left\{ \ln \left[\frac{1+\gamma^{FLAT}}{1+\gamma^{PROG}} \right] \frac{1-\beta}{1-\beta^T} \sum_{t=0}^{T} \beta^t t \right\} - 1. \tag{79}$$

References

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