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# Market Size, Productivity, Entrepreneurship and Income Inequality in a Model a'la Melitz

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## Abstract

We develop a monopolistic-competition model of closed two-sector one-factor economy, where agents are (continuously) heterogeneous in their entrepreneurship abilities and choose between being employees or entrepreneurs. The sufficient conditions in terms of *variable* elasticity of substitution are found for expanding or shrinking entrepreneurial sector if the market size changes. The influence of market size change on equilibrium parameters in the diversified sector (consumptions, outputs, prices and profits) crucially depends on measure of concavity local utility function. Particular there are pro- and anticompetitive effects: if the relative love for varieties grows under market expansion the prices decline and profit inequality increase and visa verse.

Keywords: monopolistic competition, heterogeneity, entrepreneurship, relative love to variety, income inequality

Jel Classification: D43, L11, L13, L26

## 1 Introduction

**Heterogeneity and productivity.** Traditional monopolistic competition models (see review by Brakman and Heijra (2004)) [1] emphasized economies of scale and related welfare gains for bigger markets, including gains from trade when economies integrate under trade liberalization. These gains were mainly

due to increase in product diversification, and/or to decrease in prices generated by utilities with variable elasticity of substitution (VES) like in Ottaviano et al.(2002) [6] and Zhelobodko et al. (2010) [8]. More modern models - heterogeneous models - emphasize also gains from the *productivity* increase due to selection of the best firms into the pool of exporters (Melitz, 2003) [4], and the direct selection effect generated by VES utilities, like in Melitz et al. (2008) [5] and in Zhelobodko et al. (2011) [9], that extends Melitz’s approach to VES. We can formulate our motivation for further developing such models as follows. Melitz’s approach suggests that each type of firm (having certain marginal cost  $c$ ) has as much copies as the average (expected) profitability of the market allows for. Thereby, a bigger market generates a proportionally bigger number  $N$  of copies of each type (in Melitz, 2003 [4]). This outcome follows from assuming the probabilistic entry and rises some doubts: are really new good and bad business ideas born to life *proportionally to profitability* of the market?

When we interpret heterogeneity, i.e., comparative advantages of some entrepreneurs as their idiosyncratic entrepreneurial abilities, non-Melitz heterogeneity concept also looks reasonable: the number of copies of able and less able businessmen must be *proportional to the population size* rather than to average market profitability. Such entrepreneurship idea dates back to Lucas (1978) [3]. In monopolistically competitive heterogenous setting it is implemented in Oyama et al. (2011) [7], where given population is endogenously splitting into employees and entrepreneurs, depending upon comparison between current wages and profits. The “cutoff” entrepreneur is one whose personal productivity makes him indifferent between these two options. Thus, the more profitable is the market, the more entrepreneurs remain. Importantly, the share of entrepreneurs in the population is shown to behave non-monotonically in response to trade openness. Sympathizing this setting, we are not satisfied only with specific (quadratic) utilities and an assumption that entrepreneurs are the *least* able part of the population, all those who are not productive workers enough to work as employees. In the similar model implemented by Kukharskyy (2011) [2] is obtained that share of income accruing to the more productive entrepreneurs increases when market expands.

**This present paper** aims to further develop Lucas’s idea: entrepreneurship and abilities selection influence productivity. We combine it with the VES version of Melitz model, like in Zhelobodko et al. (2011) [9]. Unlike Lucas, we study non-homogenous good with partially substitutable varieties, but like him, we assume that there is some given distribution of abilities among the population present. This distribution reflects just probability of being born with higher or lower entrepreneurship ability. Unlike Melitz model, here each agent knows her ability and chooses, subject to differential between current wage and potential profit, whether to become a worker or a businessmen. In our case the (in-)ability parameter is the marginal cost  $c$  of production if  $c$ -th agent start a firm. All those with  $c$  higher that some endogenous threshold  $\bar{c}$  value choose to be a worker. Should this threshold  $\bar{c}$  (the cutoff cost) increase or decrease in response to the growing market (say, market integration)? It influences average productivity, so we suppose important to finding the sufficient conditions on utilities and ability distribution for both outcomes.

Thus, our departure from Melitz (2003) [4] is two-fold: (1) We study two-sector economy: a differentiated sector and a sector with homogeneous good (numeraire, like in Oyama et al. , 2011) [7] but our agents have unspecified upper-tier and lower-tier utilities; (2) The agents know their abilities, instead of probability and expected (zero) profit, it is wage that determines the cutoff.

**Our results** concerning the response of the cutoff cost and productivity to the population size (market size) - are as follows. The entrepreneurs share in population is changing with the market size, but the direction of changing depend on changes in concavity of upper-tier and lower-tier utility functions (the Arrow-Pratt measure of concavity, called here relative love for variety - RLV). If lower-tier utility is more concave than the upper-tier one, the cutoff cost goes up under population growth. As far as

RLV is the inverse of substitution elasticity, this result says that entrepreneurs fraction (and their mass) is expanded when agents appreciate the diversity of differentiated good higher than necessarily mixing the differentiated and the homogenous good. In the opposite case the entrepreneurs share is shrinking, moreover the amount of entrepreneurs may decrease. The change in the average productivity positively relates to the variation of cutoff cost and it is influenced accordingly by the market size. Additionally we found out that entrepreneurs are diversified in their prices, outputs and profits due market expanding. Particular there are pro- and anticompetitive effects: if the relative love for varieties grows under market expansion the prices decline and profit inequality increase and visa versa. Importantly, when the upper-tier utility function is *log*, traditional CES lower-tier utility is the borderline case between increasing and decreasing productivity in response to the population size, without changes in the share of entrepreneurs. Thus VES modelling discovers new effects.

Section 2 presents the model, Section 3 displays the comparative statics w.r.t. market size in general form. Section 4 give more definite answers for special cases. Section 5 concludes and Appendix contains most proofs.

## 2 Model of entrepreneurs selection

**Labor supply** consists of big mass  $L$  of individuals differentiated in their entrepreneurship abilities. These are described by parameter  $c > 0$  - marginal cost of production if organizing a firm (the smaller  $c$  the higher entrepreneurship ability). The probability of each type  $c$  is described by some density function  $\gamma(c)$  on  $\mathbb{R}_+$ , and each  $c$ -type has  $L$  copies in the population. Each agent knows her type and chooses one of two activities: to be an entrepreneur or to be a worker. If being a worker, the agent supplies his single unit of labor to the market. Endogenous  $\bar{c}$  denotes the level of entrepreneurship abilities of “cutoff” or “marginal” agent, who is indifferent between two types of activity: to be entrepreneur or to be a worker. So, the relative size (probability) of entrepreneurship is equal to  $\int_0^{\bar{c}} \gamma(c) dc < 1$ .

Two sectors operate in the economy: a diversified (manufacturing) sectors and a homogenous (traditional) one. Their competition for labor equalizes wages  $w \equiv 1$  (normalized to one) among sectors.

**The demand side** of the economy involves  $L$  individuals, each can be a worker or entrepreneur. These consumers are identical in their quasi-linear preferences over (infinite-dimensional) consumption vector  $x : [0; \bar{c}] \rightarrow R$  of varieties and scalar  $A \geq 0$  of homogenous good. Utility maximization amounts to

$$V \left( L \int_0^{\bar{c}} u(x_c) \gamma_c dc \right) + A \rightarrow \max_{x, A}$$

$$\text{s.t. } L \int_0^{\bar{c}} p_c x_c \gamma_c dc + A = I.$$

Here  $\gamma_c \equiv \gamma(c)$  is the density value at any point  $c$ ,  $x_c \equiv x(c)$  is the consumption of a variety produced by any  $c$ -type entrepreneur,  $x \equiv (x_c)_{c \in [0; \bar{c}]}$  (induces denote functions' arguments). Further,  $p_c \equiv p(c)$  is the price for  $c$ -type commodity,  $p_A = 1$  is the price for homogeneous good, the numeraire. The income of the  $c$ -type agent, is  $I_c$ , it equals to the wage for worker:  $I_c = w = 1$  but it equals to the operating profit  $\pi_c$  (defined later on) for any  $c$ -type entrepreneur.

Lower-tier utility  $u(x_c)$  denotes the satisfaction from consuming  $x_c$  of the  $c$ -type variety. Additivity of such satisfactions (expressed by the integral) is a common assumption in monopolistic competition, allowing to define a function of indefinite-size argument  $x$ . The more convex is function  $u$  the higher is love for variety (that mirrors Arrow-Pratt theory of risk aversion). Interdependence of different varieties with the numerarie is expressed by the upper-tier utility function  $V$ ; the higher its derivative the more income is spent on differentiated good whereas too concave  $V$  means quick satisfaction with varieties.

We assume  $u$  to be sufficiently smooth, increasing, concave and satisfying Inada's conditions:

$$u(\cdot) \in C^3(\mathbb{R}_+ \mapsto \mathbb{R}_+), u'(\cdot) \geq 0, u''(\cdot) \leq 0$$

$$u(0) = 0, u'(0) = +\infty$$

Upper-tier utility is also sufficiently smooth, increasing and concave:

$$V(Y) \in C^2(\mathbb{R}_+ \mapsto \mathbb{R}), V'(Y) \geq 0, V''(Y) \leq 0$$

To derive the inverse demand  $\mathbf{p}(\cdot)$  from utility maximization, we use the first order condition for each  $c$ -type variety with market characteristic  $\lambda$  - marginal utility of expenditure for manufacturing goods. The inverse demand for type "c" commodity is:

$$\mathbf{p}(x_c) = \frac{u'(x_c)}{\lambda}, c \in [0; \bar{c}], \lambda \equiv 1/V' \left( L \int_0^{\bar{c}} u(x_c) \gamma_c dc \right). \quad (1)$$

Assumptions on  $u$  guarantee the neoclassic demand properties:  $\mathbf{p}(\cdot)$  decreases from infinity to zero.

The changing the price connected with measure of concavity of the elementary utility function  $u$ :

$$r_{uc} \equiv r_u(x_c) \equiv -\frac{p'_c x_c}{p_c} = -\frac{u''_c x_c}{u'_c} \quad (2)$$

is "relative love to variety" taken at point  $x_c$ , i.e, the Arrow-Pratt measure of concavity of the elementary utility function  $u$ .

**The supply** side includes as much as  $L \int_0^{\bar{c}} \gamma_c dc$  producers of all types. Any  $c$ -type producer correctly anticipates the demand function  $\mathbf{p}(\cdot)$ , the market characteristic  $\lambda$  and wage  $w$ , when maximizing the economic profit, i.e., operational profit  $\pi_c$  (the difference between total revenue and total cost) net of foregone wage:

$$\Pi_c = \pi_c - w = (\mathbf{p}(x_c) - c) L x_c - 1 \rightarrow \max_{x_c},$$

We can derive profit-maximizing output and price (standardly, maximizing w.r.t. price is equivalent). The first order condition is

$$\mathbf{p}'(x_c) x_c + \mathbf{p}(x_c) = c$$

This equality let us get the expression for the  $c$ -type producer mark-up  $M_c = \frac{p_c - c}{c}$ :

$$M_c = r_{uc} \quad (3)$$

which gives us the relation among price-consumption patterns for various types (see **Appendix B** for details):

$$\frac{(1 - r_u(x_{c_1})) p_{c_1}}{c_1} = \frac{(1 - r_u(x_{c_2})) p_{c_2}}{c_2} = 1. \quad (4)$$

**Equilibrium.** The *Entry Condition* is defined by formulating the zero-profit condition which says that the cutoff agent (one having entrepreneurship ability  $1/\bar{c}$ ) is indifferent between operational profit and wage:

$$\Pi_{\bar{c}} = \pi_{\bar{c}} - w = 0 \Leftrightarrow \frac{\bar{r}_u \bar{x}}{1 - \bar{r}_u} = \frac{1}{L\bar{c}} \quad (5)$$

where  $\bar{r}_u = r_u(x_{\bar{c}})$  is relative love for variety at point  $\bar{x} = x_{\bar{c}}$ .

The *equilibrium* is the bundle  $(\bar{c}, P, \{p_c; x_c\}_{c \in [0; \bar{c}]})$  such that consumption  $x$  maximizes each consumer's utility under price vector  $\{p_c = \mathbf{p}(x_c)\}_{c \in [0; \bar{c}]}$  and solves each producer's problem under  $\bar{c}, P, \mathbf{p}(\cdot)$ , the zero-profit condition holds and  $\lambda = V' \left( L \int_0^{\bar{c}} u(x_c) \gamma_c dc \right)$ . For a given equilibrium, consumption of the numerarie  $A_c$  for each type follows from the budget constraint, that entails also the labor balance under our normalization  $w = 1$ .

### 3 Impact of market size

The elasticity of any function  $f(x, y)$  w.r.t.  $x$  at point  $\hat{x}, y$  will be denoted

$$\mathcal{E}_x f = \mathcal{E}_x f(\hat{x}, y) \equiv \frac{\hat{x} f'(\hat{x}, y)}{f}.$$

Through differentiating the equilibrium equations w.r.t. the market size (population) and expressing the elasticities of all variables, we get the comparative statics of equilibria. First we get the elasticity of marginal cost for the cutoff agent. For this task, we denote as  $r_V \equiv -\frac{Y V''(Y)}{V'(Y)}$  the relative love for variety (concavity measure) for the upper-tier utility at the equilibrium point,  $\bar{\gamma} = \gamma(\bar{c})$  is the distribution density at point  $\bar{c}$ ,  $J = \frac{Y}{L} = \int_0^{\bar{c}} u(x_c) \gamma_c dc$  is the value of consumption of the composite good per capita at the equilibrium,  $J_0 = \int_0^{\bar{c}} \frac{u'(x_c) x_c}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u_c}'} \gamma_c dc$ .

Denoting  $\bar{\Gamma} = \int_0^{\bar{c}} \gamma_c dc$  - share of entrepreneurs in population and  $\tilde{u} = \frac{J}{\bar{\Gamma}}$  - conditional average of utilities from consumption of a variety, we can express elasticity of the cutoff as follows

**Proposition 1.** *The elasticity of the cutoff cost to the market size for marginal agent is determined as*

$$\mathcal{E}_{L\bar{c}} = \frac{\bar{r}_u}{1 - \bar{r}_u} \frac{\frac{1}{r_V} - \frac{1}{\bar{r}_u} + \frac{J_0}{J}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}}{\bar{c}}} \in \left( -1; \frac{\bar{r}_u}{1 - \bar{r}_u} \right) \quad (6)$$

*Proof.* see **Appendix D** □

*Remark.* Here lower bound  $\mathcal{E}_{L\bar{c}} > -1$  says that the cutoff cost can not decrease more intensively than the market size expands. The upper bound  $\frac{\bar{r}_u}{1 - \bar{r}_u}$  says that  $\bar{c}$  can not grow too fast. The interval for elasticity fluctuations is wider when the relative love for variety is bigger.

**Proposition 2.** *The criterion for growth of the cutoff cost under growing market compares concavity of the upper-tier and lower-tier utility as follows:*

$$\mathcal{E}_L \bar{c} \underset{<}{\overset{\geq}{\approx}} 0 \Leftrightarrow \frac{J_0}{J} \underset{<}{\overset{\geq}{\approx}} \frac{1}{\bar{r}_u} - \frac{1}{r_V}. \quad (7)$$

**Corollary.** *For increasing cutoff cost in response to the growing market size, any of the following two conditions is sufficient; either concavity for lower-tier utility  $u$  exceeds concavity of upper-tier utility  $V$ :*

$$1) \quad \bar{r}_u > r_V \Rightarrow \mathcal{E}_L \bar{c} > 0,$$

or concavity of  $u$  and concavity of  $\ln u$  behave as:

$$2) \quad r'_u \leq 0, \quad r_{\ln u} \geq 1 \Rightarrow \mathcal{E}_L \bar{c} \geq 0.$$

*Proof.* First statement is obviously:  $\bar{r}_u \geq r_V \Leftrightarrow \frac{1}{\bar{r}_u} - \frac{1}{r_V} \leq 0 \leq \frac{J_0}{J} \Leftrightarrow \mathcal{E}_L \bar{c} \geq 0$

The second statement can be proved using some usefull properties of *RLV*

$r_{\ln u} \geq 1 \Leftrightarrow \mathcal{E}_x u \geq 1 - r_u \Leftrightarrow \frac{\mathcal{E}_x u}{r_u} \geq \frac{1}{r_u} - 1$  (see **Appendix A**, Lemma 2). In the case  $r'_u \leq 0$  last means that  $\frac{\mathcal{E}_x u}{r_u} \geq \frac{1}{\bar{r}_u} - 1$ . In the same case we have  $\frac{1-r_{u_c}}{2-r_{u'_c}} \geq 1$  (see Appendix 1, Lemma 1, Corollary 2).

It leads us to the next inequality:  $\frac{J_0}{J} = \frac{\int_0^{\bar{c}} \frac{u'(x_c)x_c}{r_{u_c}} \cdot \frac{1-r_{u_c}}{2-r_{u'_c}} \gamma_c dc}{\int_0^{\bar{c}} u(x_c) \gamma_c dc} = \frac{\int_0^{\bar{c}} \frac{u'(x_c)x_c}{u(x_c)r_{u_c}} \cdot \frac{1-r_{u_c}}{2-r_{u'_c}} u(x_c) \gamma_c dc}{\int_0^{\bar{c}} u(x_c) \gamma_c dc} \geq \frac{\int_0^{\bar{c}} (\frac{1}{\bar{r}_u} - 1) u(x_c) \gamma_c dc}{\int_0^{\bar{c}} u(x_c) \gamma_c dc} = \frac{1}{\bar{r}_u} - 1 \geq \frac{1}{\bar{r}_u} - \frac{1}{r_V}$   $\square$

**Proposition 3.** *When population increases, the marginal utility of expenditure for manufacturing goods increases also, but not too fast:*

$$\mathcal{E}_L \lambda = \frac{1 + \frac{\bar{r}_u}{1-\bar{r}_u} \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1-\bar{r}_u} \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}} > 0$$

$$\mathcal{E}_L \lambda \leq \max \{ \bar{r}_u; r_V \}$$

*Proof.* see the **Appendix D**  $\square$

*Remark.* The elasticity of competitive intensive can be decomposed into two component: the direct influence of market size on that intensity  $\mathcal{E}_L^d \lambda = \bar{r}_u > 0$  and indidirect one  $\mathcal{E}_L^i \lambda = -(1 - \bar{r}_u) \mathcal{E}_L \bar{c}$  (see the Appendix D, expression (34)) . The first equals to the mark-up for marginal firm, which is non-negativ, it means that the market expanding enlarges the competitive intensity and income's shadow price, and last one is the negative product cutoff elasticity w.r.t. market size and the ratio of marginal cost and price charged by the marginal entrepreneur, i.e. the indirect effect is opposite changing of share of entrepreneurs in the population - the growing that share of entrepreneurs declines the competitive intensity. Anyway , as a result sum of these effects is positive - the bigger market size the bigger competition among entrepreneurs, the stronger influence on the charged prices.

Now we turn to finding who produce more amount with less price in equilibrium: more able or less able entrepreneurs? We compare outputs  $Lx_c$  and prices of different entrepreneurs and thereby related consumptions  $x_c$  and prices  $p_c$ . It allow us to compare profits for different producers later

**Proposition 4.** *Higher entrepreneurial ability implies bigger output and smaller price and vice verse:*

$$Lx_{c_1} \begin{matrix} \geq \\ \cong \\ \leq \end{matrix} Lx_{c_2} \Leftrightarrow c_1 \begin{matrix} \leq \\ \cong \\ \geq \end{matrix} c_2$$

$$p_{c_1} \begin{matrix} \geq \\ \cong \\ \leq \end{matrix} p_{c_2} \Leftrightarrow c_1 \begin{matrix} \geq \\ \cong \\ \leq \end{matrix} c_2$$

*Proof.* **FOC** for Producer problem (3) in the form  $\frac{(1-r_u(x_c))p_c}{c} = 1$  and **FOC** for Consumer problem  $p(x_c) = \frac{u'(x_c)}{\lambda}$  lead us to the equality, which is true for any  $c$ -type entrepreneur:

$$\frac{(1-r_{u_c})u'(x_c)}{c} = \lambda \geq 0$$

Taking the elasticity w.r.t.  $c$  from both sides and applying the Lemma 3 (see Appendix A) and the chain rule to the last equality we have:

$$-\frac{2-r_{u'}(x_c)}{1-r_u(x_c)}r_u(x_c)\mathcal{E}_c(x_c)-1=0$$

The last expression means that

$$\begin{aligned} \mathcal{E}_c(x_c) &= -\frac{1}{r_u(x_c)}\frac{1-r_u(x_c)}{2-r_{u'}(x_c)} \leq 0 \\ &\Rightarrow \left[ Lx_{c_1} \begin{matrix} \geq \\ \cong \\ \leq \end{matrix} Lx_{c_2} \Leftrightarrow c_1 \begin{matrix} \leq \\ \cong \\ \geq \end{matrix} c_2 \right] \end{aligned}$$

Taking into consideration the First Order Condition for Consumer Problem  $p(x_c) = \frac{u'(x_c)}{\lambda}$  and keeping in mind that  $u'(x)$  is decreasing w.r.t.  $x$  and  $x_c$  is increasing w.r.t. level of entrepreneurial abilities we get that price is increasing w.r.t. entrepreneurial abilities:

$$p_{c_1} \begin{matrix} \geq \\ \cong \\ \leq \end{matrix} p_{c_2} \Leftrightarrow c_1 \begin{matrix} \geq \\ \cong \\ \leq \end{matrix} c_2$$

□

**Proposition 5.** *Higher entrepreneurial ability implies bigger profit and vice verse:*

$$\pi_{c_1} \begin{matrix} \geq \\ \cong \\ \leq \end{matrix} \pi_{c_2} \Leftrightarrow c_1 \begin{matrix} \geq \\ \cong \\ \leq \end{matrix} c_2$$

*Proof.* see **Appendix C** (1)

□

We can explain last result in the next way: in the equilibrium less ability entrepreneur charges high price as compensation for high marginal cost, but high price declines individual consumption that variety. And change of consumption more bigger than the change of price, as a result: less entrepreneurial ability less profit.

Now, we get elasticities of consumptions w.r.t. to market expansion, and later - outputs and prices.

**Proposition 6.** *Elasticity of individual consumption of each variety (including the cutoff variety) w.r.t. market size is negative but it's bounded from below:*

$$\mathcal{E}_L\bar{x} = -\frac{1-\bar{r}_u}{2-\bar{r}_{u'_c}} \cdot (1 + \mathcal{E}_L\bar{c}) \in \left( -\frac{1}{2-\bar{r}_{u'_c}}; 0 \right) \quad (8)$$



$$\mathcal{E}_L x_c = \frac{1 - \bar{r}_u}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}} \cdot \left( \mathcal{E}_L \bar{c} - \frac{\bar{r}_u}{1 - \bar{r}_u} \right) \in \left( -\frac{1}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}}; 0 \right), \quad (9)$$

and both can be expressed through characteristics of utility as

$$\begin{aligned} \mathcal{E}_L \bar{x} &= -\frac{1}{2 - \bar{r}_{u'}} \cdot \frac{\frac{1}{r_V} - 1 + \frac{J_0}{J} + \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}} \\ \mathcal{E}_L x_c &= -\frac{1 - r_{u_c}}{2 - r_{u'_c}} \cdot \frac{\bar{r}_u}{r_{u_c}} \cdot \frac{\frac{1}{r_V} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}} \end{aligned}$$

*Proof.* See **Appendix C** (2), (3) □

*Remark.* Depending on the direction of changing entrepreneurial share in population we can set up the bounds more precisely. Strictly speaking we can find the bounds more precisely for different directions of entrepreneurial share changing .

In the case increasing share of entrepreneurs  $\left( \frac{1}{\bar{r}_u} \leq \frac{1}{r_V} + \frac{J_0}{J} \right)$  the bounds are:

$$\mathcal{E}_L x_c \in \left( -\frac{1}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}}; -\frac{\bar{r}_u}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}} \right]$$

in opposite case  $\left( \frac{1}{\bar{r}_u} \geq \frac{1}{r_V} + \frac{J_0}{J} \right)$  we have the bounds:

$$\mathcal{E}_L x_c \in \left[ -\frac{\bar{r}_u}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}}; 0 \right)$$

The change in output of an entrepreneur includes two effects: the change in everybody's output and the change in the personality (identity) of the marginal entrepreneur. Therefore the cutoff elasticity has two components. First one presents the direct influence of the market size on the individual consumptions. This part doesn't depend on the changing marginal agent when the market expands. Mathematically that components are the partial elasticities w.r.t.  $L$  without change the marginal agent:  $\mathcal{E}_L^d x = \varepsilon_L x$ . For the cutoff variety it equals to  $\mathcal{E}_L^d \bar{x} = -\frac{1 - \bar{r}_u}{2 - \bar{r}_{u'_c}}$ , and for the non-marginal variety it equals to  $\mathcal{E}_L^d x_c = -\frac{\bar{r}_u}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}}$ . Direct influence is negative anyway.

The second component presents the indirect influence of the market size on the individual consumptions. This part depend on the changing marginal agent when the market expands. For the marginal variety it equals to  $\mathcal{E}_L^i \bar{x} = -\frac{1 - \bar{r}_u}{2 - \bar{r}_{u'_c}} \mathcal{E}_L \bar{c}$ , and for the non-marginal variety it equals to  $\mathcal{E}_L^i x_c = \frac{1 - \bar{r}_u}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}} \mathcal{E}_L \bar{c}$ . Mathematically these elasticities equal to product of partial elasticities w.r.t.  $c$  and the elasticity of the cutoff  $\bar{c}$  w.r.t.  $L$ . The indirect influence shifts the individual consumption in opposite direction of changing entrepreneurship fraction for marginal variety and in the same direction for non-marginal variety.

Anyway, the direct effect is stronger than indirect one, and as a result, consumption is reduced when the market size expands.

**Proposition 7.** *Individual consumption of any  $c$ -type variety under direct effect of expanding market changes more than proportionally to population - if and only if concavity measure  $r_u$  is decreasing:*

$$|\mathcal{E}_L^d x_c| \geq 1 \Leftrightarrow r'_u \leq 0$$

*Proof.* First note, that  $\frac{\bar{r}_u}{r_{uc}} \geq 1$  if and only if  $r'_u \leq 0$  because consumption of non-marginal variety bigger than consumption of marginal variety due Proposition 2.

Then we note, that  $\frac{1-r_{uc}}{2-r_{u'c}} \geq 1$  if and only if  $r'_u \leq 0$  (see **Appendix A**/ Lemma 1/ Corollary 2).

As a result we have for nonmarginal agent  $|\mathcal{E}_L^d x_c| = \frac{\bar{r}_u}{r_{uc}} \cdot \frac{1-r_{uc}}{2-r_{u'c}} \geq 1 \Leftrightarrow r'_u \leq 0$  as well for marginal one  $|\mathcal{E}_L^d \bar{x}| = \frac{1-\bar{r}_u}{2-\bar{r}_{u'c}} \geq 1 \Leftrightarrow \bar{r}'_u \leq 0$   $\square$

This statement above is an analog of big market effect: if a firm faces agents' preferences with decreasing relative love to varieties the direct effect of expanding market enlarges firm's output more than proportionally to the market size change and vice versa.

As for an indirect influence market size to outputs of firm, we can analyze the case for firm with quite low productivity similar the productivity of the marginal firm. Due the changing marginal agent when the market size expands the limit of elasticities ratio  $\frac{x_c}{\bar{x}}$  of consumptions studied in the following proposition is nontrivial (need not equal 1). When this characteristic of equilibrium increases, it means that the difference in outputs among producers increases, they become more diversified in behavior, otherwise they become more similar.

**Proposition 8.** *The ratio  $\frac{Lx_c}{L\bar{x}}$  of outputs for firms with productivity close to cut-off productivity - increases if and only if the entrepreneurial sector expands:*

$$\lim_{c \rightarrow \bar{c}} \mathcal{E}_L \frac{Lx_c}{L\bar{x}} = \frac{1}{\bar{r}} \cdot \frac{1-\bar{r}}{2-\bar{r}_{u'}} \cdot \mathcal{E}_L \bar{c} \geq 0 \Leftrightarrow \mathcal{E}_L \bar{c} \geq 0,$$

and we have bounds for this limit as:

$$\lim_{c \rightarrow \bar{c}} \mathcal{E}_L \frac{Lx_c}{L\bar{x}} \in \left[ -\frac{1}{\bar{r}} \frac{1-\bar{r}}{2-\bar{r}_{u'}}; \frac{1}{2-\bar{r}_{u'c}} \right]$$

*Proof.* See the **Appendix C** (4)  $\square$

That property is also an analog of big market effect for firm with small level of productivity. They change its output respect to the change of market size relative to change of marginal agent output

This statement allows us to describe consumption of varieties produced by the entrepreneurs with productivity likes the marginal that. Namely: if a marginal cost of a firm is in small neighborhood of  $\bar{c}$  then its output will increase relate output of marginal producer if and only if the entrepreneurial sector enlarges.

Finally, we turn to price elasticities, which appear more interesting than outputs.

**Proposition 9.** *The elasticity of price w.r.t. market size is:*

$$\mathcal{E}_L \bar{p} = \bar{r}_u \cdot \frac{1-\bar{r}_u}{2-\bar{r}_{u'}} (1 + \mathcal{E}_L \bar{c}) - \mathcal{E}_L \lambda$$

$$\mathcal{E}_L p_c = -(1-\bar{r}_u) \frac{1-r_{uc}}{2-r_{u'c}} \left( \mathcal{E}_L \bar{c} - \frac{\bar{r}_u}{1-\bar{r}_u} \right) - \mathcal{E}_L \lambda$$

and both can be expressed through characteristics of utility as

$$\mathcal{E}_{L\bar{p}} = \frac{\bar{r}_u}{2 - \bar{r}_{u'}} \cdot \frac{\frac{1}{r_V} - 1 + \frac{J_0}{J} + \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}} - \frac{1 + \frac{\bar{r}_u}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}} = \frac{\bar{r}_u}{2 - \bar{r}_{u'}} \cdot \frac{\left( \frac{1}{r_V} - \frac{1}{\bar{r}_u} + \frac{J_0}{J} \right) - \bar{r}'_u \left( \frac{1}{\bar{r}_u} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}} \right)}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}}$$

$$\mathcal{E}_{Lp_c} = \bar{r}_u \cdot \frac{1 - r_{u_c}}{2 - r'_{u_c}} \cdot \frac{\frac{1}{\bar{r}_u} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}} - \frac{1 + \frac{\bar{r}_u}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}} = -r'_{u_c} \frac{\bar{r}_u}{2 - r'_{u_c}} \frac{\frac{1}{\bar{r}_u} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}} \stackrel{\geq}{\leq} 0 \Leftrightarrow r'_{u_c} \stackrel{\leq}{\geq} 0$$

*Proof.* Recalling that  $p_c = \frac{u'(x_c)}{\lambda}$  we get  $\mathcal{E}_{Lp_c} = -r_u(x_c) \mathcal{E}_{Lx_c} - \mathcal{E}_L \lambda$ . Then applying results from Proposition 1, Proposition 3 and Proposition 5 we hold the statement.  $\square$

*Remark.* Note that the direction of price changing for non-marginal producer is opposite to direction of  $RLV$  changing. It means that pro- and anticompetitive effects are hold for non-marginal agent in that specification. It means that the decision about price changing depend on structure of agent's preferences for variety produced by non-marginal entrepreneur. The direction of price changing for marginal agent depend on both the agent's preferences for variety produced by marginal entrepreneur and direction of cutoff change, because that elasticity of charged price can be decomposed in the next way:

$$\mathcal{E}_{L\bar{p}} = \frac{\bar{r}_u}{2 - \bar{r}_{u'}} \cdot \left( \mathcal{E}_{L\bar{c}} - \bar{r}'_u \cdot \frac{\frac{1}{\bar{r}_u} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}} \right)$$

We can estimate bounds for price elasticity more accuracy in both case expanding or shrinking share of entrepreneurs in the Economy. Note, that  $CES$  case is borderline for price behavior in expanding Economy.

$\mathcal{E}_{Lp_c}$	$\mathcal{E}_{L\bar{c}} < 0$	$\mathcal{E}_{L\bar{c}} = 0$	$\mathcal{E}_{L\bar{c}} > 0$
$\bar{r}'_u < 0$	$> -r'_{u_c} \frac{\bar{r}_u}{2 - r'_{u_c}}$	$= -r'_{u_c} \frac{\bar{r}_u}{2 - r'_{u_c}}$	$\left( 0; -r'_{u_c} \frac{\bar{r}_u}{2 - r'_{u_c}} \right)$
$\bar{r}'_u = 0$	$= 0$	$= 0$	$= 0$
$\bar{r}'_u > 0$	$< -r'_{u_c} \frac{\bar{r}_u}{2 - r'_{u_c}}$	$= -r'_{u_c} \frac{\bar{r}_u}{2 - r'_{u_c}}$	$\left( -r'_{u_c} \frac{\bar{r}_u}{2 - r'_{u_c}}; 0 \right)$

Like in proposition about outputs, the change a price charged bu an entrepreneur includes two effects: the direct one and indirect one. Taking into consideration fact that elasticity of market statistics can be decomposed in the same components as  $\mathcal{E}_L \lambda = \bar{r}_u - (1 - \bar{r}_u) \mathcal{E}_{L\bar{c}}$  we get that the first (direct) effects of market size on the price are equal to  $\mathcal{E}_{L\bar{p}}^d = \bar{r}_u \frac{1 - \bar{r}_u}{2 - \bar{r}_{u'}} - \bar{r}_u = -\frac{\bar{r}_u \bar{r}'_u}{2 - \bar{r}_{u'}}$  for the marginal firm price and  $\mathcal{E}_{Lp_c}^d = \bar{r}_u \cdot \frac{1 - r_{u_c}}{2 - r'_{u_c}} - \bar{r}_u = -\frac{\bar{r}_u r'_{u_c}}{2 - r'_{u_c}}$  for marginal one. These effects are the close each to other and difference between them tends to zero under  $c \rightarrow \bar{c}$ . The direct effects holds pro- and anticompetitive behavior of firms on the market with the constatnt share of entrepreneurs .

**Proposition 10.** *A price for any c-type variety goes up under direct effect of market expanding if and only if concavity measure  $r_u$  is increasing:*

$$\mathcal{E}_{Lp_c}^d \stackrel{\geq}{\leq} 0 \Leftrightarrow r'_u \stackrel{\leq}{\geq} 0$$

*Proof.* The proof comes directly from expressions for direct effects.  $\square$

Indirect effects are different for marginal and non-marginal agents. These effects are equal to  $\mathcal{E}_L^i \bar{p} = \left( \bar{r}_u \frac{1-\bar{r}_u}{2-\bar{r}_u} + 1 - \bar{r}_u \right) \mathcal{E}_L \bar{c} = \frac{1-\bar{r}_u}{2-\bar{r}_u} (1 + \bar{r}'_u) \mathcal{E}_L \bar{c}$ ,  $\mathcal{E}_L^i p_c = \left( 1 - \bar{r}_u - (1 - \bar{r}_u) \frac{1-r_{uc}}{2-r_{uc}} \right) \mathcal{E}_L \bar{c} = \frac{1-\bar{r}_u}{2-r_{uc}} r'_{uc} \mathcal{E}_L \bar{c}$ . The indirect effect depends on directions of changing both  $RLV$  and share of entrepreneurs in the population. For analyzing of indirect effect we can state that the limit of ratio  $\frac{\bar{p}}{p_c}$  is nontrivial. When the difference in prices among existing producers increases, they become more diversified in behavior, otherwise they become more similar.

**Proposition 11.** *The limit of elasticity of price ratio  $\frac{\bar{p}}{p_c}$  can be bigger or smaller than zero conditional upon expanding/shrinking entrepreneurs fraction, as follows*

$$\lim_{c \rightarrow \bar{c}} \mathcal{E}_L \frac{\bar{p}}{p_c} = \frac{1 - \bar{r}}{2 - \bar{r}'_u} \mathcal{E}_L \bar{c} \begin{matrix} \geq \\ \leq \end{matrix} 0 \Leftrightarrow \mathcal{E}_L \bar{c} \begin{matrix} \geq \\ \leq \end{matrix} 0$$

$$\lim_{c \rightarrow \bar{c}} \mathcal{E}_L \frac{\bar{p}}{p_c} = \frac{1 - \bar{r}}{2 - \bar{r}'_u} \mathcal{E}_L \bar{c} = \frac{\bar{r}_u}{2 - \bar{r}'_u} \frac{\frac{1}{r_V} - \frac{1}{\bar{r}_u} + \frac{J_0}{J}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1-\bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\gamma \bar{c}}{\bar{\Gamma}}} \in \left( -\frac{1 - \bar{r}}{2 - \bar{r}'_u}; \frac{\bar{r}_u}{2 - \bar{r}'_u} \right)$$

*Proof.* Considering difference between two price elasticities and keeping in mind that difference between direct effects is neglect in case  $c \rightarrow \bar{c}$  we find out that the limit equals to difference between two indirect effects.  $\square$

This proposition says that producers with low productivity manage the price relate to the price of marginal entrepreneur using as market signal only direction of changing fraction of entrepreneurs in population. Namely, if a marginal cost of a firm is in small neighborhood of  $\bar{c}$  then its price will decreases relate price of marginal producer if and only if the entrepreneurial sector enlarges.

Now we consider impact of market size on firm profit.

First of all note that elasticity of firm profit which is got from an agent w.r.t. his consumption is:

$$\mathcal{E}_{x_c} \frac{\pi_c}{L} = \mathcal{E}_{x_c} M_c + \mathcal{E}_{x_c} \frac{TR_c}{L} = \mathcal{E}_{x_c} r_{uc} + \mathcal{E}_{x_c} x_c p_c = r'_{uc} + 1 - r_{uc} = 2 - r_{uc}$$

Taking into consideration results from Proposition 6 we get

**Proposition 12.** *The elasticities of firm profit for marginal and non-marginal producers w.r.t. market size are:*

$$\mathcal{E}_L \bar{\pi} = \bar{r}_u - (1 - \bar{r}_u) \cdot \mathcal{E}_L \bar{c}$$

$$\mathcal{E}_L \pi_c = 1 - \frac{\bar{r}_u}{r_{uc}} (1 - r_{uc}) + (1 - \bar{r}_u) \cdot \frac{1 - r_{uc}}{r_{uc}} \cdot \mathcal{E}_L \bar{c}$$

and both can be expressed through characteristics of utility as

$$\mathcal{E}_L \bar{\pi} = \frac{1 + \frac{\bar{r}_u}{1-\bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\gamma \bar{c}}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1-\bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\gamma \bar{c}}{\bar{\Gamma}}} \in (0; 1) \quad (10)$$

$$\mathcal{E}_L \pi_c = 1 - \frac{1 - r_{uc}}{r_{uc}} \bar{r}_u \frac{\frac{1}{r_V} + \frac{1}{1-\bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\gamma \bar{c}}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1-\bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\gamma \bar{c}}{\bar{\Gamma}}} = \frac{\frac{1}{r_V} - \frac{1}{r_{uc}} + \frac{J_0}{J} + 1 + \frac{1+\bar{r}_u - \frac{\bar{r}_u}{r_{uc}}}{1-\bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\gamma \bar{c}}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1-\bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\gamma \bar{c}}{\bar{\Gamma}}} \in \left( \frac{2r_{uc} - 1}{r_{uc}}; 1 \right) \quad (11)$$

*Proof.* Using chain rule and the note from above we have  $\mathcal{E}_L \pi_c = \mathcal{E}_L \frac{\pi_c}{L} L = \mathcal{E}_L \frac{\pi_c}{L} + 1 = \mathcal{E}_{x_c} \frac{\pi_c}{L} \cdot \mathcal{E}_L x_c + 1 = (2 - r_{u'_c}) \mathcal{E}_L x_c + 1$ . Taken expression for elasticities of individual consumption and its bound from Proposition 6 we finish the proof.  $\square$

*Remark.* Remark from Proposition 6 allow us determine bounds for profit more precisely for cases of expanding/shrinking market. In the case increasing share of entrepreneurs  $\left(\frac{1}{\bar{r}_u} \leq \frac{1}{r_V} + \frac{J_0}{J}\right)$  the bounds for profit elasticity are:

$$\mathcal{E}_L \pi_c \in \left( \frac{2r_{u_c} - 1}{r_{u_c}}; 1 + \bar{r}_u - \frac{\bar{r}_u}{r_{u_c}} \right]$$

in opposite case  $\left(\frac{1}{\bar{r}_u} \geq \frac{1}{r_V} + \frac{J_0}{J}\right)$  we have the next bounds:

$$\mathcal{E}_L \pi_c \in \left[ 1 + \bar{r}_u - \frac{\bar{r}_u}{r_{u_c}}; 1 \right)$$

Particular, if measure of concavity exceed 0.5 market expanding influence to the profit positively.

**Corollary.** *Expanding market reduces profit inequality between producers if and only if concavity measure  $r_u$  is decreasing:*

*Proof.* Considering difference between profit elasticities for producers with different entrepreneurial abilities ( $c_1 < c_2$ ) and keeping in mind fact  $x_{c_1} > x_{c_2}$  in this case we get using (11):

$$\mathcal{E}_L \frac{\pi_{c_2}}{\pi_{c_1}} = \left( \frac{1}{r_{u_{c_2}}} - \frac{1}{r_{u_{c_1}}} \right) \bar{r}_u \frac{\frac{1}{\bar{r}_u} + \frac{\bar{u} \bar{\gamma}^c}{\bar{u} \bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{\bar{u} \bar{\gamma}^c}{\bar{u} \bar{\Gamma}}} \begin{matrix} \geq 0 \\ \leq 0 \end{matrix} \Leftrightarrow r_{u_{c_1}} \begin{matrix} \geq \\ \leq \end{matrix} r_{u_{c_2}} \Leftrightarrow r'_{u_c} \begin{matrix} \geq \\ \leq \end{matrix} 0$$

$\square$

## 4 Impact of market size under special cases of utilities

In this part we consider three important examples of specification of utility function for agent preferences for illustration of effects of market size expand on endogenous variables in our model.

First of all we gather all formulas for evaluation endogenous variables. And then we apply them to certain specifications.

### Formulas

Share of entrepreneurs in the population:

$$e = \bar{\Gamma} = \int_0^{\bar{c}} \gamma_c dc$$

Cumulative utility relate to the market size:

$$J = \int_0^{\bar{c}} u_c \gamma_c dc$$

Conditional average of local utility across all varieties:

$$\tilde{u} = \frac{J}{\bar{\Gamma}} = \frac{\int_0^{\bar{c}} u_c \gamma_c dc}{\int_0^{\bar{c}} \gamma_c dc}$$

Weighted reverse absolute risk averse:

$$J_0 = \int_0^{\bar{c}} \frac{u'(x_c) x_c}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}} \gamma_c dc = - \int_0^{\bar{c}} \frac{u'(x_c)}{u''(x_c)} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}} \gamma_c dc$$

Elasticity of cutoff:

$$\mathcal{E}_{L\bar{c}} = \frac{\bar{r}_u}{1 - \bar{r}_u} \frac{\frac{1}{r_V} - \frac{1}{\bar{r}_u} + \frac{J_0}{J}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}} \in \left( -1; \frac{\bar{r}_u}{1 - \bar{r}_u} \right]$$

Elasticity of share of entrepreneurs in the population:

$$\mathcal{E}_{Le} = \frac{\bar{\gamma}^c}{\bar{\Gamma}} \mathcal{E}_{L\bar{c}} = \frac{\bar{\gamma}^c}{\bar{\Gamma}} \frac{\bar{r}_u}{1 - \bar{r}_u} \frac{\frac{1}{r_V} - \frac{1}{\bar{r}_u} + \frac{J_0}{J}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}} \in \left( -\frac{\bar{\gamma}^c}{\bar{\Gamma}}; \frac{\bar{\gamma}^c}{\bar{\Gamma}} \frac{\bar{r}_u}{1 - \bar{r}_u} \right]$$

Amount of entrepreneurs:

$$E = L\bar{\Gamma} = L \int_0^{\bar{c}} \gamma_c dc$$

Elasticity of amount of entrepreneurs:

$$\mathcal{E}_L E = 1 + \frac{\bar{\gamma}^c}{\bar{\Gamma}} \mathcal{E}_{L\bar{c}} \in \left( 1 - \frac{\bar{\gamma}^c}{\bar{\Gamma}}; 1 + \frac{\bar{\gamma}^c}{\bar{\Gamma}} \frac{\bar{r}_u}{1 - \bar{r}_u} \right]$$

Elasticity of market statistic (elasticity of income shadow price):

$$\mathcal{E}_L \lambda = \frac{1 + \frac{\bar{r}_u}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}} \in (0; \max\{\bar{r}_u; r_V\})$$

Elasticity of individual consumption

- produced by marginal agent:

$$\mathcal{E}_L \bar{x} = -\frac{1 - \bar{r}_u}{2 - r_{u'_c}} \cdot (1 + \mathcal{E}_{L\bar{c}}) = -\frac{1}{2 - r_{u'_c}} \frac{\frac{1}{r_V} - 1 + \frac{J_0}{J} + \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}} \in \left( -\frac{1}{2 - r_{u'_c}}; 0 \right]$$

- produced by the non-marginal agent:

$$\mathcal{E}_L x_c = \frac{1 - \bar{r}_u}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}} \cdot \left( \mathcal{E}_{L\bar{c}} - \frac{\bar{r}_u}{1 - \bar{r}_u} \right) = -\frac{\bar{r}_u}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}} \frac{\frac{1}{r_V} + \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}} \in \left( -\frac{1}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}}; 0 \right]$$

Elasticity of price for variety

- charged by marginal entrepreneur:

$$\mathcal{E}_L \bar{p} = \bar{r}_u \cdot \frac{1 - \bar{r}_u}{2 - \bar{r}_{u'}} (1 + \mathcal{E}_L \bar{c}) - \mathcal{E}_L \lambda = \frac{\bar{r}_u}{2 - \bar{r}_{u'}} \cdot \frac{\left( \frac{1}{r_v} - \frac{1}{\bar{r}_u} + \frac{J_0}{J} \right) - \bar{r}'_u \left( \frac{1}{\bar{r}_u} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}} \right)}{\frac{1}{r_v} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}}$$

- charged by nonmarginal entrepreneur:

$$\mathcal{E}_L p_c = - (1 - \bar{r}_u) \frac{1 - r_{u_c}}{2 - r_{u'_c}} \left( \mathcal{E}_L \bar{c} - \frac{\bar{r}_u}{1 - \bar{r}_u} \right) - \mathcal{E}_L \lambda = -r'_{u_c} \frac{\bar{r}_u}{2 - r_{u'_c}} \frac{\frac{1}{\bar{r}_u} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}}{\frac{1}{r_v} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}}$$

Elasticity of profit

- has been got by marginal entrepreneur:

$$\mathcal{E}_L \bar{\pi} = (2 - \bar{r}_{u'}) \mathcal{E}_L \bar{x} + 1 = 1 - (1 - \bar{r}_u) \cdot (1 + \mathcal{E}_L \bar{c}) = \frac{1}{\frac{1}{r_v} + \frac{J_0}{J} + \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}} \in (0; 1)$$

- has been got by nonmarginal entrepreneur:

$$\begin{aligned} \mathcal{E}_L \pi_c &= (2 - r_{u'_c}) \mathcal{E}_L x_c + 1 = 1 + (1 - \bar{r}_u) \cdot \frac{1 - r_{u_c}}{r_{u_c}} \cdot \left( \mathcal{E}_L \bar{c} - \frac{\bar{r}_u}{1 - \bar{r}_u} \right) = \\ &= 1 - \bar{r}_u \cdot \frac{1 - r_{u_c}}{r_{u_c}} \frac{\frac{1}{\bar{r}_u} + \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}}{\frac{1}{r_v} + \frac{J_0}{J} + \frac{\bar{u}}{\bar{\Gamma}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}} \in \left( \frac{2r_{u_c} - 1}{r_{u_c}}; 1 \right] \end{aligned}$$

## Linear upper utility function

Under this utility specification the consumer's program is:

$$\begin{aligned} &L \int_0^{\bar{c}} u_c \gamma_c dc + A \rightarrow \max_{\mathbf{x}}, \\ \text{s.t. } &L \int_0^{\bar{c}} p_c x_c \gamma_c dc + A = I \end{aligned}$$

In this specification we have:  $r_v \equiv 0$

The main results when market size grows are summarized in the table bellow.

$z$	$\mathcal{E}_L z$	bounds	behavior	comments
$\bar{c}$	$\frac{\bar{r}}{1-\bar{r}}$	$(0; +\infty)$	$\nearrow$ in $\bar{r}$	under $\bar{r} > 0.5$ cutoff growth
$\bar{x}$	$-\frac{1}{2-r_{u'_c}}$	$(-\infty; -1)$	$\searrow$ in $r_{u'_c}$	consumption reduction
$\bar{y} = L\bar{x}$	$\frac{1-r_{u'_c}}{2-r_{u'_c}}$	$(-\infty; 0)$	$\searrow$ in $r_{u'_c}$	under $r_{u'_c} > 1.5$ output is fastly reduced with market size
$x_c$	0		<i>const</i>	consumption remains unchanged
$y_c = Lx_c$	1		<i>const</i>	output is proportional to the market size
$\lambda$	0		<i>const</i>	price index is unchanged
$\bar{p}$	$\frac{\bar{r}}{2-r_{u'_c}}$	$(0; +\infty)$	$\nearrow$ in $\bar{r}$ $\nearrow$ in $r_{u'_c}$	price absolute value is changing faster than consumption; under $\bar{r}' > 0$ price is changed faster than output; under $\bar{r}' = 0$ price is proportional to output, under $\bar{r}' < 0$ price is changed slower than output
$p_c$	0			price remains unchanged

This specification implicates independence of demands for varieties. As a result population growth doesn't influence on the manufacture sector's price index due that index presents price index for whole economy in this specification. It is obviously that price index is unchanged in closed economy including traditional sector without scale effect and mobile agents between sectors. Another sequence of this specification is quasi-linear of agents' preferences respect to the concern variety as for composite good, it implies that consumption of commodity produced a firm and price charged by it are unchanged respect to the market size, except marginal entrepreneur's ones, who is changed when the population increase. A entrepreneurs fraction and amount can increase dramatically when measure of concavity of lower-tier utility function closes to the unit.

## Lower-tier utility function is CES, $r_V < 1$ for upper-tier utility

Under this specification the consumer program is:

$$\begin{aligned}
V \left( L \int_0^{\bar{c}} x_c^\rho \gamma_c dc \right) + A &\rightarrow \max_{x_c}, c \in [0; \bar{c}] \\
s.t. L \int_0^{\bar{c}} p_c x_c \gamma_c dc + A &= I
\end{aligned}$$



In this specification we have:  $r_u \equiv 1 - \rho$  at any  $x$ ,  $\frac{J_0}{J} = \frac{\rho}{1-\rho}$

The main results when market size grows are summarized in the table bellow.

$z$	$\mathcal{E}_L z$	bounds	behavior	comments
$\bar{c}$	$\frac{\left(\frac{1}{r_V} - 1\right) \frac{1-\rho}{\rho}}{\frac{1}{r_V} + \frac{\rho}{1-\rho} + \frac{1}{\rho} \frac{\bar{u}\gamma\bar{c}}{J}}$	$\left(0; \frac{1-\rho}{\rho}\right)$	$\searrow$ in $r_V$	under $\rho \rightarrow 0$ , upper boundary expands; under $\rho > 0.5$ growth is slow
$\bar{x}$	$-\frac{\frac{1}{\rho} \left(1 + \frac{1}{r_V} - \frac{\rho}{1-\rho} + \frac{\bar{u}\gamma\bar{c}}{J}\right)}{\frac{1}{r_V} + \frac{\rho}{1-\rho} + \frac{1}{\rho} \frac{\bar{u}\gamma\bar{c}}{J}}$	$\left(-\frac{1}{\rho}; -1\right)$	$\nearrow$ in $r_V$	consumption declines speedily; lower boundary expands respect to the $\rho$
$\bar{y} = L\bar{x}$	$-\frac{\left(\frac{1}{r_V} - 1\right) \frac{1-\rho}{\rho}}{\frac{1}{r_V} + \frac{\rho}{1-\rho} + \frac{1}{\rho} \frac{\bar{u}\gamma\bar{c}}{J}}$	$\left(-\frac{1-\rho}{\rho}; 0\right)$	$\nearrow$ in $r_V$	output is reciprocal to the marginal cost of producing; under $\rho \rightarrow 0$ lower boundary expands; under $\rho > 0.5$ output declines slowly
$x_c$	$-\frac{\frac{1}{1-\rho} + \frac{1}{\rho} \frac{\bar{u}\gamma\bar{c}}{J}}{\frac{1}{r_V} + \frac{\rho}{1-\rho} + \frac{1}{\rho} \frac{\bar{u}\gamma\bar{c}}{J}}$	$(-1; 0)$	$\searrow$ in $r_V$	consumption is reduced slowly; by absolute values it is changed slower than market size expands
$y_c = Lx_c$	$\frac{\frac{1}{r_V} - 1}{\frac{1}{r_V} + \frac{\rho}{1-\rho} + \frac{1}{\rho} \frac{\bar{u}\gamma\bar{c}}{J}}$	$(0; 1)$	$\searrow$ in $r_V$	output is increased slowly, but it is changed slower than market size expands
$\lambda$	$\frac{1 + \frac{1-\rho}{\rho} \frac{\bar{u}\gamma\bar{c}}{J}}{\frac{1}{r_V} + \frac{\rho}{1-\rho} + \frac{1}{\rho} \frac{\bar{u}\gamma\bar{c}}{J}}$	$(0; 1 - \rho)$	$\nearrow$ in $r_V$	level of price index increases slowly, upper boundary shrinks with $\rho$
$\bar{p}$	$\frac{\frac{1-\rho}{\rho} \left(\frac{1}{r_V} - 1\right)}{\frac{1}{r_V} + \frac{\rho}{1-\rho} + \frac{1}{\rho} \frac{\bar{u}\gamma\bar{c}}{J}}$	$\left(0; \frac{1-\rho}{\rho}\right)$	$\searrow$ in $r_V$	price grows if and only if concavity of upper-tier utility is pretty small
$p_c$	0	<i>const</i>		price is unchanged

As a rule a specification with *CES* is borderline in models like ours, but as can be seen from table above the cutoff cost and consequently entrepreneurs fraction, amount of firms and average productivity move up if the market size expand. It implies that consumption produced by an ordinal firm decreases, although the firm's output increases, but less proportional to the market size. Interesting that the price charged by non-marginal entrepreneur doesn't changed. As for marginal entrepreneur, his output decreases and price increases because the marginal agent's identity (number) is changed and she becomes less efficient when economy expands. As a result market characteristic (price index) increase too.

## Upper-tier utility function is logarithm

In this specification consumer problem is:

$$\ln \left( L \int_0^{\bar{c}} u_c \gamma_c dc \right) + A \rightarrow \max_{x_c}, c \in [0; \bar{c}]$$

$$s.t. L \int_0^{\bar{c}} p_c x_c \gamma_c dc + A = I$$

In this specification we have:  $r_v \equiv 1$

This specification is interesting because the concavity of upper-tier utility is the maximum as possible. In this case we can observe both effects of expanding market size: pro- and anti-competitive ones. As in Zhelobodko et al.[8] these effects correspond to the behavior of concavity measure of lower-tier utility.

The result about expand/shrinking entrepreneur sector when population grows is displayed in the table:

$\mathcal{E}_{L\bar{c}}$	$r'_{u_c} < 0$	$r'_{u_c} = 0$	$r'_{u_c} > 0$
$r_{\ln(u_c)} < 1$	?	< 0	< 0
$r_{\ln(u_c)} = 1$	> 0	= 0	
$r_{\ln(u_c)} > 1$		> 0	?

Note, that CES local utility function this model is borderline as in the others numerous models like ours. Also note, that the case with constant love for variety is nontrivial unlike Zhelobodko et al.[8], where agents are homogeneous.

These conditions might be rewritten using elasticity terms:

$\mathcal{E}_{L\bar{c}}$	$(\mathcal{E}_{x_c u_c})' > 0$	$(\mathcal{E}_{x_c u_c})' = 0$	$(\mathcal{E}_{x_c u_c})' < 0$
$(\mathcal{E}_{x_c u_c})' > 0$	?	< 0	< 0
$(\mathcal{E}_{x_c u_c})' = 0$	> 0	= 0	
$(\mathcal{E}_{x_c u_c})' < 0$		> 0	> 0

When the concavity of lower-tier utility doesn't change the expanding or shrinking of entrepreneurs fraction is related to behavior of utility's elasticity. It means that ratio of marginal and average utility is important for comparison of equilibriums in economies differening in population size.

The estimates of elasticities of consumption and output of the commodity produced by the marginal entrepreneur are displayed in the following table:

$\mathcal{E}_{L\bar{x}}$	$\mathcal{E}_{L\bar{y}}$	$r'_{u_{\bar{c}}} < 0$	$r'_{u_{\bar{c}}} = 0$	$r'_{u_{\bar{c}}} > 0$
$r_{\ln(u_{\bar{c}})} < 1$	?		> -1	> -1
			> 0	> 0
$r_{\ln(u_{\bar{c}})} = 1$			< -1	< 0
			< 0	
$r_{\ln(u_{\bar{c}})} > 1$			< -1	?
			< 0	

Thus we have finished comparative statics for special cases.

# Conclusion

Our analysis shows that the structure of employment/entrepreneurship can be influenced by the economy size. Population growth changes equilibrium variables - entrepreneurs fraction, amount of firms, consumptions and price/output composition of firms - depending on preferences of agents. For their increase or decrease, the substitution between manufactured and traditional goods matter, as well as the substitution between (differentiated) varieties, which is rather new in the literature (instead of CES and other specific utilities). Our results on comparative statics can be used for econometric testing on cross-sectional data to detect which industries display increasing or decreasing RLV.

This approach can be extended in several directions: 1) comparison of equilibria with social optimum, 2) modeling of trade, 3) studying similar model without quasi-linearity of preferences, that allow for natural income effects.

# Appendix

## Appendix A (Lemmas)

**Lemma 1.**  $\mathcal{E}_x[\mathcal{E}_x f(x)] = 1 + \mathcal{E}_x f'(x) - \mathcal{E}_x f(x)$

*Proof.* this result is derived with direct evaluation

$$\begin{aligned} \mathcal{E}_x[\mathcal{E}_x f(x)] &= \frac{d\mathcal{E}_x f(x)}{dx} \frac{x}{\mathcal{E}_x f(x)} = \left( \frac{f'(x)x}{f(x)} \right)' \frac{f(x)}{f'(x)x} x = \\ &= \left( \frac{f''(x)x}{f(x)} + \frac{f'(x)}{f(x)} - \frac{f'^2(x)x}{f^2(x)} \right) \frac{f(x)}{f'(x)} = \frac{f''(x)x}{f'(x)} + 1 - \frac{f'(x)x}{f(x)} = \\ &= 1 + \mathcal{E}_x f'(x) - \mathcal{E}_x f(x) \end{aligned}$$

□

**Corollary 1.**  $\mathcal{E}_x r_u(x) = 1 - r_{u'}(x) + r_u(x)$

*Proof.* Taking into the consideration that  $r_u(x) = -\mathcal{E}_x u'(x)$  and  $r_{u'}(x) = -\mathcal{E}_x u''(x)$ , and fact  $\mathcal{E}_x f(x) = \mathcal{E}_x(-f(x))$  we get the statement of corollary just alapplying Lemma's statement for  $f(x) \equiv u'(x)$  □

**Corollary 2.**  $\frac{2-r_{u'}(x)}{1-r_u(x)} \geq 1 \Leftrightarrow r'_u(x) \geq 0$

*Proof.*  $\mathcal{E}_x r_u(x) = 1 - r_{u'}(x) + r_u(x) = [2 - r_{u'}(x)] - [1 - r_u(x)] \geq 0 \Leftrightarrow r'_u(x) \geq 0$  It follows that  $\frac{2-r_{u'}(x)}{1-r_u(x)} \geq 1 \Leftrightarrow r'_u(x) \geq 0$  □

**Lemma 2.**  $r_{\ln u}(x) = -\mathcal{E}_x u'(x) + \mathcal{E}_x u(x) = r_u(x) + \mathcal{E}_x u(x)$

*Proof.* Note firstly, that  $r_{\ln u}(x) = -\mathcal{E}_x \ln' u(x)$

$$r_{\ln u}(x) = -\mathcal{E}_x \ln' u(x) = -\frac{d\ln' u(x)}{dx} \frac{x}{\ln' u(x)} = -\left( \frac{u'(x)}{u(x)} \right)' \frac{u(x)x}{u'(x)} =$$

$$\begin{aligned}
&= - \left( \frac{u''(x)}{u(x)} - \frac{u'^2(x)x}{u^2(x)} \right) \frac{u(x)x}{u'(x)} = - \frac{u''(x)x}{u'(x)} - \frac{u'(x)x}{u(x)} = -\mathcal{E}_x u'(x) + \mathcal{E}_x u(x) = \\
&= r_u(x) + \mathcal{E}_x u(x)
\end{aligned}$$

□

**Corollary.**  $\mathcal{E}_x[\mathcal{E}_x u(x)] = 1 - r_{\ln u}(x)$

*Proof.* Applying the statement of the Lemma 1 to the  $f(x) \equiv u(x)$  and take into the consideration  $r_u(x) = -\mathcal{E}_x u'(x)$  we have:

$$\mathcal{E}_x[\mathcal{E}_x u(x)] = 1 + \mathcal{E}_x u'(x) - \mathcal{E}_x u(x) = 1 - r_u(x) - \mathcal{E}_x u(x) = 1 - r_{\ln u}(x)$$

□

**Lemma 3.**  $\mathcal{E}_x[(1 - r_u(x)) u'(x)] = -\frac{2 - r_{u'}(x)}{1 - r_u(x)} r_u(x)$

*Proof.* Let's take the derivate w.r.t.  $x$ :

$$\mathcal{E}_x[(1 - r_u(x)) u'(x)] = -\frac{r_u(x)}{1 - r_u(x)} \mathcal{E}_x r_u(x) - r_u(x)$$

Using the Corollary 1 from the Lemma 1 we have:

$$\mathcal{E}_x[(1 - r_u(x)) u'(x)] = -\frac{2 - r_{u'}(x)}{1 - r_u(x)} r_u(x)$$

□

## Appendix B (Producer Problem)

**Producer problem** for  $c$ -type entrepreneur is

$$\Pi_c = (\mathbf{p}(x_c) - c) L x_c - w \rightarrow \max$$

First order condition is:  $\frac{d\Pi_c}{dx_c} = 0$

$$(p(x_c) - c) L + p'_c L x_c = 0$$

$$-\frac{p'_c x_c}{p_c} = \frac{p_c - c}{p_c} = M_c \in (0; 1) \quad (12)$$

here:  $M_c$  is a mark-up for  $c$ -type producer.

$$p'_c = \frac{\partial p_c}{\partial x_c} = V' u''_c \quad (13)$$

$$\frac{p'_c x_c}{p_c} = \frac{V' u''_c x_c}{V' u'_c} \equiv -r_u(x_c) \equiv -r_{u_c} \quad (14)$$

Using (12) and (14) we get (15) and (16):

$$1 - r_u(x_c) = \frac{c}{p_c} \quad (15)$$

$$c = (1 - r_u) p_c \quad (16)$$

Second order condition is:

$$S.O.C. \frac{d^2 \Pi_c}{dx_c^2} = 2p'_c + p''_c x_c = 2V' u''_c + V' u'''_c x_c < 0$$

It's equals to

$$r_{u'_c} \equiv r_{u'}(x_c) \equiv \frac{u'''_c x_c}{u''_c} < 2$$

It means that the **FOC** determines the maximum of profit.

## Appendix C (Impact of Market Size on Outputs and Consumptions)

(1) *Higher entrepreneurial ability implies bigger profit and vice versa:*

$$\pi_{c_1} \begin{matrix} \geq \\ \leq \end{matrix} \pi_{c_2} \Leftrightarrow c_1 \begin{matrix} \geq \\ \leq \end{matrix} c_2$$

*Proof.* Let's write profit expression  $\pi_c = (p_c - c) L x_c$

Then let's take elasticities from both sides of last

$$\begin{aligned} \frac{d\pi_c}{dc} &= L \left[ \frac{dp_c}{dc} x_c - x_c + (p_c - c) \frac{dx_c}{dc} \right] \\ c \frac{d\pi_c}{dc} &= L \left[ \frac{dp_c}{dc} \frac{c}{p_c} p_c x_c - c x_c + (p_c - c) \frac{dx_c}{dc} \frac{c}{x_c} x_c \right] = (\mathcal{E}_c p_c + \mathcal{E}_c x_c) \pi_c \end{aligned}$$

$$\mathcal{E}_c \pi_c = \mathcal{E}_c p_c + \mathcal{E}_c x_c$$

Keeping in mind **FOC** for consumer problem (1) we have the next relation between elasticities of price and consumption:

$$\mathcal{E}_c p_c = -r_u(x_c) \mathcal{E}_c x_c$$

From Proposition 4 we have  $\mathcal{E}_c x_c$  and as a result:

$$\begin{aligned} \mathcal{E}_c \pi_c &= \mathcal{E}_c x_c (1 - r_u) \leq 0 \\ \Rightarrow \left[ \pi_{c_1} \begin{matrix} \geq \\ \leq \end{matrix} \pi_{c_2} \Leftrightarrow c_1 \begin{matrix} \geq \\ \leq \end{matrix} c_2 \right] \end{aligned}$$

□

(2) *Elasticity of individual consumption of cutoff variety w.r.t. the market size is negative, but it is bounded from below:*

$$\mathcal{E}_L \bar{x} = -\frac{1 - \bar{r}_u}{2 - \bar{r}_{u'_c}} \cdot (1 + \mathcal{E}_L \bar{c}) \in \left( \frac{1}{2 - \bar{r}_{u'_c}}; 0 \right]$$

$$\mathcal{E}_L \bar{x} = -\frac{1}{2 - \bar{r}_{u'_c}} \frac{\frac{1}{r_V} - 1 + \frac{J_0}{J} + \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}}$$

*Proof.* The **Entry Condition** (5)  $\frac{\bar{r}_u \bar{x}}{1 - \bar{r}_u} = \frac{1}{L \bar{c}}$  determines the individual consumption of  $\bar{x}$  as a function of two variables  $\bar{x}(\bar{c}; L)$

Taking the partial elasticity of  $\bar{x}$  w.r.t.  $\bar{c}$  we get:

$$\mathcal{E}_{\bar{c}} \left[ \frac{\bar{r}_u \bar{x}}{1 - \bar{r}_u} \right] = \mathcal{E}_{\bar{c}} \left[ \frac{1}{L \bar{c}} \right]$$

$$\left( \mathcal{E}_{\bar{x}} \bar{r}_u + \frac{\bar{r}_u}{1 - \bar{r}_u} \mathcal{E}_{\bar{x}} \bar{r}_u + 1 \right) \mathcal{E}_{\bar{c}} \bar{x} = -1$$

Using the corollary from Lemma 1 we have:

$$\left( \frac{1 - \bar{r}_{u'} + \bar{r}_u}{1 - \bar{r}_u} + 1 \right) \mathcal{E}_{\bar{c}} \bar{x} = -1$$

$$\mathcal{E}_{\bar{c}} \bar{x} = -\frac{1 - \bar{r}_u}{2 - \bar{r}_{u'}} \leq 0 \quad (17)$$

By the symmetry of the expression  $\frac{\bar{r}_u \bar{x}}{1 - \bar{r}_u} = \frac{1}{L \bar{c}}$  the partial elasticity of  $\bar{x}$  w.r.t.  $\bar{c}$  we get:

$$\mathcal{E}_L \bar{x} = -\frac{1 - \bar{r}_u}{2 - \bar{r}_{u'}} \leq 0 \quad (18)$$

Combining 17 and 18 we get the equation from the Theorem statement:

$$\mathcal{E}_L \bar{x} = -\frac{1 - \bar{r}_u}{2 - \bar{r}_{u'}} \cdot (1 + \mathcal{E}_L \bar{c}) \quad (19)$$

Consider the fact  $\mathcal{E}_L \bar{c} \in \left[ -1; \frac{\bar{r}_u}{1 - \bar{r}_u} \right]$ , which will be proved later (see the **Appendix D** (32) and (33)).

It will let us estimate boundaries:  $\mathcal{E}_L \bar{x} \in \left( \frac{1}{2 - \bar{r}_{u'_c}}; 0 \right]$ .

Taking into consideration the expression for the  $\bar{c}$  (see the **Appendix D** and (31)) we set up the expression for that elasticity:

$$\mathcal{E}_L \bar{x} = -\frac{1}{2 - \bar{r}_{u'}} \frac{\frac{1}{r_V} - 1 + \frac{J_0}{J} + \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}} \leq 0$$

□

(3) *Elasticity of individual consumption of non-cutoff variety w.r.t. the market size is negative:*

$$\mathcal{E}_L x_c = \frac{1 - r_{uc}}{r_{uc}} \cdot \frac{1 - \bar{r}_u}{2 - r_{u'_c}} \cdot \left( \mathcal{E}_L \bar{c} - \frac{\bar{r}_u}{1 - \bar{r}_u} \right) \in \left( -\frac{1}{r_{uc}} \cdot \frac{1 - r_{uc}}{2 - r_{u'_c}}; 0 \right]$$

$$\mathcal{E}_L x_c = -\frac{\bar{r}_u}{r_{uc}} \cdot \frac{1 - r_{uc}}{2 - r_{u'_c}} \frac{\frac{1}{\bar{r}_u} + \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}}$$

*Proof.* From (16) we have the relation between non-marginal and marginal producers:

$$\frac{(1 - r_u(x_c)) u'(x_c)}{c} = \lambda = \frac{(1 - \bar{r}_u) \bar{u}'}{\bar{c}}$$

Taken the elasticity of the left side w.r.t.  $L$  and used result from Lemma 3 and the chain rule we have:

$$\begin{aligned} \mathcal{E}_L \left[ \frac{(1 - r_u(x_c)) u'(x_c)}{c} \right] &= \mathcal{E}_{x_c} [(1 - r_u(x_c)) u'(x_c)] \cdot \mathcal{E}_L x_c = \\ &= -\frac{2 - r_{u'}(x_c)}{1 - r_u(x_c)} r_u(x_c) \cdot \mathcal{E}_L x_c \end{aligned}$$

The elasticity of the left side we evaluate using the same way, but taken into consideration nonzero elasticity cutoff w.r.t. market size and the expression for  $\mathcal{E}_L \bar{x}$  from the (19):

$$\begin{aligned} \mathcal{E}_L \left[ \frac{(1 - \bar{r}_u) \bar{u}'}{\bar{c}} \right] &= \mathcal{E}_{\bar{c}} \left[ \frac{(1 - \bar{r}_u) \bar{u}'}{\bar{c}} \right] \cdot \mathcal{E}_L \bar{c} = \mathcal{E}_{\bar{x}} [(1 - \bar{r}_u) \bar{u}'] \cdot \mathcal{E}_L \bar{x} \cdot \mathcal{E}_L \bar{c} - \mathcal{E}_L \bar{c} = \\ &= -\mathcal{E}_L \bar{c} + \frac{2 - \bar{r}_{u'}}{1 - \bar{r}_u} \bar{r}_u \cdot \frac{1 - \bar{r}_u}{2 - \bar{r}_{u'}} \cdot (1 + \mathcal{E}_L \bar{c}) = \bar{r}_u - (1 - \bar{r}_u) \mathcal{E}_L \bar{c} \end{aligned}$$

Elasticities from both sides are equal, so we have the statement:

$$\begin{aligned} -\frac{2 - r_{u'}(x_c)}{1 - r_u(x_c)} r_u(x_c) \cdot \mathcal{E}_L x_c &= \bar{r}_u - (1 - \bar{r}_u) \mathcal{E}_L \bar{c} \\ \mathcal{E}_L x_c &= \frac{1 - \bar{r}_u}{r_{uc}} \cdot \frac{1 - r_u(x_c)}{2 - r_{u'}(x_c)} \cdot \left( \mathcal{E}_L \bar{c} - \frac{\bar{r}_u}{1 - \bar{r}_u} \right) \end{aligned} \quad (20)$$

Consider the fact  $\mathcal{E}_L \bar{c} \in \left[ -1; \frac{\bar{r}_u}{1 - \bar{r}_u} \right]$ , which will be proved later (see the **Appendix D** (32) and (33)). It will lead that  $\mathcal{E}_L x_c \in \left( -\frac{1}{r_{uc}} \cdot \frac{1 - r_{uc}}{2 - r_{u'_c}}; 0 \right]$ .

Taking into consideration the expression for the  $\bar{c}$  (see the **Appendix D** (33)) we set up the expression for that elasticity:

$$\mathcal{E}_L x_c = -\frac{1 - r_{uc}}{2 - r_{u'_c}} \cdot \frac{\bar{r}_u}{r_{uc}} \frac{\frac{1}{\bar{r}_u} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma} \bar{c}}{\bar{\Gamma}}}$$

□

(3) *The elasticity of marginal utility from consumption of variety w.r.t. is:*

$$\mathcal{E}_L \bar{u}' = \bar{r}_u \cdot \frac{1 - \bar{r}_u}{2 - \bar{r}_{u'}} (1 + El_L \bar{c}) \geq 0 \quad (21)$$

$$\mathcal{E}_L u' = -(1 - \bar{r}_u) \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}} \cdot \left( \mathcal{E}_L \bar{c} - \frac{\bar{r}_u}{1 - \bar{r}_u} \right) \geq 0 \quad (22)$$

*Proof.* Using the chain rule for elasticities we have:  $\mathcal{E}_L u'(x) = \mathcal{E}_x u'(x) \cdot \mathcal{E}_L x$ . From (1) and (2)  $\mathcal{E}_x p(x)$  we have:

$$\mathcal{E}_L u'(x) = \mathcal{E}_x u'(x) \cdot \mathcal{E}_L x = \mathcal{E}_x p(x) \cdot \mathcal{E}_L x = -r_u(x) \cdot \mathcal{E}_L x$$

Applying last equality to the expressions (8) and (9) we finish the proofing.  $\square$

(4) *The ratio  $\frac{Lx_c}{L\bar{x}}$  of outputs for firms with productivity closed to the productivity of marginal firm increases if and only if the entrepreneur sector expands:*

$$\lim_{c \rightarrow \bar{c}} \mathcal{E}_L \frac{Lx_c}{L\bar{x}} = \frac{1}{\bar{r}} \cdot \frac{1 - \bar{r}}{2 - \bar{r}_{u'}} \cdot \mathcal{E}_L \bar{c} \geq 0 \Leftrightarrow \mathcal{E}_L \bar{c} \geq 0,$$

and we have also bounds for this limit:

$$\lim_{c \rightarrow \bar{c}} \mathcal{E}_L \frac{Lx_c}{L\bar{x}} \in \left[ -\frac{1}{\bar{r}} \frac{1 - \bar{r}}{2 - \bar{r}_{u'}}; \frac{1}{2 - \bar{r}_{u'}} \right]$$

*Proof.* We can decompose that elasticity into two terms: one of them linked with direct effect of expanding market and second one linked with indirect that.

$$\mathcal{E}_L \frac{x_c}{\bar{x}} = \mathcal{E}_L^d \frac{x_c}{\bar{x}} + \mathcal{E}_L^i \frac{x_c}{\bar{x}}$$

The limit for the first term equals zero:

$$\begin{aligned} \lim_{c \rightarrow \bar{c}} \mathcal{E}_L^d \left( \frac{Lx_c}{L\bar{x}} \right) &= \lim_{c \rightarrow \bar{c}} (\mathcal{E}_L^d x_c - \mathcal{E}_L^d \bar{x}) = \\ &= \lim_{c \rightarrow \bar{c}} \left( -\frac{\bar{r}_u}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}} + \frac{1 - \bar{r}_u}{2 - \bar{r}_{u'_c}} \right) = -\frac{\bar{r}_u}{\bar{r}_u} \cdot \frac{1 - \bar{r}_u}{2 - \bar{r}_{u'_c}} + \frac{1 - \bar{r}_u}{2 - \bar{r}_{u'_c}} = 0 \end{aligned}$$

On the other hand indirect effects for consumption of marginal and non-marginal variety are different and we have non-trivial limit:

$$\begin{aligned} \lim_{c \rightarrow \bar{c}} \mathcal{E}_L^i \left( \frac{Lx_c}{L\bar{x}} \right) &= \lim_{c \rightarrow \bar{c}} (\mathcal{E}_L^i x_c - \mathcal{E}_L^i \bar{x}) = \\ &= \lim_{c \rightarrow \bar{c}} \left( \frac{1 - \bar{r}_u}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}} + \frac{1 - \bar{r}_u}{2 - \bar{r}_{u'_c}} \right) \mathcal{E}_L \bar{c} = \\ &= \left( \frac{1 - \bar{r}_u}{\bar{r}_u} \cdot \frac{1 - \bar{r}_u}{2 - \bar{r}_{u'_c}} + \frac{1 - \bar{r}_u}{2 - \bar{r}_{u'_c}} \right) \mathcal{E}_L \bar{c} = \frac{1}{\bar{r}_u} \frac{1 - \bar{r}_u}{2 - \bar{r}_{u'_c}} \mathcal{E}_L \bar{c} \end{aligned}$$

Taking into consideration the upper bound for elasticity of cutoff  $\mathcal{E}_L \bar{c} \in \left[ -1; \frac{\bar{r}_u}{1 - \bar{r}_u} \right]$  (see **Appendix D**) we have:



$$\lim_{c \rightarrow \bar{c}} \mathcal{E}_L^i \frac{Lx_c}{L\bar{x}} = \frac{1}{\bar{r}_u} \frac{1 - \bar{r}_u}{2 - \bar{r}_{u'_c}} \mathcal{E}_L \bar{c} \in \left[ -\frac{1}{\bar{r}} \frac{1 - \bar{r}}{2 - \bar{r}_{u'}}; \frac{1}{2 - \bar{r}_{u'_c}} \right]$$

□

## Appendix D (Elasticity of lambda and cutoff)

Using the (16) and (1) we get:

$$\mathcal{E}_L \bar{c} = \mathcal{E}_L [\bar{u}' (1 - \bar{r}_u)] + \mathcal{E}_L V' \quad (23)$$

$$\mathcal{E}_L [\bar{u}' (1 - \bar{r}_u)] = \mathcal{E}_{\bar{x}} [\bar{u}' (1 - \bar{r}_u)] \mathcal{E}_L \bar{x}$$

From the result from Lemma 3 and (19) we have:

$$\mathcal{E}_L [\bar{u}' (1 - \bar{r}_u)] = \bar{r}_u (1 + \mathcal{E}_L \bar{c}) \quad (24)$$

Consider elastisity  $\mathcal{E}_L V' (Y)$  :

$$\mathcal{E}_L V' (Y) = \mathcal{E}_Y V' (Y) \mathcal{E}_L Y$$

$$\mathcal{E}_Y V' (Y) = -r_V$$

$$Y = L \int_0^{\bar{c}} u_c \gamma_c dc$$

$$\mathcal{E}_L Y = 1 + \mathcal{E}_L J (\bar{c}, L)$$

here:  $J (\bar{c}, L) = \int_0^{\bar{c}} u (x (c; \bar{c} (L); L)) \gamma_c dc$

As a result we have:

$$\mathcal{E}_L V' (Y) = -r_V - r_V \mathcal{E}_L J (\bar{c}, L) \quad (25)$$

Consider elastisity  $\mathcal{E}_L J (\bar{c}, L)$  :

$$\mathcal{E}_L J (\bar{c}, L) = \varepsilon_L J (\bar{c}, L) + \varepsilon_{\bar{c}} J (\bar{c}, L) \mathcal{E}_L \bar{c} \quad (26)$$

$$\varepsilon_{\bar{c}} J (\bar{c}, L) = \frac{\partial J}{\partial \bar{c}} \cdot \frac{\bar{c}}{J} = \frac{\bar{u} \bar{\gamma} \bar{c}}{J} = \frac{\bar{u} \bar{\gamma} \bar{c}}{\tilde{u} \bar{\Gamma}} \quad (27)$$

here:  $\tilde{u} = \frac{J}{\bar{\Gamma}}$ ,  $\bar{\Gamma} = \int_0^{\bar{c}} \gamma_c dc$

$$\varepsilon_L J (\bar{c}, L) = \frac{\partial J}{\partial L} \cdot \frac{L}{J} = \int_0^{\bar{c}} u'_c \mathcal{E}_L x_c \frac{x_c}{L} \gamma_c dc \cdot \frac{L}{J}$$

Using (20) we have:

$$\begin{aligned}
\varepsilon_L J(\bar{c}, L) &= \frac{1}{J} \int_0^{\bar{c}} u'_c \cdot \mathcal{E}_L x_c \cdot \frac{x_c}{L} \cdot \gamma_c dc = \\
&= \frac{1}{J} \int_0^{\bar{c}} u'_c \cdot x_c \cdot \frac{1 - \bar{r}_u}{r_{uc}} \cdot \frac{1 - r_{uc}}{2 - r_{u'_c}} \cdot \left( \mathcal{E}_L \bar{c} - \frac{\bar{r}_u}{1 - \bar{r}_u} \right) \gamma_c dc = (1 - \bar{r}) \frac{J_0}{J} \left( El_L \bar{c} - \frac{\bar{r}_u}{1 - \bar{r}_u} \right)
\end{aligned} \tag{28}$$

here:  $J_0 = \int_0^{\bar{c}} \frac{u'_c x_c}{r_{uc}} \cdot \frac{1 - r_{uc}}{2 - r_{u'_c}} \gamma_c dc$

Using equations 27 and (28) we can express the equality (26) in the form:

$$\mathcal{E}_L J(\bar{c}, L) = \left( (1 - \bar{r}_u) \frac{J_0}{J} + \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}} \right) \cdot \mathcal{E}_L \bar{c} - \frac{J_0}{J} \bar{r}_u \tag{29}$$

Using (25) we've got:

$$\mathcal{E}_L V'(Y) = -r_V \left( (1 - \bar{r}_u) \frac{J_0}{J} + \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}} \right) \cdot \mathcal{E}_L \bar{c} + \frac{J_0}{J} \bar{r}_u r_V - r_V \tag{30}$$

Using (23), (24) and (30) we have:

$$\begin{aligned}
\mathcal{E}_L \bar{c} &= \bar{r}_u + \bar{r}_u \mathcal{E}_L \bar{c} - r_V \left( (1 - \bar{r}_u) \frac{J_0}{J} + \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}} \right) \cdot \mathcal{E}_L \bar{c} + \frac{J_0}{J} \bar{r}_u r_V - r_V \\
\mathcal{E}_L \bar{c} &= \frac{\bar{r}_u - r_V + \frac{J_0}{J} r_V \bar{r}_u}{1 - \bar{r}_u + \frac{J_0}{J} r_V (1 - \bar{r}_u) + r_V \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}}
\end{aligned}$$

As a result we have:

$$\mathcal{E}_L \bar{c} = \frac{\bar{r}_u}{1 - \bar{r}_u} \frac{\frac{1}{r_V} - \frac{1}{\bar{r}_u} + \frac{J_0}{J}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}} \tag{31}$$

if  $r_V < 1$  then:

$$\mathcal{E}_L \bar{c} + 1 = \frac{1}{1 - \bar{r}_u} \frac{\frac{1}{r_V} - 1 + \frac{J_0}{J} + \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}} \geq 0 \tag{32}$$

$$\mathcal{E}_L \bar{c} - \frac{\bar{r}_u}{1 - \bar{r}_u} = -\frac{\bar{r}_u}{1 - \bar{r}_u} \frac{\frac{1}{\bar{r}_u} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}} \leq 0 \tag{33}$$

$$\mathcal{E}_L \bar{c} \geq 0 \Leftrightarrow \frac{1}{\bar{r}_u} - \frac{1}{r_V} \leq \frac{J_0}{J}$$

Put the expression (31) into the equation (23) and using (24) we obtained the elasticity for market statistics w.r.t. population in the following form:

$$\mathcal{E}_L \lambda = -\mathcal{E}_L V' = \bar{r}_u - (1 - \bar{r}_u) \mathcal{E}_L \bar{c} = \frac{1 + \frac{\bar{r}_u}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u} \bar{\gamma} \bar{c}}{\bar{u} \bar{\Gamma}}} > 0 \tag{34}$$

Put the expressions (32) and (33) into the expressions (19) and (20) we get:

$$\mathcal{E}_L \bar{x} = -\frac{r_V}{2 - \bar{r}_{u'}} \frac{\frac{1}{r_V} - 1 + \frac{J_0}{J} + \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}}$$

$$\mathcal{E}_L x_c = -\frac{1 - r_{u_c}}{2 - r_{u'_c}} \cdot \frac{\bar{r}_u}{r_{u_c}} \frac{\frac{1}{\bar{r}_u} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1 - \bar{r}_u} \frac{\bar{u}}{\bar{u}} \frac{\bar{\gamma}^c}{\bar{\Gamma}}}$$

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