

Biscaia, Ricardo; Sarmento, Paula

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Location Decisions in a Natural Resource Model of Cournot Competition

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Location Decisions in a Natural Resource Model of Cournot Competition

Ricardo Biscaia^{a,b,c,*} (ricardo_biscaia@hotmail.com)

Paula Sarmento^{a,c} (sarmento@fep.up.pt)

a - FEP – Faculty of Economics of University of Porto, Rua Dr. Roberto Frias, 4200-464 Porto, Portugal

b - CIPES, Center for Research in Higher Education Policies, Rua 1º Dezembro, 399, 4450-227 Matosinhos, Portugal

c – CEF.UP - Center for Economics and Finance at UP, Rua Dr. Roberto Frias, 4200-464 Porto, Portugal

*Corresponding author

Ricardo Biscaia

E-mail Address: ricardo_biscaia@hotmail.com

Telephone: +351914725316

Address: Rua Dr. Roberto Frias, 4200-464 Porto, Portugal

Abstract

This article focuses on the location decision of firms when competing in a spatial Cournot duopoly. Our original contribution is that firms are dependent on a natural resource input, which is assumed to be located in one of the extremes of the market, to be able to produce the output sought by the consumers, and that natural resource is controlled by an independent monopolist. We solve a three stage location game, where in the first stage downstream firms choose their location, and in the next stages upstream and downstream choose how many quantities they sell in the market, assuming that downstream firms must sell their product in all points of the linear city. We conclude that downstream firms agglomerate independently of the unit input transportation cost. In addition, increases in the unit transportation cost bring the plants closer to the natural resource location. Moreover, the upstream firm loses more profit than the downstream firms when the input transportation conditions deteriorate. When we consider the problem of a social planner, we conclude that the location that firms choose is nearly the same than the location that maximizes total welfare in the economy.

Keywords: Spatial Competition; Vertical Markets; Duopoly Studies; Game Theory

JEL Codes: D43, L13, R12

1. Introduction

The question of “where should firms locate?” has been an interesting subject in the science of Economics. Broadly speaking, location choice of all sorts of economic agents is crucial to the attainment of their objectives in a different variety of situations. For instance, one of the earliest location phenomena we all have to live in our lives is location choice in the classroom. Seating on the front or back-row location has different consequences regarding how active you are in the classroom, on your grading results, on your behavior in the classroom, and on the teacher-student relationship (see, for instance, Stires (1980)). Moreover, location in the classroom is usually a tool used by the teachers to manage misbehaved students, or students that are not performing so well during the academic year. Many other location decisions affect our everyday lives, some less relevant, as location choice in a theater, concert, or cinema; or location choice when choosing where to park your car; and others more relevant, like the place you choose to live and how close it is to other amenities, such as the place where you work, the place where your children study, and how close you are from important places like a supermarket, an hospital, or to the town landfill, which in this case you would want to avoid.

When it comes to businesses, this variable plays a dramatic role. There are many advantages and disadvantages associated with each location choice when choosing where to locate your business, and those perks depend as well on the type of business/industry in question. Two locations that are a few meters distant from each other may have significant differences in terms of visibility which may determine the difference between a success and a failure of a business. With globalization and the overall lowering of unit transportation costs in both labor and capital might have led, on the one hand, to the diminishing importance of location choice in the success of a business. However, the location possibilities of an agent are also amplified with these changes. Firms can be located, with more or less barriers, within any location in the world, which enhanced the heterogeneity of advantages and disadvantages that each location can provide to a certain type of business. Location is a significant factor in determining, to name only a few: the rent paid for land usage; the visibility of the business to consumers; the reputation of the firm; the wages you will be able to pay to your workers, the price you may be able to set for your goods, on the working conditions of your staff, in terms of environment or accessibility in terms of parking spaces or public transportations, and so on. However, none of these are factors we are analyzing directly in this model. Nevertheless, location choice is an important subject in businesses, as well in our everyday lives.

In this article we are more interested in assessing the location choice of two competing firms that sell a good in a market. However, in order to produce this good, firms need to acquire an essential input that is set in a specific location, and it is costly to

transport it from its extraction/location point to the transforming industry. Moreover, the sale of the natural resource is controlled by a monopolist that is not related with any of the two firms, which introduces another component to the strategy of the location decision process.

As an example of industries that share part of these location problems, one can think of products that are dependent on other commodities, such as iron or wood, whose final goods have to be transported to cities after being produced. Another interpretation would be of a location resource that can only be acquired through one transportation breaking point that is being controlled by an intermediary. For instance, acquired raw materials stored in ports, whose seller faces a local monopoly towards transforming industries.

We deal with the question of location using the linear city framework created by Hotelling (1929). We use an adaptation of the model created to analyze competition by quantities, developed by Hamilton et al. (1989) and Anderson and Neven (1991). The next section presents the theoretical background of the article; section 3 details the assumptions of the model and solves the game attached to our problem; section 4 analyzes the results of the model; and section 5 concludes.

2. Theoretical Background

The original game involving a location stage in the linear city concept is due to Hotelling (1929). In a two-stage game, firms first decide their location in the linear city and then both firms set the prices simultaneously. Hotelling concluded that firms would locate in the city center, given his assumptions about the market. Fifty years later, d'Aspremont et al. (1979) revolutionized the field by assuming quadratic instead of linear transportation costs, which eliminated discontinuities in demand that the original Hotelling model had. d'Aspremont and others concluded that firms would prefer to be located one at each extreme of the market, in order to soften price competition. This original result, allied with the new mathematical tractability of the model, originated a significant expansion in the field through the late 80s and 90s (Biscaia and Mota, 2013), in which many authors tried to restore the original minimum differentiation result from Hotelling.

Amongst the immense literature on the subject, some papers introduce valuable insights regarding location theory. Ziss (1993) allowed for different marginal costs of production, and concluded that if one firm has a significant advantage over the other, location equilibrium ceases to exist. Anderson et al. (1997) changed the assumption of uniform distribution of consumers in the linear city, and concluded that firms may have asymmetric location configurations, even if the distribution of consumers is symmetric towards the center and firms are homogenous. Irmen and Thisse (1998) considered a market with n dimensions in which firms can differentiate, and conclude

that if one dimension is sufficiently more important than the others, then firms choose to differentiate only on that dimension in order to soften price competition. Firms decide to remain homogenous on all other dimensions, in a result that mirrors both Hotelling and d'Aspremont solutions.

However, this article is about quantity competition in the spatial setting. Anderson and Neven (1991) firstly formulated a two-stage game similar to Hotelling, but where the price stage is replaced by a quantity stage, and some assumptions were changed in order for this framework to be tractable. The authors concluded that when the demand is linear, transportation costs are convex and are not high enough, such that firms are able to sell the product in all points of the market, agglomeration in the city center occurs. Gupta et al. (1997) changed consumer density functions as well, and concluded that agglomeration occurs if the population density is sufficiently "thick" in all market points of the city. Mayer (2000) extended this analysis by introducing different production costs along the city, and concluded that when the convexity of the production cost function holds, firms agglomerate between the minimum cost location and the city center. This article by Mayer has some similarities to ours, as we develop further on.

Although we do not investigate directly the vertical relationships between firms in the market, (that is, the possibility of integration or foreclosure, see Rey and Tirole (2007) for a review) we do consider the existence of an upstream firm that sells inputs to downstream firms. We introduce that analysis in the spatial framework of Hotelling which, to our knowledge, is limited to a few papers¹: Matsushima (2004) analyses the location decision of downstream firms by fixing the upstream's location at the extremes. Matsushima (2009) analyses the effects of integration in the location outcome of firms, and extends the endogenous location decision to both upstream and downstream firms; Kouranti and Vettas (2010) compare the location outcomes depending on when the upstream and downstream firms choose their locations, and conclude that when upstream firms choose their first, firms location becomes closer to the center which intensifies competition; Matsushima and Mizuno (2012) conclude that firms after integrating locate farther from the opponent, and that larger firms are more likely to integrate than smaller ones. However, all these approaches involve price competition between firms.

Mukherjee and Zanchettin (2012), on the other hand, analyses quantity competition instead of price competition. However, the authors do not work on the Hotelling framework of differentiation, and model product differentiation as the degree of

¹ We are referring to articles in which the location choice of downstream firms is endogenous, not those papers who consider horizontal differentiation but in which location of the firms is exogenously determined (e.g. Colangelo (1995); Chen (2001); Hackner (2003))

substitutability. So, our paper is a rather novel approach to the existence of upstream firms towards the equilibrium location outcome of the downstream firms.

Moreover, our problem is also related to the classical problem of industrial locations researched by Weber (1929). Firms have to decide where to locate given the position of the raw materials and the markets. However, our problem differs in the sense that the market is a continuum of points instead of a single location, and the raw material's extraction is controlled by a monopolist.

3. The Model

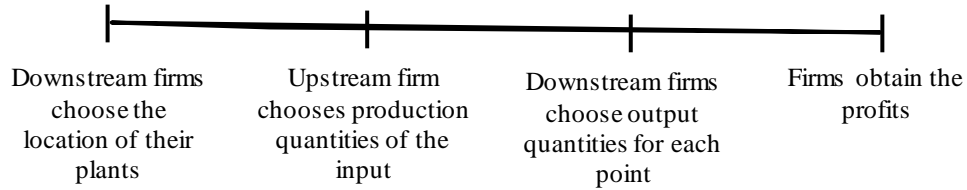
There are two downstream firms which compete in a market that is spatially differentiated. The market is composed by a continuum of markets distributed evenly in a linear city of length $[0, L]$, and we assume that $L=1$ without loss of generality. Each location in the linear city is assumed to have an inverse demand function, which is linear, and is defined by $P=10-Q$, similarly to Anderson and Neven (1991). In order to being able to produce one unit of the good, downstream firms must acquire one unit of a natural resource (fixed coefficients technology). We assume that the natural resource is located in the extreme of the linear market, as it would be the case when thinking of raw materials such as wood or iron. We assume that one upstream firm, not related with the downstream firms, managed to get the full extraction rights of this resource, having therefore a monopoly position. This firm is located in the same place as the natural resource, that is, in the extraction point.

Downstream firms have to transport the raw resource from the extraction point to their production plant in order to be able to produce the output. After the good is produced, these firms transport the goods throughout the market points of the linear city, in order to sell it to the consumers. The transportation costs are assumed to be linear with respect to the different points of the product space, and the unit transportation cost of the natural resource is given by t , while the unit transportation cost (given by T) of the output good is fixed to 1. The value of output transportation costs is constrained by the dimension of the market, since it allows both firms to sell in all points of the city, independently of any combination of locations that may arise, which is an assumption that is common in the literature of Cournot Spatial Competition (see Anderson and Neven (1991) for an example). Both downstream firms sell in all the markets if the sum of both unit transportation costs are relatively small, that is, if $T + t < \frac{2}{7}A$, where t and T are the transportation costs and A the dimension of the market.² Downstream firms are assumed to bear both transportation costs, that

² To obtain this condition, we test what are the minimum values for t , T and A for which a firm located in $x=1$ sells the good at the point $x=0$, where the input firm and the rival is located. If the firm can sell a positive quantity at $x=0$, then it can sell in all points of the market, since $x=0$ is the worst condition that a firm could face to sell its product. This condition is obtained similarly in the baseline case of Anderson and Neven (1991).

is, the transportation costs of bringing the natural resource to the production plant, and the transportation costs of the distribution of goods through the markets in the city. All three firms are assumed, without loss of generality, to have no marginal costs of either extracting the natural resource, or transforming the natural resource into an output good. The timing of the game follows.

Figure 1 – Timing of the game



In the first stage, the downstream firms choose simultaneously their location x_1 and x_2 in the linear city, which are restricted to be inside of it, that is, x_1 and $x_2 \in [0,1]$. We assume without loss of generality that firm 1 will never choose a location “to the right” of firm 2, that is, $x_1 \geq x_2$. In the second stage, the upstream firm chooses the quantities to sell of their input good. We assume, similarly to Clementi (2011), that the input price is formed due to a mechanism, in which “the downstream firms submit an aggregate input demand schedule to a walrasian auctioneer, while simultaneously the input provider submits his aggregate supply schedule. This auctioneer matches the input demand and supply and finds the market clearing price” (Clementi, 2011, p.6). Finally, in the third stage, firms choose simultaneously their quantity schedule, that is, firms choose the quantity they are going to supply to each market in the city. We seek a Perfect Subgame Nash Equilibrium, and we solve the game by backward induction.

3.1 Output Quantities Stage

First, we calculate the optimal quantity decision in each point of the city. Downstream firms will have the following profit in point x :

$$\Pi_{i,x} = (10 - q_{i,x} - q_{j,x} - I - (|x_i - x|) - tx_i)q_{i,x} \quad (1)$$

Where $\Pi_{i,x}$ is the profit of firm i in point x , $q_{i,x}$ and $q_{j,x}$ are the quantities chosen by the firm and its opponent for point x , respectively, I is the input price set by the upstream firm, and the following parts of the equation represent the transportation costs: The first part is the transportation cost of the output between the location in which the firm set its plant and the consumer in point x ; while the second part is the transportation cost of the raw material to the location chosen by the firm in the first stage. Summing up, the unitary profit in each point is given by the price of the good minus the input price and both transportation costs.

Firm i maximizes its profit by choosing the optimal quantities for each point. After satisfying the first and second-order conditions of both firms, these quantities are given by:

$$q_{i,x}^* = \frac{|x_j - x|}{3} - 2\frac{|x_i - x|}{3} + \frac{10 - I - 2tx_i + tx_j}{3} \quad (2)$$

Where $q_{i,x}^*$ is the optimal quantity chosen by firm i on market point x . To obtain all the quantities supplied by the downstream firms, we have to sum the quantities offered to all the points. However, due to the existence of the absolute value for the output transportation costs, the integral has to be separated in three different parts, one for each of the combinations of points that are “at the left” of firm 1; “between” firm 1 and 2 and “at the right” of firm 2, in order to remove the absolute value from the integral expression (e.g. see Anderson and Neven, 1991).

$$Q_i = \int_0^{x_1} (q_{i,x}^*)dx + \int_{x_1}^{x_2} (q_{i,x}^*)dx + \int_{x_2}^1 (q_{i,x}^*)dx \quad (3)$$

After computing the integral, we obtain the following total quantity for firm i .

$$Q_i^* = \frac{2(x_i - x_i^2) + (x_j^2 - x_j)}{3} + \frac{(tx_j - 2tx_i - I)}{3} + \frac{19}{6} \quad (4)$$

3.2 Input Quantities Stage

If we sum the quantities demanded for both firms, we get the demand function of the upstream firm, depending on the location choice of the downstream firms, of the input transportation costs and of the input price. Solving the equation in order of the input price, we obtain the inverse demand function of the input, which is given by:

$$I = \frac{x_1 + x_2 - x_1^2 - x_2^2 - tx_1 - tx_2 - 3Q_u + 19}{2} \quad (5)$$

Where Q_u is the quantity demanded of the input, obtained by summing both downstream's firms demand. The profit function of the upstream firm is simply given by multiplying the input price by the quantities sold, since the upstream does not pay any production or transportation cost. Maximizing it with respect to the quantity (FOC and SOC respected), the optimal quantity chosen by the upstream firm is:

$$Q_u^* = \frac{x_1 + x_2 - x_1^2 - x_2^2 - tx_1 - tx_2 + 19}{6} \quad (6)$$

Replacing Q_u^* in the above inverse demand function (equation 5) we get the input price with respect to the input transportation cost and the location choice of both firms. Replacing the input price on the firms' demanded quantities (equation 4), we

observe that this input price clears the market, since the quantities supplied equals the quantities demanded.

The profit of the upstream firm is therefore given by:

$$\Pi_u = \frac{(x_1 + x_2 - tx_1 - tx_2 - x_1^2 - x_2^2 + 19)^2}{24} \quad (7)$$

3.3 Location Stage

After knowing what their input price will be, firms are now left with the decision of choosing where to locate in the linear city. The steps to solve the problem involve very long mathematical expressions, but can be obtained after calculating the profit obtained by both firms in each market point, and summing all market points using the three-step integral that was presented in the previous subsection. Then, after obtaining this expression, the profit functions are maximized regarding the location variables x_1 and x_2 .

The resulting solution is given by:

$$x_1 = x_2 = \frac{7t+9}{18} + \theta + \frac{91t^2 + 42t - 771}{\theta}$$

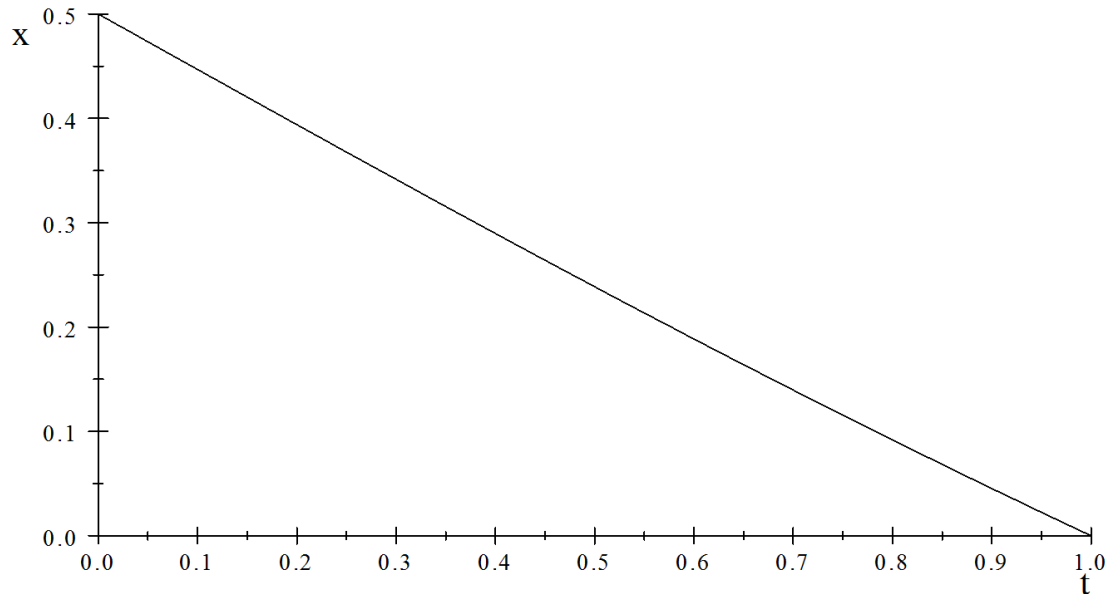
Where θ is an amount that depends non-linearly on t .³ This solution satisfies the second-order conditions, and is better understandable in Figure 2. The solution is equal to both firms, therefore firms agglomerate whatever the cost of input transportation.

Proposition 1: Given that there is an input firm selling their goods at the extreme of the market and that downstream firms bear all transportation costs in an extension of Anderson and Neven (1991), firms choose to agglomerate, independently of the value of the unit transportation cost.

Proof: Solution of the maximization problem of optimal location for the firms.

Figure 2 – Optimal location decision for both firms

³ The value of θ is displayed in the appendix.



So, as the unit input transportation costs increase, firms start to locate closer to the raw material location. If there are no input transportation costs, and firms have to pay only the cost for the acquisition of the input and the transportation cost of the output, both firms choose to locate in the middle of the linear city, since it is the one that minimizes transportation costs for a firm that is capable of selling in all points of the market (e.g. Anderson and Neven, 1991; Biscaia and Sarmento, 2012).⁴

This location result is in line with the work of Mayer (2000). The author extended the framework of Anderson and Neven (1991) by considering different production costs in different points of the city. Mayer concluded that the firms will agglomerate somewhere between the location that minimizes production cost (in our case, in the extreme where the input firm is located) and the location that minimizes transportation cost (the middle of the city). In fact, our model can be seen as an extension to Mayer, in the sense that it gives an endogenous explanation for the occurrence of different production costs that may occur in the city.

4. Discussion of the results

4.1 Location

So why does this happen to the optimal location choice of both firms? We proceed to the decomposition of different effects on the location decision, to better understand what pushes the firms to the edge of the market.

We divide the profits of the downstream firms between four components: 1) the input acquisition cost 2) The input transportation cost 3) The output transportation cost 4)

⁴ Note however, that in spite of what it looks like in Figure 2, the optimal location is not linear with respect to the input transportation cost.

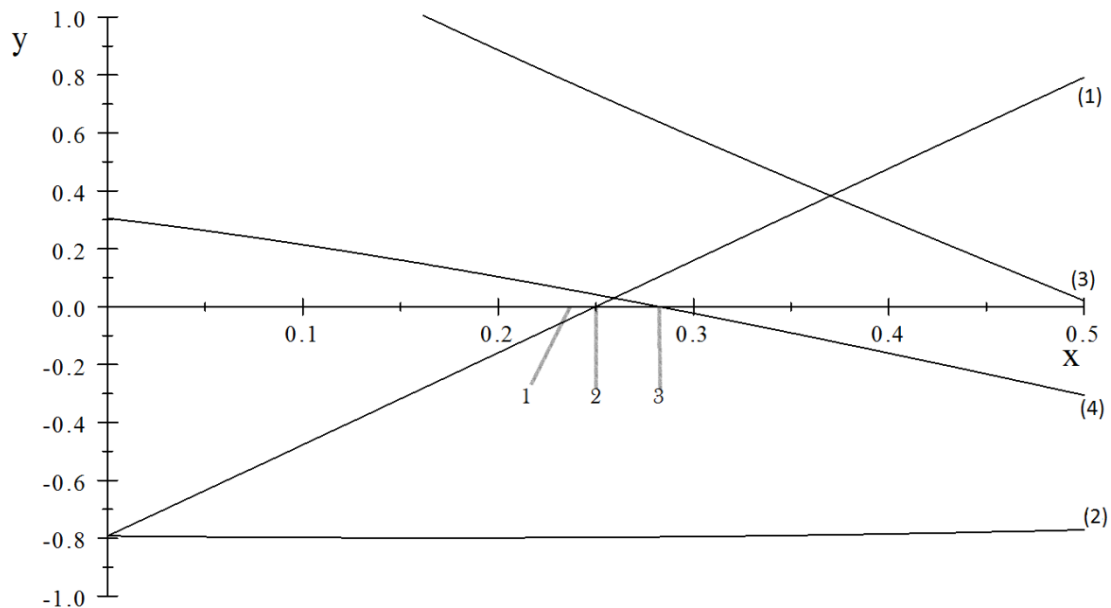
and the sales revenue. We represent the costs as their contribution to the profit function, and therefore they are negative throughout all domain. We do the analysis knowing what will be the optimal decisions of firms at the input and output quantities stage, to focus only on the location consequences. Moreover, since we already know beforehand that both firms will choose to locate in the same point of the market, we set $x_1 = x_2$ for this first part of the analysis.

We will look at the specific case where the unit input transportation cost is fixed ($t=0.5$). Then, we decompose the profit in its four components (each depending on x), and we calculate the derivative with respect the location for each case. These derivatives are presented on Table 1 and Figure 3.

Table 1 – The Four Components of the profit function at $t=0.5$

Component	Derivative of the profit component with respect to Location
Input Purchasing (1)	$-\frac{1}{3}x^3 + \frac{1}{4}x^2 + \frac{25}{8}x - \frac{19}{24}$
Input Transportation (2)	$\frac{1}{4}x^2 - \frac{1}{12}x - \frac{19}{24}$
Output Transportation (3)	$-\frac{2}{3}x^3 + \frac{5}{4}x^2 - \frac{7}{2}x + \frac{37}{24}$
Sales Revenue (4)	$\frac{2}{3}x^3 - \frac{7}{6}x^2 - \frac{29}{36}x + \frac{11}{36}$

Figure 3 – The derivatives of each component of the profit function of firm i with respect to location



The optimal solution for the location of both firms in this parameter is $x \cong 0.23865$. We can see that the effects have very different magnitudes. At the optimal solution, which is represented by point 1, it is shown the pressure that both input and output transportation costs exert over the location of the firms, that is, when the firm is located in point 1, deviating from this point would have a bigger effect in terms of their output and input transportation costs, compared to the input purchasing costs and output sales. By moving towards its right, both firms are expected to save from output transportation costs at the expense of the input transportation costs. The effects of the input purchasing and of the sales revenue are smaller at the equilibrium point. Note that the sales revenue is not maximized at the city center as one would initially expect (point 3), since the input purchasing price varies as well with location, which harms the sales possibilities of both firms in the downstream market. Note also the nature of the input purchasing costs: Given firms' current optimal position, these costs would be at their maximum at point 2. However, the costs decrease both at the left and at the right locations. This means that the downstream firms are located closer to where the input firm would choose them to be (input purchasing costs are equal to upstream profits).

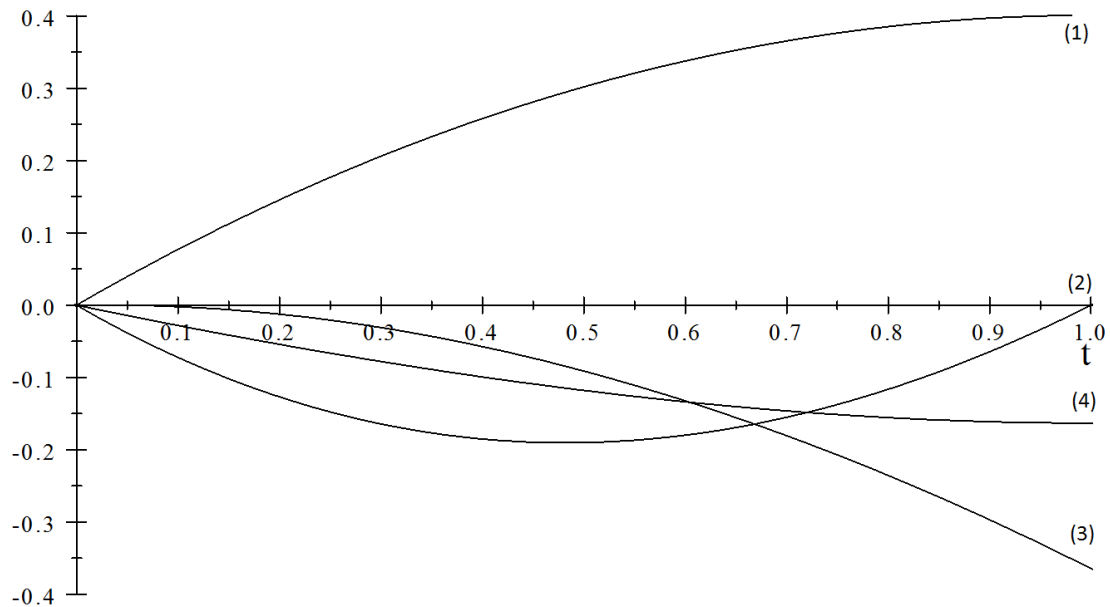
Remember we are assuming that $t=0.5$ in this example. However, the effects are similar given changes in t , but they change significantly their magnitude: As t increases the input purchasing costs curve goes up, meaning there's a pressure for firms to go to the center. The input transportation costs curve goes down, meaning that the incentive to locate closer to the upstream firm increases. The output transportation costs curve does not change, and the sales revenue curve goes down. The combination of these effects result in the outcome shown in figure 2.

4.2 Transportation Costs

If we fix the optimal location of firms for every value of the unit input transportation cost we get a different picture of the problem, namely about the effect that transportation costs have on the different components along the optimal solution path. Figure 4 presents the variation of the four components relatively to the point where the unit input transportation costs are equal to zero. Therefore, the figure details what happens to the four components as the unit input transportation cost increases: as soon as the unit input transportation cost increases, firms "travel" closer to the extreme, and both transportation costs increase – The output costs increase since the firm is now on average more distant from its consumers, in spite of selling less quantities overall; the input costs increase directly due to the increase in its unit transportation costs. However, when the unit input costs become nearly half of the unit output costs, the total input transportation costs start to decrease, as the firm becomes relatively closer to natural resource location. The output transportation cost increases in an increasing fashion because further movements of the firms to the

extreme of the market increases at an increasing rate the average distance to the consumers in the city, in spite of the decrease in total quantity sold. It is left to say that further increases in the unit input transportation costs do not bring any effects to any of the firms' decisions, leaving the results of the model unchanged.

Figure 4 – Variation of the 4 components along the optimal path



The sales revenue and the input purchasing costs evolve differently as well. The sales revenue decreases, and there are two effects determining that change: A stronger, negative effect which is the dislocation from the city center, which leads to an average sale of less quantities in each market point. And a positive, weaker, but surprising effect, which is the lowering of both firms "unit production costs", given by the sum of the input price with the input transportation price per unit (as shown in figure A.1 in the Appendix). Which leads us to analyze what happens in the input purchasing costs (or equivalently, to the upstream profit): These get lower with the increase of the unit transportation cost, since two negative effects occur: Downstream firms purchase less quantities and the input price diminishes.⁵ We can conclude, summing the 4 components, that the profit decreases with an increase in the unit input transportation costs.

Another interesting relationship is the consequences of an increase in the unit input transportation costs (on the optimal path) on the input price set by the upstream firm. After the occurrence of an increase in the unit input transportation cost, there are two effects affecting the input price determination: 1) the marginal cost of the downstream

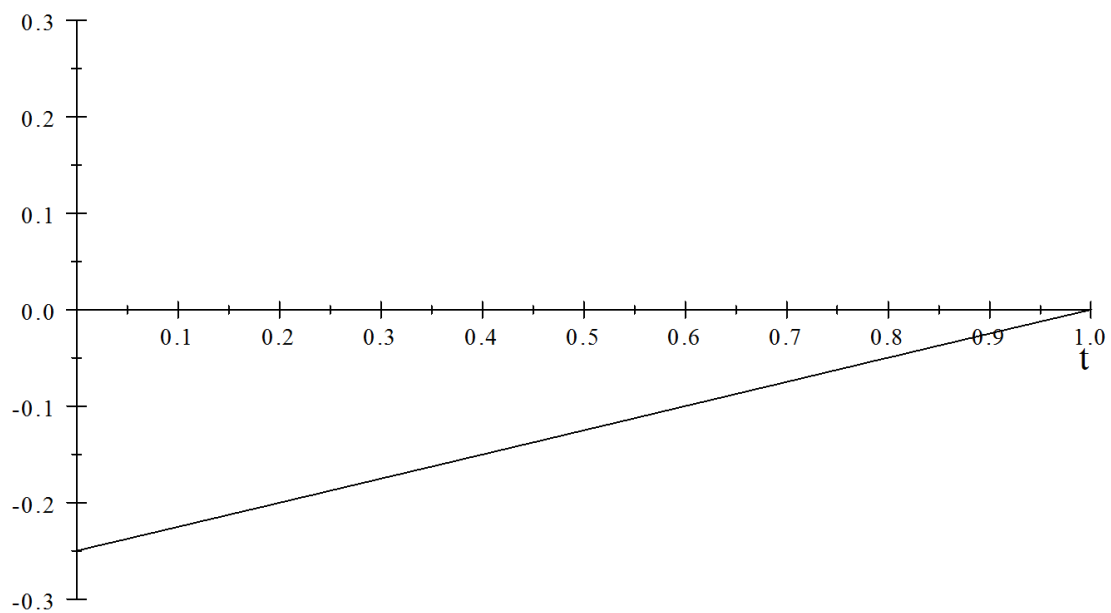
⁵ Remember that costs are represented as a negative function. When the variation is negative, this means that the costs are increasing, and vice-versa.

firm, which is an ambiguous effect, depending on the product between the unit input transportation cost itself and the distance to the natural resource. This effect is negative to the input prices for lower values of the unit transportation cost, but becomes positive after a certain threshold; and 2) the demand for the output good, which becomes lower since the firms become farther from the market center, which decreases the demand for the input good. The second effect is always larger than the first, which leads to a decrease in the input market price at a slower rate with the increase in the unit input transportation cost.

Proposition 2: Given the conditions of our model, the input price I is decreasing with an increase in the unit input transportation costs, until these become irrelevant ($t > 1$).

Proof: We are unable to do the proof analytically due to the complicated mathematical expressions resulting from the location result. We present the solution for all the values of parameter t . Figure 5 shows clearly that the derivative of the input price with respect to the unit input transportation costs is negative, except for the case where $t=1$.

Figure 5 – Derivative of the input price with respect to the unit input transportation costs



So, the model is a particular endogenous case of Mayer (2000), since the production costs in each location are a function of the location of both downstream firms, and can be divided in two parts: The first part is the input price, which is determined by the quantities that both firms are able to sell in the downstream market, as well as is determined on the value of the unit input transportation cost itself. The second part is the input transportation cost, which is paid by downstream firms, which depends on

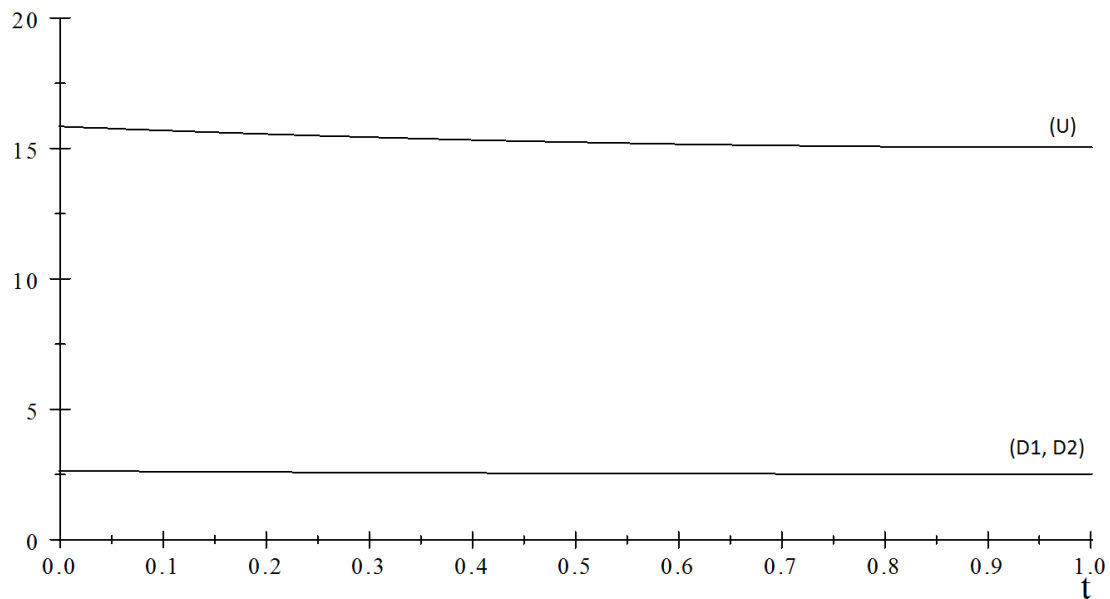
the unit input transportation cost, and the distance of the own firm relatively to the supplier.

4.2 Profits

4.2.1 Long run analysis

Naturally, the profit of all three firms decreases when facing higher unit input transportation costs (Figure 6). The red line indicates the profit for the upstream firm, while the black line indicates the profit of each of the downstream firms, which have the same profit. We conclude a bit surprisingly that the upstream firm suffers more with this increase than the downstream firms, which profit remains relatively unaltered. The surprise comes from the fact that the downstream firms are the ones supporting these transportation costs, so they could have been more affected by those.

Figure 6 – Long-Run Profit of the upstream and downstream firms depending on the input transportation cost.



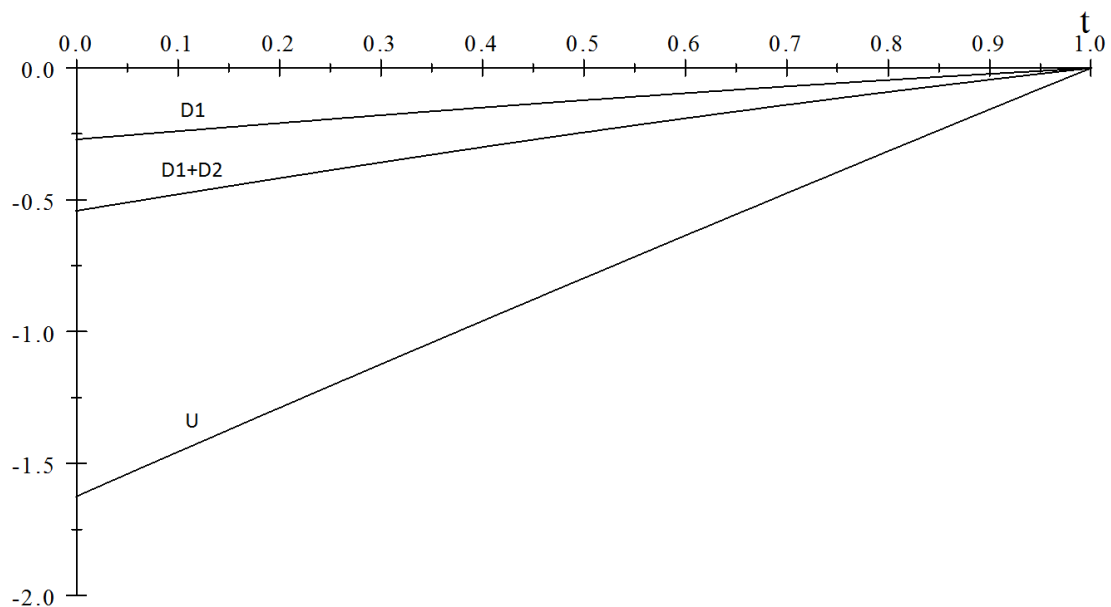
This result happens due to the effect detailed in the previous section: The unit input price decreases at a slower rate, while the quantity sold in the input market also decreases. Downstream firms, on the other hand, lose profits because they are progressively farther from the majority of its consumers and because of the increase in the total transportation costs. This difference in the decrease happens mainly because the overall quantities sold in the market decrease (remember, the input quantities sold equal the output quantities), as the profit margin the upstream firm has is larger, since it is monopolist on the market. Moreover, downstream firms benefit from the

abovementioned reduction of the input price, which reduces the effect the total transportation costs have on their profit.

Proposition 3: Given an increase in the unit input transportation cost, the upstream profit decreases more than the profit of each downstream firm.

Proof: Similarly to the previous proposition, the proof we present is based in the simulation for all possible values of the unit input transportation cost. Figure 7 compares both derivatives, and shows that the upstream firm is the one that loses more profit with an increase in the transportation cost.

Figure 7 – Comparison between the derivatives of the upstream/downstream profits with respect to the unit input transportation costs.



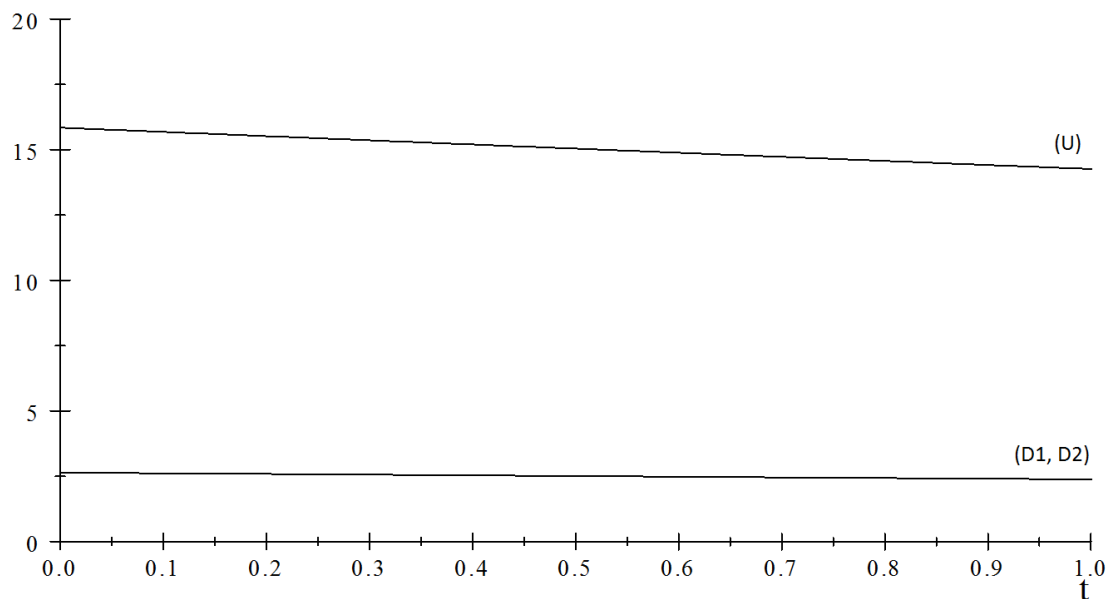
However, Figure 6 also shows one of the main weaknesses of the Cournot framework to analyze this issue: The effects of the change in the input transportation costs on the profits of the firms are very small, or close to irrelevant. We can see that both firms do not have their position in the market at risk, or do not even suffer too much if they do not follow the optimal location decision. This happens because of the assumption that the output transportation costs cannot be too big in relation to the market size. Then the effect of moving towards the extreme of the linear city has on the firms' profits is low. Abandoning this assumption proves to be a very complicated task (e.g. Chamorro-Rivas, 2000; Benassi et al., 2007).

4.2.2 Short run analysis

Location, by its nature, is something that is very expensive to change in the short-run, due to the high fixed costs that are associated with that. We assume implicitly that the unit input transportation costs do not change in the short-run, since firms are able to

see that unit cost before choosing the location of their plant. However, if we assume as an example the case where both firms cannot change their location (departing from the case where there were no unit input transportation costs: $x_1 = x_2 = 0.5$, the effects on the profits of the firms are amplified, as shown in figure 8.

Figure 8 – Short-Run Profit of the upstream and downstream firms depending on the input transportation cost.



We assume that initially both firms were in an industry where there were no input transportation costs, or there was a natural resource in the center of the city. However, for some unexpected reason, the unit input transportation costs were raised, without firms having the possibility to change the location of their firm. The profits are naturally lower for all the firms involved, but the effect is not much different from the long-run case, except, of course for values of the unit transportation cost higher than 1, in which further decreases have an effect that does not exist in this short-run case.

We can see that under this assumption that profit is not too sensitive to changes in the transportation costs or changes in the location of firms. That is one of the reasons why profits are not usually analyzed in the context of this model: Smaller changes in transportation costs may bring different outcomes, but the consequences of not changing location itself in the profits are very slim. In other words, the crucial determinant of the amount of profits firms get is the dimension of the output good demand.

4.3 Social Planner

Next, we considered what would be the solution for the social planner if he controlled the location chosen by both firms. The solution we seek is therefore a second-best, since we only allow the planner to control the first stage of the abovementioned game.

So, knowing what the output and input quantities are going to be in future stages, as previously shown in equations (2) and (6), the social planner is left to maximize the total surplus of this market, that is, the sum of the profits of the upstream firm, the downstream firms and the consumer surplus.

$$TS = \Pi_U + \Pi_1 + \Pi_2 + CS$$

To find the consumer surplus, we need to calculate the consumer surplus in one point x , then using an integral similar to the one that is used in previous subsections, that is, an integral broken in three parts due to the existence of two different absolute values. The profits of the three firms are the same used on previous calculations. The solution is given by:

$$x_1 = x_2 = \frac{5t+3}{6} + \phi + \frac{35t^2 + 10t - 187}{36\phi}$$

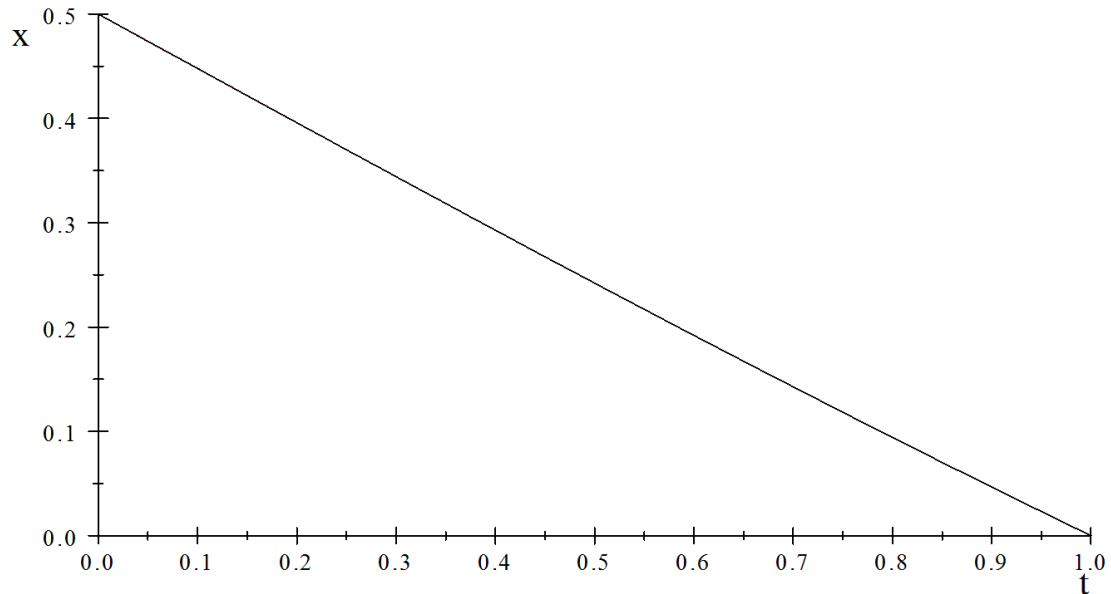
Where ϕ is a value that depends non-linearly on t .⁶ And the Second-order Conditions for this solution are met. Figure 9 shows the result of both centralized and uncentralized equilibrium in the product specification. We can see that, a bit surprisingly, the social planner equilibrium is very similar to the one found before, which leads us to conclude that firms, when thinking about maximizing their own profit, are choosing a location very close to what it would be the location chosen by a central planner.⁷ This result goes in line with the one found by Matsumura and Shimizu (2005). In addition, the social planner also chooses the same location for firm 1 and 2. Note that this result arises from the fact that both firms, by assumption, are obliged to sell on every point of the market.⁸

Figure 9 – Location Choice for Firms and Social Planner

⁶ The value for this parameter is displayed in the appendix.

⁷ Note that, in spite of being very close in Figure 7, the locations chosen in both cases are not exactly equal.

⁸ It is unsure though whether the social planner would prefer a different solution. If the sole purpose was minimization of the input and output transportation costs, then probably the social planner would distribute both firms along the product space.



This happens because firms and social planner have similar rationales for their objectives, which is the minimization of transportation costs. Firms intend to minimize their transportation costs in order to provide cheaper different goods in each market point. That way, they are able to provide more quantities of each good, therefore maximizing their profit. The social planner, on the other hand, is interested in having the highest quantities possible of every good in every market point, and that is only possible if firms find a way to minimize their transportation costs, such that they can be competitive in every point.⁹

The small difference between the two cases is justified by the worry that the social planner has with both the consumer surplus and upstream firm's profits. By staying closer to the center, consumers will get a larger quantities of goods, and this means as well that the upstream firm will have a higher demand for its input. However, these effects are very small compared to the importance of a correct location choice for both the quantities sold in the downstream markets and therefore, the profits that the downstream firms will receive. The importance of this effect, which is an objective for both firms and regulator, justifies the proximity of the result.

Since the social planner only controls the location stage, the results towards the profit of the three firms are similar as it was analyzed in the previous section. The total surplus does not differ significantly between both cases, even though the one resulting from social planner maximization is naturally superior.

Conclusion

⁹ Note that the social planner is interested in minimizing transportation costs not only because it is a form of inefficiency (in classical price competition models with horizontal differentiation this is the only source of inefficiency the social planner faces), but also because better placed firms are capable of selling more quantities, contributing to total surplus in every point of the market.

In the context of spatial competition, few articles have analyzed the implications of a vertical relationship in forming the marginal costs that firms have to face. In this article, the presence of an input that is required for firms to be in the market, and the subsequent problems of acquiring and transporting the good for the business to be successful, along with the strategic duopoly interaction makes this model suitable to explain the behavior of industries that are very dependent on vertical relationship to be successful.

In the framework of spatial competition by quantities developed by Anderson and Neven (1991), we conclude that the transportation costs are crucial to the location decision of both firms, in an almost linear relationship between the location chosen and the unit input transportation costs. However, by analyzing the resulting profits and the social welfare, input transportation costs seem to have a minor role. Additionally, both firms agglomerate, independently of the unit transportation costs. This happens because of the strategic substitutability nature of quantity competition, which makes firms concentrate more on being better located relatively to the demand than relatively to its opponent. This is the reason why we find that the location outcome chosen by a social planner is close to the solution chosen by the firms themselves: The main concern in both cases is with the transportation costs, and this is the most important driver for the location decision.

We also conclude that an increase in unit input transportation costs cripples more the profit of the upstream monopolist than the profit of the downstream firms, even though the latter supports the transportation costs. The reason is that the downstream firms sell less in the downstream market, which means a decrease of demand in the upstream market, leading to a decrease in the quantities sold, and in the price as well.

Most importantly, this article may be the starting point for an interesting analysis of the consequences of vertical relationships in the spatial competition literature. Even though the framework of Anderson and Neven (1991) has a very interesting nature, this model may not be the most suitable to the development of this vertical analysis, given the (mathematical) requirement that both firms must sell in all market points. Breaking this assumption would induce competition between firms, which would have been forced to choose their location with the concern of a better coverage of the market respectively to their opponent. However, leaving this assumption implies finding deep mathematical problems that would probably undermine a good analysis of vertical relationship's implications on the downstream firms' profits, quantities sold and location decisions.

References

- Anderson S, Neven D (1991) Cournot competition yield spatial agglomeration. *International Economic Review* 32: 793-808.
- Anderson S, Goeree J, Ramer R (1997) Location, Location, Location. *Journal of Economic Theory* 77: 102-127.
- Biscaia R, Sarmiento P (2012) Spatial Competition and Firms' Location Decision under Cost Uncertainty. FEP Working Papers 445.
- Chamorro-Rivas J (2000) Spatial dispersion in Cournot competition. *Spanish Economic Review* 2: 145-152.
- Chen Y (2001) On Vertical Mergers and Their Competitive Effects. *RAND Journal of Economics* 32 (4): 667-685.
- Clementi L (2011) Vertical Integration and Foreclosure Strategies in Bilateral Oligopoly. Job Market Paper.
- Colangelo G (1995) Vertical vs. Horizontal Integration: Pre-Emptive Merging. *Journal of Industrial Economics* 43 (3): 323-337.
- d'Aspremont C, Gabzewicz JJ, Thisse JF (1979) On Hotelling's stability in competition. *Econometrica* 47 (5): 1045-1050.
- Gupta B, Pal D, Sarkar J (1997) Spatial Cournot competition and agglomeration in a model of location choice. *Regional Science and Urban Economics* 27: 261-282.
- Hackner J (2003) Vertical Integration and Competition Policy. *Journal of Regulatory Economics* 24 (2): 213-222.
- Hamilton J, Thisse JF, Weskamp A (1989) Spatial Discrimination: Bertrand vs. Cournot in a model of Location Choice. *Regional Science and Urban Economics* 19: 87-102.
- Hotelling H (1929) Stability in competition. *Economic Journal* 39: 41-57.
- Irmen A, Thisse JF (1998) Competition in Multi-characteristics Spaces: Hotelling Was Almost Right. *Journal of Economic Theory*, Vol.78, No.1, pp 76-102.
- Matsumura T, Shimizu D (2005) Spatial Cournot Competition and Economic Welfare: A Note. *Regional Science and Urban Economics* 35: 658-670.
- Matsushima N (2004) Technology of Upstream Firms and Equilibrium Product Differentiation. *International Journal of Industrial Organization* 22: 1091-1114.
- Matsushima N (2009) Vertical Mergers and Product Differentiation. *Journal of Industrial Economics* 57 (4): 812-834.

Matsushima N, Mizuno T (2012) Why do Large Firms Tend to Integrate Vertically? *Bulletin of Economic Research* 64 (S1): S1-S21.

Mayer T (2000) Spatial Cournot Competition and Heterogeneous Production Costs across Locations. *Regional Science and Urban Economics* 30: 325-352.

Mukherjee A, Zanchettin P (2012) Vertical Integration and Product Differentiation. Department of Economics, University of Leicester Working Paper No. 12/17.

Kouranti F, Vettas N (2010) Endogenous Spatial Differentiation with Vertical Contracting. CEPR Discussion Paper No. 7948.

Rey P and Tirole J (2007) 'A Primer on Foreclosure,' in Armstrong, M. and Porter, R. H. (eds.), *Handbook of Industrial Organization*, Vol. 3 (Elsevier, Amsterdam, The Netherlands).

Stires L (1980) Classroom Seating Location, Student Grades, and Attitudes: Environment or Self-Selection? *Environment and Behavior* 12 (2): 241-254.

Weber A (1929) *Alfred Weber's theory of the location of industries*. Chicago University Press.

Ziss S (1993) Entry Deterrence, Cost Advantage and Horizontal Product Differentiation. *Regional Science and Urban Economics*, 23: 523-543.

Appendix

$$\theta = \sqrt[3]{\sqrt{-\frac{1715t^6}{419904} + \frac{9947t^5}{629856} - \frac{383033t^4}{1259712} - \frac{2401t^3}{3888} + \frac{7725809t^2}{1259712} - \frac{462343t}{209952} + \frac{16974593}{1259712} + \frac{98t^3}{729} + \frac{14t^2}{81} - \frac{266t}{81}}$$

$$\phi = \sqrt[3]{\sqrt{-\frac{2856t^6}{46656} + \frac{625t^5}{2592} - \frac{73625t^4}{15552} - \frac{38875t^3}{11664} + \frac{527585t^2}{15552} - \frac{174845t}{7776} + \frac{6539203}{46656} + \frac{25t^3}{27} + \frac{5t^2}{9} - \frac{95t}{9}}}$$

Figure A.1 – Input price (Black) and unit production costs (red) for the optimal path.

