

Garloff, Alfred; Werth, Stefan

**Conference Paper**

## Characterizing unemployment duration data with stock sample measures

53rd Congress of the European Regional Science Association: "Regional Integration: Europe, the Mediterranean and the World Economy", 27-31 August 2013, Palermo, Italy

**Provided in Cooperation with:**

European Regional Science Association (ERSA)

*Suggested Citation:* Garloff, Alfred; Werth, Stefan (2013) : Characterizing unemployment duration data with stock sample measures, 53rd Congress of the European Regional Science Association: "Regional Integration: Europe, the Mediterranean and the World Economy", 27-31 August 2013, Palermo, Italy, European Regional Science Association (ERSA), Louvain-la-Neuve

This Version is available at:

<https://hdl.handle.net/10419/124063>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Characterizing unemployment duration data with stock sample measures

Alfred Garloff (IAB) and  
Stefan Werth (BA)<sup>1</sup>

01 March 2013

**Abstract:** In this paper, we describe the usefulness of stock sample measures for average unemployment durations, when the parameter of interest is the expected unemployment duration. If both job separations and job accessions follow a Poisson process which are constant over time, the stock sample measure and the parameter of interest are identical. We discuss deviations from this benchmark in a theoretical framework and show how the stock sample measure develops compared to the parameter of interest. Over time the stock sample measure is useful (ie, moves in the same direction as the parameter of interest), when (the rate of) job accessions change whereas it is not useful when (the rate of) job separations changes. Comparing groups, the stock sample measure only provides consistent rankings under certain conditions on the moments of the underlying duration distribution. We propose a new measure for unemployment durations (an inflow corrected stock sample measure of unemployment duration) that can be easily calculated from stock sample data and takes account of the fact that it is primarily the changes of the inflow rates that invalidates the stock sample measure.

Using a large administrative microdataset for Germany, we assess the performance of stock sample measures in predicting changes in the actual duration distribution and show that while the levels of the stock sample measures are positively related to the parameter of interest, the differences are strongly negatively correlated and thus the stock sample measure points not in the same direction as the parameter of interest. However, we also show that changes to the previous year of the stock sample measures are stable and positively related to changes of a moving average

---

<sup>1</sup> We would like to thank Bernd Fitzenberger, Pierre Guericke, Katrin Sommerfeld, Christian Zeman, the participants of Fitzenberger's doctoral workshop and of the meeting of the Regional Research Net of the IAB for helpful comments and suggestions.

Correspondence: Alfred Garloff, Institute for Employment Research (IAB), Saonestrasse 2-4, 60528 Frankfurt am Main, E-mail: [alfred.garloff@iab.de](mailto:alfred.garloff@iab.de), Tel: +49 69 6670 518

Stefan Werth, Federal employment agency (BA), Saonestrasse 2-4, 60528 Frankfurt am Main, E-mail: [stefan.werth@arbeitsagentur.de](mailto:stefan.werth@arbeitsagentur.de)

of expected durations. Thus, empirically, we can interpret the stock sample measures in terms of parameters of interest. In terms of comparisons between groups, we demonstrate for the example of men and women that the stock sample measure predicts the sign of the difference correctly only in about 60 percent of the cases.

*Keywords: unemployment duration, stock sample measure, micro data analysis*

*JEL classification: J64*

Introduction .....	4
The parameter of interest .....	5
What do real measures measure?.....	6
How do real measures evolve? .....	8
Literature .....	10
The baseline model.....	11
Comparisons across time: Deviations from stationarity .....	12
Manipulating delta.....	12
Manipulating lambda.....	14
Usefulness of ET for comparisons over time.....	16
Comparison of different groups .....	16
Simulation Results.....	17
Proposing alternative measures.....	21
Assessing the performance of the measures in micro data.....	22
The dataset and calculation of the measures of interest .....	23
Results.....	24
Conclusion .....	26
Literature.....	29

# Introduction

Unemployment is a severe problem in almost all developed countries. The same unemployment rate can be made up by few people being unemployed for a long time or by many people being unemployed only shortly. Thus, the unemployment rate is not sufficient to characterize the performance of a particular labour market. The average duration of unemployment is another important datum.

Different compositions of unemployment can have very different meanings for labour market policies: whereas a high unemployment rate that is caused by a high incidence and a low duration might be problematic from a point of view that people have a preference for planning security, a high unemployment rate that is caused by a high percentage of long term unemployment might be of more concern for policy.

So it is important to measure unemployment durations accurately to be able to characterize the average dismay caused by an average unemployment spell. In addition, changes in the duration of unemployment are important.

Many statistical offices such as the Federal Employment Agency (FEA) in Germany calculate stock sample measures to give an assessment on the duration of unemployment. A priori, it is not clear which parameter these measure(s) actually measure and whether this measure is useful in a sense that is related under certain conditions in a certain way to some parameter of interest.

In this paper, we discuss the usefulness of these measure(s) in discussing under which conditions (in the sense of deviations from stationarity) we can interpret the stock sample measure in a meaningful way.

In addition, we discuss under which conditions the average previous duration can be meaningfully used to compare groups.

We suggest an alternative measure, the inflow corrected average previous duration in a stock sample that is almost as easy to calculate as the stock sample measures and show under which conditions this alternative measure is meaningful.

Using a large micro dataset, we calculate the stock sample measure(s) including our proposed measure and the expected duration and show that our measure performs somewhat better than the one used by the FEA but still not satisfactory.

## **The parameter of interest**

How should we actually measure average unemployment duration? The first thing to know is, what is parameter that we would like to know given we are not constrained through measurement problems.

From a point of view of policy, probably, the expected duration at entrance in unemployment is the most interesting parameter. If one was to plan the use of active labour market measures this would be the perfect thing to know, in the sense that you could plan at the beginning of the spell over the use and timing of active labor market measures if one was to use any. A drawback might be, however, that neither measuring this, especially at the time a spell begins, is straightforward nor is it likely that the distribution of the ex ante expected unemployment duration(s) equals the (distribution of) ex post realized unemployment duration(s). This is the case, since to estimate the unemployment duration at the beginning of an unemployment spell one needs assumptions about the development of the world and these are unlikely to be exactly matched through real developments.

By the same, it is also interesting to know the average remaining duration of unemployment for those unemployed at a certain point in time. There are several drawbacks to measuring this, however. First, all those unemployed at a certain point in time, a stock sample, is not a random sample of the underlying population. It is a random sample only for those that entered just before and a length-biased sample of those that entered at all times before; namely those that survived to the moment we observe them.<sup>2</sup> The average remaining unemployment duration of this mixture or the change of the unemployment duration of this mixture of persons is more difficult to interpret. Even if we could guarantee that the composition of this mixture of persons was the same across observation points, it was not straightforward to interpret

---

<sup>2</sup> See for example Lancaster (1990), chapter 5 and 8 for a formal description of the length-biased sampling problem.

changes in the so defined average remaining unemployment durations, since an unchanged measure could well mean that for a part of the persons unemployment duration rose, while for another part the duration decreased and this would probably have different implications for policy.

In addition, it has to be noted, that given the knowledge on the expected unemployment duration at entrance for every entry cohort, the above stock sample measure can be calculated, but not vice versa. Thus the information contained in the second measure is only a subset of the information contained in the average unemployment duration at entrance.

Admittedly, the second measure contains more information in the sense that after a certain duration has elapsed, more information could be known about the average remaining unemployment duration of each entry cohort. Thus, a third measure of interest could be for each entry cohort the average remaining duration after  $x$  months. But on which duration should one concentrate? Again, it is more general to set  $x$  to zero.

Given this discussion, we hope to convince the reader that the expected duration at entrance is the most interesting parameter. Because of this and because it is conceptually the easiest parameter, we use it in this paper as the parameter of interest and we will use this as the benchmark when judging how good a certain measure performs.

## **What do real measures measure?**

Now, what do statistical offices actually measure? First, we have not found any discussion published by the FEA or any other statistical office that uses stock sample measures on what their parameter of interest is.

If they wanted to measure the expected unemployment duration at entrance (which we call later on for simplicity only expected unemployment duration) they would have to use some past measure and some stationarity assumption. Instead, the FEA publishes two measures that are based on what the statistical office can directly observe at the moment of measurement: the first measure is the average interrupted

unemployment duration in the stock sample. The second measure is the average unemployment duration of those spells ending in a particular time interval (monthwise).

Both measures do not measure the expected unemployment duration for the following reasons. The first measure is the average of a stock sample and is thus not a random sample of the duration distribution. More specifically, in a stock sample there are comparatively many long spells and comparatively few short spells (so called length-biased sampling). Thus, averaging over those spells, even if all were drawn from the same distribution, we would tend to overestimate the expected unemployment duration. The second mistake this stock sample measure makes in comparison to the parameter of interest is the fact that this measure uses the previous unemployment duration; but those spells are not finished yet. Thus, a random sample of previous unemployment durations would underestimate the expected unemployment duration.

The second measure of the FEA is more difficult to judge. First, the stock at the beginning of the time interval is a stock sample with an overrepresentation again of long spells. Those ending in a particular time interval overestimate the expected unemployment duration in the sense that since we have in the stock sample too many long spells those ending will be on average too long. On the other hand the measure ignores the fact that there are still individuals at risk at the end of the period, which last definitely longer than so far. In that sense the measure underestimates the expected duration. In addition, for a few persons in the sample, we have even a random sample, namely those that enter unemployment in the period we look at (the particular month). For them, with the data at hand, one could construct consistently the hazard rate for the first few days in unemployment.

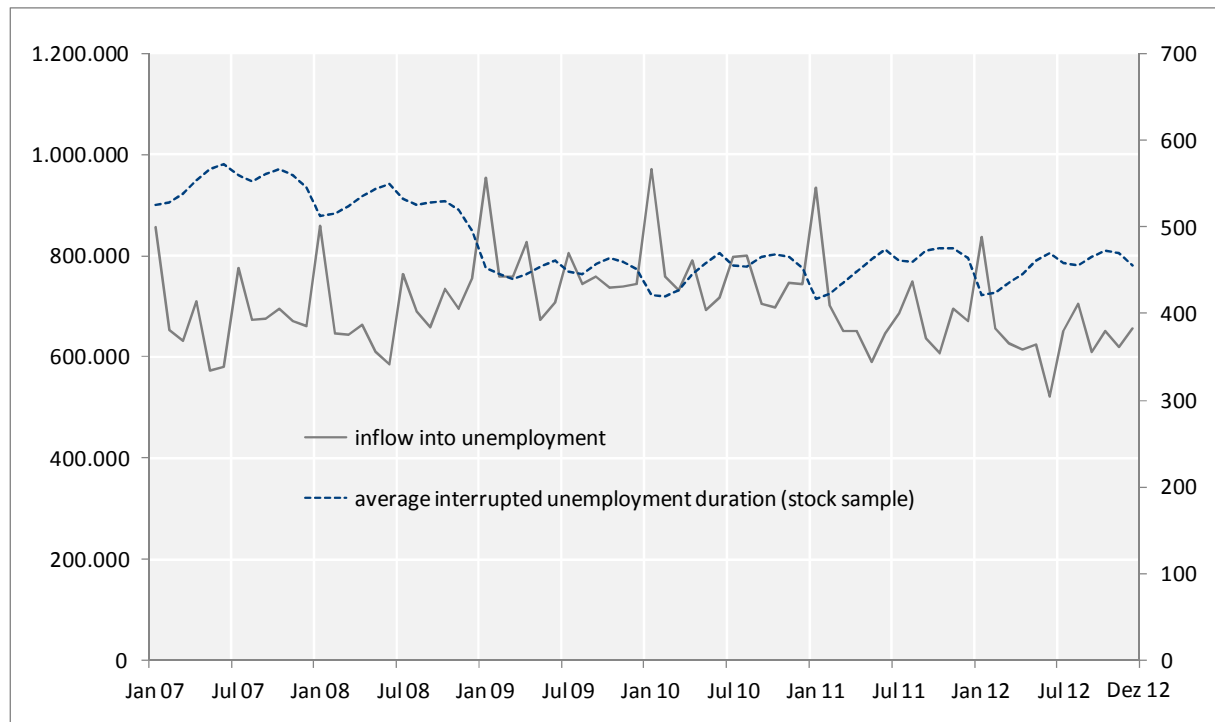
In what follows, we mostly concentrate on the first measure of unemployment, namely the average previous unemployment duration of a stock sample, and its relationship to the parameter of interest.



## How do real measures evolve?

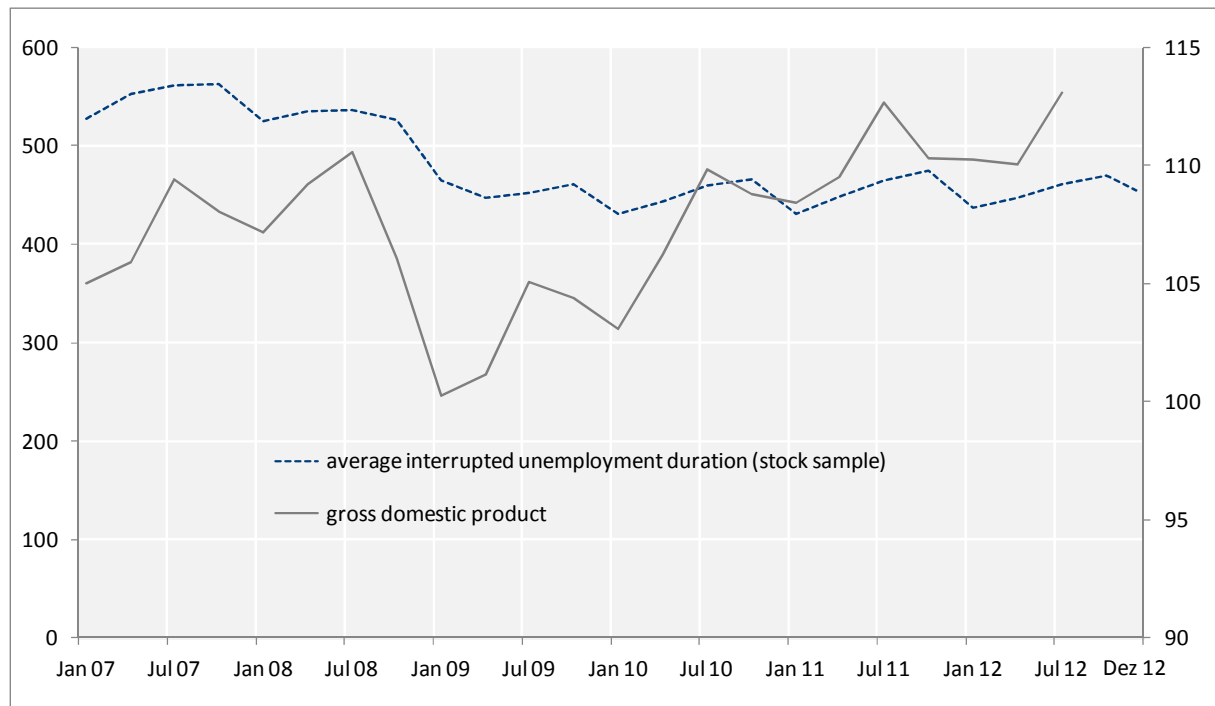
We start by presenting evidence on the actual development of the measures that the FEA uses and discuss shortly its quality when considering the development over time. Figure 1 demonstrates that the interrupted unemployment duration is particularly short in January 2009 where inflows are very large, where however, the expected unemployment duration is probably relatively long. Generalising this impression, looking at situations where the business cycle moves in a certain direction, we find that in general the stock sample measures move in a different direction as we expect the actual duration to move, since the overall development of the measure is dominated by changes in the inflows into unemployment, when inflows in and outflows out of unemployment move together (see Figure 1). Figure 2 demonstrates the relationship between GDP and the average interrupted unemployment duration. Again, for the case of 2009, where the German economy was hit by the world economic crises, the parameter of interest, i.e., the expected unemployment duration was likely to increase, whereas the average interrupted unemployment duration of the stock sample was decreasing, since there were so many newly unemployed persons with a short unemployment duration, thereby leading to a decreasing interrupted unemployment duration in the stock sample.

Figure 1: Interrupted unemployment duration in the stock sample and entries into unemployment (Germany, January 2007 to December 2012)



Source: FEA

Figure 2: Interrupted unemployment duration in the stock sample (moving average) and the gross domestic product (Germany, January 2007 to December 2012)



Source: FEA and Statistical office of Germany

# Literature

It is well known in the literature that the average of previous unemployment durations in a stock sample is not a correct measure of expected unemployment duration. Other statistical offices, for example the CPS, do also calculate this measure since it is readily calculated. Thus, literature has been discussing this problem already quite a bit. One of the first to note and discuss this measurement issue is Kaitz (1970). He actually proposes the second measure that is used by the FEA, described above. In addition, he gives a few results on the relationship between expected duration and the measure. Salant (1977) generalises these results and provides a formula that links the moments of the expected duration (S) and the previous duration of a stock sample (T):

$$\frac{E(T)}{E(S)} = \frac{1}{2} \left[ \frac{Var(s)}{E^2(s)} + 1 \right] \quad (1)$$

This formula applies if the environment is stationary. I.e., if inflow rates into unemployment are constant over time and if outflows are drawn from the same distribution across time. In fact, this formula implies that when the inflow rates are constant over time and the distribution of outflows is exponential for which  $ES = Var(S)$ , the average previous duration of a stock sample equals the expected duration, i.e., the measure measures what it is supposed to measure. That implies that the two errors described above exactly offset in this case.

Later Research, however, demonstrated that this result is not of much help since these conditions are obviously not met by real data. Kiefer, Lundberg and Neumann (1985) show that “the best inference that can be made about the actual distribution using cps-like data are seriously biased” (p.3).

In what follows, we will discuss this assertion in detail in discussing the supposed performance of the stock sample measure in a model, which allows us to observe deviations from the baseline case which gives rise to equation (1).

# The baseline model

We start by characterizing the relationship between the expected duration  $s$  (distribution  $F_s(s)$ ) and the average previous duration in a stock sample  $t$  (distribution  $G_t(t)$ ) in a general way. Let,  $p(r)$  denote the inflow at time  $r$  and  $s_0$  denote the point in time where one wants to measure things (i.e. today). The probability that a person, that entered unemployment at time  $l$  is still unemployed today, is  $1 - F_s(s_0 - l)$ . Then, the density of the previous interrupted unemployment duration is:<sup>3</sup>

$$g_t(t) = \frac{p(s_0 - t)(1 - F_s(t))}{\int_{-\infty}^{s_0} p(l)(1 - F_s(s_0 - l))dl} \quad (2)$$

To be able to describe the relationship between expected unemployment duration and the previous unemployment duration of a stock sample, we choose a very simple baseline model, in which deviations from the baseline model are easily discussed.

We discuss deviations from stationarity when in- or outflow rates change.

Let us begin by laying out the simple model we are using to discuss the performance of the FEA measure “average previous unemployment duration”. For simplicity at the outset, we assume that all individuals are identical, i.e., all individuals starting an unemployment spell at a certain point in time (given the information at that time) have the same expected unemployment duration. Assume in addition, that the inflow in unemployment is constant over time. For simplicity, assume that there are only two states in this labour market: people are either employed ( $E$ ) or unemployed ( $U$ ). We can state the above assumptions in formulating hazard rates for the two groups. Let  $\delta$  denote the constant (Poisson) rate at which jobs end and let  $\lambda$  denote the constant (Poisson) rate at which individuals start jobs out of unemployment. These assumptions imply a stationary unemployment rate  $u = \delta/(\delta + \lambda)$ .<sup>4</sup>

---

<sup>3</sup> Note, that  $l$  and  $s_0$  refer to certain points on the time axis, whereas  $t$  is a duration (the distance between two points on the time axis). The entry probabilities pertain to certain points on the time axis, whereas the distribution function for durations is cast in term of durations (or the differences of points in time).

<sup>4</sup> For a more extensive treatment of search models, see Garloff (2010) and the textbook of Cahuc and Zylberberg (2004), chapter 3.

It is easily shown that under these assumptions equation (2) simplifies to the probability density function (pdf) of the exponential distribution. Thus, the expectation of the distribution of interrupted unemployment duration(s) in a stock sample is identical to the expectation of average expected unemployment duration upon entry, thereby confirming equation (1) for the case of the exponential.

## **Comparisons across time: Deviations from stationarity**

The FEA publishes monthly information for unemployment durations as measured in the stock sample. This is probably meant to enable the observer to track the development of the unemployment duration over time, i.e., to answer the question whether the unemployment duration has increased or decreased as compared to the last month. The two subsections that follow try to answer the question whether and under what conditions, a certain observation for the stock sample implies something precise for the actual unemployment duration. Or to put it more clearly: when the stock sampled measured unemployment duration increases, are individuals indeed longer unemployed on average? In what follows we answer this question in the theoretical model outlined above distinguishing whether inflow rates or outflow rates or both change over time.

### **Manipulating delta**

Let us first deviate from the stationarity assumptions imposed above in manipulating  $\delta$ . The simplest possible manipulation is a permanent change of its value to another value. Assume, for example that starting at some moment in time  $a$ , for some reason firms have better expectations and therefore prefer keeping their workforce longer on average, thereby permanently decreasing  $\delta$  ( $\delta_2 < \delta_1$ ) at time  $a$ . We assume in addition that the labour market jumps immediately to its new stationary value, i.e., the new stationary unemployment rate is attained immediately.<sup>5</sup> The parameter of interest does not change through the change of  $\delta$ , since the expected duration upon entrance is given by  $ES = 1/\lambda$  and is thus unaffected by any change of the inflow rate to unemployment  $\delta$ .

---

<sup>5</sup> This will clearly not be the case in reality but makes the handling of the formulas a bit easier. Thus, in our simulations above, we do not invoke this assumption and allow for a slow adaption process to the new equilibrium.

Intuitively, it is clear that the stock sample measure is affected by this change, since the parameter change changes the composition of the stock sample. A few periods after the parameter change, we have comparatively few short interrupted durations and comparatively many long interrupted durations, because the inflows in the last few periods were smaller than previously. Thus the stock sample measure of the average interrupted durations will increase, although the true measure is not altered. At some point, the direction of the change must revert, since, as  $a$  goes to infinity, the expectation returns to the true value.<sup>6</sup> In what follows we formalize this assessment.

The expectation of the stock sample of interrupted unemployment durations is given by

$$ET = \int_{-\infty}^a r g_{\delta, \lambda, (r > s_0 - a)}(r) dr + \int_a^{s_0} r g_{\delta, \lambda, (r \leq s_0 - a)}(r) dr \quad (3)$$

, where  $r$  is the interrupted duration,  $a$  is the moment of the parameter change,  $s$  is the moment of observation and  $g_{\delta, \lambda, r}$  is the pdf of those unemployment spells that started either before ( $r > s - a$ ) or after ( $r \leq s - a$ ) the parameter change. The density depends on  $\delta$  and  $\lambda$ , since the former (uniquely) determines the inflows, whereas the latter determines the outflows, together determining the composition of the stock sample. To calculate this expectation, we look at the two densities in turn.

We start with the density for  $r \leq s - a$ :

$$\begin{aligned} g_{\delta, \lambda, (t \leq s_0 - a)}(t) &= \frac{p(s_0 - t)(1 - F_s(t))}{\int_{-\infty}^{s_0} p(l)(1 - F_s(s_0 - l))dl} = \frac{p(s_0 - t) \exp(-\lambda t)}{\int_{-\infty}^{s_0} p(l) \exp(-\lambda(s_0 - l))dl} \\ &= \frac{\delta_2(1 - u_2) \exp(-\lambda t)}{\int_a^{s_0} \delta_2(1 - u_2) \exp(-\lambda(s_0 - l))dl + \exp(-\lambda(s_0 - a)) \int_{-\infty}^a \delta_1(1 - u_1) \exp(-\lambda(a - l))dl} \end{aligned}$$

The numerator are the inflows  $t$  time units ago ( $t \leq s - a$ ) times the probability that these inflows survive till today, i.e., all those that are in the stock sample for exactly  $t$  time units (at time  $s_0$ ). The denominator sums all individuals that are in the stock

---

<sup>6</sup> The direction of the change is reverted at some distance from  $a$  because a) the effect of the composition dies out over time and b) the true duration is smaller than the one measured by the wrongly composed sample. Thus from some point the composition effect is dominated by the reversal to the true value, because the measured duration has increased through the composition effect over its true value

sample at time  $s_0$ . The first term are all those that arrived in unemployment after the parameter change (in  $a$ ) and that survived till  $s_0$ . The second term are those that arrived in unemployment before  $a$  and that survived till  $a$  times the probability of surviving between  $a$  and  $s_0$ .

Now, we can calculate the density for  $r > s - a$  as well. It is given by

$$g_{\delta, \lambda, (t > s_0 - a)}(t) = \frac{\delta_1(1 - u_1) \exp(-\lambda t)}{\int_a^{s_0} \delta_2(1 - u_2) \exp(-\lambda(s_0 - l)) dl + \exp(-\lambda(s_0 - a)) \int_{-\infty}^a \delta_1(1 - u_1) \exp(-\lambda(a - l)) dl}$$

The only difference to the previous density being that the inflows happen earlier thus slightly changing the numerator.

Now, plugging the two densities back into equation (3) and after a few manipulations we obtain:

$$ET = \frac{\frac{u_2}{\lambda} + (u_1 - u_2) \exp(-\lambda(s_0 - a)) \left( (s_0 - a) - \frac{1}{\lambda} \right)}{u_2 + (u_1 - u_2) \exp(-\lambda(s_0 - a))}$$

This is the expected interrupted duration in the stock sample, thus the parameter we are measuring, given the change in lambda at time  $a$ .

The derivative of this expression with respect to  $(s_0 - a)$  is positive for small  $(s_0 - a)$  and decreases with  $(s_0 - a)$ . At some point it becomes negative and returns to zero for very large  $(s_0 - a)$ .

Thus, after a permanent decrease in delta, the average interrupted unemployment duration first increases and then decreases again, although the average expected unemployment duration at entry in unemployment does not react. Below we simulate the size of the effect, and show that this effect is neither small nor short lived.

## Manipulating lambda

Next, consider a change in  $\lambda$ . Assume the change was in the same direction as above, i.e.,  $\lambda$  increases at time  $a$ , where  $\lambda_2 > \lambda_1$ , the labour market performance becomes better, the unemployment rate decreases.

Formula (3) still applies, but we have to rewrite the densities. For  $t \leq s_0 - a$ , we have:

$$g_{\delta, \lambda, (t \leq s_0 - a)}(t) = \frac{\delta(1 - u_2) \exp(-\lambda_2 t)}{\int_a^{s_0} \delta(1 - u_2) \exp(-\lambda_2(s_0 - l)) dl + \exp(-\lambda_2(s_0 - a)) \int_{-\infty}^a \delta(1 - u_1) \exp(-\lambda_1(a - l)) dl}$$

For  $t > s_0 - a$ , we obtain

$$g_{\delta, \lambda, (t > s_0 - a)}(t) = \frac{\delta(1 - u_1) \exp(-\lambda_1(t - (s_0 - a))) \exp(-\lambda_2(s_0 - a))}{\int_a^{s_0} \delta(1 - u_2) \exp(-\lambda_2(s_0 - l)) dl + \exp(-\lambda_2(s_0 - a)) \int_{-\infty}^a \delta(1 - u_1) \exp(-\lambda_1(a - l)) dl}$$

and thus we obtain as expectation

$$ET = \frac{\frac{u_2}{\lambda_2} + \exp(-\lambda_2(s_0 - a)) \left( u_1 \left( s_0 - a + \frac{1}{\lambda_1} \right) - u_2 \left( s_0 - a + \frac{1}{\lambda_2} \right) \right)}{u_2 + (u_1 - u_2) \exp(-\lambda_2(s_0 - a))} \quad (4)$$

This is the expected interrupted unemployment duration in the stock sample, when  $\lambda$  changes in  $a$ . Note that this equation is easily generalized to the case where  $\lambda$  and  $\delta$  change at the same time  $a$ , simply by using the new  $\delta$  for the calculation of the unemployment rates  $u_1$  and  $u_2$  in Formula (4).

We show in the simulations below that the first derivative of  $ET$  with respect to  $(s_0 - a)$  after a change of  $\lambda$  is negative. Thus the measure moves in the same direction as the parameter of interest.



## Usefulness of ET for comparisons over time

Summing up, we find that for permanent changes of  $\lambda$  the stock sample measure moves in the same direction as the parameter of interest, attaining the new correct value as  $(s_0 - a)$  goes to infinity.

For permanent parameter changes of  $\delta$ , there is no change of the parameter of interest, whereas the stock sample measure first increases and then decreases when  $\delta$  decreases.

We define usefulness in the sense that with parameter changes the measure moves in the same direction as the parameter of interest.

From this, we conclude that *ET* is useful only in the case when  $\lambda$  changes not when  $\delta$  changes. Or formulated more clearly, if the measured unemployment duration in the stock sample increases, we can deduce that the expected unemployment duration is in fact increasing if the increase stems from a change in  $\lambda$ ,  $\delta$  being constant. On the contrary, when the measured unemployment duration is increasing and when this change stems from a change in  $\delta$  or when we simply do not know where it comes from, we cannot be sure, that the actual duration is in fact increasing.

Since in real world, we can probably never be sure, where the change comes from and because real world situations tend to be more complex than the model demonstrated here, we cannot learn much from the shift of the stock sample measure about changes of the underlying duration distribution.

The measure we propose below answers at least partly to these facts.

## Comparison of different groups

Above we have shown how the stock sample measures develop given a small deviation from the baseline model. We have seen that the stock sample measure seems somehow useful, when the outflow rate changes, but is not helpful when either the inflow rate (into unemployment) changes or if both rates change at the same time.

Thus, for comparisons across time the measure can be useful or not. What about comparisons between groups, say occupations? Can we expect that an occupation

that has an higher unemployment duration (as measured by the stock sample measure) will exhibit a higher expected duration as the other occupation as well? The unfortunate answer is no or at least not always.

To demonstrate this, we go back to the simple case of stationarity, i.e., the in- and outflow rates for each group are drawn from the same distribution independently of when it happens. Under these conditions formula (1) applies for each group. Let A be the occupation with the higher expected unemployment duration. It is easily shown that if

$$\frac{Var(s_A)}{2E(s_A)} - \frac{Var(s_B)}{2E(s_B)} < E(T_A) - E(T_B)$$

holds, A has indeed the higher (stock sample) measured unemployment duration, as is true for the actual unemployment duration. Otherwise however, the stock sample measure will be higher for the group with the lower actual duration. Note that in the case of exponentially distributed durations this cannot happen, since as described above for this case the expectation equals the variance and both durations of A and B are correctly measured by the stock sample.

To put it more clearly, when we observe in the stock sample that nurses have smaller unemployment durations than engineers, we cannot conclude that nurses are indeed unemployed for a shorter time on average than engineers. In other words, the comparison is useless. Below with the micro data evidence, we show that the above inequality was likely to be violated in the past at least for some groups.

## Simulation Results

In what follows we use calibrated values for the German labour market to demonstrate the results above and to estimate the size and the duration of the bias introduced by the stock sample measures. In addition, we demonstrate that the first derivative of the expectation of the interrupted durations in the stock sample for the change of  $\lambda$  is negative.

Further, we modify the above model to allow the unemployment rate to adjust sequentially. I.e., in this modification the unemployment rate does not jump immediately to its new equilibrium value (as assumed for the analytical description above) but adjusts slowly to its new value: every day the new unemployment rate is

calculated from outflows minus inflows.<sup>7</sup> We use the following values for our calibration exercise. These are taken from real labour market data for Germany from the year 2011.

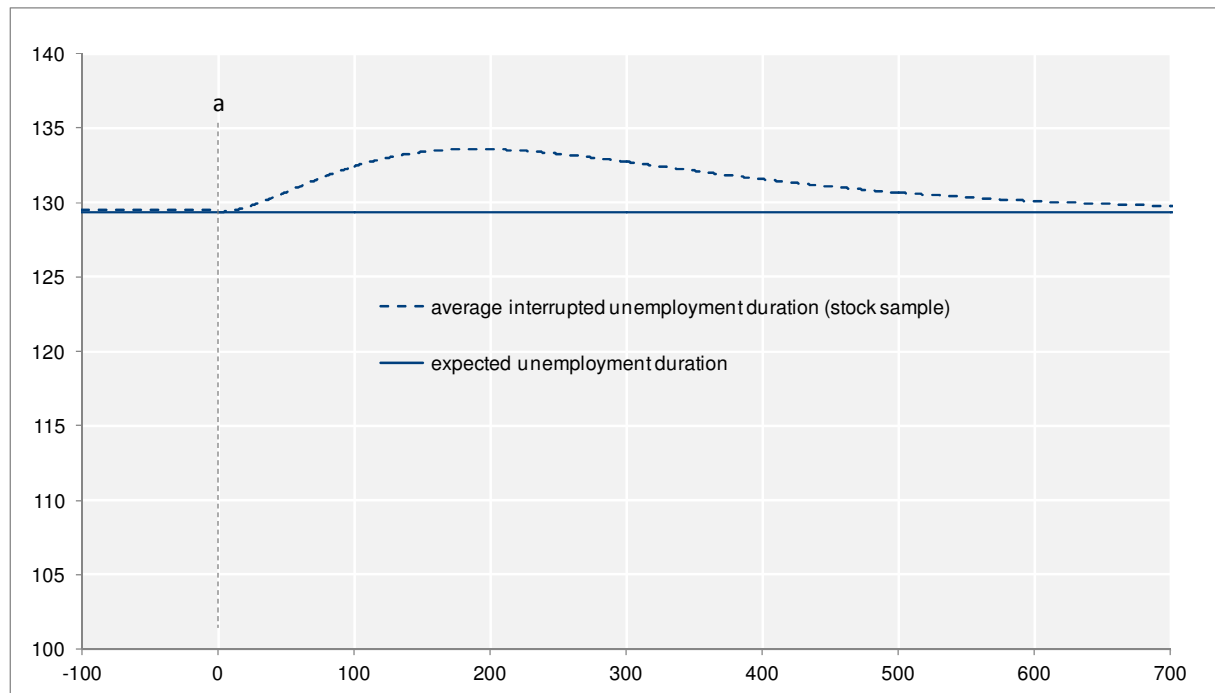
Table 1: Calibration values (Germany, 2011)

Variable	Value
Daily separation rate, $\delta_1$ (number of newly unemployed, divided by the number of jobs)	0,079% (=8.2mio/28.4mio*365)
Fictitious new daily separation rate, $\delta_2$	0,070% (~value for May 2012)
Daily accession rate, $\lambda_1$ (number of persons leaving unemployment, divided by the number of unemployed)	0,773% (= 494.000/178.000*365)
Fictitious new accession rate, $\lambda_2$	0,889% (~value for Sept 2012)
Expected unemployment duration $ET_1$ , given stationary accession rate	~129 days
Expected unemployment duration $ET_2$ , given new accession rate	~112 days
Stationary unemployment rate from flow measures	9,3%
Actual unemployment rate	7,1%
New stationary unemployment rate, given new separation rate	8,3%
New stationary unemployment rate, given new accession rate	8,2%
New stationary unemployment rate, given both rates change	7,3%

Once we change the daily separation rate  $\delta_1$  to its new lower value  $\delta_2$  (a reduction of a bit more than 15 percent from the yearly average), which was the lowest monthly average value in 2012 (June), the stock sample measure of the unemployment duration starts to increase. It attains its maximum after about 140 days and is then 139,8 days or about 6,1 percent higher than the real value. After a bit less than 600 days the difference between the measured value and the real value has fallen to below one percent. Figure 1 demonstrates the development of  $ET$ .

<sup>7</sup> Using a discrete daily adjustment process, it takes almost one (half) year for the actual unemployment rate to deviate less than one (four) percent from the new stationary unemployment rate.

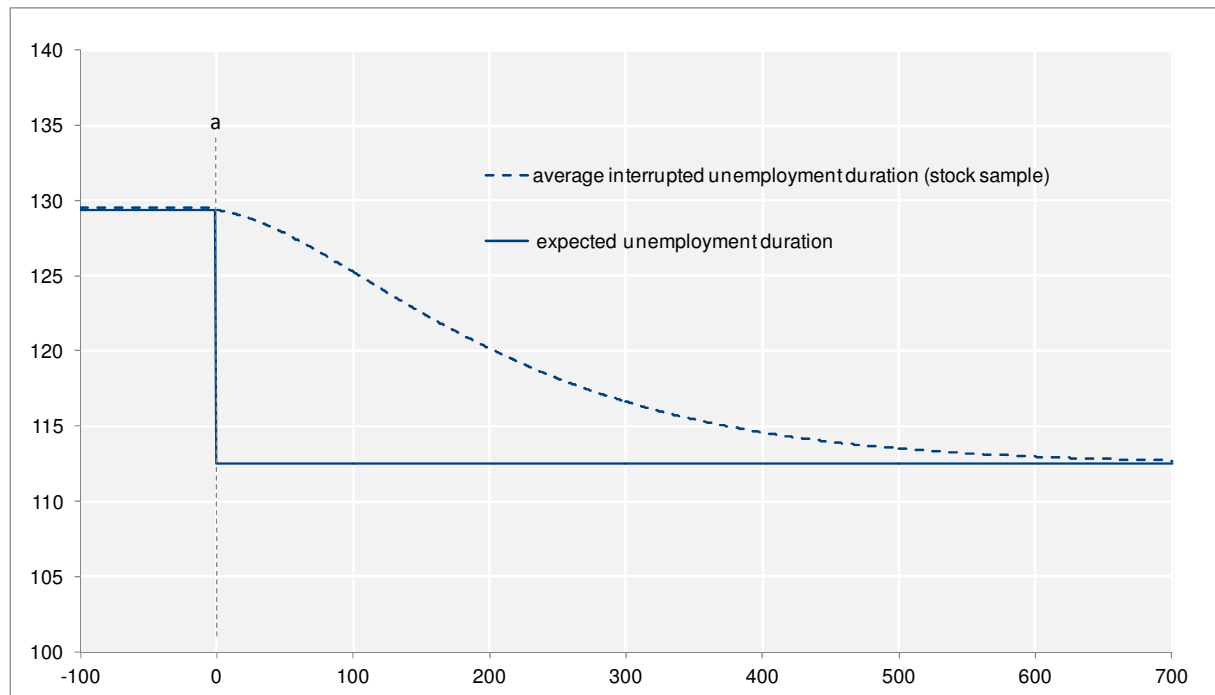
Figure 1: Development of the expected unemployment duration and the average interrupted unemployment duration after a shock to inflows in  $a=0$



Source: Own calculations

Once we change the daily accession rate  $\lambda_1$  to its new value  $\lambda_2$  (an increase of about 15% from the yearly average), which corresponds to the highest monthly average value in 2012 (August), the stock sample measure immediately begins to drop and moves (albeit slow) in the same direction as the expected unemployment duration. The difference between the measured and the actual duration drops to below one (five) percent only after 500 (260) days. Figure 2 demonstrates this.

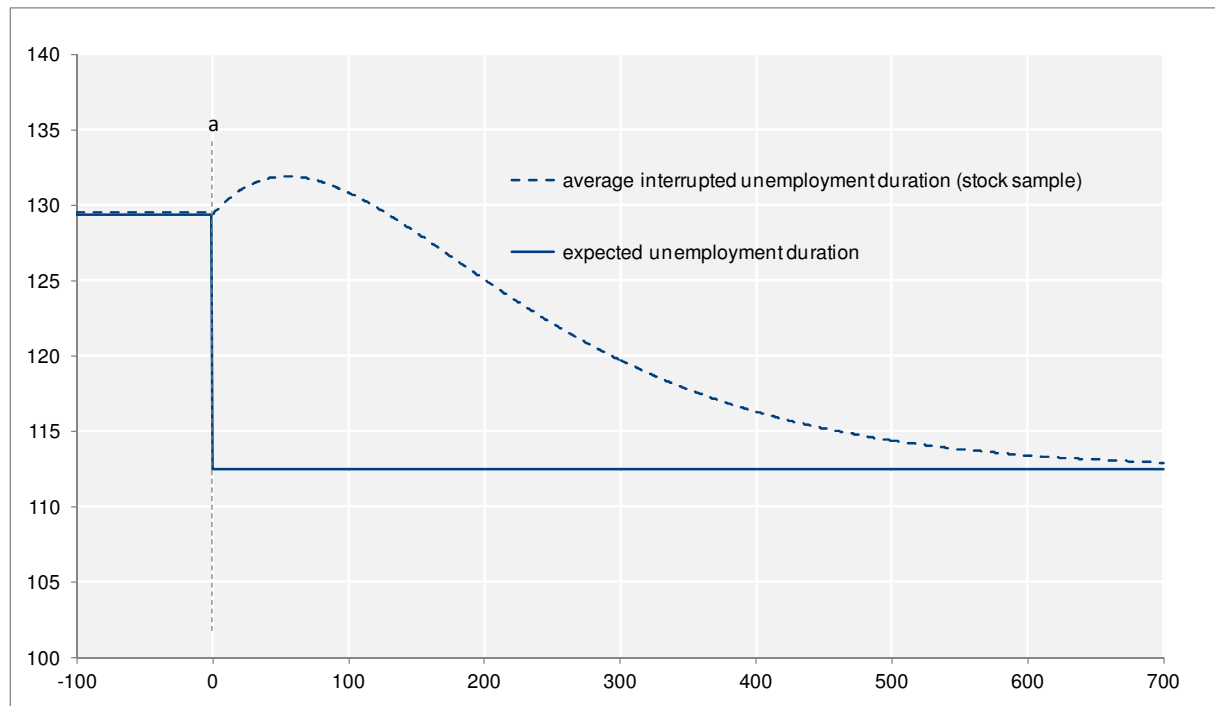
Figure 2: Development of the expected unemployment duration and the average interrupted unemployment duration after a shock to outflows in  $a=0$



Source: Own calculations

Finally, we manipulate the two values at the same time to their new values  $\delta_2$  and  $\lambda_2$ . Now, the stock sample measure increases immediately for around 70 days, where the maximum value is about 136 days (almost 20 percent higher than the true value), then it starts to decrease. The difference is below 5% for the first time after a bit more than a year, and below one percent after more than 600 days. This is probably the worst result in our simulations: although the (true) expected duration decreases, the stock sample measure increases for more than 2 months. Figure 3 shows this unsatisfactory behaviour.

Figure 3: Development of the expected unemployment duration and the average interrupted unemployment duration after a shock to in- and outflows in  $a=0$



Source: Own calculations

## Proposing alternative measures

As far as comparisons over time are concerned, the problem of incorrectly estimating movements of the duration variable stem from variations in the inflow variable. Thus, it seems natural to hold inflows constant thereby making it more likely that the stock sample measure moves in the correct direction. There are three options available to make inflows constant over time: either one takes the average inflows at each time, requiring however to move parts of the observations randomly on the time axis, randomly assigning counterfactual durations to them. I.e., if the actual inflow is larger as the average, one would randomly sample from those inflows away to make the inflows average and then randomly assigning the sampled inflows to the past to make every inflow time size average. This clearly requires that there are no systematic differences across inflow cohorts (and exponentially distributed durations). Otherwise, we introduce a new source of error in the stock sample measure. An alternative that requires weaker assumptions to not introduce new errors but which still relies on the average inflows is to sample away above average inflows and to not use those sampled away and to resample from those inflows and to use the same

spells several times for filling below average inflows to the average. Given the random sampling this correction should not introduce a new source of error and corrects for changes in inflows. The third option avoids the resampling and double use of observations and simply picks the minimum of the inflows (across time) to calculate the stock sample measure.

Another alternative relies on a completely different way of thinking. It uses the most recent information available to estimate the expected duration. The idea is to use daily (or monthly) hazards from the past to estimate the expected duration of newly entrants. I.e., for those entering unemployment today, the estimated probability of leaving tomorrow is estimated from those entering yesterday and having left today, adjusted by season, trend, cohort (if estimable?) and other factors. Or one could average over the last 30 1 day observations for the one day hazard, over the last 30 2 days observation for the two day hazard and so on. From the hazard rate  $\lambda$ , we can calculate the Survivor function by using  $\lambda = -d\ln[S(t)]/dt$  or  $S(t) = \exp[-\int_0^t \lambda(t)dt]$ . From this, we can use that the expected duration is equal to the area under the survival curve, i.e.,  $ES = \int_0^\infty S(t)dt$ . In discrete time the integrals are to be replaced by sums and the survivor can be calculated by multiplying over all  $(1 - \lambda_t)$  up to  $t$  (Cameron/Trivedi, 2005; Kalbfleisch/Prentice, 2002).

## **Assessing the performance of the measures in micro data**

We assess the performance of the measures used by statistical offices, namely the average interrupted unemployment duration in the stock sample and the average unemployment duration of completed spells in a stock sample, as well as measures we propose above, namely the inflow adjusted average interrupted unemployment duration of the stock sample and the inflow adjusted completed average unemployment duration in the stock sample in predicting the parameter of interest. To do this, we use a large administrative dataset for Germany (the SIAB). Using this dataset, we demonstrate a) how the stock sample measures perform in predicting the development of the parameter of interest over time, b) how our proposed measures perform in this respect and c) whether the condition for comparison between groups

holds, i.e., whether - given real data - from comparing stock sample measures of different groups we can learn something about the moments of the underlying duration distribution.

## **The dataset and calculation of the measures of interest**

The SIAB is a 2 percent random sample of all individuals that have been at least once subject to social security employment in Germany in the time period considered. It stretches from 1975 – 2008 and covers both employment and unemployment spells of the individuals considered.<sup>8</sup>

We calculate the true unemployment duration from those entering unemployment in each month of the observation period by following them over the complete durations of their first spell<sup>9</sup> up to the maximum, which corresponds to the end of the observation period (31.12.2008).<sup>10</sup> Therefore, we end our comparison period by the end of 2005 to allow at least three complete years to be observed for all observation months.

We also cut the observation period backward on January, 1<sup>st</sup>, 1978 since for calculating the stock sample measures, we need a certain time span for calculating the previous inflows and thus the stock sample correctly.<sup>11</sup> We calculate both measures suggested by the FEA, ie, the average interrupted unemployment duration in the stock sample and the average terminated unemployment duration of those spells ending in a certain month.

---

<sup>8</sup> For a thorough description of the dataset, see Dorner et al. (2010).

<sup>9</sup> The first spell of unemployment can end for different reasons. Unemployed individuals may find employment, leave the dataset for some reason (e.g., becoming self-employed or being pregnant) or participate in active labour market programs. Although one might argue that the latter does not interrupt unemployment, it is in accordance with the definition of unemployment durations of the FEA.

<sup>10</sup> We calculate monthly values since this corresponds to the statistic of the FEA, which classifies durations into monthly intervals.

<sup>11</sup> Clearly, the choice of three years forward and backward is arbitrary. Note however, that the choice (1, 2, 3, 4 or 5 years) does not affect our results in any substantive way. In addition, note that when considering a Poisson process, assuming an expected daily outflow rate of 0.01, i.e. an expected duration of 100 days, only about 2/100.000 persons would be left after three years; an expected error of about 0.02 percent for the measured true duration, and a maximum error of about 0.2 percent, when the two spells ended not as expected in the next year, but in the next 30 years.



# Results

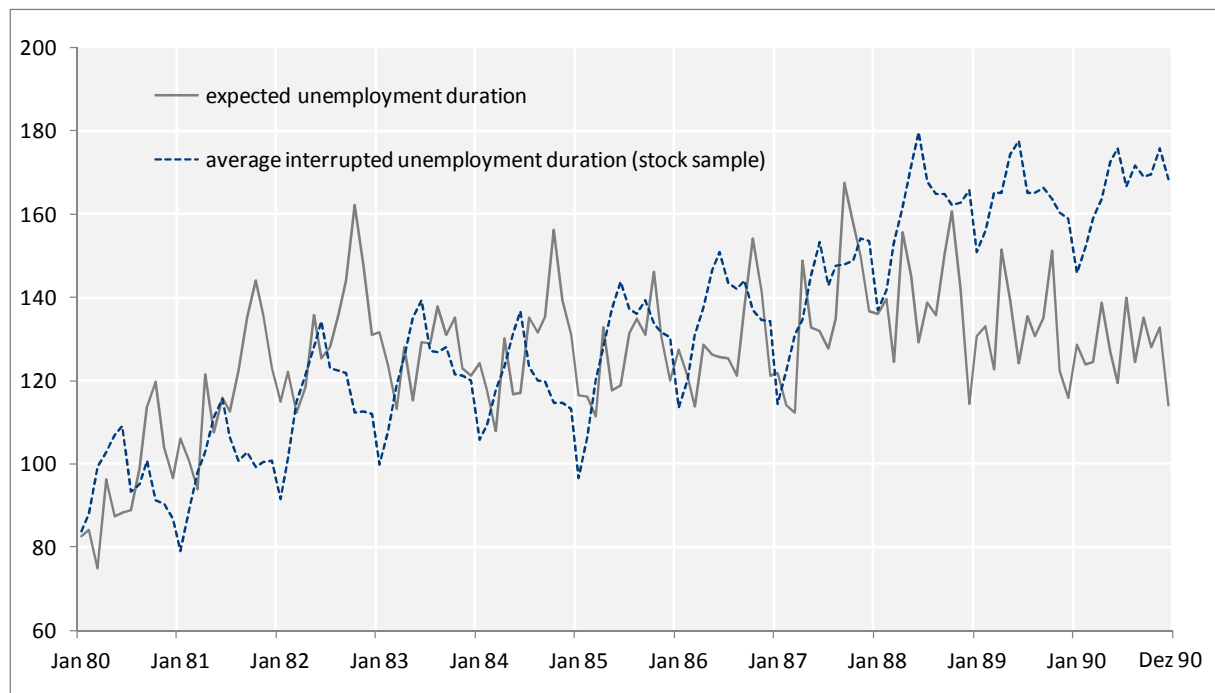
We regress the expected duration on the respective measure either in levels or in differences including month dummies to control for the strong seasonal structure in the data (see figure 4 and 5). We find that both stock sample measures on interrupted durations and the stock sample measure for completed durations (as well as the inflow corrected measures we propose) are positively correlated with the true unemployment duration<sup>12</sup>, whereas the first differences of both variables are significantly and highly negative correlated with the first differences of the true duration. In other words, when the (stock sample) measured unemployment duration increases, in most cases the expected duration in unemployment decreases. Table 1 in the Appendix gives a few of the regression results. Visual inspection of the two time series in figure 4 confirms this result: the two variables are developing contrary to each other. Unfortunately, the same is true for the inflow-corrected measure(s) that we propose (see figure 5). While it is true that the inflow corrected measure explains a little more of the variance of the expected duration, the size of the coefficients in level and first differenced regressions are similar.

We also checked whether the year-to-year differences of any of the stock sample measures are interpretable with respect to the year-to-year change of the parameter of interest. They are not: for the one year difference of the four stock sample measures we find insignificant relationships to the one year difference of the parameter of interest.

Figure 4: Expected unemployment duration and average interrupted unemployment duration (stock sample) (Germany, January 1980 – December 1990)

---

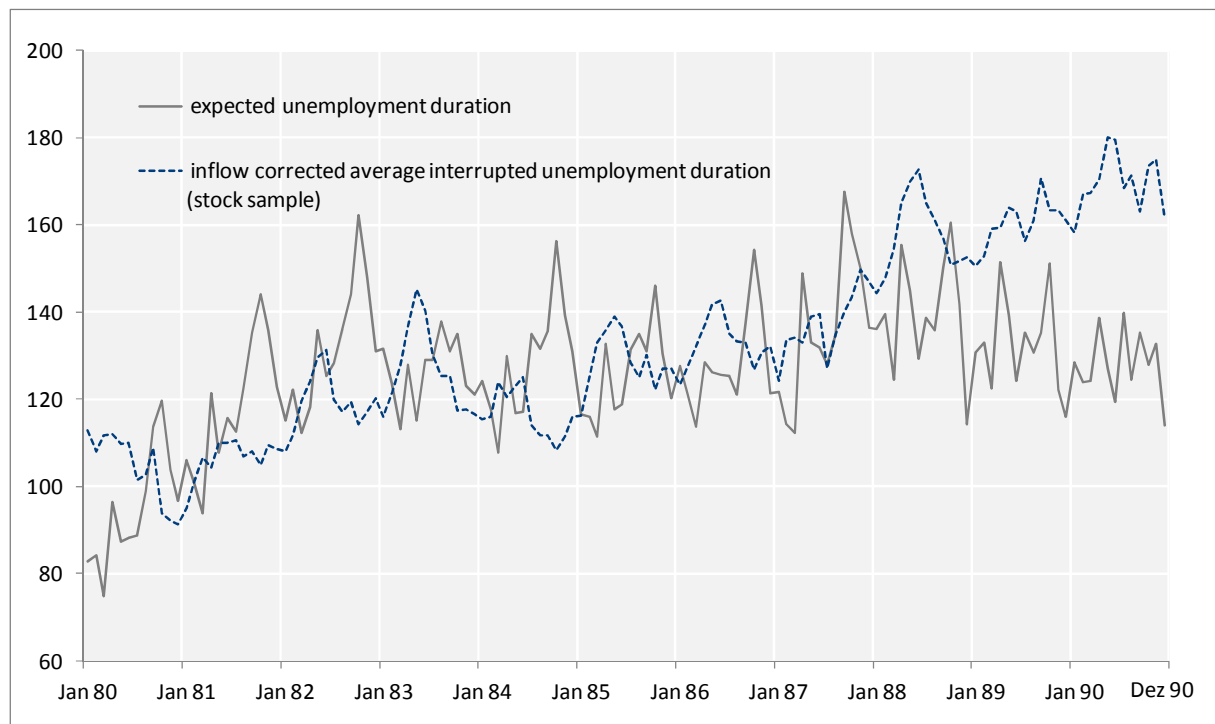
<sup>12</sup> Note that, using an augmented Dickey-Fuller test, we can reject the hypothesis of a unit root both for the expected duration and the average interrupted duration but not for inflow corrected measure. Thus, the level regression for the latter has to be interpreted with caution. Note that for the completed duration measures there is also no unit root.



Source: SIAB 1975 -2008, own calculations

Interestingly, there is a stable, positive and significant relationship between the one year change of the stock sample measured unemployment duration and the one year change of the 6/12/24 months moving average of the expected unemployment duration. The relationship is the stronger, the longer the time span over which the moving average is built. The month to month changes of the stock sample measure and the moving average on the contrary are not related. Thus, at least in our dataset, we can deduce from changes of the stock sample measure to the previous year, that the true unemployment duration of the average of the entry cohorts of the last 24 months has changed (on average) in the same direction. Unfortunately this is not true for month to month comparisons. In addition we are not able to show that this relationship has a theoretical justification. In that sense, we cannot be sure that this relationship is stable.

Figure 5: Expected unemployment duration and inflow corrected average interrupted unemployment duration (stock sample) (Germany, January 1980 – December 1990)



Source: SIAB 1975 -2008, own calculations

A further question of interest was, whether we can compare groups with the stock sample measure and deduce that the parameters of interest for the groups behave accordingly. Above we have given a condition (equation (2)) for the expectation and the variance for the two groups that guarantees that indeed the difference between the two groups has the same sign as the difference for the stock sample measure. The empirical results for comparing for example men and women are stunning. In around 40 percent of the cases the signs of the differences between the stock sample measures and the expected durations are different. Thus, when observing that one group has a larger (stock sample) duration than another group, this does not at all imply that this group has indeed the larger duration.

## Conclusion

In this paper, we show that stock sample measures are difficult to interpret. First, we show theoretically under what conditions we can expect the stock sample measure to move in the same direction as the parameter of interest. Second, calibrated with values for the Hessian labour market, we simulate the development of the stock

sample measure when manipulating the rates of in- and outflow in and out of unemployment. We show that it is specifically the inflows that alter the stock sample measure in an unacceptable way. This leads us to propose (stock sample based) measures that equalizes inflows across time, thus excluding this source of error.

Using microdata for the German labour market, we calculate monthwise the expected duration, the stock sample measures, and the stock sample measure that we propose. Unfortunately, the stock sample measures have no good performance in predicting the parameter of interest, especially when we concentrate on changes of these measures. One notable exception is the development of the stock sample measure in the one year comparison, which we can relate to the one year comparison of a moving average of the true duration. The stock-sample measure we propose performs a bit better in terms of the explained variance than the stock sample measure that the FEA uses and slightly better in terms of comparisons of groups, but the results are also not convincing for our measure.

We have to conclude that one cannot deduce from changes in stock sample measures of unemployment duration that expected unemployment duration has changed as well and if so whether it is in the same direction. By the same, a finding that the (stock-sampled) unemployment duration is larger for one group than for another does not mean that this group is in fact longer unemployed on average.

Saying this, does not mean that the unemployment duration of the stock sample could not convey valuable information to some recipients. For example, it might be interesting for the FEA to have averaged information about the composition of those that are currently unemployed. However, one has to be extremely careful with the interpretation of all kinds of comparisons over time and groups.

We can transfer our results in addition to the discussion about labour shortages. Namely, vacancy durations play an important role in the discussion of the development of labour shortages. For example, increasing vacancy durations are often interpreted as a sign for increasing labour shortages. However, the truth is that from this we do not know whether or not the real (expected) vacancy duration increases, when the stock sample measure has increased. Again the same is

possibly true when comparing groups, i.e. when for example the vacancy duration of nurses is longer than that for accountants, it does not necessarily mean that these vacancies are actually harder to fill.

# Literature

Cameron, Colin A. and Pravin K. Trivedi (2005): Microeconometrics: Methods and Applications, Cambridge: Cambridge University Press.

Cahuc, Pierre and André Zylberberg (2004): Labour Economics, Cambridge: MIT Press.

Dorner, Matthias; Heinig, Jörg; Jacobebbinghaus, Peter and Stefan Seth (2010): Sample of Integrated Labour Market Biographies (SIAB) 1975 – 2008, FDZ data reports, No. 01/2010, Nürnberg.

Garloff, Alfred (2010): Minimum Wages, Wage Dispersion and Unemployment in Search Models. A Review, Journal of Labour Market Research, 43, 2, pp. 145-167.

Kalbfleisch, John D. and Ross L. Prentice (2002): The Statistical Analysis of Failure Time Data, 2<sup>nd</sup> edition, Wiley: Hoboken.

Kaitz, Hyman B. (1970): Analyzing the length of spells of unemployment, Monthly Labor Review, 43, pp. 11-20.

Kiefer, Nicholas M; Lundberg, Shelly J. and George R. Neumann (1985): How long is a spell of unemployment?: Illusions and Biases in the Use of CPS Data, NBER working papers, No. 1467, NBER: Cambridge.

Lancaster, Tony (1990): The Econometric Analysis of Transition Data, Cambridge University Press: Cambridge.

Salant, Stephen W. (1977): Search Theory and Duration Data: A Theory of Sorts, in: The Quarterly Journal of Economics, Vol. 91, No. 1, pp.39-57.

# Appendix

Table 1: Regression results from regression of the (first difference of the) expected duration E(s) (dE(s)) on RHS and month dummies (years 1978-2005), Standard errors in parentheses

RHS	E(T)	dE(T)	E(T) (prop)	dE(T) (prop)	E(T) (compl)	DE(T) (compl)	E(T) (prop, compl)	dE(T) (prop, compl)
LHS E(s)	0,476 (0,032)	--	0,534 (0,032)	--	0,627 (0,039)	--	0,709 (0,039)	--
LHS dE(s)	--	-0,991 (0,089)	--	-1,003 (0,133)	--	-0,277 (0,045)	--	-0,343 (0,066)
Adj R <sup>2</sup>	0,424	0,635	0,490	0,569	0,470	0,546	0,527	0,533

Source: SIAB, own calculations, d refers to the first difference (change to previous month), “prop” refers to the inflow corrected measures we propose, “compl” refers to the measure of the completed durations.