Rouwendal, Jan; van Duijn, Mark

Conference Paper

Diverging house prices

53rd Congress of the European Regional Science Association: "Regional Integration: Europe, the Mediterranean and the World Economy", 27-31 August 2013, Palermo, Italy

Provided in Cooperation with:
European Regional Science Association (ERSA)

Suggested Citation: Rouwendal, Jan; van Duijn, Mark (2013) : Diverging house prices, 53rd Congress of the European Regional Science Association: "Regional Integration: Europe, the Mediterranean and the World Economy", 27-31 August 2013, Palermo, Italy, European Regional Science Association (ERSA), Louvain-la-Neuve

This Version is available at:
https://hdl.handle.net/10419/124062

Diverging house prices

Jan Rouwendal and Mark van Duijn
Department of Spatial Economics
Free University
De Boelelaan 1105
1081 HV Amsterdam
The Netherlands

This version: January 4, 2013

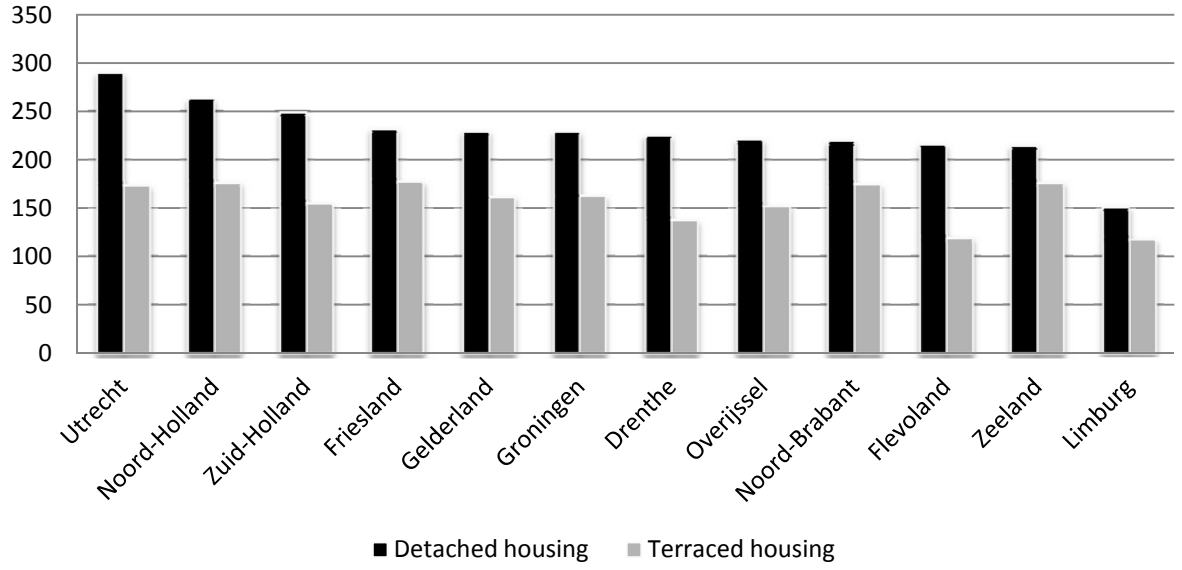
Keywords: housing demand, fixed heterogeneous housing supply, income shocks
JEL Classification Codes: R31, R21, D40

Abstract
This paper provides a model for the well-known empirical phenomenon that houses of different quality experience different price developments. The typical pattern is that luxury houses appreciate more in boom periods and depreciate more during busts. The standard model of housing demand treats housing as a quantity of 'housing services', an imaginary homogeneous commodity that is available in arbitrary quantities at a constant price per unit. This model is unable to explain differential development of house prices. However, a simple variant that treats the number of houses offering a given number of housing services as fixed is able to do this. This is shown by means of a formal analysis of a model in which households that differ in income are allocated over a given housing stock. In particular, the model predicts that the price of housing as a function of quality becomes more convex after a proportional increase in all incomes. Earlier explanations of this phenomenon relied on down payment effects, but since diverging house price developments are also observed in countries where these effects are negligible, this provides only a partial explanation. Empirical analysis of house prices in Amsterdam confirms the predictions of the model.

1 Introduction

There exists abundant evidence that the prices of different types of housing evolve differently over time. Typically, luxury housing appreciates more than other types during booms, and depreciates more during busts. Table 1 illustrates this phenomenon for detached and terraced housing in the Netherlands during a long period of house price increases that lasted from 1995 to 2007. The figures refer to existing owner-occupied dwellings (new construction is excluded) and are published by CBS and the Dutch Land Register (in Dutch: Kadaster). In all provinces except one (Limburg) the prices of detached houses more than tripled: the price increase was more than 200%. However, for terraced houses prices never tripled, although in all cases they doubled.[1]

Figure 1. Appreciation of detached and single family housing in Dutch provinces 1995-2007



The figures show the ratio of prices in 2007 to 1995 minus 1, multiplied by 100. Source: Statistic Netherlands/Kadaster. See: *www.kadaster.nl/kadaster/wat_doen_we/waardeindex.html*

The differences in appreciation rates between various types of housing do not fit easily with the use of 'housing services' as an explanatory device for the functioning of the housing market. Housing services are an imaginary commodity introduced by Muth (1960) to facilitate the use of standard micro-economic tools for housing market analysis and it has been very

[1] Since 2007 house prices in the Netherlands have on average decreased, but very modestly.

successful.[2] The typical application of this approach considers the housing stock as a large number of housing services that can be distributed arbitrarily over households. The convenient consequence of this approach is that there is a single price of housing services, but the flipside of this coin is clearly that differences between price developments of different housing types are excluded.

The explanations for diverging house prices that have been put forward in the literature have therefore relaxed the housing services concept by distinguishing between two (or more) types of dwellings without imposing proportionality of the prices. Differences in price developments following a shock in income are then shown the be related to the down-payment constraint and the effects that initial (modest) price increases on low quality houses have on the possibility for leverage owners to switch to higher quality houses (see Ortalo, Magné and Rady, 2006). However, the differential development of prices of different housing types can also be observed in countries like the Netherlands where the down-payment constraint does hardly play a role, as was shown in Figure 1.[3]

In this paper, a model is developed that explains the differential development of house prices following an income shock without a down-payment constraint. The model remains very close to the conventional housing services approach and can be derived from it by introducing some restrictions. What we do here is abandon the assumption of perfect malleability of housing capital. Instead, we think of houses as a given quality level (that is, producing a given number of housing services in each period). If the number of available houses of each quality level is given (in the short run), the price per unit of housing services can differ for houses of different quality and differences in price movements between houses of different types become possible. In this setup the housing stock can be described as a distribution function of houses that differ in quality. This housing stock has to be distributed over a set of households that differ in income. To focus on one important aspect – the relationship between income shocks causing housing market booms and differential price development – we assume that demand for housing depends only on household income. We show that if housing is normal, the ranking of housing consumption follows that of incomes. This allows us to find the matching between incomes and houses. This

[2] See Rouwendal (1998) for an examination of the micro-economic foundations of the concept.

[3] In the Netherlands, a low priced mortgage insurance (the Nationale Hypotheek Garantie) is available for first-time buyers. It allows them to borrow as much as 100% of the value of the house. To be eligible for the insurance the mortgage payment to income ratio should not exceed a threshold value of approximately 30%. However, this constraint does not have the same effects as a down-payment constraint following an income shock.

matching must be facilitated by the price mechanism, and this requirement implies a relationship between house price and quality, that can be viewed as a (one dimensional) hedonic price function.

The conditions under which the house price function – that describes the price of housing as a function of quality – is locally convex, linear or concave are made precise. The curvature of this house price function at a particular quality level is shown to be related to the ratio between the number of houses with that quality level and the number of households with the corresponding income (i.e. the income at which this level of housing quality is demanded at the prevailing hedonic price function). Intuitively, if this ratio is large, the house price function must be locally convex to prevent demand from increasing 'too fast' with income. If the ratio is small, the house price function must be concave. The situation in which the house price function is locally linear can be interpreted as an equilibrium in the sense that the number of available houses with a given quality matches the number of household demanding that exactly that quality.

The results of the analysis are most clear-cut when the slopes of the demand and Engel curves for housing services do not change as a consequence of the income shock. For this case the analysis implies that an equal (absolute) increase of all incomes leaves the curvature of the hedonic price function unchanged, although the (marginal) price of housing services may change. However, a proportional increase in all incomes will make the hedonic price function more convex, which implies the phenomenon of differential price development. This conclusion is reached under the assumption of rigid supply, while demand shifts towards houses of higher quality. In the longer run changes in the housing stock will counteract this initial price reaction, although it should be noted that asymmetric adjustment (see Glaeser and Gyourko, 2005) makes it probable that the impact of income shocks may last for a prolonged period of time.

To show that the model explains the phenomenon of interest we derive the conditions under which a proportional change in all incomes results in a more convex housing price function. A special case in which a closed form solution of the housing price function can be derived occurs if the distributions of income and housing quality are uniform and demand for housing services is linear.

Our model implies that houses that provide a larger number of housing services will always command a higher prices. The ranking based on prices therefore coincides with the

ranking based on housing services. In our empirical application we use this property of the model to estimate the number of housing services as a function of housing characteristics. With this function in hand, we can investigate the development of the housing price as a function of the number of housing services over time. We fund for the Amsterdam region that this function became increasingly convex during the long boom period that lasted from 1995 to 2007, and less convex in the recession that followd.

The paper is organized as follows. The next section introduces the model and the main theoretical results. We start with a discussion of the setup, then derive some initial results that characterize the matching of households over housing and go on to derive the curvature of the house price function. Section 3 discusses the implications of the model for the effect of income shocks and illustrates them in various ways. Section 4 provides some preliminary empirical evidence. This section is incomplete and will be extended in the coming weeks. Section 5 concludes.

2 The model

2.1 Introduction

We consider a market with a fixed supply of a heterogeneous commodity: housing. Houses are available in a continuum of varieties, and each variety is characterized by a number of housing services. This number is interpreted as a scalar index of housing quality. The consumers that demand housing all have identical tastes, but differ in incomes. The housing stock is fixed in the short run.

Formally, we define housing quality as the number of housing services q offered by a house. The only departure from Muth's (1960) framework is that we treat q as fixed for each house and allow the price per unit of housing services to differ over houses.[4] The price (rent) $p(q)$ of a house that offers q units of housing services per period is therefore not necessarily equal to the product of q and a unit price that is equal for all housing qualities.

[4] The model of the present paper is related somewhat to Braid's (1981, 1984) analysis of rental housing markets, which built on Sweeny (1974).

The housing stock is described by the distribution function of the quality of housing, $G(q)$. G is assumed to be differentiable and to have support on an interval $[q^{min}, q^{max}]$. The stock of houses is S, $S = G(q^{max})$. The density function associated with G is denoted as g.

The function $p(q)$ can be interpreted as a simple hedonic price function. It gives the rent or user cost of a house as a function of its quality. We assume that the hedonic price function is twice differentiable. The marginal price of housing services, π, is the first derivative of the hedonic price function:

$$\pi = \frac{\partial p}{\partial q}. \tag{1}$$

Clearly, the marginal price of housing services is a constant if and only if the hedonic price function is linear, that is if: $p(q) = \mu + \pi q$. In the familiar Muth case the house price is proportional to the number of housing services: $\mu = 0$. In the model we develop here, the hedonic price function will in general be nonlinear.

The stock of houses is used by a population of households. As said, we assume that they all have identical tastes that can be described by a utility function u:

$$u(q, c). \tag{2}$$

The two arguments of this function are housing consumption q, which is equal to the number of housing services offered by the dwelling in which the household lives, and other consumption c, which is summarized in the number of units of a composite good. The utility function is assumed to be two times differentiable, increasing in its two arguments, and to have convex indifference curves.

Consumers differ in income. The distribution of income is $F^*(y)$, which has positive support on an interval $[y^{min}, y^{max}]$. We treat income as a continuous variable and assume that F^* is differentiable and denote the density function as f. The total number of households equals B, where $B = B(y^{max})$.

Although we have emphasized that the hedonic price function should be expected to be nonlinear, we will make extensive use of the demand function, which is defined for a linear budget constraint. We denote the demand function as:

$$q = q(\pi, y), \tag{3}$$

where π denotes the – constant - marginal price for housing services.

The budget constraint for a household is:

$$c + p(q) = y. \tag{4}$$

Maximization of the utility function (2) subject to condition (4) leads to the familiar first-order condition:

$$\frac{\partial u}{\partial q}\Big/\frac{\partial u}{\partial c} = \frac{\partial p}{\partial q}. \tag{5}$$

This condition says that in the optimum an indifference curve touches the nonlinear budget line, as is illustrated in Figure 1.

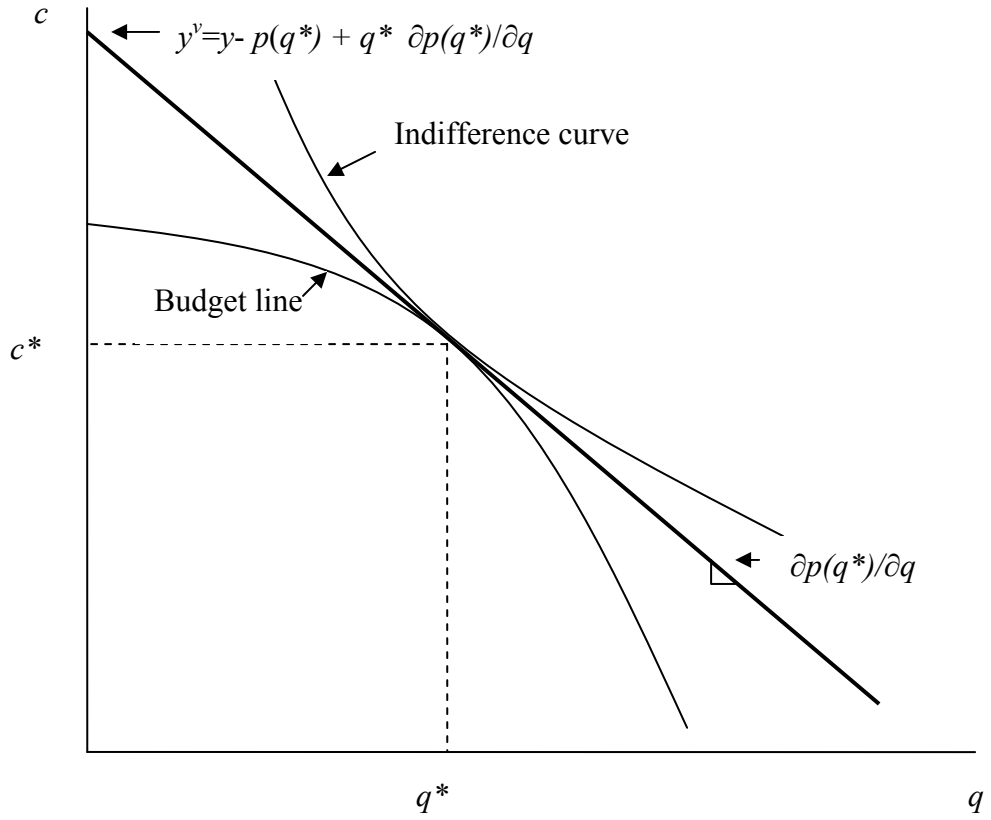


Figure 1. Linearizing the budget constraint

We can describe consumer choice behavior in terms of the conventional demand function by linearizing the budget line at the optimum of the consumer. This implies that we use the marginal price of housing services, $\partial p/\partial q$, in the optimum as the first argument of the demand function and virtual income y^v, which is defined as:

$$y^v = y - p(q) + q\frac{\partial p}{\partial q}, \tag{6}$$

as its second argument. Demand function (3) is therefore rewritten as:

$$q = q(\partial p/\partial q, y^v). \tag{7}$$

In market equilibrium, each household must be on a demand function (7), with y^v given as in (6). Note that the arguments of this demand function are determined by the choice the household makes on the housing market. That is, both the marginal price and virtual income are functions of the chosen amount of housing services q.

2.2 Two preliminary results

In this subsection we establish two elementary properties of the hedonic price function. The first one is that the hedonic price function is increasing in the number of housing services. To see this, suppose that the hedonic price function is not increasing in the number of housing services. Then there is at least one pair of housing services, say q^1 and q^2 with $q^2 > q^1$ and $p(q^2) < p(q^1)$. Since all consumers are utility maximizers, there will then be no demand for housing with quality q^1. The existence of such a pair is therefore incompatible with price equilibrium. Hence the user cost function must be increasing in the number of housing services.

The second result is that in a market equilibrium housing consumption must be increasing in income if housing is a normal good. This sounds a bit trivial since normal goods are defined as goods whose consumption increases with income, but remember that this definition refers to a situation in which the unit price of the good is constant. That is, it refers to the special case $p(q) = \pi q$ only, and what we will show now is that it also holds with a nonlinear hedonic price function. Fortunately, this is easy to do since with a nonlinear budget constraint exactly the same logic applies. Housing is normal if and only if the marginal rate of substitution between housing and the composite commodity increases in the consumption of the composite commodity, that is if:

$$\frac{\partial}{\partial c}\left(\frac{\partial u/\partial q}{\partial u/\partial c}\right) > 0. \tag{8}$$

If the budget line shifts upward, its slope remains unchanged for any given level of housing consumption. This is true for a linear as well as a nonlinear budget line. However, the slope of the indifference curve through the point of the budget line corresponding to this given level of housing consumption gets steeper, if inequality (7) holds. This implies that the optimal level of

housing consumption must be larger after the vertical shift of the budget line than it was before. The same reasoning applies of course to a downward shift.

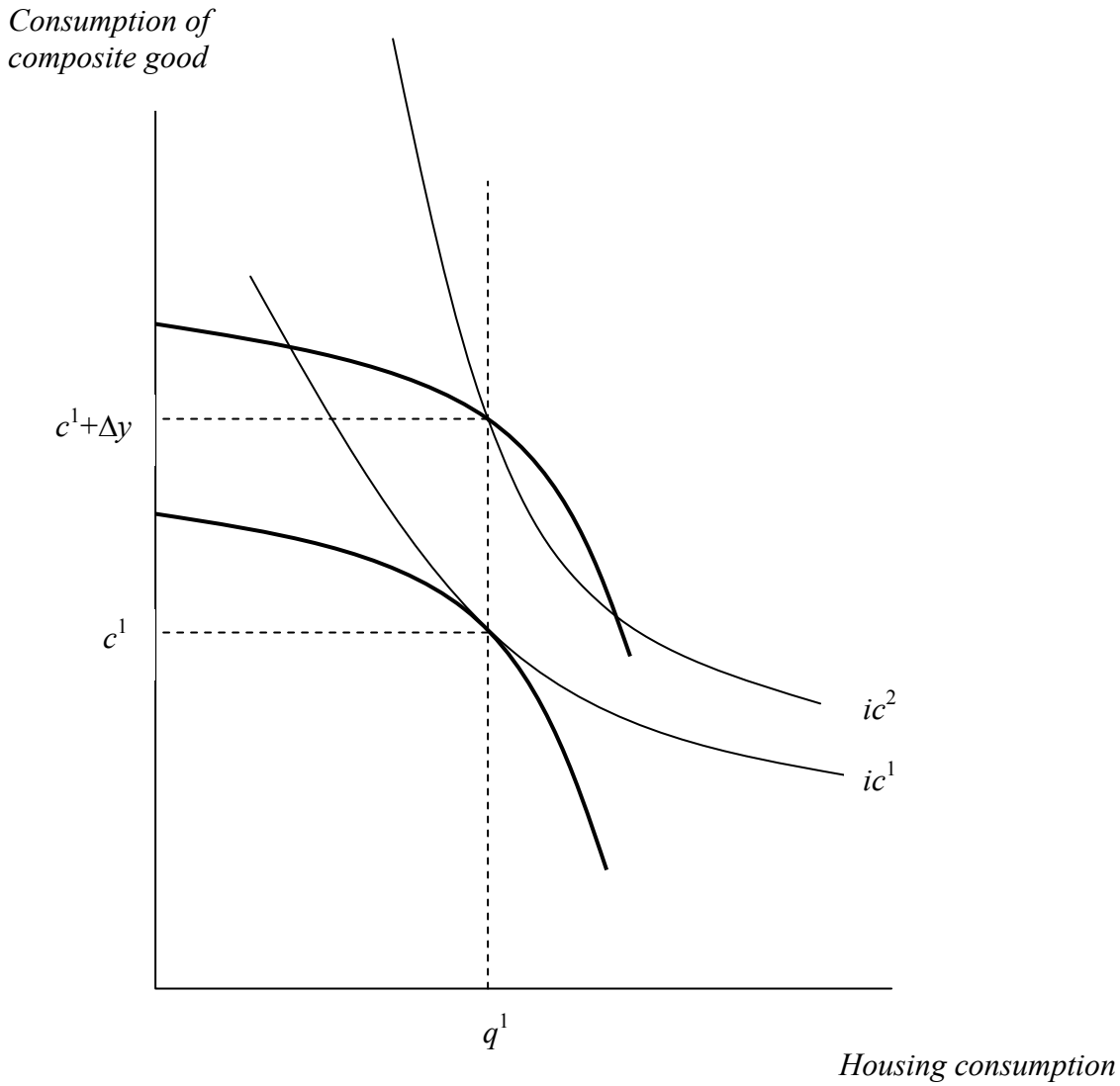


Figure 2. Normal goods and a nonlinear budget line.

This is illustrated in Figure 2. In that figure two budget lines are drawn as $q = y - p(q)$ for a nonlinear hedonic price function. The lowest budget line touches the indifference curve ic^1. For given housing consumption, for instance q^1, the slopes of the two budget lines are equal. If the slopes of the indifference curves crossing or touching the two budget lines would also be equal,

housing could not be a normal good.[5] Indifference curve ic^2 must therefore be steeper than ic^1 when housing consumption equals q^1 and optimal housing consumption at the higher income level must exceed q^1. This second result ensures that, in equilibrium, a household's position in the income distribution is reflected in its position in the housing stock, even if the housing price function is nonlinear.

Earlier in this section we introduced the income and housing distributions. The result just reached tells us that there is an intimate relationship between the two. Consider the situation in which the number of households is at least as large as the housing stock: $B \geq S$. Then only the households with the highest income will be able to live in a house. The B-S remaining households can be interpreted as potential households, that will only be formed if the situation on the housing market permits. Alternatively, the housing stock S may refer to a part of the housing market only, for instance owner-occupied dwellings. We will use the latter example in what follows.

Let y^c be the lowest income of households with an owner-occupied house. The results just derived imply that the household with this income lives in the house of the lowest quality q^{min} and pays the lowest price $p(q^{min})$. Similarly, the household with the highest income y^{max} lives in the house with the highest quality and pays the highest price for housing. More generally, we can order the incomes of the homeowners from low to high and we can similarly order the quality of the houses from low to high. The order of the incomes must be the same as the order of the housing qualities. We can therefore determine the pairs of incomes and housing qualities that must match. We denote the income y that is associated with housing quality q as $y(q)$.

The relationship between income and housing consumption implies:

$$F^*(y(q)) - F^*(y^c) = G(q), \tag{9}$$

which follows from our earlier result that housing consumption is increasing in income. We use the more convenient notation $F(y) = F^*(y) - F^*(y^c)$ for the part of the income distribution that refers to households with positive housing consumption. Using this notation, we can rewrite (9) as:

$$y(q) = F^{-1}\big(G(q)\big). \tag{10}$$

This gives the relationship between income and housing consumption in this model. Note that it could be determined on the basis of some general properties of the allocation process and that the role of prices is not yet made explicit.

[5] Note that for this conclusion the nonlinearity of the budget constraint does not matter.

For later reference, we note that (10) implies:

$$\frac{dy(q)}{dq} = \frac{g(q)}{f(y(q))},$$

where $g(q) = \partial G/\partial q$ and $f(y) = \partial F/\partial y$, that is g and f are the densities associated with the distributions G and F, respectively.

2.3 Market equilibrium and the curvature of the hedonic price function

In a market equilibrium each household must be on demand curve (7) and the implied combination of income and housing consumption should satisfy (10). That is, in market equilibrium we can rewrite (7) as:

$$q^*(y) = q\left(\frac{\partial p}{\partial q}, y(q) - p(q) + \frac{\partial p}{\partial q}q\right). \tag{11}$$

Substitution of (10) into (11) gives:

$$q^*(y) = q\left(\frac{\partial p}{\partial q}, F^{-1}\big(G(q)\big) - p(q) + \frac{\partial p}{\partial q}q\right). \tag{12}$$

This equation defines the market equilibrium in the model.

We will now characterize the nonlinearity of the hedonic price function. To do so, we focus on its second derivative of the hedonic price function, which gives the change in the marginal price of housing. With a linear hedonic price function, this second derivative equals 0, but in general it will, of course, be nonzero. Our main result is the following:

Proposition 1 In market equilibrium the second derivative of the hedonic price function is:

$$\frac{\partial^2 p}{\partial q^2} = \frac{\frac{\partial q}{\partial y}\frac{g(q)}{f(y(q))} - 1}{-\left(\frac{\partial q}{\partial \pi} + q\frac{\partial q}{\partial y}\right)}, \tag{13}$$

with $\pi = \partial p(q)/\partial q$, the marginal price of housing services.

To show this, we differentiate the equilibrium demand equation (12) with respect to q. The result is:

$$dq = \frac{\partial q}{\partial y}\left(\frac{g(q)}{f(y(q))}dq + \left(-\frac{\partial p}{\partial q} + \frac{\partial p}{\partial q} + q\frac{\partial^2 p}{\partial q^2}\right)dq\right) + \frac{\partial q}{\partial \pi}\frac{\partial^2 p}{\partial q^2}dq. \tag{14}$$

After removing the terms that cancel and collecting the remaining terms, this gives:

$$1 - \frac{\partial q}{\partial y}\frac{g(q)}{f(y(q))} = \left(\frac{\partial q}{\partial \pi} + q\frac{\partial q}{\partial y}\right)\frac{\partial^2 p}{\partial q^2}. \tag{15}$$

Solving this equation for $\partial^2 p/\partial q^2$ gives (13).

To interpret (13), observe that the expression between brackets in the denominator is the Slutsky term of the demand equation for housing. It is negative if the demand for housing is consistent with utility theory. Assuming this condition is satisfied, we conclude that the proposition says that the hedonic price function is linear when:

$$\frac{\partial G}{\partial q}\frac{\partial q}{\partial y} = \frac{\partial F}{\partial y}. \tag{16}$$

This can be interpreted as a local equilibrium condition that holds when the housing stock and the income distribution are balanced: the density of households with a particular income level y is matched perfectly with the density of houses that have the quality level q demanded by these households at the prevailing marginal price of housing.

The hedonic price function is (strictly) convex when

$$\frac{\partial G}{\partial q}\frac{\partial q}{\partial y} > \frac{\partial F}{\partial y}, \tag{17}$$

and strictly concave when:

$$\frac{\partial G}{\partial q}\frac{\partial q}{\partial y} < \frac{\partial F}{\partial y}. \tag{18}$$

To see what this means, observe that the densities on left-hand sides of equations (17) and (18) give numbers of houses and the densities on the right-hand side numbers of households. The slopes of the Engel curve, that also appear on the left-hand sides translate the number of houses into corresponding numbers of households. The houses whose number is indicated on the left are those demanded by the households whose number is indicated on the right, and if the translation of houses into households results in equal numbers on both sides of the equation, the hedonic price function is linear. If not, the hedonic price function must be nonlinear in order to match all households to houses.

If (17) holds there are more dwellings available than needed for the households to be on their demand curve if the marginal price is fixed. Equilibrium can therefore only be realized in this part of the stock when the marginal price changes. More precisely, the marginal price must increase in order to slow down the increase of demand with income so that all houses in this part of the stock will be demanded. Indeed, equation (13) implies that $\partial^2 p/\partial q^2 > 0$ in this case. In the alternative case (18), analogous reasoning shows that the hedonic price function is concave.

Since we have assumed that housing is a normal good, and $\partial y(q)/\partial q$ is nonnegative, the value of the numerator on the right hand side of (12) has -1 as its lower bound. This implies that there

is also a bound on the possible concavity of the hedonic price function (i.e. on the absolute value of $\partial^2 p/\partial q^2$ whenever it is negative), whereas there is no such upper bound on the convexity. To see what the upper bound on the concavity implies, we consider the Hicksian demand curve for housing $q^H = q(\pi, u)$. If we move along this demand curve, we have: $dq = (\partial q^H/\partial \pi)d\pi$ or $d\pi/dq = 1/(\partial q^H/\partial \pi)$. Now observe that π is the slope of the indifference curve corresponding to the Hicksian demand, and that $d\pi/dq$ is the second derivative of this indifference curve. This second derivative equals $1/(\partial q^H/\partial \pi)$, which is minus the upper bound of of $\partial^2 p/\partial q^2$. We conclude therefore that the concavity of the hedonic price function is bounded by the convexity of the indifference curve. That is, $-p(q)$ cannot be more convex than the indifference curve to which it is tangent.

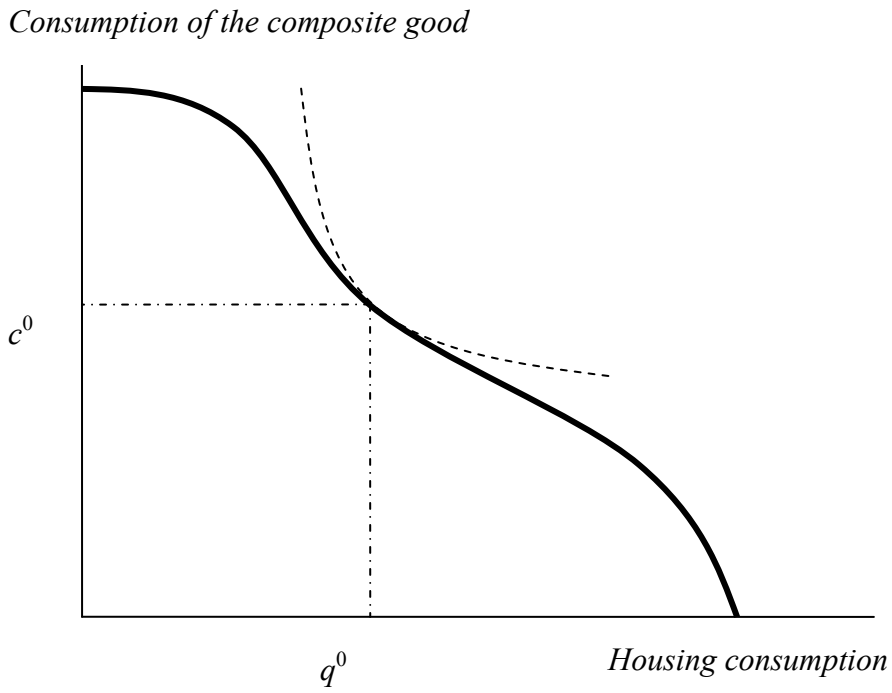


Figure 3. A locally concave hedonic price function for housing

This is illustrated in Figure 3. The figure shows a non-linear budget line, which is partly convex, because the hedonic price function is partly concave. However, in the optimum, the convexity of the budget line is less than that of the indifference curves. The highest indifference curve that can

be reached touches the budget line: the two have just a single point in common. The budget line is less convex than the indifference curve.

3 Income shocks and house prices

3.1 General discussion

To see what the model implies about the effects of income shocks, we consider a change in the distribution from $G^0(y)$ to $G^1(y)$, while we assume that the housing stock remains unchanged. We consider a shift to the right of the income distribution to investigate the possibility of the model to explain the phenomenon that motivated this paper. The case of interest is one which all incomes change by the same percentage, but we start by considering the somewhat simpler one in which all incomes change by the same amount. We assume that the housing stock remains unchanged.

With an equal change in all incomes $F^1(y+\Delta) = F^0(y)$, where Δ denotes the common change in income and we have used super fixes to distinguish the two density functions. The matching of households to houses requires that households with income $y+\Delta$ now inhabit houses formerly used by households with income y. Since $f^1(y+\Delta) = f^0(y)$ the ratio $\frac{g(q)}{f(y(q))}$ in (13) remains unchanged for all q. The curvature of the hedonic price function may nevertheless change when the higher income (at a given value of q) affects the slopes of the Engel curve or the demand curve (or both). Although general statements cannot be made it seems likely that the absolute value of the Slutsky term will decrease, which would imply more curvature of the hedonic price function: if it was concave it becomes more concave, if it was convex it becomes more convex. If the slope of the Engel curve also decreases, this would strengthen the impact on concavity, and counteract the impact on convexity.

Now consider the situation in which all incomes increase with the same percentage: $F^1(ky) = F^0(y)$ for some k>1. Matching of the households to the housing stock now requires that households with income ky occupy the houses formerly inhabited by households with income y. Moreover, we must have $f^1(ky) = \frac{f^0(y)}{k} < f^0(y)$, which tells us that the term $\frac{g(q)}{f(y(q))}$ in (13) now increases. This makes the hedonic price function more convex in the sense that it increases the value of the second derivative of this function.

The slope of the Engel curve and the Slutsky term may change also in this case, and this complicates the picture of course. It seems likely that the absolute value of the Slutsky term decreases when income changes, but the slope of the Engel curve may also decrease. Since the latter phenomenon counteracts the movement towards a more convex price function we look at it in some detail. The net change in the numerator of the right-hand side of (13) remains positive after all incomes increase with a factor k>1 if $\frac{\partial q(ky)}{\partial y} > \frac{1}{k}\frac{\partial q(y)}{\partial y}$. It can be shown that this inequality is fulfilled if $\frac{\partial^2 q(ky)}{\partial y^2} > -\frac{\partial q(y)}{\partial y}\frac{1}{y}$. This shows that some concavity of the Engel curve for housing is compatible with a price function that becomes more convex after an income shock. It is not difficult to verify that the linear and loglinear Engelcurves satisfy this criterion. Concluding, we may state:

Proposition 2 If the absolute value of the Slutsky term is non-increasing in income and the Engel curve for housing services is not too concave in the sense that $\frac{\partial^2 q(ky)}{\partial y^2} > -\frac{\partial q(y)}{\partial y}\frac{1}{y}$, then a proportional increase in all incomes causes the second derivative of the house price function to increase everywhere.

3.2 A linear example

To illustrate the model further, we consider an example. Assume that preferences are such that the demand function for housing is linear:

$$q = a + b\pi + cy, \tag{19}$$

and that the distributions of income and housing stock are uniform:

$$F(y) = \frac{y}{y^{max} - y^c}, \tag{20}$$

$$G(q) = \frac{q}{q^{max} - q^{min}}. \tag{21}$$

The maximum income should be small enough to keep the Slutsky term of the linear demand equation (19) negative, as is required by economic theory. Equation (13) implies:

$$\frac{\partial^2 p}{\partial q^2} = \frac{c\frac{y^{max} - y^c}{q^{max} - q^{min}} - 1}{-(b + qc)}. \tag{22}$$

Differential equation (22) can be solved as:

$$p(q) = p\left(q^{min}\right) + \left(\left(C + \pi\left(q^{min}\right)\right) - \frac{1}{c}\right)\left(q - q^{min}\right) + \tag{23}$$

$$\frac{1}{c^2}(1-cC)(b+cq)\ln\left(\frac{b+cq}{b+cq^{min}}\right),$$

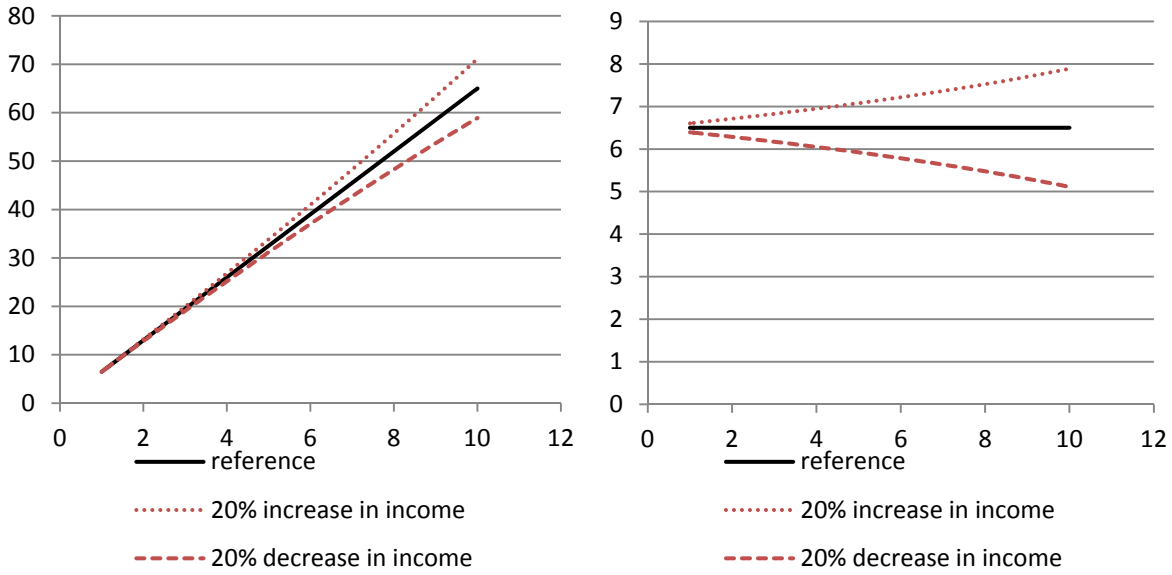
where $C=\frac{y^{max}-y^{c}}{q^{max}-q^{min}}$. It is clear from (23) that the second derivative of the hedonic price function equals 0 if $cC=1$, and in that case (23) simplifies to:

$$p(q)=p\left(q^{min}\right)+\pi\left(q^{min}\right)\left(q-q^{min}\right). \tag{24}$$

We can compute the value of $\pi\left(q^{min}\right)$ from the requirement that the owner-occupying household with the lowest income chooses the house with the lowest quality:

$$q^{min}=a+b\,\pi\left(q^{min}\right)+cy^{min}. \tag{25}$$

This gives $\pi\left(q^{min}\right)=\left(q^{min}-a-cy^{min}\right)/b$. The value of $p\left(q^{min}\right)$ is determined by the requirement that the owner-occupying household with the lowest income should be able to reach the same level of utility in rental housing.



a) The hedonic price function b) The marginal price of housing services

Figure 4 Hedonic price functions and marginal prices

The linear hedonic is, of course, a special case. If $cC>1$ the coefficient for $\left(q-q^{min}\right)$ in the second term of $p(q)$ is a constant that is larger than $\pi\left(q^{min}\right)$, and the third term is non-zero. If $cC>1$ this third term is negative and convex. If $cC<1$ the coefficient for $\left(q-q^{min}\right)$ is smaller than $p(q)$ and the third term is positive and concave.

A simple numerical example can be constructed as follows. The parameters of the demand function are chosen as: a=13, b=-2, c=0.01. Incomes are between y^{min}=10 and y^{max}=100. This implies that the Slutsky term $b + cy$ varies between -1.9 and -1. Housing quality varies between q^{min}=1 and q^{max}=10. The market is equilibrated by a linear hedonic price function that passes through the origin. The price per unit of housing services equals 6.5.

If all incomes increase with 1 unit, the market is equilibrated by a unit price 6.55 for housing services. This requires that the price of the owner-occupied house of minimum quality now also has a price 6.55. This might be due to an increase in rent that parallels the increase in user costs. If rents remain unchanged, and the price of the lowest quality owner-occupied house is constant at 6.5, the new marginal price of housing is slightly higher: 6.5526. The hedonic price function is still a straight line, but it does not pass through the origin.

If all incomes increase by 5%, the hedonic price function is no longer linear. The marginal price increases from 6.525 for q=q^{min} to 6.846 for q=q^{max} when it is assumed that the user cost of the smallest owner occupied house also increases to 6.525. Again, results are slightly different when the price of this house is kept constant. The results for decreases in incomes are, of course, similar but in the opposite direction.

Figure 3 illustrates the model for the 20% changes in income and all other parameters identical to those we just discussed. The upper panel shows the hedonic price functions in the original situation (in which it is linear) and with the higher and lower incomes, whereas the lower panel pictures the marginal prices in each of the three situations.

The results just shown for a specific case can be generalized to arbitrary linear demand curves. First consider a change in the income distribution by which all incomes grow with the same absolute number Δy. The income change implies that the demand for housing quality of each household increases with $c\Delta y$. This implies that demand for the lowest quality houses disappears completely, while there is now demand for houses of a somewhat higher quality than the maximum currently available in the market. The old equilibrium thus no longer holds. To find the new one, note first that y^{max} and y^{c} both increase by Δy, which implies that C will not change. This tells us that if the hedonic price function were linear in the original situation, it will again be so in the new equilibrium. Also if it were convex or concave, this will not change.

Assuming a linear hedonic price function in the original situation, we know that the new equilibrium price π^{**} must satisfy $q^{min} = a + b\pi^{**} + c(y^{c} + \Delta y)$. From this it is easy to

compute that $\pi^{**} = \pi^* + (c/-b)\Delta y$, where π^* denotes the original equilibrium price. This means that the prices of all housing qualities increase proportional to their quality. In other words, incomes change by the same number but house prices with the same percentage. Note also that in this example all households remain in the same dwelling. All that changes is that a higher price has to be paid for these dwellings. And there is, of course, a wealth effect for the owners of the houses.

Now consider the effect of a proportional change in all incomes: all incomes change by the same percentage. This means that the difference between y^{max} and y^c increases, and therefore the value of C changes. If the hedonic price function is linear in the original situation, it will be convex in the new one when incomes increase, and concave when incomes decrease. Proportional changes in incomes will therefore lead to changes in house prices that are not proportional to quality. The relative change in the housing price will be largest for the highest quality dwellings. This will probably stimulate the supply of high quality dwellings.

3.3 Solving the model in the general case

To see how the model can be used with an arbitrary demand curve, return to (11), which we repeat here:

$$q^*(y) = q\left(\frac{\partial p}{\partial q}, y(q) - p(q) + \frac{\partial p}{\partial q}q\right). \tag{26}$$

We assume that the distributions of income and housing are known. This allows us to find the matching function $y(q)$ and therefore the income that corresponds to the housing of minimum quality: $y^* = y\left(q^{min}\right)$. At this minimum income a household must be indifferent between the owner occupied housing of minimum quality and its substitute, for instance rental housing. This allows us to determine the price of the lowest quality housing $p\left(q^{min}\right)$. Imposing the condition that this household is on its demand curve gives the marginal price $\pi\left(q^{min}\right)$. This brings us in a position in which we can use standard methods for solving differential equations, for instance Euler's method, to trace out the complete hedonic price function $p(q)$.

3.4 Heterogeneity in preferences

Until now we have only considered heterogeneity in incomes. To deal with a situation in which actors can also differ in tastes we now generalize the model to a situation in which the utility

function is $u(q, c; \varepsilon)$, where ε is a possible vector valued variable that indicates taste heterogeneity. We assume a simultaneous density function $f^*(y, \varepsilon)$. Demand for housing can be written as $q = q(y^v, \pi, \varepsilon)$. The consumer is a homeowner when the maximum utility of owning exceeds that of renting and we denote the set of combinations $(y,)$ for which this is the case with a given hedonic price function as $O(p(q))$.

The distribution of the demand for housing at a given hedonic price function will be denoted as $H(q; p(q))$. It is defined as:

$$H\big(q; p(q)\big) = \iint_{\substack{(y,\varepsilon)\epsilon O(p(q)) \\ q(y^v,\pi,\varepsilon)\leq q}} f^*(y, \varepsilon) dy d\varepsilon$$

The distribution of houses is denoted as before as $G(q)$. A price equilibrium is a housing price function $p(q)$ for which:

$H\big(q; p(q)\big) = G(q)$ for all $q\epsilon[q^{min}, q^{max}]$.

This implies:

$h\big(q; p(q)\big) = g(q)$ for all $q\epsilon[q^{min}, q^{max}]$.

where $h(.) = \partial H/\partial q$. A given demand for housing services q can be generated by different combinations of y and ε and we can write the income that generates q as a function of ε by inverting the demand function:

$y = p(q) - \pi q + z(\varepsilon, \pi; q)$.

Using this, we can write:

$$h\big(q; p(q)\big) = \int_{\substack{(y,\varepsilon)\epsilon O(p(q)) \\ q(y^v,\pi,\varepsilon)\leq q}} f^*(p(q) - \pi q + z(\varepsilon, \pi; q), \varepsilon) d\varepsilon.$$

This can be used to find an expression for $h(.)$ from a demand function and the simultaneous distribution of income and the taste heterogeneity parameter. Numerical techniques can then be used to find the equilibrium price function.

For the special case of a linear demand function we can introduce taste heterogeneity as a random intercept:

$q = a + \varepsilon + b\pi + cy.$

This allows one to summarize all heterogeneity in a scalar $\mu = y + \frac{1}{\alpha}\varepsilon$. The distribution of μ can be derived from the simultaneous density $f^*(y, \varepsilon)$, and then one can proceed as in the example given above. However, in this model there is no longer a strict one-to-one relationship between income and housing consumption.

4 Diverging house prices in Amsterdam

To estimate the divergence of house prices, we focus on what we regard as a crucial property of the model developed above: that the ranking of houses on the basis of housing services is identical to that on the basis of prices. This ranking therefore reveals information about the housing services that we will exploit this information to develop a measure of housing services. Once we have this measure, we can compare the price increases for any level of housing services. The data we use are provided by the Dutch association of realtors, abbreviated in Dutch as NVM. They contain information on transaction prices and housing characteristics of houses sold by members of this association. The majority of Dutch realtors is a member. We focus on the transactions in the municipality of Amsterdam over the period 1995-2009. During most of this period the Dutch economy was growing and house prices increased.

4.1 Estimation strategy

A major difficulty in applying the model developed in the previous section is that we cannot observe housing services. The purchase price of a house is, in this framework, the product of a unit price and a number of housing services. If, in a given market and period, the unit price is equal for all houses, differences in the purchase price are proportional to differences in the quantity of housing services. Changes in the price over time or space can be estimated by comparing house prices of similar houses in different periods or markets as is done with 'hedonic' price indices. However, if the assumption of a constant unit price in a given market and period is dropped, things become less clear.

We assume that housing services are a function of observed and unobserved housing characteristics:

$$q = q(h) + \xi. \tag{30}$$

In this equation h is a vector of observed housing characteristics, and ξ is a random variable that reflects the unobserved characteristics. An elementary property of out model is that in each market and in each period the house prices is a monotone increasing function of the number of housing services provided by the house. This implies that the ranking of houses on the basis of price reflects the ranking on the basis of the number of housing services, although the strict proportionality of the Muth model is lost. We thus have:

$$p_i > p_j \iff q(h_i) + \xi_i > q(h_j) + \xi_j \quad (31)$$

where the suffixes *i* and *j* denote arbitrary houses observed on the same market and in the same period.

The be able to estimate the function q that links housing characteristics to housing services, we assume that ξ_j is Extreme Value type I distributed and apply the results of Beggs, Cardell and Hausman (1981). We have observations on prices and housing characteristics for a number of years t=1..T and we order the observations within each year on the basis of their prices: the most expensive house in year t is indexed 1,t, et cetera. The likelihood of observing the actual ranking of these houses on the basis of the prices in year t is then given as:

$$L_t = \frac{e^{q_{1,t}}}{\sum_{i\geq 1} e^{q_{i,t}}} \frac{e^{q_{2,t}}}{\sum_{i\geq 2} e^{q_{i,t}}} \frac{e^{q_{3,t}}}{\sum_{i\geq 3} e^{q_{i,t}}} \cdots \frac{e^{q_{n(t)-1,t}}}{e^{q_{n(t)-1,t}} + e^{q_{n(t),t}}} \quad (32)$$

where $n(t)$ denotes the number of observations in year t. We pool the observations for all years and maximize the likelihood of all observations:

$$L = \prod_t L_t \ . \quad (33)$$

This means that we use the same specification of the housing services function $q(.)$ in all periods. Moreover, we specify $q(.)$ as being linear in the parameters to be estimated:

$$q(h) = \sum_{k=1}^{K} \beta_k h_k. \quad (34)$$

This specification of housing quality is consistent with what is often used in hedonic price equations.

However, it can be argued that this is a bit too restrictive, because the attractiveness of neighborhoods, which is part of the housing services, can change over time due to changes in household composition, shopping possibilities et cetera. We have therefore estimated two variants of the one: one in which all coefficients are assumed to be constant over time, and a second in which we allow the coefficients for neighborhood dummies to be year-specific.

Once we have a measure of housing services, we can proceed to estimate the housing price function $p(q)$. One difficulty that emerges is that the proper argument of this function is $\sum_{k=1}^{K} \beta_k h_k + \xi_i$, while we do not have information about the last term, ξ_i. Since the price function is in general nonlinear, it is not of much help that we can assume that the expected value of ξ equals 0. However, it helps if we can assume that the *median* of this variable equals 0,

because the median of $p(\sum_{k=1}^{K}\beta_k h_k + \xi\ )$ equals $p(\sum_{k=1}^{K}\beta_k h_k)$. We will thus estimate $p(q)$ by quantile (median) regression.

Table 1. Summary statistics for house transactions in Amsterdam

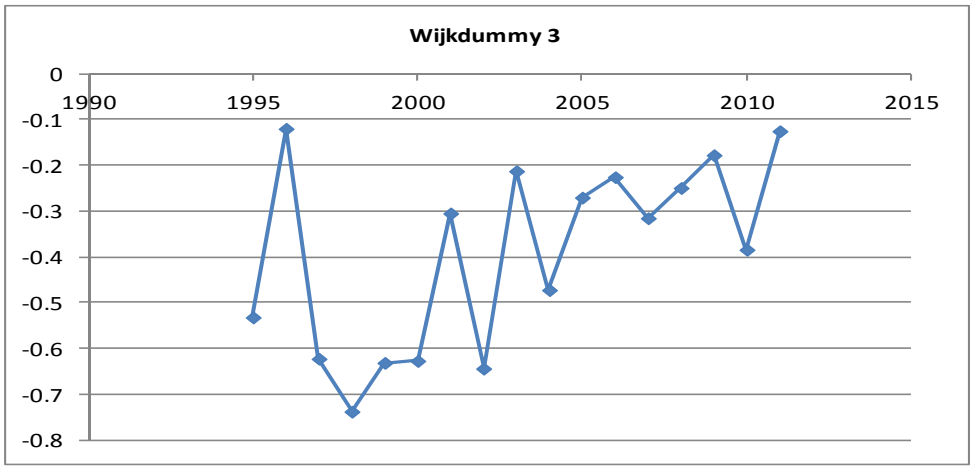
Variables	Mean	Std. Dev.	Min	Max
Transactions (in euros)	254,975	170,158	25,900	1,500,000
Floor space (m^2)	90.10	43.38	10	919
Rooms (#)	3.27	1.30	1	10
Distance to city center (km)	3.89	2.18	0.26	11.66
Detached house (ref: standard house)	0.01	0.09	0	1
Corner house (ref: standard house)	0.02	0.15	0	1
Semidetached house (ref: standard house)	0.01	0.09	0	1
Apartment (ref: standard house)	0.87	0.34	0	1
Balcony	0.52	0.50	0	1
Dormer	0.02	0.14	0	1
Terrace	0.11	0.31	0	1
Private parking	0.08	0.27	0	1
Garden	0.99	0.12	0	1
Well-maintained garden	0.08	0.27	0	1
Bad inside maintenance	0.11	0.32	0	1
Bad outside maintenance	0.04	0.20	0	1
Monument	0.03	0.18	0	1
1 Centrum	0.16	0.36	0	1
2 Slotervaart en Overtoomse Veld	0.05	0.21	0	1
3 Zuidoost	0.06	0.24	0	1
4 Oost en Watergraafsmeer	0.06	0.24	0	1
5 Amsterdam Oud-Zuid	0.04	0.20	0	1
6 Zuideramstel	0.05	0.22	0	1
7 Westerpark	0.07	0.26	0	1
8 Oud-West	0.03	0.18	0	1
9 Zeeburg	0.05	0.22	0	1
10 Bos en Lommer	0.04	0.20	0	1
11 De Baarsjes	0.06	0.24	0	1
12 Amsterdam-Noord	0.07	0.26	0	1
13 Geuzenveld en Slotermeer	0.16	0.37	0	1
14 Osdorp	0.09	0.28	0	1

4.4 Results: Housing services and housing characteristics

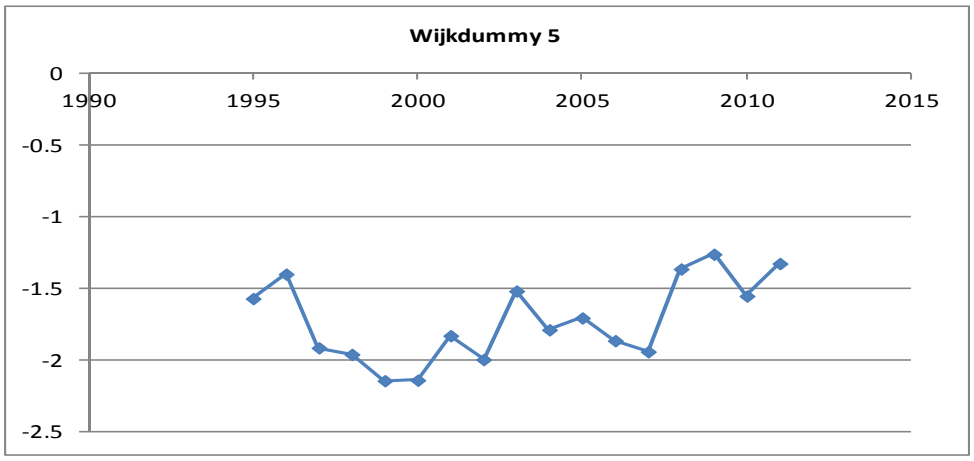
Estimation results for the housing services function $h(q)$ are reported in Table 1. The observations refer to the years 1995-2011. The number of available observations increased gradually over the years. In order to keep estimation tractable, we imposed a maximum of 3,000 on the number of observations to be used per year. If the number of available observations was larger, we randomly drawn fraction of the available observations. All coefficients for housing characteristics have the expected sign and most of them are highly significant.

Table 6.2. Estimation results for housing services

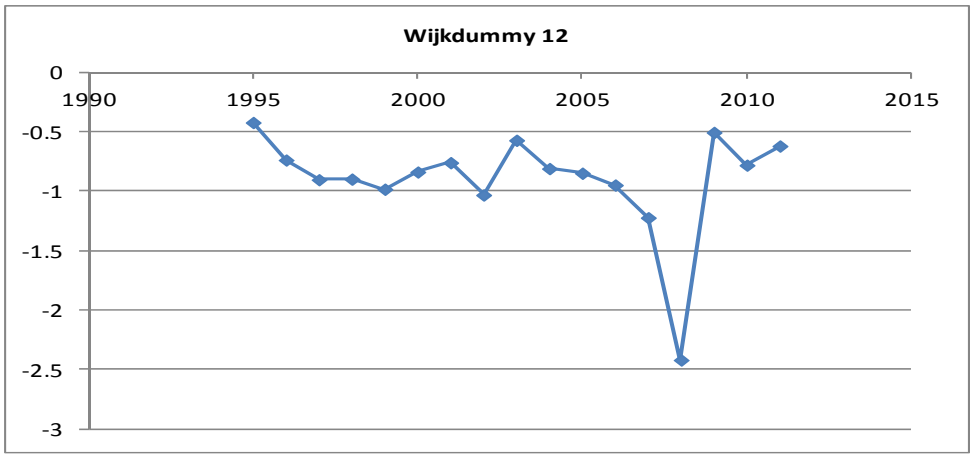
Variables		(1)			(2)	
Floor space (m^2)	0.020	(0.0002)	***	0.019	(0.0001)	***
Rooms (#)	0.326	(0.006)	***	0.334	(0.0056)	***
Distance to city center (km)	-0.052	(0.0067)	***	-0.028	(0.0063)	***
Detached house (ref: standard house)	0.143	(0.0593)	**	0.144	(0.0538)	***
Corner house (ref: standard house)	0.263	(0.0387)	***	0.302	(0.0355)	***
Semidetached house (ref: standard house)	1.023	(0.0641)	***	1.058	(0.0605)	***
Apartment (ref: standard house)	-0.272	(0.0226)	***	-0.217	(0.0209)	***
Balcony	0.011	(0.012)		0.010	(0.0111)	
Dormer	0.287	(0.0401)	***	0.202	(0.036)	***
Terrace	0.557	(0.02)	***	0.499	(0.019)	***
Private parking	0.479	(0.0219)	***	0.586	(0.0209)	***
Garden	0.818	(0.0457)	***	0.735	(0.043)	***
Well-maintained garden	0.596	(0.0215)	***	0.502	(0.0198)	***
Bad inside maintenance	-0.669	(0.0186)	***	-0.662	(0.0169)	***
Bad outside maintenance	-0.649	(0.0274)	***	-0.594	(0.0242)	***
Monument	0.252	(0.0305)	***	0.307	(0.0275)	***
Neighborhood dummies		YES			-	
Neighborhood * Year dummies		-			YES	
Log likelihood		-202,999			-205,557	
Observations		34,351			34,351	



a) Amsterdam Zuid Oost



b) Amsterdam Oud Zuid



c) Amsterdam North

Figure 5 Time specific estimates of neighborhood effects

The estimation results for the neighborhood dummies also show patterns that confirm expectations based on prior knowledge. We used the center as the reference in each period. Figure 5 shows three examples. Amsterdam Zuid Oost is a residential area that was developed in the heydays of modernism with a large share of rental housing. It gained a bad reputation in the 1980s and in the 1990s there was a large restructuring effort, some of the high-rise buildings were turned down and high quality owner-occupied housing was constructed. Panel a) suggests that the operation was to some extend successful. Amsterdam Oud-Zuid dates back to the late 19-th and early 20-th century. Notwithstanding the age of the houses, it is still a popular residential area and a reputation that is constant over time, as panel b) confirms. The area to the north of the river IJ was mainly industrial until the 1960s. Recently plans to connect the neighborhood better to the part of the city below the IJ river appear to make the neighborhood more attractive. The large negative outlier for the year 2007 is remarkable.

4.5 Results: the housing price function

To investigate the relationship with convexity of the house price function, we carried out a median regression using a quadratic specification of the housing price function.[6] We use the estimation results (specification 2 of Table 6.2) to compute the estimated value of housing services $\hat{q}(h) = \sum_{k=1}^{K} \beta_k h_k$, and use this as a regressor for the housing price function. The results are given in Table 6.3. The coefficient of the quadratic term indicates the convexity of the housing price function. It shows a clear upward trend which is summarized in Figure 6.7 that depicts the four year moving average of the coefficient for the quadratic term.

In an attempt to get an even clearer picture, we have also carried out local linear quantile regressions of the housing price function. Bandwidth selection is based on minimizing the mean squared error. The results are shown in Figure 6.8. Since the impact of the recession that started in 2007 is clearly indicated, we split the result in two panels. The first refers to the years 1995 to 2007 and clearly shows the tendency of more luxury housing to increase more in price than more decent types of housing, which results in a strong increase in the convexity of the hosing price function throughout the period. The second panel shows that the convexity diminished after the year 2007, when the great recession started. Convexity decreased in 2008 and 2009, a modest

[6] See, for example, Chapter 7 of Koenker (2005).

Table 3 Quadratic specification of the housing price function

VARIABLES	(1) 1995	(2) 1996	(3) 1997	(4) 1998	(5) 1999	(6) 2000	(7) 2001	(8) 2002	(9) 2003
Housing services	11,580***	26,263***	17,858***	16,178***	15,883***	13,062***	7,765***	17,040***	18,117***
	(1,393)	(1,796)	(1,132)	(1,276)	(1,522)	(2,435)	(2,400)	(2,151)	(2,780)
(Housing services)^2	3,816***	1,923***	4,226***	6,197***	8,200***	10,698***	11,553***	9,202***	7,820***
	(209.4)	(257.2)	(182.2)	(214.2)	(246.6)	(434.3)	(383.1)	(371.2)	(422.7)
Constant	34,885***	25,450***	54,601***	69,827***	95,057***	113,187***	130,607***	131,513***	110,810***
	(2,103)	(2,777)	(1,550)	(1,659)	(2,019)	(3,040)	(3,299)	(2,715)	(4,049)
Observations	2,458	2,680	3,007	3,354	3,807	1,871	1,919	1,920	1,874

VARIABLES	(10) 2004	(11) 2005	(12) 2006	(13) 2007	(14) 2008	(15) 2009	(16) 2010	(17) 2011
Housing services	20,430***	11,970***	14,872***	28,933***	10,266***	7,928***	13,153***	3,255
	(2,566)	(2,988)	(2,303)	(2,620)	(2,075)	(2,181)	(2,304)	(2,687)
(Housing services)^2	8,623***	11,789***	12,933***	14,788***	16,447***	14,809***	15,293***	16,674***
	(419.0)	(504.2)	(415.6)	(530.7)	(417.2)	(393.0)	(455.9)	(467.2)
Constant	115,022***	125,195***	128,880***	141,616***	155,197***	138,547***	144,929***	140,025***
	(3,507)	(3,871)	(2,861)	(2,808)	(2,371)	(2,767)	(2,730)	(3,548)
Observations	1,989	1,923	1,900	1,941	1,929	1,940	1,967	1,872

recovery followed in 2010, but in 2011 the level of house prices decreased in combination with an increase in convexity.

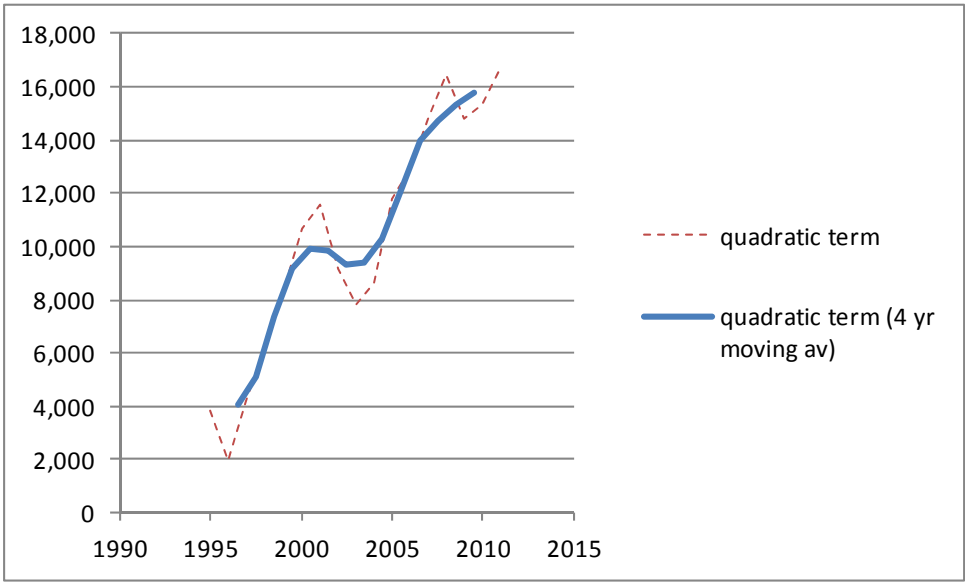
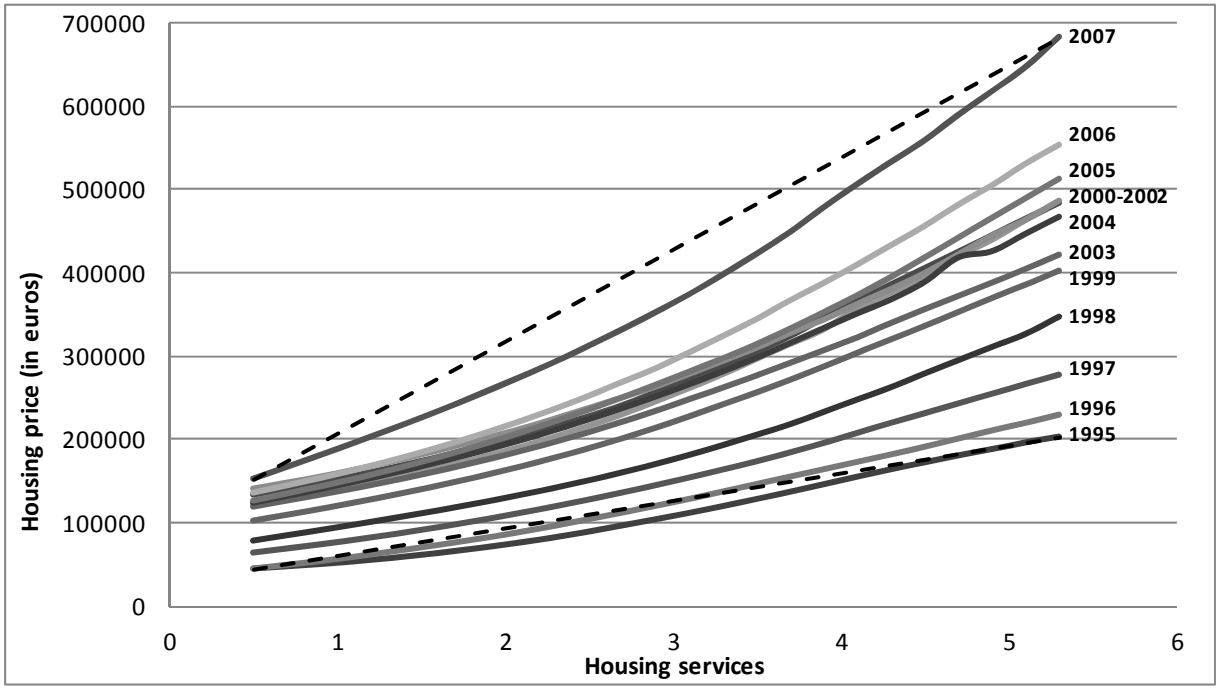


Figure 6 Increasing convexity of the housing price function over time

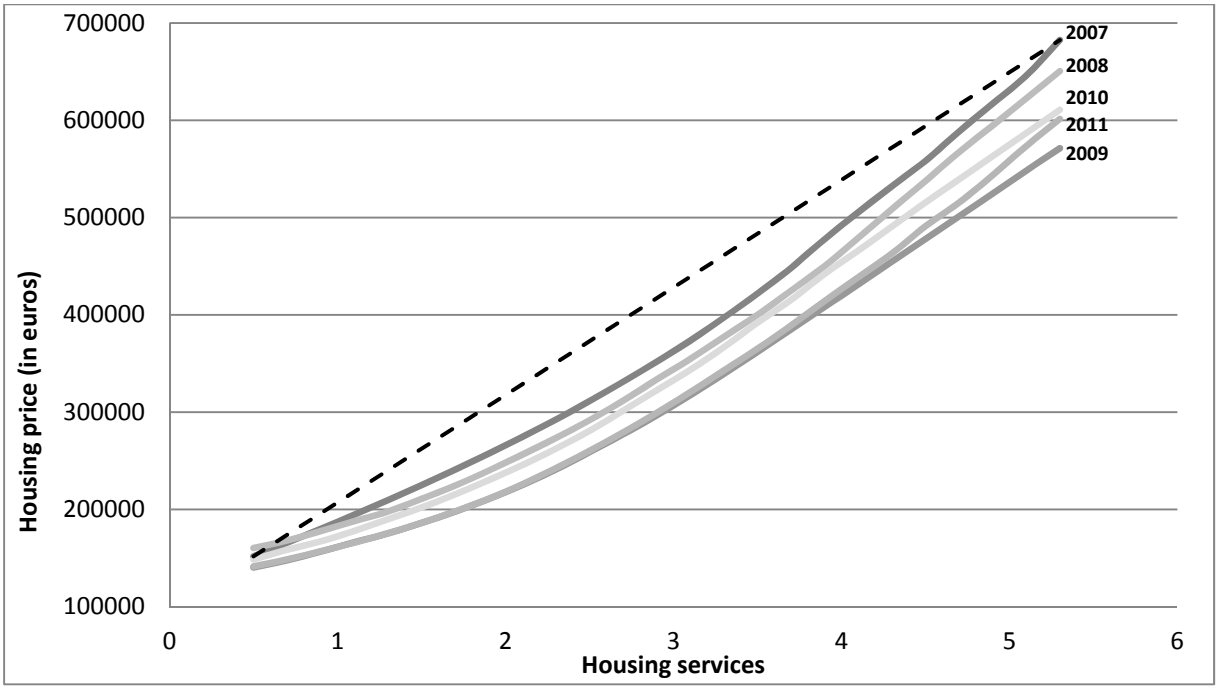
4.5 Discussion

Our estimation results show that during the period 1995-2007 house in Amsterdam increased while the housing price function tended to become more and more convex. After 2007 house prices decreased two years, then followed a slight recovery and a new drop. The model developed in this paper suggests that this development could be caused by income shocks. To investigate this issue we should realize that economic theory suggests that it is not the current income as well as the permanent income that should be viewed as a determinant of housing demand. Permanent income reflects expectations with respect to future developments of income and it is generally thought that the development of consumption expenditure provides a better indication of the development of permanent income than does current income. Figure 4 shows the annual changes in consumption volume in the Netherlands in the period 1995-2011. For the period 1995-2007 it shows positive numbers except for the year 2003. A close inspection of panel a) of Figure 7 shows that this is reflected in an exceptional downward movement of the housing price function. The drop in consumption expenditure in 2006 is not reflected in a drop in house prices. And the drop in house prices in 2008 does not reflect a drop in consumption expenditure. However, in the

years 2009-2011 there is close correspondence between the development of consumption expenditure and house prices.



a) 1995-2007



b) 2007-2011

Figure 7 Annual housing price functions 1995-2011

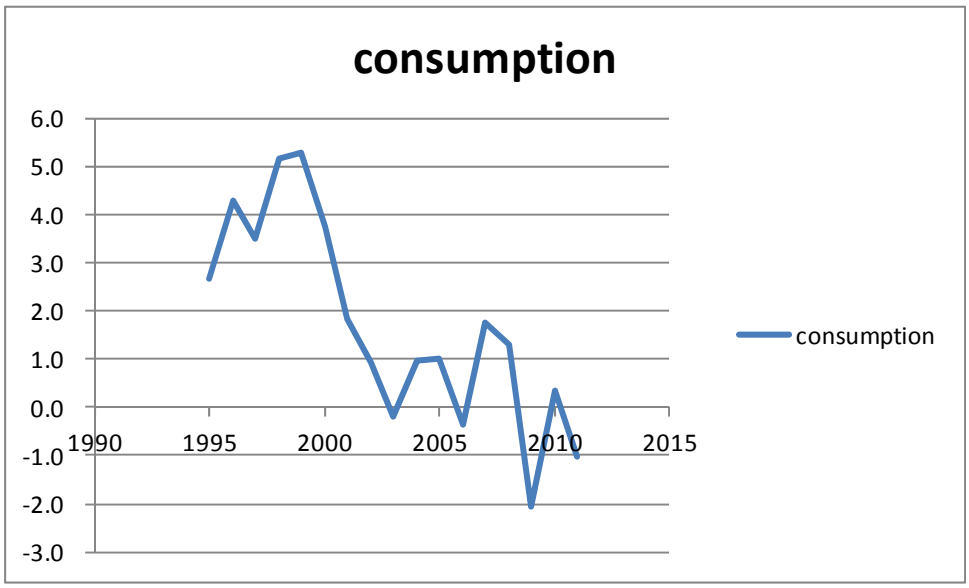


Figure 7 Annual changes in consumption volume

5 Conclusion

This paper has proposed an explanation of the well-known phenomenon of diverging house prices by imposing a restriction on the malleability of housing capital in the conventional Muth model of housing services. Instead of a single market there is now a continuum of markets for all the possible quality levels of housing. The housing price is an increasing function of the number of housing services and its curvature is determined by local supply and demand conditions. General conditions under which a proportional change in all incomes causes increasing convexity of this function were derived.

Our empirical application assumed that house prices are a stable function of housing characteristics, but neighborhood quality was allowed to change over time. Our model implies that the ranking of houses on the basis of price reflects the ranking on the basis of housing services and we used this property to estimate the number of housing services as a function of the housing characteristics. Using the results of this analysis, we investigated the development of the convexity of the housing price function. We found a gradual increase in the boom period 1995-2007 and a decrease followed by a weak recovery in the period 2007-2011.

There is a close correspondence between the development of the convexity of the housing price function and that of consumption volume, which supports the hypothesis that the deve,opment of permanent income drives that of hose prices.

A potentially important implication of the analysis of this paper is that it suggests that analysis of policies that effect house prices via repeat sale analysis can give biased results if the houses that are 'treated' by the policy are not a random sample from the housing stock but belong more than proportionally to the high or low quality segments. If the time window of the analysis coincides with a boom or bust period, the findings of such an analysis may be seriously biased by the diverging trends of the prices of different parts of the housing stock.

References

Beggs, S., S. Cardell and J. Hausman (1981) Assessing the potential demand for electric cars. *Journal of Econometrics*, **17**, 1-19.

Braid, R.M. (1981) The short run comparative statics of a rental housing market. *Journal of Urban Economics*, **10**, 286-310.

Braid, R.M. (1984) Effects of government housing policies in a vintage filtering model. *Journal of Urban Economics*, **16**, 272-296.

Glaeser and Gyourko (2005) Urban decline and durable housing. *Journal of Political Economy*, **113**, 345-375.

Koenker, R. (2005) *Quantile regression*. Cambridge.

Muth (1960) The demand for non-farm housing. In: A.C. Harberger (ed.) *The Demand for Durable Goods*, Chicago, 29-96.

Ortalo-Magné and Rady (2006) *Review of Economic Studies*

Rouwendal, J. (1998) On housing services. *Journal of Housing Economics*, **7**, 218-242.

Stein (1995) Prices and trading voluime in the housing market: A model with down-payment effects. *Quarterly Journal of Economics*, **110**, 379-406.

Sweeny, J.L. (1974) A commodity hierarchy model of the rental housing market. *Journal of Urban Economics*, **1**, 288-323.