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Institutional Differences, Economic Integration and Regional Growth

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1 Introduction

Regions consistently differ in their economic performance and development². This results in differences in the level of income, employment and welfare. Such differences are seen as neither socially nor politically desirable. Economic integration, is seen as a strategy to reduce such regional differences³. Cooperation between regions to enhance the process of economic integration can be beneficial.

In reality, however, although many cross-border barriers have been removed and factor mobility has increased, we nevertheless still observe great differences between regions and there economic development. The Euro crisis gives us with ample examples. What might be the reason for this development? It is often pointed out that markets, like the labour market, do not work well and are relatively rigid. We suspect that unvarying differentials in income and welfare between regions are also caused by other factors. Although some regions have comparative ‘physical’ advantages, all kind of intangibles can also lead to comparative advantages (the legal structure, the public administration, and the public health system). These intangibles form a part of the institutional structure of a region and have public good characteristics (high level of non-excludability and non-rivalry).

In this paper we analyse the effects of differences in institutional structure on economic integration and economic development. We try to get an answer on the question: “What are the consequences of regional integration for economic development and efficiency when regions differ in their institutional structure”. It is assumed that institutional structure generates externalities. Next to that we assume there is a positive relation between public

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³ See for example the motivation for regional policy in Baldwin and Wyplosz e.a (2013).
capital and institutional setting (maintenance of property rights and so on). We incorporate public capital as one of its arguments, in addition to labour and private capital in the production function which generates positive externalities in line with the suggestions made previously. These are the unpaid production factors (public capital).

The paper is organized as follows. First, in section 2, we introduce the model used to compare regions with a difference in the productivity of their public capital. To assess the effects of economic integration, in section 3 we first look at the economic development of the regions separately (autarky situation), without integration. Next (section 4) we introduce economic integration, where we assume mobility of labour and private capital, but immobility of public capital. Labour commutes, if the wage rate in the other region is higher, workers will work in that other region but will still spend their income in their resident region. In section 5 the effect of integration of regions on overall income, production and efficiency is discussed. Section 6 summarizes and concludes.

2 Public / Private Capital and Externalities

In this section we describe the model used to compare the development of two different regions. Both regions are identical except for the productivity of public capital. In the regions there are \(n\) firms, operating under perfect competition. We start with the production function of a representative firm.

2.1 Public Capital

Besides labour and private capital, public capital has a positive effect on output of the firm but at a decreasing rate. The more a firm makes use of public capital, the smaller the productivity of public capital. To take account of this phenomenon the ratio of public capital and aggregate private capital is incorporated in the production function, in line with Barro and Sala-i-Martin (1992). They distinguish three versions of publicly provided goods: publicly provided private goods, which are rival and non-excludable, publicly provided goods which are non-rival and non-excludable and publicly provided goods that are subject to congestion, which are rival and to some extend non-excludable (highways, water and sewage systems, and law courts). This last type fits best with our purpose, to include institutions as a productive resource. Below we give the production function for the representative firm \(j\):
The variables bearing the subscript $j$ are specific to a representative firm; the other variables are exogenous to a single firm. $K_j$ represents the capital stock and $L_j$ represents the number of workers of the $j$-th firm. The variable $K$ represents the total private capital stock of the region, whereas $\hat{k} = K/L$ is the region-wide capital intensity, which is exogenous to the firm.

The variable $P$ represents public capital. The parameters $0 < \alpha < 1$ and $0 < \beta < 1$ indicate the productivity of private and public capital.

The positive externalities stem from the region-wide stock of capital and the capital intensity of public capital. Labour productivity increases if the region-wide capital intensity increases (unpaid production factor)$^4$. On the other hand, it generates negative externalities due to the more intensive usage of public capital. There is a kind of rivalry with respect to the use of public capital. Note that the public capital stock generates positive externalities, but at a decreasing rate$^5$. The production function is linear-homogenous in the firm-specific variables $(K_j, L_j)$, and has the following properties: $F_v > 0, F_{vv} < 0, \nu$ , where $\nu = K_j, L_j, \frac{K}{L}, \frac{P}{K}$

Next we move to the firm trying to maximize its profit.

$$\max_{K_j, L_j}(Y_j - w_j L_j - R_j K_j)$$

$\textit{2.2 Firm behavior, Private Capital and Externalities}$

Because of the assumption of perfect competition we take one firm to resemble all $n$ firms’ in the economy. Firm $j$ faces the following maximization problem:

$\text{max}_{K_j, L_j}(Y_j - w_j L_j - R_j K_j)$

$^4$ See Barro & Sala-i-Martin (1992) and (1995) for details and explanation of this mechanism.

$^5$ Output increases as the ratio public capital / private capital increase or : $\frac{\partial Y_j}{\partial \left( \frac{P}{K} \right)} = \frac{Y_j}{\frac{P}{K}} = \frac{\beta}{\frac{P}{K}} Y_j > 0$.

As the ratio, $\left( \frac{P}{K} \right)$, increases the contribution of public capital decreases.
The production costs consist of labour costs and costs of capital. The costs of capital consist of depreciation, which is assumed to be 100% (depreciation rate of capital) and interest costs.

The wage rate \( w_j \) and return on capital \( R_j \) can be found by the first order condition:

\[
\frac{\partial Y_j}{\partial K_j} = \alpha K_j^{-\alpha} (\hat{k}_j L_j)^{1-\alpha} \left( \frac{P}{K} \right)^\beta - R_j = 0 \quad \text{and} \\
\frac{\partial Y_j}{\partial L_j} = (1-\alpha) K_j^{-\alpha} \hat{k}_j^{1-\alpha} L_j^{-\alpha} \left( \frac{P}{K} \right)^\beta - w_j = 0
\]

In a fully competitive environment (labour market, capital market and final goods market) the wage rate and return on capital are the same. \( R_j = R \) and \( w_j = w \). The region wide equilibrium aggregate capital stock and labour is the total of capital and labour of all \((n)\) firms \( (K = nK_j \text{ and } L = nL_j) \). In equilibrium the capital intensity \( \hat{k} = \frac{K}{L} = \frac{K_j}{L_j} \), for films and regions are the same, thus return on capital equals:

\[
R = \alpha \left( \frac{P}{K} \right)^\beta \quad \text{(3)}
\]

and wage rate:

\[
w = (1-\alpha) \hat{k} \left( \frac{P}{K} \right)^\beta \quad \text{(4)}
\]

To find the gross regional output we have to aggregate over all \( n \) (identical) firms:

\[
Y_j = nY_j = (nK_j)^\beta (\hat{k}_j nL_j)^{1-\beta} \left( \frac{P}{K} \right)^\beta = K_j^{1-\alpha} \left( \frac{K_j}{L_j} \right)^{\beta} \left( \frac{P}{K} \right)^\beta \quad \text{so} \quad Y = K \left( \frac{P}{K} \right)^\beta.
\]
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\[ Y = K\left(\frac{P}{K}\right)^\beta, \text{ or } Y = K^{1-\beta} P^\beta, \]  

(5)

In equilibrium, regional output depends on the aggregate private capital stock and the ratio of public / private capital stock. The social rate of return on private capital equals the marginal costs:

\[ \frac{\partial Y}{\partial K} = (1 - \beta)\left(\frac{P}{K}\right)^\beta = R_{\text{social}}^{\beta} \]  

(3B)

Comparing equation (3) with (3B) we see the difference between private and social return on capital. The social rate of return, \( R_{\text{social}}^{\beta} \), is lower, (i.e. \( \alpha > (1 - \beta) \)), equal to (i.e. \( \alpha = (1 - \beta) \)) or higher, (i.e. \( \alpha < (1 - \beta) \)) than the private return on capital. If the social rate of return exceeds that of the private rate of return, private investments results in positive externalities and otherwise negative externalities.

After the factor prices are known we can calculate the income share of labour and capital. Total income consists of wage income and capital income.\(^9\)

\[ Y = wL + RK = (1 - \alpha)K\left(\frac{P}{K}\right)^\beta + \alpha K\left(\frac{P}{K}\right)^\beta \]  

(6)

The income share of both labor and capital is fixed because of a Cobb-Douglas type of the production function.

### 2.3 Private and Public Capital Accumulation

We restrict the analyses to two periods, the period before (autarky) and after economic integration. We compare the regions in autarchy with economic integration. First we look how the region evolves over time without integration. We introduce a time index \( t \) for all endogenous variables. See section 2.1 & 2.2

\[ Y = wL + RK = (1 - \alpha)\frac{K}{L}\left(\frac{P}{K}\right)^\beta L + \alpha\left(\frac{P}{K}\right)^\beta L = K\left(\frac{P}{K}\right)^\beta. \]
Below we look at the development of the economy in autarky, that means the equation of motions of public and private capital.

**Equation of Motion for Private Capital**

We use an OLG model with two generations to determine savings.\(^\text{10}\) The young generation supplies labour, resulting in a labour income. In our case this equals \(w_tL = (1-\alpha)Y_t\). Total savings in period \(t\) are \(S_t = \hat{s}w_tL\) where \(\hat{s} = s(1-\tau)\) is the after tax saving rate and \(\tau\) the average tax rate on wage income. Total savings are thus;

\[
S_t = \hat{s}(1-\alpha)Y_t
\]  

Next we determine, \(K_{t+1}\), the private capital stock in period \(t+1\). This capital stock equals total savings in the previous period \(t\). Current savings are the next period’s private capital stock, that is \(K_{t+1} = S_t\). (Capital is fully depreciated after one period) This results in the following private capital stock period \(t+1\):\(^\text{11}\)

\[
K_{t+1} = S_t = \hat{s}(1-\alpha)K_t\left(\frac{P_t}{K_t}\right)^\beta
\]  

**Equation of Motion for Public Capital**

\(^{\text{10}}\) In this type of OLG model with a log-linear utility function the saving rate equals \(s = \frac{1}{(2 + \rho)}\) where \(\rho\) is the rate of time preference.

\(^{\text{11}}\) \(w_t = (1-\alpha)\frac{K_t}{L}\left(\frac{P_t}{K_t}\right)^\beta\) and \(w_tL = (1-\alpha)K_t\left(\frac{P_t}{K_t}\right)^\beta\) so \(S_t = \hat{s}(1-\alpha)K_t\left(\frac{P_t}{K_t}\right)^\beta\).
The public capital stock in period $t+1$, $P_{t+1}$, is financed by taxes and equals the tax revenues of the previous period $t$. Total tax revenues in period $t$ are given by $T_t = \tau \omega_t L$. If taxes are levied on labour income, the public capita stock in period $t+1$ amounts to:

$$P_{t+1} = T_t = \tau (1 - \alpha) K_t \left( \frac{P_t}{K_t} \right)^{\beta}$$

Using the result in (8) and (9) the ratio between public and private capital equals:

$$\frac{P_{t+1}}{K_{t+1}} = \frac{\tau}{\hat{s}}$$

The ratio depends on two parameters: the overall tax rate and the saving rate; that is $\tau$ and, $\hat{s}$ and is constant. Therefore the return on private capital is constant in equilibrium. Now the production function can be rewritten as follows:

$$Y_t = \left( \frac{\tau}{\hat{s}} \right)^{\beta} K_t \text{ or } Y_t = AK_t$$

where $A = \left( \frac{\tau}{\hat{s}} \right)^{\beta}$. The resulting production function, (11), resembles that of the AK production functions used in endogenous growth theory and exhibits constant returns to scale.

Using aggregate savings and the equilibrium ratio between public and private capital we find the following income relation:\footnote{\textsuperscript{12}}

$$Y_{t+1} = \hat{s}(1 - \alpha) \left( \frac{\tau}{\hat{s}} \right)^{\beta} Y_t$$

The growth factor can now be calculated quite easily from equation (12). It is:

$${\frac{Y_{t+1}}{Y_t} = \hat{s}(1 - \alpha) \left( \frac{\tau}{\hat{s}} \right)^{\beta}}$$

\footnote{\textsuperscript{12} $Y_{t+1} = K_{t+1} \left( \frac{\tau}{\hat{s}} \right)^{\beta}$ because $S_t = K_{t+1}$ we have $Y_{t+1} = S_t \left( \frac{\tau}{\hat{s}} \right)^{\beta}$ and $S_t = \hat{s}(1 - \alpha)Y_t$ we have $Y_{t+1} = \hat{s}(1 - \alpha)Y_t \left( \frac{\tau}{\hat{s}} \right)^{\beta}$. From this the growth rate can be derived, which equals $$(1 + g) = \frac{Y_{t+1}}{Y_t} = \hat{s}(1 - \alpha) \left( \frac{\tau}{\hat{s}} \right)^{\beta}.$$}
1 + g = \frac{Y_{t+1}}{Y_t} = \hat{s}(1-\alpha)\left(\frac{\tau}{s}\right)^\beta \quad (13)

Economic growth depends not only on the propensity to save, \(\hat{s}(1-\alpha)\) and the tax rate, \(\tau\), but also on the productivity of public capital, \(\beta\) and the ratio between tax and saving rate from wage income, \(\left(\frac{\tau}{s}\right)\). Assuming no population growth this is also the per capita growth.

The growth rates of income, consumption, private and public capital formation and per capita consumption are the same\(^{13}\).

Next we introduce difference in productivity of capital and analyse the consequence for economic development in autarky.

3 Difference in Productivity of Public Capital: the Autarchy Case

In this section we examine the autarchy case of the regions and how they develop when there is no labour or capital mobility. The only difference between the two regions is that they differ in efficiency of their public capital.

3.1 Regional Differences in Public Capital

We start where region 1 is more efficient in its public capital. We take \(h \geq 1\) as a scale parameter for the productivity / efficiency of public capital. For the least productive region we set the value of \(h\) as 1. For any firm \(j\) we have the following adjusted production functions for the 2 regions:

\(^{13}\)From (8) and (10) \(K_{t+1} = \hat{s}(1-\alpha)K_t\left(\frac{\tau}{s}\right)^\beta\). The growth rate equals \(\frac{K_{t+1}}{K_t} = \hat{s}(1-\alpha)A = (1 + g)\).

From (10) \(\frac{P_{t+1}}{K_{t+1}} = \frac{P_t(1 + g_P)}{K_t(1 + g)} = \frac{\tau}{s}\) thus \(g = g_P\) and \(\frac{P_{t+1}}{P_t} = (1 + g)\). The resource constraint equals \(Y_t = C_t + K_{t+1} + P_{t+1}\) and \((1 + g)Y_t = Y_{t+1} = C_{t+1} + (1 + g)K_{t+1} + (1 + g)P_{t+1}\). This results in \(C_{t+1} = (1 + g)C_t\) or \(\frac{C_{t+1}}{C_t} = (1 + g)\).
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\[ Y_{1,j} = K_{1,j}^{\alpha} \left( \frac{P_{1,j}}{K_{1,j}} \right)^{\beta} \]  
\[ Y_{2,j} = K_{2,j}^{\alpha} \left( \frac{P_{2,j}}{K_{2,j}} \right)^{\beta} \]

The region-wide aggregated production functions can be rewritten as follows:

\[ Y_{1,t} = h^\beta K_{1,t} \left( \frac{P_{1,t}}{K_{1,t}} \right)^{\beta} \]  
\[ Y_{2,t} = K_{2,t} \left( \frac{P_{2,t}}{K_{2,t}} \right)^{\beta} \]

We see that income of region 1 exceeds that of region 2, \((h^\beta > 1)\), and the wage rate and the return on capital in region 1 will therefore be higher;

\[ R_{1,t} = \alpha h^\beta \left( \frac{P_{1,t}}{K_{1,t}} \right)^{\beta} > R_{2,t} = \alpha \left( \frac{P_{2,t}}{K_{2,t}} \right)^{\beta} \]

\[ w_{1,t} = (1 - \alpha)h^\beta k_{1,t} \left( \frac{P_{1,t}}{K_{1,t}} \right)^{\beta} > w_{2,t} = (1 - \alpha)k_{2,t} \left( \frac{P_{2,t}}{K_{2,t}} \right)^{\beta} \]

Below we summarize the differences of the two regions in the case of autarchy:

\[ R_{1,t} = h^\beta R_{2,t}, \quad w_{1,t} = h^\beta w_{2,t}, \quad Y_{1,t} = h^\beta Y_{2,t} \]

Now we need to assess capital accumulation of the two regions.

**3.2 Private and Public Capital Accumulation in autarky**

Next we move to the development of public and private capital of the two different regions in autarky.

**Equations of Motion for Public Capital**

We assume that the local government have a balanced budget and that public investment is financed by taxes on labour income \((P_{1,t+1} = T_{1,t} \quad \text{and} \quad P_{2,t+1} = T_{2,t})\) Knowing the total tax
revenues from the previous period, the level of public investment in region 1 and 2 are respectively:

\[
P_{1,t+1} = \tau(1-\alpha)Y_{1,t} \quad \text{and} \\
P_{2,t+1} = \tau(1-\alpha)Y_{2,t}
\]  

(21)  

(22)

Comparing the two regions we can see that public investment in region 1 exceeds that in region 2 by a factor \( h^\beta \) \( (Y_{1,t} = h^\beta Y_{2,t}) \), see equation (16) and (17)). This is due to higher tax revenues from a higher wage income.

**Equations of Motion for Private Capital**

For private capital formation market clearing in region 1 and 2., \( (K_{1,t+1} = S_{1,t} \) and \( K_{2,t+1} = S_{2,t} \), result in the equations of motion for private capital and the ratio of public and private capital:

\[
K_{1,t+1} = \dot{s}(1-\alpha)Y_{1,t} \quad \text{and} \quad \frac{P_{1,t+1}}{K_{1,t+1}} = \left(\frac{\tau}{\dot{s}}\right) \quad \text{and} \\
K_{2,t+1} = \dot{s}(1-\alpha)Y_{2,t} \quad \text{and} \quad \frac{P_{2,t+1}}{K_{2,t+1}} = \left(\frac{\tau}{\dot{s}}\right)
\]

(23)  

(24)

As with public capital, the private capital stock in region 1 is higher than in region 2. Next we move to the consequents for regional income.

### 3.3 Income / Output and Growth in Autarky

Knowing private capital and the ratio public / private capital ((23) & (24)) gives us the development of income over time in the two regions ((16) and (17))

\[
Y_{1,t+1} = h^\beta \dot{s}(1-\alpha)\left(\frac{\tau}{\dot{s}}\right)^\beta Y_{1,t} \quad \text{and} \\
Y_{2,t+1} = \dot{s}(1-\alpha)\left(\frac{\tau}{\dot{s}}\right)^\beta Y_{2,t}
\]

(25)  

(26)
The development of income in region 1 is $h^\beta$ times greater compared to the region 2. The corresponding growth factors of region 1 and 2 are given by:

$$1 + g_1 = \frac{Y_{1,t+1}}{Y_{1,t}} = h^\beta \hat{s}(1 - \alpha) \left( \frac{\tau}{s} \right)^\beta.$$  \hspace{1cm} (27)

$$1 + g_2 = \frac{Y_{2,t+1}}{Y_{2,t}} = \hat{s}(1 - \alpha) \left( \frac{\tau}{s} \right)^\beta.$$  \hspace{1cm} (28)

We summarize differences between the two regions below:

$$P_{1,t+1} = h^\beta P_{2,t+1}$$
$$K_{1,t+1} = h^\beta K_{2,t+1}$$  \hspace{1cm} (20B)
$$Y_{1,t+1} = h^\beta Y_{2,t+1}$$
$$(1 + g_1) = h^\beta (1 + g_2)$$

It is clear that private and public capital formation, income development and the growth in region 1 is $h^\beta$ times greater than in region 2. This means that the development of both regions diverges due to the difference in the productivity of public capital resulting from different institutions which is evidently. In autarchy there will of course be no convergence between the two regions.

As was mentioned above, autarchy is simply a reference case. The question we are interested in is what will be the effect of economic integration with free mobility of factors of production? This question is addressed in the next section.

### 4 Economic Integration: from Autarchy to Factor Mobility

To investigate the consequences of economic integration we now allow mobility of labour and capital, but public capital remains immobile. We assume that both regions have an identical quantity of resources (capital and labour) from the first period ($t$) and that factor mobility occurs, just after period $t$, in period ($t+1$). The tax and saving rates are unchanged after integration. Factor mobility results in equalizing wages and return on private capital through arbitrage between the two regions after integration.
We first look at the implication of the non-arbitrage conditions for the capital and the labour market in the two regions and then the resulting effects on income and growth.

### 4.1 The Arbitrage Process after Integration

The wage rate and the return on capital in region 1 exceed those of region 2. Therefore both capital and labour will flow from region 2 to region 1 until the wage rate and the return on capital are equal in the two regions. The two non-arbitrage conditions are then met and the flow of labour and capital will stop.

This means that in equilibrium we have \( R_{1,t+1} = R_{2,t+1} = R_{r+1} \). This leads to the following non-arbitrage condition for the capital market:\(^14\)

\[
h \frac{P_{1,t+1}}{K_{1,t+1}} = \frac{P_{2,t+1}}{K_{2,t+1}} \quad \text{or} \quad K_{1,t+1} = h \frac{P_{1,t+1}}{P_{2,t+1}} K_{2,t+1}
\]

(Capital between the two regions will be divided in line with the overall productivity.

Integration also leads to equalized wage rates, thus \( w_{1,t+1} = w_{2,t+1} = w_{r+1} \). This results in the equation below:\(^15\):

\[
\frac{K_{1,t+1}}{L_{1,t+1}} = \frac{K_{2,t+1}}{L_{2,t+1}} = \hat{k}_{r+1}
\]

It indicates that, after integration, in equilibrium, the capital-labour ratio is the same for the two regions. What the consequences are with respect to capital accumulation after integration is analysed in the next section.

\(^14\) Using equation (18) to find \( R_{1,t+1} = R_{2,t+1} \) results in \( \alpha \left( \frac{h P_{1,t+1}}{K_{1,t+1}} \right) = \alpha \left( \frac{P_{2,t+1}}{K_{2,t+1}} \right) \) or \( h \frac{P_{1,t+1}}{P_{2,t+1}} K_{2,t+1} \). From this we find \( K_{1,t+1} = h \frac{P_{1,t+1}}{P_{2,t+1}} K_{2,t+1} \).

\(^15\) We use (19) to equate the wage rates which gives us: \( (1 - \alpha) \hat{k}_{1,t+1} \left( \frac{h P_{1,t+1}}{K_{1,t+1}} \right)^\beta = (1 - \alpha) \hat{k}_{2,t+1} \left( \frac{P_{2,t+1}}{K_{2,t+1}} \right)^\beta \).

From (29) we know that \( \frac{h P_{1,t+1}}{K_{1,t+1}} = \left( \frac{P_{2,t+1}}{K_{2,t+1}} \right) \) thus \( \hat{k}_{1,t+1} = \hat{k}_{2,t+1} = \hat{k} \).
4.2 Private and Public Capital after Integration

Both regions have identical tax rates and taxes are used to finance own public capital. The budgets are balanced.

Equations of Motion of Public Capital after Integration

The public capital stocks for regions 1 and 2 are given by: $P_{1,t+1} = \tau(1-\alpha)Y_{t,1}$ and $P_{2,t+1} = \tau(1-\alpha)Y_{t,2}$, similar to the autarchy case.

For private capital formation, however this is not the case. The introduction of capital mobility leads to market clearing for both regions simultaneously. After integration has taken place, aggregate savings are the sum of savings in the autarchy case, $S_t = S_{t,1} + S_{t,2}$ where $S_{t,1} = \delta(1-\alpha)Y_{t,1}$ and $S_{t,2} = \delta(1-\alpha)Y_{t,2}$. The non-arbitrage condition (30) gives us the relation between capital formation in the two regions in period $(t+1)$ after integration. Using equation (20B) we to derive the ratio between public capital of the two regions:

$$\frac{P_{1,t+1}}{P_{2,t+1}} = h^\beta$$

(31)

The non-arbitrage condition for the capital market, equation (29) and the relation between public capitals, equation (31) gives us the relation between private capital of the two regions.\(^{16}\)

$$K_{1,t+1} = h^{(\beta+1)}K_{2,t+1}$$

(32)

It tells us how capital will be divided between the two regions in case of factor mobility. Capital stock in region 1 exceeds that of the autarchy case. In case of autarchy, capital stock in region 1 is $h^\beta$ (20B) times higher than the capital stock in region 2. After integration it is $h^{(\beta+1)}$ times higher (32). This is caused by a higher return on capital in region 1 compared with region 2.

\(^{16}\) Non-arbitrage condition capital market (30) $K_{1,t+1} = \frac{P_{1,t+1}}{P_{2,t+1}} K_{2,t+1}$ and inserting $\frac{P_{1,t+1}}{P_{2,t+1}} = h^\beta$ (31) results in equation (32).
4.3 Aggregate (Private) Savings and Private Capital

After integration, capital mobility allows aggregate savings to move where rates of return are highest. Below we analyse the consequences of capital mobility for the two regions.

Equations of Motions for Private Capital after Integration

After integration, the aggregate savings of the previous period will be distributed over private capital in the two regions in line with equation (32). Total saving depends on total income of region 1 or 2 (*Y*_t = *Y*_1,t + *Y*_2,t).

\[ S_t = \hat{s}(1 - \alpha)Y_t \]  \hspace{1cm} (33)

To compare autarchy with integration we assume that in period *t* income and production in region 1 and 2 are the same, that is, *Y*_1,t = *Y*_2,t. The same can be done with the total private capital stock. Market clearing ensures that aggregate savings equals capital formation in region 1 and 2. We use equation (32) to find the capital stock of the two regions after integration:

\[ K_{1,t+1}^M = \hat{s}(1 - \alpha)\left(\frac{1}{1 + h^{1+\beta}}\right)Y_t \hspace{1cm} \text{and} \]
\[ K_{2,t+1}^M = \hat{s}(1 - \alpha)\left(\frac{h^{1+\beta}}{1 + h^{1+\beta}}\right)Y_t \]  \hspace{1cm} (34)

\[ K_{1,t+1}^M > K_{1,t+1} \hspace{1cm} \text{and} \hspace{1cm} K_{2,t+1}^M < K_{2,t+1} \]  \hspace{1cm} (35)

The superscript *M* refers to the situation of factor mobility and integration. We conclude that

\[ K_{1,t+1}^M > K_{1,t+1} \hspace{1cm} \text{and} \hspace{1cm} K_{2,t+1}^M < K_{2,t+1} \]  \hspace{1cm} (34)

\[ 0 < \left(\frac{1}{1 + h^{1+\beta}}\right) < 1/2 \hspace{1cm} \text{and} \hspace{1cm} 1/2 < \left(\frac{h^{1+\beta}}{1 + h^{1+\beta}}\right). \]  \hspace{1cm} (35)

---

17 We have \( K_{t+1} = K_{1,t+1} + K_{2,t+1} \). Inserting (32) \( K_{1,t+1} = h^{(\beta+1)}K_{2,t+1} \) results in \( K_{t+1} = (1 + h^{(\beta+1)})K_{2,t+1} \). The equating of motion \( S_t = K_{t+1} \) results in: \( \hat{s}(1 - \alpha)Y_t = K_{1,t+1} + K_{2,t+1} \). Substituting (32) results in \( \hat{s}(1 - \alpha)Y_t = h^{(\beta+1)}K_{2,t+1} + K_{2,t+1} \), finally leading to (34) and because (32) \( K_{1,t+1} = h^{(\beta+1)}K_{2,t+1} \) to (35).

18 By assumption \( Y_{1,t} = Y_{2,t} \) and \( Y_{1,t} + Y_{2,t} = Y_t \) thus we have \( Y_{1,t} = Y_{2,t} = \frac{1}{2}Y_t \) \( K_{1,t+1} = \hat{s}(1 - \alpha)\frac{1}{2}Y_t \) and \( K_{2,t+1} = \hat{s}(1 - \alpha)\frac{1}{2}Y_t \). See equation (23) and (24).
Institutional Differences and Productivity of Public Capital

As might be expected, part of the savings from region 2 are used for capital accumulation in region 1. Because if the stock of private capital changes and public capital remains the same, the ratio of the two also changes.\(^{19}\)

\[
\frac{P_{1,t+1}}{K_{1,t+1}^M} = \frac{\tau(1-\alpha)Y_{1,t}}{\hat{s}(1-\alpha)\left(\frac{h^{1+\beta}}{1+h^{1+\beta}}\right)Y_t} = \frac{\tau}{\hat{s}}\left(\frac{1+h^{1+\beta}}{2h^{1+\beta}}\right)
\]

and

\[
\frac{P_{2,t+1}}{K_{2,t+1}^M} = \frac{\tau(1-\alpha)Y_{2,t}}{\hat{s}(1-\alpha)\left(\frac{1}{1+h^{1+\beta}}\right)Y_t} = \frac{\tau}{\hat{s}}\left(\frac{1+h^{1+\beta}}{2}\right)
\]

In region 1, the ratio between the public and private capital increases (an increase in private capital, income and savings) while in region 2 it decreases (a decrease in the private capital stock, income and savings). Comparing the ratio with the autarky results in:

\[
\frac{P_{1,t+1}}{K_{1,t+1}^M} > \frac{P_{1,t}}{K_{1,t}^M} \quad \text{and} \quad \frac{P_{2,t+1}}{K_{2,t+1}^M} < \frac{P_{2,t}}{K_{2,t}^M} \quad \text{because} \quad K_{1,t+1}^M > K_{1,t+1} \quad \text{and} \quad K_{2,t+1}^M < K_{2,t+1}^M
\]

or alternatively

\[
0 < \left(\frac{1+h^{1+\beta}}{2h^{1+\beta}}\right) < 1 \quad \text{and} \quad 1 < \left(\frac{1+h^{1+\beta}}{2}\right)
\]

Workers of region 2 move to region 1 to work there because wages are higher. Employment in region 2 also will decrease.\(^{20}\)

Summarizing we can say that integration has two opposite effects for region 1. First, private capital accumulation increases due to higher return on (private) capital and secondly the productivity of public capital decreases due to increased private capital formation, which causes congestion. The opposite effects occur in region 2. Private capital formation decreases but there is a simultaneous increase in the productivity of public capital due to reduced congestion. However, integration leads to a more efficient allocation of private capital and an increase in the productivity of public capital. In the next section we will investigate the effects on income and growth of economic integration.

\(^{19}\) We have substituted \(Y_{1,t} = Y_{2,t} = \frac{1}{2}Y_t\).

\(^{20}\) The non-arbitrage condition of the labour market tells us that the capital-labour ratio is equal across the regions, so we have \(\frac{K_{1,t+1}^M}{L_{1,t+1}^M} = \frac{K_{2,t+1}^M}{L_{2,t+1}^M} = \hat{k}\) and because \(K_{2,t+1}^M < K_{2,t+1}\) we have \(L_{2,t+1}^M < L_{2,t+1}\).
4.4 Income and Growth after Integration

Now we can determine private capital formation and the production and income of the two regions in period $t+1$. We use the results from the section 3, The aggregate savings are allocated differently between the two regions after integration, which results in:

$$Y_{1,t+1}^M = h^\beta \hat{s}(1-\alpha) \left( \frac{2h^{1+\beta}}{1+h^{1+\beta}} \right)^{1-\beta} \left( \frac{\tau}{s} \right)^\beta Y_{1t}, \quad \text{and}$$

$$Y_{2,t+1}^M = \hat{s}(1-\alpha) \left( \frac{2}{1+h^{1+\beta}} \right)^{1-\beta} \left( \frac{\tau}{s} \right)^\beta Y_{2t}. \quad (37)$$

It is clear that region 1 benefits from economic integration. If we compare autarchy Equations (25), (26), with integration we see that income increases, because $Y_{1,t+1}^M > Y_{1,t+1} \left( \frac{2h^{1+\beta}}{1+h^{1+\beta}} \right)^{1-\beta} > 1$). Income decreases in the less productive region 2, $Y_{2,t+1}^M < Y_{2,t+1} \left( \frac{2}{1+h^{1+\beta}} \right)^{1-\beta} < 1$). Income difference between the two regions increases. If we express region 1’s output in terms of region 2’s output we find the following result:

$$Y_{1,t+1}^M = h^{1-\beta} Y_{2,t+1}^M. \quad (39)$$

---

21 Using equation (16) and (17) and inserting (34) and (35) and the public / private capital ratios of (36) gives;

$$Y_{1,t+1} = h^\beta K_{1,t+1} \left( \frac{P_{1,t+1}}{K_{1,t+1}} \right)^\beta = h^\beta \hat{s}(1-\alpha) \left( \frac{h^{1+\beta}}{1+h^{1+\beta}} \right)^{1-\beta} \left( \frac{\tau}{2s} \right)^\beta Y_{1t}. \quad \text{Next we can substitute } Y_t = 2Y_{1t}$$

resulting in: $Y_{1,t+1} = h^\beta \hat{s}(1-\alpha) \left( \frac{2h^{1+\beta}}{1+h^{1+\beta}} \right)^{1-\beta} \left( \frac{\tau}{s} \right)^\beta Y_{1t}$.

For region 2 we have $Y_{2,t+1} = K_{2,t+1} \left( \frac{P_{2,t+1}}{K_{2,t+1}} \right)^\beta = \hat{s}(1-\alpha) \left( \frac{1}{1+h^{1+\beta}} \right)^{1-\beta} \left( \frac{\tau}{2s} \right)^\beta Y_{2t}$ substituting $Y_t = 2Y_{2t}$

results in: $Y_{2,t+1} = \hat{s}(1-\alpha) \left( \frac{2}{1+h^{1+\beta}} \right)^{1-\beta} \left( \frac{\tau}{s} \right)^\beta Y_{2t}$. Note that $\left[ \frac{\tau}{2s} \right]^\beta 2Y_{1,t} = \left[ \frac{\tau}{s} \right]^\beta 2^{-1-\beta} Y_{1t}$.

22 The factor equals $\frac{2h^{1+\beta}}{1+h^{1+\beta}} = h^{\beta+(1+\beta)(1-\beta)} = h^{\beta+1-\beta^2}$ because $0 < \beta < 1$ and $0 < \beta^2 < 1$ we have $\beta < \beta + 1 - \beta^2 < \beta + 1$. 

16
The income in region 1 is $h^{1+\beta-\beta^2}$ times higher than in region 2, where in autarchy, income is $h^\beta$ time higher. Income differences are increasing after integration. Additionally we can calculate the growth rates of the two regions after integration:

\[
(1 + g_{M1}) = h^\beta \hat{s}(1 - \alpha) \left(\frac{2h^{1+\beta}}{1 + h^{1+\beta}}\right)^{1-\beta} \left(\frac{\tau}{\hat{s}}\right) \beta \quad \text{and} \quad (40)
\]

\[
(1 + g_{M2}) = \hat{s}(1 - \alpha) \left(\frac{2}{1 + h^{1+\beta}}\right)^{1-\beta} \left(\frac{\tau}{\hat{s}}\right) \beta \quad \text{(41)}
\]

Of course differences in growth rates of the two regions are also increasing.

\[
(1 + g_{M1}) = h^{1+\beta-\beta^2} \left(1 + g_{M2}\right) \quad \text{(42)}
\]

Summarizing the results we have:

\[
P_{1,r+1} = P_{2,r+1} \\
K_{M1,r+1} = h^{1+\beta} K_{M2,r+1} \quad \text{(20C)}
\]

\[
Y_{M1,r+1} = h^{1+\beta-\beta^2} Y_{M2,r+1}
\]

\[
(1 + g_1) = h^{1+\beta-\beta^2} \left(1 + g_2\right)
\]

The result brings us to the proposition below.

**Proposition 1:**

Economic integration of two regions, with difference in productivity of public capital reflecting differences in institutions, leads to divergence of economic development of regions.

**Proof:**

We prove by comparing the levels of output of the two regions in autarchy and integration. Because $0 < \beta < 1$ and thus $h^{1+\beta} > h^{1+\beta-\beta^2} > h^\beta > 1$ we have $Y_{M1,r+1} > Y_{1,r+1}$ and $Y_{2,r+1} > Y_{M2,r+1}$. Furthermore, we know that $Y_{1,r+1} > Y_{2,r+1}$ and this results in $Y_{M1,r+1} > Y_{1,r+1} > Y_{2,r+1} > Y_{M2,r+1}$.

---

23 Equation (20B) where $Y_{1,r} = h^\beta Y_{2,r}$. 

17
Integration improves the overall efficiency of the two regions and leads to higher output and income on aggregate. Coordination between the two regions might be needed to internalize the efficiency effects of integration. Below we investigate the efficiency effects.

5 Economic Integration and Efficiency

To analyse the efficiency effects we compare aggregate output / income of the two regions in autarky with integration.

The aggregate income of the two regions is \( Y_t = Y_{1,t} + Y_{2,t} \). We assume that in autarky income and output are the same for both regions that is \( Y_{1,t} = Y_{2,t} = \frac{1}{2} Y_t \) in period \( t \). For the next period \( (t+1) \) we first look at autarky and secondly at integration \( (M) \).

5.1 Aggregate Income / Output in Autarky and after Integration

Overall income can be found by aggregating the income of the two regions, equations (25) and (26), which results in the overall income development and growth rate:

\[
Y_{t+1} = Y_{1,t+1} + Y_{2,t+1} = (1 + h^\theta) \frac{\delta(1 - \alpha)}{2} \left( \frac{\tau}{s} \right)^\beta Y_t \quad \text{and} \\
(1 + g) = (1 + h^\theta) \frac{\delta(1 - \alpha)}{2} \left( \frac{\tau}{s} \right)^\beta
\]

The share of income between the two regions equals \( \frac{1}{1 + h^\theta} \) for region 2 and \( \frac{h^\theta}{1 + h^\theta} \) for region 1. If there were no differences in public capital \( (h = 1) \), then the share of income for both regions equals 50%.

Using equation (20C) gives us the income development and growth rate in case of integration:

\[ Y_{1,t} = Y_{2,t} = \frac{1}{2} Y_t \quad \text{we find} \quad Y_{t+1} = (1 + h^\theta) \frac{\delta(1 - \alpha)}{2} \left( \frac{\tau}{s} \right)^\beta Y_t, \quad \text{aggregate output in autarky case.} \]

\[ \text{Total output equals} \quad Y_{t+1} = Y_{1,t+1} + Y_{2,t+1}. \quad \text{Adding up (25) and (26) after we have substituted} \\
Y_{1,t} = Y_{2,t} = \frac{1}{2} Y_t \quad \text{we find} \quad Y_{t+1} = (1 + h^\theta) \frac{\delta(1 - \alpha)}{2} \left( \frac{\tau}{s} \right)^\beta Y_t, \quad \text{aggregate output in autarky case.} \]
Institutional Differences and Productivity of Public Capital

\[ Y^M_{t+1} = Y^M_{1,t+1} + Y^M_{2,t+1} = (1 + h^\beta) \hat{s}(1 - \alpha) \left( \frac{\tau}{\hat{s}} \right)^\beta Y_t \quad \text{and} \]
\[ (1 + g^M) = (1 + h^\beta) \hat{s}(1 - \alpha) \left( \frac{\tau}{\hat{s}} \right)^\beta \]  

Next we compare economic development in autarky with integration.

5.3 Comparing Income / Output in Autarky with Integration

Comparing the result of autarky given the restriction on the parameters, \(0 \leq \beta \leq 1\) and \(h \geq 1\), leads to the following proposition:

**Proposition 2:**

Economic integration of two regions that differ in institutions (public capital in this case), is efficient because it leads to an overall increase in aggregate income and output.

**Proof:**

The proof is by comparing the aggregate output of the two regimes (autarky and integration).

From equation (44A) and (45A) it follows that (for \(0 < \beta < 1\)) \(2^\beta < 2\) which implies that:

\[ Y^M_{t+1} = (1 + h^\beta) \hat{s}(1 - \alpha) \left( \frac{\tau}{\hat{s}} \right)^\beta Y_t > (1 + h^\beta) \hat{s}(1 - \alpha) \left( \frac{\tau}{\hat{s}} \right)^\beta Y_t = Y_{t+1}. \]

\[ Y^M_{t+1} = (1 + h^\beta) \hat{s}(1 - \alpha) \left( \frac{\tau}{\hat{s}} \right)^\beta Y_t > (1 + h^\beta) \hat{s}(1 - \alpha) \left( \frac{\tau}{\hat{s}} \right)^\beta Y_t = Y_{t+1}. \]

---

Total output equals \( Y^M_{t+1} = Y^M_{1,t+1} + Y^M_{2,t+1} \). Using equations (37) and (38) substituting

\[ Y_{1,t} = Y_{2,t} = \frac{1}{2} Y_t \] results in:

\[ Y^M_{1,t+1} = h^\beta \hat{s}(1 - \alpha) \left( \frac{2h^{1+\beta}}{1 + h^{1+\beta}} \right) \left( \frac{\tau}{\hat{s}} \right)^\beta Y_t \]

Adding up results in:

\[ Y^M_{2,t+1} = \hat{s}(1 - \alpha) \left( \frac{2}{1 + h^{1+\beta}} \right) \left( \frac{\tau}{\hat{s}} \right)^\beta Y_t \]

Adding up results in:

\[ Y^M_{t+1} = (1 + h^\beta) \hat{s}(1 - \alpha) \left( \frac{\tau}{\hat{s}} \right)^\beta Y_t \]

This simplifies by bringing 2 out of the brackets which results in:

\[ Y^M_{t+1} = (1 + h^\beta) \hat{s}(1 - \alpha) \left( \frac{\tau}{\hat{s}} \right)^\beta Y_t \]

leaves \( Y^M_{t+1} = (1 + h^\beta) \hat{s}(1 - \alpha) \left( \frac{\tau}{\hat{s}} \right)^\beta Y_t \), which reduces to:

\[ Y^M_{t+1} = (1 + h^\beta) \frac{\hat{s}(1 - \alpha) \left( \frac{\tau}{\hat{s}} \right)^\beta}{2^{1-\beta}} Y_t. \]
Income differences between regions are smaller in autarchy than in integration. Aggregate income in integration exceeds that in case of autarchy. Economic integration has losers and winners. The region with a less efficient institutional structure and thus public capital stock will see a decrease of income and output. The more productive region benefits from the integration process due to an increase in income. However, the integration process is not a zero sum game. On aggregate, overall income and output will increase.

6 Summary of the Results and Conclusions

We may conclude that if regions have different institutional settings, resulting in difference in overall productivity this will result in different economic development in autarchy and economic integration. As an indicator of institutional efficiency we used the ratio between public and private capital. in the regions’ production function

If regions integrate we observe a movement of capital from the less productive region to the more productive one, resulting from a difference in return on private capital. This has two consequences. For the more productive region return on private capital decreases together with return on public capital, but output increases. For the less productive region the reverse happens: return on private and public capital increases but output decreases. This arbitrage process equalizes private returns on capital between the two regions, but it results in a divergence in the economic development after integration. The more productive region gains and the less productive region loses from integration. The overall picture, though, is that integration increases efficiency and the overall economic development on aggregate. However, only one of the regions will gain and the other will lose. It is questionable whether the two regions would be willing to integrate. There is no incentive for the losing region to co-operate in the integration process. This leads to a co-ordination problem is addressed by the authors in another paper.
References:


