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## Mortality Risk Reduction Benefit Measurement By Overlapping Generations Model

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MORTALITY RISK REDUCTION BENEFIT MEASUREMENT  
BY OVERLAPPING GENERATIONS MODEL

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## ABSTRACT

This study proposes a method of measuring the mortality risk reduction benefit and the value of a statistical life (VSL) by applying the overlapping generation model with perpetual youth where the mortality risk is described by the Poisson process with a constant mortality probability per unit of time. This study starts with supposing that at time 0 the steady state changes induced by the change in mortality probability. Then the annual welfare gain ( as the Equivalent Variation ) of mortality risk reduction at time  $t$  is defined as the change in total consumption level at time  $t$  which equal to the steady state per capita consumption multiplied by endogenous population size at time  $t$ . The steady state consumption and capital stock level is determined by two simultaneous equations, Euler equation and market clearance given the mortality risk level. The mortality risk reduction does decrease the per capita consumption because of population growth as well as through the change in per capita capital stock due to the population increase and the change in capital accumulation path, which is different from the static analysis that does not arrow for capital accumulation nor endogenous population. Although the mortality risk reduction creates negative impacts on the per capita consumption which seems to be contradictory, the resulting change in total consumption level goes from negative to positive along the time due to the population growth. Thus the present value of benefit PV and the average annual benefit AEV turn out to a positive figure, which leads to a positive figure of per capita average annual benefit, per capital AEV and VSL. Finally this study measures the impacts of mortality reduction of  $10^{-6} \sim 10^{-3}$  and VSL by applying the overlapping generation model to the Japanese economy. Estimated value of a statistical life (VSL) is \$1.1-5.4 million per life saved, increasing as the pure time preference rate decreases, and slightly increasing as the change range in mortality rate increases. We believe that it is the first time to estimate the VSL based on an overlapping generation model, which has an advantage in a sense that it is a revealed preference approach rather than the state preference approach.

Keywords: mortality risk reduction benefit, Value of a statistical life(VSL), overlapping generation model, cost benefit analysis. JEL code:H43

## 1. INTRODUCTION

A typical conventional measurement method of mortality risk reduction benefit and Value of a statistical life (VSL) is to estimate the willingness to pay by contingent valuation method which asks people the value to a hypothetical market situation. Thus it is a stated preference approach which is its weak point. Another typical method is the revealed preference approach which estimate the wage premium associated with the fatality risk at work. Thus it treats a very specific situation which is its weak point. See for survey, for example, Boardman, Greenberg, Vining and Weimer (2001) chapter 15, Ashenfelter (2006), and Aldy and Viscusi (2008). It is desired to find a reveal preference approach which takes into account the common and typical risk situation. For such a common and typical risk situation this study proposes to look at the economy described by the overlapping generation model with perpetual youth where the mortality risk is described by the Poisson process with a constant mortality probability per unit of time (Barro and Sala-i-Martin(2004) chapter 3.6, and Acemoglu(2009) chapter 9.7 ). Its very useful feature is the introduction of the mortality risk described by the Poisson process with a constant mortality probability per unit of time. The steady state consumption and capital stock level is determined by two simultaneous equations, Euler equation and market clearance, given the mortality risk level. The mortality risk reduction does decrease the per capita consumption because of population growth as well as through the change in per capita capital stock due to the population increase and the change in capital accumulation path, which is different from the static analysis that does not allow for capital accumulation. Although the mortality risk reduction creates negative impacts on the per capita consumption which seems to be contradictory (see appendix for proof), the resulting change in total consumption level goes from negative to positive along the time due to the population growth. Thus the present value of benefit PV and the average annual benefit AEV turn out to a positive figure, which leads to a positive figure of per capita average annual benefit AEV and VSL.

This study is composed of description of the overlapping generation model with perpetual youth where the mortality risk is described by the Poisson process with a constant mortality probability per unit of time in next section, the definition of mortality risk reduction benefits and derivation of measurement formula in section 3, and case study for Japan in section 4.

## 2. OVERLAPPING GENERATION MODEL WITH PERPETUAL YOUTH

### 2.1 Demographics, Mortality Risk and Insurance

Suppose that each individual faces a Poisson death rate of  $\nu$ , and a constant birth rate of  $n$ . Demographics in this economy are as follows: Let the population at time  $t$  be  $L(t)$ . Then the Poisson death rate of  $\nu$  implies that a total flow of  $\nu L(t)$  individuals dies at  $t$ . As new households arrive at the exponential rate  $n > \nu$ , so that aggregate population dynamics are given by

$$\dot{L}(t) = (n - \nu)L(t) \quad (1)$$

The number of individuals of the cohort born at time  $\tau < t$  who alive at time  $t$  is

$$L(t | \tau) = n \exp\{-\nu(t - \tau)\} + (n - \nu)\tau \quad (2)$$

Here it is assumed that at  $t=0$ , the economy starts with the population of  $L(0) = 1$  who are all new born at that point.

Since individuals face an uncertain death, there may be accidental bequests which are positive asset positions because the debts are not allowed. Yaari (1965) and Blanchard (1985) introduced life insurance or annuity market, where competitive life insurance firms make payments to individuals (as a function of their asset level) in return for receiving their positive assets when they die. The instantaneous profits of a life insurance company providing annuity payment of  $z(a(t | \tau) | t, \tau)$  at time  $t$  for an individual born at time  $\tau$  with asset  $a(t | \tau)$  is

$$\pi(a(t | \tau) | t, \tau) = \nu a(t | \tau) - z(a(t | \tau) | t, \tau)$$

,since the individual will die and leave his assets to the life insurance company at the flow rate  $\nu$ . The zero profits condition now implies

$$z(a(t | \tau) | t, \tau) = \nu a(t | \tau)$$

Thus the annuity payment rate equals to the mortality rate.

### 2.2 Households

Let  $c(t | \tau)$  be the consumption and  $a(t | \tau)$  the asset at time  $t$  for a person born at time  $\tau$ . It is assumed that productivity is independent of age, so that the wage rate  $w(t)$  is the same for all  $\tau$ . Starting from the born time  $\tau$ , the household maximizes the present value of expected utility evaluated at  $\tau$ , given by

$$E_\tau U = E_\tau \left[ \int_\tau^\infty e^{-\rho(t-\tau)} \log c(t | \tau) dt \right] = \int_\tau^\infty e^{-(\rho+\nu)(t-\tau)} \log c(t | \tau) dt \quad (3)$$

,where it is assumed that the utility function is  $u(c)=\log c$  at being alive and  $u(c)=0$  after the death. The second equality comes from that  $e^{-\nu(t-\tau)}$  is the probability of being alive, so that the present value of expected utility at  $t$  is the present value of utility at  $t$  multiplied by the probability of being alive  $e^{-\nu(t-\tau)}$ . Thus  $\rho+\nu$  is the effective rate of time preference in the context of an uncertain lifetime. The households provide labor services in exchange for wages  $w(t)$ , receive interest and annuity payment income on assets  $(r(t)+\nu)a(t | \tau)$ , purchase goods for consumption  $c(t | \tau)$ , and save by accumulating assets. Therefore the flow budget constraint for cohort  $\tau$  at time  $t$  is

$$\begin{aligned} \dot{a}(t | \tau) &= r(t)a(t | \tau) - c(t | \tau) + w(t) + z(a(t | \tau) | t, \tau) \\ &= (r(t)+\nu)a(t | \tau) - c(t | \tau) + w(t) \end{aligned} \quad (4)$$

,where the second equality comes from that the annuity payment rate equals to the mortality rate,  $z(a(t | \tau) | t, \tau) = \nu a(t | \tau)$ .

Each household maximizes the present value of expected utility evaluated at  $\tau$  given by (3), under the flow budget constraint for cohort  $\tau$  at time  $t$  given by (4). Its necessary condition is the well-known Euler equation as

$$\frac{\dot{c}(t | \tau)}{c(t | \tau)} = r(t) + \nu - (\rho + \nu) = r(t) - \rho \quad (5)$$

It is also well-known that (5) and transversality condition gives the following consumption function for an individual of cohort  $\tau$ :

$$c(t | \tau) = (\rho + \nu)(a(t | \tau) + \omega(t)) \quad (6)$$

,where

$$\omega(t) = \int_t^\infty e^{-\bar{r}(s,t)+\nu} w(s) ds \quad (7)$$

,where

$$\bar{r}(s,t) = \frac{1}{s-t} \int_t^s r(x) dx$$

Now integrating (6) across cohorts and using (2), average per capita consumption at time  $t$  is then obtained as

$$c(t) = (\rho + \nu)(a(t) + \omega(t)) \quad (8)$$

,where

$$c(t) = \frac{\int_{-\infty}^t c(t | \tau) L(t | \tau) d\tau}{\int_{-\infty}^t L(t | \tau) d\tau} = \frac{\int_{-\infty}^t c(t | \tau) L(t | \tau) d\tau}{L(t)} \quad (9)$$

$$a(t) = \frac{\int_{-\infty}^t a(t|\tau)L(t|\tau)d\tau}{\int_{-\infty}^t L(t|\tau)d\tau} = \frac{\int_{-\infty}^t a(t|\tau)L(t|\tau)d\tau}{L(t)} \quad (10)$$

Finally differentiating (8) yields

$$\dot{c}(t) = (\rho + \nu)(\dot{a}(t) + \dot{\alpha}(t)) \quad (11)$$

The law of motion of assets per capita can be written as

$$\dot{a}(t) = (r(t) - (n - \nu))a(t) + w(t) - c(t) \quad (12)$$

, which is derived by differentiating (10) and substituting (4).

And it can be shown that human wealth per capita satisfies

$$(r(t) + \nu)\alpha(t) = \dot{\alpha}(t) + w(t) \quad (13)$$

Substituting these two laws of motion above into (11), yields,

$$\begin{aligned} \dot{c}(t) &= (\rho + \nu)[(r(t) - (n - \nu))a(t) + \alpha(t) - c(t) + (r(t) + \nu)\alpha(t) - w(t)] \\ &= (\rho + \nu)[(r(t) + \nu)(a(t) + \alpha(t)) - na(t) - c(t)] \\ &= (\rho + \nu) \left[ \frac{(r(t) + \nu)}{\rho + \nu} c(t) - na(t) - c(t) \right] \\ &= (r(t) - \rho)c(t) - (\rho + \nu)na(t) \end{aligned} \quad (14)$$

, where the third line uses (8).

## 2.3 Firms

Firms maximize the profit at any point

$$\pi = F(K, L) - (r + \delta)K - wL = L[f(k) - (r + \delta)k - w] \quad (15)$$

, where

$\delta$ : depreciation rate of capital stock.

$F(K, L)$ : production function with constant returns to scale, and

$$y = Y/L = F(k, 1) \equiv f(k)$$

The first order conditions of this problem are

$$f'(k) = r + \delta \quad (16)$$

And

$$f(k) - (r + \delta)k = w \quad (17)$$

Therefore zero profit of (15) holds.

## 2.4 Equilibrium

$a = k$  with (12), (14), (16), (17) determine the equilibrium value of variables  $c, k, w, r$ . Substituting (16) and (17) into (12) and (14), gives the equilibrium path of  $(k, c)$  of the following two differential equations:

$$\dot{k}(t) = f(k(t)) - (n - \nu + \delta)k(t) - c(t) \quad (18)$$

$$\frac{\dot{c}(t)}{c(t)} = f'(k(t)) - \delta - \rho - (\rho + \nu)n \frac{k(t)}{c(t)} \quad (19)$$

## 2.5 The Steady State and Comparative Statics

$\dot{k} = 0$ ,  $\dot{c} = 0$  in (18) and (19) leads to the steady state which is expressed as

$$c = f(k) - (n - \nu + \delta)k \quad (20)$$

$$f'(k) - \delta = r = \rho - (\rho + \nu)n \frac{k}{c} \quad (21)$$

The graph of (20) and (21) are shown in Figure 1 and the steady state for the mortality rate  $\nu_0$  is indicated at the point  $\nu_0$ .

Now suppose that mortality rate decreases from  $\nu_0$  to  $\nu_1$ . Then the steady state changes from point  $\nu_0$  to point  $\nu_1$  in Figure 1 with the increase or decrease in capital stock per capita  $k$  and the decrease in consumption per capita  $c$  in the steady state. The characteristics that the increase or decrease in capital stock per capita  $k$  and the decrease in consumption per capita  $c$  with mortality rate decreases from  $\nu_0$  to  $\nu_1$ , is proved in the appendix. The level variables are the total consumption  $C = e^{(n-\nu)t}c$  and the total capital stock  $K = e^{(n-\nu)t}k$ . The impact on the total consumption of the mortality decrease depends on the change in population due to the increase in net population growth rate.



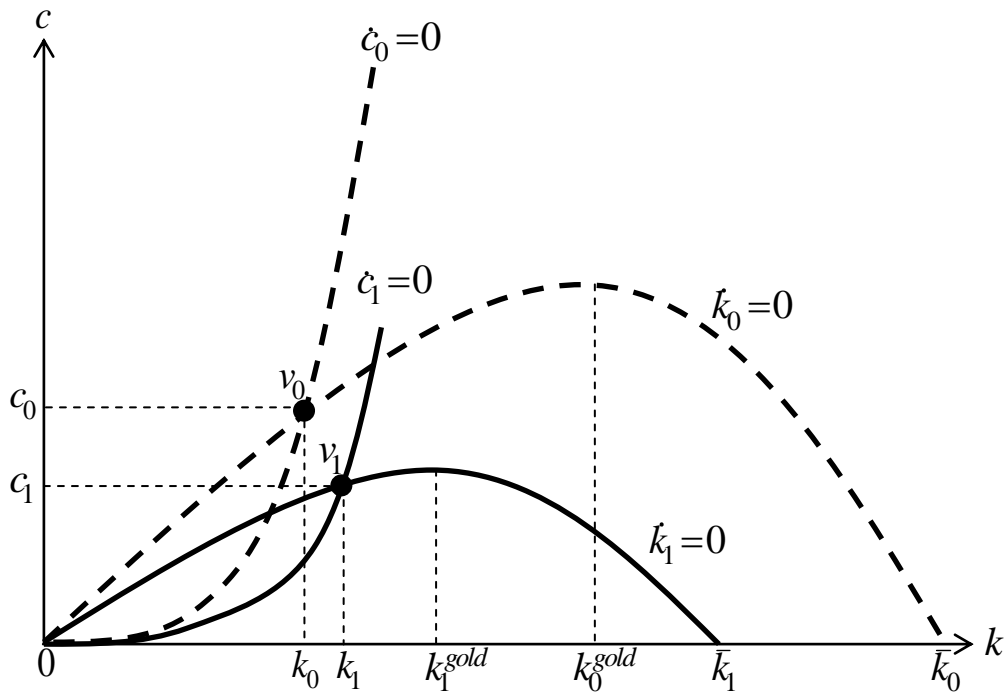


Figure 1 Steady State

### 3. BENEFIT of MORTALITY RISK REDUCTION and VALUE OF A STATISTICAL LIFE

Suppose that at present time  $t=0$ , the economy is at steady state with the mortality rate of  $\nu_0$ . The level of total consumption at time  $t$  is  $C(t; \nu^0) = e^{(n-\nu^0)t} c^*(\nu^0)$  where  $c^*(\nu^0)$  is the per capita consumption at steady state with the mortality rate of  $\nu_0$ . Now suppose that the mortality rate decreases from  $\nu_0$  to  $\nu^1 < \nu_0$  so that the per capita consumption at steady state decreases from  $c^*(\nu^0)$  to  $c^*(\nu^1) < c^*(\nu^0)$  (see appendix for proof), accordingly the level of total consumption at time  $t$  changes from  $C(t; \nu^0) = e^{(n-\nu^0)t} c^*(\nu^0)$  to  $C(t; \nu^1) = e^{(n-\nu^1)t} c^*(\nu^1)$ . It is defined as the annual benefit  $EV(t)$  of mortality rate change at time  $t$  by the difference in total consumption level such that

$$\begin{aligned} C(t; \nu^0) + EV(t) &= C(t; \nu^1) \\ e^{(n-\nu^0)t} c^*(\nu^0) + EV(t) &= e^{(n-\nu^1)t} c^*(\nu^1) \\ EV(t) &= e^{(n-\nu^1)t} c^*(\nu^1) - e^{(n-\nu^0)t} c^*(\nu^0) \end{aligned} \quad (22)$$

As the first term is less than the second term of R.S.H of the third line of (22) above at time  $t=0$ , the  $EV(t)$  starts at the negative value despite the good thing of mortality risk reduction. But as the net population growth of the first term is greater than the second of R.H.S of 3<sup>rd</sup> line of (22),  $EV(t)$  is an increasing function of time  $t$  and turns from the negative to the positive value at a certain time and increases forever as shown in Figure 2. Therefore the Present Value of  $EV(t)$ , PV, and the average annual benefit, AEV, defined by the followings, are all positive.

$$\begin{aligned} PV &= \int_0^{\infty} e^{-\rho t} EV(t) dt \\ &= \int_0^{\infty} \left\{ e^{-\rho t} e^{(n-\nu^1)t} c^*(\nu^1) - e^{-\rho t} e^{(n-\nu^0)t} c^*(\nu^0) \right\} dt \end{aligned} \quad (24)$$

$$\begin{aligned} &= \frac{c^*(\nu^1)}{\rho - (n - \nu^1)} - \frac{c^*(\nu^0)}{\rho - (n - \nu^0)} \\ AEV &= \rho PV = \frac{c^*(\nu^1)}{1 - \frac{n - \nu^1}{\rho}} - \frac{c^*(\nu^0)}{1 - \frac{n - \nu^0}{\rho}} \end{aligned} \quad (25)$$

The value of a statistical life, VSL, is obtained from the per capita annual benefit, per capita AEV, divided by the change in mortality probability.

$$VSL = AEV / (v^1 - v^0) = \rho PV / (v^1 - v^0)$$

$$VSL = \left( \frac{c^*(v^1)}{1 - \frac{n-v^1}{\rho}} - \frac{c^*(v^0)}{1 - \frac{n-v^0}{\rho}} \right) / (v^1 - v^0) \quad (26)$$

Note that all the figures above are the per capita base of population at  $t=0$  because population at  $t=0$ ,  $L(0)$ , is fixed at 1. Otherwise the per capita figures be obtained by dividing the population size at  $t=0$ ,  $L(0)$ . Note also the value of a statistical life, VSL, is a decreasing function of pure time preference rate  $r$  because the annual benefit, AEV, is a decreasing function of pure time preference rate  $r$ . The latter fact is not yet recognized as far as this study is concerned. The previous studies did not forecast the population change due to mortality rate change nor the change in willingness to pay due to the economic growth.

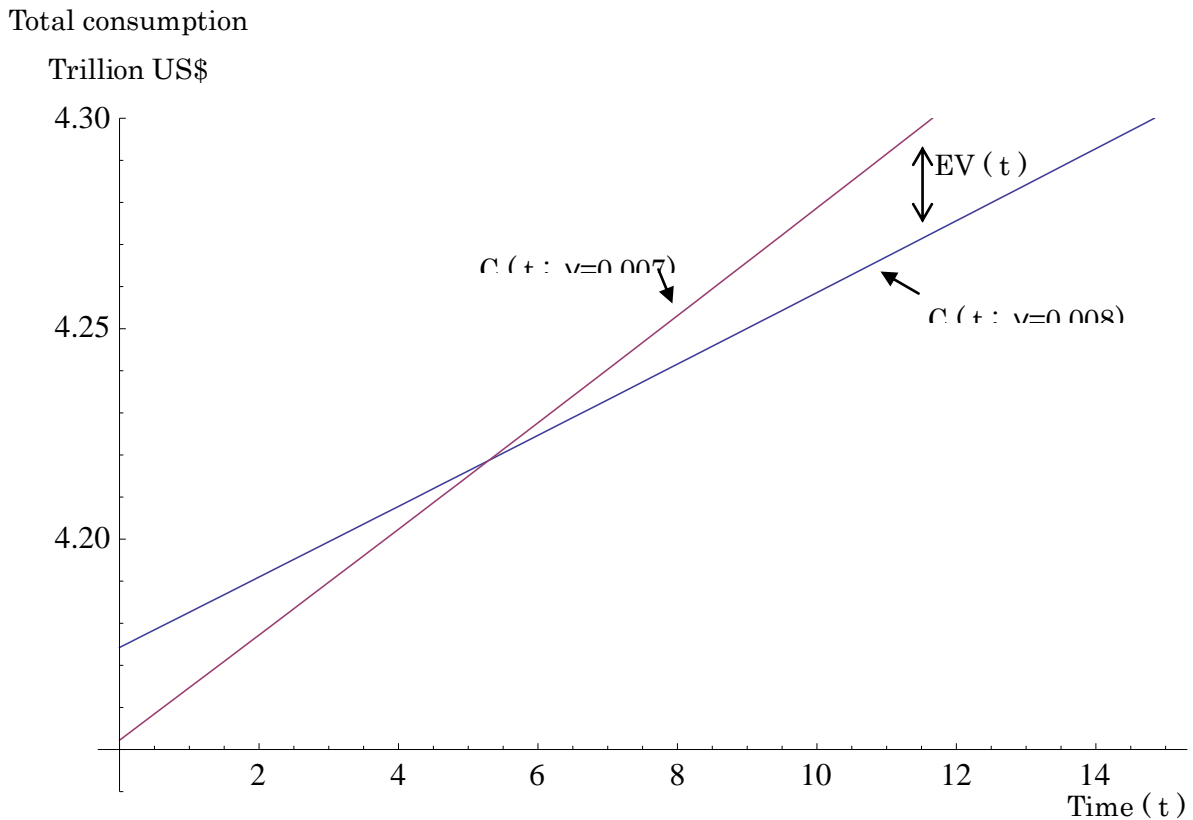


Figure 2 Total Consumption and  $EV(t)$  over Time

#### 4. ESTIMATION of VALUE of a STATISTICAL LIFE (VSL) in JAPAN

##### 4.1 Japan Economy at the Present and Calibration

The present Japanese economy is such that the capital stock  $K$ , output  $Y$ , and population  $L$ , are, respectively,

$$K_{t=0} = \text{US\$18trillion}, Y_{t=0} = \text{US\$5trillion}, L_{t=0} = 120\text{million} \quad (27)$$

,and that birth rate  $n$ , death rate  $\nu$  and depreciation rate  $\delta$ , are, respectively,

$$n = 0.014, \nu = 0.008, \delta = 0.05 \quad (28)$$

Assume that production function of Cobb-Douglas type as

$$Y_t = B(L_0 e^{(n-\nu)t})^{1-\alpha} K_t^\alpha \quad (29)$$

Thus the per capita (per present population  $L_0$ ) production function is

$$y = f(k) = Bk^\alpha \quad (30)$$

Based on the previous studies, it is estimated as  $\alpha = 0.3$ , so that  $B$  is calibrated as 2.101 from (30).

##### 4.2 Solutions for Steady State

Substituting (30) into (20) and (21), the steady state can be expressed as

$$Bk^\alpha - (n - \nu + \delta)k - c = 0 \quad (31)$$

$$(\alpha Bk^{\alpha-1} - \delta - \rho)c - (\rho + \nu)nk = 0 \quad (32)$$

Furthermore substituting  $c$  in (31) into (32), and arranging,

$$\alpha X^2 - \{\delta + \rho + \alpha(n - \nu + \delta)\}X + (\delta + \rho)(n - \nu + \delta) - (\rho + \nu)n = 0 \quad (33)$$

,where  $X = Bk^{\alpha-1}$ . (33) can be solved with respect to  $X$  as,

$$X^* = \frac{\delta + \rho + \alpha(n - \nu + \delta) + \sqrt{\{\delta + \rho + \alpha(n - \nu + \delta)\}^2 - 4\alpha\{(\delta + \rho)(n - \nu + \delta) - (\rho + \nu)n\}}}{2\alpha} \quad (34)$$

,where  $\{\delta + \rho + \alpha(n - \nu + \delta)\}^2 - 4\alpha\{(\delta + \rho)(n - \nu + \delta) - (\rho + \nu)n\} \geq 0$

Thus the steady state can be solved as

$$k^* = \left( \frac{X^*}{B} \right)^{\frac{1}{1-\alpha}} \quad (35)$$

$$y^* = B(k^*)^\alpha = k^* X^* \quad (36)$$

$$c^* = B(k^*)^\alpha - (n - \nu + \delta)k^* \quad (37)$$

Note that the smaller solution of X in (33),

$$X = \frac{\delta + \rho + \alpha(n - \nu + \delta) - \sqrt{\{\delta + \rho + \alpha(n - \nu + \delta)\}^2 - 4\alpha\{(\delta + \rho)(n - \nu + \delta) - (\rho + \nu)n\}}}{2\alpha}$$

is not feasible because it gives the negative consumption level  $c^* < 0$ .

### 4.3 Steady State by Mortality Risk

We calculate steady states for the following cases:

- (1) Base run: for each case of pure time preference rate  $\rho = 0.01, 0.02, 0.03$ , set  $B = 2.101, \alpha = 0.3, \delta = 0.05, n = 0.01, \nu = 0.008$  as the base run representing the steady state for the present Japan economy with the present mortality rate  $\nu = 0.008$  (Hayashi and Prescott (2002)).
- (2) Simulation runs: for each case of pure time preference rate  $\rho = 0.01, 0.02, 0.03$ , set  $B = 2.101, \alpha = 0.3, \delta = 0.05, n = 0.01$  as the simulation runs representing alternative steady states for the present Japan economy, where the alternatives are the changes in mortality risk to  $\nu = 0.008 \pm 10^{-6}, 0.008 \pm 10^{-5}, 0.008 \pm 10^{-4}, 0.008 \pm 10^{-3}$

### 4.4 Base Run

Table 1 shows the base run representing the steady state for the present Japan economy with the present mortality rate  $n = 0.008$ , for each case of pure time preference rate  $\rho = 0.01, 0.02, 0.03$ , and  $B = 2.101, \alpha = 0.3, \delta = 0.05, n = 0.01$ .

As the present output is \$5trillion and capital stock is \$18trillion, estimations are all higher than the present economy. The closest one is the case of  $\rho = 0.03$ .

Table 1  
Base Run Steady State

Variables	Value at the Steady State		
	$\rho=0.01$	$\rho=0.02$	$\rho=0.03$
Consumption ( $c$ )	4.25256	4.17425	4.07952
Outputs ( $y$ )	5.70787	5.33911	5.04196
Capital Stock ( $k$ )	27.9868	22.401	18.5085

(trillion US\$)

Note: Parameters are set as  $\nu=0.008$ ,  $\alpha=0.3$ ,  $\delta=0.05$ ,  $\rho=0.02$ ,  $n=0.01$ ,  $B=2.101$

#### 4.5 Simulation Runs

Figure 3-5 show the steady states for simulation runs which are for each case of pure time preference rate  $\rho=0.01, 0.02, 0.03$ , and with  $B=2.101$ ,  $\alpha=0.3$ ,  $\delta=0.05$ ,  $n=0.01$ , representing alternative steady states for the present Japan economy, where the alternatives are the changes in mortality risk from 0.008 to  $\nu=0.008\pm 10^{-6}$ ,  $0.008\pm 10^{-5}$ ,  $0.008\pm 10^{-4}$ ,  $0.008\pm 10^{-3}$ .

Note, first, the values of central vertical line in every Figure show the base run with mortality rate of 0.008. Toward left (right) from the central vertical line indicates the mortality reduction (increase) case as an alternative for simulation runs. Second, the steady state consumption level increases as the mortality rate increases (Figure 3), while the capital stocks (Figure 5) as well as the outputs (Figure4) change very little. The former is already justified by theoretical inspection in section 3 and appendix. Third, all figures are higher as the pure time preference rate are lower.

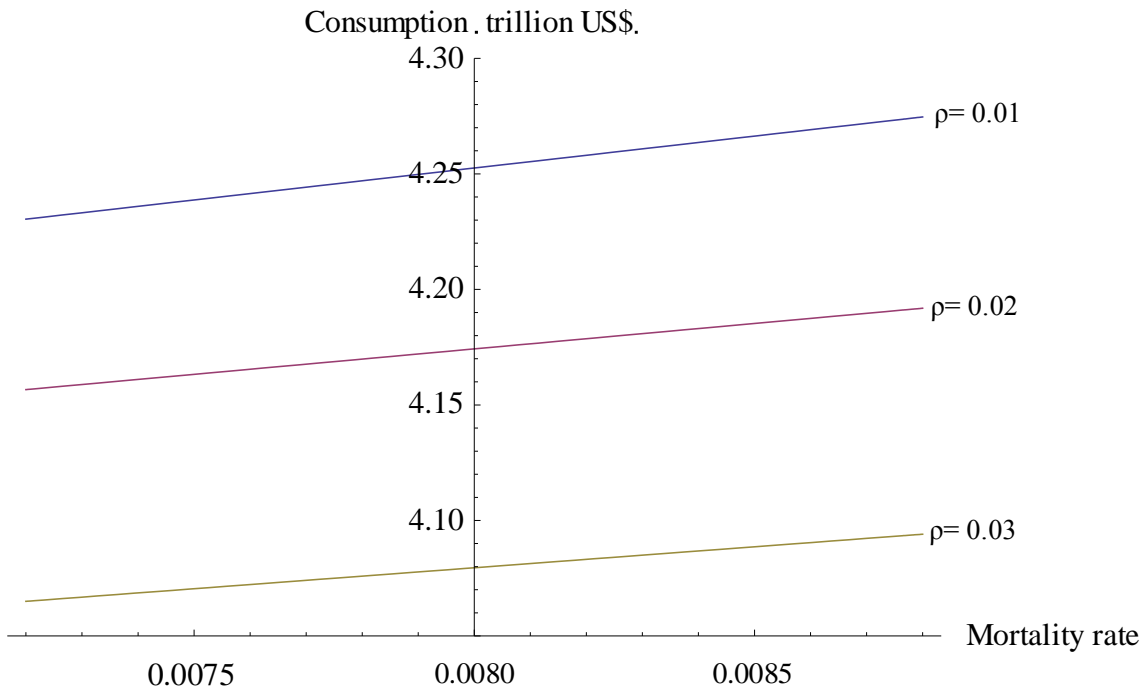


Figure 3: Consumption at Steady State by Mortality Rate

Note: Parameters except  $\nu$  and  $\rho$  are set as  $\alpha=0.3, \delta=0.05, n=0.01, B=2.101$ .

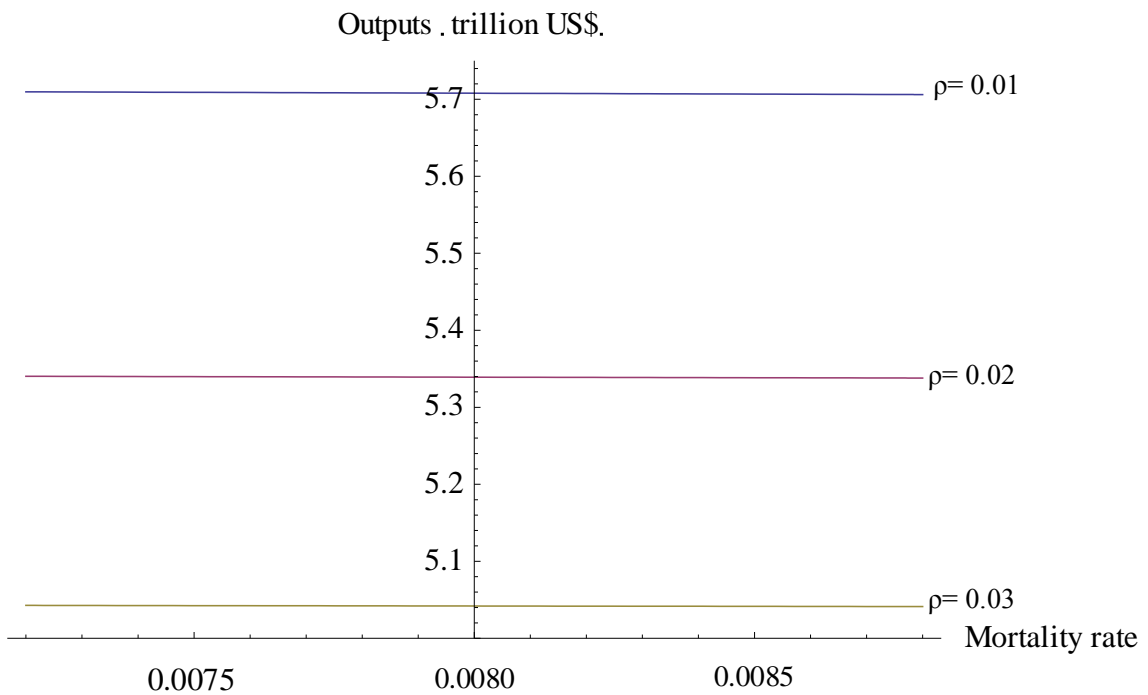


Figure 4: Outputs at Steady State by Mortality Rate

Note: Parameters except  $\nu$  and  $\rho$  are set as  $\alpha=0.3, \delta=0.05, n=0.01, B=2.101$

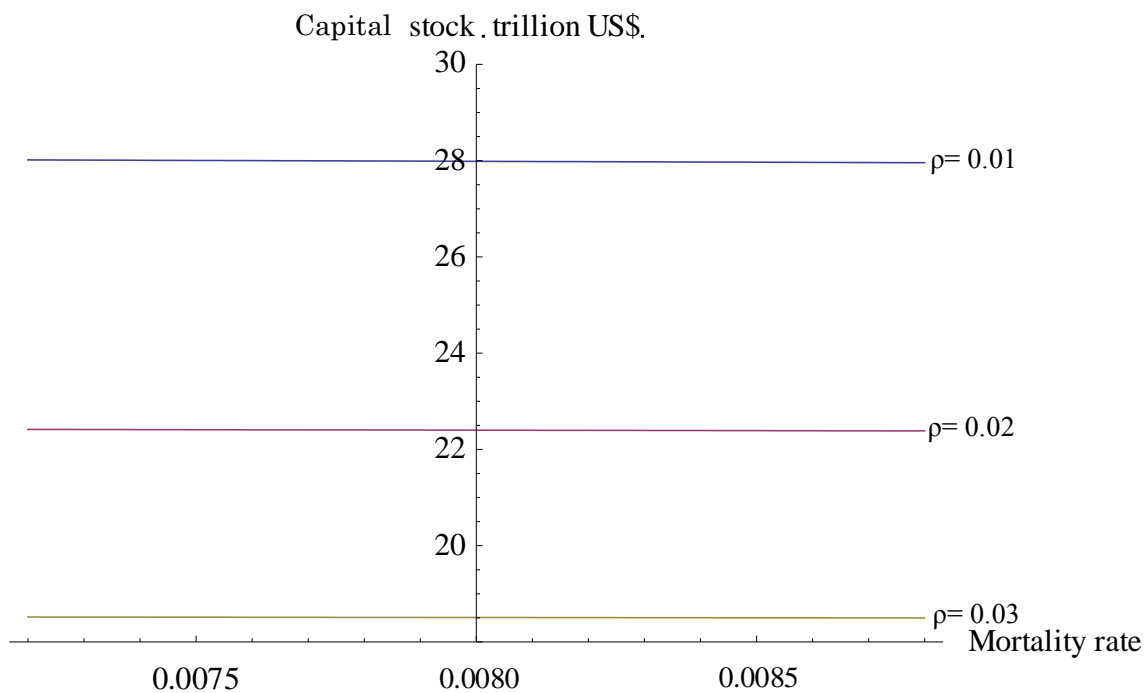


Figure 5: Capital Stock at Steady State by Mortality Rate

Note: Parameters except  $\nu$  and  $\rho$  are set as  $\alpha=0.3$ ,  $\delta=0.05$ ,  $n=0.01$ ,  $B=2.101$

#### 4.6 Estimation of Benefit, Value of Statistical Life (VSL) for Mortality Risk Reduction

The Present Value of EV ( $t$ ), PV, and the average annual benefit, AVE for the present population of 120million now can be calculated by substituting the steady state consumption level into (24) and (25), respectively, and shown as Figure 6 and 7. As expected both PV and AVE are positive (negative) when the mortality rate decreases (increases). Both PV and AVE are convex increasing functions with respect to mortality rate reduction as well as pure time preference rate decrease.



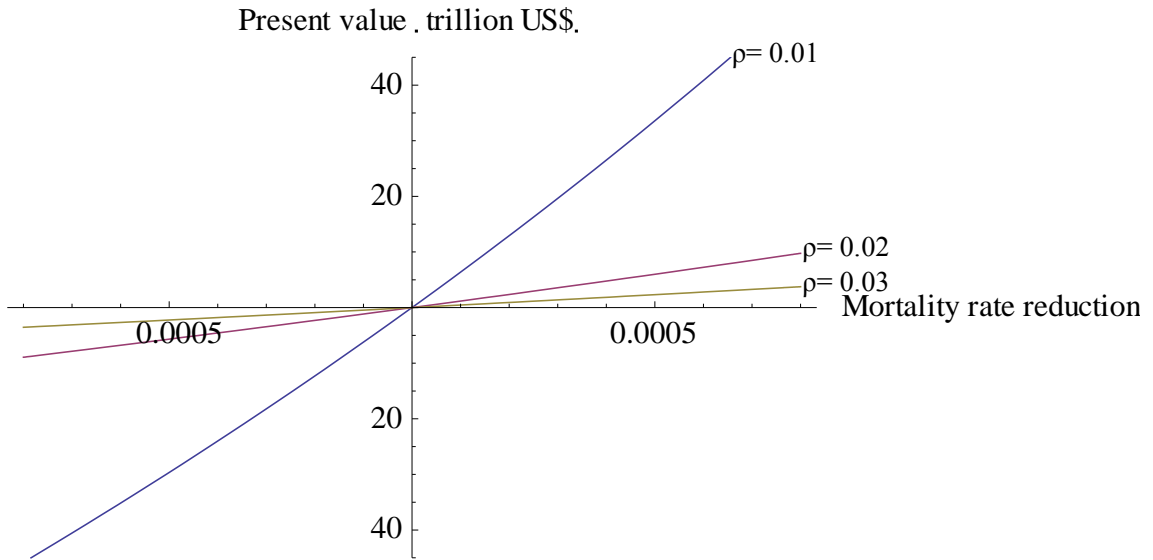


Figure 6: Present Value PV

Note: Parameters except  $\nu$  and  $\rho$  are set as  $\alpha=0.3$ ,  $\delta=0.05$ ,  $n=0.01$ ,  $B=2.101$ .

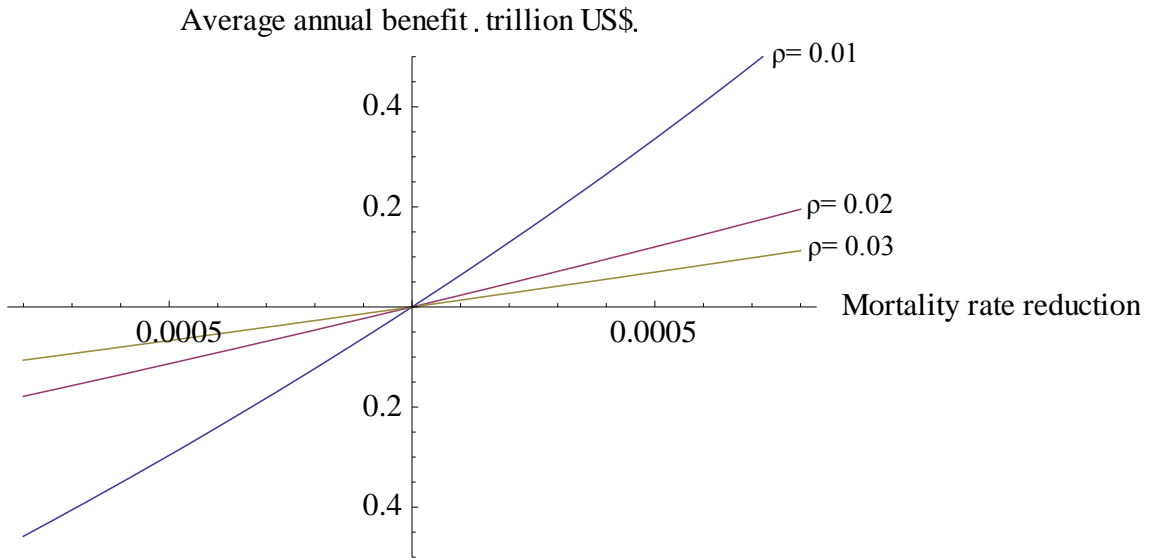


Figure 7: Average Annual Benefit

Note: Parameters except  $\nu$  and  $\rho$  are set as  $\alpha=0.3$ ,  $\delta=0.05$ ,  $n=0.01$ ,  $B=2.101$ .

Because the above average annual benefit AEV is per the present population of 120 million, the per capita average annual benefit AEV is obtained as AEV divided by population which is shown in Figure 8 and Table 2. The Value of a statistical life (VSL) is obtained by dividing per capita average annual benefit AEV by the change in mortality rate which is shown in Figure 9 and Table 2.

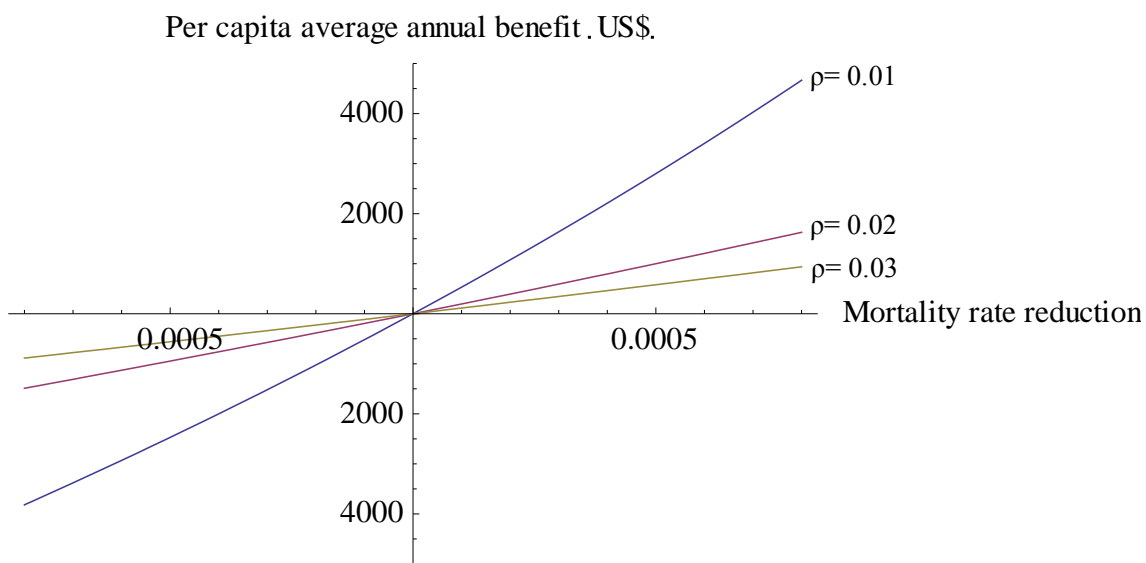


Figure 8: Per Capita Average Annual Benefit

Note: Parameters except  $\nu$  and  $\rho$  are set as  $\alpha=0.3$ ,  $\delta=0.05$ ,  $n=0.01$ ,  $B=2.101$ .

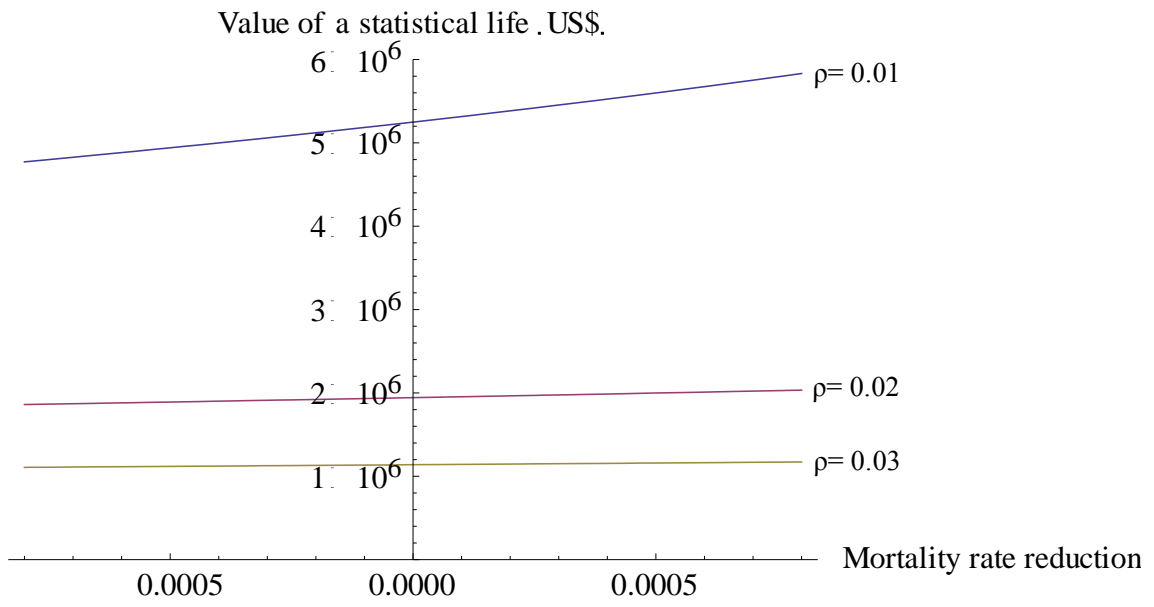


Figure 9 Value of a Statistical Life

Table 2  
Value of a Statistical Life

Case 1 : pure time preference rate = 0.01				
(US\$)				
Change in Mortality rate reduction	$-10^{-4}$	$-10^{-6}$	$10^{-6}$	$10^{-4}$
Per Capita Average Annual Benefit	-518.442	-5.24854	5.24985	531.56
Value of a Statistical Life	$5.18442 \times 10^6$	$5.24854 \times 10^6$	$5.24985 \times 10^6$	$5.3156 \times 10^6$
Case 2 : pure time preference rate = 0.02				
(US\$)				
Change in Mortality rate reduction	$-10^{-4}$	$-10^{-6}$	$10^{-6}$	$10^{-4}$
Per Capita Average Annual Benefit	-193.27	-1.94331	1.94353	195.426
Value of a Statistical Life	$1.9327 \times 10^6$	$1.94331 \times 10^6$	$1.94353 \times 10^6$	$1.95426 \times 10^6$
Case 3 : pure time preference rate = 0.03				
(US\$)				
Change in Mortality rate reduction	$-10^{-4}$	$-10^{-6}$	$10^{-6}$	$10^{-4}$
Per Capita Average Annual Benefit	-113.471	-1.13871	1.1388	114.283
Value of a Statistical Life	$1.13471 \times 10^6$	$1.13871 \times 10^6$	$1.1388 \times 10^6$	$1.14283 \times 10^6$

The calculation above says that

- (1) The value of life calculated is \$1.1-5.3 million per life saved, increasing as the pure time preference rate decreases, and slightly increasing (decreasing) as the change range in mortality rate reduction (increase).
- (2) A previous study (Boardman, Greenberg, Vining and Weimer (2001), p.395) suggests a plausible range for the value of a statistical life (VSL) saved is between \$2.5

million and \$4.0 million dollars. PIARC C9 (2004) says the range for the VSL adopted for road project cost benefit analysis manuals for is \$1-4 million dollars. Thus our estimation fits into the practical range.

- (3) For the pure time preference rate of 1, 2, 3%, VSL is approximately \$5.2 million, \$2.0 million, \$1.1 million dollars, respectively. Previous studies did not at all discuss this fact. Because previous studies did not forecast the population change due to mortality rate change nor the change in willingness to pay due to the economic growth.
- (4) For the change range of mortality reduction, the estimated VSL marginally *increases* as the change range of mortality reduction increases. This fact is contrast to the previous studies which shows VSL marginally *decreases* as the change range of mortality reduction increases. This contrast comes from the shape of average annual benefit function, AEV, which of this study is convex while the previous studies are concave due to the same reason above (3).

## 5. CONCLUDING REMARKS

This study proposes a method of measuring the mortality risk reduction benefit and the value of a statistical life (VSL) by applying the overlapping generation model with perpetual youth where the mortality risk is described by the Poisson process with a constant mortality probability per unit of time. This study starts with supposing that at time 0 the steady state changes induced by the change in mortality probability. Then the annual welfare gain ( as the Equivalent Variation ) of mortality risk reduction at time t is defined as the change in total consumption level at time t which equal to the steady state per capita consumption multiplied by endogenous population size at time t. The steady state consumption and capital stock level is determined by two simultaneous equations, Euler equation and market clearance given the mortality risk level. The mortality risk reduction does decrease the per capita consumption because of population growth as well as through the change in per capita capital stock due to the population increase and the change in capital accumulation path, which is different from the static analysis that does not allow for capital accumulation nor endogenous population. Although the mortality risk reduction creates negative impacts on the per capita consumption which seems to be contradictory, the resulting change in total consumption level goes from negative to positive along the time due to the population growth. Thus the present value of benefit PV and the average annual benefit AEV turn out to a positive figure, which leads to a positive figure of per capita average annual benefit AEV and VSL.

Finally this study measures the impacts of mortality reduction of  $10^{-6}$  ~  $10^{-3}$  and VSL by applying the constructed growth model to the Japanese economy. The calculation says that:

The value of life calculated is \$1.1-5.2 million per life saved, increasing as the pure time preference rate decreases, and slightly increasing(decreasing) as the change range in mortality rate reduction(increase).

A previous study (Boardman, Greenberg, Vining and Weimer (2001), p.395) suggests a plausible range for the value of a statistical life (VSL) saved is between \$2.5 million and \$4.0 million dollars. PIARC C9 (2004) says the range for the VSL adopted for road project cost benefit analysis manuals for is \$1-4 million dollars. Thus our estimation fits into the practical range.

For the pure time preference rate is 1, 2, 3%, VSL is approximately \$5.2 million, \$2.0 million, \$1.1 million dollars, respectively. Previous studies did not at all discuss this fact. Because previous studies did not forecast the population change due to mortality

rate change nor the change in willingness to pay due to the economic growth.

For the change range of mortality reduction, the estimated VSL marginally *increases* as the change range of mortality reduction increases. This fact is contrast to the previous studies which shows VSL marginally *decreases* as the change range of mortality reduction increases. This contrast comes from the shape of average annual benefit function, AEV, which of this study is convex while the previous studies are concave due to the same reason above.

We believe that it is the first time to estimate the VSL based on an overlapping generation model, which has an advantage in a sense that first, it is a revealed preference approach rather than the state preference approach, and second, it deals with a common phenomenon of statistical mortality rate unlike the risky job wage premium.

There are several works remaining for future. First, the utility function be more general as CIES (Constant Intertemporal Elasticity Substitution) type. Second, the technological progress be introduced. Third the more exact definition of benefit be examined for endogenous population case.

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## APPENDIX

### Proposition to be proved

The steady state is stable. Furthermore

$$\text{If } f'(k^*) - \rho - \delta - n < 0, \text{ then } \frac{dk}{dv} < 0, \frac{dc}{dv} > 0$$

$$\text{If } f'(k^*) - \rho - \delta - n > 0, \text{ then } \frac{dk}{dv} > 0, \frac{dc}{dv} > 0$$

### Proof:

The equilibrium path is

$$\dot{k} = f(k) - (n - \nu + \delta)k - c \quad (\text{A1})$$

$$\dot{c} = (f'(k) - \delta - \rho)c - (\rho + \nu)nk \quad (\text{A2})$$

The steady state implies

$$\dot{k} = f(k^*) - (n - \nu + \delta)k^* - c^* \equiv 0 \quad (\text{A3})$$

$$\dot{c} = (f'(k^*) - \delta - \rho)c^* - (\rho + \nu)nk^* \equiv 0 \quad (\text{A4})$$

Making (A1) and (A2) a Taylor's expansion around the steady state  $(k^*, c^*)$ ,

$$\begin{aligned} \dot{k} &= f(k) - (n - \nu + \delta)k - c \\ &\approx \{f(k^*) - (n - \nu + \delta)k^* - c^*\} + \{f'(k^*) - (n - \nu + \delta)\}(k - k^*) - (c - c^*) \\ &= \{f'(k^*) - (n - \nu + \delta)\}(k - k^*) - (c - c^*) \end{aligned} \quad (\text{A5})$$

$$\begin{aligned} \dot{c} &= (f'(k) - \delta - \rho)c - (\rho + \nu)nk \\ &\approx \{(f'(k^*) - \delta - \rho)c^* - (\rho + \nu)nk^*\} \\ &\quad + \{(f''(k^*)c^* - (\rho + \nu)n)\}(k - k^*) + (f'(k^*) - \delta - \rho)(c - c^*) \\ &= \{(f''(k^*)c^* - (\rho + \nu)n)\}(k - k^*) + (f'(k^*) - \delta - \rho)(c - c^*) \end{aligned} \quad (\text{A6})$$

We will show stability, first, by showing the negative determinant of (A5) and (A6) as

$$\begin{aligned} |D| &= \det \begin{bmatrix} f'(k^*) - (n - \nu + \delta) & -1 \\ f''(k^*)c^* - (\rho + \nu)n & f'(k^*) - \delta - \rho \end{bmatrix} \\ &= \{f'(k^*) - (n - \nu + \delta)\}\{f'(k^*) - \delta - \rho\} + \{f''(k^*)c^* - (\rho + \nu)n\} \end{aligned} \quad (\text{A7})$$

From (A3) and (A4)

$$n - \nu + \delta = \frac{f(k^*)}{k^*} - \frac{c^*}{k^*} \quad (\text{A8})$$

$$f'(k^*) - \delta - \rho = (\rho + \nu)n \frac{k^*}{c^*} \quad (>0) \quad (\text{A9})$$

Substituting (A8), (A9) into (A7)

$$\begin{aligned} |D| &= \det \begin{bmatrix} f'(k^*) - (n - \nu + \delta) & -1 \\ f''(k^*)c^* - (\rho + \nu)n & f'(k^*) - \delta - \rho \end{bmatrix} \\ &= \{f'(k^*) - (n - \nu + \delta)\} \{f'(k^*) - \delta - \rho\} + \{f''(k^*)c^* - (\rho + \nu)n\} \\ &= \left( f'(k^*) - \frac{f(k^*)}{k^*} + \frac{c^*}{k^*} \right) (\rho + \nu)n \frac{k^*}{c^*} + \{f''(k^*)c^* - (\rho + \nu)n\} \\ &= (\rho + \nu)n \frac{k^*}{c^*} \left\{ f'(k^*) - \frac{f(k^*)}{k^*} + \frac{c^*}{k^*} + \frac{f''(k^*)c^* - (\rho + \nu)n}{(\rho + \nu)n \frac{k^*}{c^*}} \right\} \\ &= (\rho + \nu)n \frac{k^*}{c^*} \left\{ f'(k^*) - \frac{f(k^*)}{k^*} + \frac{c^*}{k^*} + \frac{f''(k^*)c^*}{(\rho + \nu)n \frac{k^*}{c^*}} - \frac{(\rho + \nu)n}{(\rho + \nu)n \frac{k^*}{c^*}} \right\} \\ &= (\rho + \nu)n \frac{k^*}{c^*} \left\{ f'(k^*) - \frac{f(k^*)}{k^*} + \frac{c^*}{k^*} + \frac{f''(k^*)(c^*)^2}{(\rho + \nu)nk^*} - \frac{c^*}{k^*} \right\} \\ &= (\rho + \nu)n \frac{k^*}{c^*} \left\{ f'(k^*) - \frac{f(k^*)}{k^*} + \frac{f''(k^*)(c^*)^2}{(\rho + \nu)nk^*} \right\} < 0 \end{aligned} \quad (\text{A10})$$

The negative determinant above is the proof of stability.

Next in order to show the sign of derivatives of  $(k^*, c^*)$  with respect to  $\nu$ , applying a total differentiation to (A3) and (A4) with respects to  $k, c, \nu$ ,

$$\begin{bmatrix} f'(k^*) - (n - \nu + \delta) & -1 \\ f''(k^*)c^* - (\rho + \nu)n & f'(k^*) - \delta - \rho \end{bmatrix} \begin{bmatrix} dk \\ dc \end{bmatrix} = \begin{bmatrix} -k^* \\ nk^* \end{bmatrix} d\nu$$

And solving with respect to  $dk$ ,

$$\begin{aligned} \frac{dk}{d\nu} &= \frac{1}{|D|} \begin{vmatrix} -k^* & -1 \\ nk^* & f'(k^*) - \delta - \rho \end{vmatrix} \\ &= \frac{1}{|D|} \{-k^*(f'(k^*) - \delta - \rho) + nk^*\} \\ &= \frac{-k^*}{|D|} (f'(k^*) - \rho - \delta - n) \end{aligned}$$

,where  $|D| < 0$  due to (A10). Thus the proof on the sign of  $\frac{dk}{dv}$  is finished.

We will show the sign of  $\frac{dc}{dv}$  by case.

Case 1)  $f'(k^*) - \rho - \delta - n > 0$

$$\begin{aligned} \frac{dc}{dv} &= \frac{1}{|D|} \begin{vmatrix} f'(k^*) - (n - v + \delta) & -k^* \\ f''(k^*)c^* - (\rho + v)n & nk^* \end{vmatrix} \\ &= \frac{1}{|D|} \left[ \{f'(k^*) - (n - v + \delta)\}nk^* + k^* \{f''(k^*)c^* - (\rho + v)n\} \right] \end{aligned}$$

Substituting  $n - v + \delta = \frac{f(k^*)}{k^*} - \frac{c^*}{k^*}$ , form (A8)

$$\begin{aligned} \frac{dc}{dv} &= \frac{1}{|D|} \left[ \left\{ f'(k^*) - \frac{f(k^*)}{k^*} + \frac{c^*}{k^*} \right\} nk^* + k^* \{ f''(k^*)c^* - (\rho + v)n \} \right] \\ &= \frac{1}{|D|} \left[ \left\{ f'(k^*) - \frac{f(k^*)}{k^*} \right\} nk^* + nc^* + k^* f''(k^*)c^* - k^* (\rho + v)n \right] \end{aligned}$$

Furthermore substituting  $k^* (\rho + v)n = c^* (f'(k^*) - \delta - \rho)$ , form (A9)

$$\begin{aligned} \frac{dc}{dv} &= \frac{1}{|D|} \left[ \left\{ f'(k^*) - \frac{f(k^*)}{k^*} \right\} nk^* + nc^* + k^* f''(k^*)c^* - c^* (f'(k^*) - \delta - \rho) \right] \\ &= \frac{1}{|D|} \left[ \left\{ f'(k^*) - \frac{f(k^*)}{k^*} \right\} nk^* + k^* f''(k^*)c^* - c^* (f'(k^*) - \delta - \rho - n) \right] > 0 \end{aligned}$$

Case2)  $f'(k^*) - \rho - \delta - n < 0$

$$\begin{aligned} \frac{dc}{dn} &= \frac{1}{|D|} \begin{vmatrix} f'(k^*) - (n - n + d) & -k^* \\ f''(k^*)c^* - (r + n)n & nk^* \end{vmatrix} \\ &= \frac{1}{|D|} \left[ \{f'(k^*) - (n - n + d)\}nk^* + k^* \{f''(k^*)c^* - (r + n)n\} \right] \\ &= \frac{k^*}{|D|} \left[ \{f'(k^*) - (n - n + d)\}n + \{f''(k^*)c^* - (r + n)n\} \right] \\ &= \frac{k^*}{|D|} \left[ \{f'(k^*) - (n - n + d)\}n + f''(k^*)c^* - (r + n)n \right] \\ &= \frac{k^*}{|D|} \left[ \{f'(k^*) - (n + r + d)\}n + f''(k^*)c^* \right] > 0 \end{aligned}$$

The end of proof.