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# The role of labor market in urban sprawl\*

Vincent Boitier<sup>†</sup>

## Abstract

The goal of this paper is to specify the link between urban sprawl and labor market. To this fashion, I build a urban labor model with housing consumption, with a social planer problem and where spatial allocation of workers is directed by a Nash equilibrium In the context of a Potential game. Two results appear. First, I demonstrate the existence and the uniqueness of a market equilibrium where an endogenous closed form solution for the urban sprawl is obtained and where the role of each labor parameter on sprawl is unambiguous. Second, I underscore that the urban sprawl in the decentralized economy is inefficient (*i.e.* the degree of urban sprawl is to high) because firms compensate workers too much for their spatial costs.

JEL Codes : D83, J64, R14.

*Keywords:* urban sprawl, search and spatial frictions, efficiency.

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# 1 Introduction

Urban sprawl is experienced by many cities.<sup>1</sup> For instance, as reported by Brueckner (2000), between 1970 and 1990, in Chicago, the number of inhabitants increased by 4 % while the urbanized land increased by more than 45 %. At the same time, in Cleveland, the population declined by 9 % whereas the urban growth was about 30 %.<sup>2</sup>

This robust empirical fact gives birth to two debates. On the one hand, it is valuable to know what accounts for this phenomenon. Several studies have highlighted, among others, the role of geographical constraints (Burchfield *et al.* (2004)), the role of the interaction between income, transport costs and agricultural rent (Wheaton (1974), Brueckner (1981), Brueckner and Fansler (1983), Margo (1992), Mc Grath (2005), Mc Millen (2006), Wassmer (2009)), the role of policy determinants (Brueckner and Kim (2003), Wassmer (2009)), the role of local amenities (Brueckner, Thisse and Zenou (1999), Adams *et al.* (1996), Cullen and Levitt (1999)), the role of racial segregation (Yinger (1993), Powell (2000), Power (2001), Glarser and Khan (2003)) and the role of social interactions (Putnam (2000)). On the other hand, another part of the literature questions its optimality. Indeed, an important claim of the latter is that cities exhibit too large degrees of urban sprawl. This inefficiency stems from the fact that sprawl implies too long commute, too high pollution (Hivert (2007)) and too low social interaction (Putnam (2000), Mossay and Picard (2011)) to mention only a few.

Surprisingly, there is no clear understanding of the role of labor market in these two issues. Empirically, as far I know, only Patacchini and Zenou (2009) show a positive correlation between employment rate and urban expansion. Moreover, theoretically, no models undertake to characterize the relationship between urban sprawl and labor market for European cities. In urban models (see among others Alonso (1964), Mills (1967), Muth (1969), Wheaton (1974), Kanemoto (1980), Fujita (1969), Fujita and Ogawa (1980, 1982), Brueckner (1987), Fujita and Thisse (2002), Lucas and Rossi-Hansberg (2002)) unemployment is nowhere to be found because labor market is not explicitly model. In urban labor models

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<sup>1</sup>In this article, urban sprawl is defined as low density urban development as in Galster *et al.* (2001, P. 685): "*Sprawl (n.) is a pattern of land use in an urbanized area that exhibits low levels of some combinations of eight distinct dimensions: density, continuity, concentration, clustering, centrality, nuclearity, mixed uses, and proximity.*" or, more precisely, as in Nechyba and Walsh (2004, p. 178): "*By sprawl, we will mean the tendency toward lower city densities as city footprints expand*". Accordingly, there is no negative connotation in this term, contrary to Brueckner (2000, p. 66) "*urban sprawl will be defined as spatial growth of cities that is excessive relative to what is socially desirable*" even if I will show that the degree of urban sprawl in the decentralized city is too large (*i.e.* urban sprawl is inefficient) from the point of view of the social planner.

<sup>2</sup>Others empirical evidences can be found for US cities in Brueckner (2000, 2001), Gleaser and Khan (2001, 2004), Nechyba and Walsh (2004), Burchfield *et al.* (2004) and for European cities in Patacchini and Zenou (2009).

(hereafter ULMs) (see within a large literature Simpson (1992), Ortega (2000), Coulson, Land and Wang (2001), Sato (2001, 2004), Wasmer and Zenou (2002, 2006), Zenou (2009a, 2009b)) urban sprawl is inexistent since these models consider exogenous uniform spatial distribution of agents and the size of city as exogenously fixed, except for Smith and Zenou (2003) and Zenou (2009a, Chap 1, 2009b) but where the study of urban sprawl is avoided and optimality is not tackled.

The purpose of the present theoretical article is to understand the interplay between urban sprawl and labor market. More precisely, it aims at answering the following questions: What is the role of labor market parameters in urban expansion? Do cities with higher unemployment rates experience lower degrees of urban sprawl? Is urban sprawl optimal when labor market is explicitly modeled?

To this end, I develop and examine the properties of the simplest ULM able to breed endogenous urban sprawl. To wit, I consider the Zenou (2009)'s model in which the capital market is perfect (i.e. the risk-free interest rate converges to 0), the relocation costs are high, the spatial allocation of workers and the land prices are ruled by an urban use equilibrium and where the labor market is governed by a search and matching equilibrium.<sup>3</sup> I introduce in the latter three new features.

First, I allow for endogenous housing consumption, as in Zenou (2003), Smith and Zenou (2003), Zenou (2009, Chap 1) and Zenou (2009), in order to generate urban sprawl. Notice that, in accordance with Zenou (2003), I suppose that workers are endowed with quasi-linear preferences with respect to the numeraire good. In the case of non quasi-linear utilities (i.e. Cobb-Douglas, CES, CARA, CRRA preferences), the obtained model is such cumbersome that it is only numerically solvable and existence and uniqueness occurs rarely.

Second, I specify a social welfare problem in which a planner chooses a degree of sprawl in order to maximize the global welfare of the economy defined as the sum of the workers' welfare, landlords' welfare and firms' welfare.

Third, I show that a urban land use equilibrium can be reformulated as a Nash equilibrium in the context of a Potential game. By doing so, I provide a new origin for the existence and the uniqueness of an urban land use equilibrium and I fully characterize the social welfare problem.<sup>4</sup>

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<sup>3</sup>The labor market follows a search and matching model in the sense that job seekers and vacant jobs are pooled by a matching function, wages are determined according to a generalized Nash bargaining game and new jobs and equilibrium unemployment are conducted by a job creation equation and a Beveridge curve. Another important feature of this standard model is that firms evolve in perfect information (*i.e.* they observe workers' locations). As a consequence, they compensate workers for their spatial costs and, therefore, distance to jobs has a positive impact on workers' wages.

<sup>4</sup>Note that the notion of Potential game is widespread in economics and can be viewed as a particular

Within this new framework, two patent results emerge. First, I prove the existence and the uniqueness of a market equilibrium where an endogenous closed form solution for the urban sprawl is obtained and where the effect of each parameter on sprawl is well established. Namely, I stress that the unemployment benefits, the separation rate, the cost of a vacancy, the preference for land and the elasticity of the matching function increase urban expansion while the worker's productivity, the efficiency of the labor market, the transport costs, the worker's search efficiency and bargaining power decreases the degree of urban sprawl. I also emphasize that the relationship between unemployment rate and sprawl depends on the parameters. Indeed, cities where the transport costs are high (*respectively* low) expose high (*respectively* low) unemployment rates but low (*respectively* large) urban expansion while cities with high (*respectively* low) unemployment benefits, separation rate, worker's search efficiency, cost of a vacant job and elasticity of the matching function (*respectively* worker's productivity and efficiency of the matching process) exhibit both high (*respectively* low) unemployment rate and urban sprawl.

Second, solving the welfare social problem, I find that the urban sprawl in the decentralized economy is inefficient. Especially, I show that the degree of urban sprawl is too high and I also underscore that this inefficiency comes from the fact that workers do not care about their negative impact on firms' welfare. Indeed, workers in the decentralized economy are spatially too spread out because, from the point of view of the social planner, the distance between jobs and place of residence of agents is too long. Correspondingly, firms compensate too much workers for their spatial costs. This lowers firm's profit and firm's welfare. For this reason, the city needs to be shortened with a less degree of urban sprawl.

The article is organized as follows. Section 2 presents the new ULM. Section 3 gives a conclusion.

## 2 Spatial equilibrium with unemployment and urban sprawl

The setting considered hereafter is the Zenou (2009)'s model with endogenous housing consumption, with a social planner problem and where the urban land use equilibrium is written as a Nash equilibrium.

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case of a nonatomic game à la Mas Collé (1983) (see Boulogne (2009) for an explicit demonstration). It was introduced by Monderer and Shapley (1996), generalized to infinite number of agents by Sandholm (2001, 2009) and Ui (2007) and applied to densities by Lions (2008), Blanchet and Carlier (2012) and Blanchet, Mossay and Santambrogio (2012).

## 2.1 Environment

### 2.1.1 Basics

Let  $\mathcal{X} = [0, D]$  be a linear featureless space composed of a continuum of locations denoted by  $x \in \mathcal{X}$  and let  $\mathcal{K} = [0, \check{x}]$  be a linear city with endogenous city size denoted by  $\check{x}$  such that  $0 < \check{x} \leq D$ . The city is assumed to be monocentric, that is to say,  $x = 0$  is considered as the Central Business District (hereafter CBD) where all firms are exogenously located.<sup>5</sup> Consequently,  $x$  also represents distance to city center and access to jobs. I suppose a land market and a labor market in a steady state which are populated with three types of agents: firms, workers and absent landlords. The land market is governed by a Walrasian equilibrium while the labor market is driven by a search and matching equilibrium. The allocation of workers in space follows a Nash equilibrium. Time is continuous, the capital market is perfect (*i.e.* interest rate converges to 0) and the relocation costs are high.<sup>6</sup>

### 2.1.2 Job matching

The labor market under study gathers a continuum of homogenous and infinitely lived unemployed with mass  $u \in [0, 1]$  and a continuum of identical and infinitely lived employed represented by a mass  $e = 1 - u \in [0, 1]$ .<sup>7</sup> As a consequence,  $u$  (*respectively*  $e$ ) stands for the unemployment (*respectively* employment) rate. These workers are spatially dispersed

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<sup>5</sup>I follow the literature (see Zenou (2009a, 2009b)). While this assumption is not empirically observed for all cities across the world as cities tend to be more and more polycentric (see Mieskowski and Mills (1993), Anas, Arnott and Small (1993), Mc Millen and Mc Donald (1998) and Mc Millen and Smith (2003)), some empirical studies demonstrate that the predictions made by the standard monocentric model are still valid (see Mc Grath (2005), Mc Millen (2006) and Spivey (2008)). Additionally, this hypothesis is also very useful since: it "*greatly simplifies the analysis: for example, commuting trips can be exactly specified if the residential locations are known, and with the assumption of a linear or circular city, the spatial characteristics of each location in the city can be described simply by the distance from the CBD*" in Fujita and Owaga (1980, p. 455). Lastly, it is possible to consider a city with exogenous multiple centers as in Smith and Zenou (1997) and Brueckner and Zenou (2003). This alternative delivers results in line with the baseline model.

<sup>6</sup>Note that, when interest rate converges to zero, "*workers have no intrinsic preference for the present so that they only care about the fraction of time they spend employed and unemployed. Therefore, the expected utilities are not state dependent. We will thus consider the average expected utility of a worker rather than, as in the previous section, the lifetime expected utilities of employed and unemployed workers. At any moment of time, the disposable utility of a worker is thus equal to that worker's average utility over the job cycle.*" in Zenou (2003, p.14). The notion of high relocation costs comes from Zenou (2003, 2006, 2009). The idea is to "*assume that mobility costs are so high that once someone is located somewhere the worker never moves. As a result, a worker's residential location and housing consumption remain fixed as he/she enters and leaves unemployment*" in Zenou (2003, p.14).

<sup>7</sup>Unemployed workers are considered as job seekers (*i.e.* no on-the-job search).

within the city following three endogenous distributions  $\mu_U, \mu_W, \mu \in \mathcal{M}(\mathcal{X})$  defined as

$$\begin{cases} \mu_U : \mathcal{X} \rightarrow \mathbb{R}_+ \\ \int_{\mathcal{X}} \mu_U(x) dx = u \\ \mu_W : \mathcal{X} \rightarrow \mathbb{R}_+ \\ \int_{\mathcal{X}} \mu_W(x) dx = 1 - u \\ \mu : \mathcal{M}(\mathcal{X}) \rightarrow \mathbb{R}_+ \\ \int_{\mathcal{X}} \mu(x) dx = 1 \end{cases} \quad (1)$$

where  $\mathcal{M}(\mathcal{X})$  is the set of absolutely continuous spatial distributions over  $\mathcal{X}$  with respect to the Lebesgue measure,  $\mu_U(x)$  is the density of unemployed workers located in  $x$ ,  $\mu_W(x)$  is the density of employees residing in  $x$  and  $\mu(x) = \mu_U(x) + \mu_W(x)$  is the residential density. There also exists a continuum of vacant jobs exogenously located in the CBD with mass  $v \in [0, 1]$  where  $v$  is referred to as the vacancy rate. Job seekers find a job and vacancies are filled according to two random processes. These processes are governed by a Cobb-Douglas matching function with constant return to scales leading to the following aggregate number of contacts per unit of time<sup>8</sup>

$$m(\bar{s}u, v) = \vartheta (\bar{s}u)^\alpha v^{1-\alpha} \quad (2)$$

with  $\vartheta$  a constant capturing the efficiency of the matching process,  $0 < \alpha < 1$  the elasticity of the matching function respect to unemployment in intensity units and  $\bar{s}$  the exogenous average search intensity of unemployed workers. Thus, in this city, the filling rate is

$$q(\theta) = \frac{m(\bar{s}u, v)}{v} = \vartheta \theta^{-\alpha} \quad (3)$$

with  $\theta = \frac{v}{\bar{s}u}$  the labor market tightness in intensity units and observe that  $q'(\theta) < 0$ .<sup>9</sup> Similarly, the finding rate is

$$f(\theta) = \frac{s}{\bar{s}} \frac{m(\bar{s}u, v)}{u} = \vartheta \theta^{1-\alpha} s \quad (4)$$

where  $0 < s < 1$  is the exogenous search intensity of unemployed and such that  $f'(\theta) > 0$ .

### 2.1.3 Firm

A firm consumes no space, is exogenously located in  $x = 0$  and can remain in two different situations: either productive or unproductive.

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<sup>8</sup>The model remains true and tractable for general matching function  $m(\bar{s}u, v)$ . I choose the standard Cobb-Douglas specification because it helps to examine the impact of the efficiency of the matching process and the effect of the unemployment stock through the elasticity of the matching function.

<sup>9</sup>Throughout this article,  $h'(\bullet)$  denotes the first derivative of  $h$  with respect to  $\bullet$ .

**Productive firm** If the firm is productive, she is associated with a worker residing in location  $x$  and makes the following instantaneous profit

$$J(x) = y - \omega(x) \quad (5)$$

with  $y \in \mathbb{R}_+$  the worker's productivity,  $\omega(x) \in \mathbb{R}_+$  the worker's wage and such that, for every  $x$  in  $Supp(\mu_W)$ ,  $y > \omega(x)$ . As jobs are destroyed according to an exogenous rate  $\delta \in \mathbb{R}_+$  referred to as the separation rate, the lifetime expected profit of a productive firm (i.e. a filled job) that employs a worker located in  $x$  denoted by  $\mathcal{J}(x)$  satisfies a Bellman equation

$$r\mathcal{J}(x) = J(x) - \delta[\mathcal{J}(x) - \mathcal{V}] \quad (6)$$

where  $r$  is the risk-free interest rate,  $\mathcal{V}$  is the lifetime expected profit of an unproductive firm and  $\mathcal{J}(x) - \mathcal{V}$  is the local firm's surplus. Using equation (6) and equation (14) involves

$$\mathcal{J}(x) - \mathcal{V} = \frac{J(x) - V}{r + \delta + q(\theta)} \quad (7)$$

Integrating this result in equation (6) yields

$$r\mathcal{J}(x) = \frac{r + q(\theta)}{r + \delta + q(\theta)}J(x) + \frac{\delta}{r + \delta + q(\theta)}V \quad (8)$$

with  $\frac{r+q(\theta)}{r+\delta+q(\theta)}$  (respectively  $\frac{\delta}{r+\delta+q(\theta)}$ ) the fraction of time that a firm spends filled (respectively unfilled). Taking the limit  $r \rightarrow 0$ , I find that

$$\lim_{r \rightarrow 0} r\mathcal{J}(x) = \mathcal{F}(x) = \frac{q(\theta)}{\delta + q(\theta)}J(x) + \frac{\delta}{\delta + q(\theta)}V \quad (9)$$

Notice that the dynamics of the vacancy rate is given by

$$\dot{v} = \delta(1 - v) - vq(\theta) \quad (10)$$

with  $\dot{v}$  the variation of vacancy with respect to time. In steady state, flows are equal such that

$$v = \frac{\delta}{\delta + q(\theta)} \quad (11)$$

Using equation (11), the average expected profit of a productive firm that employs a worker residing in location  $x$  becomes

$$\mathcal{F}(x) = (1 - v)J(x) + vV = (1 - v)[y - \omega(x)] - v\kappa \quad (12)$$

where  $1 - v$  (respectively  $v$ ) can be re-interpreted as the fraction of time that a firm spends filled (respectively unfilled).



**Unproductive firm** If the firm is unproductive, she is unfilled by a worker. As a result, she posts a unique vacancy at cost  $\kappa \in \mathbb{R}_+$ . Since the job vacant is filled at rate  $q(\theta)$ , the instantaneous profit for an unproductive firm is

$$V = -\kappa \quad (13)$$

and the lifetime expected profit of an unproductive firm (i.e. a vacancy) is

$$r\mathcal{V} = V + q(\theta)(\mathcal{J} - \mathcal{V}) \quad (14)$$

Following the same mathematical reasoning outlined above, I obtain that

$$r\mathcal{V} = \frac{q(\theta)}{r + \delta + q(\theta)}J(x) + \frac{r + \delta}{r + \delta + q(\theta)}V \quad (15)$$

and the average expected profit of an unproductive firm is

$$\lim_{r \rightarrow 0} r\mathcal{V} = \mathcal{F}(x) = (1 - v)J(x) + vV = (1 - v)[y - \omega(x)] - v\kappa \quad (16)$$

Equation (9) and equation (16) clearly state that, when capital market is perfect and relocation costs are high, productive and unproductive firms have the same preferences: the average expected utility of a firm matters rather than the lifetime expected profits of a productive and an unproductive firm.

#### 2.1.4 Worker

A worker has to choose how much to consume, a place of residence and can remain in two different states: either employed or unemployed.

**Employed worker** If the worker is employed, he is endowed with the following hyperbolic utility function à la Mossay and Picard (2011)<sup>10</sup>

$$\mathcal{Z}_W(\sigma_W(x), \zeta_W(x)) = \sigma_W(x) - \frac{\phi}{2\zeta_W(x)} \quad (17)$$

with  $\sigma_W(x) \in \mathbb{R}_+$  (respectively  $\zeta_W(x) \in \mathbb{R}_+$ ) the amount of composite good (respectively land) consumed by an employed person residing in  $x$  and  $\phi \in \mathbb{R}_+$  a parameter catching the

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<sup>10</sup>Hyperbolic preferences are a special case of quasi-linear preferences. This means that the income effect in the land consumption is eliminated (i.e. indifference curves are parallel). I use these particular preferences for the sake of simplicity. Indeed, they have the convenient property that the instantaneous indirect utility linearly depends on residential density of workers (see equation (37) and equation (38)). This simplification helps to derive the important analytical results of this article. Notice that my analysis remains true with general quasi-linear preferences. However, with non quasi-linear preferences (i.e. Cobb Douglas, log-linear, CES, CARA, CRRA preferences), it is impossible to obtain explicit solutions and the existence and the uniqueness of a market equilibrium occurs rarely.

preference for land. He is also endowed with one unit of labor, a level of productivity  $y$ , earns a wage  $\omega(x)$ , faces a rate  $\delta$  of losing his job, commutes to the CBD to work incurring a linear transport cost  $t_W \in \mathbb{R}_+$  per unit of distance and pays  $R(x)$  per unit of land to absent landlords.<sup>11</sup> Therefore, his budget constraint is

$$\sigma_W(x) + R(x)\zeta_W(x) + t_W x = \omega(x) \quad (18)$$

and the lifetime expected utility of an employed in  $x$  denoted by  $\mathcal{W}(x)$  satisfies the following Bellman equation

$$r\mathcal{W}(x) = \mathcal{Z}_W(\sigma_W(x), \zeta_W(x)) - \delta [\mathcal{W}(x) - \mathcal{U}(x)] \quad (19)$$

with  $\mathcal{W}(x) - \mathcal{U}(x)$  the local worker's surplus. Using equation (19) and equation (30) gives

$$\mathcal{W}(x) - \mathcal{U}(x) = \frac{\mathcal{Z}_W(\sigma_W(x), \zeta_W(x)) - \mathcal{Z}_U(\sigma_U(x), \zeta_U(x))}{r + \delta + f(\theta)} \quad (20)$$

Plugging above equation (20) in equation (19) entails

$$r\mathcal{W}(x) = \frac{r + f(\theta)}{r + \delta + f(\theta)} \mathcal{Z}_W(\sigma_W(x), \zeta_W(x)) + \frac{\delta}{r + \delta + f(\theta)} \mathcal{Z}_U(\sigma_U(x), \zeta_U(x)) \quad (21)$$

with  $\frac{r+f(\theta)}{r+\delta+f(\theta)}$  (respectively  $\frac{\delta}{r+\delta+f(\theta)}$ ) the fraction of time spend by a worker in employment (respectively unemployment). Taking  $r \rightarrow 0$ , I obtain

$$\lim_{r \rightarrow 0} r\mathcal{W}(x) = \mathcal{A}(x) = \frac{f(\theta)}{\delta + f(\theta)} \mathcal{Z}_W(\sigma_W(x), \zeta_W(x)) + \frac{\delta}{\delta + f(\theta)} \mathcal{Z}_U(\sigma_U(x), \zeta_U(x)) \quad (22)$$

Remark that the dynamics of the unemployment rate is

$$\dot{u} = \int_{\text{Supp}(\mu_W)} \delta \mu_W(x) + \int_{\text{Supp}(\mu_U)} f(\theta) \mu_U(x) dx \quad (23)$$

By integration, I have

$$\dot{u} = \delta(1 - u) - f(\theta)u \quad (24)$$

with  $\dot{u}$  the variation of unemployment with respect to time,  $\delta(1-u)$  the number of employed workers entering in unemployment and  $f(\theta)u$  the number of unemployed workers finding a job. In steady state, the flows are equal such that

$$u = \frac{\delta}{\delta + f(\theta)} = \frac{\delta}{\delta + \vartheta\theta^{1-\alpha_S}} \quad (25)$$

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<sup>11</sup>Transport costs are independent of residential density: the city is free of congestion. In addition, for analytical convenience, they are supposed to be linear. However, the model remains true and tractable for continuous general functions  $t_W(x)$  and  $t_U(x)$ .

Equation (25) is the modified Beveridge curve showing an inverse relationship between the unemployment rate and the vacancy rate. Using the Beveridge curve, the average expected utility of an employed residing in  $x$  becomes

$$\mathcal{A}(x) = e\mathcal{Z}_W(\sigma_W(x), \zeta_W(x)) + u\mathcal{Z}_U(\sigma_U(x), \zeta_U(x)) \quad (26)$$

where  $e$  (*resp.*  $u$ ) can be re-interpreted as the fraction of time that a worker spends in employment (*resp.* unemployment). Within this structure, an employed person located in  $x$  has the following maximization program

$$\max_{x, \zeta(x), \sigma(x)} \mathcal{A}(x) \quad (27)$$

subject to constraint (18) and constraint (29).

**Unemployed worker** If the worker is unemployed, he is associated with the same hyperbolic utility function than the employed<sup>12</sup>

$$\mathcal{Z}_U(\sigma_U(x), \zeta_U(x)) = \sigma_U(x) - \frac{\phi}{2\zeta_U(x)^\varphi} \quad (28)$$

with  $\sigma_U(x) \in \mathbb{R}_+$  (*respectively*  $\zeta_U(x) \in \mathbb{R}_+$ ) the amount of composite good (*respectively* land) consumed by an unemployed person locating in  $x$ . He also earns a level of benefits  $z \in \mathbb{R}_+$  such that  $y > z$ , goes to the CBD to look for a job incurring a linear transport cost  $t_U$  such that  $t_W \geq t_U$ , faces a rate  $f(\theta)$  to have a job and pays  $R(x)$  per unit of land to absent landlords. In this case, his budget constraint is

$$\sigma_U + \zeta_U(x)R(x) + t_U x = z \quad (29)$$

and the expected utility of an unemployed in  $x$  denoted by  $\mathcal{U}(x)$  is determined by the following Bellman equation

$$r\mathcal{U}(x) = \mathcal{Z}_U(\sigma_U(x), \zeta_U(x)) + f(\theta) [\mathcal{W}(x) - \mathcal{U}(x)] \quad (30)$$

Following the same mathematical reasoning developed above, I find

$$\lim_{r \rightarrow 0} r\mathcal{U}(x) = \mathcal{A}(x) = e\mathcal{Z}_W(\sigma_W(x), \zeta_W(x)) + u\mathcal{Z}_U(\sigma_U(x), \zeta_U(x)) \quad (31)$$

So, when  $r \rightarrow 0$  and relocation costs are high, employed and unemployed workers have the same preferences, that is to say, the average expected utility of a worker matters rather than the lifetime expected utilities of an employed and an unemployed person. Given this environment, an unemployed person residing in  $x$  has to solve the following problem

$$\max_{x, \zeta(x), \sigma(x)} \mathcal{A}(x) \quad (32)$$

subject to constraint (18) and constraint (29).

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<sup>12</sup>This simplifying assumption (i.e. workers have identical preferences for good and land) is standard in urban labor economics (see Smith and Zenou (2003), Zenou (2009a, 2009b)) and does not determine the nature of the results.

### 2.1.5 Consumption choice

Maximizing equation (27) with respect to  $\zeta(x)$  subject to constraint (18) and constraint (29) gives the Marshallian (uncompensated) demand function of an employed located in  $x$

$$\zeta^*(x) = \zeta_W^*(x) = \sqrt{\frac{\phi}{2R(x)}} \quad (33)$$

Equivalently, I find that the Marshallian (uncompensated) demand function of an unemployed residing in  $x$  is

$$\zeta^*(x) = \zeta_U^*(x) = \sqrt{\frac{\phi}{2R(x)}} \quad (34)$$

Furthermore, in the land market, demand and supply are equals such that

$$\mu_U(x)\zeta_U^*(x) + \mu_W(x)\zeta_W^*(x) = \mu(x)\zeta^*(x) = \iota \quad (35)$$

with  $\mu_U(x)\zeta_U^*(x)$  (*respectively*  $\mu_W(x)\zeta_W^*(x)$ ) the demand for land made by unemployed (*respectively* employed) workers and  $\iota = 1$  the land intensity.<sup>13</sup> Using equation (33) and equation (34), I obtain

$$R^*(x) = \frac{\phi}{2}\mu(x)^2 \quad (36)$$

such that  $R^*(x) \geq R_A$  where  $R_A = 0$  is the exogenous agricultural rent. Integrating this result in equation (17) and assuming  $t_W = t$ , the instantaneous indirect utility of a employee residing in  $x$  is

$$W(x) = \omega(x) - tx - \phi\mu(x) \quad (37)$$

Likewise, using equation (28) and assuming that  $t_U = st$ , the instantaneous indirect utility of an unemployed person located in  $x$  is

$$U(x) = z - stx - \phi\mu(x) \quad (38)$$

Thence, using equation (37) and equation (38), a worker living in  $x$  has the following average expected utility

$$\mathcal{A}(x) = eW(x) + uU(x) = e[\omega(x) - tx - \phi\mu(x)] + u[z - stx - \phi\mu(x)] \quad (39)$$

### 2.1.6 Wage setting

Firms evolve in perfect information meaning that they observe workers' locations (*i.e.* worker's locations are possible to verify). Thence, once the match is made, the total local surplus  $S(x) = \mathcal{A}(x) - U(x) + \mathcal{J}(x) - V$  is negotiated according to a generalized Nash bargaining game

$$\omega(x) = \operatorname{argmax} [\mathcal{A}(x) - U(x)]^\gamma [\mathcal{F}(x) - V]^{1-\gamma} \quad (40)$$

<sup>13</sup>This assumption (*i.e.*  $\iota = 1$ ) is standard in urban labor economics (see Zenou (2009a, 2009b)).

with  $\gamma$  the worker's bargaining power. Maximizing (40), the following sharing rule appears

$$\gamma e [\mathcal{F}(x) - V] = (1 - \gamma)(1 - v) [\mathcal{A}(x) - U(x)] \quad (41)$$

Using above equations, I find the modified wage equation for all  $x$  in  $Supp(\mu_W)$

$$\omega(x) = (1 - \gamma) [z + (1 - s)tx] + \gamma(y + \kappa) \quad (42)$$

where  $z + (1 - s)tx$  is the reservation wage and  $y + \kappa$  is the outside effect. Plugging equation (42) in equation (37) entails

$$W(x) = (1 - \gamma)z + \gamma(y + \kappa) - [s + (1 - s)\gamma]tx - \phi\mu(x) \quad (43)$$

Distance seems to have two opposite effects. On the other hand, it decreases instantaneous utility as transport costs raise. On the other hand, it also increases this instantaneous utility since it increases the wage compensation. Nevertheless, equation (43) shows that the net effect is negative (*i.e.*  $-\tau + \gamma(1 - \tau)tx$ ) and decreases with the worker's bargaining power and the worker's search efficiency. Integrating equation (38) and equation (43) in equation (39), a worker decides his residential location maximizing

$$\mathcal{A}(x) = (1 - \gamma e)z + \gamma(y + \kappa)e - [s + (1 - s)\gamma e]tx - \phi\mu(x) \quad (44)$$

The location of workers, and so urban sprawl, is directed by three channels. To wit, workers want to live near the city center in order to maximize their average expected incomes (*i.e.*  $(1 - \gamma e)z + \gamma(y + \kappa)e$ ) and minimize their average expected transport costs (*i.e.*  $-et + e(1 - \gamma)(1 - s)t - ust = [s + (1 - s)\gamma e]t$ ) but they also live farther the others to pay low rent and consume more units of land (*i.e.*  $\phi\mu(x)$ ). This is the standard trade-off (*i.e.* Alonso-Mills-Muth trade-off) between accessibility and space. This equation also shows that the employment rate of the city and the worker's bargaining power have equivocal effects on the average expected utility of a worker. Indeed, a city with a larger employment rate and worker's bargaining power betokens that average expected revenue increases as well as average expected transport costs. The net effect is positive if the outside option is larger than the reservation wage (*i.e.*  $y + k > z + (1 - s)tx$ ).

## 2.2 Market equilibrium $(\mu^*, \theta^*, u^*)$

### 2.2.1 An informal definition

A market equilibrium is composed of two partial equilibria: a spatial equilibrium and a labor market equilibrium.<sup>14</sup> A spatial equilibrium determines an allocation of workers in

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<sup>14</sup>I exclude from this definition the land market equilibrium because the latter is already explicitly solved in equation (35) and equation (36).

space denoted by  $\mu^*$  according to a Nash equilibrium. A labor market equilibrium leads to a labor market tightness index denoted by  $\theta^*$  and an unemployment rate denoted by  $u^*$  according to a search and matching equilibrium (matching function, generalized Nash bargaining game, job creation equation and Beveridge curve).<sup>15</sup>

## 2.2.2 Spatial equilibrium $\mu^*$

Assume that the labor market is in equilibrium.

### 2.2.2.1 Definition, existence and uniqueness

Microeconomically, a worker decides his residential location so that

$$\max_{x \in \mathcal{X}} \mathcal{A}(x) \quad (45)$$

Hence, the worker's choice is strategic as he selects a residential location in accordance with his preferences and the strategies of others summarized by the endogenous densities  $\mu_U(x)$  and  $\mu_W(x)$ . Therefore, macroeconomically, the suited spatial equilibrium is a Nash equilibrium. However, with infinite number of agents and interactions summarized by densities, it is shown that the Nash equilibrium takes the following form<sup>16</sup>

**Definition 1** *A spatial equilibrium is a spatial distribution of workers  $\mu^* \in \mathcal{M}(\mathcal{X})$  such that*

$$\text{Supp}(\mu^*) \subset \arg \max_{x \in \mathcal{X}} \mathcal{A}(x) \quad (46)$$

In order to prove the existence and the uniqueness of a spatial equilibrium, I need the following definition

**Definition 2**  *$\mathcal{A}$  is a differential of a potential functional  $\mathcal{S} : \mathcal{M}(\mathcal{X}) \rightarrow \mathbb{R}$  (i.e.  $\mathcal{S}$  is a "primitive" of  $\mathcal{A}$ ) if for all  $\mu$  and  $\mu'$  in the set  $\mathcal{M}(\mathcal{X})$*

$$\lim_{\epsilon \rightarrow 0^+} \frac{\mathcal{S}[\mu + \epsilon(\mu' - \mu)] - \mathcal{S}[\mu]}{\epsilon} = \int_{\mathcal{X}} \mathcal{A}(x) [\mu(x) - \mu'(x)] dx \quad (47)$$

and the following problem

$$\max_{x \in \mathcal{X}} \mathcal{S}[\mu] \quad (48)$$

Using Definition 1, Definition 2 and program (48), I find that<sup>17</sup>

**Proposition 1** *For a given labor market equilibrium  $(\theta^*, u^*)$ , there exists a unique spatial equilibrium  $\mu^*$  if and only if  $\mathcal{A}$  is a differential of a strictly concave potential functional  $\mathcal{S}$  (i.e.  $\mathcal{A}$  admits a strictly concave "primitive").*

<sup>15</sup>A complete definition of a market equilibrium is given in Section 2.2.4

<sup>16</sup>See Lions (2008), Lachapelle (2011), Achdou (2011), Cardaliaguet (2012) to mention only a few.

<sup>17</sup>Proofs of this article are gathered in Appendix A.

This proposition is obtained using Blanchet, Mossay and Santambrogio (2012, Theorem 1). More precisely, the idea is to link equation (46) with equation (48) showing that the spatial equilibrium is the maximizer of the potential functional  $\mathcal{S}$ . By doing so, I obtain the following convenient property stating that if  $\mathcal{S}$  admits a unique maximizer, then, there exists a unique spatial equilibrium.

**Proposition 2** *For a given labor market equilibrium  $(\theta^*, u^*)$ , there exists a unique spatial equilibrium  $\mu^*$ .*

This proposition is very intuitive. Indeed, since  $\mathcal{X}$  is compact and  $\mathcal{A}$  is continuous,  $\mathcal{A}$  is a differential of

$$\mathcal{S}[\mu] = \int_{\mathcal{X}} \{(1 - \gamma e^*)z + \gamma(y + \kappa)e^* - [s + (1 - s)\gamma e^*]t\} \mu(x) - \phi \frac{\mu(x)^2}{2} dx \quad (49)$$

Moreover, as  $\mathcal{A}$  decreases with respect to the residential density  $\mu(x)$

$$\frac{\partial \mathcal{A}}{\partial \mu(x)} = -\phi < 0 \quad (50)$$

$\mathcal{S}$  is strictly concave. Let assume now that workers have quasi-linear preferences such that

$$\mathcal{Z}_i(\sigma_i(x), \zeta_i(x)) = \sigma_i(x) - \rho(\zeta_i(x)) \quad (51)$$

with  $i \in \{\mathcal{W}, \mathcal{U}\}$ ,  $\rho'(\zeta_i(x)) > 0$  and  $\rho''(\zeta_i(x)) < 0$ . Using Section 2.1, the expected average utility becomes

$$\mathcal{A}(x) = (1 - \gamma e)z + \gamma(y + \kappa)e - [s + (1 - s)\gamma e]tx - \frac{1}{\mu(x)}\rho'\left(\frac{1}{\mu(x)}\right) + \rho\left(\frac{1}{\mu(x)}\right) \quad (52)$$

Within this framework, there exists a unique spatial equilibrium because  $\mathcal{A}$  is the differential of a potential functional defined as

$$\mathcal{S}[\mu] = \int_{\mathcal{X}} \{(1 - \gamma e^*)z + \gamma(y + \kappa)e^* - [s + (1 - s)\gamma e^*]t\} \mu(x) - \frac{1}{\mu(x)}\rho\left(\frac{1}{\mu(x)}\right) dx \quad (53)$$

and  $\mathcal{A}$  decreases with respect to the residential density

$$\frac{\partial \mathcal{A}}{\partial \mu(x)} = \frac{1}{\mu(x)^3}\rho''\left(\frac{1}{\mu(x)}\right) < 0 \quad (54)$$

Thus, uniqueness emerges due a congestion effect which is a fairly robust result found in other literatures: (among others) Cooper and Johns (1988), Behrens et al. (2006) and Lasry and Lions (2007).

### 2.2.2.2 Equivalence between spatial equilibrium and urban land use equilibrium

An urban land use equilibrium (when relocation costs are high, capital market is perfect and with endogenous housing consumption) is defined as

**Definition 3** An urban land use equilibrium is  $(\tilde{R}(x), \tilde{x}, \tilde{A})$  so that

$$\tilde{R}(x) = \max \left\{ \Psi(x, \tilde{A}), R_A \right\} = \max \left\{ \Psi(x, \tilde{A}), 0 \right\} \quad (55)$$

$$\Psi(\tilde{x}, \tilde{A}) = R_A = 0 \quad (56)$$

$$\int_0^{\tilde{x}} \frac{1}{\tilde{\zeta}(x)} dx = 1 \quad (57)$$

with  $\Psi(x, \tilde{A})$  the bid rent of workers living in  $x$ ,  $\tilde{A}$  the common utility reached by workers in equilibrium and  $\tilde{\zeta}(x)$  the Marshallian demand for housing decided by workers living in  $x$ .

From Proposition 1, I can deduce that

**Proposition 3** A spatial equilibrium is a urban land use equilibrium.

Reformulate the spatial equilibrium as a Nash equilibrium in the context of a Potential game allows to study existence and uniqueness of a land use equilibrium by a different approach. Namely, the existence and the uniqueness stem from the simply property that the utility of agents admits a strictly concave primitive.

### 2.2.2.3 Endogenous degree of urban sprawl $\mu^*(x)$

Solving equation (46), I find a closed form solution for the endogenous degree of urban sprawl.

**Proposition 4** For a given labor market equilibrium  $(\theta^*, u^*)$ , the endogenous degree urban sprawl denoted by  $\mu^*(x)$  is

$$\mu^*(x) = \begin{cases} \sqrt{\frac{2[s+(1-s)\gamma e^*]t}{\phi}} - \frac{[s+(1-s)\gamma e^*]t}{\phi}x & \forall x \in [0, \tilde{x}] \text{ if } \tilde{x} < D \\ \frac{1}{D} + \frac{[s+(1-s)\gamma e^*]tD}{2\phi} - \frac{[s+(1-s)\gamma e^*]t}{\phi}x & \forall x \in [0, D] \text{ if } \tilde{x} = D \end{cases} \quad (58)$$

where  $\sqrt{\frac{2[s+(1-s)\gamma e^*]t}{\phi}}$  and  $\frac{1}{D} + \frac{[s+(1-s)\gamma e^*]tD}{2\phi}$  are the density of workers living in the city center,  $\frac{[s+(1-s)\gamma e^*]t}{\phi}$  is the density gradient and  $\tilde{x} = \sqrt{\frac{2\phi}{[s+(1-s)\gamma e^*]t}}$  is the endogenous city size.



Several short remarks are in order. First, observe that only the average expected transport costs effect and the congestion effect determine directly urban sprawl. This suggests that under hyperbolic preferences (and more generally under quasi-linear utilities) the effect of the average expected transport costs always outweighs the average expected income (see Appendix A, Proof 3). Second, this model generates an endogenous wage dispersion (a feature absent from ULMs where wage distribution is exogenously uniform) defined as  $\mu_W^*(x) = e^* \mu^*(x)$  on the support  $[\omega(0), \omega(\tilde{x})]$ . Third, the impact of each parameter on urban sprawl is well settled (see Figure 1).

[Insert Figure 1]

When preference for land is larger, housing consumption and the size of dwellings are greater. This prompts workers to disperse. Therefore, urban density is higher. An increase in transport costs, by improving spatial costs, incites workers to live closer to the city center. As a result, the BD is more inhabited, the density gradient is larger and the city size diminishes. The impacts of worker's bargaining and worker's search efficiency are the same. If these parameters pull up, workers when they are employed are less compensated by firms for their transport costs because reservation wage decreases. This reinforces the negative effect of transport costs and leads to smaller urban sprawl. Finally, urban expansion is lowered by higher employment rate. Indeed, larger employment rate points to, on average, workers will spend more time of their life being employed. Since, transport costs bear by employed are higher than unemployed, an increase of employment rate is associated with an increase of transport costs.

### 2.2.3 Labor market equilibrium $(\theta^*, u^*)$

Assume that the spatial market is in equilibrium.

#### 2.2.3.1 Modified job creation equation

Unproductive firms do not know the location of their future workers when they post their vacancies, but they expect the average wage given by

$$\bar{\omega} = \int_{Supp(\mu^*)} \omega(x) \mu^*(x) dx = (1 - \gamma)z + \gamma(y + \kappa) + (1 - \gamma)(1 - s)t\bar{x} \quad (59)$$

with  $\bar{x} = \int_{Supp(\mu^*)} x \mu^*(x) dx$  the average location of workers and  $z + (1 - s)t\bar{x}$  the average reservation wage. By integration, I get

$$\bar{x} = \begin{cases} \frac{\tilde{x}}{3} = \frac{1}{3} \sqrt{\frac{2\phi}{[s+(1-s)\gamma e]t}} & \text{if } \tilde{x} < D \\ \frac{D}{2} - \frac{[s+(1-s)\gamma e^*]tD^3}{12\phi} & \text{if } \tilde{x} = D \end{cases} \quad (60)$$

Introducing this result in the average wage leads to

$$\bar{w} = \begin{cases} (1 - \gamma)z + \gamma(y + \kappa) + \frac{(1-\gamma)(1-s)}{3} \sqrt{\frac{2\phi t}{[s+(1-s)\gamma e]}} & \text{if } \tilde{x} < D \\ (1 - \gamma)z + \gamma(y + \kappa) + (1 - \gamma)(1 - s) \left( \frac{D}{2} - \frac{[s+(1-s)\gamma e^*]tD^3}{12\phi} \right) & \text{if } \tilde{x} = D \end{cases} \quad (61)$$

The unemployed benefits, the worker's productivity and the cost of a vacancy play the same role on the average wage than on the wage. The worker's bargaining has a positive impact on the average wage if and only if  $y + \kappa > z + (1 - s)t\frac{\partial \bar{x}}{\partial \gamma}$ . Furthermore, the average wage is improved by the preference for land and the worker's search efficiency since the latter increases city size (i.e. average distance between jobs and workers) and lowers average reservation wage. On contrary, transport cost seems to have an indefinite impact as two opposite forces are at play: they pull up the average wage by raising the average reservation wage but they also decrease the latter by lowering the city size. Nonetheless, the net effect is positive (see equation (61)).

Under this set-up, firms post vacancies until the free entry condition holds (i.e.  $\mathcal{F}(\bar{x}) = 0$  with  $\mathcal{F}(\bar{x})$  the average profit made by firms). This implies that the following modified job creation equation stating the value of the labor market tightness

$$\frac{y - \bar{w}}{\delta} = \frac{\kappa}{q(\theta)} \quad (62)$$

Equation (56), showing an inverse relationship between the labor market tightness and the average wage, emphasizes that the average benefit of a filled job is equal to the average search cost of a vacancy. Equation (3), equation (61) and equation (63) yields

$$\begin{cases} y - (1 - \gamma)z - \gamma(y + \kappa) - \frac{(1-\gamma)(1-s)}{3} \sqrt{\frac{2\phi t}{s+(1-s)\gamma e}} = \frac{\delta\kappa\theta^\alpha}{\vartheta} & \text{if } \tilde{x} < D \\ y - (1 - \gamma)z - \gamma(y + \kappa) - (1 - \gamma)(1 - s) \left( \frac{D}{2} - \frac{[s+(1-s)\gamma e^*]tD^3}{12\phi} \right) = \frac{\delta\kappa\theta^\alpha}{\vartheta} & \text{if } \tilde{x} = D \end{cases} \quad (63)$$

From equation (25) and equation (63), the following simple comparative static analysis can be performed (see Figure 2 and Figure 3).

[Insert Figure 2 and Figure 3]

Urban sprawl affects negatively the unemployment rate. The intuitive underlying mechanism is as follows. Higher sprawl increases the average reservation wage since workers live farther to jobs. For that reason, the average wage set by firms pulls up and the expected gain of hiring a new worker decreases. To restore the equilibrium, the expected cost of looking for a job has to decrease too. This leads to a fall in the labor market tightness and an increase in the unemployment rate. Likewise, because unemployment benefits, preference for land and transport costs improve the average reservation wage of workers, this

implies a decline in the labor market tightness and so a higher unemployment rate. The worker's productivity seems to play an ambiguous role on the labor market equilibrium: it raises wage pressure, but it also improves production. Nonetheless, the net effect on the expected average profit is positive since  $\gamma < 1$  and so, an increase in the productivity lowers the unemployment rate. By definition, an increase in the cost of a vacancy leads to greater expected average cost. This reduces the labor market tightness and increases the unemployment rate. On the one hand, greater worker's search efficiency pulls up the finding rate. This suggests that workers will experience a shorter unemployment spell. According to Section 2.2.2.3, this lowers the city size. On the other hand, this parameter lowers average wage as stated earlier. Thus, the worker's search efficiency has a positive effect on the labor market tightness, as well as on the unemployment rate. An increase in the efficiency of the matching process has the same impact of an increase of the worker's search efficiency. As the worker's search efficiency, it improves the finding rate and city size and it also increases the average cost of posting a job vacant. Naturally, this prompts the labor market tightness to decrease and the unemployment rate to increase. The effect of the separation rate is identical to a decrease in the efficiency of the labor market, which is a standard result in search and matching models (see Cahuc and Zylberberg (2004, Chap 9)). Finally, a higher elasticity of the matching function increases the competition of unemployed in their looks for a job. Accordingly, the probability of finding a job becomes smaller. This also designates that workers will spend more time being unemployed that increases the city size as well as the average wage. The combination of these two effects leads to a drop in the labor market tightness and an increase in the unemployment rate.

### 2.2.3.2 Definition, existence and uniqueness

To complete the model, I define and prove the existence and the uniqueness of a labor market equilibrium.

**Definition 4** *A labor market equilibrium consists in finding a labor market tightness index  $\theta^*$  solving the job creation equation (63) and an unemployment rate  $u^*$  solving the Beveridge curve (25).*

**Proposition 5** *For a given land market equilibrium  $\mu^*$ , there exists a unique labor market equilibrium  $(\theta^*, u^*)$  if and only if*

$$y - (1 - \gamma)z - \gamma(y + \kappa) > \frac{(1 - \gamma)(1 - s)}{3} \sqrt{\frac{2\phi t}{s}} > (1 - \gamma)(1 - s) \left( \frac{D}{2} - \frac{stD^3}{12\phi} \right) \quad (64)$$

## 2.2.4 Definition, existence and uniqueness

**Definition 5** A market equilibrium  $(\mu^*, \theta^*, u^*)$  is such that a spatial equilibrium  $\mu^*$  and a labor market equilibrium  $(\theta^*, u^*)$  are solved for simultaneously.

Proposition 1 and Proposition 5 insure the existence and the uniqueness of a market equilibrium. Indeed,

**Proposition 6** There exists a unique market equilibrium  $(\mu^*, \theta^*, u^*)$  if and only if

$$y - (1 - \gamma)z - \gamma(y + \kappa) > \frac{(1 - \gamma)(1 - s)}{3} \sqrt{\frac{2\phi t}{s}} > (1 - \gamma)(1 - s) \left( \frac{D}{2} - \frac{stD^3}{12\phi} \right) \quad (65)$$

## 2.3 Urban sprawl and labor market

### 2.3.1 What is the role of labor market in urban sprawl?

Using the Beveridge curve in equation (25), the endogenous degree of sprawl in equation (58) and the job creation equation (63), the following results emerge (see Figure 4).

[Insert Figure 4]

Since they (only) affect positively (*respectively* negatively) the endogenous unemployment rate, the unemployment benefits, the separation rate, the cost of a vacant job and the elasticity of the matching function (*respectively* the worker's productivity and the efficiency of the matching process) increases (*respectively* decreases) the degree of urban sprawl. The effect of the preference for land is also unequivocal: the latter leads to higher degree of sprawl because it increases housing consumption and because it lowers the employment rate of the economy. Moreover, an increase in the worker's search efficiency lowers urban expansion because it increases the negative effect of transport costs and it reduces the unemployment rate. Nevertheless, the impact of the bargaining power of workers is uncertain (see Section 2.2). In the same way, the effect of the transport costs also seems to be indefinite. This indeterminacy stems from the fact that these parameters affect the urban sprawl and the unemployment rate in opposite sign. However, the net effect is negative since

$$s + (1 - s)\gamma e^* + (1 - s)\gamma \frac{\partial e^*}{\partial t} > 0 \quad (66)$$

### 2.3.2 Do cities with higher unemployment rates experience lower degrees of urban sprawl?

Because the unemployment rate is an endogenous variable, the relationship between the latter and urban sprawl are contingent upon the relative importance of each parameter (see Table 1).

[Insert Table 1]

Indeed, cities where the transport costs are high (*respectively* low) expose high (*respectively* low) unemployment rates but low (*respectively* large) urban expansion while cities with high (*respectively* low) unemployment benefits, separation rate, worker's search efficiency, cost of a vacant job and elasticity of the matching function (*respectively* worker's productivity and efficiency of the matching process) exhibit both high (*respectively* low) unemployment rate and urban sprawl.

## 2.4 Optimal degree of urban sprawl

### 2.4.1 Full characterization of the optimality problem

A level of urban sprawl in a decentralized economy is said to be optimal if it is identical to the one determined by a social planner. For a given unemployment rate  $u^*$  and a given labor market tightness  $\theta^*$ , the latter chooses a degree of sprawl in order to maximize the global welfare of the economy denoted by  $\Omega$  so that<sup>18</sup>

$$\max_{\mu \in \mathcal{M}(\mathcal{X})} \Omega[\mu] \quad (67)$$

Using Proposition 1 and Proposition 3, I find that

**Proposition 7** *The degree of urban sprawl in a decentralized economy is optimal if and only if*

$$\Omega[\mu] = \int_{\mathcal{X}} \{\xi - [s + (1-s)\gamma e^*]tx\} \mu(x) - \frac{\phi}{2} \mu(x)^2 dx$$

with  $\xi$  a constant.

Write the urban land use equilibrium as a Nash equilibrium in the context of a Potential game has the convenient property to full characterize the social welfare problem: it determines the only criterion leading to optimality. This answers to the assertion of Fujita (1989, p 63): "*Exactly what optimal land use is, of course, depends on how the objective function is specified*".

### 2.4.2 Benthamite social planner

The social planner is considered as Benthamite in the sense that he maximizes the sum of workers' welfare, absent landlords' welfare and productive firms' welfare such that

$$\Omega[\mu] = \int_{\mathcal{X}} U(x)\mu_U(x)W(x)\mu_W(x) + R(x)\zeta(x)\mu(x)dx + J(x)\mu_W(x)dx - \kappa v \quad (68)$$

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<sup>18</sup>In Appendix C, I consider another social problem where, for a given degree of urban sprawl  $\mu^*(x)$ , the social planner chooses an unemployment rate and a labor market tightness to maximize the welfare of the economy. From this problem, I underline a new Hosios-Pissarides condition which is a generalization of the one of Zenou (2009).

that is, using equation (35), equation (37) and equation (38)

$$\Omega[\mu] = \int_{\mathcal{X}} \mathcal{A}(x)\mu(x) + R(x)\zeta(x)\mu(x)dx + \mathcal{F}(x)\mu_W(x)dx - \kappa v \quad (69)$$

or using equation (5), equation (36) and equation (44)

$$\Omega[\mu] = \int_{\mathcal{X}} [ye^* + zu^* - (e^* + su^*)tx] \mu(x) - \phi \frac{\mu(x)^2}{2} dx \quad (70)$$

Following Proposition 7, the urban sprawl is inefficient. More precisely, solving program (68), I find the following optimal degree of urban sprawl denoted by  $\check{\mu}(x)$  is  $\forall x \in \text{Supp}(\check{\mu}) = [0, \check{x}]$

$$\check{\mu}(x) = \sqrt{\frac{2(e^* + su^*)t}{\phi}} - \frac{(e^* + su^*)t}{\phi}x \quad (71)$$

where  $e^* + su^* > s + (1 - s)\gamma e^*$ . Noting that  $s + (1 - s)\gamma e^* = e^* + su^* + (1 - \gamma)(1 - s)e^*$ , the degree of urban density in the decentralized economy is

$$\mu^*(x) = \sqrt{\frac{2(e^* + su^* + (1 - \gamma)(1 - s)e^*)t}{\phi}} - \frac{(e^* + su^* + (1 - \gamma)(1 - s)e^*)t}{\phi}x \quad (72)$$

meaning that the degree of urban sprawl is systematically too high.

[Insert Figure 5]

This result is straightforward to analyze. This inefficiency comes from the fact that workers do not anticipate the fallouts of their locations on firms' profit. Because the distance between jobs and place of residence of agents is too long, firms compensate too much workers for their spatial costs. This lowers firm's profit and firm's welfare.

### 2.4.3 Imperfect information

In order to show that the inefficiency comes from the interplay between the wages and the distance, I extent the basic model presented in Section 2.1 to imperfect information where the wages are not related with the locations of workers. Following Zenou (2009, Appendix B), the wage equation is given by

$$\omega = (1 - \gamma)z + \gamma(y + \kappa) \quad (73)$$

where  $y + \kappa > z$ , the average expected utility of a worker residing in  $x$  is

$$\mathcal{A}(x) = (1 - \gamma e)z + \gamma(y + \kappa)e - (e + su)tx - \phi\mu(x) \quad (74)$$

Thusly, the endogenous degree of sprawl is

$$\mu^*(x) = \sqrt{\frac{2(e^* + su^*)t}{\phi}} - \frac{(e^* + su^*)t}{\phi}x \quad (75)$$

$\mathcal{A}$  is a differential of the following potential

$$\mathcal{S}[\mu] = \int_{\mathcal{X}} [(1 - \gamma e)z + \gamma(y + \kappa)e^* - (e^* + su^*)tx] \mu(x) - \frac{\phi}{2} \mu(x)^2 dx \quad (76)$$

the criterion of the social planner is

$$\mathcal{S}[\mu] = \int_{\mathcal{X}} [ye^* + zu^* - (e^* + su^*)tx] \mu(x) - \frac{\phi}{2} \mu(x)^2 dx \quad (77)$$

and the optimal degree of sprawl is

$$\check{\mu}(x) = \sqrt{\frac{2(e^* + su^*)t}{\phi}} - \frac{(e^* + su^*)t}{\phi} x \quad (78)$$

Henceforth, the degree of sprawl in the decentralized economy is optimal since the spatial distribution of employed workers does not imply a increase in the wages and firms' profits are not worse off.

### 3 Conclusion

Introducing housing consumption, a social planner problem and Nash equilibrium concept in a standard ULM with high relocation costs, I delimit the role of labor market in urban sprawl: I prove the existence and the uniqueness of a market equilibrium where an endogenous closed form solution for the urban sprawl is obtained and where the impact of each labor parameter on sprawl is unambiguous and I stress that the degree of urban sprawl in the decentralized economy is too high because firms compensate workers too much for their spatial costs.

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## A Proofs

**Proof 1** *The idea of the proof is to link the maximization problem (48) with the spatial equilibrium (46) showing that the latter is a maximizer of  $\mathcal{S}$  for two different specifications of  $\mathcal{A}$ . Consider that the labor market is in equilibrium.*

• **Hyperbolic preferences:** *assume that  $\mathcal{A}$  is defined as in equation (44). Let rewrite the spatial equilibrium as a variational inequality (see Lions (2008))*

$$\int_{\mathcal{X}} \mathcal{A}(x) [\mu(x) - \mu'(x)] dx \geq 0$$

$\forall \mu, \mu' \in \mathcal{M}(\mathcal{X})$  and let also define the potential functional  $\mathcal{S}$  as (see Blanchet, Mossay and Santambrogio (2012))

$$\mathcal{S}[\mu] = \mathcal{S}_1[\mu] + \mathcal{S}_2[\mu]$$

with

$$\mathcal{S}_1[\mu] = \int_{\mathcal{X}} \{(1 - \gamma e^*)z + \gamma(y + \kappa)e^* - [s + (1 - s)\gamma e^*]tx\} \mu(x) dx$$

and

$$\mathcal{S}_2[\mu] = - \int_{\mathcal{X}} \frac{\phi}{2} \mu(x)^2 dx$$

Following Blanchet, Mossay and Santambrogio (2012, Appendix A.1), let  $\mu \in \mathcal{M}(\mathcal{X})$  be a maximizer of  $\mathcal{S}$ ,  $\mu' \in \mathcal{M}(\mathcal{X})$  be some admissible spatial distribution of workers and  $\mu_\epsilon = (1 - \epsilon)\mu + \epsilon\mu'$  be a family of perturbations with  $0 \leq \epsilon \leq 1$ . Since  $\mu$  maximizes  $\mathcal{S}$ ,

$$\frac{d}{d\epsilon} \mathcal{S}[\mu_\epsilon]_{|\epsilon=0} = \frac{d}{d\epsilon} \mathcal{S}_1[\mu_\epsilon]_{|\epsilon=0} + \frac{d}{d\epsilon} \mathcal{S}_2[\mu_\epsilon]_{|\epsilon=0} \geq 0$$

Observe that

$$\frac{d}{d\epsilon} \mathcal{S}_1[\mu_\epsilon]_{|\epsilon=0} = \int_{\mathcal{X}} \{(1 - \gamma e^*)z + \gamma(y + \kappa)e^* - [s + (1 - s)\gamma e^*]tx\} [\mu(x) - \mu'(x)] dx \geq 0$$

and

$$\frac{d}{d\epsilon} \mathcal{S}_2[\mu_\epsilon]_{|\epsilon=0} = - \int_{\mathcal{X}} \phi \mu(x) [\mu(x) - \mu'(x)] dx$$

Using the two above results, I get

$$\frac{d}{d\epsilon} \mathcal{S}[\mu_\epsilon]_{|\epsilon=0} = \int_{\mathcal{X}} \{(1 - \gamma e^*)z + \gamma(y + \kappa)e^* - [s + (1 - s)\gamma e^*]tx\} [\mu(x) - \mu'(x)] dx \geq 0$$

that is

$$\frac{d}{d\epsilon} \mathcal{S}[\mu_\epsilon]_{|\epsilon=0} = \int_{\mathcal{X}} \mathcal{A}(x) [\mu(x) - \mu'(x)] dx \geq 0$$

Thus, a spatial equilibrium is a local maximizer of  $\mathcal{S}$ .

• **Quasi-linear preferences:** assume that  $\mathcal{A}$  is defined as in equation (52).  $\mathcal{S}$  is re-written with

$$\mathcal{S}_2[\mu] = - \int_{\mathcal{X}} \frac{1}{\mu(x)} \rho \left( \frac{1}{\mu(x)} \right) dx$$

and notice that

$$\frac{d}{d\epsilon} \mathcal{S}_2[\mu_\epsilon]_{|\epsilon=0} = - \int_{\mathcal{X}} \left[ \frac{1}{\mu(x)} \rho' \left( \frac{1}{\mu(x)} \right) + \rho \left( \frac{1}{\mu(x)} \right) \right] \mu(x) [\mu(x) - \mu'(x)] dx$$

Using the above reasoning, I can conclude that a spatial equilibrium is a local maximizer of  $\mathcal{S}$ .

Thus, for a given labor market equilibrium  $(\theta^*, u^*)$ , there exists a unique spatial equilibrium  $\mu^*$  if and only if  $\mathcal{A}$  is a differential of a strictly concave potential functional  $\mathcal{S}$  (i.e.  $\mathcal{A}$  admits a strictly concave "primitive").  $\diamond$

Notice that Proposition 1 holds for a general form  $\mathcal{A}$ . Indeed, the Lagrangian of problem (48) is

$$\mathcal{L} = \mathcal{S} + \int_{\mathcal{X}} \psi^*(x) \mu(x) dx + \mathcal{A}^* \left( \int_{\mathcal{X}} \mu(x) dx - 1 \right)$$

Hence, the Kuhn and Tucker first order necessary conditions are

$$\begin{cases} \mathcal{A}(x) = \mathcal{A}^* - \psi^*(x) \\ \int_{\mathcal{X}} \mu^*(x) dx = 1 \\ \psi^*(x) \geq 0 \\ \psi^*(x) \mu^*(x) = 0 \end{cases}$$

Using Sandholm (2001, Proposition 3.1), a spatial equilibrium is a local maximizer of  $\mathcal{S}$ .

**Proof 2** Assume that the labor market is in equilibrium.

• **Hyperbolic preferences:** assume that  $\mathcal{A}$  is defined as in equation (44). Since  $\mathcal{X}$  is compact and  $\mathcal{A}$  is continuous,  $\mathcal{A}$  is a differential of  $\mathcal{S}$  given by

$$\mathcal{S}[\mu] = \int_{\mathcal{X}} \left\{ (1 - \gamma e^*) z + \gamma(y + \kappa) e^* - [s + (1 - s)\gamma e^*] t \right\} \mu(x) - \phi \frac{\mu(x)^2}{2} dx$$

Moreover, as  $\mathcal{A}$  decreases with respect to the residential density  $\mu(x)$ , that is,

$$\frac{\partial \mathcal{A}}{\partial \mu(x)} = -\phi < 0$$

$\mathcal{S}$  is strictly concave.

• **Quasi-linear preferences:** assume that  $\mathcal{A}$  is defined as in equation (52). Since  $\mathcal{X}$  is compact and  $\mathcal{A}$  is continuous,  $\mathcal{A}$  is the differential of a potential functional defined as

$$\mathcal{S}[\mu] = \int_{\mathcal{X}} \{(1 - \gamma e^*)z + \gamma(y + \kappa)e^* - [s + (1 - s)\gamma e^*]t\} \mu(x) - \frac{1}{\mu(x)} \rho\left(\frac{1}{\mu(x)}\right) dx$$

Furthermore, as  $\mathcal{A}$  decreases with respect to the residential density, that is,

$$\frac{\partial \mathcal{A}}{\partial \mu(x)} = \frac{1}{\mu(x)^3} \rho''\left(\frac{1}{\mu(x)}\right) < 0$$

$\mathcal{S}$  is strictly concave.

Thus, for a given labor market equilibrium  $(\theta^*, u^*)$ , there exists a unique spatial equilibrium  $\mu^*$ .  $\diamond$

**Proof 3** Assume that the labor market is in equilibrium.

• **Hyperbolic preferences:** assume that  $\mathcal{A}$  is defined as in equation (44). The Marshallian (uncompensated) demand function is

$$\tilde{\zeta}(x) = \sqrt{\frac{\phi}{2R(x)}} = \zeta^*(x)$$

Plugging this result in equation (26) and using constraints (18) and (29) gives

$$\tilde{\mathcal{A}}(x) = (1 - \gamma e^*)z + \gamma(y + \kappa)e^* - [s + (1 - s)\gamma e^*]tx - R(x) \sqrt{\frac{\phi}{2R(x)}} - \frac{\phi}{2\sqrt{\frac{\phi}{2R(x)}}}$$

Following Zenou (2009, Appendix A),  $\Psi$  is determined solving  $\tilde{\mathcal{A}}(x) = \tilde{\mathcal{A}}^*$ , that is to say,

$$\Psi(x, \tilde{\mathcal{A}}^*) = \frac{2 \left( (1 - \gamma e^*)z + \gamma(y + \kappa)e^* - [s + (1 - s)\gamma e^*]tx - \tilde{\mathcal{A}}^* \right)^2}{4\phi}$$

Integrating the above equation in condition (57) leads

$$\tilde{\mu}(x) = \frac{1}{\tilde{\zeta}(x)} = \frac{1}{\sqrt{\frac{\phi}{2\Psi(x, \tilde{\mathcal{A}}^*)}}} = \frac{(1 - \gamma e^*)z + \gamma(y + \kappa)e^* - \tilde{\mathcal{A}}^* - [s + (1 - s)\gamma e^*]tx}{\phi}$$

Using condition (56), the endogenous city size is

$$\Psi(\tilde{x}, \tilde{\mathcal{A}}^*) = R_A = 0 \Leftrightarrow \tilde{x} = \frac{2 \left( (1 - \gamma e^*)z + \gamma(y + \kappa)e^* - \tilde{\mathcal{A}}^* \right)}{[s + (1 - s)\gamma e^*] t}$$

therefore

$$\int_0^{\tilde{x}} \tilde{\mu}(x) dx = 1 \Leftrightarrow \tilde{\mathcal{A}}^* = (1 - \gamma e^*)z + \gamma(y + \kappa)e^* - \sqrt{2\phi [s + (1 - s)\gamma e^*] t} = \mathcal{A}^*$$

$\tilde{\mu}(x) = \mu^*(x)$  and  $\tilde{x} = \tilde{x}$ . Thus, a spatial equilibrium  $\mu^*$  is a urban land use equilibrium.

• **Quasi-linear preferences:** assume that  $\mathcal{A}$  is defined as in equation (52). The Marshallian (uncompensated) demand function is

$$\tilde{\zeta}(x, \tilde{\mathcal{A}}) = \rho'^{-1} \left( \frac{1}{R(x)} \right) = \zeta^*(x)$$

with  $\rho'^{-1}$  the inverse function of  $\rho'$ . The bid rent is defined as the maximum rent per unit of land that the household can pay residing at distance  $x$  while enjoying a utility level  $\tilde{\mathcal{A}}^*$

$$\Psi(x, \tilde{\mathcal{A}}^*) = \max_{\zeta(x), \sigma(x)} \left\{ \frac{e[\omega(x) - tx] + u(z - stx) - \sigma(x)}{\zeta(x)} \mid Z(\sigma(x), \zeta(x)) = \tilde{\mathcal{A}}^* \right\}$$

Noting that

$$\sigma(x) = Z(\zeta(x), \tilde{\mathcal{A}}^*)$$

the amount of composite good that is necessary to achieve utility level  $\tilde{\mathcal{A}}^*$  when the lot size of the house is  $\zeta(x)$ . The program can be re-written as

$$\Psi(x, \tilde{\mathcal{A}}^*) = \max_{\zeta(x)} \frac{e[\omega(x) - tx] + u(z - stx) - Z(\zeta(x), \tilde{\mathcal{A}}^*)}{\zeta(x)}$$

The first order condition of the above problem is

$$-\frac{1}{\tilde{\zeta}(x)} \frac{\partial Z(\tilde{\zeta}(x), \tilde{\mathcal{A}}^*)}{\partial \tilde{\zeta}(x)} - \frac{I(x) - Z(\tilde{\zeta}(x), \tilde{\mathcal{A}}^*)}{\tilde{\zeta}(x)^2} = 0$$

that is

$$\Psi(x, \tilde{\mathcal{A}}^*) = -\frac{\partial Z(\tilde{\zeta}(x), \tilde{\mathcal{A}}^*)}{\partial \tilde{\zeta}(x)}$$

Since

$$Z(\zeta(x), \tilde{\mathcal{A}}^*) = \tilde{\mathcal{A}}^* - \rho(\zeta(x))$$

I find that

$$\Psi(x, \tilde{\mathcal{A}}^*) = \rho' \left( \frac{1}{\mu(x)} \right) = R^*(x)$$

This means that

$$\tilde{\mathcal{A}}(x) = (1-\gamma e)z + \gamma(y+\kappa)e - [s + (1-s)\gamma e]tx - \frac{1}{\mu(x)}\rho' \left( \frac{1}{\mu(x)} \right) + \rho \left( \frac{1}{\mu(x)} \right) = \mathcal{A}(x)$$

and this implies that  $\tilde{\mu}(x) = \mu^*(x)$ ,  $\tilde{x} = \check{x}$  and  $\tilde{\mathcal{A}} = \mathcal{A}^*$ . Thus, a spatial equilibrium  $\mu^*$  is a urban land use equilibrium.  $\diamond$

**Proof 4** Assume that the labor market is in equilibrium.

• **Hyperbolic preferences:** assume that  $\mathcal{A}$  is defined as in equation (44). Solving equation (58) consists in finding a degree of urban sprawl denoted by  $\mu^*(x)$  such that

$$\begin{cases} \mathcal{A}(x) = \mathcal{A}^* & \forall x \in \mathcal{X} \\ \mu^*(x) \geq 0 & \forall x \in \mathcal{X} \\ \int_{\text{Supp}(\mu^*)} \mu^*(x) dx = 1 \end{cases}$$

Solving the first equation the above system using equation (44), I obtain

$$\mu^*(x) = \frac{(1-\gamma e^*)z + \gamma(y+\kappa)e^* - \mathcal{A}^* - [s + (1-s)\gamma e^*]tx}{\phi}$$

Observe that this distribution necessarily decreases with distance.

▷ If  $\check{x} \geq D$ , then I assume that  $\check{x} = D$ . Integrating the distribution  $\mu^*$  on its endogenous support  $\text{Supp}(\mu^*) = [0, D] = \mathcal{X}$  and using the third equation of the above system yields

$$\int_{\mathcal{X}} \mu^*(x) dx = 1 \Leftrightarrow \frac{\{(1-\gamma e^*)z + \gamma(y+\kappa)e^* - \mathcal{A}^*\} D}{\phi} - \frac{[s + (1-s)\gamma e^*] t D^2}{2\phi} = 1$$

or

$$\mathcal{A}^* = (1-\gamma e^*)z + \gamma(y+\kappa)e^* - \frac{\phi}{D} - \frac{[s + (1-s)\gamma e^*] t D}{2}$$

Therefore, the endogenous degree of urban sprawl is

$$\mu^*(x) = \frac{1}{D} + \frac{[s + (1-s)\gamma e^*] t D}{2\phi} - \frac{[s + (1-s)\gamma e^*] t}{\phi} x$$

with  $\frac{1}{D} + \frac{[s+(1-s)\gamma e^*]tD}{2\phi}$  is the endogenous density of workers living in the city center.

▷ If  $\tilde{x} < D$ , the endogenous city size is determined as

$$R(\tilde{x}) = R_A \Leftrightarrow \mu^*(\tilde{x}) = 0 \Leftrightarrow \tilde{x} = \frac{(1 - \gamma e^*)z + \gamma(y + \kappa)e^* - \mathcal{A}^*}{[s + (1 - s)\gamma e^*]t}$$

Integrating the distribution  $\mu^*$  on its endogenous support  $\text{Supp}(\mu^*) = [0, \tilde{x}]$  and using the third equation of the above system yields

$$\int_0^{\tilde{x}} \mu^*(x) dx = 1 \Leftrightarrow \frac{\{(1 - \gamma e^*)z + \gamma(y + \kappa)e^* - \mathcal{A}^*\}^2}{2\phi[s + (1 - s)\gamma e^*]t} = 1$$

or

$$\mathcal{A}^* = (1 - \gamma e^*)z + \gamma(y + \kappa)e^* - \sqrt{2\phi[s + (1 - s)\gamma e^*]t}$$

Therefore, the endogenous city size becomes

$$\tilde{x} = \sqrt{\frac{2\phi}{[s + (1 - s)\gamma e^*]t}}$$

and the endogenous degree of urban sprawl is

$$\mu^*(x) = \sqrt{\frac{2[s + (1 - s)\gamma e^*]t}{\phi}} - \frac{[s + (1 - s)\gamma e^*]t}{\phi}x$$

with  $\sqrt{\frac{2[s + (1 - s)\gamma e^*]t}{\phi}}$  is the endogenous density of workers living in the city center.

• **Quasi-linear preferences:** assume that  $R_A > 0$ ,  $N \neq 1$  and  $\mathcal{A}$  is defined as in equation (52). Let re-write the latter as

$$\mathcal{A}(x) = (1 - \gamma e)z + \gamma(y + \kappa)e - [s + (1 - s)\gamma e]tx - \Phi(\mu(x))$$

with

$$\Phi(\mu(x)) = \frac{1}{\mu(x)}\rho' \left( \frac{1}{\mu(x)} \right) - \rho \left( \frac{1}{\mu(x)} \right)$$

Solving the first equation the above system, I have

$$\mu^*(x) = \Phi^{-1}((1 - \gamma e^*)z + \gamma(y + \kappa)e^* - \mathcal{A}^* - [s + (1 - s)\gamma e^*]tx)$$

with  $\Phi^{-1}$  the inverse function of  $\Phi$ . Notice that this distribution necessarily decreases with distance because  $\frac{\partial \Phi(\mu(x))}{\partial \mu(x)} < 0$  (see equation (54)).

▷ If  $\tilde{x} \geq D$ , then I assume that  $\tilde{x} = D$ . Integrating the distribution  $\mu^*$  on its endogenous support  $\text{Supp}(\mu^*) = [0, D] = \mathcal{X}$  gives

$$\int_{\mathcal{X}} \mu^*(x) dx = N \Leftrightarrow \int_{\mathcal{X}} \Phi^{-1}((1 - \gamma e^*)z + \gamma(y + \kappa)e^* - \mathcal{A}^* - [s + (1 - s)\gamma e^*]tx) dx = N$$



meaning that  $\mathcal{A}^*$  is determined solving the above equation.

▷ If  $\tilde{x} < D$ , the endogenous city size is determined as

$$R(\tilde{x}) = R_A \Leftrightarrow \rho'^{-1} \left( \frac{1}{\mu^*(\tilde{x})} \right) = R_A \Leftrightarrow \tilde{x} = \frac{(1 - \gamma e^*)z + \gamma(y + \kappa)e^* - \mathcal{A}^* - \Phi \left( \rho'^{-1} \left( \frac{1}{R_A} \right) \right)}{[s + (1 - s)\gamma e^*]t}$$

$$\int_0^{\tilde{x}} \mu^*(x) dx = N \Leftrightarrow \int_0^{\tilde{x}} \Phi^{-1} \left( (1 - \gamma e^*)z + \gamma(y + \kappa)e^* - \mathcal{A}^* - [s + (1 - s)\gamma e^*]tx \right) dx = N$$

meaning that  $\mathcal{A}^*$  is determined solving the above equation.

▷ For instance, if the preferences are log, as in Beckmann (1976),

$$\mathcal{Z}_i(\sigma_i(x), \zeta_i(x)) = \sigma_i(x) + \phi \ln(\zeta_i(x))$$

$$\text{then } \forall x \in \left[ 0, \frac{\ln \left( \frac{[s+(1-s)\gamma e^*]tN + e^{\frac{1}{\phi}} \frac{R_A}{\phi}}{\frac{R_A}{\phi}} \right) - \ln \left( \frac{R_A}{\phi} \right)}{[s+(1-s)\gamma e^*]t} \right]$$

$$\mu^*(x) = e^{\ln \left( \frac{[s+(1-s)\gamma e^*]tN + e^{\frac{1}{\phi}} \frac{R_A}{\phi}}{\frac{R_A}{\phi}} \right) - \frac{[s+(1-s)\gamma e^*]t}{\phi} x}$$

If the preferences are square root, as in Zenou (2003),

$$\mathcal{Z}_i(\sigma_i(x), \zeta_i(x)) = \sigma_i(x) + \phi \sqrt{\zeta_i(x)}$$

$$\text{then } \forall x \in \left[ 0, \frac{\left( \frac{\phi}{2} \right)^2 \left( \frac{1}{R_A + [s+(1-s)\gamma e^*]tN} \right) + \left( \frac{\phi}{2} \right) \left( \frac{1}{R_A} \right)}{[s+(1-s)\gamma e^*]t} \right]$$

$$\mu^*(x) = \left( \frac{\phi}{2} \right)^2 \left[ \frac{1}{\left( \frac{\phi}{2} \right)^2 \left( \frac{1}{R_A + [s+(1-s)\gamma e^*]tN} \right) + [s + (1 - s)\gamma e^*]tx} \right]^2$$

◇

The result, under hyperbolic preferences, can be generalized to  $R_A > 0$  and  $N > 1$  such that the endogenous city size becomes

$$\tilde{x} = \sqrt{\frac{2\phi N}{[s + (1 - s)\gamma e^*]t}}$$

and the endogenous degree of urban sprawl is

$$\mu^*(x) = \sqrt{\frac{2N[s + (1 - s)\gamma e^*]t}{\phi}} + \frac{\sqrt{2R_A}}{\phi^{\frac{3}{2}}} - \frac{[s + (1 - s)\gamma e^*]t}{\phi} x$$

with  $\sqrt{\frac{2N[s+(1-s)\gamma e^*]t}{\phi}} + \frac{\sqrt{2R_A}}{\phi^{\frac{3}{2}}}$  is the endogenous density of workers living in the city center.

**Proof 5** Assume that the spatial market is in equilibrium.

- If  $\tilde{x} = D$ , equation (25) and equation (63) yields

$$y - (1 - \gamma)z - \gamma(y + \kappa) - (1 - \gamma)(1 - s) \left\{ \frac{D}{2} - \frac{[s + (1 - s)\gamma \left(1 - \frac{\delta}{\delta + \vartheta\theta^{1 - \alpha_s}}\right)] tD^3}{12\phi} \right\} = \frac{\delta\kappa\theta^\alpha}{\vartheta}$$

Let  $g_1$  be a continuous function defined as  $g_1(\theta) = \frac{\delta\kappa\theta^\alpha}{\vartheta}$  and notice that

$$\begin{cases} g_1(0) = 0 \\ \lim_{\theta \rightarrow +\infty} g_1(\theta) = +\infty \\ \frac{\partial g_1(\theta)}{\partial \theta} > 0 \\ \frac{\partial g_1(\theta)}{\partial \theta \partial \theta} < 0 \end{cases}$$

Let  $g_2$  be a continuous function defined as  $g_2(\theta) = A_2 - B_2 \left\{ \frac{D}{2} - \frac{[s + (1 - s)\gamma \left(1 - \frac{\delta}{\delta + \vartheta\theta^{1 - \alpha_s}}\right)] tD^3}{12\phi} \right\}$  with  $A_2 = y - (1 - \gamma)z - \gamma(y + \kappa)$  and  $B_2 = (1 - \gamma)(1 - s)$  and notice that

$$\begin{cases} g_2(0) = A_2 - B_2 \left( \frac{D}{2} - \frac{stD^3}{12\phi} \right) \\ \lim_{\theta \rightarrow +\infty} g_2(\theta) = A_2 - B_2 \left\{ \frac{D}{2} - \frac{[s + (1 - s)\gamma] tD^3}{12\phi} \right\} > g_2(0) \\ \frac{\partial g_2(\theta)}{\partial \theta} > 0 \\ \frac{\partial g_2(\theta)}{\partial \theta \partial \theta} < 0 \end{cases}$$

The characteristics of functions  $g_1$  and  $g_2$  ensure that there exists a unique labor market tightness index denoted by  $\theta^*$  if and only if  $g_2(0) > 0$ , that is, if and only if

$$y - (1 - \gamma)z - \gamma(y + \kappa) > (1 - \gamma)(1 - s) \left( \frac{D}{2} - \frac{stD^3}{12\phi} \right)$$

Using equation (25), if there exists a unique labor market tightness index, this implies the existence of a unique unemployment rate denoted by  $u^*$ .

- If  $\tilde{x} < D$ , plugging equation (25) in equation (63) gives

$$y - (1 - \gamma)z - \gamma(y + \kappa) - \frac{(1 - \gamma)(1 - s)}{3} \sqrt{\frac{2\phi t}{s + (1 - s)\gamma \left(1 - \frac{\delta}{\delta + \vartheta\theta^{1 - \alpha_s}}\right)}} = \frac{\delta\kappa\theta^\alpha}{\vartheta}$$

Let  $g_3$  be a continuous function defined as  $g_3(\theta) = A_3 - B_3 \sqrt{\frac{2\phi t}{s + (1-s)\gamma \left(1 - \frac{\delta}{\delta + \theta \theta^{1-\alpha_s}}\right)}}$  with  $A_3 = y - (1 - \gamma)z - \gamma(y + \kappa)$  and  $B_3 = \frac{(1-\gamma)(1-s)}{3}$  and notice that

$$\begin{cases} g_3(0) = A_3 - B_3 \sqrt{\frac{2\phi t}{s}} \\ \lim_{\theta \rightarrow +\infty} g_3(\theta) = A_3 - B_3 \sqrt{\frac{2\phi t}{s + \gamma(1-s)}} > g_3(0) \\ \frac{\partial g_3(\theta)}{\partial \theta} > 0 \\ \frac{\partial g_3(\theta)}{\partial \theta \partial \theta} < 0 \end{cases}$$

The characteristics of functions  $g_1$  and  $g_3$  ensure that there exists a unique labor market tightness index denoted by  $\theta^*$  if and only if  $g_3(0) > 0$ , that is, if and only if

$$y - (1 - \gamma)z - \gamma(y + \kappa) > \frac{(1 - \gamma)(1 - s)}{3} \sqrt{\frac{2\phi t}{s}}$$

Using equation (25), if there exists a unique labor market tightness index, this implies the existence of a unique unemployment rate denoted by  $u^*$ .

Thus, for a given land market equilibrium  $\mu^*$ , there exists a unique labor market equilibrium  $(\theta^*, u^*)$  if and only if

$$y - (1 - \gamma)z - \gamma(y + \kappa) > \frac{(1 - \gamma)(1 - s)}{3} \sqrt{\frac{2\phi t}{s}}$$

◇

**Proof 6** Using Proof 3 and Proof 4, there exists a unique market equilibrium  $(\mu^*, \theta^*, u^*)$  if and only if

$$y - (1 - \gamma)z - \gamma(y + \kappa) > \frac{(1 - \gamma)(1 - s)}{3} \sqrt{\frac{2\phi t}{s}} > (1 - \gamma)(1 - s) \left( \frac{D}{2} - \frac{stD^3}{12\phi} \right)$$

◇

**Proof 7** From Proof 1 and equation (67), it is simple to conclude that the degree of urban sprawl in a decentralized economy is optimal if and only if

$$\Omega[\mu] = \mathcal{S}[\mu]$$

Nevertheless, Proof 2 clearly states that, under hyperbolic preferences and more generally under quasi-linear preferences, the average expected income (i.e.  $(1 - \gamma e^*)z + \gamma(y + \kappa)e^*$ ) does not matter in the explicitely formulation of the

residential density  $\mu^*(x)$ . As a consequence,

- **Hyperbolic preferences:** the degree of urban sprawl in a decentralized economy is optimal if and only if

$$\Omega[\mu] = \int_x \{\xi - [s + (1-s)\gamma e^*]tx\} \mu(x) - \frac{\phi}{2} \mu(x)^2 dx$$

with  $\xi$  a constant.

- **Quasi-linear preferences:** the degree of urban sprawl in a decentralized economy is optimal if and only if

$$\Omega[\mu] = \int_x \left\{ \tilde{\xi} - [s + (1-s)\gamma e^*]tx \right\} \mu(x) - \frac{1}{\mu(x)} \rho' \left( \frac{1}{\mu(x)} \right) + \rho \left( \frac{1}{\mu(x)} \right) dx$$

with  $\tilde{\xi}$  a constant.  $\diamond$

## B New spatial Hosios-Pissarides condition

Hosios (1990) and Pissarides (2000) show that the standard search and matching model is inefficient due to the presence of search externalities. However, if the bargaining power of workers is equal to the elasticity of the matching function, the equilibrium is restored. This result is known as the Hosios-Pissarides condition. Moreover, the literature of ULMs without high relocation costs demonstrates that introducing space into a search and matching model does not add inefficiency because the Hosios-Pissarides condition remains the same. Nevertheless, when high relocation costs exist, this result is no longer true. Especially, Zenou (2009) finds the following new spatial Hosios-Pissarides condition

$$\gamma = \frac{[\delta + \theta q(\theta)] \eta(\theta)}{\delta + q(\theta) [1 - \eta(\theta)(1 - \theta)]}$$

The natural question is to know if the introduction of endogenous housing consumption creates additional market failures. In order to do so, I consider a new social problem where the social planner chooses an unemployment rate  $u$  and a labor market tightness  $\theta$  for a given distribution of workers  $\mu^*$  in order to maximize the global welfare of the economy defined as the total production of the city. In other words, the planner solves the following program

$$\Gamma = \int_{\text{Supp}(\mu^*)} (y - tx) e \mu^*(x) dx + \int_{\text{Supp}(\mu^*)} (z - stx) u \mu^*(x) dx - \kappa \theta u$$

under the following constraint

$$u = \frac{\delta}{\delta + f(\theta)}$$

By integration, I obtain

$$\Gamma = ye + zu - (e + su)t\bar{x} - \kappa\theta u$$

Using the constraint of the problem, I have the results

$$\frac{\partial ye}{\partial \theta} = \frac{q(\theta) + \theta q'(\theta)}{[\delta + \theta q(\theta)]^2} y = \frac{1 - \eta(\theta)}{[\delta + \theta q(\theta)]^2} y$$

$$\frac{\partial zu}{\partial \theta} = -\frac{1 - \eta(\theta)}{[\delta + \theta q(\theta)]^2} z$$

$$\frac{\partial (e + su)t}{\partial \theta} = \frac{1 - \eta(\theta)}{[\delta + \theta q(\theta)]^2} (1 - s)t$$

and

$$\frac{\partial \kappa\theta u}{\partial \theta} = \frac{\kappa}{[\delta + \theta q(\theta)]^2} \left[ \frac{\delta}{q(\theta)} + \theta\eta(\theta) \right]$$

Assume  $\tilde{x} < D$ . Then, I find

$$\frac{\partial \bar{x}}{\partial \theta} = \frac{(1 - s)\gamma t\bar{x} [1 - \eta(\theta)]}{2[s + (1 - s)\gamma e] t [\delta + \theta q(\theta)]^2}$$

Integrating these computations in the welfare equation  $\Gamma$  gives

$$y - z = \left[ \frac{\delta}{q(\theta)} + \theta\eta(\theta) \right] \frac{\kappa}{1 - \eta(\theta)} + (1 - s)t\bar{x} - \frac{(e + su)\gamma(1 - s)t}{2[s + (1 - s)\gamma e]} \bar{x}$$

If  $D = \tilde{x}$ , I obtain

$$\frac{\partial \bar{x}}{\partial \theta} = \frac{2D^3 [1 - \eta(\theta)]}{3[\delta + \theta q(\theta)]^2}$$

Integrating these computations in the welfare equation  $\Gamma$  implies

$$y - z = \left[ \frac{\delta}{q(\theta)} + \theta\eta(\theta) \right] \frac{\kappa}{1 - \eta(\theta)} + (1 - s)t\bar{x} - \frac{2D^3(e + su)t}{3}$$

Rewriting the job creation equation such that

$$y - z = \left[ \frac{\delta}{q(\theta)} + \gamma \right] \frac{\kappa}{1 - \gamma} + (1 - s)t\bar{x}$$

I obtain a new (non trivial) Hosios-Pissarides condition:  $\exists \gamma$  such that

$$\left[ \frac{\delta}{q(\theta)} + \theta\eta(\theta) \right] \frac{\kappa}{1 - \eta(\theta)} - \frac{(e + su)\gamma(1 - s)t}{2[s + (1 - s)\gamma e]} \bar{x} = \left[ \frac{\delta}{q(\theta)} + \gamma \right] \frac{\kappa}{1 - \gamma}$$

if  $\tilde{x} > D$ ,  $\exists \gamma$  such that

$$\left[ \frac{\delta}{q(\theta)} + \theta\eta(\theta) \right] \frac{\kappa}{1 - \eta(\theta)} - \frac{2D^3(e + su)t}{3} = \left[ \frac{\delta}{q(\theta)} + \gamma \right] \frac{\kappa}{1 - \gamma}$$

This indicates that the introduction of endogenous housing consumption leads to additional inefficiency.

## C Table

	Low degree of urban sprawl	High degree of urban sprawl
Low unemployment rate	high $y, \vartheta$ or/and low $z, \delta, \kappa, \alpha, s$	low $t$
High unemployment rate	high $t$	low $y, \vartheta$ or/and high $z, \delta, \kappa, \alpha, s$

Table 1: Urban sprawl and unemployment

## D Figures

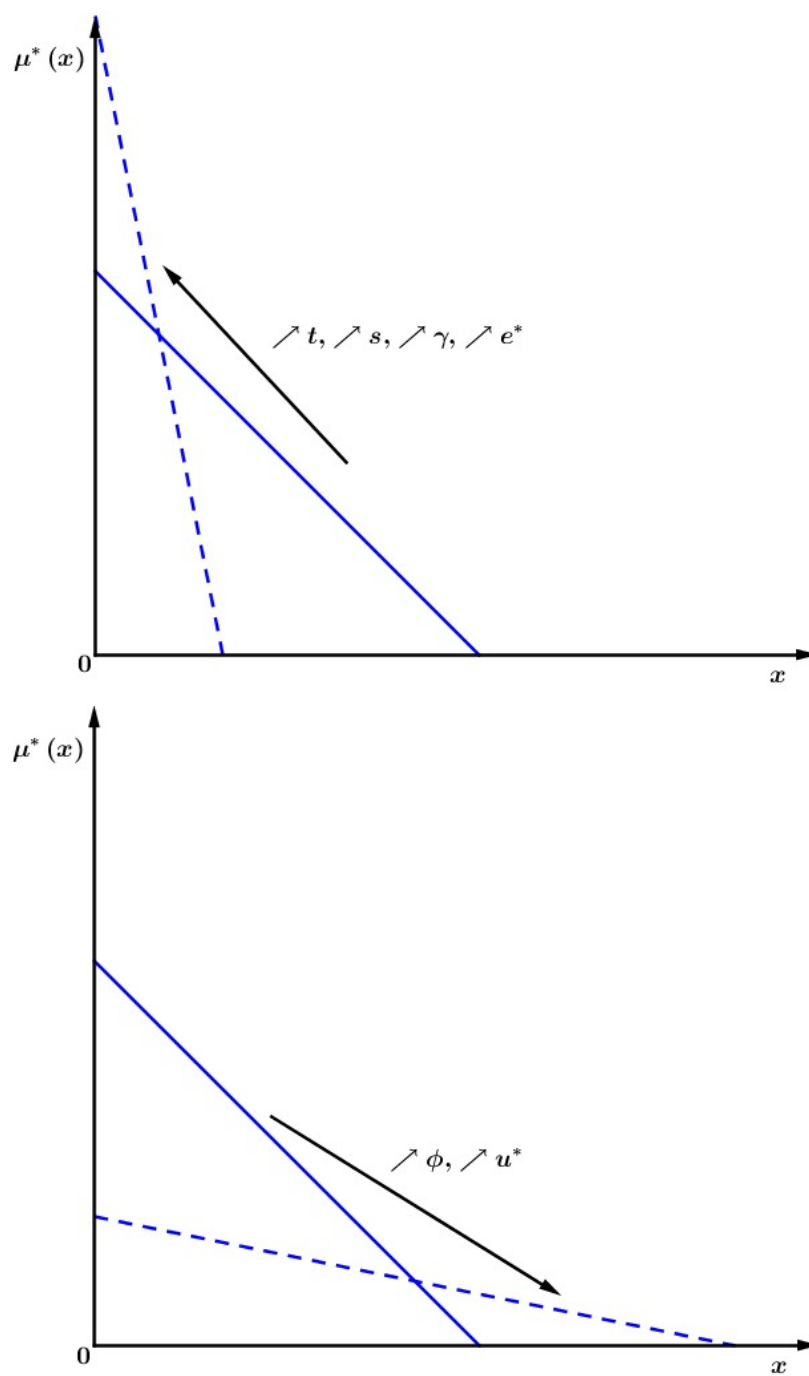


Figure 1: Effects of parameters on urban sprawl (partial equilibrium)

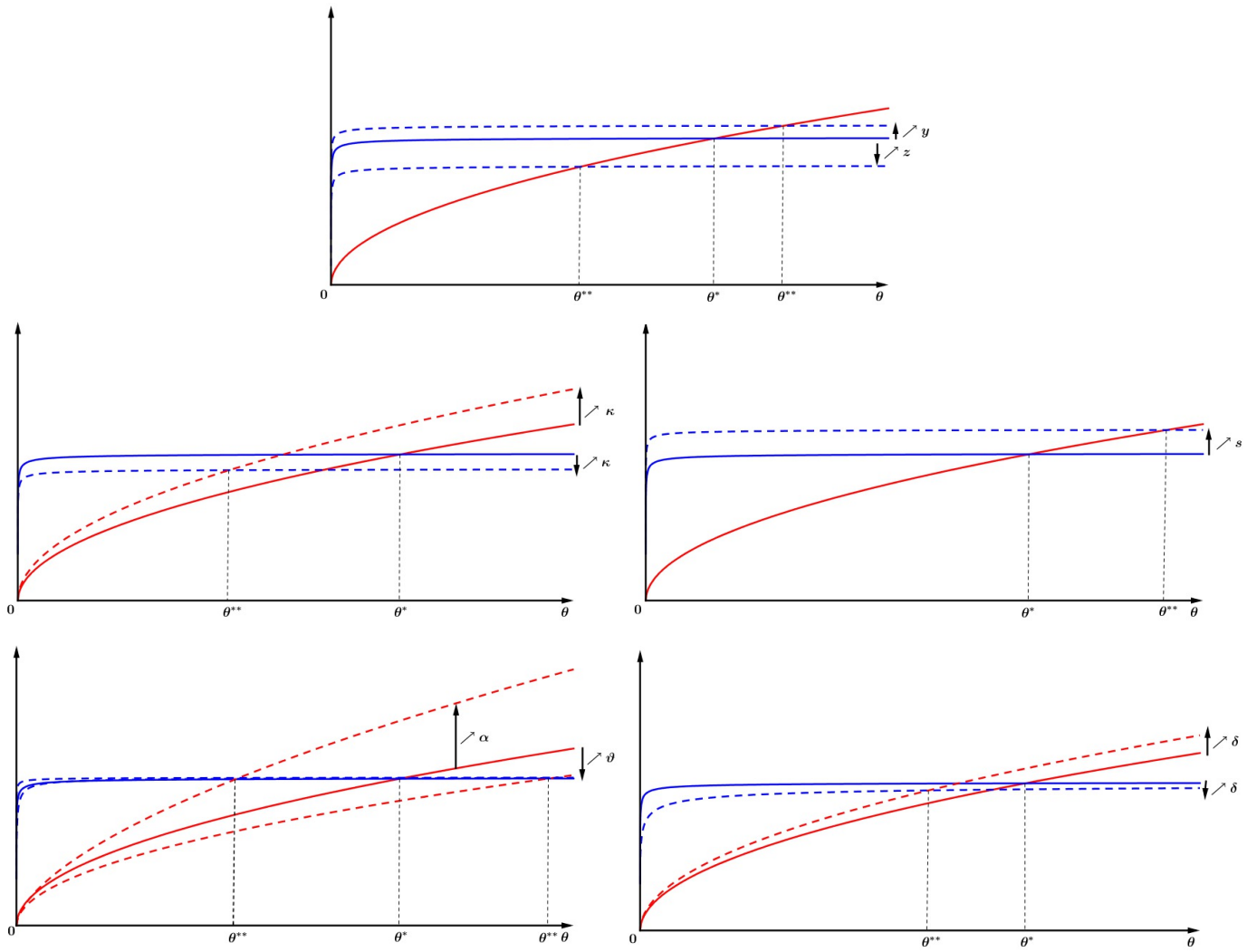


Figure 2: Effects of parameters on labor market tightness



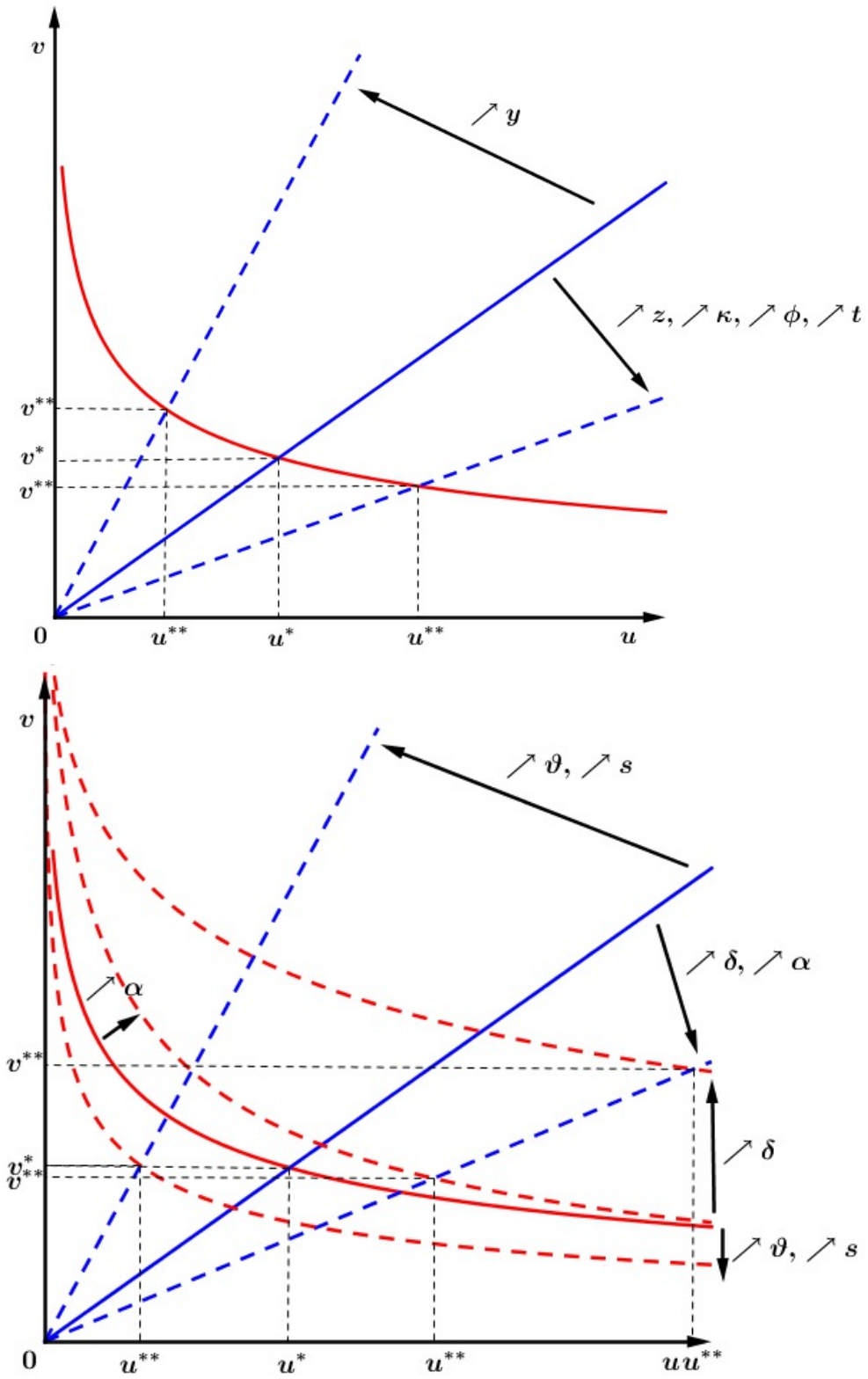


Figure 3: Effects of parameters on unemployment rate

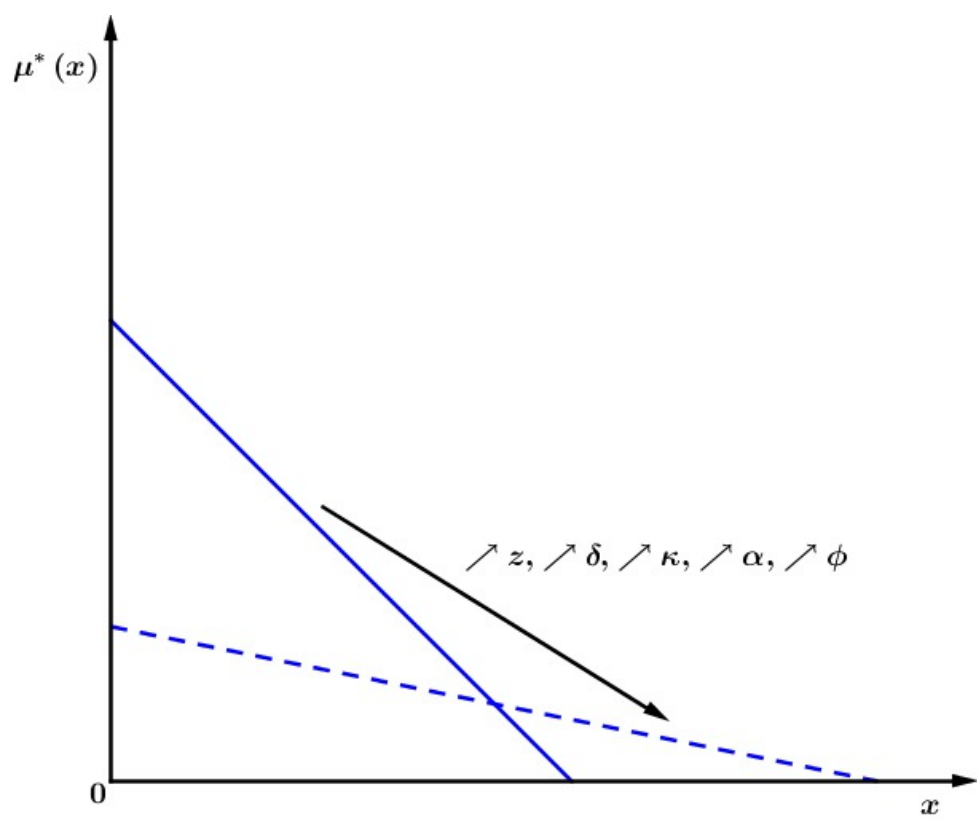
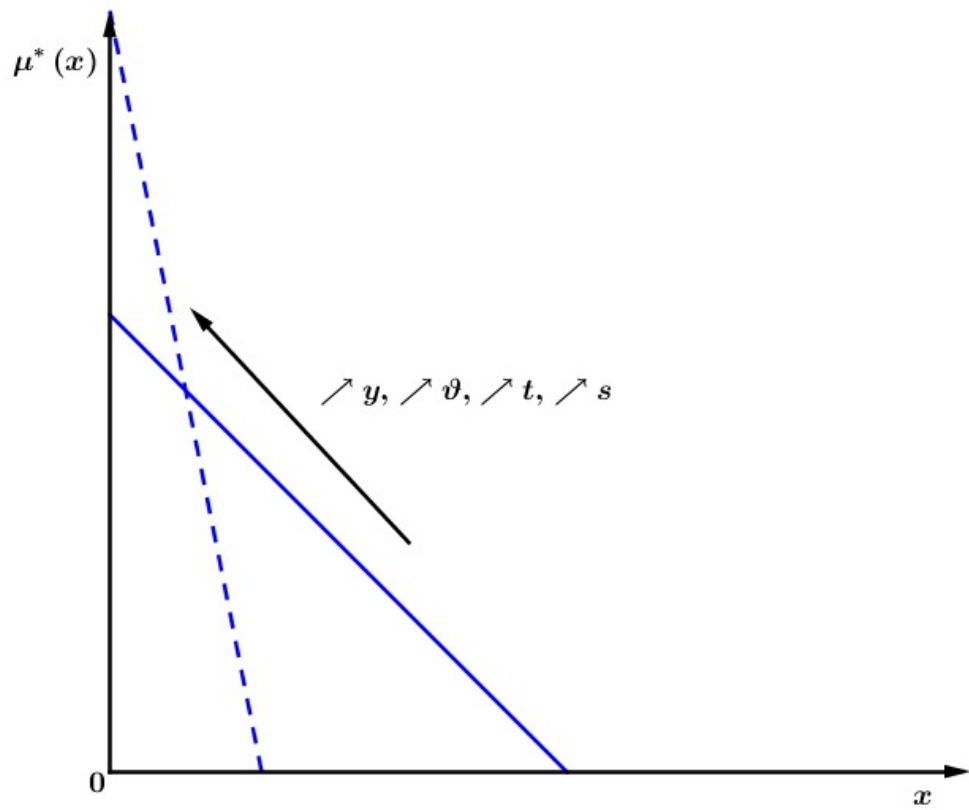


Figure 4: Effects of parameters on urban sprawl (general equilibrium)

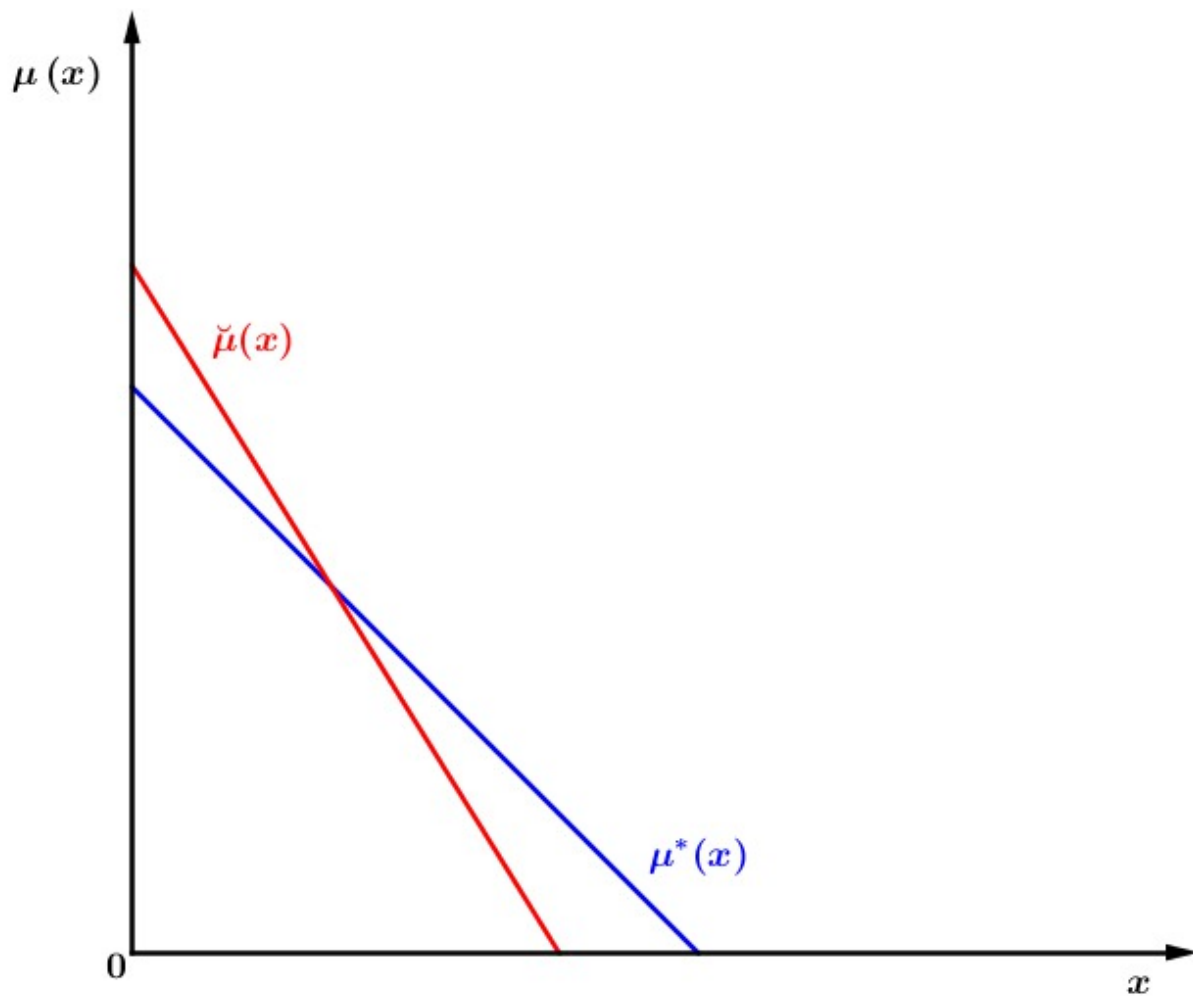


Figure 5: Optimal degree of sprawl and degree of sprawl in the decentralized economy