

Orszag, Jonathan Michael; Snower, Dennis J.

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A macro theory of employment vouchers

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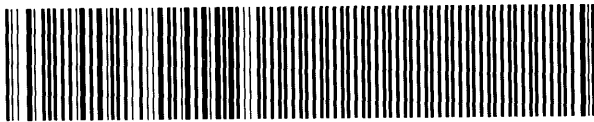
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**A MACRO THEORY OF
EMPLOYMENT VOUCHERS**

J Michael Orszag and Dennis J Snower

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Centre for Economic Policy Research
25–28 Old Burlington Street
London W1X 1LB
Tel: (44 171) 878 2900

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ABSTRACT

A Macro Theory of Employment Vouchers*

This paper explores the optimal design of subsidies for hiring unemployed workers ('employment vouchers' for short) in the context of a simple macroeconomic model of the labour market. Focusing on the short-term and long-term effects of the vouchers on employment and unemployment, the analysis shows how the optimal policy depends on the rates of hiring and firing, and on the problems of displacement and deadweight. It also examines the roles of the government budget constraint and of the level of unemployment benefits in optimal policy design.

JEL Classification: J23, J24, J31, J32, J64

Keywords: employment policy, unemployment benefits, government budget constraint

J Michael Orszag and Dennis J Snower
Department of Economics
Birkbeck College
7-15 Gresse Street
London W1P 1PA
UK
Tel: (44 171) 631 6415/6408

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NON-TECHNICAL SUMMARY

This paper explores the optimal design of subsidies for hiring unemployed workers ('employment vouchers' for short). In so doing, the analysis suggests that it is useful to view these subsidies as an alternative to existing unemployment benefit systems. Specifically, when employment vouchers reduce unemployment, they create revenue for the government, since the government no longer has to pay unemployment benefits to the newly hired workers. This 'voucher revenue' is a source of funds that may be used, partially or wholly, to finance the costs of the subsidies, which we call 'voucher cost'. In short, employment vouchers may be seen as a way of redirecting government funds away from support for the jobless and towards providing market incentives for work.

In recent years, policy-makers have come increasingly to recognize the potential importance of subsidizing the jobs of currently unemployed people. But despite the growing interest in the design of such policies, there has been little dynamic analysis of the optimal policies and their employment effects. This paper provides a simple analytical framework for doing so. It analyses the short- and long-term effects of employment vouchers on unemployment, identifies the major channels whereby this policy works, examines the main obstacles inhibiting the effectiveness of the policy, and investigates the role of the government budget constraint on policy formation.

It is often claimed that the two main factors limiting the effectiveness of employment vouchers are 'deadweight' (vouchers given to people who would have found jobs anyway) and 'displacement' (subsidized employees displacing current employees who are not subsidized). Practical policy evaluations of employment voucher schemes consequently give these two factors special attention. We argue that, although these factors are important, they are far from constituting a comprehensive account of the main obstacles to this policy; thus policy-makers who focus predominantly on them will gain a misleading picture of the underlying problem and will be led to an inappropriate policy response. This paper provides a simple macroeconomic framework of analysis that permits a more balanced assessment of the channels whereby employment vouchers reduce unemployment and the constraints on these channels.

Our analysis concentrates on six major determinants of optimal employment vouchers: (i) *deadweight* (the hiring rate in the absence of the vouchers); (ii) *hiring responsiveness* (the effect of the vouchers on the hiring rate); (iii)

autonomous job loss (the flow from employment into unemployment in the absence of the vouchers); (iv) *displacement* (the effect of the vouchers on the flow from employment into unemployment); (v) *unemployment benefits*; and (vi) the *budgetary allocation* for policy (the government budget deficit or surplus that is to be generated through the policy).

The analysis shows how the trade-off between voucher revenue and voucher cost determines the magnitude of optimal voucher. We indicate that the dynamic repercussions of employment vouchers are likely to be important in practice, since incumbent employees' probabilities of being retained usually exceed the unemployed workers' probabilities of being hired. Consequently, when the vouchers stimulate hiring, they improve people's longer-term job prospects. We capture the dynamic effects arising from the difference between the retention and hiring probabilities through a model of the labour market in which workers' transitions between employment and unemployment are governed by a Markov process. In this context, the paper focuses on a simple, useful policy problem, namely, to find the magnitude of employment vouchers that minimize the level of unemployment, subject to a government budget constraint.

Our analysis shows how the government budget constraint makes the optimal employment vouchers depend positively on the existing unemployment benefits and how this relation is conditioned by the problems of deadweight and displacement. The analysis relates the optimal employment policies to the economy's underlying hiring and firing activities and it specifies the conditions under which these policies can be self-financing.

1. INTRODUCTION

Over the past two decades, subsidies for hiring unemployed workers have become an increasingly favored tool for dealing with unemployment. The subsidies may be granted to employers or employees and they may be implemented through a wide variety of policy instruments, such as tax breaks, grants, and so on. Since these policies all have analogous effects on labor market activities and government budgetary outlays, this paper groups them together under the broad heading of “employment vouchers”.

Employment vouchers have some well-known advantages in comparison with other policy instruments to tackle high unemployment. First, the vouchers are an appropriate way of dealing with a wide variety of market failures that lead to excessive real wages and thereby depress labor demand. When the cost of labor is inefficiently high, employment vouchers are a straightforward instrument to reduce labor costs, regardless of whether the excessive costs are due to, say, efficiency wage, insider-outsider, or labor union considerations. Second, employment vouchers tend to be less costly than subsidies that are awarded to all employees, since the former are more narrowly targetted at the group requiring jobs. Third, employment vouchers are a more flexible tool for tackling unemployment than discretionary subsidies to groups of workers with particular characteristics. For example, if unskilled service sector workers have the highest unemployment rate initially and subsequently unemployment rises among semiskilled manufacturing labor, then the targetting of the employment vouchers will automatically shift from the first group to the second.

This paper explores the optimal design of employment vouchers in the context of a simple macroeconomic model of the labor market. We aim to analyze the short- and long-term effects of this policy on employment and unemployment, identify the major channels whereby this

policy works, examine the main obstacles inhibiting the effectiveness of the policy, and investigate the role of the government budget constraint on policy formation.

It is often claimed that the two main factors limiting the effectiveness of employment vouchers are *deadweight* (vouchers given to people who would have found jobs anyway) and *displacement*¹ (subsidized employees displacing current employees who are not subsidized). Practical policy evaluations of employment voucher schemes consequently give these two factors special attention. We will argue that although these factors are important, they are far from constituting a comprehensive account of the main obstacles to this policy; thus policy makers who focus predominantly on them will gain a misleading picture of the underlying problem and will be led in inappropriate policy responses. This paper provides a simple macroeconomic framework of analysis that permits a more balanced assessment of the channels whereby employment vouchers reduce unemployment.

Our analysis concentrates on six major determinants of optimal employment vouchers: (1) *deadweight* (represented by the hiring rate in the absence of vouchers), (2) *hiring responsiveness* (the effect of vouchers on the hiring rate), (3) *autonomous job loss* (depicted by the flow from employment into unemployment in the absence of the vouchers), (4) *displacement* (represented by the effect of the vouchers on the flow from employment to unemployment), (5) *unemployment benefits*, and (6) the *budgetary allocation* for the voucher policy (the government budget deficit or surplus that is to be generated through the policy).

Surprisingly enough, the existing theoretical literature on the macroeconomic effects of subsidizing employment has paid scant attention to

¹We define displacement broadly to cover not only the replacement of incumbent employees by subsidized recruits *within a particular firm*, but also *inter-firm displacement* that arises when vouchers promote employment at labor-intensive firms at the expense of dismissals in capital-intensive firms.

the inter-related roles of these factors in employment policy formulation. Even deadweight and displacement, on which so much practical policy attention has been lavished, have received little formal analysis.

Furthermore, the literature has focused largely on the effect of subsidizing employment under static conditions.² However, the dynamic repercussions of these subsidies are likely to be important in practice. The main reason is that incumbent employees' probabilities of being retained usually exceed the unemployed workers' probabilities of being hired. Consequently, when employment vouchers stimulate hiring, they improve people's longer-run job prospects and these long-term effects could dwarf the short-term ones.³

The existing macro literature on subsidizing employment has also tended to ignore the full effects of these subsidies on the government's budget. It is standard to assume that the aggregate amount the government spends on the subsidies must be equal to its aggregate tax receipts, e.g. receipts from payroll taxes.⁴ This approach is seriously incomplete, for a major cost of unemployment to the government comes from unemployment benefits and other associated welfare state entitlements, and when the subsidies reduce unemployment, the resulting reduction in the government's unemployment benefit payments must be included in the government's budget constraint as well.

This paper covers these important neglected issues and, in so doing, provides analytical guidelines for the design of employment policy. The dynamic effects arising from the difference between retention and hiring probabilities can be captured straightforwardly through a model of the labor market in which workers' transitions between employment and unemployment are governed by a Markov process. We concentrate

²See, for example, (Layard et al. [1991], pp. 490-2) and Snower [1994].

³Of course, employment vouchers may also stimulate firing, due to displacement, and this influence will limit the improvement in people's long-term job prospects.

⁴See, for example, Layard et al. [1991], p. 490.

primarily on steady states of the labour market.⁵ In this context, the paper focuses on a simple, useful policy problem, namely, to find the magnitude of employment vouchers that minimize the level of unemployment, subject to a government budget constraint. We begin by concentrating on self-financing employment vouchers, i.e. ones whose cost to the government does not exceed the corresponding amount saved on unemployment benefits. We then examine how the optimal policy is affected by a change in the government budget constraint, viz, a switch from a self-financing policy to vouchers on which the government does not spend more than a fixed amount, which could be positive (implying budget deficit from the voucher policy) or negative (implying a surplus).

A central assumption of our analysis is that the employment vouchers stimulate hiring, even in the long run when the full influence of the vouchers on wages has worked itself out. Some have argued that in the long-run that any wage subsidy leads to an equal increase in the wage that employees receive, and consequently wage subsidies have no effect on long-run labor costs or long-run employment.⁶ The argument is that if the wage is the outcome of a Nash bargain and if the subsidy falls in equal proportions on the employees' take-home pay and on their fall-back position, then the subsidy can be factored out of the Nash maximand, leaving the wage paid by the firm unchanged. This argument, however, is unlikely to hold in practice, particularly for subsidies to previously unemployed workers. First, these subsidies may be expected to induce people to move from inactivity to active job search,

⁵For practical policy purposes, this limitation is not as serious as it may appear at first sight, since in general it is politically and institutionally infeasible to devise detailed rules whereby employment vouchers vary through time in response to changing labor market conditions.

⁶Layard et al. [1991] (p. 108) uses a version of this argument, applied to taxes on labor.

thus raising the supply of labor, reducing wages, and raising employment. Second, the fall-back position of previously unemployed people depends on unemployment benefits, minimum wages and welfare state entitlements, and the latter need not necessarily rise in proportion to the subsidies. Third, in the transition to the long run, the subsidies may be expected to raise the number of employees relative to the number of unemployed people, and if (as is generally the case) the retention rate of employees tends to exceed the hiring rate for the unemployed — at any given real wage — the subsidies will then raise the long-run employment rate. Finally, the subsidies will generally raise the recruits' take-home pay relative to their non-wage income and thereby induce them to work harder, shirk less, and be less motivated to quit, thereby reducing the profit-maximizing efficiency wage. For these various reasons, we will assume here that there is a positive long-run equilibrium relation between the hiring rate and the magnitude of the employment voucher.

The paper is organized as follows. Section 2 presents our Markov model of the labor market and describes the government's budget constraint. To focus attention on the most important dynamic implications of employment vouchers, we consider some simple functional relations between the Markov transition probabilities and the vouchers and we then derive the optimal long-run, self-financing vouchers. Section 3 does so in the case in which the hiring probability depends linearly on the subsidy and the firing probability is constant, so that there is no displacement. This analysis highlights the role of deadweight in the design of employment vouchers. Section 4 clarifies the importance of dynamic considerations in policy design by comparing the optimal long-run and short-run (myopic) policies. Section 5 solves the policy problem when both the hiring and firing probabilities depend linearly

on the subsidy. This model sheds light on the joint role of displacement and deadweight costs in subsidy design. Then Section 6 derives bounds for the vouchers when the hiring and firing probabilities have more general functional forms. Section 7 moves beyond self-financing employment vouchers by deriving the optimal policy when the government runs a specified policy-induced deficit or surplus. Finally, Section 8 concludes.

2. THE UNDERLYING MODEL

Time is discrete and workers can be in one of two states, employment or unemployment. Let h be the probability that an unemployed worker will be hired, and f be the probability that an employed worker will become unemployed (e.g., be “fired”). The labor force L is assumed constant through time. Let N_t be level of employment in period t and U_t be the level of unemployment in that period. Thus:

$$(1) \quad N_t + U_t = L$$

2.1. The Employment Equation. The change in employment ($\Delta N_t = N_t - N_{t-1}$) is the difference between the number of people hired and the number of people fired: $\Delta N_t = hU_{t-1} - fN_{t-1}$. Conversely, the change in unemployment $\Delta U_t = U_t - U_{t-1}$ is $\Delta U_t = fN_{t-1} - hU_{t-1}$. Thus the evolution of the system may be described by the following system:

$$(2) \quad S_t = TS_{t-1}$$

where S_t is a vector of labor market states:

$$(2.A) \quad S_t = \begin{pmatrix} N_t \\ U_t \end{pmatrix}$$

and T is the Markov matrix of transition probabilities:

$$(2.B) \quad T = \begin{pmatrix} 1-f & h \\ f & 1-h \end{pmatrix}$$

We now turn to the effect of employment vouchers on this system. These vouchers may be granted either to the firms hiring previously unemployed workers or to the previously unemployed workers who find jobs. In either case,⁷ the voucher is assumed to stimulate hiring,⁸

$$(3.A) \quad h = h(v), \quad h'(v) > 0$$

where the voucher v is measured in units of national income. In addition, the voucher may stimulate firing by encouraging the displacement of some existing employees by the subsidized new recruits:⁹

$$(3.B) \quad f = f(v), \quad f'(v) \geq 0.$$

By Eqs. (1) – (3) above, we obtain the following employment equation, showing how the government can affect the long-run unemployment level by varying the voucher v :

$$(4.A) \quad N(v) = \frac{h(v)}{f(v) + h(v)}L,$$

⁷If the vouchers are granted to the firms, they generally have a direct effect on labor costs; if they are granted to the employees, they generally reduce labor costs indirectly by leading to reduced wages paid by the firms.

⁸Models of the microeconomic foundations of the hiring and firing functions lie beyond the scope of this paper. The existing literature contains a number of them, e.g. Phelps [1994]. Eq. (3.A) is to be interpreted as a long-run relationship.

⁹In practice, the degree to which existing employees are displaced is a matter of policy choice, for the policy maker can grant the vouchers under the condition that there is no displacement. The greater the degree to which anti-displacement provisions are monitored and enforced, the less the firing rate will depend on the vouchers and, since these provisions generally raise the cost of recruitment, they also reduce the responsiveness of the hiring rate to the vouchers.

and the corresponding unemployment equation is:

$$(4.B) \quad U(v) = \frac{f(v)}{f(v) + h(v)}L.$$

The voucher will stimulate steady-state (long-run) employment so long as it raises the hiring rate by proportionately more than it raises the firing rate. To see this, differentiate Eq. (4.A),

$$(5) \quad \left. \frac{\partial N}{\partial v} \right|_{LR} = \frac{[h'(v)(f(v) + h(v)) - h(v)(h'(v) + f'(v))]}{(f(v) + h(v))^2}L$$

and observe that, for the numerator of Eq. (5) to be positive,

$$(6) \quad \frac{h'(v)}{h(v)} > \frac{f'(v)}{f(v)}.$$

Thus, as long as the percentage increase in hiring is greater than the percentage increase in firing due to the rise in the voucher, the rise in the voucher will lead to higher employment. In what follows, we will plausibly assume this to be the case.

Observe that the long-run employment effect of the voucher will clearly differ from the short-run effect. The long run effect is given by Eq. (5) above. By contrast, the short-run level of employment is $N_t = h(v)U_{t-1} + (1 - f(v))N_{t-1}$ (by Eqs. 2, so that short-run effect of the voucher (starting from a steady state, using Eq. (4.A) and Eq. (4.B)) is:

$$(7) \quad \left. \frac{\partial N}{\partial v} \right|_{SR} = L \frac{h'(v)f(v) - f'(v)h(v)}{f(v) + h(v)}.$$

If condition (6) holds (so that the proportional increase in hiring exceeds the proportional increase in firing in response to the voucher), then the long-run and short-run employment effects, Eq. (5) and Eq. (7), will both be positive. Furthermore, if the retention rate $(1 - f(v))$ exceeds the hiring rate $h(v)$ — which holds whenever current employees have some degree of job security which currently unemployed people

do not share — the long-run employment effect of the voucher (Eq. (5)) will exceed the short-run effect (Eq. (7)).

2.2. The Government Budget Constraint. As noted, the government's policy problem is to find the magnitude of the voucher that minimizes the level of unemployment in the long run, subject to the government budget constraint. We specify this constraint straightforwardly as follows. Let $U(v)$ be the long-run unemployment level, as a function of the voucher v . Then, since the number of unemployed people hired in each period is $h(v)U(v)$, the total cost of vouchers to the government is $v h(v)U(v)$.

This cost must be set against the "voucher revenue", which is the total amount that the government saves on unemployment benefits due to the voucher-induced rise in the employment level. In particular, let $U(v)$ and $U(0)$ be the long-run unemployment levels in the presence and absence of the voucher v ($v > 0$), respectively. Let the unemployment benefit b be a positive constant, measured in units of national income. Then the amount that the employment vouchers enable the government to save on unemployment benefit disbursements is $b(U(0) - U(v))$.

Finally, let G be the maximum lump-sum cost of the employment policy to the government, measured in units of national income per capita, where G could be positive, zero or negative. Consequently, the government budget constraint (GBC) is:

$$(8) \quad v h(v)U(v) \leq GL + b(U(0) - U(v))$$

i.e. the cost of the employment vouchers $v h(v)U(v)$ must not exceed the maximum cost of the policy to the government GL plus voucher revenue $b(U(0) - U(v))$ from reduced unemployment.¹⁰

¹⁰Another interpretation of the GBC is obtained when it is rewritten as:

$$N(v) \geq \frac{vh(v)L + bN(0) - GL}{b + vh(v)}.$$

Net government spending on the employment vouchers, $vhU + b(U(v) - U(0))$, need not be *not* monotonic in v : At low enough levels of the voucher v (and high enough levels of the unemployment benefit b) a rise in the voucher may actually *reduce* government spending, since the rise in the voucher may reduce employment sufficiently and to generate more voucher revenue $b(U(0) - U(v))$ than voucher cost (vhU). But provided that voucher cost rises faster with the level of the voucher than does voucher revenue, then at higher levels of the voucher (and lower levels of the unemployment benefit) an increase in the voucher will of course raise government spending, and at the policy optimum — when unemployment is minimized — the optimal employment voucher $v = v^*$ is such that the government budget constraint Eq. (8) holds as an equality:

$$(8^*) \quad v^*h(v^*)U(v^*) = b(U(0) - U(v^*)) + GL.$$

2.3. The Optimal Employment Voucher. Observe that the greater is net government spending G , the greater is the maximum voucher that satisfies the government budget constraint, and consequently the greater is the level of employment generated by the policy. To avoid trivial solutions to the government's policy problem, we will assume that G is sufficiently large ($G \geq \bar{G}$) so that it is feasible to increase employment through a positive employment voucher. In other words, G is assumed to be large enough so that there exists a voucher $v > 0$ such that constraint Eq. (10) is satisfied and:

$$(9) \quad \frac{h(v)}{h(v) + f(v)}L = N(v) > N(0).$$

This means that any given voucher *must* generate at least the level of employment $N(v)$ to be fundable within the government budget constraint. Clearly, as the maximum government spending G increases, the level of employment need not be as high to sustain funding.

Then, by the employment equation Eq. (2) and the government budget constraint Eq. (8'), the optimal employment voucher (v^*) is given by:

$$(10) \quad \left[\frac{h(v^*)}{f(v^*) + h(v^*)} \right] L = \frac{v^* h(v^*) L + bN(0) - GL}{b + v^* h(v^*)}.$$

Eq. (10) defines optimal voucher policy implicitly. In the next two sections, we derive the optimal policy explicitly for particular parametric forms of the hiring function $h(v)$ and firing function $f(v)$.

3. OPTIMAL EMPLOYMENT VOUCHERS IN THE ABSENCE OF DISPLACEMENT

We focus on the case of balanced-budget voucher programs¹¹: $G = 0$. We assume that:

- hire rates are given by the linear equation:

$$(11.A) \quad h(v) = \eta_0 + \eta_1 v, \quad \eta_0 > 0 \quad \eta_1 > 0$$

- fire rates are independent of vouchers (i.e., there is no displacement) :

$$(11.B) \quad f(v) = \lambda_0, \quad \lambda_0 > 0$$

where “ η ” stands for “employment” and “ λ ” stands for “job loss”. The coefficient η_0 stands for *deadweight* (the hiring rate in the absence of the voucher), η_1 is *hiring responsiveness* (the effect of the voucher on the hiring rate); and η_0 is *autonomous job loss* (the rate at which employees become unemployed).

Substituting the hiring function Eq. (11.A) and the firing function Eq. (11.B) into the unemployment equation Eq. (4.A), we obtain the following expression for the level of unemployment:

$$(12) \quad U(v) = \frac{\lambda_0}{\lambda_0 + \eta_0 + \eta_1 v} L.$$

¹¹The case of $G > 0$ involves more algebra which can obscure some of the economic insights; thus, we will deal with it separately in Section 7.

Thus, the *voucher cost* may be expressed as:

$$(13) \quad v \cdot h(v)U(v) = v \cdot (\eta_0 + \eta_1 v) \cdot \frac{\lambda_0}{\lambda_0 + \eta_0 + \eta_1 v} L$$

and the *voucher revenue* becomes:

$$(14) \quad b \cdot [U(0) - U(v)] = b \cdot \left[\frac{\lambda_0}{\lambda_0 + \eta_0} - \frac{\lambda_0}{\lambda_0 + \eta_0 + \eta_1 v} \right] L.$$

For a balanced budget policy ($G = 0$), the government budget constraint simply provides that the voucher cost must not exceed voucher revenue (by (13) and (14)):

$$(15) \quad v \cdot (\eta_0 + \eta_1 v) \cdot \frac{\lambda_0}{\lambda_0 + \eta_0 + \eta_1 v} L \leq b \cdot \left[\frac{\lambda_0}{\lambda_0 + \eta_0} - \frac{\lambda_0}{\lambda_0 + \eta_0 + \eta_1 v} \right] L.$$

Expressing these terms as magnitudes per unemployed person (i.e., dividing both sides of Eq. (15) by $\left(\frac{\lambda_0}{\lambda_0 + \eta_0 + \eta_1 v} L\right)$), the government budget constraint becomes:

$$\begin{aligned} v(\eta_0 + \eta_1 v) &\leq b \cdot \left(\frac{\lambda_0 + \eta_0 + \eta_1 v}{\lambda_0 + \eta_0} \right) - b \\ &= \frac{b\eta_1 v}{\lambda_0 + \eta_0} \end{aligned}$$

and expressing these magnitudes in units of the voucher (dividing both sides by v), we obtain:

$$(16) \quad \eta_0 + \eta_1 v \leq \frac{b\eta_1}{\lambda_0 + \eta_0}$$

i.e., the voucher cost (per unemployed person, in voucher units) is not greater than the voucher revenue (measured in the same terms).

The voucher cost may be divided into two components: (i) *deadweight*, η_0 (the cost of providing vouchers for people who would have become employed anyway) and (ii) the voucher cost of induced hiring $\eta_1 v$. By Eq. (16), this means the the voucher cost of induced hiring must not exceed voucher revenue minus deadweight:

$$(17) \quad \eta_1 v \leq \frac{b\eta_1}{\lambda_0 + \eta_0} - \eta_0$$

which, of course, implies that:

$$(18) \quad v \leq \frac{b}{\lambda_0 + \eta_0} - \frac{\eta_0}{\eta_1}$$

Naturally, there are two further constraints on the size of the voucher, namely a non-negativity constraint:

$$(19) \quad v \geq 0$$

and a constraint specifying that the hiring rate cannot exceed unity ($h = \eta_0 + \eta_1 v \leq 1$), so that:

$$(20) \quad v \leq \frac{1 - \eta_0}{\eta_1}$$

By Eq. (1) and Eq. (12), the employment equation becomes:

$$(21) \quad N(v) = \frac{\eta_0 + \eta_1 v}{\lambda_0 + \eta_0 + \eta_1 v} L$$

Differentiating this equation, we find that the voucher stimulates employment:

$$(22) \quad \frac{\partial N}{\partial v} = \frac{\eta_1 \lambda_0}{(\lambda_0 + \eta_0 + \eta_1 v)^2} L > 0$$

Thus, the policy maker's problem of minimizing unemployment through a balanced budget voucher policy reduces the problem of finding the highest voucher that satisfies the constraints Eqs. (16), (19), and (20). Thus, the optimal voucher is:¹²

$$(23) \quad v^* = \min \left[\max \left(\frac{b}{\lambda_0 + \eta_0} - \frac{\eta_0}{\eta_1}, 0 \right), \frac{1 - \eta_0}{\eta_1} \right].$$

If we ignore the corner point solutions for the optimal voucher (i.e., if we ignore constraints (19) and (20)), then Eq. (23) implies that the optimal voucher depends:

- negatively on deadweight,
- positively on the unemployment benefit.
- positively on hiring responsiveness, and

¹²Orszag and Snower [1996] achieve the same result for a much more complex model involving an infinite number of states but constant transition rates.

- negatively on autonomous job loss.

The intuition underlying these results is clarified in Figs. (1).¹³ Here the C curve represents voucher cost (Eq. (13)) and the R curve stands for voucher revenue (Eq. (14)), both as a function of the voucher. The voucher that minimizes unemployment is the maximal voucher for which voucher revenue does not fall short of voucher cost. Thus the optimal voucher lies at the intersection of the C and R curves.

An increase in deadweight shifts the voucher cost curve upwards,¹⁴ since this causes more people to qualify for the voucher. It also shifts the voucher revenue curve downwards, since it reduces the difference between unemployment in the absence and presence of the voucher. Consequently, as shown in Fig. (1a) the optimal voucher falls.

An increase in hiring responsiveness raises voucher revenue (since unemployment in the presence of the voucher rises relative to unemployment in its absence) and raises the voucher cost curve (since more people get the voucher); however, the former effect dominates (see Appendix A for the details) so that the optimal voucher increases as shown in Fig. (1b).

Along the same lines, an increase in autonomous job loss raises the voucher cost curve (since some of the extra people who lose their jobs get the voucher) and also raises the voucher revenue curve (since it

¹³The horizontal axis represents the voucher and the vertical axis represents the cost/revenue. Parameter values used for drawing the figures are: $b = 0.45$, $\eta_0 = 0.5$, $\eta_1 = 1.0$, $\lambda_0 = 0.1$. In Fig. (1a), the deadweight loss parameter increases to 0.55. In Fig. (1b), the hiring responsiveness parameter rises to 1.15. In Fig. (1c), the autonomous job loss parameter rises to 0.20. In Fig. (1d), the unemployment benefit parameter increases to 0.50. Appendix A justifies these effects analytically.

¹⁴In Fig. (1a) this shift is very small. The small size of the shift in the cost curve is explained in Appendix A.

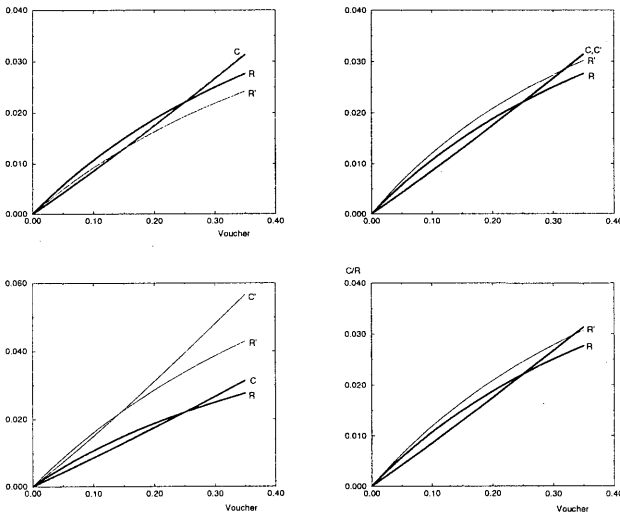


FIGURE 1. Comparative statics. Fig. (1a) (upper left) shows the effect of an increase in deadweight loss. Fig (1b) (upper right) shows the the effect of an increase in voucher effectiveness. Fig. (1c) (lower left) shows the effect of an increase in autonomous job loss. Fig. (1d) (lower right) shows the effect of an increase in unemployment benefits.

increases the difference between unemployment in the absence and presence of the voucher). The former effect dominates so that the optimal voucher decreases as illustrated in Fig. (1c).

Finally an increase in unemployment benefits raises the voucher revenue curve while leaving the voucher cost curve unchanged. Thus, the optimal voucher increases (Fig. (1d)).

4. SHORT-RUN VERSUS LONG-RUN POLICY

We now examine how the optimal policy depends on whether the policy maker has a long-run or a short-run time horizon. Specifically, we analyze how the optimal long-run policy described above differs from the optimal short-run (myopic) policy, in which the policy maker chooses the balanced-budget policy that minimizes unemployment in the current time period. We show how the deadweight, unemployment benefits and the firing rate influence the differential between the optimal long- and short-run vouchers. To make the two vouchers comparable, we evaluate both at the long-run steady state levels of employment and unemployment.

By Eqs. (2), (11.A) and (11.B), the short-run employment level is:

$$\begin{aligned} N_t &= h \cdot U_{t-1} + (1 - f) \cdot N_{t-1} \\ &= (\eta_0 + \eta_1 v) U_{t-1} + (1 - \lambda_0) N_{t-1} \end{aligned}$$

Thus, the effect of the voucher in the first period is $\partial N_t / \partial v = \eta_1 U_{t-1}$ and evaluating this expression at the long-run unemployment level, we obtain:

$$(24) \quad \left. \frac{\partial N}{\partial v} \right|_{SR} = \frac{\eta_1 \lambda_0}{\eta_0 + \eta_1 v + \lambda_0} L$$

From Eq. (22) and Eq. (24), the long-run effect of vouchers exceeds the short-run effect if the retention rate ($1 - f = 1 - \lambda_0$) exceeds the hiring rate ($h = \eta_0 + \eta_1 v$).

We now turn to the difference between the optimal voucher in the long- and short-run. The short-run unemployment level is:

$$U_t(v) = f(v) N_{t-1} + (1 - h(v)) U_{t-1}.$$

Substituting this expression into the government budget constraint Eq. (8), setting $G = 0$, and evaluating the expression at the steady

state where $N_{t-1} = N(0)$ and $U_{t-1} = U(0)$ we obtain the one-period-ahead government budget constraint:

$$(25) \quad vh(v)U(0) \leq bU(0) - bU_t(v).$$

Substituting the hiring function Eq. (11.A) and the firing function Eq. (11.B) into Eq. (25), we find:

$$(26) \quad vh(v) - \eta_1 vb \leq 0$$

The other constraints on the size of the voucher are the non-negativity constraint Eq. (19) and the hiring constraint Eq. (20). Thus, the optimal short-run voucher is:

$$(27) \quad v^0 = \min \left[\max \left(b - \frac{\eta_0}{\eta_1}, 0 \right), \frac{1 - \eta_0}{\eta_1} \right].$$

Let us ignore the corner point solutions Eq. (19) and Eq. (20) and concentrate instead on the interior solution in which the optimal voucher defined by condition (26) holding as equality. Assuming that the retention rate exceeds the hiring rate, observe that the short- and long-run vouchers have the same voucher cost but the long-run voucher yields greater voucher revenue than the short-run voucher. Thus, the long-run voucher exceeds the short-run voucher:

$$v^* - v^0 = b \cdot \frac{(1 - \lambda_0 - \eta_0)}{(\lambda_0 + \eta_0)} > 0.$$

This equation indicates that the voucher differential ($v^* - v^0$) is greater:

- the smaller is the deadweight λ_2 .
- the smaller is the firing rate in the absence of the voucher (λ_1),
and
- the greater is the unemployment benefit b .

5. INTRODUCING DISPLACEMENT

We now consider the influence of displacement on the optimal employment voucher policy. Having assumed thus far that the voucher affects the hiring but not the firing rate, the analysis above has excluded the possibility of displacement. We now relax this assumption and suppose, instead, that both the hiring and firing rate depend positively and linearly on the size of the voucher:

$$(11.A') \quad h(v) = \eta_0 + \eta_1 v$$

$$(11.B') \quad f(v) = \lambda_0 + \lambda_1 v$$

where η_0 , η_1 , λ_0 and λ_1 are positive constants.

The usual definition of policy-induced displacement is simply the number of people who lose their jobs on account of the policy. Our dynamic analysis offers a richer account of displacement than is possible within the standard static framework, since it draws attention to the important fact that when the hiring rate (h) of the unemployed is less than the retention rate ($1 - f$) of the employed, displacement in the short run will be greater than displacement in the long run.

In the short run, displacement may be measured by the policy-induced change in the probability that a currently employed person will be fired:

$$(28) \quad \frac{df(v)}{dv} = \lambda_1 > 0.$$

Over the long run, the corresponding measure is the policy-induced change in the probability that a person will be unemployed in the long run steady state:¹⁵

¹⁵When the transitions between labor market states are described by a Markov process, a person's long-run probability of being unemployed does not depend on initial employment status.

$$\frac{dU(v)}{dv} \frac{1}{L} = \frac{f'h - h'f}{(h + f)^2}$$

Under our assumption that the voucher has greater proportional effect on hiring than on firing (condition (6)), this magnitude is *negative*. To fix ideas, however, we will define displacement according to Eq. (28) in the analysis that follows.

The long-run unemployment level is:

$$(29) \quad U(v) = \frac{\lambda_0 + \lambda_1 v}{\lambda_0 + \lambda_1 v + \eta_0 + \eta_1 v} L$$

by Equations (12), (11.A') and (11.B'). Thus the voucher cost is:

$$(30) \quad v h(v) U(v) = v \cdot (\eta_0 + \eta_1 v) \cdot \frac{\lambda_0 + \lambda_1 v}{\lambda_0 + \lambda_1 v + \eta_0 + \eta_1 v} L.$$

The voucher revenue is:

$$(31) \quad b[U(0) - U(v)] = b \left[\frac{\lambda_0}{\lambda_0 + \eta_0} - \frac{\lambda_0 + \lambda_1 v}{\lambda_0 + \lambda_1 v + \eta_0 + \eta_1 v} \right] L.$$

Thus, expressing both voucher revenue and voucher cost as magnitudes per unemployed person, the government budget constraint, under a balanced budget policy ($G = 0$), becomes:

$$v(\eta_0 + \eta_1 v) \leq b \left[\frac{\lambda_0}{\lambda_0 + \eta_0} - \frac{\lambda_0 + \lambda_1 v}{\lambda_0 + \lambda_1 v + \eta_0 + \eta_1 v} \right] \frac{\lambda_0 + \lambda_1 v + \eta_0 + \eta_1 v}{\lambda_0 + \lambda_1 v}$$

by Eqs. (29) and (8). Assuming $v \geq 0$, this government budget constraint simplifies to the condition that the deadweight loss must not exceed the voucher revenue (in voucher units) per unemployed person:

$$(\eta_0 + \eta_1 v) \leq b \left[\frac{\lambda_0 \eta_1 - \eta_0 \lambda_1}{(\lambda_0 + \eta_0)(\lambda_0 + \lambda_1 v)} \right].$$

The above equation implies that, if the non-negativity constraint on the voucher (Eq. (19)) is redundant and the constraint that the hiring

rate less than 1 (Eq. (20)) is not binding,¹⁶ the optimal voucher is the largest root of the equation:

$$(32) \quad \eta_1 \lambda_1 v^2 + (\eta_1 \lambda_0 + \lambda_1 \eta_0)v + \left[\eta_0 \lambda_0 - b \frac{\lambda_0 \eta_1 - \eta_0 \lambda_1}{\lambda_0 + \eta_0} \right] = 0.$$

The optimal voucher will be positive whenever the third term is negative:

$$(33) \quad -b \left[\frac{\lambda_0 \eta_1 - \eta_0 \lambda_1}{\lambda_0 + \eta_0} \right] + \eta_0 \lambda_0 < 0.$$

For sufficiently large b , the optimal voucher will be positive if:

$$\frac{\eta_1}{\eta_0} > \frac{\lambda_1}{\lambda_0}$$

which is equivalent to Eq. (6). In other words, for a sufficiently large benefit level, a positive voucher will reduce unemployment if the voucher-induced proportional increase in hiring is sufficiently large relative to the associated proportional increase in firing, implying that the voucher revenue is sufficiently large.

Solving Eq. (32) and taking into account the non-negativity constraint (Eq. (19)) and the hiring rate constraint (Eq. (20)), we obtain the optimal voucher explicitly:

$$(34) \quad v^* = \min \left[\max \left[\left(\frac{1}{2\eta_1 \lambda_1} \left[-(\eta_1 \lambda_0 + \lambda_1 \eta_0) + \sqrt{Z} \right] \right), 0 \right], \frac{1 - \eta_0}{\eta_1} \right]$$

where:

$$(34.A) \quad Z = (\eta_1 \lambda_0 + \lambda_1 \eta_0)^2 - 4\eta_1 \lambda_1 \left[\eta_0 \lambda_0 - b \frac{\lambda_0 \eta_1 - \eta_0 \lambda_1}{\lambda_0 + \eta_0} \right].$$

¹⁶The fire rate must also be less than one, which implies $v \leq \frac{1-\lambda_0}{\lambda_1}$. However, we have assumed that the proportional effect of a voucher on hiring is greater than that on firing, which implies: $\frac{\eta_1}{\eta_0} > \frac{\lambda_1}{\lambda_0}$. If the initial hire rate is greater than the initial fire rate (as in every major industrialized country), this condition implies that the hire rate restriction is binding first.

One interesting property of this optimal v^* is that the voucher rises with the square root of benefits rather than benefits. In the Appendix B, we show that the optimal voucher is lowered by:

- An increase in deadweight (η_0).
- A decrease in unemployment benefits (b).
- An increase in displacement (λ_1).

6. NONPARAMETRIC BOUNDS

To shed some light on the properties of optimal employment vouchers in more general contexts where the hire and fire rates are nonlinear functionals of the voucher, this section derives non-parametric bounds for the voucher, applicable for broad classes of the hiring and firing functions.

6.1. An Upper Bound When the Maximum Hiring Rate is Known. We denote the hire function by $h(v)$ and the fire function by $f(v)$. We assume balanced budget policies. The government budget constraint is then:

$$(vh(v) + b) \frac{f(v)}{h(v) + f(v)} \leq b \frac{f(0)}{h(0) + f(0)}$$

or:

$$(vh(v) + b) f(v) \leq b \frac{f(0)}{h(0) + f(0)} (h(v) + f(v))$$

Hence:

$$\left(vf(v) - b \frac{f(0)}{h(0) + f(0)} \right) h(v) \leq bf(v) \left(\frac{f(0)}{h(0) + f(0)} - 1 \right)$$

or:

$$(35) \quad \left(vf(v) - b \frac{f(0)}{h(0) + f(0)} \right) h(v) \leq bf(v) \frac{-h(0)}{h(0) + f(0)}$$

The right hand side is negative and $h(v) > 0$ which implies that the term:

$$\left(vf(v) - b \frac{f(0)}{h(0) + f(0)} \right)$$

is negative. Let $h^* = \sup_v h(v)$ then:

$$\left(vf(v) - b \frac{f(0)}{h(0) + f(0)} \right) \leq b \frac{f(v)}{h^*} \frac{-h(0)}{h(0) + f(0)}$$

Let $f^* = \inf_v f(v) = f(0)$ then:

$$\left(vf(v) - b \frac{f(0)}{h(0) + f(0)} \right) \leq b \frac{f(0)}{h^*} \frac{-h(0)}{h(0) + f(0)}$$

so that:

$$\begin{aligned} vf(v) &\leq b \frac{f(0)}{h(0) + f(0)} + b \frac{h(0)}{h^*} \frac{-f(0)}{h(0) + f(0)} \\ (36) \quad &\leq b \left(1 - \frac{h(0)}{h^*} \right) \frac{f(0)}{h(0) + f(0)} \end{aligned}$$

Since $vf(0) < vf(v)$:

$$v \leq b \left(1 - \frac{h(0)}{h^*} \right) \frac{1}{h(0) + f(0)}.$$

For cases in which voucher effectiveness is known, a better bound is provided below. We proceed to extend the bounds for cases in which fire rates depend on vouchers.

6.2. An Upper Bound by the Mean Value Theorem. By the mean value theorem for some $\tilde{v} \in [0, v]$:

$$f(v) = f(0) + f'(\tilde{v})v$$

Hence in Eq. (36):

$$(f(0) + f'(\tilde{v})v)v \leq b \left(1 - \frac{h(0)}{h^*} \right) \frac{f(0)}{h(0) + f(0)}$$

This implies that:

$$(37) \quad v \leq \frac{1}{2\bar{f}} \left[-f(0) + \sqrt{f(0)^2 + 4\bar{f}b \left(1 - \frac{h(0)}{h^*} \right) \frac{f(0)}{h(0) + f(0)}} \right]$$

where $\tilde{f} = f'(\tilde{v}) > 0$. This reproduces our result in Section 4 that optimal vouchers grow at most with the square root of unemployment benefits.

6.3. An Upper Bound in the Absence of Displacement. We assume that $f(v) = f(0)$. By the mean value theorem again:

$$h(v) = h(0) + h'(\tilde{v})v$$

We assume that $f(v) = f(0)$ so that fire rates are constant and do not depend on vouchers. Recalling Eq. (35):

$$h(v) \left[v - \frac{b}{f(0) + h(0)} \right] \leq b \left[\frac{-h(0)}{h(0) + f(0)} \right]$$

we have:

$$[h(0) + h'(\tilde{v})v]v - \tilde{h}v \frac{b}{f(0) + h(0)} \leq 0$$

where $\tilde{h} = h'(\tilde{v}) > 0$. This leads to the optimal v^* :

$$(38) \quad v^* \leq \max \left(0, -\frac{h(0)}{\tilde{h}} + \frac{b}{f(0) + h(0)} \right)$$

which is the result obtained in Section 3 where \tilde{h} is the constant voucher effectiveness η_1 .

7. POLICIES THAT GENERATE A BUDGET DEFICIT OR SURPLUS

In this section, we extend the analysis to employment voucher policies which generate a specified net deficit or surplus to the government. To keep the analysis simple, we focus on the case where vouchers do not affect firing (e.g., $\lambda_1 = 0$). In this case the government budget constraint is:

$$[v(\eta_0 + \eta_1 v) + b] \leq [\eta_0 + \eta_1 v + \lambda_0] \left[\frac{b}{\eta_0 + \lambda_0} + \frac{G}{\lambda_0} \right].$$

This implies that the optimal v is the largest root of the equation:

$$\eta_1 v^2 + \left(\eta_0 - b \frac{\eta_1}{\eta_0 + \lambda_0} - \frac{G\eta_1}{\lambda_0} \right) - \frac{\eta_0 + \lambda_0}{\lambda_0} G$$

which is:

$$(39) \quad v^* = \frac{1}{2} \left[B + \sqrt{B^2 + 4C} \right]$$

where:

$$(39.A) \quad B = -\frac{\eta_0}{\eta_1} + \frac{b}{\eta_0 + \lambda_0} + \frac{G}{\lambda_0}$$

$$(39.B) \quad C = \frac{\eta_0 + \lambda_0}{\eta_1 \lambda_0} G > 0$$

To interpret this result simply, we note that since $C > 0$, the optimal voucher is bounded below by B :

$$(40) \quad v^* \geq \frac{b}{\lambda_2 + \lambda_0} - \frac{\eta_0}{\eta_1} + \frac{G}{\lambda_0}$$

which agrees with the formula in Eq. (23) when $G = 0$. Since the expected time workers spend employed after receiving a voucher is $\frac{1}{\lambda_0}$, the third term on the right hand side of Eq. (40) may be interpreted as a government spending multiplier. When $\lambda_1 > 0$, we posit that vouchers are a sublinear function of G . In Fig. (2), we show a plot of the optimal voucher as a function of λ_0 and G where the expected behavior appears.¹⁷

8. CONCLUSION

Unemployment benefit systems have become a costly obligation for many governments. Since these systems can provide a substantial safety net against job insecurity without substantial government expenditures only when the unemployment rate is low, it is not surprising that these systems have come under attack in the two decades of high unemployment experienced in many European countries and elsewhere. What has made unemployment benefit systems particularly difficult to defend when unemployment is high is that they discourage job search

¹⁷We set $b = 1.45$, $\eta_0 = 0.85$, $\eta_1 = 3.0$ and $\lambda_1 = 0.4$ and computed the optimal solution numerically by nonlinear programming.

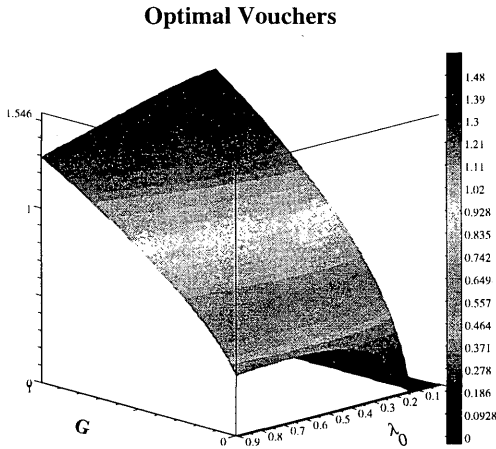


FIGURE 2. Optimal voucher as a function of government spending and λ_0 .

and thereby augment the problem they are meant to address. The analysis of this paper suggests an alternative approach to these systems: instead of seeing unemployment benefit payments merely as support given to people on the condition that they remain jobless, they can be used as a source of funding for employment-creating policies. We have explored how employment vouchers, in reducing unemployment,

create “voucher revenue” for the government (saving in terms of unemployment benefits) and how this revenue can be used to finance the vouchers themselves (wholly or partially). Thereby unemployment benefits become less of a drag on government finances and on labor market performance, and turn into a useful resource instead.

In recent years, policy makers have come increasingly to recognize the potential importance of subsidizing the jobs of currently unemployed people. But despite the growing interest in the design of such policies, there has been little dynamic analysis of the optimal policies and their short- and long-term employment effects. This paper provides a simple analytical framework for doing so.

Our analysis highlights how the optimal design of these employment policies depends on three factors that limit their effectiveness: deadweight (vouchers received by those who would have found jobs anyway), displacement (subsidized new recruits displacing unsubsidized incumbent employees), and a limited government budget. In the context of a simple Markov model, we have shown how the government budget constraint makes the optimal employment vouchers depend positively on the existing unemployment benefits and how this relation is conditioned by deadweight and displacement. Our analysis relates the optimal employment policies to the economy’s underlying hiring and firing activities and it specifies the conditions under which these policies can be self-financing. Thereby the analysis takes a first step towards investigating how the formulation of employment policies should depend on the government’s budget and flows of labor into and out of employment.

APPENDIX A: COMPARATIVE STATICS IN THE ABSENCE OF
DISPLACEMENT

This appendix reviews some comparative statics results for the basic model with displacement (Sec. 4 above) which are shown graphically in Figs. (1)-(4). Assuming Eq. (18) holds as an equality, we have:

Deadweight Changes

The overall effect is:

$$(41) \quad \frac{dv^*}{d\eta_0} = \frac{-b}{(\lambda_0 + \eta_0)^2} - \frac{1}{\eta_1} < 0$$

The effect on costs (using Eq. (13) is:

$$(42) \quad \frac{d \ln C}{d\eta_0} = \frac{1}{\eta_0 + \eta_1 v} - \frac{1}{\lambda_0 + \eta_0 + \eta_1 v} > 0$$

In Fig. (1a) the cost curve did not shift very much with a shift in η_0 . When the autonomous fire rate is small (as is likely to be the case in practice), the effect on costs is not likely to be large as the terms on the right hand side of Eq. (42) will be roughly the same.

The effect on revenues (using Eq. (14) is:

$$(43) \quad \frac{dR}{d\eta_0} = b \left[-\frac{\lambda_0}{(\lambda_0 + \eta_0)^2} + \frac{\lambda_0}{(\lambda_0 + \eta_0 + \eta_1 v)^2} \right] L < 0$$

Voucher Effectiveness

The overall effect is:

$$(44) \quad \frac{dv^*}{d\eta_1} = \frac{\eta_0}{\eta_1^2} > 0.$$

The effect on costs (using Eq. (13) is:

$$(45) \quad \frac{d \ln C}{d \eta_1} = \frac{v}{\eta_0 + \eta_1 v} - \frac{v}{\lambda_0 + \eta_0 + \eta_1 v} > 0.$$

In Fig. (1b) the cost curve did not shift very much with a shift in η_1 . When the autonomous fire rate is small (as is likely to be the case in practice), the effect on costs is not likely to be large as the terms on the right hand side of Eq. (45) will be roughly the same.

The effect on revenues (using Eq. (14) is:

$$(46) \quad \frac{dR}{d\eta_1} = b \cdot \left[\frac{\lambda_0 v}{(\lambda_0 + \eta_0 + \eta_1 v)^2} \right] L > 0.$$

Autonomous Firing

The overall effect is:

$$\frac{dv}{d\lambda_0} = \frac{-b}{(\lambda_0 + \eta_0)^2} < 0$$

The effect on costs (using Eq. (13) is:

$$\frac{d \ln C}{d\lambda_0} = \frac{1}{\lambda_0} - \frac{1}{\lambda_0 + \eta_0 + \eta_1 v} > 0$$

The effect on revenues (using Eq. (14) is somewhat complex. The reason for this is that a change in autonomous firing effects both the unemployment rate with and without vouchers. The revenue expression can be rewritten in terms of employment levels:

$$(47) \quad R = b \cdot [E(v) - E(0)] = b \cdot \left[-\frac{\eta_0}{\lambda_0 + \eta_0} + \frac{\eta_0 + \eta_1 v}{\lambda_0 + \eta_0 + \eta_1 v} \right] L.$$

Differentiating the rhs of Eq. (47):

$$(48) \quad \frac{dR}{d\lambda_0} = bL \left[\frac{\eta_0}{(\lambda_0 + \eta_0)^2} - \frac{\eta_0 + \eta_1 v}{(\lambda_0 + \eta_0 + \eta_1 v)^2} \right],$$

which has the sign of the term in brackets. Consider the function:

$$(49) \quad G(z) = \frac{z}{(a+z)^2}$$

which has derivative:

$$(50) \quad G'(z) = \frac{1}{(a+z)^2} \left[1 - \frac{2z}{a+z} \right]$$

which is positive as long as $\frac{z}{a+z} < \frac{1}{2}$. In our case $z = \eta_0 + \eta_1 v$ and $a = \lambda_0$ so the condition means the employment rate with vouchers is less than 50%. We expect the opposite to occur and therefore the second term in brackets will be smaller and the voucher revenue curve will shift up.

Unemployment Benefits

The overall effect is:

$$\frac{dv}{db} = \frac{1}{\lambda_0 + \eta_0} > 0.$$

Unemployment benefits do not enter Eq. (13) so that a change in b does not effect voucher cost. From Eq, (14):

$$\frac{dR}{db} = U(0) - U(v) > 0.$$

APPENDIX B: COMPARATIVE STATICS IN THE PRESENCE OF
DISPLACEMENT

This appendix reviews some comparative statics results for the basic model with displacement (Sec. 5 above). Consider Eq. (32):¹⁸

$$(32) \quad \eta_1 \lambda_1 v^2 + (\eta_1 \lambda_0 + \lambda_1 \eta_0)v + \left[\eta_0 \lambda_0 - b \frac{\lambda_0 \eta_1 - \eta_0 \lambda_1}{\lambda_0 + \eta_0} \right] = 0.$$

Totally differentiating Eq. (32) with respect to v and η_0 yields:

$$(51) \quad \begin{aligned} & [2\eta_1 \lambda_1 v + \eta_1 \lambda_0 + \lambda_1 \eta_0] dv + \\ & \left[\lambda_1 v + \lambda_0 + \frac{b\lambda_0(\lambda_1 + \eta_1)}{(\lambda_0 + \eta_0)^2} \right] d\eta_0 = 0 \end{aligned}$$

Since both terms in brackets in Eq. (51) are positive, we have $\frac{dv}{d\eta_0} < 0$.

Similarly:

$$(52) \quad \begin{aligned} & [2\eta_1 \lambda_1 v + \eta_1 \lambda_0 + \lambda_1 \eta_0] dv + \\ & \left[\eta_1 v^2 + \eta_0 v + \frac{b\eta_0}{\lambda_0 + \eta_0} \right] d\lambda_1 = 0 \end{aligned}$$

Both terms in brackets are positive and $\frac{dv}{d\lambda_1} < 0$.

¹⁸We write v instead of v^* in the Appendix to simplify notation since it should be clear that we are referring to the *optimal* voucher.

To determine the effect of benefits on the optimal voucher we have:

$$(53) \quad [2\eta_1\lambda_1v + \eta_1\lambda_0 + \lambda_1\eta_0] dv + \left[\frac{\lambda_1\eta_0 - \lambda_0\eta_1}{\lambda_0 + \eta_0} \right] db = 0.$$

The last term has a sign which depends on the quantity:

$$(54) \quad \lambda_1\eta_0 - \lambda_0\eta_1$$

This expression is negative if:

$$(55) \quad \frac{\eta_1}{\eta_0} > \frac{\lambda_1}{\lambda_0}$$

or the proportional effect of a voucher on hiring is greater than on firing (Eq. (6)). Under this condition, it follows that $\frac{dv}{db} > 0$ so that the optimal voucher is increasing in the size of unemployment benefits.

REFERENCES

- Richard Layard, Stephen Nickell, and Richard Jackman. *Unemployment: Macroeconomic Performance and the Labour Market*. Oxford University Press, 1991.
- J. Michael Orszag and Dennis Snower. Optimal structure of employment vouchers and the long-term unemployment problem. Birkbeck College, Univ. of London, 1996.
- Edmund Phelps. *Structural Slumps: The Modern Equilibrium Theory of Unemployment, Interest and Assets*. Harvard, 1994.
- Dennis Snower. Converting unemployment benefits into employment subsidies. *American Economic Review Papers and Proceedings*, pages 65–70, 1994.